

¹ **A Comparative Study of Ligra and Ligra+:**
² **Shared-Memory Graph Processing with and**
³ **without Compression**

⁴ Your Name

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⁶ —— **Abstract** ——

⁷ Today’s shared-memory machines offer hundreds of gigabytes to several terabytes of RAM, making
⁸ it possible to process graphs with billions of edges on a single host. Despite this, many graph
⁹ algorithms remain bottlenecked by memory bandwidth rather than computation. Ligra addresses
¹⁰ this imbalance through a lightweight, data-parallel interface built around two primitives—**VERTEXMAP**
¹¹ and **EDGEMAP**—that dynamically switch between sparse and dense traversal modes. Ligra+ extends
¹² this design with compression techniques such as delta encoding and compact variable-byte formats,
¹³ reducing memory footprint while improving throughput on multi-core systems. This report synthe-
¹⁴ sizes the central ideas of both frameworks, reconstructs their algorithmic design principles, contrasts
¹⁵ their empirical performance, and analyzes how compression reshapes memory–computation trade-offs
¹⁶ in shared-memory graph processing.

¹⁷ **2012 ACM Subject Classification** Theory of computation → Shared memory algorithms; Information
¹⁸ systems → Graph-based data models

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²⁰ algorithms

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²² **1 Introduction**

²³ Large graphs appear routinely in social networks, web infrastructure, biological interaction
²⁴ networks, and simulations of physical systems. While distributed graph-processing frameworks
²⁵ such as Pregel and GraphLab were historically dominant, modern commodity servers now
²⁶ provide enough RAM to store ten billion edges or more on a single machine. Shared-
²⁷ memory systems reduce communication delays, simplify programming, and often outperform
²⁸ distributed solutions for many workloads [5].

²⁹ Ligra exploits this hardware trend by offering a minimal, efficient interface for data-parallel
³⁰ graph traversal. Rather than relying on message passing, Ligra organizes computations around
³¹ dynamically changing *frontiers*—sets of currently active vertices. The system automatically
³² switches between sparse and dense evaluation modes depending on frontier size.

³³ Ligra+ extends the same interface but compresses adjacency lists using delta-encoded
³⁴ and variable-length formats [6]. Earlier compression systems achieved space savings at the
³⁵ cost of slower runtime [1]; Ligra+ demonstrates conditions under which compression yields
³⁶ speedups instead, particularly under memory bandwidth pressure on multi-core machines.

³⁷ This report explains the design choices behind Ligra and Ligra+, analyzes their per-
³⁸ formance characteristics, and compares how compression modifies algorithmic behavior in
³⁹ shared-memory graph workloads.

⁴⁰ **2 Background and Graph Model**

⁴¹ Both Ligra and Ligra+ operate on directed graphs $G = (V, E)$ where vertices are numbered
⁴² $0, \dots, |V| - 1$. For each vertex v , the tools store outgoing neighbors $N^+(v)$ and incoming



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43 neighbors $N^-(v)$ alongside their degrees [5]. Computation uses fork-join parallelism with
44 atomic instructions such as compare-and-swap (CAS) ensuring correctness under concurrent
45 writes.

46 The fundamental abstraction is the `vertexSubset`, which represents an active frontier
47 either as a sparse list of vertex identifiers or as a dense bitmap. The representation is selected
48 dynamically based on frontier size.

49 Ligra exposes two core parallel primitives:

50 ■ **VERTEXMAP(U, F):** Applies a Boolean function F to each $u \in U$ and returns those
51 for which F succeeds.

52 ■ **EDGEMAP(G, U, F, C):** Applies an update function F to edges leaving vertices in
53 U , generating candidate vertices, which are filtered by condition C .

54 EDGEMAP automatically selects sparse traversal over outgoing edges or dense traversal
55 scanning all vertices and their incoming edges, depending on the number of active edges.

56 Ligra+ preserves this API but replaces raw adjacency lists with compressed, delta-encoded
57 representations and partitions large neighbor lists for better load balancing [6].

58 3 Ligra

59 3.1 Motivation

60 Ligra aims to provide a lightweight shared-memory alternative to distributed frameworks.
61 Systems like Pregel [4] or GraphLab [3] incur synchronization and communication overhead,
62 whereas Ligra is optimized for uniform memory access with fast, shared RAM.

63 3.2 Core Techniques

64 Ligra's performance hinges on adaptively switching between sparse and dense modes when
65 frontier size crosses a threshold proportional to $|E|$ [5]. Sparse traversal examines only
66 outgoing edges from active vertices; dense traversal checks all vertices using incoming edges.
67 Graphs are stored in flat arrays for both in-edges and out-edges, minimizing overhead.
68 The `vertexSubset` abstraction simplifies algorithms such as BFS and Brandes' betweenness
69 centrality [2].

70 3.3 Applications

71 Common algorithms implemented in Ligra include:

- 72 ■ Breadth-First Search (BFS)
- 73 ■ Betweenness centrality via Brandes' algorithm
- 74 ■ Connected components
- 75 ■ Graph radius approximation
- 76 ■ PageRank
- 77 ■ Bellman-Ford shortest paths

78 All implementations retain textbook asymptotic complexity (e.g., BFS in $O(|V| + |E|)$)
79 while achieving near-linear speedup up to 40 cores [5].

80 **4 Ligra+**

81 **4.1 Motivation**

82 On modern systems, graph processing is frequently memory-bandwidth-bound: fetching
 83 adjacency lists dominates runtime relative to arithmetic. Ligra+ tests whether compressed
 84 representations—decoded on the fly—reduce bandwidth consumption enough to offset de-
 85 compression costs.

86 **4.2 Compressed Representation**

87 Adjacency lists in Ligra+ are sorted and stored as sequences of delta values encoded using:

- 88 ■ variable-byte codes,
- 89 ■ run-length-coded byte sequences,
- 90 ■ nibble (4-bit) codes.

91 Run-length encoding groups values requiring equal byte lengths, reducing branching
 92 during decoding [6].

93 Decoding uses two core operations:

- 94 ■ **FirstEdge**: decodes the first neighbor;
- 95 ■ **NextEdge**: decodes subsequent neighbors from the last offset.

96 High-degree vertices are partitioned into chunks of size at most T , allowing parallel
 97 decoding with independent starting points, improving load balance on multi-core hardware.

98 **4.3 Empirical Behavior**

99 Across real and synthetic datasets, Ligra+ uses roughly half the memory of Ligra. Compre-
 100 ssion benefits scale with repeating structural patterns.

101 Single-threaded runtime is often slightly slower due to decompression. However, on
 102 40-core machines, byte-coded Ligra+ achieves:

- 103 ■ up to 2× speedup,
- 104 ■ occasionally up to 10% slowdown,
- 105 ■ on average 14% faster than Ligra.

106 These gains appear when reduced memory traffic outweighs decoding overhead.

107 **5 Comparative Analysis**

108 **5.1 Abstraction and API**

109 Ligra and Ligra+ share the same programming interface; any algorithm written for Ligra
 110 runs on Ligra+ unchanged. The key difference lies entirely in edge storage and traversal cost.

111 **5.2 Space–Time Trade-offs**

112 Ligra maximizes per-edge traversal speed but consumes more memory. Ligra+ cuts memory
 113 by nearly half, often resulting in faster performance for memory-bound workloads or large
 114 core counts.

115 For small graphs or lightly threaded execution, Ligra often wins. For large graphs or
 116 memory-constrained settings, Ligra+ is typically superior.

117 **5.3 Algorithmic Sensitivity**

118 Algorithms like BFS that perform little computation per edge benefit most from compression.
 119 Algorithms with heavier per-edge arithmetic, such as PageRank or Bellman–Ford, may hide
 120 decoding overhead, sometimes making Ligra+ faster in practice.

121 **6 Conclusion**

122 Ligra demonstrates that a simple shared-memory framework can support a broad class of
 123 graph traversal algorithms both efficiently and expressively. Ligra+ extends this model
 124 by showing that lightweight compression can simultaneously reduce memory usage and
 125 improve performance on modern multicore processors. Together, both systems highlight
 126 the importance of data layout, memory bandwidth, and threading decisions in designing
 127 high-performance graph processing systems.

128 ————— **References** —————

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