# M U N D A R I J A

# 1 - bob. Haqiqiy sonlar

## 1.1 Natural va butun sonlar

O'nli sanoq sistemasidagi sonlarni yozish uchun o'nta belgidan foydalaniladi. Bular $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ dir. Bu belgilar *raqamlar deyiladi.* Masalan: $8, 18, 524, 2815$, $62703$. 8 - bu ham raqam ham son. 18 - bu raqam emas, u 1 (bir) va 8 (sakkiz) raqamlaridan tashkil topgan son. Yuqorida keltirilgan sonlarning yozuvidagi har bir raqam o'zi egallagan o'rniga qarab turli ma'noga egadir. Xususan, 524 (besh yuz yigirma to'rt) yozuvida 4 raqami bu sonda to'rtta bir borligini, 2 raqami bu sonda ikkita o'n borligini, 5 raqami bu sonda beshta yuz borligini bildiradi. Ya'ni $524 = 5 \cdot 100 + 2 \cdot 10 + 4 \cdot 1$, $62703 = 6 \cdot 10000 + 2 \cdot 1000 + 7 \cdot 100 + 0 \cdot 10 + 3 \cdot 1$.

Sonlarning o'nta raqam yordamida yozishning bunday usuli "o'nli sanoq sistemasi" deyiladi. Predmetlarni sanashda ishlatiladigan sonlar natural sonlar deyiladi. Natural sonlar to'plami $N$ harfi bilan belgilanadi, ya'ni $N = \{1, 2, 3, \ldots, n, \ldots\}$. Butun sonlar to'plami $Z = \{\ldots -2, -1, 0, 1, 2, \ldots\}$ harfi bilan belgilanadi. Butun sonlar to'plamida qo'shish, ayirish va ko'paytirish amallari aniqlangan. Ular quyidagi xossalarga ega.

1. $a + (-b) = a - b$.

2. $-(-a) = a$.

3. $(a + b) + c = a + (b + c)$.

4. $-(a + b - c) = -a - b + c$.

5. $a \cdot (-b) = (-a) \cdot b = -ab$.

6. $(ab) \cdot c = a \cdot (bc) = b \cdot (ac)$.

7. $(a + b) \cdot c = a \cdot c + b \cdot c$.
   Natural ko'rsatkichli darajaning ba'zi xossalarini keltiramiz. $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$ va hokazo $a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n}$. Bu yerda $a$ asos, $n$ daraja ko'rsatkich deyiladi. Ixtiyoriy $a, b$ va $n, m$ natural sonlar uchun quyidagi tengliklar o'rinli:

8. $a^n \cdot a^m = a^{n+m}$.

9. $(ab)^n = a^n \cdot b^n$.

10. $\dfrac{a^n}{a^m} = a^{n-m}$.

11. $a^0 = 1$.

Qo'shish va ayirish birinchi tartibli amallar, ko'paytirish va bo'lish ikkinchi tartibli amallar, darajaga ko'tarish va ildiz chiqarish uchinchi tartibli amallar hisoblanadi. Yuqori tartibli amallar oldin bajariladi. Bir xil tartibli amallar qatnashgan ifodalarda birinchi kelgan amal birinchi bajariladi. Agar ifodada qavslar bo'lsa, dastlab qavs ichidagi amallar bajariladi.

### 1.1.1 Hisoblashga oid misollar

1. Amallarning bajarilish tartibiga e'tibor qilib ifodaning qiymatini hisoblang.

$$18 - 6 : 2 + 3 \cdot 4$$

A) 3     B) 27     C) 18     D) 33

**Yechish:** Ifodada birinchi tartibli va ikkinchi tartibli amallar qatnashgan. Dastlab biz ikkinchi tartibli (ko'paytirish va bo'lish) amallarini bajaramiz.

$$18 - 6 : 2 + 3 \cdot 4 = 18 - 3 + 12.$$

Endi faqat birinchi tartibli amallar qoldi. Ularni navbati bilan bajaramiz.

$$18 - 3 + 12 = 15 + 12 = 27.$$

**Javob:** 27 (B).

2. (96-3-1) Ifodaning qiymatini toping:
$12 - 6 : 3 + 2 \cdot 4$
A) 16     B) 10     C) 18     D) 48

3. (96-11-1) Ifodaning qiymatini toping:
$15 - 9 : 3 + 4 \cdot 3$
A) 24     B) 18     C) 48     D) 12

4. (96-12-1) Ifodaning qiymatini toping:
$18 - 12 : 2 + 5 \cdot 3$
A) 18     B) 24     C) 4     D) 27

5. Ifodaning qiymatini toping:
$24 - 6 : 3 + 5 \cdot 2$
A) 16     B) $-3$     C) 32     D) 22

6. Ifodaning qiymatini toping:
$8 : 2^2 + 4 \cdot 3 - 10$
A) 6     B) 4     C) 18     D) 12

**Yechish:** Dastlab uchinchi bosqich amali, darajaga ko'tarish amalini, keyin esa ikkinchi bosqich amallari bo'lish va ko'paytirishni bajaramiz:

$$8 : 2^2 + 4 \cdot 3 - 10 = 8 : 4 + 4 \cdot 3 - 10 = 2 + 12 - 10.$$

Endi birinchi bosqich amallarini bajarib $2 + 12 - 10 = 14 - 10 = 4$ ni olamiz. **Javob:** 4 (B).

7. Ifodaning qiymatini toping:
$4^2 : 2 + 3 \cdot 4 - 5$
A) 11     B) 39     C) 15     D) 12

8. Ifodaning qiymatini toping:
$3 \cdot 5 + 3 \cdot 2^3 - 25$
A) 206     B) 14     C) 119     D) 12

9. Ifodaning qiymatini toping:
$24 : 2 \cdot 5 - 3 \cdot 2^4 - 7$
A) 19     B) $-52,6$     C) $-1243$     D) 5

10. Ifodaning qiymatini toping:
$27 : 3^3 + 2 \cdot 3^2 - 15$
A) 22     B) 4     C) 15     D) 732

11. $(2 \cdot 6 + 8) \cdot 2 - 2$ ifodaning qiymatini toping:
A) 16      B) 0      C) 54      D) 38

**Yechish:** Dastlab qavs ichidagi ifodaning qiymatini hisoblaymiz: $2 \cdot 6 + 8 = 12 + 8 = 20$. Endi ko'paytirishni keyin ayirishni bajaramiz: $20 \cdot 2 - 2 = 40 - 2 = 38$. **Javob:** 38 (D).

12. $(2 + 8 \cdot 6) \cdot 2 - 2 \cdot 7$ ifodaning qiymatini toping:
A) 86      B) 0      C) 54      D) 38

13. $(5 + 3 \cdot 6) \cdot 2 - 2 \cdot 23$ ifodaning qiymatini toping:
A) 18      B) 0      C) 50      D) 13

14. $3 + 3 \cdot 2(7 \cdot 2 - 4) : 3$ ifodaning qiymatini toping:
A) 40      B) 20      C) 23      D) 35

15. (96-7-1) Hisoblang:
$21 \cdot 18 - 19 \cdot 18 + 18 \cdot 17 - 17 \cdot 16 + 16 \cdot 15 - 15 \cdot 14$
A) 50      B) 100      C) 98      D) 24

**Yechish:** Umumiy ko'paytuvchini qavsdan tashqariga chiqarish yordamida hisoblaymiz:

$21 \cdot 18 - 19 \cdot 18 + 18 \cdot 17 - 17 \cdot 16 + 16 \cdot 15 - 15 \cdot 14 =$
$= 18(21 - 19) + 17(18 - 16) + 15(16 - 14) =$
$= 18 \cdot 2 + 17 \cdot 2 + 15 \cdot 2 = 2(18 + 17 + 15) =$
$= 2 \cdot 50 = 100$. **Javob:** 100 (B).

16. (97-7-1) Hisoblang:
$36 \cdot 24 - 33 \cdot 24 + 17 \cdot 11 - 14 \cdot 11 + 18 \cdot 16 - 15 \cdot 16$
A) 166      B) 155      C) 180      D) 153

17. (97-10-1) Hisoblang:
$27 \cdot 23 - 24 \cdot 23 + 21 \cdot 19 - 18 \cdot 19 + 17 \cdot 11 - 14 \cdot 11$
A) 165      B) 159      C) 143      D) 203

18. (00-5-4) Hisoblang:
$139 \cdot 15 + 18 \cdot 139 + 15 \cdot 261 + 18 \cdot 261$
A) 13200      B) 14500      C) 15100      D) 16200

19. (96-1-1) Ifodaning qiymatini toping:
$26 \cdot 25 - 25 \cdot 24 + 24 \cdot 23 - 23 \cdot 22 - 12 \cdot 8$
A) 106      B) 1      C) 54      D) 0

### 1.1.2 Tub va murakkab sonlar

*Agar $a$ va $b$ natural sonlari uchun $a : b = c$ ham natural son bo'lsa, u holda $a$ soni $b$ songa bo'linadi yoki $a$ soni $b$ soniga karrali deyiladi. $b$ soni $a$ sonining bo'luvchisi deyiladi. $c$ soni bo'linma deyiladi.* Masalan, 24 soni 1,2,3,4,6,8,12 va 24 sonlariga bo'linadi. *1 va o'zidan boshqa bo'luvchisi bo'lmagan natural sonlar tub sonlar deyiladi. 1 va o'zidan boshqa bo'luvchilarga ega bo'lgan natural sonlar murakkab sonlar deyiladi.* 1 soni tub ham murakkab ham emas. $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, \ldots$ sonlari – tub sonlardir. $4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, \ldots$ sonlari – murakkab sonlardir.

1. (99-7-10) 30 dan kichik tub sonlar nechta?
A) 11      B) 9      C) 10      D) 12

**Yechish:** 30 dan kichik tub sonlar $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$. Ular soni 10 ta. **Javob:** 10 (C).

2. (98-5-8) 50 dan kichik tub sonlar nechta?
A) 10      B) 15      C) 17      D) 9

3. 30 va 50 sonlari orasida nechta tub son bor?
A) 4      B) 3      C) 5      D) 7

4. (02-5-4) 1; 2; 3; 15; 17; 23; 24; 169; 289; 361 sonlar ketma-ketligida nechta tub son bor?
A) 3      B) 4      C) 5      D) 7

5. 2; 3; 15; 17; 21; 23; 29; 39; 51; 57 sonlar ketma-ketligida nechta murakkab son bor?
A) 3      B) 4      C) 5      D) 7

6. (99-3-7) Quyidagi sonli ketma-ketliklardan qaysilari tub sonlardan iborat?
1) 0; 3; 5; 7; 11;      2) 1; 3; 5; 7; 13;      3) 3; 5; 7; 9; 11;      4) 2; 3; 5; 7; 17;      5) 3; 5; 17; 19; 381
A) 1; 2      B) 2; 4      C) 5      D) 4

### 1.1.3 Eng katta umumiy bo'luvchi – EKUB va eng kichik umumiy karrali – EKUK

*$a$ va $b$ natural sonlarining eng katta umumiy bo'luvchisi, ya'ni EKUBi deb, ularning har ikkisini bo'luvchi sonlar ichidan eng kattasiga aytiladi va $B(a; b)$ shaklda yoziladi. $a$ va $b$ natural sonlarining eng kichik umumiy karralisi, ya'ni EKUKi deb ularning har ikkisiga karrali natural sonlar ichidan eng kichigiga aytiladi va $K(a; b)$ shaklda yoziladi.* Agar $a$ va $b$ natural sonlarining 1 dan boshqa umumiy bo'luvchisi bo'lmasa, ular o'zaro tub sonlar deyiladi. Ikki $a$ va $b$ sonlarning minimumi deb, ularning kichigiga aytiladi va $min\{a, b\}$ shaklda yoziladi. Ikki $a$ va $b$ sonlarning maksimumi deb, ularning kattasiga aytiladi va $max\{a, b\}$ shaklda yoziladi. Masalan, $min\{0, 2\} = 0$, $max\{1, 3\} = 3$.

1. **$a = 2^m \cdot 3^n \cdots p^k$ tub ko'paytuvchilarga ajratilgan $a$ sonining bo'luvchilari soni $(m + 1)(n + 1) \cdots (k + 1)$ ga teng.**

2. **$a = 2^m \cdot 3^n \cdots p^k$ tub ko'paytuvchilarga ajratilgan $a$ sonining bo'luvchilari yig'indisi:**

$$Y(a) = \frac{2^{m+1} - 1}{2 - 1} \cdot \frac{3^{n+1} - 1}{3 - 1} \cdots \frac{p^{k+1} - 1}{p - 1}.$$

3. **$a$ va $b$ sonlarning umumiy bo'luvchilari soni ular EKUBining bo'luvchilari soniga teng.**

4. **$B(a; b) \cdot K(a; b) = a \cdot b$ tenglik o'rinli.**

   **Agar $a$ va $b$ natural sonlari $a = 2^{m_1} \cdot 3^{n_1} \cdots p^{k_1}$ va $b = 2^{m_2} \cdot 3^{n_2} \cdots p^{k_2}$ tub ko'paytuvchilarga ajratilgan bo'lsa, u holda**

5. **$B(a; b) = 2^{min\{m_1, m_2\}} \cdot 3^{min\{n_1, n_2\}} \cdots p^{min\{k_1, k_2\}}$.**

6. **$K(a; b) = 2^{max\{m_1, m_2\}} \cdot 3^{max\{n_1, n_2\}} \cdots p^{max\{k_1, k_2\}}$.**

7. **O'zaro tub $a$ va $b$ sonlar uchun $B(a; b) = 1$ va aksincha.**

1. (02-5-5) 36 ning natural bo'luvchilari nechta?
A) 5      B) 7      C) 8      D) 9

**Yechish:** 36 ni tub ko'paytuvchilarga ajratamiz. $36 = 2^2 \cdot 3^2$. 1-qoidaga ko'ra, 36 ning bo'luvchilari soni $(2 + 1) \cdot (2 + 1) = 9$. **Javob:** 9 (D).

2. 100 ning natural bo'luvchilari sonini toping.
   A) 4    B) 6    C) 9    D) 8

3. 480 ning natural bo'luvchilari sonini toping.
   A) 14    B) 24    C) 48    D) 32

4. 900 ning natural bo'luvchilari sonini toping.
   A) 27    B) 36    C) 49    D) 28

5. 1000 ning natural bo'luvchilari nechta?
   A) 24    B) 16    C) 28    D) 32

6. (03-10-11) $8^{n+2} \cdot 12^{n-3}$ ko'paytmaning natural bo'luvchilari soni 42 ga teng bo'lsa, $n$ nechaga teng bo'ladi?
   A) 4    B) 3    C) 2    D) 5

7. (00-5-6) 48 sonining barcha natural bo'luvchilari yig'indisini toping.
   A) 123    B) 100    C) 108    D) 124

   **Yechish.** 48 sonini tub ko'paytuvchilarga ajrata-miz. $48 = 2^4 \cdot 3$. 2-qoidaga ko'ra 48 sonining barcha natural bo'luvchilari yig'indisi $Y(48) = (2^{4+1} - 1) : (2-1) \cdot (3^{1+1} - 1) : (3-1) = 31 \cdot 4 = 124$. **Javob:** 124 (D).

8. 12 sonining barcha natural bo'luvchilari yig'indisini toping.
   A) 12    B) 28    C) 32    D) 24

9. 24 sonining barcha natural bo'luvchilari yig'indisini toping.
   A) 48    B) 58    C) 60    D) 54

10. 20 sonining barcha natural bo'luvchilari yig'indisini toping.
    A) 48    B) 42    C) 38    D) 58

11. 28 sonining barcha natural bo'luvchilari yig'indisini toping.
    A) 44    B) 58    C) 62    D) 56

12. (96-3-2) 8 va 6 sonlarining eng kichik umumiy karralisi, ya'ni EKUKini toping.
    A) 16    B) 24    C) 12    D) 8

    **Yechish:** Bu sonlarni tub ko'paytuvchilarga ajra-tamiz. $8 = 2^3 \cdot 3^0$; $6 = 2 \cdot 3$. 6-qoidaga ko'ra $K(8; 6) = 2^3 \cdot 3 = 24$.   **Javob:** 24 (B).

13. (96-12-2) 6 va 4 sonlarining EKUKini toping.
    A) 6    B) 14    C) 24    D) 12

14. (96-11-2) 10 va 8 sonlarining EKUKini toping.
    A) 80    B) 10    C) 18    D) 40

15. 36 va 48 sonlarining EKUBini toping.
    A) 36    B) 14    C) 24    D) 12

16. 480 va 600 sonlarining EKUBini toping.
    A) 160    B) 300    C) 240    D) 120

17. (96-9-1) 594 va 378 ning umumiy bo'luvchilari nechta?
    A) 8    B) 7    C) 9    D) 5

**Yechish:** Ikki sonning umumiy bo'luvchilari soni 3-qoidaga ko'ra ular EKUBining umumiy bo'luvchi-lari soniga teng.

$$594 = 2 \cdot 3^3 \cdot 7^0 \cdot 11, \quad 378 = 2 \cdot 3^3 \cdot 7 \cdot 11^0$$

bo'lgani uchun 594 va 378 sonlarining EKUBi 5-qoidaga ko'ra $B(594; 378) = 2^1 \cdot 3^3 \cdot 7^0 \cdot 11^0 = 2^1 \cdot 3^3$ ga teng. 1-qoidaga ko'ra bu sonning umumiy bo'luvchilari soni $n = (1+1)(3+1)(0+1)(0+1) = 8$ ga teng.   **Javob:** 8 (A).

18. (99-7-1) 56 va 16 sonlarining umumiy bo'luvchilari nechta?
    A) 4    B) 3    C) 2    D) 5

19. (96-3-61) 630 va 198 ning umumiy bo'luvchilari nechta?
    A) 5    B) 6    C) 4    D) 7

20. (96-13-1) 420 va 156 ning umumiy bo'luvchilari nechta?
    A) 7    B) 5    C) 6    D) 4

21. (98-2-2) 8 va 12 sonlari eng kichik umumiy kar-ralisining natural bo'luvchilari nechta?
    A) 6    B) 7    C) 8    D) 9

22. (98-10-1) 21 va 35 sonlarining EKUKi va EKUBi-ning yig'indisini toping.
    A) 108    B) 110    C) 112    D) 109

    **Yechish:** 21 va 35 sonlarining EKUBi va EKUKi 5 va 6-qoidalarga ko'ra

    $$21 = 3 \cdot 5^0 \cdot 7, \quad 35 = 3^0 \cdot 5 \cdot 7$$

    tenglikdan $B(21; 35) = 3^0 \cdot 5^0 \cdot 7 = 7$, $K(21; 35) = 3 \cdot 5 \cdot 7 = 105$. Ularning yig'indisi $7 + 105 = 112$. **Javob:** 112 (C).

23. (00-3-5) 72 va 96 sonlari EKUKining EKUBiga nisbatini toping?
    A) 10    B) 0,1    C) 9    D) 12

24. (98-11-2) 270 va 300 sonlari EKUKining 4 va 6 sonlari EKUKiga nisbatini toping.
    A) 25    B) 45    C) 225    D) 95

25. (00-10-2) 108 va 135 sonlari EKUKini 12 va 54 sonlari EKUKiga nisbatini toping?
    A) 8    B) 5    C) 12    D) 6

26. (99-2-4) 24, 18 va 30 sonlari EKUKining EKUBi-ga nisbatini toping.
    A) 90    B) 72    C) 48    D) 60

27. (00-7-7) 9, 10, 22 va 25 sonlari orasida nechta o'zaro tub sonlar jufti bor?
    A) 4    B) 3    C) 2    D) 6

    **Yechish.** 7-qoidaga ko'ra o'zaro tub sonlarning EKUBi 1 ga teng. $B(9; 10) = 1$, $B(9; 22) = 1$, $B(9; 25) = 1$, $B(10; 22) = 2$, $B(10; 25) = 5$, $B(22; 25) = 1$. Demak, o'zaro tub sonlar jufti 4 ta ekan. **Javob:** 4 (A).

28. (03-4-3) [4; 8] kesmada nechta o'zaro tub sonlar jufti bor?
    A) 5     B) 6     C) 4     D) 7

29. (01-12-1) Dastlabki 30 ta natural sonlar ichida 6 soni bilan o'zaro tub bo'lgan sonlar nechta?
    A) 7     B) 8     C) 9     D) 10

30. (97-5-10) Qaysi juftlik o'zaro tub sonlardan iborat?
    A) (8;14)   B) (11;22)   C) (12;35)   D) (12;34)

31. (97-9-10) Qaysi juftlik o'zaro tub sonlardan iborat?
    A) (21;14)   B) (21;10)   C) (12;15)   D) (10;15)

32. (99-8-19) Ikki sonning ko'paytmasi 294 ga, ularning eng katta umumiy bo'luvchisi 7 ga teng. Bu sonlarning eng kichik umumiy karralisini toping.
    A) 42     B) 52     C) 56     D) 49

    **Yechish.** 4-qoidaga ko'ra $a \cdot b = B(a; b) \cdot K(a; b)$. Mos qiymatlarni qo'yib $294 = 7 \cdot K(a; b)$ ni olamiz. Bu yerdan $K(a; b) = 42$. **Javob:** 42 (A).

33. Ikki sonning ko'paytmasi 192 ga, ularning EKUKi 48 ga teng. Bu sonlarning EKUBini toping.
    A) 4     B) 6     C) 5     D) 8

34. (00-7-2) 18 va 12 sonlari EKUKi va EKUBi ko'paytmasini toping?
    A) 220     B) 218     C) 214     D) 216

35. 7 va $a$ sonlarining EKUKi va EKUBining ko'paytmasi 126 ga teng. $a$ ni toping.
    A) 18     B) 16     C) 25     D) 36

36. 12 va 15 sonlari EKUKi va EKUBi ko'paytmasini toping.
    A) 180     B) 160     C) 250     D) 360

### 1.1.4 Bo'linish belgilari

Raqam - bu sonlarni yozishda ishlatiladigan shartli belgilardir. Ular $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ lardir. Nol barcha sonlarga bo'linadi deb hisoblanadi. $a_1, a_2, \ldots, a_n$ raqamlaridan tashkil topgan $n$ xonali sonni $\overline{a_1 a_2 \ldots a_n}$ ko'rinishda yozamiz.

1. **Oxirgi raqami 0,2,4,6,8 lar bilan tugaydigan sonlar va faqat ular 2 ga bo'linadi. 2 ga bo'linadigan sonlar juft sonlar deyiladi. Ular $2, 4, 6, \ldots, 2n, \ldots$. 2 ga bo'lganda 1 qoldiq beradigan sonlar toq sonlar deyiladi. Ular $1, 3, 5, \ldots, 2n - 1, \ldots$.**

2. **Oxirgi ikki raqamidan tashkil topgan ikki xonali son 4 ga (25 ga) bo'linsa, berilgan sonning o'zi ham 4 ga (25 ga) bo'linadi.**

3. **Oxirgi uchta raqami 0 bilan tugasa yoki oxirgi uchta raqamidan tashkil topgan uch xonali son 8 ga bo'linsa, berilgan sonning o'zi ham 8 ga bo'linadi.**

4. **Oxirgi raqami 0 yoki 5 bilan tugagan sonlar va faqat ular 5 ga bo'linadi.**

5. **Raqamlari yig'indisi 3 ga bo'linadigan sonlar 3 ga bo'linadi va aksincha.**

6. **Raqamlari yig'indisi 9 ga bo'linadigan sonlar 9 ga bo'linadi va aksincha.**

7. **Agar $N = \overline{a_1 a_2 \ldots a_n}$ ko'rinishdagi natural son uchun $\overline{a_1 a_2 \ldots a_{n-1}} - 2 \cdot a_n$ soni 7 ga bo'linsa berilgan $N$ natural soni ham 7 ga bo'linadi.**

8. **Agar sonning juft o'rinda turgan raqamlar yig'indisi bilan toq o'rinda turgan raqamlar yig'indisining ayirmasi 11 ga bo'linsa, bunday sonlar 11 ga bo'linadi.**

9. **Agar $N = \overline{a_1 a_2 \ldots a_n}$ ko'rinishdagi natural son uchun $\overline{a_1 a_2 \ldots a_{n-1}} + 2 \cdot a_n$ soni 19 ga bo'linsa berilgan $N$ natural soni ham 19 ga bo'linadi.**

10. **Agar $n$ natural son uchun $n = kl$ ($k$ va $l$ o'zaro tub sonlar) tenglik o'rinli bo'lsa, u holda $k$ va $l$ ga bo'lingan sonlar $n$ ga bo'linadi va aksincha. Bu belgini murakkab sonlarga bo'linish belgisi sifatida qabul qilish mumkin. Masalan, $45 = 5 \cdot 9$ bo'lgani uchun 5 ga va 9 ga bo'lingan sonlar 45 ga ham bo'linadi.**

11. **Agar $A$ soni $n$ ga, $B$ soni $m$ ga bo'linsa, u holda $A \cdot B$ soni $n \cdot m$ ga bo'linadi.**

12. **Agar $A$ va $B$ sonlarning har biri $n$ ga bo'linsa, u holda $A + B$ ham $n$ ga bo'linadi.**

13. **Agar $A$ va $B$ sonlardan birortasi $n$ ga bo'linsa, u holda $A \cdot B$ ham $n$ ga bo'linadi.**

1. 2 ga bo'linadigan sonni toping.
   A) 1205     B) 7806     C) 9321     D) 6843

   **Yechish.** 1-alomatga ko'ra, oxirgi raqami 6 bo'lgan 7806 soni 2 ga bo'linadi. Qolgan 1205, 9321, 6843 sonlar 2 ga bo'linmaydi. **Javob:** 7806 (B).

2. 2 ga bo'linmaydigan sonni toping.
   A) 3456     B) 5842     C) 7648     D) 8641

3. 5 ga bo'linadigan sonni toping.
   A) 6348     B) 8951     C) 3965     D) 5554

4. 5 ga bo'linmaydigan sonni toping.
   A) 6665     B) 3335     C) 4440     D) 5554

5. 2 ga ham 5 ga ham bo'linadigan sonni toping.
   A) 5522     B) 2255     C) 3840     D) 5258

6. 4 ga bo'linmaydigan sonni toping.
   A) 1100     B) 1520     C) 130     D) 1008

   **Yechish.** 2-alomatga ko'ra, oxirgi ikki raqamidan tashkil topgan ikki xonali sonni 4 ga bo'linishini tekshirishimiz kerak. C) javobdagi 130 sonining oxirgi ikki raqamidan tashkil topgan ikki xonali son 30 bo'lib, u 4 ga bo'linmaydi. Demak, 130 soni 4 ga bo'linmaydi. **Javob:** 130 (C).

7. 4 ga bo'linadigan sonni toping.
   A) 582      B) 674      C) 804      D) 442

8. 25 ga bo'linadigan sonni toping.
   A) 2540      B) 8800      C) 2552      D) 4520

9. 25 ga bo'linmaydigan sonni toping.
   A) 6300      B) 8975      C) 6850      D) 9855

10. 3 ga bo'linadigan sonni toping.
    A) 326      B) 213      C) 475      D) 739

    **Yechish.** 5-alomatga ko'ra, raqamlar yig'indisining 3 ga bo'linishini tekshiramiz. A) javobda $3 + 2 + 6 = 11$ soni 3 ga bo'linmaydi, B) javobda $2 + 1 + 3 = 6$ soni 3 ga bo'linadi. Demak, 213 soni ham 3 ga bo'linadi. **Javob:** 213 (B).

11. 3 ga bo'linmaydigan sonni toping.
    A) 6825      B) 8937      C) 5841      D) 3133

12. 9 ga bo'linadigan sonni toping.
    A) 881      B) 672      C) 432      D) 763

13. 9 ga bo'linmaydigan sonni toping.
    A) 8082      B) 4365      C) 1791      D) 2654

14. 7 ga bo'linadigan sonni toping.
    A) 114      B) 235      C) 315      D) 370

    **Yechish.** Misolni yechishda 7-alomatdan foydalanamiz. A) javobda $11 - 2 \cdot 4 = 3$ soni 7 ga bo'linmaydi, B) javobda $23 - 2 \cdot 5 = 13$ soni 7 ga bo'linmaydi. C) javobda $31 - 2 \cdot 5 = 21$ soni 7 ga bo'linadi. Demak, 315 soni 7 ga bo'linadi. **Javob:** 315 (C).

15. 7 ga bo'linmaydigan sonni toping.
    A) 514      B) 635      C) 828      D) 546

16. 8 ga bo'linadigan sonni toping.
    A) 1140      B) 2350      C) 3700      D) 3152

17. 8 ga bo'linmaydigan sonni toping.
    A) 5408      B) 3600      C) 7000      D) 8148

18. 11 ga bo'linadigan sonni toping.
    A) 1540      B) 2350      C) 3712      D) 8152

    **Yechish.** Misolni yechishda 8-alomatdan foydalanamiz. A) javobda keltirilgan sinning juft o'rindagi raqamlar yig'indisi $5 + 0 = 5$, toq o'rindagi raqamlar yig'indisi $1 + 4 = 5$, ularning ayirmasi $5 - 5 = 0$ soni 11 ga bo'linadi. Demak, 1540 soni ham 11 ga bo'linadi. **Javob:** 1540 (A).

19. 11 ga bo'linmaydigan sonni toping.
    A) 2332      B) 4554      C) 6798      D) 1011

20. 19 ga bo'linadigan sonni toping.
    A) 323      B) 266      C) 456      D) 319

21. 19 ga bo'linadigan sonni toping.
    A) 2140      B) 1653      C) 3751      D) 5152

22. (97-9-61) $n$ raqamining qanday qiymatlarida $50 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
    A) 3      B) 5      C) 3; 9      D) 1; 9

    **Yechish.** Ma'lumki, tub sonlar eng kam tub ko'paytuvchilarga ajraladi. $n$ raqam bo'lganligi uchun, u $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ qiymatlarni qabul qiladi. U holda $50 + n$ ifoda $50, 51, \ldots, 59$ qiymatlarni qabul qiladi. Bu sonlar ichida ikkitasi, ya'ni 53 va 59 lar tub sonlardir. Demak, $n = 3$ yoki $n = 9$ bo'lsa, $50 + n$ ifoda eng kam tub ko'paytuvchilarga ajraydi. **Javob:** 3; 9 (C).

23. $n$ raqamining qanday qiymatlarida $40 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
    A) 1; 3; 7      B) 1; 5      C) 3; 9      D) 1; 9

24. $n$ raqamining qanday qiymatlarida $30 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
    A) 1; 3; 7      B) 1; 7      C) 3; 9      D) 1; 9

25. $n$ raqamining nechta qiymatida $25 + n$ tub son bo'ladi?
    A) 1      B) 7      C) 3      D) 2

26. $n$ raqamining nechta qiymatida $39 + n$ tub son bo'ladi?
    A) 1      B) 2      C) 3      D) 4

27. (97-9-72) $n$ raqamining qanday qiymatlarida $\overline{7851n}$ soni 9 ga qoldiqsiz bo'linadi?
    A) 2      B) 4      C) 6      D) 9

    **Yechish:** 6-qoidaga ko'ra biror son 9 ga qoldiqsiz bo'linishi uchun, uning raqamlari yig'indisi 9 ga bo'linishi zarur va yetarli. $7 + 8 + 5 + 1 + n = 21 + n$ soni 9 ga bo'linishi uchun $n = 6$ bo'lishi kerak. **Javob:** 6 (C).

28. (97-4-12) $n$ raqamining qanday qiymatlarida $\overline{6134n}$ soni 3 ga qoldiqsiz bo'linadi?
    A) 1      B) 4      C) 2      D) 1; 4; 7

29. (02-4-28) 246*013579 soni 9 ga bo'linishi uchun yulduzchaning o'rniga qanday raqam qo'yilishi kerak?
    A) 0      B) 4      C) 7      D) 8

30. $n$ raqamining qanday qiymatlarida $\overline{27n}$ soni 7 ga qoldiqsiz bo'linadi?
    A) 3      B) 4      C) 2      D) 7

31. $n$ raqamining qanday qiymatlarida $\overline{6134n}$ besh xonali son 11 ga qoldiqsiz bo'linadi?
    A) 6      B) 4      C) 2      D) 7

32. (98-10-2) Quyidagi sonlardan qaysi biri 36 ga qoldiqsiz bo'linmaydi?

    A) 2016      B) 3924      C) 1782      D) 8244

    **Yechish:** 4 va 9 o'zaro tub sonlar bo'lib, ularning ko'paytmasi 36 ga teng. 10-qoidaga ko'ra, 36 ga qoldiqsiz bo'lingan sonlar 4 va 9 ga ham bo'linadi. 1782 sonining oxirgi ikki raqamidan tashkil topgan son 82 bo'lib, u (2-qoidaga qarang) 4 ga bo'linmaydi. Shuning uchun 1782 soni 36 ga bo'linmaydi. **Javob:** 1782 (C).

33. (00-7-3) 752 sonining o'ng tomoniga qanday raqam yozilsa, hosil bo'lgan son 36 ga qoldiqsiz bo'linadi?
    A) 0      B) 2      C) 6      D) 4

34. (97-4-2) 17827516 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?
    A) 3      B) 10      C) 4      D) 5

35. (97-9-62) 41582637 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?
    A) 4      B) 9      C) 5      D) 10

36. (98-2-3) Quyidagi sonlardan qaysi biri 15 ga karrali emas?
    A) 6525      B) 3105      C) 4620      D) 6145

37. (98-9-4) Quyidagi sonlardan qaysi biri 12 ga qoldiqsiz bo'linmaydi?
    A) 9216      B) 13626      C) 12024      D) 18312

38. 19 ga karrali sonni toping.
    A) 297      B) 399      C) 405      D) 810

39. (96-7-2) Berilgan sonlardan qaysilari 15 ga qoldiqsiz bo'linadi?
    $x = 220350$, $y = 321000$, $z = 1024145$.
    A) faqat $x$      B) faqat $z$
    C) $y$ va $z$      D) $x$ va $y$

    **Yechish:** 10-qoidaga ko'ra biror son 15 ga qoldiqsiz bo'linishi uchun u 3 ga ham 5 ga ham bo'linishi kerak. 4-qoidaga ko'ra berilgan sonlarning uchalasi ham 5 ga bo'linadi. $x$ sonining raqamlari yig'indisi: $2 + 2 + 0 + 3 + 5 + 0 = 12$, $y$ sonining raqamlari yig'indisi: $3 + 2 + 1 + 0 = 6$, $z$ sonining raqamlari yig'indisi: $1 + 0 + 2 + 4 + 1 + 4 + 5 = 17$. 5-qoidaga ko'ra $z$ soni 3 ga bo'linmaydi, $x$ va $y$ lar esa 3 ga bo'linadi. Shuning uchun $x$ va $y$ sonlari 15 ga bo'linadi. **Javob:** $x$ va $y$ (D).

40. (98-10-51) Berilgan $p = 10189144$, $q = 396715256$ va $r = 78901644$ sonlardan qaysilari 8 ga qoldiqsiz bo'linadi?
    A) hech qaysisi      B) $p$ va $q$      C) $p$ va $r$      D) $p$

41. (97-3-2) $x = 30112$, $y = 330000$ va $z = 102588$ sonlardan qaysilari 12 ga qoldiqsiz bo'linadi?
    A) faqat $y$      B) faqat $x$
    C) $x$ va $z$      D) $y$ va $z$

42. (97-7-2) Quyidagi sonlardan qaysilari 18 ga qoldiqsiz bo'linadi?
    $x = 10842$, $y = 54900$, $z = 306198$
    A) faqat $x$      B) faqat $y$
    C) $x$ va $y$      D) $y$ va $z$

43. (97-10-2) Quyidagi sonlardan qaysilari 6 ga qoldiqsiz bo'linadi?
    $x = 123386$, $y = 402108$, $z = 261000$
    A) faqat $x$      B) faqat $y$      C) faqat $z$      D) $y$ va $z$

44. (01-11-2) Quyidagi ko'paytmalardan qaysi biri 45 ga qoldiqsiz bo'linadi?
    A) $42 \cdot 85$      B) $35 \cdot 61$      C) $80 \cdot 123$      D) $36 \cdot 20$

**Yechish:** 36 soni 9 ga, 20 soni esa 5 ga bo'linadi. 11-qoidaga ko'ra, $36 \cdot 20$ ko'paytma $9 \cdot 5 = 45$ ga bo'linadi. **Javob:** $36 \cdot 20$ (D).

45. (02-12-21) Quyidagi ko'paytmalardan qaysi biri 45 ga qoldiqsiz bo'linadi?
    A) $42 \cdot 85$      B) $35 \cdot 61$      C) $80 \cdot 123$      D) $243 \cdot 80$

46. Quyidagi ko'paytmalardan qaysi biri 18 ga qoldiqsiz bo'linadi?
    A) $42 \cdot 15$      B) $25 \cdot 61$      C) $80 \cdot 23$      D) $43 \cdot 20$

47. Quyidagi ko'paytmalardan qaysi biri 12 ga qoldiqsiz bo'linadi?
    A) $11 \cdot 15$      B) $25 \cdot 30$      C) $80 \cdot 21$      D) $43 \cdot 20$

48. Quyidagi ko'paytmalardan qaysi biri 21 ga qoldiqsiz bo'linadi?
    A) $11 \cdot 15$      B) $14 \cdot 30$      C) $20 \cdot 27$      D) $31 \cdot 20$

49. Quyidagi ko'paytmalardan qaysi biri 35 ga qoldiqsiz bo'linadi?
    A) $18 \cdot 15$      B) $28 \cdot 40$      C) $50 \cdot 27$      D) $49 \cdot 56$

### 1.1.5 Qoldiqli bo'lish

1. **Qoldiqli bo'lish formulasi** $a = n \cdot m + r$, $0 \le r < n$. **Bu yerda** $a-$ **bo'linuvchi,** $n-$ **bo'luvchi,** $m-$ **bo'linma,** $r-$ **qoldiq.**

2. **Agar** $A = n \cdot m_1 + r_1$ **va** $B = n \cdot m_2 + r_2$ **sonlarining** $n$ **ga bo'lgandagi qoldiqlari yig'indisi** $r_1 + r_2 = n$ **bo'lsa, u holda** $A + B$ **soni** $n$ **ga bo'linadi.**

3. $A = n \cdot m_1$ **va** $B = n \cdot m_2 + r$ **ko'rinishdagi sonlar bo'lsa, u holda** $A+B$ **va** $B$ **sonlarining** $n$ **ga bo'lgandagi qoldiqlari teng bo'ladi.**

1. (98-7-3) Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
   1) $43 = 9 \cdot 5 - 2$      2) $43 = 8 \cdot 5 + 3$
   3) $43 = 7 \cdot 5 + 8$      4) $43 = 21 \cdot 2 + 1$
   A) 1; 2; 4      B) 2; 3; 4      C) 2; 4      D) 3; 4

   **Yechish:** Qoldiqli bo'lish (1 ga qarang) qoidasiga ko'ra qoldiq $r$ bo'luvchi $n$ dan kichik va u $n \cdot m$ ga qo'shiladi. 1) da 2 ayrilgan, 3) da esa qoldiq 8 bo'luvchi 7 dan katta. 2) va 4) lar qoldiqli bo'lishni ifodalaydi. **Javob:** 2; 4 (C).

2. (98-12-3) Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
   1) $47 = 4 \cdot 11 + 3$      2) $47 = 6 \cdot 6 + 11$
   3) $47 = 9 \cdot 5 + 2$      4) $47 = 7 \cdot 7 - 2$
   A) 1; 3      B) 1; 2; 3      C) 1; 4      D) 2; 3

3. Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
   1) $45 = 2 \cdot 23 - 1$      2) $45 = 8 \cdot 6 - 3$
   3) $45 = 7 \cdot 6 + 3$      4) $45 = 11 \cdot 4 + 1$
   A) 1; 2; 4      B) 2; 3; 4      C) 2; 4      D) 3; 4

4. Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
   1) $25 = 2 \cdot 12 + 1$      2) $25 = 8 \cdot 3 + 1$
   3) $25 = 4 \cdot 6 + 1$      4) $25 = 3 \cdot 9 - 2$
   A) 1; 2; 3      B) 3; 4      C) 2; 4      D) 1; 3

5. 723 ni 6 ga bo'lgandagi qoldiqni toping.
   A) 4      B) 3      C) 1      D) 2

   **Yechish:** 723 sonini $723 = 6 \cdot 120 + 3$ ko'rinishda yozish mumkin. 1-qoidaga ko'ra, $r = 3$. Demak, qoldiq 3 ga teng ekan. **Javob:** 3 (B).

6. 500 ni 7 ga bo'lgandagi qoldiqni toping.
   A) 6      B) 3      C) 1      D) 2

7. 790 ni 8 ga bo'lgandagi qoldiqni toping.
   A) 4      B) 3      C) 6      D) 2

8. 893 ni 9 ga bo'lgandagi qoldiqni toping.
   A) 6      B) 3      C) 7      D) 2

9. Quyidagi yig'indilarning qaysi biri 6 ga bo'linadi?
   A) 86+628   B) 75+412   C) 83+623   D) 76+214

   **Yechish:** 86+628 sonini 6 ga bo'linishini 2-qoida yordamida tekshiramiz: $86 = 6 \cdot 14 + 2$, $628 = 6 \cdot 104 + 4$. Birinchisida qoldiq $r_1 = 2$, ikkinchisida qoldiq $r_2 = 4$. 2-qoidaga ko'ra $r_1 + r_2 = 2 + 4 = 6$ soni 6 ga bo'linadi. **Javob:** 86+628 (A).

10. Quyidagi yig'indilarning qaysi biri 7 ga bo'linadi?
    A) 47+701   B) 64+218   C) 76+189   D) 85+216

11. Quyidagi yig'indilarning qaysi biri 8 ga bo'linadi?
    A) 58+794   B) 68+215   C) 76+316   D) 91+217

12. Quyidagi yig'indilarning qaysi biri 9 ga bo'linadi?
    A) 48+368   B) 60+543   C) 84+692   D) 78+216

13. (98-6-7) $3^{20}$ ni 7 ga bo'lgandagi qoldiqni toping.

    A) 6      B) 3      C) 1      D) 2

    **Yechish:** 3-qoidaga ko'ra $(mk + r)^n$ sonni $k$ ga bo'lgandagi qoldiq $r^n$ ni $k$ ga bo'lgandagi qoldiqqa teng, ya'ni $(mk+r)^n = m_1 k + r^n$ $(m_1-$ bo'linma) bo'lgani uchun $3^{20} = 9^{10} = (7+2)^{10} = 7n + 2^{10} = 7n + 32^2 = 7n + (4 \cdot 7 + 4)^2 = 7n + 7n_1 + 4^2 = 7(n + n_1) + 16 = 7(n + n_1 + 2) + 2$ ekanini hosil qilamiz. Demak, qoldiq 2 ga teng ekan. **Javob:** 2 (D).

14. (98-11-57) $9^{10}$ ni 7 ga bo'lgandagi qoldiqni toping.
    A) 1      B) 3      C) 2      D) 6

15. (99-3-6) $4^{12}$ ni 9 ga bo'lganda qoldiq necha bo'ladi?
    A) 1      B) 2      C) 4      D) 7

16. $5^{40}$ ni 8 ga bo'lganda qoldiq necha bo'ladi?
    A) 1      B) 2      C) 4      D) 5

17. $13^9$ ni 5 ga bo'lganda qoldiq necha bo'ladi?
    A) 3      B) 2      C) 4      D) 1

18. $2002^{2002}$ sonini 4 ga bo'lganda qoldiq nimaga teng?
    A) 0      B) 1      C) 2      D) 4

19. $2011^{2010}$ sonini 5 ga bo'lgandagi qoldiqni toping.
    A) 0      B) 1      C) 2      D) 4

20. (96-6-2) 243 ni qandaydir songa bo'lganda bo'linma 15 ga, qoldiq 3 ga teng chiqdi. Bo'luvchini toping.
    A) 17      B) 16      C) 18      D) 19

    **Yechish:** 1-qoidaga ko'ra $243 = x \cdot 15 + 3$ tenglik o'rinli. Bu yerdan $15x = 243 - 3 \Longleftrightarrow x = 240 : 15 = 16$. **Javob:** 16 (B).

21. (97-2-2) 215 ni 19 ga bo'lganda, qoldiq 6 bo'ladi. Bo'linma nechaga teng?
    A) 13      B) 12      C) 9      D) 11

22. (97-8-2) 358 ni qanday songa bo'lganda bo'linma 17 va qoldiq 1 bo'ladi?
    A) 19      B) 21      C) 22      D) 20

23. (00-7-4) 624 ni qanday songa bo'lganda bo'linma 41 ga qoldiq esa 9 ga teng bo'ladi?
    A) 16      B) 17      C) 13      D) 15

24. (97-12-2) Natural sonni 18 ga bo'lganda, bo'linma 15 ga, qoldiq 3 ga teng bo'ldi. Bo'linuvchini toping.
    A) 173      B) 243      C) 253      D) 273

25. (99-1-3) $7 + 69 + 671 + 6673 + 66675$ sonni 6 ga bo'lgandagi qoldiqni toping.
    A) 1      B) 4      C) 3      D) 5

    **Yechish:** 3-qoidaga ko'ra $7 + 69 + 671 + 6673 + 66675 = 7+60+9+660+11+6660+13+66660+15$ son bilan $7 + 9 + 11 + 13 + 15 = 55$ sonning 6 ga bo'lgandagi qoldig'i teng. 55 ni 6 ga bo'lgandagi qoldig'i esa 1 ga teng. **Javob:** 1 (A).

26. $1 + 93 + 995 + 9997 + 9999 + 999901$ sonni 9 ga bo'lgandagi qoldiqni toping.
    A) 1      B) 4      C) 8      D) 5

27. $27 + 1029 + 10031 + 100033 + 1000035$ sonni 25 ga bo'lgandagi qoldiqni toping.
    A) 1      B) 4      C) 8      D) 5

28. (99-3-5) 36455478354 sonni 2, 4, 5, 9, 10 va 25 ga bo'lganda hosil bo'ladigan qoldiqlar yig'indisini toping?
    A) 18      B) 16      C) 15      D) 14

29. (99-3-5) 36455468 sonni 2, 4, 5, 10 va 25 ga bo'lganda hosil bo'ladigan qoldiqlar yig'indisini toping?
    A) 18      B) 29      C) 15      D) 14

30. (00-2-10) 3 ga bo'linmaydigan natural sonning kubini 9 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
    A) 1 yoki 8      B) 0 yoki 1
    C) 0 yoki 8      D) 3 yoki 6

    **Yechish:** Berilgan natural son 3 ga bo'linmaganligi uchun u quyidagi ko'rinishda $n = 3k + r, r = 1$ yoki $r = 2$ bo'ladi. Shuning uchun $n^3 = (3k + r)^3 = 27k^3 + 3 \cdot 9k^2 \cdot r + 3 \cdot 3k \cdot r^2 + r^3$. 3-qoidaga ko'ra bu son bilan $r^3$ ning 9 ga bo'lgandagi qoldiqlari teng. $r = 1$ yoki $r = 2$ bo'lgani uchun $r^3 = 1$ yoki $r^3 = 8$ bo'ladi. **Javob:** 1 yoki 8 (A).

31. (99-8-25) $a$ va $b$ natural sonlarini 5 ga bo'lganda, mos ravishda 1 va 3 qoldiq hosil bo'ladi. Bu sonlar kvadratlarining yig'indisini 5 ga bo'lganda, qoldiq nechaga teng bo'ladi?
    A) 4     B) 1     C) 2     D) 0

32. Toq natural sonning kubini 4 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
    A) 1     B) 2     C) 1 yoki 3     D) 3

33. (99-8-25) Natural sonni 3 ga bo'lganda 1 qoldiq hosil bo'ladi. Bu sonning kvadratini 6 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
    A) 1 yoki 4     B) 1 yoki 3     C) 2     D) 5

34. (03-4-5) Nechta ikki xonali son 15 ga qoldiqsiz bo'linadi?
    A) 4     B) 5     C) 7     D) 6

    **Yechish:** Ma'lumki, ikki xonali son 10 dan boshlanib 99 bilan tugaydi. Demak, $15 = 1 \cdot 15, 30 = 2 \cdot 15, 45 = 3 \cdot 15, \ldots, 90 = 6 \cdot 15$. 6 ta ikki xonali son 15 ga qoldiqsiz bo'linadi. **Javob:** 6 (D).

35. Nechta ikki xonali son 9 ga qoldiqsiz bo'linadi?
    A) 9     B) 10     C) 11     D) 8

36. Nechta uch xonali son 50 ga qoldiqsiz bo'linadi?
    A) 19     B) 20     C) 17     D) 18

37. (01-6-2) Barcha uch xonali sonlar ichida 45 ga qoldiqsiz bo'linadiganlari nechta?
    A) 19     B) 20     C) 18     D) 21

38. (99-2-2) 821 ga qanday eng kichik musbat sonni qo'shganda, yig'indi 6 ga qoldiqsiz bo'linadi?
    A) 4     B) 1     C) 5     D) 7

## 1.1.6   Oxirgi raqam

1. $0^n = 0, 10^n = \ldots 0, 850^n = \ldots 0$

2. $1^n = 1, 21^n = \ldots 1, 271^n = \ldots 1$

3. $2^{4k+1} = \ldots 2, \quad 2^{4k+2} = \ldots 4,$
   $2^{4k+3} = \ldots 8, \quad 2^{4k} = \ldots 6.$

4. $3^{4k+1} = \ldots 3, \quad 3^{4k+2} = \ldots 9,$
   $3^{4k+3} = \ldots 7, \quad 3^{4k} = \ldots 1.$

5. $4^{2k} = \ldots 6, \quad 4^{2k+1} = \ldots 4.$

6. $5^n = \ldots 5, \quad 275^n = \ldots 5$

7. $6^n = \ldots 6, \quad 2756^n = \ldots 6$

8. $7^{4k+1} = \ldots 7, \quad 7^{4k+2} = \ldots 9,$
   $7^{4k+3} = \ldots 3, \quad 7^{4k} = \ldots 1.$

9. $8^{4k+1} = \ldots 8, \quad 8^{4k+2} = \ldots 4,$
   $8^{4k+3} = \ldots 2, \quad 8^{4k} = \ldots 6.$

10. $9^{2k} = \ldots 1, \quad 9^{2k+1} = \ldots 9.$

1. (96-13-11) $2^{100}$ ning oxirgi raqamini toping?
   A) 2     B) 0     C) 4     D) 6
   **Yechish:** 2 soni darajalarining oxirgi raqamlari har $4-$ darajadan keyin takrorlanib keladi. 3-ga qarang. 2 ning daraja ko'rsatkichi 4 ga bo'linsa, u son 6 raqami bilan tugaydi. **Javob:** 6 (D).

2. $21^{1964}$ ning oxirgi raqamini toping.
   A) 3     B) 1     C) 7     D) 9

3. $15^{1994}$ ning oxirgi raqamini toping.
   A) 3     B) 1     C) 7     D) 5

4. (96-3-71) $8^{99}$ ning oxirgi raqamini toping.
   A) 0     B) 2     C) 4     D) 6

5. $3^{2010}$ ning oxirgi raqamini toping.
   A) 3     B) 1     C) 7     D) 9

6. $6^{1991}$ ning oxirgi raqamini toping.
   A) 2     B) 6     C) 8     D) 4

7. $7^{1971}$ ning oxirgi raqamini toping.
   A) 7     B) 9     C) 3     D) 1

8. $9^{2009}$ soni qanday raqam bilan tugaydi?
   A) 0     B) 1     C) 3     D) 9

9. $24^{2011}$ soni qanday raqam bilan tugaydi?
   A) 0     B) 6     C) 4     D) 8

10. (97-11-2) Yig'indining oxirgi raqamini toping.
    $$15 \cdot 25 \cdot 37 \cdot 43 + 34 \cdot 48 \cdot 77$$
    A) 4     B) 9     C) 0     D) 5
    **Yechish:** Birinchi ko'paytmaning oxirgi raqami $5 \cdot 5 \cdot 7 \cdot 3 = 525$ ko'paytmaning oxirgi raqami bilan bir xil, ya'ni 5 ga teng. Ikkinchi ko'paytmaning oxirgi raqami $4 \cdot 8 \cdot 7 = 224$ esa 4 raqami bilan tugaydi. Yig'indining oxirgi raqami $5 + 4 = 9$, ya'ni 9 bilan tugaydi. **Javob:** 9 (B).

11. (97-1-2) Ayirma qanday raqam bilan tugaydi?
    $$17 \cdot 28 \cdot 41 \cdot 35 - 24 \cdot 12 \cdot 87$$
    A) 0     B) 2     C) 4     D) 6

12. (97-6-2) Yig'indining oxirgi raqamini toping.
    $$16 \cdot 27 \cdot 38 \cdot 19 + 22 \cdot 43 \cdot 98$$
    A) 8     B) 6     C) 4     D) 2

13. (99-6-7) Ifodaning qiymati qanday raqam bilan tugaydi?
    $$11^6 + 14^6 - 13^3 - 8$$
    A) 1     B) 2     C) 3     D) 4

14. (99-6-11) Yig'indi qanday raqam bilan tugaydi?
    $$9^{1996} + 9^{1997}$$
    A) 0     B) 1     C) 2     D) 3

15. (01-2-4) $43^{43} - 17^{17}$ ayirmaning oxirgi raqamini toping.
   A) 5     B) 2     C) 1     D) 0

16. $41^{14} + 56^{65} + 75^{57}$ ning oxirgi raqamini toping.
   A) 5     B) 2     C) 1     D) 0

### 1.1.7  Butun sonlar

Butun sonlar to'plami $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ harfi bilan belgilanadi. Quyidagi tengliklar o'rinli.

1. $(-1)^{2n} = 1$,            $(-1)^{2n-1} = -1$

2. $m(-n) = -m \cdot n$,       $m : (-n) = -m : n$.

3. $-m(-n) = m \cdot n$,       $-m : (-n) = m : n$.

1. (97-12-7) Quyidagi ifodalarning qaysi biri 1 ga teng?
   A) $(-(-1)^2)^3$       B) $((-1)^3)^3$
   C) $(-(-1)^4)^5$       D) $((-1)^3)^4$

   **Yechish:** 1-ga ko'ra $-1$ ning juft darajasi 1 ga teng. Shuning uchun $((-1)^3)^4 = (-1)^{12} = 1$.
   **Javob:** (D).

2. (96-6-9) Quyidagi ifodalardan qaysi biri $-1$ ga teng?
   A) $((-1)^2)^3$       B) $(-(-1)^2)^3$
   C) $((-1)^3)^2$       D) $(-(-1)^3)^3$

3. (97-2-9) Quyidagi ifodalardan qaysi biri 1 ga teng?
   A) $(-(-1)^2)^3$       B) $((-1)^3)^5$
   C) $-((-1)^5)^4$       D) $((-1)^3)^4$

4. (97-8-9) Quyidagilardan qaysi biri 1 ga teng?
   A) $((-1)^3)^2$           B) $(-(-1)^3)^6$
   C) $(-(-1)^2)^4$          D) $-((-1)^3)^4$

5. (96-12-6) $8 + 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
   A) 99     B) 15     C) 33     D) 27

   **Yechish:** Dastlab ikkinchi tartibli amallarni, yani ko'paytirish va bo'lishni 2 va 3-qoidalar asosida bajaramiz. $8 + 6 : (-2) - 2 \cdot (-11) = 8 - 3 + 22 = 5 + 22 = 27$.   **Javob:** 27 (D).

6. (96-3-6)  $-8 + 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
   A) 23     B) 11     C) $-11$     D) $-10$

7. (96-11-6)  $-8 - 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
   A) 17     B) $-55$     C) 55     D) 77

8. Hisoblang.

$$13 \cdot 58 - 83 \cdot 42 - 58 \cdot 15 + 42 \cdot 81$$

   A) $-100$     B) $-200$     C) 100     D) $-10$

9. (99-3-2)* Hisoblang.

$$1 - 3 + 5 - 7 + 9 - 11 + \cdots + 97 - 99$$

   A) $-46$     B) $-48$     C) $-50$     D) $-52$

10. (01-1-2)* Hisoblang.

$$4 - 7 + 8 - 11 + 12 - 15 + \cdots + 96 - 99$$

   A) $-75$     B) $-80$     C) $-72$     D) $-63$

11. Hisoblang.

$$199 - 198 + 197 - 196 + \cdots + 3 - 2 + 1$$

   A) 75     B) 80     C) 100     D) 99

## 1.2  Ratsional sonlar. Kasrlar

*Bir butunning bir yoki bir nechta teng qismlarini ifodalovchi son kasr deyiladi. Ixtiyoriy $n \in N$, $m \in Z$ uchun $\dfrac{m}{n}$ ifoda oddiy kasr deyiladi. Bu yerda $m-$ kasrning surati, $n-$ kasrning maxraji deyiladi, o'rtadagi chiziq esa kasr chizig'i deyiladi. Agar $n = 1$ bo'lsa, $\dfrac{m}{1} = m$ bo'ladi. Demak, har qanday butun sonni maxraji bir bo'lgan oddiy kasr ko'rinishida yozish mumkin. Oddiy kasrga ratsional son deyiladi.* Ratsional sonlar to'plami $Q$ harfi bilan belgilanadi. Demak, $Q = \{\dfrac{m}{n} : n \in N, m \in Z\}$. *Agar $\dfrac{a}{b}$ va $\dfrac{c}{d}$ kasrlar uchun $ad = bc$ tenglik bajarilsa, berilgan kasrlar teng kasrlar deyiladi.* Masalan, $\dfrac{1}{2} = \dfrac{3}{6}$, chunki $1 \cdot 6 = 2 \cdot 3$ yoki $\dfrac{1}{3} = \dfrac{2}{6}$, chunki $1 \cdot 6 = 2 \cdot 3$. Umumiy holda

$$\frac{a}{b} \quad \text{va} \quad \frac{an}{bn}$$

kasrlar tengdir, chunki $abn = ban$ tenglik o'rinlidir. *Demak, kasrning surat va maxrajini bir xil natural songa ko'paytirsak yoki bo'lsak berilgan kasrga teng kasr hosil bo'ladi. Bu xossa kasrlarning asosiy xossasi deyiladi. Agar kasrning surat va maxraji o'zaro tub bo'lsa, bunday kasr qisqarmas kasr deyiladi.* Kasrlarning asosiy xossasidan foydalanib, maxrajlari har xil bo'lgan kasrlarning maxrajlarini bir xil qilish mumkin. *Kasrlarning maxrajlarini tenglashtirishdagi bunday almashtirishga kasrlarni umumiy maxrajga keltirish deyiladi.* Masalan, $\dfrac{1}{2}$ va $\dfrac{1}{3}$ kasrlarini olaylik, ularga teng kasrlar $\dfrac{3}{6}$ va $\dfrac{2}{6}$ kasrlardir. Bu kasrlar umumiy maxrajga (bir xil maxrajga) keltirildi. *Agar kasrning surati maxrajidan kichik $(a < b)$ bo'lsa, u to'g'ri kasr deyiladi. Aksincha, surati maxrajidan katta yoki teng $(a \geq b)$ bo'lsa, kasr noto'g'ri kasr deyiladi.* Masalan, $\dfrac{1}{2}$ va $\dfrac{1}{3}$ to'g'ri kasrlardir. $\dfrac{3}{3}$ va $\dfrac{8}{5}$ noto'g'ri kasrlardir.

### 1.2.1  Oddiy kasrlar

1. $\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$ **bir xil maxrajli kasrlarni qo'shish**

2. $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+bc}{bd}$ **kasrlarni qo'shish**

3. $\dfrac{a+b-c}{m} = \dfrac{a}{m} + \dfrac{b}{m} - \dfrac{c}{m}$ **kasrni kasrlarga ajratish**

4.  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$  **kasrni kasrga ko'paytirish**

5.  $a \cdot \frac{b}{c} = \frac{ab}{c}$  **butun sonni kasrga ko'paytirish**

6.  $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$  **ishoralar bilan ishlash**

7.  $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$  **kasrni kasrga bo'lish**

8.  $c : \frac{a}{b} = \frac{cb}{a}$  **butun sonni kasrga bo'lish**

9.  $\frac{a}{b} : c = \frac{a}{bc}$  **kasrni butun songa bo'lish**

1.  (96-3-12) Hisoblang. $-\frac{1}{2} - \frac{1}{3}$
    A) $\frac{5}{6}$      B) $-\frac{2}{5}$      C) $\frac{2}{5}$      D) $-\frac{5}{6}$

    **Yechish:** Kasrlarni umumiy maxrajga keltiramiz, keyin qo'shamiz.

    $$-\frac{1}{2} - \frac{1}{3} = -\frac{3}{6} - \frac{2}{6} = -(\frac{3}{6} + \frac{2}{6}) = -\frac{3+2}{6} = -\frac{5}{6}.$$

    **Javob:** $-\frac{5}{6}$ (D) .

2.  (96-3-13) Ayirmani toping. $\frac{1}{2} - \frac{2}{3}$
    A) $\frac{1}{6}$      B) 1      C) $-\frac{1}{3}$      D) $-\frac{1}{6}$

3.  (96-11-13) Hisoblang. $-\frac{1}{4} - \frac{1}{5}$
    A) $-\frac{9}{20}$      B) $-\frac{2}{9}$      C) $-\frac{1}{10}$      D) $\frac{9}{20}$

4.  (96-11-14) Ayirmani toping. $\frac{1}{4} - \frac{4}{5}$
    A) $-\frac{11}{20}$      B) $-1$      C) $-\frac{7}{20}$      D) $\frac{13}{20}$

5.  (96-12-13) Hisoblang. $-\frac{1}{3} - \frac{1}{4}$
    A) $-\frac{2}{7}$      B) $-\frac{7}{12}$      C) $\frac{1}{6}$      D) $-\frac{1}{6}$

6.  (96-3-9) Hisoblang. $-\frac{3}{15} + \frac{1}{5} - \frac{1}{3}$
    A) $-\frac{19}{30}$      B) $-\frac{1}{3}$      C) $\frac{19}{30}$      D) $\frac{1}{3}$

    **Yechish:** Birinchi kasrning surat va maxrajini 3 ga qisqartiramiz. $-\frac{1}{5} + \frac{1}{5} - \frac{1}{3} = -\frac{1}{3}$. **Javob:** $-\frac{1}{3}$ (B).

7.  (96-11-9) Hisoblang. $-\frac{3}{15} + \frac{1}{5} + \frac{1}{3}$
    A) $-\frac{1}{3}$      B) $\frac{2}{15}$      C) $\frac{7}{15}$      D) $\frac{1}{3}$

8.  (96-12-9) Hisoblang. $\frac{3}{15} - \frac{1}{5} - \frac{1}{3}$
    A) $\frac{1}{3}$      B) $-\frac{3}{10}$      C) $\frac{3}{10}$      D) $-\frac{1}{3}$

9.  Hisoblang. $-\frac{5}{15} + \frac{1}{5} + \frac{1}{3}$
    A) $-\frac{1}{3}$      B) $\frac{2}{15}$      C) $\frac{1}{5}$      D) $\frac{1}{3}$

10.  (96-12-9) Hisoblang. $\frac{5}{15} - \frac{1}{5} - \frac{1}{3}$
    A) $\frac{1}{3}$      B) $-\frac{1}{5}$      C) $\frac{3}{10}$      D) $-\frac{1}{3}$

11.  (00-6-16) Hisoblang.

    $$\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \frac{1}{11 \cdot 14} + \frac{1}{14 \cdot 17}$$

    A) $\frac{15}{34}$      B) $\frac{5}{17}$      C) $\frac{5}{34}$      D) $\frac{16}{173}$

    **Yechish:** Har bir maxrajdagi ko'paytuvchilar 3 ga farq qiladi. Ixtiyoriy $n$ natural son uchun $\frac{1}{n(n+3)} = \frac{1}{3} \cdot (\frac{1}{n} - \frac{1}{n+3})$ ekanligidan

    $$\frac{1}{3} \cdot (\frac{1}{2} - \frac{1}{5}) + \frac{1}{3} \cdot (\frac{1}{5} - \frac{1}{8}) + \frac{1}{3} \cdot (\frac{1}{8} - \frac{1}{11}) + \frac{1}{3} \cdot (\frac{1}{11} - \frac{1}{14}) +$$

    $$+ \frac{1}{3} \cdot (\frac{1}{14} - \frac{1}{17}) = \frac{1}{3} \cdot (\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \frac{1}{11} -$$

    $$- \frac{1}{14} + \frac{1}{14} - \frac{1}{17}) = \frac{1}{3}(\frac{1}{2} - \frac{1}{17}) = \frac{5}{34}$$

    ni hosil qilamiz. **Javob:** $\frac{5}{34}$ (C).

12.  Hisoblang.

    $$\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11} + \frac{1}{11 \cdot 13}$$

    A) $\frac{5}{34}$      B) $\frac{5}{39}$      C) $\frac{5}{33}$      D) $\frac{5}{78}$

13.  Hisoblang.

    $$\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \frac{1}{15 \cdot 19} + \frac{1}{19 \cdot 23}$$

    A) $\frac{5}{69}$      B) $\frac{7}{96}$      C) $\frac{1}{12}$      D) $\frac{7}{94}$

14.  Hisoblang.

    $$\frac{1}{3 \cdot 8} + \frac{1}{8 \cdot 13} + \frac{1}{13 \cdot 18} + \frac{1}{18 \cdot 23} + \frac{1}{23 \cdot 28}$$

    A) $\frac{5}{84}$      B) $\frac{7}{96}$      C) $\frac{25}{84}$      D) $\frac{15}{84}$

15.  Hisoblang.

    $$\frac{1}{3 \cdot 10} + \frac{1}{10 \cdot 17} + \frac{1}{17 \cdot 24} + \frac{1}{24 \cdot 31} + \frac{1}{31 \cdot 38}$$

    A) $\frac{5}{114}$      B) $\frac{7}{104}$      C) $\frac{5}{104}$      D) $\frac{7}{114}$

16.  (98-12-62)* Hisoblang.

    $$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{999 \cdot 1000}$$

A) $0,750$   B) $1,125$   C) $0,998$   D) $0,999$

**Yechish:** Yuqorida keltirilgan yechish usulidan foydalanamiz. Etibor qilgan bo'lsangiz yig'indida birinchi qo'shiluvchi minus oxirgisi qoladi. Ixtiyoriy $n \in N$ uchun $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$ dan

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{999} - \frac{1}{1000}\right) =$$

$$= 1 - \frac{1}{1000} = \frac{999}{1000} = 0,999$$

ni olamiz. **Javob:** $0,999$ (D).

17. (00-3-15)* Hisoblang.

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{13 \cdot 15}$$

A) $\dfrac{11}{15}$   B) $\dfrac{7}{30}$   C) $\dfrac{8}{15}$   D) $\dfrac{7}{15}$

18. (00-8-57)* Hisoblang.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100}$$

A) $\dfrac{1}{9}$   B) $\dfrac{1}{10}$   C) $\dfrac{1}{100}$   D) $\dfrac{99}{100}$

19. (03-7-43)* Hisoblang.

$$\frac{2}{5 \cdot 7} + \frac{2}{7 \cdot 9} + \frac{2}{9 \cdot 11} + \cdots + \frac{2}{73 \cdot 75}$$

A) $\dfrac{16}{75}$   B) $\dfrac{28}{75}$   C) $\dfrac{1}{5}$   D) $\dfrac{14}{75}$

20. (03-8-29)* Hisoblang.

$$1 + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} + \frac{1}{12 \cdot 13} + \frac{1}{13 \cdot 14} + \frac{1}{14 \cdot 15}$$

A) $1\dfrac{3}{80}$   B) $1,16$   C) $1\dfrac{1}{30}$   D) $1\dfrac{7}{80}$

21. (03-1-9)* Hisoblang.

$$\frac{1}{8} + \frac{1}{24} + \frac{1}{48} + \frac{1}{80}$$

A) $0,1$   B) $0,2$   C) $0,4$   D) $0,6$

**Yechish:** Yuqorida yechilgan 11-misol usulidan foydalanamiz: $\dfrac{1}{8} + \dfrac{1}{24} + \dfrac{1}{48} + \dfrac{1}{80} = \dfrac{1}{2 \cdot 4} + \dfrac{1}{4 \cdot 6} + \dfrac{1}{6 \cdot 8} + \dfrac{1}{8 \cdot 10} = \dfrac{1}{2}\left(\dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{4} - \dfrac{1}{6} + \dfrac{1}{6} - \dfrac{1}{8} + \dfrac{1}{8} - \dfrac{1}{10}\right) = \dfrac{1}{2}\left(\dfrac{1}{2} - \dfrac{1}{10}\right) = \dfrac{1}{2} \cdot \dfrac{4}{10} = \dfrac{1}{5} = 0,2.$
**Javob:** $0,2$ (B).

22. (00-2-4)* Hisoblang.

$$\frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} + \frac{1}{195}$$

A) $\dfrac{4}{15}$   B) $\dfrac{7}{15}$   C) $\dfrac{17}{45}$   D) $\dfrac{2}{15}$

23. (99-5-1)* Hisoblang.

$$\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots + \frac{1}{182}$$

A) $\dfrac{11}{42}$   B) $\dfrac{10}{33}$   C) $\dfrac{1}{4}$   D) $\dfrac{12}{35}$

24. (00-9-10)* Hisoblang.

$$\frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \cdots + \frac{1}{255}$$

A) $\dfrac{7}{51}$   B) $\dfrac{2}{15}$   C) $\dfrac{2}{25}$   D) $\dfrac{3}{35}$

25. (96-11-11) Hisoblang.

$$\left(-\frac{1}{3}\right) \cdot \frac{2}{7} : \frac{5}{42}$$

A) $\dfrac{4}{5}$   B) $\dfrac{5}{441}$   C) $-\dfrac{4}{5}$   D) $\dfrac{10}{882}$

**Yechish:** Ko'paytirish (4-ga qarang) va bo'lish (7-ga qarang) qoidalaridan foydalanamiz:

$$\left(-\frac{1}{3}\right) \cdot \frac{2}{7} : \frac{5}{42} = -\frac{1}{3} \cdot \frac{2}{7} \cdot \frac{42}{5} = -\frac{1}{3} \cdot \frac{2}{7} \cdot \frac{2 \cdot 3 \cdot 7}{5} = -\frac{4}{5}.$$

**Javob:** $-\dfrac{4}{5}$ (C).

26. (96-12-10) Hisoblang.

$$-\frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \frac{5}{42}$$

A) $\dfrac{5}{441}$   B) $\dfrac{4}{5}$   C) $-\dfrac{5}{441}$   D) $-\dfrac{4}{5}$

27. Hisoblang.

$$\frac{1}{17} : \left(-\frac{5}{51}\right) \cdot \frac{4}{15}$$

A) $\dfrac{5}{441}$   B) $\dfrac{4}{25}$   C) $-\dfrac{5}{24}$   D) $-\dfrac{4}{25}$

28. Hisoblang.

$$\frac{3}{8} : \frac{3}{4}\left(-\frac{2}{5}\right) : \frac{1}{20}$$

A) $\dfrac{5}{4}$   B) $4$   C) $-4$   D) $-\dfrac{4}{25}$

29. (98-7-5) Hisoblang. $243 : (9 : 11)$

A) $27$   B) $2\dfrac{5}{11}$   C) $\dfrac{11}{27}$   D) $297$

30. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} - \frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{5}{42}\right)$$

A) $\dfrac{7}{10}$   B) $-\dfrac{3}{10}$   C) $\dfrac{5}{42}$   D) $\dfrac{3}{35}$

**Yechish:** Ko'paytirish (4-ga qarang) va bo'lish (7-ga qarang) qoidalaridan foydalanamiz:
$\dfrac{3}{8} : \dfrac{3}{4} - \dfrac{1}{3} \cdot \left(-\dfrac{2}{7}\right) : \left(-\dfrac{5}{42}\right) = \dfrac{3}{8} \cdot \dfrac{4}{3} - \dfrac{1}{3} \cdot \dfrac{2}{7} \cdot \dfrac{42}{5} = \dfrac{1}{2} - \dfrac{4}{5} = \dfrac{5-8}{10} = -\dfrac{3}{10}.$ **Javob:** $-\dfrac{3}{10}$ (B).

31. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} - \frac{1}{3} : \left(-\frac{2}{9}\right)$$

A) 10    B) −1    C) 2    D) 3, 5

32. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} + \frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{5}{42}\right)$$

A) $1\frac{1}{10}$    B) $-\frac{3}{10}$    C) $\frac{5}{42}$    D) $\frac{3}{35}$

33. Hisoblang.

$$\frac{7}{8} : \frac{7}{4} + \frac{1}{5} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{4}{35}\right)$$

A) 10    B) 0    C) $\frac{3}{35}$    D) 3

34. Hisoblang.

$$\frac{5}{8} : \frac{5}{4} + \frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{35}{42}\right)$$

A) $1\frac{5}{6}$    B) $\frac{9}{10}$    C) $\frac{5}{6}$    D) $\frac{7}{42}$

35. (96-9-58) $\frac{3}{4}$ va $\frac{8}{9}$ sonlari orasida maxraji 36 ga teng bo'lgan nechta kasr son bor?

A) 1    B) 2    C) 3    D) 4

**Yechish:** Berilgan kasrlarni 36 maxrajga keltiramiz. $\frac{3}{4} = \frac{27}{36}$ va $\frac{8}{9} = \frac{32}{36}$. Bu sonlar orasida maxraji 36 ga teng bo'lgan $\frac{28}{36}, \frac{29}{36}, \frac{30}{36}, \frac{31}{36}$ sonlar bor. **Javob:** 4 (D).

36. $\frac{3}{5}$ va $\frac{11}{15}$ sonlari orasida maxraji 45 ga teng bo'lgan nechta kasr son bor?

A) 5    B) 2    C) 3    D) 4

37. $\frac{3}{4}$ va $\frac{9}{10}$ sonlari orasida maxraji 40 ga teng bo'lgan nechta kasr son bor?

A) 6    B) 5    C) 3    D) 4

38. (96-1-8) $\frac{9}{11}$ va 1 sonlari orasida maxraji 33 ga teng bo'lgan nechta kasr son bor?

A) 2    B) 1    C) 5    D) 6

39. (96-10-8) $\frac{2}{3}$ dan katta va $\frac{5}{6}$ dan kichik bo'lgan, maxraji 30 ga teng bo'lgan nechta kasr mavjud?

A) 1    B) 2    C) 4    D) 5

### 1.2.2 Aralash kasrlar

Bizga $\frac{a}{n}$, $a \geq n$ noto'g'ri kasr berilgan bo'lsin. Qoldiqli bo'lish (1.1.5-dagi 1-ga qarang) qoidasiga ko'ra, shunday $m$ natural va $r$ ($0 \leq r < n$) butun sonlar mavjud bo'lib, bular uchun $a = nm + r$ tenglik o'rinli bo'ladi. U holda $\frac{a}{n} = \frac{nm + r}{n} = \frac{nm}{n} + \frac{r}{n} = m + \frac{r}{n}$. Demak, $\frac{a}{n}$

noto'g'ri kasrni natural son $m$ bilan to'g'ri kasr $\frac{r}{n}$ ning yig'indisi shaklida yozish mumkin. Bunday amal noto'g'ri kasrdan butun ajratish deyiladi.

Masalan, $\frac{12}{5} = \frac{10 + 2}{5} = \frac{10}{5} + \frac{2}{5} = 2 + \frac{2}{5}$. Natural son bilan to'g'ri kasrning yig'indisini qo'shish amalisiz yozish qabul qilingan, ya'ni $2 + \frac{2}{5}$ o'rniga $2\frac{2}{5}$ ko'rinishda yoziladi. Bu son aralash kasr deyiladi. Endi musbat kasrlarni taqqoslash haqida qoidalar beramiz.

1. **Bir xil maxrajli** $\frac{a}{b}$ **va** $\frac{c}{b}$ **kasrlarning surati kattasi katta bo'ladi, ya'ni** $a > c \Longleftrightarrow \frac{a}{b} > \frac{c}{b}$.

2. **Suratlari bir xil** $\frac{a}{b}$ **va** $\frac{a}{c}$ **kasrlarning maxraji kattasi kichik bo'ladi, ya'ni** $b < c \Longleftrightarrow \frac{a}{b} > \frac{a}{c}$.

3. **Ikki aralash kasrning butun qismi kattasi katta bo'ladi. Agar ularning butun qismlari teng bo'lsa, u holda kasr qismi kattasi katta bo'ladi.**

4. $0 < a < b < c \Longleftrightarrow \frac{1}{a} > \frac{1}{b} > \frac{1}{c}$.

5. $a$ **soniga teskari son** $\frac{1}{a}$. **Masalan,** $a = 0,8$ **bo'lsa, unga teskari son** $\frac{1}{0,8} = 1,25$.

6. $a$ **soniga qarama-qarshi son** $-a$. **Masalan,** $a = 0,8$ **bo'lsa, unga qarama-qarshi son** $-0,8$.

1. (97-5-9) Amalni bajaring. $1\frac{3}{5} - 3\frac{1}{5}$

A) $-1\frac{2}{5}$    B) $1\frac{2}{5}$    C) $1\frac{3}{5}$    D) $-1\frac{3}{5}$

**Yechish:** Aralash kasrlarni ayirishda butun qismidan butun qismi, kasr qismidan kasr qismi ayriladi. Demak, $1\frac{3}{5} - 3\frac{1}{5} = (1 - 3) + (\frac{3}{5} - \frac{1}{5}) = -2 + \frac{2}{5} = -(2 - \frac{2}{5}) = -\frac{10 - 2}{5} = -\frac{8}{5} = -1\frac{3}{5}$. **Javob:** $-1\frac{3}{5}$. (D).

2. (97-9-9) Amalni bajaring. $3\frac{4}{7} - 5\frac{2}{7}$

A) $-1\frac{5}{7}$    B) $1\frac{4}{7}$    C) $1\frac{5}{7}$    D) $-\frac{4}{7}$

3. Amalni bajaring. $3\frac{4}{9} - 2\frac{5}{18}$

A) $-1\frac{1}{6}$    B) $1\frac{5}{6}$    C) $1\frac{1}{6}$    D) $-1\frac{1}{18}$

4. Amalni bajaring. $2011\frac{1}{9} - 2010\frac{11}{18}$

A) $-0,1$    B) $0,5$    C) $0,6$    D) $-0,5$

5. (96-1-7) $0,6$ ga teskari sonni toping.

A) $-0,6$    B) $1\frac{2}{3}$    C) $0,4$    D) $-6$

**Yechish:** Teskari son ta'rifiga ko'ra (5-ga qarang), $0,6 = \frac{6}{10}$ ga teskari son $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$ dir. **Javob:** $1\frac{2}{3}$ (B).

6. (96-9-57) $0,8$ songa teskari sonni toping.

A) $-0,8$     B) $8$     C) $-\dfrac{5}{4}$     D) $1,25$

7. (96-10-7) $-1,5$ soniga teskari sonni toping.

A) $1,5$     B) $-0,75$     C) $-\dfrac{2}{3}$     D) $\dfrac{2}{3}$

8. (00-2-6) $\dfrac{11}{25}$ va $4\dfrac{6}{11}$ sonlariga teskari sonlar ko'paytmasini toping.

A) $\dfrac{1}{2}$     B) $1$     C) $\dfrac{3}{4}$     D) $2$

9. (06-111-15) O'zaro teskari sonlarni aniqlang:

1) $3 - \sqrt{2}$ va $3 + \sqrt{2}$;     2) $\dfrac{\sqrt{5}}{3}$ va $\dfrac{3\sqrt{5}}{5}$;

3) $\dfrac{2\sqrt{3}}{5}$ va $\dfrac{5\sqrt{3}}{6}$;     4) $\sqrt{2} + 1$ va $\sqrt{2} - 1$;

A) $1; 2; 3$     B) $1; 3; 4$     C) $1; 3$     D) $2; 3; 4$

10. $2,5$ ga qarama-qarshi sonni toping.

A) $0,4$     B) $-0,4$     C) $-2,5$     D) $2$

**Yechish:** Qarama-qarshi son ta'rifiga ko'ra (6-ga qarang), $2,5$ ga qarama-qarshi son $-2,5$ dir.
**Javob:** $-2,5$ (C).

11. $-1,6$ ga qarama-qarshi sonni toping.

A) $0,125$     B) $1,6$     C) $0,8$     D) $16$

12. $0,4$ ga qarama-qarshi bo'lgan songa teskari sonni toping.

A) $0,4$     B) $-0,4$     C) $-2,5$     D) $2$

13. (03-1-56) $0,8$ ga teskari bo'lgan songa qarama-qarshi sonni toping.

A) $-0,8$     B) $1,25$     C) $-1,25$     D) $-1,2$

14. (98-3-5) Agar $a = \dfrac{5}{11}$, $b = \dfrac{3}{7}$, $c = \dfrac{6}{13}$ bo'lsa, $a, b$ va $c$ ni o'sish tartibida joylashtring.

A) $a < b < c$          B) $b < a < c$
C) $b < c < a$          D) $c < b < a$

**Yechish:** Berilgan sonlarga teskari sonlarni taqqoslaymiz. $\dfrac{1}{a} = \dfrac{11}{5} = 2\dfrac{1}{5}$, $\dfrac{1}{b} = \dfrac{7}{3} = 2\dfrac{1}{3}$, $\dfrac{1}{c} = \dfrac{13}{6} = 2\dfrac{1}{6}$. Bu kasrlarning butun qismlari (2 ga) teng. Ularning kasr qismlari 2-qoida bo'yicha taqqoslanadi va biz $\dfrac{1}{c} < \dfrac{1}{a} < \dfrac{1}{b}$ ekanligini olamiz. 4-qoidaga ko'ra, $c > a > b$ ekan. **Javob:** $b < a < c$ (B).

15. (98-10-7) Sonlarni o'sish tartibida joylashtiring?

$$a = \dfrac{49}{150}; \quad b = \dfrac{102}{300}; \quad c = \dfrac{22}{75}$$

A) $a < c < b$          B) $b < c < a$
C) $c < a < b$          D) $b < c < a$

16. (98-10-53) Sonlarni o'sish tartibida joylashtiring?

$$a = \dfrac{5}{11}; \quad b = \dfrac{6}{13}; \quad c = \dfrac{7}{15}$$

A) $a < b < c$     B) $b < a < c$
C) $b < c < a$     D) $c < b < a$

17. (99-4-10) Sonlarni kamayish tartibida joylashtiring?

$$a = \dfrac{7}{36}; \quad b = \dfrac{11}{34}; \quad c = \dfrac{7}{32}; \quad d = \dfrac{9}{25}$$

A) $a > b > c > d$          B) $b > a > d > c$
C) $d > a > b > c$          D) $d > b > c > a$

18. (02-3-3) $\dfrac{3}{7}, \dfrac{4}{17}, \dfrac{21}{23}$ sonlariga bo'lganda, butun son chiqadigan eng kichik natural sonni toping.
A) $84$     B) $36$     C) $42$     D) $63$

**Yechish:** $\dfrac{3}{7}, \dfrac{4}{17}, \dfrac{21}{23}$ sonlariga bo'lganda, butun son chiqadigan eng kichik natural sonni $n$ bilan belgilaymiz. Demak, $n : \dfrac{3}{7} = n \cdot \dfrac{7}{3}$, $n : \dfrac{4}{17} = n \cdot \dfrac{17}{4}$, $n : \dfrac{21}{23} = n \cdot \dfrac{23}{21}$ sonlari butun bo'lishi uchun $n$ soni $3$, $4$ va $21$ sonlariga bo'linishi kerak. Minimallik shartidan $n$ ularning EKUKi ekanligi kelib chiqadi. EKUK topish qoidasiga ko'ra, $K(3; 4; 21) = 84$.
**Javob:** $84$ (A).

19. (03-7-6) $\dfrac{2}{7}, \dfrac{4}{11}, \dfrac{6}{13}$ sonlariga bo'linganda, bo'linma butun son chiqadigan eng kichik natural sonni toping.
A) $6$     B) $12$     C) $18$     D) $24$

20. (00-5-7) $\dfrac{1}{30}$ va $\dfrac{1}{45}$ kasr umumiy maxrajining barcha natural bo'luvchilari soni nechta?
A) $11$     B) $7$     C) $12$     D) $13$

21. (03-6-5) Agar $\dfrac{29}{31} + \dfrac{38}{41} + \dfrac{47}{51} = a$ bo'lsa,

$$\dfrac{2}{31} + \dfrac{3}{41} + \dfrac{4}{51}$$

quyidagilardan qaysi biriga teng?
A) $3 - a$     B) $4 - a$     C) $5 - a$     D) $3 - \dfrac{a}{2}$

**Yechish:** Izlanayotgan sonni $x$ bilan belgilaymiz. $\dfrac{29}{31} + \dfrac{38}{41} + \dfrac{47}{51} = a$ va $\dfrac{2}{31} + \dfrac{3}{41} + \dfrac{4}{51} = x$ sonlarni (chap tomoniga chap tomonini, o'ng tomoniga o'ng tomonini) qo'shamiz. Natijada $1 + 1 + 1 = a + x$ ni olamiz. Bu yerdan $x = 3 - a$. **Javob:** $3 - a$ (A).

22. Agar $\dfrac{3}{5} + \dfrac{7}{15} + \dfrac{21}{25} = a$ bo'lsa,

$$\dfrac{2}{5} + \dfrac{8}{15} + \dfrac{4}{25}$$

quyidagilardan qaysi biriga teng?
A) $3 - a$     B) $4 - a$     C) $5 - a$     D) $3 - \dfrac{a}{2}$

23. (03-7-8) Agar $\dfrac{29}{31} + \dfrac{38}{41} + \dfrac{47}{51} + \dfrac{56}{61} = a$ bo'lsa,

$$\dfrac{2}{31} + \dfrac{3}{41} + \dfrac{4}{51} + \dfrac{5}{61}$$

quyidagilardan qaysi biriga teng?
A) $3 - a$     B) $4 - a$     C) $5 - a$     D) $3 - \dfrac{a}{2}$

24. Agar $\dfrac{3}{10} + \dfrac{5}{20} + \dfrac{7}{30} + \dfrac{9}{40} = a$ bo'lsa,

$$\dfrac{2}{10} + \dfrac{5}{20} + \dfrac{8}{30} + \dfrac{11}{40}$$

quyidagilardan qaysi biriga teng?
A) $3 - a$    B) $4 - a$    C) $2 - a$    D) $3 - 2a$

25. (97-10-6) Hisoblang.

$$1\dfrac{8}{17} \cdot 3\dfrac{2}{5} : \dfrac{11}{12} \cdot 2\dfrac{1}{5} : \dfrac{4}{9}$$

A) $2,7$    B) $24\dfrac{3}{17}$    C) $27$    D) $29\dfrac{1}{9}$

**Yechish:** Aralash kasrlarni noto'g'ri kasrga kelti-rib so'ngra bo'lish amalini 1.2.1-dagi 7-qoidaga ko'ra ko'paytmaga almashtiramiz, keyin ularning surat va maxrajlarini qisqartiramiz:

$$1\dfrac{8}{17} \cdot 3\dfrac{2}{5} : \dfrac{11}{12} \cdot 2\dfrac{1}{5} : \dfrac{4}{9} = \dfrac{25}{17} \cdot \dfrac{17}{5} \cdot \dfrac{12}{11} \cdot \dfrac{11}{5} \cdot \dfrac{9}{4} = 27.$$

**Javob:** 27 (C).

26. (96-7-6) Hisoblang.

$$5\dfrac{5}{7} : 2\dfrac{2}{5} \cdot 5\dfrac{1}{4} : 1\dfrac{1}{6} \cdot \dfrac{2}{3}$$

A) $7\dfrac{1}{7}$    B) $8\dfrac{1}{7}$    C) $6\dfrac{6}{7}$    D) $5\dfrac{5}{7}$

27. (97-3-6) Hisoblang.

$$\dfrac{3}{4} \cdot 1\dfrac{1}{7} : \dfrac{2}{15} \cdot 12\dfrac{1}{4} : 7\dfrac{1}{2}$$

A) $10\dfrac{1}{2}$    B) $11$    C) $9\dfrac{1}{4}$    D) $7\dfrac{1}{2}$

28. (97-7-6) Hisoblang.

$$\dfrac{42}{95} \cdot 1\dfrac{3}{14} : \dfrac{3}{5} : 2 \cdot 4\dfrac{3}{4}$$

A) $\dfrac{13}{8}$    B) $1\dfrac{3}{8}$    C) $2\dfrac{1}{8}$    D) $1\dfrac{5}{7}$

29. (98-3-8) Hisoblang.

$$3\dfrac{1}{3} \cdot 2\dfrac{1}{4} \cdot \left(-\dfrac{1}{2}\right) \cdot \dfrac{4}{5}$$

A) $3$    B) $-3$    C) $2,5$    D) $-2,5$

30. (98-10-56) Hisoblang.

$$2\dfrac{2}{3} : 1\dfrac{1}{7} \cdot 3\dfrac{3}{7} \cdot \left(-\dfrac{1}{4}\right)$$

A) $4$    B) $3$    C) $-2$    D) $\dfrac{2}{7}$

31. (07-108-1) Hisoblang.

$$\dfrac{15}{56} \cdot 1\dfrac{1}{7} : \dfrac{2}{15} \cdot 24\dfrac{1}{2} : 7\dfrac{1}{2}$$

A) $11$    B) $10\dfrac{1}{2}$    C) $7\dfrac{1}{2}$    D) $21$

32. (96-7-9) Hisoblang.

$$\left(7\dfrac{1}{3} - 6\dfrac{7}{8}\right) : \dfrac{3}{4} + 8\dfrac{8}{9} \cdot 2\dfrac{1}{80}$$

A) $17\dfrac{2}{3}$    B) $18\dfrac{1}{2}$    C) $21\dfrac{1}{2}$    D) $16\dfrac{1}{3}$

**Yechish:** Dastlab qavs ichidagi ayirmani hisoblay-miz.

$$7\dfrac{1}{3} - 6\dfrac{7}{8} = 7 - 6 + \dfrac{1}{3} - \dfrac{7}{8} = 1 + \dfrac{8 \cdot 1 - 3 \cdot 7}{24} =$$
$$= 1 - \dfrac{13}{24} = \dfrac{24 - 13}{24} = \dfrac{11}{24}.$$

Aralash kasrlarni noto'g'ri kasrlarga aylantirib, amallarni bajaramiz:

$$\dfrac{11}{24} \cdot \dfrac{4}{3} + \dfrac{80}{9} \cdot \dfrac{161}{80} = \dfrac{11}{18} + \dfrac{161}{9} = \dfrac{11 + 322}{18} =$$
$$\dfrac{333}{18} = \dfrac{37}{2} = 18\dfrac{1}{2}. \quad \textbf{Javob: } 18\dfrac{1}{2} \text{ (B)}.$$

33. (97-1-3) Hisoblang.

$$1\dfrac{1}{4} + \dfrac{5}{12} : \left(\dfrac{1}{3} \cdot 2\dfrac{1}{2} - \dfrac{7}{8}\right)$$

A) $11\dfrac{1}{4}$    B) $-1\dfrac{1}{4}$    C) $9\dfrac{1}{4}$    D) $-8\dfrac{3}{4}$

34. (97-3-9) Hisoblang.

$$\left(5\dfrac{3}{4} - 4\dfrac{8}{9}\right) \cdot 2 + 67\dfrac{1}{2} : 2\dfrac{1}{7}$$

A) $24\dfrac{1}{3}$    B) $33\dfrac{2}{9}$    C) $36\dfrac{1}{9}$    D) $31\dfrac{1}{3}$

35. (97-6-3) Hisoblang.

$$\left(\dfrac{5}{9} - 1\dfrac{1}{6} \cdot \dfrac{1}{2}\right) : \dfrac{5}{9} + \dfrac{1}{3}$$

A) $\dfrac{3}{20}$    B) $\dfrac{17}{60}$    C) $\dfrac{7}{30}$    D) $-\dfrac{7}{60}$

36. (97-7-9) Hisoblang.

$$\left(4\dfrac{1}{10} - 3\dfrac{4}{15}\right) \cdot \dfrac{5}{6} + 4\dfrac{1}{10} : 1\dfrac{1}{5}$$

A) $3\dfrac{5}{9}$    B) $4\dfrac{1}{9}$    C) $5\dfrac{2}{3}$    D) $2\dfrac{3}{5}$

37. (97-10-9) Hisoblang.

$$\left(12\dfrac{1}{9} - 10\dfrac{2}{5}\right) : 38\dfrac{1}{2} + 2\dfrac{8}{9} \cdot 18$$

A) $24\dfrac{1}{15}$    B) $32\dfrac{7}{45}$    C) $38\dfrac{3}{5}$    D) $52\dfrac{2}{45}$

38. (97-11-3) Hisoblang.

$$\left(3\dfrac{17}{36} - 5\dfrac{7}{12}\right) : \dfrac{2}{9} - \dfrac{3}{26} \cdot 4\dfrac{1}{3}$$

A) $-9$    B) $8\dfrac{1}{2}$    C) $9$    D) $-10$

39. (07-107-1) Hisoblang.

$$8\dfrac{3}{4} + \dfrac{5}{12} : \left(\dfrac{1}{3} \cdot 2\dfrac{1}{2} - \dfrac{7}{8}\right)$$

A) $-1\dfrac{1}{4}$    B) $-6\dfrac{3}{4}$    C) $-8\dfrac{3}{4}$    D) $9\dfrac{1}{4}$

### 1.2.3  O'nli kasrlar

**1. Agar kasrning maxrajini 10 va uning darajalari ko'rinishida tasvirlash mumkin bo'lsa, bunday kasrga o'nli kasr deyiladai.**

**2.** $a \cdot 10^{-n} = \dfrac{a}{10^n}$ **ni o'nli kasrga aylantirish uchun** $a$ **sonida chapdan o'ngga tomon** $n$ **ta raqamdan so'ng vergul qo'yiladi.** $a$ **da raqamlar soni** $n$ **tadan kam bo'lsa, oldiga nol raqamlari qo'yiladi.**
**Masalan,** $\dfrac{2345}{10^3} = 2,345,\quad \dfrac{23}{10^4} = 0,0023.$

1. (96-3-64) Hisoblang.
$$2,701 \cdot 10^{-4} + 3,205 \cdot 10^{-3}.$$
A) $5,906 \cdot 10^{-3}$      B) $5,906 \cdot 10^{-4}$
C) $3,4751 \cdot 10^{-3}$     D) $3,0215 \cdot 10^{-4}$

**Yechish:** Umumiy ko'paytuvchi $10^{-3}$ ni qavsdan tashqariga chiqaramiz.

$2,701 \cdot 10^{-4} + 3,205 \cdot 10^{-3} = 10^{-3}(2,701 \cdot 10^{-1} + 3,205)$

$= 10^{-3}(0,2701 + 3,205) = 3,4751 \cdot 10^{-3}.$

**Javob:** $3,4751 \cdot 10^{-3}$ (C).

2. (06-111-2) $2,014 : 0,19 + 2,5 \cdot 0,3$ ni hisoblang.
A) $11,35$      B) $9,35$      C) $12,85$      D) $8,85$

3. (96-13-4) Ushbu
$$3,104 \cdot 10^{-2} + 1,81 \cdot 10^{-3}$$
yig'indi quyidagi sonlarning qaysi biriga teng?
A) $3,285 \cdot 10^{-3}$      B) $3,285 \cdot 10^{-2}$
C) $4,914 \cdot 10^{-2}$      D) $4,914 \cdot 10^{-3}$

4. (96-9-4) $1,011 \cdot 10^{-3} + 2,1 \cdot 10^{-4}$ ni hisoblang.
A) $3,111 \cdot 10^{-3}$      B) $3,111 \cdot 10^{-4}$
C) $3,111 \cdot 10^{-7}$      D) $1,221 \cdot 10^{-3}$

5. (96-12-62) Hisoblang.
$$1,015 \cdot 10^{-4} + 3,14 \cdot 10^{-5}$$
A) $4,155 \cdot 10^{-4}$      B) $4,155 \cdot 10^{-5}$
C) $4,155 \cdot 10^{-9}$      D) $1,329 \cdot 10^{-4}$

6. (98-12-8) Hisoblang.
$$\frac{3,21 \cdot 5,95 - 4,44}{2,21 \cdot 5,95 + 1,51}$$
A) $1$      B) $2$      C) $\dfrac{1}{2}$      D) $1\dfrac{1}{2}$

**Yechish:** Kasr suratida quyidagicha almashtirish bajaramiz.

$3,21 \cdot 5,95 - 4,44 = (2,21 + 1) \cdot 5,95 - 4,44 =$

$= 2,21 \cdot 5,95 + 5,95 - 4,44 = 2,21 \cdot 5,95 + 1,51$

ekanidan

$$\frac{3,21 \cdot 5,95 - 4,44}{2,21 \cdot 5,95 + 1,51} = \frac{2,21 \cdot 5,95 + 1,51}{2,21 \cdot 5,95 + 1,51} = 1$$

ni hosil qilamiz. **Javob:** $1$ (A).

7. (98-7-9) Hisoblang.
$$\frac{2,21 \cdot 5,95 + 1,51}{6,42 \cdot 5,95 - 8,88}$$
A) $1$     B) $\dfrac{1}{2}$     C) $1\dfrac{1}{2}$     D) $-\dfrac{62}{41}$

8. Hisoblang.
$$\frac{6,86 \cdot 4,75 - 4,62}{2,44 + 4,75 \cdot 2,43}$$
A) $1$     B) $\dfrac{1}{2}$     C) $2$     D) $-2$

9. Hisoblang.
$$\frac{1,27 \cdot 3,45 + 2,25}{4,54 \cdot 3,45 - 2,4}$$
A) $1$     B) $\dfrac{1}{2}$     C) $1\dfrac{1}{2}$     D) $-\dfrac{1}{2}$

10. (01-8-17) Hisoblang.
$$0,21 : \left(0,05 + \frac{3}{20}\right) - 2,5 \cdot 1,4$$
A) $-2,45$    B) $-2,55$    C) $-2$    D) $-3,35$

**Yechish:** Dastlab qavs ichini hisoblaymiz.

$$0,05 + \frac{3}{20} = 0,05 + \frac{15}{100} = 0,05 + 0,15 = 0,20.$$

Ikkinchi, keyin birinchi tartibli amallarni bajarib

$$0,21 : 0,20 - 2,5 \cdot 1,4 = 1,05 - 3,50 = -2,45$$

ni olamiz. **Javob:** $-2,45$ (A).

11. (01-2-11) Hisoblang.
$$4 - 3,3 : \left(2\frac{1}{7} - 1\frac{1}{5}\right)$$
A) $3,5$     B) $2,5$     C) $-1,5$     D) $0,5$

12. (00-5-17) Ifodaning qiymatini toping.
$$-2,4 + 3\frac{1}{3} - (-2,6)$$
A) $-10,6$    B) $12,5$    C) $3\dfrac{8}{15}$    D) $-12,5$

13. (00-5-18) Hisoblang.
$$\left(-\frac{3}{8}\right) \cdot (-32) + 0,5 \cdot (-8)$$
A) $8$     B) $4$     C) $6$     D) $7$

14. (96-1-5) Hisoblang.
$$\left(2,5 - 2\frac{1}{3}\right) \cdot 5,2 : 2\frac{3}{5}$$
A) $\dfrac{2}{3}$     B) $\dfrac{1}{3}$     C) $3$     D) $\dfrac{3}{7}$

15. (96-6-1) Hisoblang.

$$1,75 - \left(-1\frac{2}{7}\right) \cdot 6,5 \cdot \frac{7}{9}$$

A) $-4,75$  B) $2,15$  C) $8,25$  D) $4,75$

16. (96-9-56) Hisoblang.

$$6\frac{3}{8} - \left(2,5 - 2\frac{1}{3}\right) : 1\frac{1}{3}$$

A) $5\frac{2}{3}$  B) $6\frac{1}{4}$  C) $4\frac{1}{2}$  D) $2\frac{1}{3}$

**Yechish:** Dastlab qavs ichini hisoblaymiz:

$$2,5 - 2\frac{1}{3} = 2\frac{1}{2} - 2\frac{1}{3} = (2-2) + \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

$1\frac{1}{3}$ aralash kasrni noto'g'ri kasrga aylantirib, amallarni bajaramiz

$$6\frac{3}{8} - \frac{1}{6} : \frac{4}{3} = 6\frac{3}{8} - \frac{1}{6} \cdot \frac{3}{4} = 6 + \frac{3}{8} - \frac{1}{8} = 6\frac{1}{4}$$

ni olamiz. **Javob:** $6\frac{1}{4}$ (B).

17. (97-1-7) Hisoblang.

$$\left(\frac{1}{6} - 1\frac{1}{15} + \frac{1}{10}\right) : 0,6 + 0,4$$

A) $1\frac{11}{15}$  B) $0,88$  C) $-1\frac{1}{3}$  D) $-\frac{14}{15}$

18. (97-2-1) Hisoblang.

$$-1\frac{3}{4} \cdot 6,5 \cdot \left(-\frac{4}{7}\right) - 3,75$$

A) $-2,75$  B) $-10,25$  C) $2,75$  D) $10,25$

19. (97-8-1) Hisoblang.

$$5,8 - \frac{3}{7} \cdot 2,2 \cdot \left(-2\frac{1}{3}\right)$$

A) $3,6$  B) $-8$  C) $8$  D) $-3,6$

20. (97-11-7) Hisoblang.

$$0,2 + 1,8 \cdot \left(\frac{4}{9} - 1\frac{1}{2} + \frac{1}{6}\right)$$

A) $-1,4$  B) $1,8$  C) $0,04$  D) $-0,36$

21. (98-8-5) Hisoblang.

$$\frac{3}{16} + \frac{1}{16} \cdot (0,312 : 0,3 - 3,15 \cdot 1,6)$$

A) $\frac{1}{4}$  B) $\frac{3}{16}$  C) $-\frac{1}{16}$  D) $-\frac{1}{8}$

22. (98-1-3) Hisoblang.

$$19,9 \cdot 18 - 19,9 \cdot 16 + 30,1 \cdot 18 - 30,1 \cdot 16$$

A) $98$  B) $100$  C) $10$  D) $110$

**Yechish:** Birinchi va uchinchi qo'shiluvchilardan umumiy ko'paytuvchi 18 ni, ikkinchi va to'rtinchi qo'shiluvchilardan 16 ni qavsdan tashqariga chiqaramiz. Natijada $18(19,9 + 30,1) - 16(19,9 + 30,1) = 18 \cdot 50 - 16 \cdot 50 = 50(18-16) = 50 \cdot 2 = 100$.
**Javob:** $100$ (B).

23. (99-6-2) Hisoblang.

$$13,5 \cdot 5,8 - 8,3 \cdot 4,2 - 5,8 \cdot 8,3 + 4,2 \cdot 13,5$$

A) $42$  B) $52$  C) $50$  D) $48$

24. (00-2-1) Ifodaning qiymatini toping.

$$12,7 \cdot 64 + 173 \cdot 3,6 + 12,7 \cdot 36 + 17,3 \cdot 64$$

A) $3000$  B) $1800$  C) $2000$  D) $3600$

25. (98-8-3) Hisoblang.

$$109 \cdot 9,17 - 5,37 \cdot 72 - 37 \cdot 9,17 + 1,2 \cdot 72$$

A) $360$  B) $350$  C) $290$  D) $380$

26. (99-8-7) Ifodaning qiymatini toping.

$$79,9 - 79,8 + 79,7 - 79,6 + 79,5 - 79,4 + \cdots +$$
$$+60,3 - 60,2 + 60,1 - 60$$

A) $100$  B) $20$  C) $10$  D) $18,8$

27. (98-8-7) Hisoblang.

$$\left(\frac{5}{6} \cdot 5 - 5\right) : \frac{2}{3} - 0,5^2$$

A) $1$  B) $-1$  C) $0,5$  D) $-1,5$

**Yechish:** Dastlab qavs ichidagi amallarni bajaramiz.

$$\frac{5}{6} \cdot 5 - 5 = \frac{25}{6} - 5 = \frac{25-30}{6} = -\frac{5}{6}.$$

Agar $0,5 = \frac{1}{2}$ ekanligini hisobga olsak, $0,5^2 = \frac{1}{4}$ ekanligini olamiz. Natijada,

$$-\frac{5}{6} : \frac{2}{3} - \frac{1}{4} = -\frac{5}{6} \cdot \frac{3}{2} - \frac{1}{4} = -\frac{5}{4} - \frac{1}{4} = -\frac{6}{4} = -1,5.$$

**Javob:** $-1,5$ (D).

28. (98-1-7) Hisoblang.

$$\left(\frac{2}{3} : 3 - 1\right) \cdot 1,5^2 - 0,25$$

A) $1,5$  B) $-2$  C) $-5$  D) $-0,2$

29. (98-4-1) Hisoblang.

$$\left(1,6^2 - 2,2 \cdot \frac{3}{11}\right) : 1,4$$

A) $1,4$  B) $1,2$  C) $1,5$  D) $1,6$

30. (99-4-4) Hisoblang.

$$2,8 \cdot \left(2\frac{1}{3} : 2,8 - 1\right) + 2\frac{4}{5}$$

A) $5,6$  B) $2\frac{2}{3}$  C) $2\frac{1}{3}$  D) $2,8$

31. (00-6-2) Hisoblang.

$$(0,2 \cdot 0,1 - 0,1) : 0,25 + 0,75$$

A) $1,07$      B) $-2,45$      C) $3,95$      D) $0,43$

32. (00-6-3) Ifodaning qiymatini toping.

$$(1\frac{2}{3} \cdot 2,2 + 1) : 2\frac{1}{5} - \frac{5}{11}$$

A) $1$      B) $1,6$      C) $2\frac{1}{3}$      D) $1\frac{2}{3}$

33. (07-109-1) Hisoblang.

$$\left(3,5 - 3\frac{1}{3}\right) \cdot 10,4 : 5\frac{1}{5}$$

A) $\frac{1}{3}$      B) $\frac{2}{5}$      C) $\frac{3}{7}$      D) $\frac{1}{12}$

34. (96-1-3) Ifodaning qiymatini toping.

$$\frac{6,8 \cdot 0,04 \cdot 1,65}{3,3 \cdot 5,1 \cdot 0,16}$$

A) $6$      B) $\frac{1}{2}$      C) $\frac{2}{3}$      D) $\frac{1}{6}$

**Yechish:** Kasr suratida verguldan keyingi raqamlar sonini hisoblaymiz. Ular 5 ta. Endi kasr maxrajida verguldan keyingi raqamlar sonini hisoblaymiz. Ular 4 ta. Kasr maxrajidagi $5,1$ ni, unga teng bo'lgan $5,10$ bilan almashtiramiz, natijada kasr maxraji va suratida verguldan keyingi raqamlar soni tenglashadi. Kasr surati va maxrajini $10^5$ ga ko'paytiramiz

$$\frac{6,8 \cdot 0,04 \cdot 1,65 \cdot 10^5}{3,3 \cdot 5,10 \cdot 0,16 \cdot 10^5} = \frac{68 \cdot 4 \cdot 165}{33 \cdot 510 \cdot 16}.$$

Endi kasrni qisqatriramiz

$$\frac{68 \cdot 4 \cdot 165}{33 \cdot 510 \cdot 16} = \frac{17 \cdot 4 \cdot 4 \cdot 33 \cdot 5}{33 \cdot 17 \cdot 30 \cdot 16} = \frac{5}{30} = \frac{1}{6}.$$

**Javob:** $\frac{1}{6}$ (D).

35. (96-9-54) Ifodaning qiymatini toping.

$$\frac{0,7 \cdot 1,8 \cdot 2,6}{7,2 \cdot 7,8 \cdot 1,4}$$

A) $\frac{1}{24}$      B) $\frac{2}{5}$      C) $0,04$      D) $\frac{1}{12}$

36. (96-10-3) Ifodaning qiymati nechaga teng?

$$\frac{0,15 \cdot 1,6 \cdot 4,6}{9,2 \cdot 0,03 \cdot 6,4}$$

A) $\frac{5}{8}$      B) $\frac{2}{5}$      C) $2$      D) $0,2$

37. (99-4-3) Ifodaning qiymatini toping.

$$\frac{3,2 \cdot 0,027 \cdot 0,005}{0,09 \cdot 0,0025 \cdot 0,64}$$

A) $3$      B) $0,3$      C) $30$      D) $2$

38. (03-5-1) Hisoblang.

$$\frac{0,13}{0,00013} + \frac{0,02}{0,0005} - \frac{0,7}{0,0014}$$

A) $540$      B) $580$      C) $620$      D) $1400$

39. (03-10-3) Ifodaning qiymatini toping.

$$\frac{0,07}{0,21} + \frac{0,4}{0,06} + \frac{0,9}{0,05}$$

A) $25$      B) $20$      C) $15$      D) $30$

40. (01-6-1) Hisoblang.

$$\frac{400 - 21,5 \cdot 18,5}{1,5 \cdot 2\frac{1}{5} + 2,8 \cdot 1\frac{1}{2}}$$

A) $\frac{2}{7}$      B) $\frac{3}{5}$      C) $\frac{3}{7}$      D) $\frac{3}{10}$

**Yechish:** Kasr suratini hisoblaymiz, $400 - 21,5 \cdot 18,5 = 400 - 397,75 = 2,25$. Endi kasr maxrajini $1,5 \cdot 2,2 + 2,8 \cdot 1,5 = 1,5(2,2 + 2,8) = 1,5 \cdot 5 = 7,5$. Natijada, $2,25 : 7,5 = 0,3 = \frac{3}{10}$. **Javob:** $\frac{3}{10}$ (D).

41. (96-10-5) Hisoblang.

$$(5\frac{1}{3} - 3,2) : 2\frac{2}{3} + 1\frac{2}{5}$$

A) $2\frac{1}{2}$      B) $2,2$      C) $3,2$      D) $2$

42. (98-6-4) Hisoblang.

$$\frac{[(1,2 : 36) + 0,3] \cdot 9}{0,2}$$

A) $148,5$      B) $1,5$      C) $150$      D) $15$

43. (01-5-1) Hisoblang.

$$\frac{(6\frac{3}{5} - 3\frac{3}{14}) \cdot 5\frac{5}{6}}{(21 - 1,25) : 2,5}$$

A) $2,5$      B) $3$      C) $-2,5$      D) $4$

44. (99-2-1) Hisoblang.

$$\frac{7,4 + \frac{13}{17} \cdot 0,15 \cdot 1\frac{4}{13} \cdot 6\frac{2}{3}}{0,2 \cdot 5 - 0,16}$$

A) $10$      B) $8$      C) $12$      D) $6$

45. (00-1-1) Hisoblang.

$$\frac{\frac{5}{11} \cdot 0,006 \cdot 2\frac{1}{5} + 1\frac{1}{8} \cdot 0,004 \cdot \frac{8}{9}}{0,5 \cdot 0,0009 + 0,0001 \cdot 0,5}$$

A) $10$      B) $0,4$      C) $20$      D) $2$

46. (02-4-1) Hisoblang.

$$\left(2\frac{3}{4} - 0,25\right) \cdot 0,8 - 1\frac{2}{3} \cdot 1,8$$

A) $1$      B) $1,5$      C) $-1$      D) $-1,5$

47. (02-6-1) Hisoblang.

$$32 \cdot 0,99 \cdot 25 \cdot 1,25 + 411 + 57 \cdot 5 \cdot 0,4 \cdot 25 \cdot \frac{4}{19}$$

A) $2001$      B) $2000$      C) $1999$      D) $2002$

### 1.2.4 Cheksiz davriy o'nli kasrlar

Cheksiz o'nli kasrlarning kasr qismidagi bir yoki bir necha raqamlari bir xil tartibda ketma-ket takrorlansa, bunday kasrlar cheksiz davriy o'nli kasrlar deb, takrorlanadigan raqamlar gruppasiga shu kasrning davri deb ataladi. Davr qavsga olib yoziladi. Masalan, $0,5555\ldots = 0,(5)$; $2,1232323\ldots = 2,1(23)$. Cheksiz davriy o'nli kasr ko'rinishida tasvirlash mumkin bo'lgan sonlar ratsional sonlar deyiladi.

1. **Agar qisqarmas kasrning maxrajini tub ko'paytuvchilarga ajratganda 2 va 5 sonlaridan boshqa tub ko'paytuvchilar uchrasa, bunday kasr cheksiz davriy o'nli kasr bo'ladi.**
   **Misollar:** $\frac{3}{48} = \frac{1}{16} = \frac{1}{2^4}$ - chekli o'nli kasr.
   $\frac{5}{12} = \frac{5}{2^2 \, 3}$ - cheksiz davriy o'nli kasr.

2. **Davriy kasrlar ikki xil bo'ladi. a) agar davr verguldan keyin darhol boshlansa, bunday davriy kasr sof davriy kasr deyiladi.**
   **Misol:** $0,333\ldots = 0,(3)$, $2,161616\ldots = 2,(16)$.
   **b) agar davr verguldan keyin darhol boshlanmasa, bunday davriy kasr aralash davriy kasr deyiladi.**
   **Misol:** $0,377\ldots = 0,3(7)$, $2,81212\ldots = 2,8(12)$

3. **Sof davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo'lsa shuncha 9 dan, surati esa davrning o'zidan iborat.**
   **Masalan,** $0,(3) = \frac{3}{9} = \frac{1}{3}$, $\quad 2,(16) = 2\frac{16}{99}$.

4. **Aralash davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo'lsa shuncha 9 va verguldan keyin davrgacha nechta raqam bo'lsa shuncha 0 dan tuzilgan sondan, surati esa verguldan keyingi ikkinchi davrgacha bo'lgan raqamlardan tuzilgan sondan birinchi davrgacha bo'lgan raqamlardan tuzilgan son ayirmasidan iborat.**
   **Masalan,** $0,3(7) = \frac{37-3}{90} = \frac{34}{90} = \frac{17}{45}$,
   $2,8(12) = 2\frac{812-8}{990} = 2\frac{804}{990} = 2\frac{134}{165}$.

1. (96-1-12) Quyidagi sonlardan qaysi biri $0,(2)$ ga teng?
   A) $\frac{1}{9}$    B) $\frac{2}{9}$    C) $\frac{2}{3}$    D) $0,22$

   **Yechish:** $0,(2)$ sof davriy kasr, 3-qoidaga ko'ra $0,(2) = \frac{2}{9}$ dir. **Javob:** $\frac{2}{9}$ (B).

2. (96-9-62) Quyidagi sonlardan qaysi biri $0,(5)$ ga teng?
   A) $\frac{1}{2}$    B) $\frac{5}{9}$    C) $0,555$    D) $\frac{1}{5}$

3. (97-9-71) $8,(5)$ ni oddiy kasrga aylantiring.
   A) $8\frac{4}{9}$    B) $8\frac{5}{8}$    C) $8\frac{7}{8}$    D) $8\frac{5}{9}$

4. $0,(18)$ ni oddiy kasr shaklida yozing.
   A) $\frac{2}{11}$    B) $\frac{18}{90}$    C) $\frac{8}{99}$    D) $\frac{18}{900}$

5. (99-4-27) $0,5(6)$ soni quyidagilardan qaysi biriga teng?
   A) $\frac{56}{99}$    B) $\frac{1}{18}$    C) $\frac{17}{30}$    D) $\frac{28}{45}$

6. (01-6-22) $0,2(3)$ ni oddiy kasrga aylantiring.
   A) $\frac{7}{30}$    B) $\frac{4}{15}$    C) $\frac{3}{8}$    D) $\frac{2}{7}$

7. (03-8-27) $0,2(18)$ ni oddiy kasr shaklida yozing.
   A) $\frac{12}{55}$    B) $\frac{13}{55}$    C) $\frac{28}{99}$    D) $\frac{218}{900}$

8. (02-11-2) $3\frac{127}{495}$ ni cheksiz davriy o'nli kasr ko'rinishida yozing.
   A) $3,(127)$  B) $3,(254)$  C) $3,2(54)$  D) $3,2(56)$

9. (99-7-6) Hisoblang. $0,(5) + 0,(1)$
   A) $\frac{2}{3}$    B) $\frac{1}{3}$    C) $1,5$    D) $\frac{1}{4}$

   **Yechish:** $0,(5)$ va $0,(1)$ sof davriy kasrlardir. Ularni 3-qoidaga ko'ra oddiy kasrlarga aylantiramiz. $0,(5) = \frac{5}{9}$; $0,(1) = \frac{1}{9}$. Endi ularni qo'shamiz. $\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$. **Javob:** $\frac{2}{3}$ (A).

10. (98-5-4) $0,(8) + 0,(7)$ ni hisoblang.
    A) $0,(15)$    B) $1,(6)$    C) $1,(5)$    D) $1,(15)$

11. (01-3-39) $0,(8) + 0,(3)$ ni hisoblang.
    A) $1\frac{1}{9}$    B) $1\frac{2}{9}$    C) $1,(11)$    D) $1,(1)$

12. (02-5-2) $0,5(6) + 0,(8)$ ni hisoblang.
    A) $0,6(4)$    B) $1,3(6)$    C) $1,4(5)$    D) $1,36$

13. $0,(5) + 0,(6) + 0,(7)$ ni hisoblang.
    A) $1,(8)$  B) $1,3(6)$  C) $2$  D) $1,(18)$

14. $3,(7) + 6,(2)$ ni hisoblang.
    A) $9,(9)$    B) $\frac{80}{9}$    C) $10$    D) $\frac{89}{9}$

15. (98-11-3) Hisoblang.

$$\frac{0,8(3) - 0,4(6)}{0,(3)}$$

    A) $1,1$    B) $1\frac{1}{3}$    C) $3$    D) $0,3$

16. (96-3-68) Ushbu

$$a = 0,5(3), \quad b = \frac{47}{90}, \quad c = 1 - 0,48(1)$$

    sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?
    A) $a < b < c$    B) $b < c < a$
    C) $c < b < a$    D) $b < a < c$

    **Yechish:** Berilgan sonlarni oddiy kasrlarga aylantiramiz. $a = \frac{53-5}{90} = \frac{48}{90} = \frac{480}{900}$, $b = \frac{470}{900}$,

$c = 1 - \dfrac{481 - 48}{900} = 1 - \dfrac{433}{900} = \dfrac{467}{900}$. Bir xil maxrajli kasrlarni taqqoslash qoidasiga ko'ra $c < b < a$ munosabat o'rinli. **Javob:** $c < b < a$ (C).

17. (96-12-66) $a$, $b$ va $c$ sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?

$$a = 0,6(4), \quad b = \frac{59}{90}, \quad c = 1 - 0,36(9)$$

A) $a < c < b$      B) $a < b < c$
C) $b < a < c$      D) $c < a < b$

18. (98-1-10) Sonlarni kamayish tartibida joylashtiring.

$$a = 2,(4), \quad b = 2,5 - \frac{1}{8}, \quad c = 1,2 : 0,5$$

A) $a > b > c$      B) $a > c > b$
C) $b > a > c$      D) $c > a > b$

19. Sonlarni o'sish tartibida joylashtiring.

$$a = 0,8(87), \quad b = \frac{87}{99}, \quad c = 1 - 0,(13)$$

A) $a < c < b$      B) $a < b < c$
C) $b < a < c$      D) $c < b < a$

20. Sonlarni o'sish tartibida joylashtiring.

$$a = 0,(6) + 0,(7), \quad b = 1,(3), \quad c = 2 - \frac{7}{9}$$

A) $a < c < b$      B) $a < b < c$
C) $b < a < c$      D) $c < b < a$

21. Sonlarni o'sish tartibida joylashtiring.

$$a = -0,1(3), \quad b = -0,13(5), \quad c = -0,103(5)$$

A) $a < c < b$      B) $a < b < c$
C) $b < a < c$      D) $c < b < a$

22. Sonlarni o'sish tartibida joylashtiring.

$$a = \frac{10}{7}, \quad b = \frac{100}{77}, \quad c = \frac{1000}{777}$$

A) $a < c < b$      B) $a < b < c$
C) $b < a < c$      D) $c < b < a$

23. (96-9-3) Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilarini chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi?

1) $\dfrac{7}{32}$;   2) $\dfrac{11}{160}$;   3) $\dfrac{5}{48}$;   4) $\dfrac{5}{14}$;

A) $2;3$      B) $3;4$      C) $4;1$      D) $1;2$

**Yechish:** Berilgan kasrlarning maxrajlarini tub ko'paytuvchilarga ajratamiz. $32 = 2^5$; $160 = 2^5 \cdot 5$; $48 = 2^4 \cdot 3$ va $14 = 2 \cdot 7$. 48 va 14 sonlarining tub ko'paytuvchilari ichida 2 va 5 dan farqli 3 va 7 tub sonlari qatnashyapti. 1-qoidaga ko'ra $\dfrac{5}{48}$ va $\dfrac{5}{14}$ ni chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi. **Javob:** $3;4$ (B).

24. (96-13-3) Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilarini chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi?

1) $\dfrac{14}{625}$;   2) $\dfrac{3}{64}$;   3) $\dfrac{32}{75}$;   4) $\dfrac{11}{375}$;

A) $1;2$      B) $2;3$      C) $3;4$      D) $4;1$

25. Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilari chekli o'nli kasr bo'ladi?

1) $\dfrac{3}{48}$;   2) $\dfrac{7}{120}$;   3) $\dfrac{7}{112}$;   4) $\dfrac{3}{96}$;

A) $1;2$      B) $2;3$      C) $1;3;4$      D) $1;2;4$

26. Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$$m = 2\frac{5}{17}, \quad n = \frac{7}{32}, \quad p = \frac{2}{333}$$

A) $m, n$      B) faqat $m$      C) $n$      D) $m, p$

27. (98-12-5) Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$$m = 2,32666\ldots, \quad n = \frac{7}{99}, \quad p = \frac{5}{16},$$

$$q = 7,145222\ldots, \quad l = 3,222$$

A) $m, n$      B) $m, q$      C) $m, n, q$      D) $m, n, p$

28. (01-11-1) Hisoblang.

$$\left(6\frac{1}{3} \cdot 0,(5) + 0,(4) : \frac{3}{19}\right) \cdot 4\frac{5}{19}$$

A) $28$      B) $27,5$      C)$27$      D) $26,5$

**Yechish:** $0,(5)$ va $0,(4)$ davriy kasrlarni, $\dfrac{5}{9}$ va $\dfrac{4}{9}$ shaklda yozib, qavs ichidagi amallarni bajaramiz:

$$\frac{19}{3} \cdot \frac{5}{9} + \frac{4}{9} \cdot \frac{19}{3} = \frac{19}{3}\left(\frac{5}{9} + \frac{4}{9}\right) = \frac{19}{3} \cdot 1 = \frac{19}{3}.$$

Endi ko'paytirish amalini bajaramiz $\dfrac{19}{3} \cdot 4\dfrac{5}{19} = \dfrac{19}{3} \cdot \dfrac{81}{19} = 27$. **Javob:** 27 (C).

29. (99-10-1) Hisoblang.

$$\frac{0,48 \cdot 0,75 + 0,52 : 1\frac{1}{3}}{(0,(3) + 0,(6)) : 0,012}$$

A) $1$      B) $0,08$      C) $0,008$      D) $0,009$

30. (02-12-20) Hisoblang.

$$\left(\frac{81 \cdot 3}{567} + \frac{22}{77}\right) \cdot 24,5 - \frac{2}{3} : 0,(3)$$

A) $16,5$      B) $14,5$      C) $15,5$      D) $16,5$

31. (03-6-2) Hisoblang.

$$\frac{0,(4) + 0,(41) + 0,(42) + 0,(43)}{0,(5) + 0,(51) + 0,(52) + 0,(53)}$$

A) $\dfrac{170}{211}$      B) $\dfrac{83}{103}$      C) $\dfrac{63}{107}$      D) $\dfrac{65}{106}$

32. (03-7-4) Hisoblang.

$$\frac{0,(40) + 0,(41) + 0,(42) + 0,(43)}{0,(50) + 0,(51) + 0,(52) + 0,(53)}$$

A) $\frac{170}{211}$    B) $\frac{83}{103}$    C) $\frac{63}{107}$    D) $\frac{65}{106}$

33. (07-102-1) Hisoblang.

$$\left(2011\frac{1}{5} - 2010\frac{1}{6}\right) \cdot 1\frac{29}{31}$$

A) $2\frac{28}{29}$    B) $2\frac{29}{31}$    C) $3\frac{1}{29}$    D) $2$

34. (07-105-1) Hisoblang.

$$\frac{0,202 - 0,004}{\frac{8}{9} \cdot 81 \cdot 0,125}$$

A) $0,99$    B) $0,099$    C) $0,022$    D) $0,0099$

### 1.2.5 Protsent va proporsiya

Turmushda ko'p qo'llaniladigan kasrlar maxsus nomlarga ega. Masalan, $\frac{1}{2}$ va $\frac{1}{4}$ kasrlari yarim va chorak deb yuritiladi. $\frac{1}{100}$ kasr yoki yuzdan bir ulush tushunchasi keng qo'llaniladi. Bu kasrga maxsus nom berilgan u protsent yoki foiz deb ataladi. Protsent deb biror sonning yuzdan bir ulushiga aytiladi. Protsent odatda % belgi bilan ifodalanadi. $n\%$ yozuvi $\frac{n}{100}$ ni bildiradi. $n$ protsent $\frac{n}{100}$ oddiy kasrning boshqacha ko'rinishidir. $a$ sonining $n\%$ ni topish uchun $a$ ni $\frac{n}{100}$ kasrga ko'paytirish kerak. Masalan, $a$ sonining $10\%$ i $a \cdot \frac{10}{100} = 0,1a$ ga, $a$ ning $25\%$ i esa $0,25a$ ga teng.

$\frac{a}{b}$ yoki $a : b$ ga $a$ ning $b$ ga nisbati deyiladi. Ikkita nisbatning tengligi proporsiya deyiladi. Proporsiyaning umumiy ko'rinishi

$$a : b = c : d \quad \text{yoki} \quad \frac{a}{b} = \frac{c}{d}$$

ko'rinishda yoziladi. $a$ va $d$ lar proporsiyaning chetki hadlari $b$ va $c$ lar proporsiyaning o'rta hadlari deyiladi. Proporsiya quyidagi xossalarga ega.

**1.** $a : b = c : d \iff ad = bc$.

**2.** $a : b = c : d \iff na : b = nc : d$.

**3.** $a : b = c : d \iff a : c = b : d$.

**4.** $a : b = c : d \iff d : b = c : a$.

1. Maktab kutubxonasida 40000 ta kitob bor. Ularning $2\%$ i matematikaga oid kitoblardir. Kutubxonada matematikaga oid nechta kitob bor?

   A) 400    B) 200    C) 800    D) 1000

   **Yechish:** Sonning protsentini topish formulasiga asosan $\frac{40000 \cdot 2}{100} = 800$. **Javob:** 800 (C).

2. Maktab bog'ida 9652 tup mevali daraxt bo'lib, ularning $75\%$ olma daraxti. Maktab bog'ida necha tup olma daraxti bor?

   A) 7237    B) 7239    C) 7300    D) 7229

3. Matematika fakultetida 80 ta a'lochi talaba bo'lib, bu fakultetdagi barcha talabalarning $20\%$ ni tashkil qiladi. Fakultetdagi jami talabalar sonini toping.

   A) 400    B) 320    C) 500    D) 360

4. Viloyat olimpiadasida 80 ta o'quvchi qatnashdi. Ulardan 16 tasi barcha test savollarini yechdi. Test savollarining barchasini to'g'ri yechgan o'quvchilar olimpiada ishtirokchilarining necha foizini tashkil qiladi.

   A) 40    B) 20    C) 80    D) 10

5. SamDU matematika yo'nalishiga 70 ta talaba qabul qilinadi. Bu yo'nalishga 20 ta harbiy tavsiyanomali abituriyent hujjat topshirgan. Harbiy tavsiyanomali abituriyentlar uchun yo'nalish bo'yicha ajratilgan qabulning $20\%$ miqdorida qo'shimcha joy ajratilgan. Ko'pi bilan nechta harbiy tavsiyanomali abituriyent talabalikka tavsiya qilinmay qolishi mumkin.

   A) 14    B) 6    C) 4    D) 0

6. Bir kilogramm asal 10000 so'm turadi. Moliyaviy krizis tufayli uning narxi 12% ga arzonlashdi. Endi bir kilogramm asal qancha turadi.

   A) 9100    B) 9200    C) 8800    D) 8200

7. Quyidagi sonlar guruhlaridan 1) 7,8,14,16;   2) 1,2,3,4;   3) 3,4,15,20; qaysilari proporsiya tashkil qiladi?

   A) 1; 2    B) 1; 3    C) hammasi    D) 2; 3

   **Yechish:** 1) da $7 \cdot 16 = 8 \cdot 14$ tenglik o'rinli. Demak, 7,8,14,16; sonlar guruhi proporsiya tashkil qiladi. 2) da $1 \cdot 4 \neq 2 \cdot 3$, $1 \cdot 3 \neq 2 \cdot 4$, $1 \cdot 2 \neq 3 \cdot 4$ bo'lgani uchun bu sonlar guruhi proporsiya tashkil qilmaydi. 3) da $3 \cdot 20 = 4 \cdot 15$ tenglik o'rinli. Demak, 3,4,15,20; sonlar guruhi proporsiya tashkil qiladi. **Javob:** 1, 3 (B).

8. Piyoda 2,5 saotda 14 km yo'l bosdi. U shunday tezlik bilan yursa, 4,2 km yo'lni necha soatda bosadi.

   A) 0,7    B) 0,5    C) 0,75    D) 0,6

9. Proporsiyaning chetki hadlari 14 va 20 ga, o'rta hadlaridan biri 35 ga teng. Proporsiyaning ikkinchi o'rta hadini toping.

   A) 2    B) 8    C) 10    D) 7

10. 4, 8, 12, $a$ sonlari ko'rsatilgan tartibda proporsiya tashkil qilsa, $a$ ni toping.

    A) 20    B) 24    C) 28    D) 32

11. $21 : x = 7 : 8$ proporsiyaning noma'lum hadini toping.

    A) 21    B) 24    C) 22    D) 28

## 1.3 Irratsional sonlar

Cheksiz davriy o'nli kasrlar bilan bir qatorda cheksiz davriy bo'lmagan o'nli kasrlar ham mavjud. Masalan, $0, 10110111011110\ldots$ son cheksiz davriy bo'lmagan o'nli kasrga misol bo'ladi. Bu sonni tashkil qiluvchi raqamlar ma'lum bir qonuniyat asosida joylashgan, lekin hech bir raqamlar gruppasi davriy emas. *Davriy bo'lmagan cheksiz o'nli kasrlarga irratsional sonlar deyiladi.* Irratsional sonlarga misol qilib quyidagilarni ko'rsatish mumkin.

$$\sqrt{2} = 1,4142135\ldots, \quad \sqrt{3} = 1,7320508\ldots,$$

$$\pi = 3,1415926535\ldots, \quad e = 2,718281828459\ldots.$$

Irratsional sonlar musbat va manfiy bo'lishi mumkin. *Barcha ratsional va irratsional sonlar haqiqiy sonlar to'plamini tashkil qiladi.* Ma'lumki, ratsional sonlar to'plami $Q$ harfi bilan belgilanadi. Ratsional va irratsional sonlar to'plami o'zaro kesishmaydi. Shuning uchun irratsional sonlar to'plamini $R \setminus Q$ orqali belgilash mumkin.

1. $a$ va $b$ ratsional sonlar yig'indisi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

   **Yechish:** $a$ va $b$ ratsional sonlar bo'lganligi uchun ularni oddiy kasr ko'rinishida yozish mumkin. Oddiy kasrlar yig'indisi yana oddiy kasr, ya'ni ratsional sondir. Demak, ratsional sonlar yig'indisi doim ratsional son bo'ladi. **Javob:** doim ratsional (A).

2. $a$ va $b$ ratsional sonlar ayirmasi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

3. $a$ va $b$ ratsional sonlar ko'paytmasi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

4. $\alpha$ va $\beta$ irratsional sonlar yig'indisi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

5. $\alpha$ va $\beta$ irratsional sonlar ayirmasi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

6. $a$ ratsional son, $\alpha$ irratsional son bo'lsa, ularning yig'indisi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

7. $a$ ratsional son, $\alpha$ irratsional son bo'lsa, ularning ayirmasi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

8. $a$ noldan farqli ratsional son, $\alpha$ irratsional son bo'lsa, ularning ko'paytmasi qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

9. $\alpha$ va $\beta$ irratsional sonlar. Ularning nisbati qanday son bo'ladi?
   A) doim ratsional
   B) doim irratsional
   C) ratsional ham irratsional ham bo'lishi mumkin
   D) to'g'ri javob keltirilmagan

   **Yechish:** $\alpha$ va $\beta$ sonlar sifatida $\alpha = 2\pi$ va $\beta = \pi$ irratsional sonlarini olsak, u holda $\alpha : \beta = 2$ ratsional sonni olamiz. Agar biz $\alpha = \sqrt{6}$ va $\beta = \sqrt{3}$ irratsional sonlarini olsak, u holda $\alpha : \beta = \sqrt{2}$ irratsional sonni olamiz. **Javob:** ratsional ham irratsional ham bo'lishi mumkin (C).

10. $a$ noldan farqli ratsional son, $\alpha$ irratsional son bo'lsa, $\alpha : a$ (bo'linma) qanday son bo'ladi?
    A) doim ratsional
    B) doim irratsional
    C) ratsional ham irratsional ham bo'lishi mumkin
    D) to'g'ri javob keltirilmagan

11. $\alpha$ va $\beta$ irratsional sonlar. Ularning ko'paytmasi qanday son bo'ladi?
    A) doim ratsional
    B) doim irratsional
    C) ratsional ham irratsional ham bo'lishi mumkin
    D) to'g'ri javob keltirilmagan

12. $\alpha$ va $\beta$ irratsional sonlar bo'lib, ularning yig'indisi $\alpha + \beta$ ratsional son bo'lsin. Quyidagilardan qaysi biri doim ratsional son bo'ladi?
    A) $\alpha \cdot \beta$
    B) $\alpha + 2\beta$
    C) $\alpha^2 + \beta^2 + 2\alpha\beta$
    D) $\alpha - \beta$

13. Quyidagi sonlardan qaysilari irratsional sonlar:
    $a = 0,(123456789); \quad b = 3,12(61); \quad \alpha = \pi^2;$
    $\beta = 2,101001000100001\ldots;$
    A) $\alpha; \ b$     B) $a; \ \alpha$     C) $\alpha; \ \beta$     D) $a; \ b$

## 1.4 Haqiqiy sonlar

Yuqorida ta'kidlaganimizdek barcha ratsional va irratsional sonlar to'plami birgalikda haqiqiy sonlar to'plamini tashkil qiladi. Ma'lumki haqiqiy sonlar to'plami $R$ harfi bilan belgilanadi. Gorizontal $\ell$ to'g'ri chiziq olamiz (1.1-chizma). Unda ixtiyoriy $O$ nuqta olamiz va uni koordinata boshi deb ataymiz. $O$ nuqtaga nol sonini mos qo'yamiz. $O$ nuqtadan o'ngda $E$ nuqta tanlaymiz. $OE$ masshtab birligi deyiladi. $E$ nuqtaga 1 (bir) sonini mos qo'yamiz. $OE$ musbat yo'nalish hisoblanadi. $E$ nuqtadan bir masshtab o'ngdagi nuqtaga 2 (ikki) soni mos qo'yiladi va hokazo. $O$ nuqtadan bir masshtab chapdagi $E'$ nuqtaga $-1$ (minus bir) soni mos qo'yiladi, $E'$ nuqtadan bir masshtab chapdagi nuqtaga $-2$ (minus ikki) soni mos qo'yiladi va hokazo. Shunday qilib $R$ to'plamning elementlari bilan $\ell$ to'g'ri chiziqda joylashgan nuqtalar o'rtasida o'zaro bir qiymatli moslik o'rnatiladi. Bu holda $\ell$ koordinatalar to'g'ri chizig'i berilgan deyiladi. Ma'lumki, ixtiyoriy $r \in R$ soni uchun, $\ell$ koordinata to'g'ri chizig'ida unga mos keluvchi yagona $M$ nuqta mavjud. $r$ soni $M$ nuqtaning koordinatasi deyiladi va $M(r)$ ko'rinishda yoziladi. $\ell$ to'g'ri chiziqda koordinata boshi $O$ dan o'ng tomonda joylashgan nuqtalarga mos kelgan sonlar musbat, $O$ dan chap tomonda joylashgan nuqtalarga mos kelgan sonlar manfiy sonlar deyiladi. Nol soni musbat ham manfiy ham hisoblanmaydi. Musbat sonlar "plyus" $(+)$ ishorasi, manfiy sonlar "minus" $(-)$ ishorasi orqali yoziladi. Masalan, $+1$, $+2,5$, $+5,8,\dots$, $-1$, $-2,8$, $-8,7,\dots$. Musbat sonlar oldidagi $+$ ishorasini yozmaslikka kelishilgan, ya'ni $+1 = 1$, $+2,5 = 2,5$, $+5,8 = 5,8$.

Haqiqiy sonning moduli deb koordinata boshidan shu songa mos keluvchi nuqtagacha bo'lgan masofaga aytiladi. $a$ sonining moduli $|a|$ ko'rinishida yoziladi. Sonning moduli shu sonning absolyut qiymati deb ham ataladi. Har qanday musbat sonning moduli shu sonning o'ziga teng, manfiy sonning moduli shu sonning qarama-qarshisiga teng. Sonning modulini quyidagi formula shaklida ham yozish mimkin:

$$|a| = \begin{cases} a, & agar\ a \geq 0, \\ -a, & agar\ a \leq 0. \end{cases} \qquad (1.1)$$

$|a - b|$ miqdor $a$ va $b$ sonlariga mos keluvchi nuqtalar orasidagi masofaga teng. Agar $a$ va $b$ sonlariga mos keluvchi nuqtalar $A$ va $B$ bo'lib, $A$ nuqta $B$ dan chapda joylashgan bo'lsa, u holda $a$ soni $b$ sonidan kichik bo'ladi va aksincha. Haqiqiy sonning butun va kasr qismlari tushunchalarini keltiramiz. Butun bo'lmagan $a \in R$ sonining butun qismi deb sonlar o'qida $a$ sonidan chapda yotuvchi birinchi butun songa aytiladi va $[a]$ shaklda yoziladi. $a \in R$ sonining kasr qismi deb $a - [a]$ miqdorga aytiladi va $\{a\}$ shaklda yoziladi. Ma'lumki, butub sonning butun qismi o'ziga teng, kasr qismi esa nolga teng. Misol uchun $a = 2,34$

va $b = -2,71$ sonlarining butun va kasr qismlarini hisoblaymiz. Ta'rifga ko'ra $2,34$ dan chapda yotuvchi birinchi butun son bu 2 dir. Uning kasr qismi $a - [a] = 2,34 - 2 = 0,34$. Xuddi shunday $[b] = [-2,71] = -3$ va $\{b\} = \{-2,71\} = -2,71 - (-3) = 0,29$. Haqiqiy $b > 0$ sonining standart shakli deganda $a \cdot 10^n = b$ tushuniladi. Bu yerda $a$ sonining butun qismi 1 dan 9 gacha qiymatlardan birini qabul qiladi. Masalan, $0,01023 = 1,023 \cdot 10^{-2}$; $543,26 = 5,4326 \cdot 10^2$; $0,000026 = 2,6 \cdot 10^{-5}$. $n$ faktorial deb 1 dan $n$ gacha bo'lgan natural sonlarning ko'paytmasiga aytiladi va $n! = 1 \cdot 2 \cdots n$ ko'rinishda yoziladi. Agar $n! = 1 \cdot 2 \cdots n$ ko'paytma $k$ ta 0 raqami bilan tugasa, $k$ soni quyidagicha aniqlanadi

$$k = [\frac{n}{5}] + [\frac{n}{5^2}] + [\frac{n}{5^3}] + \cdots . \qquad (1.2)$$

Haqiqiy sonning moduli quyidagi xossalarga ega:

**1.** $|a| \geq 0$.

**2.** $|-a| = |a|$.

**3.** $|a| = |b| \Longleftrightarrow a = \pm b$.

**4.** $|a \cdot b| = |a| \cdot |b|$.

**5.** $|\frac{a}{b}| = \frac{|a|}{|b|}$, $(b \neq 0)$.

**6.** $|a|^2 = a^2$.

**7.** $|a + b| \leq |a| + |b|$.

**8.** $|a| - |b| \leq |a - b|$.

**9.** $|a| < c$, $(c > 0) \Longleftrightarrow -c < a < c$.

**10.** $|a| > c$, $(c > 0) \Longleftrightarrow \begin{bmatrix} a > c \\ a < -c. \end{bmatrix}$

1. (97-12-13) Agar $m > n > k > 0$ bo'lsa,

$$|n - m| + |n + k| - |m - k|$$

ni soddalashtiring.
A) $2k - 2m$   B) $2k - 2n$   C) $2k$   D) $2m - 2k$

**Yechish:** Ma'lumki, $|x| = \begin{cases} x, & agar\ x \geq 0 \\ -x, & agar\ x \leq 0 \end{cases}$
$m > n > k > 0$ bo'lgani uchun $n - m < 0$ shu sababli $|n - m| = -(n - m)$, $n + k > 0$ shu sababli $|n + k| = n + k$; $m - k > 0$ shu sababli $|m - k| = m - k$ bo'ladi. U holda

$$|n - m| + |n + k| - |m - k| = -(n - m) + n + k -$$

$$-(m - k) = -n + m + n + k - m + k = 2k.$$

**Javob:** $2k$ (C).

2. (98-5-9) Hisoblang.

$$\frac{|4 - 5|4 - 6| + 4|3 - 6||}{|3 - 4|7 - 5||}$$

A) 1       B) $\frac{1}{2}$       C) $1\frac{2}{5}$       D) $1\frac{1}{5}$

3. (99-7-11) Hisoblang.

$$\frac{|4 - 4 \cdot |3 - 6| - 8|}{|4 - |3 - 8| - 7|}$$

A) 2     B) 1     C) 3     D) $2,5$

4. (96-6-14) Agar $a > b > c$ bo'lsa,

$$|a - b| + |c - a| - |b - c|$$

ni soddalashtiring.
A) $a - 2b$    B) $2c$    C) $2a$    D) $2a - 2b$

5. (97-2-14) Agar $x > y > z$ bo'lsa,

$$|x - y| - |z - y| - |z - x|$$

ni soddalashtiring.
A) $2x$    B) $2y - 2x$    C) $2z - 2y$    D) $2y$

6. (97-8-14) Agar $p > q > k > 0$ bo'lsa,

$$|p + q| - |k - q| + |k - p|$$

ni soddalashtiring.
A) $2p$         B) $2p + 2q - 2k$
C) $2p + 2q + 2k$    D) $2p + 2k$

7. Agar $x > y > 0$ bo'lsa,

$$\left| xy - \frac{x^2 + y^2}{2} \right| + \left| \frac{x^2 + y^2}{2} + xy \right| - 2y^2$$

ni soddalashtiring.
A) $4xy$    B) $2(x^2 + y^2)$    C) $2y^2$    D) $2x^2$

8. (96-3-7) Agar $a = -2$ va $b = 3$ bo'lsa, rasmda $|a - b|$ ga mos to'g'ri javobni ko'rsating.
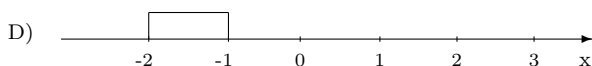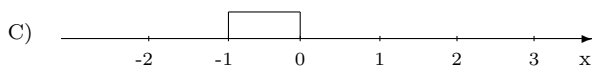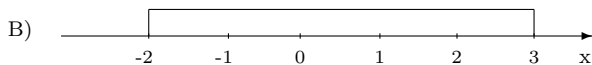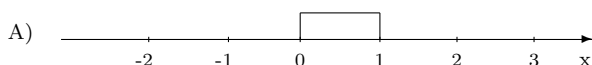


**Yechish:** Ma'lumki, $|a - b|$ miqdor $a$ va $b$ sonlarga mos keluvchi nuqtalar orasidagi masofadir. Demak, $-2$ va $3$ nuqtalar orasidagi masofa B) javobda keltirilgan. **Javob:** (B).

9. (96-12-7) Agar $a = -3$ va $b = 2$ bo'lsa, rasmda $|a - b|$ ga mos to'g'ri javobni ko'rsating.
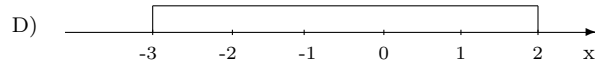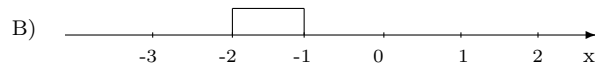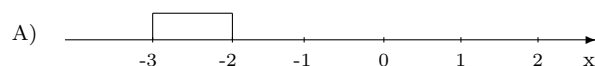


10. (97-4-9) Sonlarni kamayish tartibida yozing.
$m = |4, 8|$;     $n = |-4,(8)|$;     $p = |4\frac{3}{5}|$ va $q = |-3, 2|$
A) $n > m > p > q$     B) $m > n > p > q$
C) $m > p > q > n$     D) $p > m > q > n$

11. (97-9-69) Sonlarni kamayish tartibida yozing.
$m = |8,(8)|$;   $n = |-8, 8|$;   $p = |8\frac{7}{9}|$ va $q = |-8\frac{6}{7}|$
A) $n > m > p > q$     B) $m > n > p > q$
C) $m > q > n > p$     D) $q > m > n > p$

12. (03-2-63) $a > 0$; $b < 0$; $|a| \neq |b|$. Quyidagi ifodalardan qaysi birining qiymati musbat bo'lmasligi mumkin?
A) $a - b$     B) $|a + b|$     C) $a^3 b^2$     D) $|a| - |b|$

13. (98-5-6) $-5, 2$ bilan $10, 4$ orasida nechta butun son bor?
A) 16    B) 10    C) 15    D) 12

**Yechish:** Sonlar o'qi chiziladi va $-5, 2$ hamda $10, 4$ nuqtalar belgilanadi. Endi ular orasidagi butun sonlarni sanaymiz: Manfiylari $-5, -4, -3, -2, -1$ ya'ni 5 ta, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 musbatlari 10 ta, nol ham butun son u bilan jami 16 ta. **Javob:** 16 (A).

14. (99-7-8) Koordinatalari $-3, 2$ va $4, 2$ bo'lgan sonlar orasida nechta butun son bor?
A) 7    B) 6    C) 9    D) 8

15. (98-7-11) Son o'qida $-4$ dan $2, 3$ birlik masofada joylashgan sonlarni aniqlang.
A) $-6, 3$         B) $-6, 3$ va $1, 7$
C) $6, 3$ va $1, 7$     D) $-6, 3$ va $-1, 7$

16. (02-10-40) $-\dfrac{21}{6} + 2,(2)$ ni butun qismini toping.
A) $-2$    B) $-1$    C) $0$    D) $1$

17. $0,(4) + 1,(5) - 2,(3)$ ni kasr qismini toping.
A) $0,(5)$    B) $0,(6)$    C) $0, 6$    D) $0, 56$

18. $0,00003602$ sonini standart shaklda yozing.
A) $3, 6 \cdot 10^{-5}$        B) $0, 36 \cdot 10^{-4}$
C) $36, 02 \cdot 10^{-6}$     D) $3, 602 \cdot 10^{-5}$

### 1.4.1 Aralash tipdagi masalalar

1. $60! - 50!$ ayirmaning oxirida nechta nol bo'ladi?
   A) 14      B) 12      C) 10      D) 8

   **Yechish:** Agar biz (2) formuladan foydalansak, 50! oxirida 12 ta nol, 60! oxirida esa 14 ta nol raqami bo'ladi. Bu sonlarni tagma-tag yozib ayirsangiz, ayirmaning oxirida 12 ta nol bo'lishiga ishonch hosil qilasiz. **Javob:** 12 (B).

2. $150!$ sonining oxirida nechta nol bo'ladi?
   A) 30      B) 33      C) 36      D) 37

3. $3! + 6! + 9! + \cdots + 33!$ yig'indining oxirgi raqamini toping.
   A) 0      B) 1      C) 2      D) 6

4. $(1! + 2! + 3! + 5! + \cdots + 33!)^{33}$ yig'indining oxirgi raqamini toping.
   A) 9      B) 1      C) 2      D) 3

5. $10! + 11! + 12!$ yig'indi quyidagilarning qaysi biriga bo'linmaydi?
   A) 144      B) 350      C) 800      D) 500

6. $15!$ sonini 1001 ga bo'lgandagi qoldiqni toping.
   A) 0      B) 1      C) 11      D) 7

   **Yechish:** 11 ga bo'linish belgisi (1.1.4-band 8-alomatga qarang) ga ko'ra, 1001 ning toq o'rindagi raqamlari yig'indisi $1 + 0 = 1$ bilan, juft o'rindagi raqamlari yig'indisi $0 + 1 = 1$ teng, shuning uchun 1001 soni 11 ga bo'linadi. Bo'lishni bajarib $1001 = 11 \cdot 91$ ekanligini olamiz. Ma'lumki, 91 soni 7 ga bo'linadi va bo'linma 13 ga teng. Demak, 1001 soni quyidagi $1001 = 7 \cdot 11 \cdot 13$ ko'paytma shaklida ifodalanadi. Faktorial ta'rifiga ko'ra 15!, $1001 = 7 \cdot 11 \cdot 13$ ga qoldiqsiz bo'linadi, ya'ni qoldiq nol bo'ladi. **Javob:** 0 (A).

7. 48 va 60 sonlarining nechta tub bo'lmagan, umumiy bo'luvchilari bor.
   A) 4      B) 6      C) 3      D) 5

8. $4 \cdot 45^n$ sonining 198 ta natural bo'luvchisi bo'lsa, $n$ nechaga teng.
   A) 2      B) 3      C) 4      D) 5

9. $\dfrac{80!}{8^n}$ ifoda butun son bo'lsa, $n$ ning eng katta qiymati nimaga teng bo'ladi?
   A) 10      B) 18      C) 20      D) 26

10. $5\,200\,000$ sonining nechta natural bo'luvchisi bor?
    A) 48      B) 56      C) 64      D) 96

11. 1440 sonining barcha natural bo'luvchilari yig'indisini toping.
    A) 5225      B) 4914      C) 2317      D) 198

12. $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytma necha xonali son bo'ladi?
    A) 21      B) 18      C) 19      D) 20

    **Yechish:** $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytmani quyidagicha $4^{10} \cdot 3^3 \cdot 5^3 \cdot 25^8$ yozib olamiz. Natural ko'rsatkichli darajaning xossalaridan foydalanib, uni

    $$4^{10} \cdot 3^3 \cdot 5^3 \cdot 25^8 = 27 \cdot 5 \cdot 4^{10} \cdot 25^9 = 27 \cdot 5 \cdot 4 \cdot 100^9 =$$

    $$= 54 \cdot 10 \cdot (10^2)^9 = 54 \cdot 10 \cdot 10^{18} = 54 \cdot 10^{19}$$

    shaklga keltiramiz. Berilgan ko'paytma 54 va uning orqasida 19 ta nol bo'lgan sondan iborat. Demak, $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytma 21 xonali sondan iborat bo'lar ekan. **Javob:** 21 (A).

13. $8^{18} \cdot 5^{55}$ ko'paytma necha xonali son bo'ladi?
    A) 36      B) 54      C) 55      D) 73

14. $2^{10} \cdot 5^9 \cdot 4^6 \cdot 25^4$ ko'paytma necha xonali son bo'ladi?
    A) 21      B) 18      C) 19      D) 20

15. (02-1-3) Sonning uchdan bir qismini toping.

    $$\frac{(-2) \cdot (-3)^{17} - (-3)^{16}}{9^7 \cdot 15}$$

    A) 1      B) 3      C) 2      D) 9

16. $a$ va $b$ natural sonlar $\dfrac{5a - b}{b} = 11$ munosabatni qanoatlantirsa, $a + b$ ifodaning eng kichik qiymati nimaga teng bo'ladi?
    A) 17      B) 16      C) 14      D) 13

17. Uch xonali uchta turli natural sonning yig'indisi 349 bo'lsa, ular ichidan eng kattasini toping.
    A) 101      B) 146      C) 148      D) 147

18. $a$ soni 2 dan farqli tub son bo'lsa, quyidagilardan qaysi biri juft son bo'ladi?
    A) $a$    B) $2a - 3$    C) $a^2 + a + 1$    D) $a^3 - 3a$

19. Hisoblang.

    $$(1 + \frac{1}{2})(1 + \frac{1}{3})(1 + \frac{1}{4}) \cdots (1 + \frac{1}{2n})$$

    A) $\dfrac{1}{2n}$    B) $\dfrac{n+1}{2}$    C) $2n+1$    D) $\dfrac{2n+1}{2}$

20. Hisoblang.

    $$(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \cdots (1 - \frac{1}{100})$$

    A) $\dfrac{1}{2}$    B) $\dfrac{1}{10}$    C) 2    D) $\dfrac{1}{100}$

21. Hisoblang.

    $$\frac{\frac{0,(3)}{0,44} + \frac{19}{10}}{1,9 + \frac{0,(3)}{0,44}}$$

    A) $\dfrac{4}{9}$      B) 1      C) 3      D) $\dfrac{11}{9}$

22. $(1,(3) + 5) : (5 + 1,(333))$ ni hisoblang.
    A) $\dfrac{1}{9}$      B) 1      C) 3      D) $\dfrac{5}{9}$

23. (98-7-2) Hisoblang.

    $$\frac{488 \cdot 475 - 462}{244 + 475 \cdot 243}$$

    A) 3      B) 1      C) $0,5$      D) 2

24. $2\dfrac{5}{9} - 3,2(7) + 0,55$ ni butun qismini toping.
    A) $-2$      B) $-1$      C) 0      D) 1

25. $1 - 5,(8) - 6,(5)$ son modulining kasr qismini toping.
    A) $0,(5)$      B) $0,(4)$      C) $0,4$      D) $0,44$

# 2 - bob. Algebraik ifodalar

Agar sonli ifodaning ayrim sonlari yoki barcha sonlari harflar bilan almashtirilsa, u harfiy ifoda deyiladi. Algebrada harfiy ifodalar o'rganiladi. *Shu sababdan bunday ifodalar algebraik ifodalar deyiladi.* Odatda harfiy ifodalar orasiga ko'paytirish belgisi qo'yilmaydi. Masalan, $5 \cdot a \cdot b \cdot c^2 = 5abc^2$; $4 \cdot x \cdot y \cdot z = 4xyz$.

## 2.1 Natural ko'psatkichli daraja

Natural ko'psatkichli darajaning ayrim xossalarini keltiramiz. $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$ va hokazo $a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n}$. *Bu yerda a asos, n daraja ko'rsatkich deyiladi.* Ixtiyoriy $a > 0$, $b > 0$ va $n, m \in N$ sonlar uchun quyidagi tengliklar o'rinli:

**1.** $a^n \cdot a^m = a^{n+m}$.

**2.** $a^n : a^m = a^{n-m}$.

**3.** $(ab)^n = a^n \cdot b^n$.

**4.** $(a^n)^m = a^{nm}$.

**5.** $a^0 = 1$,

**6.** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

**7.** $a^{-n} = \dfrac{1}{a^n}$

**8.** $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

1. $2^5$ ni hisoblang.
   A) 10     B) 7     C) 32     D) 16

   **Yechish:** Daraja ta'rifiga ko'ra, $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 4 \cdot 2 \cdot 2 \cdot 2 = 8 \cdot 2 \cdot 2 = 16 \cdot 2 = 32$. **Javob:** 32 (C).

2. $3^5$ ni hisoblang.
   A) 81     B) 15     C) 243     D) 27

3. $5^4$ ni hisoblang.
   A) 20     B) 125     C) 625     D) 25

4. $4^3$ ni hisoblang.
   A) 12     B) 16     C) 64     D) 32

5. $6^3$ ni hisoblang.
   A) 12     B) 216     C) 36     D) 18

6. $2^2 \cdot 2^3$ ni 2 ning darajasi shaklida yozing.
   A) $2^4$     B) $2^3$     C) $2^5$     D) $2^6$

   **Yechish:** 1-xossaga ko'ra, $2^2 \cdot 2^3 = 2^{2+3} = 2^5$. **Javob:** $2^5$ (C).

7. $3 \cdot 3^2 \cdot 3^5$ ni 3 ning darajasi shaklida yozing.
   A) $3^7$     B) $3^8$     C) $3^5$     D) $3^6$

8. $4 \cdot 4 \cdot 4^5$ ni 4 ning darajasi shaklida yozing.
   A) $4^7$     B) $4^8$     C) $4^5$     D) $4^6$

9. $5 \cdot 5 \cdot 5^2 \cdot 5^5$ ni 5 ning darajasi shaklida yozing.
   A) $5^7$     B) $5^8$     C) $5^9$     D) $5^6$

10. $7 \cdot 7 \cdot 7^2 \cdot 7^3$ ni 7 ning darajasi shaklida yozing.
    A) $7^7$     B) $7^8$     C) $7^9$     D) $7^6$

11. $2^{30} \cdot 4^{20} \cdot 8^{10}$ ni 2 ning darajasi shaklida yozing.
    A) $2^{60}$     B) $2^{100}$     C) $2^{80}$     D) $2^{90}$

12. $30^4$ darajani tub sonlar darajasi shaklida yozing.
    A) $2^4 \cdot 3^4 \cdot 5^4$     B) $6^4 \cdot 5^4$     C) $3^4 \cdot 10^4$     D) $2^4 \cdot 15^4$

    **Yechish:** 30 ni tub ko'paytuvchilarga ajratamiz $30 = 2 \cdot 3 \cdot 5$. Demak, $30^4 = (2 \cdot 3 \cdot 5)^4$. 3-xossaga ko'ra, $30^4 = 2^4 \cdot 3^4 \cdot 5^4$. **Javob:** $2^4 \cdot 3^4 \cdot 5^4$ (A).

13. $6^8$ darajani tub sonlar darajasi shaklida yozing.
    A) $2^8 \cdot 3^8$     B) $6^4 \cdot 6^4$     C) $1^8 \cdot 6^8$     D) $2^4 \cdot 3^4$

14. $15^5$ darajani tub sonlar darajasi shaklida yozing.
    A) $3^8 \cdot 5^3$     B) $3^3 \cdot 5^2$     C) $3^5 \cdot 5^5$     D) $2^5 \cdot 3^5$

15. $18^9$ darajani tub sonlar darajasi shaklida yozing.
    A) $2^9 \cdot 3^9$     B) $2^9 \cdot 3^{18}$     C) $2^{18} \cdot 3^9$     D) $2^9 \cdot 9^9$

16. $20^7$ darajani tub sonlar darajasi shaklida yozing.
    A) $2^7 \cdot 10^7$     B) $2^{14} \cdot 5^7$     C) $4^7 \cdot 5^7$     D) $2^5 \cdot 5^2$

17. $21^6$ darajani tub sonlar darajasi shaklida yozing.
    A) $3^3 \cdot 7^3$     B) $3^4 \cdot 7^2$     C) $3^6 \cdot 7^6$     D) $3^5 \cdot 7^5$

18. $2^6 : 2^3$ bo'linmani daraja shaklida yozing.
    A) $2^3$     B) $2^2$     C) $2^6$     D) $2^1$

    **Yechish:** 2-xossaga ko'ra, $2^6 : 2^3 = 2^{6-3} = 2^3$. **Javob:** $2^3$ (A).

19. $3^8 : 3^5$ bo'linmani daraja shaklida yozing.
    A) $3^3$     B) $3^2$     C) $3^6$     D) $3^1$

20. $5^{12} : 5^7$ bo'linmani daraja shaklida yozing.
    A) $5^3$     B) $5^2$     C) $5^5$     D) $5^8$

21. $7^{12} : 7^4$ bo'linmani daraja shaklida yozing.
    A) $7^3$     B) $7^4$     C) $7^5$     D) $7^8$

22. $6^5 : 6$ bo'linmani daraja shaklda yozing.
    A) $6^3$     B) $6^4$     C) $6^5$     D) $6^2$

23. $(3^4)^5$ ni daraja shaklda yozing.
    A) $3^9$     B) $3^{11}$     C) $3^{20}$     D) $3^{15}$

    **Yechish:** 4-xossaga ko'ra, $(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$. **Javob:** $3^{20}$ (C).

24. $(2^5)^3$ ni daraja shaklda yozing.
    A) $2^8$     B) $2^{11}$     C) $2^{20}$     D) $2^{15}$

25. $(3^4)^8$ ni daraja shaklda yozing.
    A) $3^{12}$     B) $3^{11}$     C) $3^{24}$     D) $3^{32}$

26. $(5^2)^7$ ni daraja shaklda yozing.
    A) $5^9$     B) $5^{11}$     C) $5^{49}$     D) $5^{14}$

27. $(7^4)^6$ ni daraja shaklda yozing.
    A) $7^9$     B) $7^{10}$     C) $7^{24}$     D) $7^{12}$

28. $\left(\dfrac{2}{3}\right)^2$ kasrni darajaga ko'taring.
    A) $\dfrac{4}{6}$     B) $\dfrac{4}{9}$     C) $\dfrac{2}{9}$     D) $\dfrac{4}{3}$

    **Yechish:** 6-xossaga ko'ra, $\left(\dfrac{2}{3}\right)^2 = \dfrac{2^2}{3^2} = \dfrac{4}{9}$.

    **Javob:** $\dfrac{4}{9}$ (B).

29. $(\frac{2}{5})^3$ kasrni darajaga ko'taring.

   A) $\frac{6}{15}$   B) $\frac{8}{5}$   C) $\frac{8}{125}$   D) $\frac{8}{25}$

30. $(\frac{3}{7})^4$ kasrni darajaga ko'taring.

   A) $\frac{24}{56}$   B) $\frac{81}{2401}$   C) $\frac{81}{28}$   D) $\frac{81}{343}$

31. $(\frac{2}{7})^3$ kasrni darajaga ko'taring.

   A) $\frac{6}{21}$   B) $\frac{8}{21}$   C) $\frac{8}{343}$   D) $\frac{8}{49}$

32. $(\frac{2}{3})^{-7}$ ni natural ko'rsatkichli daraja shaklida yozing.

   A) $(\frac{3}{2})^7$   B) $(\frac{2}{3})^7$   C) $(\frac{2}{3})^5$   D) $(\frac{3}{2})^{14}$

   **Yechish:** 8-xossaga ko'ra, $(\frac{2}{3})^{-7} = (\frac{3}{2})^7$. **Javob:** $(\frac{3}{2})^7$ (A).

33. $(\frac{2}{5})^{-3}$ ni natural ko'rsatkichli daraja shaklida yozing.

   A) $(\frac{5}{2})^3$   B) $(\frac{2}{5})^3$   C) $(\frac{5}{3})^3$   D) $(\frac{5}{2})^5$

34. $(\frac{3}{7})^{-5}$ ni natural ko'rsatkichli daraja shaklida yozing.

   A) $(\frac{7}{5})^3$   B) $(\frac{7}{3})^5$   C) $(\frac{7}{3})^8$   D) $(\frac{5}{3})^7$

35. $(\frac{2}{9})^{-1}$ ni natural ko'rsatkichli daraja shaklida yozing.

   A) $9^2$   B) $\frac{9}{2}$   C) $(\frac{1}{2})^9$   D) $2^9$

36. (99-8-20) Soddalashtiring.

$$5 \cdot 4^{2n-3} - 20 \cdot \left(2^{n-2}\right)^4$$

   A) 2   B) $4^{2n}$   C) 4   D) 0

   **Yechish:** 4 ni $2^2$ shaklda, 20 esa $5 \cdot 2^2$ ko'rinishda yozamiz, 3 va 4-xossalardan foydalanib

$$5 \cdot 4^{2n-3} - 20 \cdot \left(2^{n-2}\right)^4 = 5 \cdot (2^2)^{2n-3} -$$
$$-5 \cdot 2^2 \cdot 2^{4n-8} = 5 \cdot 2^{4n-6} - 5 \cdot 2^{4n-6} = 0$$

   ni olamiz. **Javob:** 0 (D).

37. (98-7-25) Soddalashtiring.

$$\frac{2^{5n-3} \cdot 2^{3n+2}}{2^{4n-1}}$$

   A) $2^{3n}$   B) $2^{4n+1}$   C) $2^{4n+2}$   D) $2^{4n}$

38. (98-12-24) Soddalashtiring.

$$\frac{3^{4n+3} \cdot 3^{3n-2}}{3^{2n-1}}$$

   A) $3^{5n+2}$   B) $3^{5n+3}$   C) $3^{5n+1}$   D) $3^{5n-1}$

39. (01-3-30) Soddalashtiring.

$$\frac{2^{5n+3} \cdot 2^{3n-4}}{2^{4n+1}}$$

   A) $2^{4n-1}$   B) $2^{n-2}$   C) $2^{2n-2}$   D) $2^{4n-2}$

40. (96-10-25) Hisoblang.

$$\frac{0,5^5 \cdot 32^2}{4^3}$$

   A) 2   B) $\frac{1}{2}$   C) 4   D) $\frac{1}{4}$

   **Yechish:** $0,5$ ni $\frac{1}{2}$ shaklda 32 ni $2^5$, 4 esa $2^2$ ko'rinishda yozamiz, 4 va 6-xossalardan foydalanib

$$0,5^5 \cdot \frac{32^2}{4^3} = (\frac{1}{2})^5 \cdot \frac{(2^5)^2}{(2^2)^3} = \frac{1^5}{2^5} \cdot \frac{2^{10}}{2^6} = \frac{2^{10}}{2^{11}} = \frac{1}{2}$$

   ni olamiz. **Javob:** $\frac{1}{2}$ (B).

41. (96-1-24) Hisoblang. $\frac{9^2 \cdot 3^5}{81^2}$

   A) 1   B) 3   C) $\frac{1}{81}$   D) 9

42. (96-9-65) Hisoblang. $\frac{27^3}{3^4 \cdot 9^2}$

   A) 3   B) $\frac{1}{3}$   C) 1   D) 9

43. (98-7-24) Hisoblang.

$$\frac{5(3 \cdot 7^{15} - 19 \cdot 7^{14})}{7^{16} + 3 \cdot 7^{15}}$$

   A) 7   B) 49   C) $\frac{1}{7}$   D) $\frac{1}{49}$

44. (98-12-23) Hisoblang.

$$\frac{5 \cdot 2^{32} - 4 \cdot 2^{30}}{4^{16}}$$

   A) 4   B) 2   C) 5   D) 16

45. (99-6-1) Hisoblang.

$$\frac{10^9 \cdot 3^5}{3^3 \cdot 10^{11}}$$

   A) $0,09$   B) $0,9$   C) 9   D) $0,03$

46. (97-9-78) Hisoblang.

$$\frac{72^6 \cdot 24^4}{36^8 \cdot 8^3}$$

   A) 24   B) 32   C) 16   D) 36

47. (99-7-7) Hisoblang.

$$\frac{100^5}{(80+20)^{10}} \cdot 50^5$$

   A) $\frac{1}{32}$   B) 16   C) 8   D) $\frac{1}{64}$

## 2.2 Birhad va uning xossalari

*Birhad deb sonlar, o'zgaruvchilarning natural darajalari va ularning ko'paytmalari qatnashgan hamda sonlar va o'zgaruvchilar ustida boshqa amallarni o'z ichiga olmaydigan ifodaga aytiladi.* Son yoki bitta harf ham birhad sanaladi. Son va harflar birhadning ko'paytuvchilari deyiladi. Birhad quyidagi xossalarga ega.

1. **Birhadda uning ko'paytuvchilari o'rinlarini almashtirish mumkin. Masalan,** $ab \cdot 5xy = 5abxy$.

2. **Birhaddagi bir necha sonli ko'paytuvchilarni ularning ko'paytmasi bilan almashtirish mumkin. Masalan,** $5ab \cdot 3xy \cdot 4tz = 60abtxyz$.

3. **Birhadda bir xil harfiy ko'paytuvchilarni mos darajali ko'paytmaga almashtirish mumkin. Masalan,** $5ab \cdot a^2b^3 = 5a^{1+2}b^{1+3} = 5a^3b^4$.

4. **Birhadning ko'paytuvchilaridan biri nolga teng bo'lsa, bunday ko'paytma nolga teng. Masalan,** $5ab \cdot 0 \cdot 8xy = 0$.

5. **Birhadda 1 ko'paytuvchini tashlab yuborish mumkin. Masalan,** $4ab \cdot 0,25x^2y = 1 \cdot abx^2y = abx^2y$.

6. **Agar birhad oldiga ” + ” qo'shish belgisini qo'ysak, berilgan birhadga teng birhad hosil bo'ladi. Masalan,** $+abc = abc$.

7. **Agar birhad oldiga ” − ” ayirish belgisini qo'ysak, berilgan birhad** $-1$ **ga ko'paytirilgan hisoblanadi. Masalan,** $-a \cdot (-5)c = (-1)(-5)ac = 5ac$.

Faqat ishoralari bilan farq qiluvchi birhadlar qarama-qarshi birhadlar deyiladi. Masalan, $5xyz$ va $-5xyz$ yoki $4x^2y^3$ va $-4x^2y^3$. *Nolga teng bo'lmagan birhadda birgina sonli ko'paytuvchi birinchi o'rinda, birhaddagi harfiy ko'paytuvchilar alfavit tartibida daraja ko'rsatkichi orqali bir marta yozilgan bo'lsa, birhad standart shaklda deyiladi.* Masalan, $15a^2b^3x^5y^8$ birhad standart shaklda. Standart ko'rinishdagi birhaddagi sonli ko'paytuvchiga birhadning koeffitsiyenti deyiladi. Masalan, $5a^2b^3x^5y^8$ birhadning koeffitsiyenti 5 ga teng. *Teng birhadlar yoki faqat koeffitsiyenti bilan farq qiluvchi birhadlar o'xshash deyiladi.* Masalan, $5a^2bc$ va $-3a^2bc$ birhadlar o'xshash. Standart shakldagi birhadning darajasi deb, birhaddagi harfiy ko'paytuvchilar darajalarining yig'indisiga aytiladi. Masalan, $a^2b^3x^5y^8z$ birhadning darajasi $2+3+5+8+1 = 19$ dir.

1. Quyidagi ifodalarning qaysilari birhad.
   1) $3ab^{-2}$,    2) $a+b$,    3) $\frac{1}{2}abc$,    4) $1 : (2c)$

   A) 1    B) 2    C) 3    D) 4

   **Yechish:** 1) da manfiy daraja qatnashyapti, demak, u birhad emas. 2) da ” + ” ishora qatnashyapti, u birhad emas. 4) da ” : ” amali qatnashyapti, u ham birhad emas. 3) da son va harflar faqat ko'paytirish amali bilan bog'langan, demak u birhad. **Javob:** 3 (C).

2. Quyidagi ifodalarning qaysilari birhad.
   1) $ab^{-1}$,    2) $a-b$,    3) $\frac{1}{2}a^3b^2$,    4) $2c : 3$

   A) 1; 2    B) 2; 3    C) 3; 4    D) 2; 4

3. $3ab^2 \cdot 2xy^3$ birhadning koeffitsiyentini toping.
   A) 3    B) 2    C) 6    D) 5

4. $4ab^2 \cdot 0,25xy^3$ birhadning koeffitsiyentini toping.
   A) 3    B) 2    C) 1    D) 5

5. Birinchi darajali birhadni toping.
   A) $abc$    B) $6^3a$    C) $2^{-2}a^3$    D) $5xyz$

6. Ikkinchi darajali birhadni toping.
   A) $2abc$    B) $6^3a^2$    C) $2^2a^2b^2$    D) $2xyz$

7. Uchinchi darajali birhadni toping.
   A) $abc$    B) $3^3a$    C) $2a^3b$    D) $3x^3y^3$

8. $3^2a^3b^2xy^3$ birhadning darajasini toping.
   A) 11    B) 9    C) 8    D) 10

9. $3ab^2 \cdot 2a^3b^5$ birhadni standart shaklga keltiring.
   A) $6ab^2 \cdot a^3b^5$    B) $6a^3b^{10}$    C) $6a^4b^7$    D) $6a^3b^7$

10. $3x^2 \cdot 2x^3y^5$ birhadni standart shaklga keltiring.
    A) $6x^6y^6$    B) $6x^5y^9$    C) $6x^5y^5$    D) $5x^5y^5$

11. Standart shakldagi birhadlarni ajrating.
    1) $3x^2 \cdot 2x^3y^5$;  2) $6ab^2x^3z^5$;  3) $7x^6y^6$
    A) 1; 2    B) 1; 3    C) 2; 3    D) 1; 2; 3

12. $n$ ning qanday qiymatida $8a^2x^3y^n$ birhadning darajasi uning koeffitsiyentiga teng bo'ladi?
    A) 6    B) 5    C) 3    D) 4

    **Yechish:** Birhadning koeffitsiyenti 8 ga, uning darajasi esa $2+3+n$ ga teng. Ularni tenglashtiramiz $2+3+n = 8$. Bu yerdan $n = 3$ ni olamiz. **Javob:** 3 (C).

13. $n$ ning qanday qiymatida $5a^2x^3y^n$ birhadning darajasi uning koeffitsiyentidan 5 marta katta bo'ladi?
    A) 16    B) 15    C) 23    D) 20

14. Quyidagilar ichidan qaysi biri $10a^3b^5$ teng.
    A) $2ab \cdot 5a^2b$    B) $a \cdot 10a^2b^5$
    C) $\frac{1}{2}ab^2 \cdot 20ab^3$    D) $ab \cdot 5ab^2 \cdot 2ab^3$

15. Qarama-qarshi birhadlarni toping.
    1) $3ab$ va $\frac{1}{3ab}$    2) $a$ va $a^{-1}$
    3) $\frac{1}{2}ab^2$ va $-0,5ab^2$    4) $a+b$ va $a-b$

    A) 1    B) 2    C) 3    D) 4

16. $-0,5ab^2x^3$ ga o'xshash birhadni toping.
    A) $3abx$    B) $ba^2x^3$    C) $ab^2x$    D) $2ab^2x^3$

17. O'xshash birhadlarni ajrating.
    1) $3ab$ va $\frac{ab}{3}$    2) $ab$ va $-ab$
    3) $\frac{1}{2}ab^2$ va $-0,5ab^2$    4) $7a^2b$ va $7^{-1}a^2b$

    A) 1; 2    B) 2; 3    C) 1; 2; 3    D) 1; 2; 3; 4

## 2.3 Ko'phad va uning xossalari

*Bir necha birhadning algebraik yig'indisiga ko'phad deyiladi. Ko'phadni tashkil etuvchi birhadlar shu ko'phadning hadlari deyiladi.* Ko'phadlar ikkita, uchta va hokazo $n$ ta birhadlar yig'indisidan iborat bo'lishi mumkin. Masalan, $x^2 + 2xy + y^2, x^4 - y^4, a^2 + b - c^2 + d$ ifodalarda, birinchisi - uchhad, ikkinchisi - ikkihad, uchinchisi - to'rthad. *Ko'phadni tashkil etuvchi birhadlar standart shaklda va ular ixchamlangan bo'lsa, ko'phad standart shaklda berilgan deyiladi.* Ko'phad quyidagi xossalarga ega.

1. **Ko'phadning hadlari o'rinlarini almashtirish mumkin.** $x^2 + y^2 = y^2 + x^2$, $x^4 - y^4 = -y^4 + x^4$.

2. **Ko'phadga noldan iborat birhad qo'shsak, berilgan ko'phad o'zgarmaydi.**
$x^2 + y^2 + 0 = x^2 + y^2$, $x^4 + 0 - y^4 = x^4 - y^4$.

3. **Ko'phadda o'xshash hadlarni ixchamlash mumkin.** $x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$, $3x^2 + x - x + y^2 = 3x^2 + 0 + y^2 = 3x^2 + y^2$.

4. **Birhadni ko'phadga ko'paytirish uchun, birhad ko'phadning har bir hadiga ko'paytiriladi + va − ishoralar o'zgarishsiz qoladi.**
$x^2(x - xy + y) = x^2 x - x^2 xy + x^2 y = x^3 - x^3 y + x^2 y$.

5. **Ko'phadni ko'phadga ko'paytirish uchun, ko'phadlardan birining har bir hadi ikkinchi ko'phadning barcha hadlariga ko'paytirilib, o'xshash hadlar ixchamlanadi.** Masalan,
$(x + y)(x - y) = x(x - y) + y(x - y) =$
$= x^2 - xy + xy - y^2 = x^2 + 0 - y^2 = x^2 - y^2$.

1. (97-10-5) Soddalashtiring.
$$2\frac{2}{3} \cdot \left(1\frac{1}{2}a - 2\frac{1}{4}\right) + 1\frac{1}{5} \cdot \left(2\frac{1}{2}a - \frac{5}{6}\right)$$
A) $a + 5$    B) $7a - 7$    C) $7$    D) $3a - 5$

**Yechish:** Aralash kasrlarni noto'g'ri kasrga keltirib, qavslarni ochamiz:
$$2\frac{2}{3} \cdot \left(1\frac{1}{2}a - 2\frac{1}{4}\right) + 1\frac{1}{5} \cdot \left(2\frac{1}{2}a - \frac{5}{6}\right) =$$
$$= \frac{8}{3} \cdot \left(\frac{3}{2}a - \frac{9}{4}\right) + \frac{6}{5} \cdot \left(\frac{5}{2}a - \frac{5}{6}\right) =$$
$$= 4a - 6 + 3a - 1 = 7a - 7$$
**Javob:** $7a - 7$ (B).

2. (97-5-2) $4a - 13a + 5a$ ni soddalashtiring.
A) $4a$    B) $-4a$    C) $6a$    D) $-6a$

3. (97-9-2) $7x - 14x + 6x$ ni soddalashtiring.
A) $x$    B) $-2x$    C) $2x$    D) $-x$

4. (97-9-6) $-8 - 2(1 - b) - 2b + 1$ ni soddalashtiring.
A) $9$    B) $9 - 4b$    C) $9 + 4b$    D) $-9$

5. (98-1-14) Soddalashtiring.
$$a(b - c) + b(c - a) - c(b - a)$$
A) $-2ac$    B) $2ab$    C) $0$    D) $2$

6. (97-3-5) Soddalashtiring.
$$2\frac{1}{3} \cdot \left(\frac{6}{7}m + 3\right) - 1\frac{2}{3} \cdot \left(\frac{3}{5}m - 3\right)$$
A) $m - 2$    B) $4$    C) $m + 12$    D) $4 + m$

7. (99-4-13) Soddalashtiring.
$$\frac{4}{9} \cdot \left(4\frac{1}{2}y - 1\frac{1}{2}\right) - \frac{2}{7} \cdot \left(1\frac{1}{6} - 3\frac{1}{2}y\right)$$
A) $0, 2y - 1$    B) $2y + 1$    C) $3y - 1$    D) $y - 1$

8. (96-1-25) Ifodani ko'phadning standart shakliga keltiring.
$$2x(x - 1) - (2x - 1) \cdot (x + 1)$$
A) $4x^2 - 1$    B) $2x^2 - 3x$
C) $3x + 1$      D) $-3x + 1$

**Yechish:** Dastlab qavslarni ochamiz, keyin o'xshash hadlarni ixchamlaymiz. $2x(x - 1) - (2x - 1) \cdot (x + 1) = 2x^2 - 2x - 2x \cdot (x + 1) + 1 \cdot (x + 1) = 2x^2 - 2x - 2x^2 - 2x + x + 1 = -3x + 1$. **Javob:** $-3x + 1$ (D).

9. (99-8-24) $P$ va $Q$ ko'phadlar ayirmasini toping.
$$P = \frac{1}{3}x - \frac{1}{3}y - (x + 2y), \quad Q = \frac{1}{3}x + \frac{1}{3}y - (x - y)$$
A) $-\frac{11}{3}y$    B) $4y$    C) $-4y$    D) $\frac{13}{3}y$

10. Ko'phadlarni ko'paytiring. $(a - b)(a + b)$
A) $a^2 - b$    B) $a^2 - 2b$    C) $a^2 + b^2$    D) $a^2 - b^2$

11. Ko'phadlarni ko'paytiring. $(a - b)(a^2 + ab + b^2)$
A) $a^3 - b^3$    B) $a^2 - b^3$    C) $a^3 + b^3$    D) $a^2 - b^2$

12. Ko'phadlarni ko'paytiring. $(a + b)(a^2 - ab + b^2)$
A) $a^3 - b^3$    B) $a^2 - b^3$    C) $a^3 + 3b^3$    D) $a^3 + b^3$

13. (01-8-12) Ushbu
$$(a + 3b)(a + b + 2) - (a + b)(a + 3b + 2)$$
ko'phadni standart shaklga keltiring.
A) $2a - b$    B) $a - 2b$    C) $4a + 2b$    D) $4b$

14. (99-8-10) Agar $a + b + 3 = 10$ bo'lsa,
$$3, 8a + 7, 7 + 1, 7b + 2, 5a + 11, 2 + 4, 6b$$
ifodaning qiymatini toping.
A) $53$    B) $58$    C) $72$    D) $63$

15. (06-121-4) Ifodani soddalashtirgandan keyin hosil bo'lgan ko'phadning nechta hadi bo'ladi?
$$(y^4 - y^2 + 1)(y^2 + 1) - (y - 1)(y + 2) + y^4 + y^3$$
A) $4$    B) $3$    C) $5$    D) $6$

16. $(a^4 + a^2)(a^4 - a^2)$ ifoda standart shaklga keltirilsa, u nechta haddan iborat bo'ladi?
A) $4$    B) $3$    C) $5$    D) $2$

17. $(2a - b)(4a^2 + 2ab + b^2) - b^3$ ifoda standart shaklga keltirilsa, u nechta haddan iborat bo'ladi?
A) $4$    B) $3$    C) $5$    D) $2$

## 2.4 Qisqa ko'paytirish formulalari

**1. Ikki son yig'indisining kvadrati**
$(a + b)^2 = a^2 + 2ab + b^2$.

**2. Ikki son ayirmasining kvadrati**
$(a - b)^2 = a^2 - 2ab + b^2$.

**3. Ikki son kvadratlarining ayirmasi**
$a^2 - b^2 = (a - b)(a + b)$.

**4. Ikki son yig'indisining kubi**
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

**5. Ikki son ayirmasining kubi**
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

**6. Ikki son kublarining ayirmasi**
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

**7. Ikki son kublarining yig'indisi**
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

1. $a^2 + 4a + 4$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2a + 1)^2$     B) $(a + 2)^2$
C) $(2a + 3)^2$     D) $(a + 0,5)^2$

**Yechish:** Berilgan ko'phadni quyidagicha yozib olamiz $a^2 + 2 \cdot a \cdot 2 + 2^2$. Bu esa 1-ga ko'ra $(a + 2)^2$ dir. **Javob:** $(a + 2)^2$ (B).

2. $4a^2 + 12a + 9$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2a + 1)^2$     B) $(a + 2)^2$
C) $(2a + 3)^2$     D) $(a + 0,5)^2$

3. $4x^4 + 20x^2 + 25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2x^2 + 5)^2$     B) $(x^2 + 2)^2$
C) $(2x^2 + 3)^2$     D) $(2x^2 + 0,5)^2$

4. $9x^6 + 12x^3 + 4$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2x^3 + 5)^2$     B) $(x^2 + 2)^2$
C) $(3x^3 + 2)^2$     D) $(3x^3 + 0,5)^2$

5. $4x^2 + 2x + 0,25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2x + 5)^2$     B) $(x + 2)^2$
C) $(2x + 3)^2$     D) $(2x + 0,5)^2$

6. $25x^6 - 10x^3 + 1$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.
A) $(5x^3 - 1)^2$     B) $(x^2 - 2)^2$
C) $(3x^3 - 2)^2$     D) $(3x^3 - 0,5)^2$

**Yechish:** Berilgan ko'phadni quyidagicha yozib olamiz $25x^6 - 10x^3 + 1 = (5x^3)^2 - 2 \cdot 5x^3 \cdot 1 + 1^2$. Bu esa 2-ga ko'ra $(5x^3 - 1)^2$ ga teng. **Javob:** $(5x^3 - 1)^2$ (A).

7. $x^2 - 6x + 9$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.
A) $(x - 1)^2$     B) $(x^2 - 3)^2$
C) $(x - 3)^2$     D) $(3x - 0,5)^2$

8. $2x^2 - 4x + 2$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.
A) $(\sqrt{2}x - 1)^2$     B) $(2(x - 1))^2$
C) $(x - \sqrt{2})^2$     D) $2(x - 1)^2$

9. $4x^4 - 2x^2 + 0,25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.
A) $(2x^2 - 5)^2$     B) $(x^2 - 0,2)^2$
C) $(2x - 0,5)^2$     D) $(2x^2 - 0,5)^2$

10. $9x^4 - 12x^2 + 4$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.
A) $(3x^2 - 1)^2$     B) $(3x^2 - 2)^2$
C) $(3x - 2)^2$     D) $(3x^2 - 1)^2$

11. $27a^3 + 27a^2b + 9ab^2 + b^3$ ko'phadni ikki had yig'indisining kubi shaklida yozing.
A) $(2a + b)^3$     B) $(a + 3b)^3$
C) $(3a + b)^3$     D) $(3a - b)^3$

**Yechish:** Berilgan ko'phadni quyidagicha yozish mimkin $(3a)^3 + 3 \cdot (3a)^2 \cdot b + 3 \cdot 3a \cdot b^2 + b^3$. Bu esa 4-ga ko'ra $(3a + b)^3$ dir. **Javob:** $(3a + b)^3$ (C).

12. $1 + 3a + 3a^2 + a^3$ ko'phadni ikki had yig'indisining kubi shaklida yozing.
A) $(a + 1)^3$     B) $(1 + a^2)^3$
C) $(3a + 1)^3$     D) $(1 - a)^3$

13. $x^3 + 6x^2 + 12x + 8$ ko'phadni ikki had yig'indisining kubi shaklida yozing.
A) $(a + 3)^3$     B) $(x + 2)^3$
C) $(x - 2)^3$     D) $(a + 2)^3$

14. $8x^3 + 36x^2 + 54x + 27$ ko'phadni ikki had yig'indisining kubi shaklida yozing.
A) $(2x + 3)^3$     B) $(3x + 2)^3$
C) $(2x - 3)^3$     D) $(2a + 3)^3$

15. $y^6 + 3y^4 + 3y^2 + 1$ ko'phadni ikki had yig'indisining kubi shaklida yozing.
A) $(x^2 + 1)^3$     B) $(y^2 + 1)^3$
C) $(y^2 - 1)^3$     D) $(a^2 + 1)^3$

16. $1 - 3a^2 + 3a^4 - a^6$ ko'phadni ikki had ayirmasining kubi shaklida yozing.
A) $(1 - a^2)^3$     B) $(1 + a^2)^3$
C) $(2a + 1)^3$     D) $(1 - a)^3$

**Yechish:** Berilgan ko'phadni quyidagicha yozish mimkin $1^3 - 3 \cdot 1 \cdot a^2 + 3 \cdot 1 \cdot (a^2)^2 - (a^2)^3$. Bu esa 5-ga ko'ra $(1 - a^2)^3$ dir. **Javob:** $(1 - a^2)^3$ (A).

17. $1 - 3b + 3b^2 - b^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.
A) $(a - 1)^3$     B) $(1 - b^2)^3$
C) $(3b + 1)^3$     D) $(1 - b)^3$

18. $125z^3 - 75z^2 + 15z - 1$ ko'phadni ikki had ayirmasining kubi shaklida yozing.
A) $(5a - 1)^3$     B) $(1 - 5z)^3$
C) $(5z + 1)^3$     D) $(5z - 1)^3$

19. $1 - 3y + 3y^2 - y^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.
A) $(1 - y^2)^3$     B) $(1 + y)^3$
C) $(2y + 1)^3$     D) $(1 - y)^3$

20. $125 - 75z + 15z^2 - z^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.
A) $(5a - 1)^3$    B) $(1 - 5z)^3$
C) $(5z + 1)^3$    D) $(5 - z)^3$

21. (96-1-17) Soddalashtiring.

$$(2a - b)^2 - (2a + b)^2$$

A) 0    B) $-2b^2$    C) $-8ab$    D) $-4ab + 2b^2$

**Yechish:** 1-usul. Ikki son ayirmasining kvadrati va ikki son yig'indisining kvadrati (2 va 1 ga qarang) formulalaridan foydalanamiz:

$(2a - b)^2 - (2a + b)^2 = 4a^2 - 4ab + b^2 - (4a^2 + 4ab + b^2) =$

$= 4a^2 - 4a^2 - 4ab - 4ab + b^2 - b^2 = -8ab.$

2-usul. Ikki son kvadratlarining ayirmasi (3 ga qarang) formulasidan foydalanamiz:

$(2a - b)^2 - (2a + b)^2 = (2a - b - 2a - b)(2a - b + 2a + b) =$

$= -2b \cdot 4a = -8ab.$    **Javob:** $-8ab$ (C).

22. (96-9-68) Soddalashtiring.

$$(a - 3b)^2 - (a + b)^2$$

A) $8b^2 - 8ab$  B) $8b^2$  C) $2b^2 - 8ab$  D) $-8b^2$

23. (96-9-76) Ushbu $(4x - 3)^2 - x(4x + 1)$ ifodani ko'phadning standart shakliga keltiring.
A) $2x^2 + x - 9$    B) $12x^2 - 25x + 9$
C) $4x^2 - 13x$    D) $8x^2 - x + 7$

24. (96-10-18) Soddalashtiring.

$$(1 - 2a)^2 + (1 + 2a)(2a - 1)$$

A) $8a^2 - 4a$    B) $-2a$    C) $-2a + 2$    D) $8a^2$

25. (98-11-8) Soddalashtiring.

$$12^2 - (x + 7)^2 - (5 - x) \cdot (19 + x)$$

A) 0    B) 50    C) 140    D) 90

26. (96-11-20) $(b - c)(b^2 + bc + c^2)$ ifodaning $b = -2$ va $c = 1$ bo'lgandagi qiymatini hisoblang.
A) 7    B) 5    C) $-9$    D) $-7$

**Yechish:** 6-qoidaga (ikki son kublarining ayirmasi) ko'ra $(b - c)(b^2 + bc + c^2) = b^3 - c^3$. Endi $b = -2$ va $c = 1$ deymiz. $(-2)^3 - 1^3 = -8 - 1 = -9.$    **Javob:** $-9$ (C).

27. (96-12-20) $(x^2 + xy + y^2)(x - y)$ ifodaning $x = 1$ va $y = -2$ bo'lgandagi qiymatini hisoblang.
A) 5    B) $-9$    C) 7    D) 9

28. (00-5-21) $(2a + 3b)(4a^2 - 6ab + 9b^2)$ ifodaning $a = 2$ va $b = 1$ dagi qiymatini toping.
A) 91    B) 93    C) 96    D) 99

29. (00-5-23) $(x + 3)(x^2 - 3x + 9)$ ifodaning $x = \frac{1}{2}$ dagi qiymatini hisoblang.
A) $-26,875$    B) $\frac{343}{27}$    C) $27\frac{1}{2}$    D) $27,125$

30. (97-12-9) Ifodani soddalashtirgandan keyin nechta haddan iborat bo'ladi?

$$(y^3 - 1)^2 + (y^2 + 1)(y^4 - y^2 + 1)$$

A) 4    B) 5    C) 6    D) 3

**Yechish:** 2 va 7-qoidalardan foydalansak, berilgan ifodani quyidagicha yozish mumkin $y^6 - 2y^3 + 1 + y^6 + 1^3$. O'xshash hadlarni ixchamlab $2y^6 - 2y^3 + 2$ ni olamiz. Demak, ifoda 3 ta haddan iborat bo'lar ekan.    **Javob:** 3 (D).

31. (97-12-9) Ifodani soddalashtirgandan keyin unda nechta had qoladi?

$$(a - 5)(a + 5) - a^2$$

A) 4    B) 3    C) 2    D) 1

32. (97-12-9) Ifodani soddalashtirgandan keyin unda nechta had qoladi?

$$(x^3 - 1)(x^3 + 1) + x^6$$

A) 4    B) 3    C) 2    D) 1

33. (97-12-9) Ifodani soddalashtirgandan keyin unda nechta had qoladi?

$$(1 - 2a)(2a + 1) + 4a^2$$

A) 4    B) 3    C) 2    D) 1

34. (03-8-44)* Agar $a + a^{-1} = 3$ bo'lsa, $a^2 + a^{-2}$ ni hisoblang?
A) 7    B) 4    C) 9    D) 13

**Yechish:** $a + a^{-1} = 3$ tenglikning ikkala qismini kvadratga ko'taramiz $(a + a^{-1})^2 = 3^2$. Endi 1-qoidaga (ikki son yig'indisining kvadrati) ko'ra $a^2 + 2 \cdot a \cdot a^{-1} + a^{-2} = a^2 + 2 + a^{-2} = 9$. Bu yerdan $a^2 + a^{-2} = 9 - 2 = 7$ ni olamiz.    **Javob:** 7 (A).

35. (00-6-7)* Agar $a - \frac{1}{a} = \frac{2}{3}$ bo'lsa, $\frac{a^4 + 1}{a^2}$ ning qiymatini toping?
A) $2\frac{4}{9}$    B) $1\frac{1}{3}$    C) $1\frac{5}{9}$    D) $2\frac{5}{9}$

36. (99-6-40)* $a^2 + \frac{9}{a^2} = 22$ bo'lsa, $a - \frac{3}{a}$ nimaga teng.
A) 3    B) $-3$    C) 2    D) $\pm 4$

37. (01-8-7)* Agar $a + \frac{1}{a} = 3$ bo'lsa, $\frac{a^6 + 1}{a^3}$ ning qiymatini toping.
A) 27    B) 24    C) 18    D) $21\frac{1}{3}$

38. (02-5-7)* Agar $a - \frac{1}{a} = \sqrt{7}$ bo'lsa, $a^4 + \frac{1}{a^4}$ ning qiymatini hisoblang.
A) 81    B) 79    C) 49    D) 63

39. (02-9-6)* Agar $a + \frac{1}{a} = 3$ bo'lsa, $\frac{a^4 + 1}{2a^2}$ ning qiymati nimaga teng?
A) $3,5$    B) 4    C) $5,5$    D) 7

## 2.5 Ko'phadni ko'paytuvchilarga ajratish

Ko'phadni ko'paytuvchilarga ajratish shunday shakl al-mashtirishki, bunda ko'phad ikki yoki undan ortiq ko'p-had yoki birhaddan iborat ko'paytmaga almashtiriladi. Ko'phadlarni ko'paytuvchilarga ajratishning eng ko'p ishlatiladigan sodda usullaridan ikkitasiga to'xtalamiz.

1) *Umumiy ko'paytuvchini qavsdan tashqariga chi-qarish usuli.* Bu usulni quyidagi ikki misolda ko'pib chiqamiz. *1-misol.*

$$4x^4y^2 - 8x^3y^2 + 12x^2yz^2.$$

Berilgan ko'phadning barcha hadlari uchun umumiy ko'paytuvchi $4x^2y$ ni qavsdan tashqariga chiqarsak,

$$4x^4y^2 - 8x^3y^2 + 12x^2yz^2 = 4x^2y(x^2y - 2xy + 3z^2)$$

ga ega bo'lamiz. Shunday qilib, berilgan ko'phad ikkita ko'paytuvchiga ega bo'ldi. *2-misol.*

$$3x^2(a - b) - 8x^3y^2(a - b) + 5xy^2(a - b).$$

Ko'phaddagi hamma hadlar uchun $x(a - b)$ umumiy ko'paytuvchi bo'ladi. Shu sababli berilgan ko'phad quyidagicha ko'paytuvchilarga ajraladi.

$$x(a - b)(3x - 8x^2y^2 + 5y^2).$$

2) *Guruhlash usuli.* Bu usul ko'phadning barcha hadlari uchun umumiy ko'paytuvchi mavjud bo'lmagan holda qo'llaniladi. Quyidagi misolni qaraymiz:

$$2ac + bc - 3b - 6a.$$

Ko'phadning barcha hadlari umumiy ko'paytuvchiga ega emas. Birinchi va oxirgi hadlarining umumiy ko'pay-tuvchisi $2a$ ga, ikkinchi va uchinchi hadlarining umu-miy ko'paytuvchisi $b$ ga teng. Birinchi va oxirgi had-laridan $2a$ ni, ikkinchi va uchinchi hadlaridan $b$ ni qavs oldiga chiqaramiz. Natijada quyidagini olamiz:

$$2ac + bc - 3b - 6a = 2a(c-3) + b(c-3) = (c-3)(2a+b).$$

1. Ko'phadni ko'paytuvchilarga ajrating.

$$2x^2y - 2xy^2$$

A) $2xy(y - x^2)$     B) $2xy(x - y)$
C) $2xy(x - y^2)$     D) $2xy(y - x)$

**Yechish:** Misolni yechishda 1-usuldan foydalana-miz. Berilgan ko'phadning barcha hadlari uchun umumiy ko'paytuvchi $2xy$ ni qavsdan tashqariga chiqaramiz

$$2x^2y - 2xy^2 = 2xy(x - y).$$

**Javob:** $2xy(x - y)$ (B).

2. Ko'phadni ko'paytuvchilarga ajrating.

$$2x^2y^7 - 8x^5y^5$$

A) $2x^2y^5(y^2 - x^3)$     B) $2x^2y^5(x^2 - 4y^3)$
C) $2x^2y^5(x^2 - y^3)$     D) $2x^2y^5(y^2 - 4x^3)$

3. Ko'phadni ko'paytuvchilarga ajrating.

$$x^2y^7 - x^5y^5 + 2x^3y^5$$

A) $x^2y^5(y^2 - x^3 + xy)$     B) $x^2y^5(x^2 - y^3 + 2xy)$
C) $x^2y^5(x^2 - y^3 + 2x)$     D) $x^2y^5(y^2 - x^3 + 2x)$

4. Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b^3 - 6ab^2$$

A) $2ab^2(ab - 5)$     B) $2ab^2(ab - 3)$
C) $2ab(3 - ab)$     D) $2a^2b(ab - 3)$

5. Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b^3 - 6ab^2 + 8a^2b^2$$

A) $2ab^2(ab - 5 + 4a)$     B) $2ab^2(ab - 3 + 4a)$
C) $2ab(3 - ab + 4a)$     D) $2a^2b(ab - 3 + 4a)$

6. (98-1-18) Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b - 3a + 10ab^2 - 15b$$

A) $(2ab + 3)(a - 5b)$     B) $(a + 5b)(2ab - 3)$
C) $(3 + ab)(2a - 5b)$     D) $(2a^2 + b)(b - 5a)$

**Yechish:** Misolni yechishda 2-usul, guruhlash usulidan foydalanamiz. Berilgan ko'phadda ikkin-chi va uchinchi hadlari o'rinlarini almashtiramiz:

$$2a^2b - 3a + 10ab^2 - 15b = 2a^2b + 10ab^2 - 3a - 15b.$$

Birinchi va ikkinchi hadlaridan $2ab$ ni, uchinchi va oxirgi hadlaridan $-3$ ni qavs oldiga chiqaramiz. Natijada quyidagini olamiz:

$$2a^2b + 10ab^2 - 3a - 15b = 2ab(a + 5b) - 3(a + 5b).$$

Endi umumiy ko'paytuvchi $a + 5b$ ni qavs oldiga chiqaramiz $(a + 5b)(2ab - 3)$.
**Javob:** $(a + 5b)(2ab - 3)$ (B).

7. (98-8-18) Ushbu $2n^2 - 3an - 10n + 15a$ ko'phadni ko'paytuvchilarga ajrating.
A) $(5 - n)(3a - 2n)$     B) $(5 + n)(2n - 3a)$
C) $(3a - n)(5 - 2n)$     D) $(2n + 3a)(n + 5)$

8. (00-6-18) $4y(5x - y) - (5x - 2)(5x + 2)$ ning eng katta qiymatini toping.
A) 10     B) 5     C) 4     D) 2

9. (97-1-13) Ushbu $1 - (2x - 3)^2$ ifodani ko'paytuv-chilarga ajrating.
A) $2(x + 2)(x + 1)$     B) $3(x - 2)(x + 1)$
C) $4(2 - x)(x - 1)$     D) $2(1 - x)(x - 2)$

**Yechish:** Misolni yechishda qisqa ko'paytirish formulasining 3-dan foydalanamiz.

$$1 - (2x-3)^2 = 1^2 - (2x-3)^2 = (1-2x+3)(1+2x-3) =$$

$$= (4-2x)(2x-2) = 2(2-x) \cdot 2(x-1) = 4(2-x)(x-1).$$

**Javob:** $4(2 - x)(x - 1)$ (C).

10. (97-6-13) Ushbu $9 - (2c-1)^2$ ifodani ko'paytuvchilarga ajrating.
    A) $2(c-1)(c+2)$     B) $4(c-2)(c+1)$
    C) $(3c-1)(c+4)$     D) $4(c+1)(2-c)$

11. (97-10-18) Ko'paytuvchilarga ajrating.

$$(x^2+9)^2 - 36x^2$$

A) $(x^2-5)(x^2+4)$     B) $(x-3)^2 \cdot (x+3)^2$
C) $(x-6)^2 \cdot (x+6)^2$     D) $x^2(x^2-6)$

12. (97-11-13) Ifodani ko'paytuvchilarga ajrating.

$$1 - (8a-3)^2$$

A) $8(4a+1) \cdot (1-2a)$     B) $(16a-1) \cdot (4a-3)$
C) $4(2a+1) \cdot (4a-1)$     D) $8(1-2a) \cdot (4a-1)$

13. (96-7-18) Ko'paytuvchilarga ajrating.

$$(a^2+16)^2 - 64a^2$$

A) $(a^2-8) \cdot (a^2+4)$     B) $(a-2)^2 \cdot (a+2)^2$
C) $(a-4)^2 \cdot (a+4)^2$     D) $a^2 \cdot (a^2-60)$

14. $9a^4 - 1$ ni ko'paytuvchilarga ajrating.
    A) $(3a^2-1)(3a^2+1)$     B) $(9a^2-1)(a^2+1)$
    C) $(3a-1)(3a+1)$     D) $(9a^2-1)(a^2-1)$

    **Yechish:** Berilgan $9a^4-1$ ko'phadni quyidagicha yozib olamiz $9a^4-1 = (3a^2)^2-1^2$. Endi 3-formuladan foydalansak, $(3a^2)^2 - 1^2 = (3a^2-1)(3a^2+1)$ ni olamiz. **Javob:** $(3a^2-1)(3a^2+1)$ (A).

15. $25a^4 - 9b^2$ ni ko'paytuvchilarga ajrating.
    A) $(5a^2-3)(5a^2+3b)$     B) $(5a^2-3b)(5a^2+3b)$
    C) $(5a-3b)(5a+3b)$     D) $(25a^2-b)(a^2-9b)$

16. (01-8-8) Ko'paytuvchilarga ajrating.

$$(a+b)(a+b+2) - (a-b)(a-b-2)$$

A) $2(a+b)(b+1)$     B) $4a(b+1)$
C) $2a(b-1)$     D) $4a(b-1)$

17. $y^4 - 9$ ni ko'paytuvchilarga ajrating.
    A) $(y^2-1)(y^2+9)$     B) $(y^2-9)(y^2+1)$
    C) $(y^2-3)(y^2+3)$     D) $(y^2-3)(y^2-3)$

18. $a^6 - b^6c^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

    **Yechish:** Berilgan $a^6-b^6c^6$ ko'phadni quyidagicha yozib olamiz $a^6 - b^6c^6 = (a^3)^2 - (b^3c^3)^2$. Endi 3-formuladan foydalansak, $a^6-b^6c^6 = (a^3-b^3c^3)(a^3+b^3c^3)$ ni olamiz. Biribchi qavsga 6-formulani, ikkinchi qavsga 7-formulani qo'llaymiz. Natijada $a^6 - b^6c^6 = (a-bc)(a^2+abc+b^2c^2)(a+bc)(a^2-abc+b^2c^2)$. **Javob:** 4 (C).

19. $a^6 + b^6c^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

20. $y^6 - 64$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

21. $27^2 x^6 - y^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

22. $9^3 x^6 - y^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

23. $x^6 y^6 - 4^3$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?
    A) 2     B) 3     C) 4     D) 6

24. $(x+3y)^3 + (x-3y)^3 - 52xy^2$ ko'phadni ko'paytuvchilarga ajrating.
    A) $2x(x^2+y^2)$     B) $2y(x^2+y^2)$
    C) $2x(x^2-y^2)$     D) $x(x^2-y^2)$

25. Quyidagilardan qaysi biri $x^5 - 16x$ ko'phadning ko'paytuvchisi emas.
    A) $x$     B) $x-2$     C) $x+2$     D) $x+1$

    **Yechish:** Berilgan $x^5-16x$ ko'phadni ko'paytuvchilarga ajratish uchun umumiy ko'paytuvchi $x$ ni qavs oldiga chiqaramiz, $x^4$ ni $(x^2)^2$ deb, 16 ni esa $4^2$ deb yozib olamiz. Natijada, $x^5 - 16x = x((x^2)^2 - 4^2)$ ga ega bo'lamiz. $(x^2)^2 - 4^2$ ifodaga ikki son kvadratlarining ayirmasi uchun formulasini qo'llab $x^5 - 16x = x(x^2 - 4)(x^2 + 4) = x(x-2)(x+2)(x^2+4)$ ni olamiz. Demak, $x$, $x-2$ va $x+2$ lar $x^5-16x$ ko'phadning ko'paytuvchilari ekan. **Javob:** $x+1$ (D).

26. Quyidagilardan qaysi biri $x^5 + x^3 + x$ ko'phadning ko'paytuvchisi emas.
    A) $x$   B) $x^2-x+1$     C) $x+2$     D) $x^2+x+1$

27. (99-4-16)* Ko'paytuvchilarga ajrating.

$$(a+b+2) \cdot (a+b) - (a-b)^2 + 1$$

A) $(a+b)(2a-1)$     B) $(a+1)(b+1)$
C) $2b (a+1)$     D) $(2b+1)(2a+1)$

28. (99-10-7)* Ko'paytuvchilarga ajrating.

$$a^5 + a^4 - 2a^3 - 2a^2 + a + 1$$

A) $(a+1)^2 \cdot (a-1)^3$     B) $(a+1)^3 \cdot (a-1)^2$
C) $(a+1)^4 \cdot (a-1)$     D) $(a+1) \cdot (a-1)^4$

29. (00-6-9) Ko'paytuvchilarga ajrating.

$$b^2 + ab - 2a^2 - b + a$$

A) $(a-b)(2a-b)$     B) $(a+b)(2a-b-1)$
C) $(a-b)(2a-b-1)$     D) $(b-a)(2a+b-1)$

30. (00-10-77)* Ko'phadni ko'paytuvchilarga ajrating.

$$(x-y)^3 - (z-y)^3 + (z-x)^3$$

A) $3(x-y)(y-z)(x-z)$     B) $-3(x-y)(z-y)(x-z)$
C) $3(y-x)(y-z)(z-x)$     D) $-3(x-y)(z-y)(z-x)$

## 2.6 Algebraik kasrlar

Bizga $P = x^3 - y^3$, $Q = x - y$ ko'phadlar berilgan bo'lsin. Algebraik kasr deb $P$ ko'phadni nolga teng bo'lmagan $Q$ ko'phadga bo'lishdan hosil bo'lgan $\dfrac{P}{Q}$ bo'linmaga aytiladi. $P$ ko'phad $\dfrac{P}{Q}$ algebraik kasrning surati, $Q$ ko'phad esa maxraji deyiladi. Algebraik kasrlarga misollar keltiramiz:

$$\frac{3a^2 - 2a}{a^2 + 12a + 36}; \ \frac{3a + 16}{a^2 - 36}; \ \frac{6(a^2 - 36)}{a + 6}; \ \frac{a^2 + b}{7}.$$

$\dfrac{P}{Q}$ va $\dfrac{M}{N}$ algebraik kasrlar ustida arifmetik amallar quyidagicha bajariladi:

1. $\dfrac{P}{Q} + \dfrac{M}{N} = \dfrac{PN + QM}{QN}$.

2. $\dfrac{P}{Q} - \dfrac{M}{N} = \dfrac{PN - QM}{QN}$.

3. $\dfrac{P}{Q} \cdot \dfrac{M}{N} = \dfrac{PM}{QN}$.

4. $\dfrac{P}{Q} : \dfrac{M}{N} = \dfrac{PN}{QM}$.

1. (96-12-72) Soddalashtiring.

$$\frac{x^3 + x^2 + x + 1}{x^2 + 1}$$

A) $x - 1$    B) $x$    C) $2x$    D) $x + 1$

**Yechish:** Berilgan kasrning suratida quyidagicha shakl almashtirish qilamiz $x^3 + x^2 + x + 1 = x^3 + x + x^2 + 1 = x(x^2 + 1) + (x^2 + 1)$. Umumiy ko'paytuvchi sifatida $x^2 + 1$ ni qavs oldiga chiqaramiz, natijada

$$\frac{x(x^2 + 1) + (x^2 + 1)}{x^2 + 1} = \frac{(x^2 + 1)(x + 1)}{x^2 + 1} = x + 1.$$

**Javob:** $x + 1$ (D).

2. (96-9-15) Soddalashtiring.

$$\frac{1 - x^{-1} + x^{-2}}{1 - x + x^2}$$

A) $1$    B) $x^2$    C) $\dfrac{1}{x^2}$    D) $1 - \dfrac{1}{x}$

3. (97-4-21) Soddalashtiring.

$$\frac{a^{-3} + b^{-3}}{a^2 - ab + b^2} \cdot a^3 \cdot b^3$$

A) $(a + b)^2$    B) $1$    C) $ab$    D) $a + b$

4. (96-3-21) Kasrni qisqartiring:

$$\frac{x^2 - 3xy}{9y^2 - x^2}$$

A) $\dfrac{x}{x + 3y}$   B) $\dfrac{-x}{x + 3y}$   C) $\dfrac{x}{x - 3y}$   D) $\dfrac{-x}{x - 3y}$

5. (96-3-74) Soddalashtiring:

$$\frac{x^3 + 2x^2 + x}{(x + 1)^2}$$

A) $2x$    B) $x + 1$    C) $x + 2$    D) $x$

6. (98-11-9) Soddalashtiring.

$$\frac{x^6 - x^4}{x^3 + x^2}$$

A) $x^3 - x^2 + 1$    B) $x^3 + x^2 + 1$
C) $x^3 - x^2$    D) $x^3 + x^2$

7. (00-8-54) Soddalashtiring.

$$\frac{a^8 - a^4}{a^4 + a^2}$$

A) $a^6$   B) $a^4 - a^2$   C) $a^4 - 1$   D) $a^4 + a^2$

**Yechish:** Berilgan kasrning suratini quyidagicha yozib olamiz

$$a^8 - a^4 = (a^4)^2 - (a^2)^2 = (a^4 - a^2)(a^4 + a^2).$$

Bu yerdan

$$\frac{a^8 - a^4}{a^4 + a^2} = \frac{(a^4 - a^2)(a^4 + a^2)}{a^4 + a^2} = a^4 - a^2.$$

**Javob:** $a^4 - a^2$ (B).

8. (99-1-10) Soddalashtiring.

$$\frac{p - q}{p^3 \cdot q^2} - \frac{p + q}{p^2 \cdot q^3}$$

A) $-\dfrac{p^2 + q^2}{p^3 \cdot q^3}$    B) $\dfrac{2pq - p^2 - q^2}{p^3 \cdot q^3}$
C) $-\dfrac{2}{p^3 \cdot q^2}$    D) $-\dfrac{2}{p^3 \cdot q - p^2 \cdot q^2}$

9. (99-6-5) Soddalashtiring.

$$\left(\frac{-16x^{31}}{9y^3}\right)^3 : \left(\frac{8x^{23}}{3y^2}\right)^4$$

A) $\dfrac{-y}{x}$    B) $\dfrac{-x}{y}$    C) $\dfrac{x}{9y}$    D) $\dfrac{-x}{9y}$

10. (01-2-14) Soddalashtiring.

$$\frac{a^2 + \frac{1}{a}}{a + \frac{1}{a} - 1}$$

A) $a - 1$    B) $a^2 - a + 1$
C) $a^2 + a + 1$    D) $a + 1$

11. (02-8-2) Soddalashtiring.

$$\frac{1 - b^{-1} + b^{-2}}{1 - b + b^2}$$

A) $b^{-1}$    B) $b^{-2}$    C) $b^2$    D) $b + 1$

## 2.7 Ratsional ifodalar

Ratsional ifodalar deb shunday ifodaga aytiladiki, unda bir nechta algebraik kasrlar arifmetik amal belgilari bilan bog'langan bo'ladi va ifodada nolga teng ko'phadga bo'lish ishtirok etmaydi. Masalan,

$$\left(\frac{5m}{m+3} - \frac{14m}{m^2+6m+9}\right) : \frac{5m+1}{m^2-9} + \frac{3 \cdot (m-3)}{m+3}.$$

1. (97-10-19) Soddalashtiring:

$$\left(\frac{3a}{a+6} - \frac{2a}{a^2+12a+36}\right) : \frac{3a+16}{a^2-36} + \frac{6(a-6)}{a+6}$$

A) 6     B) $\dfrac{6}{a+6}$     C) $\dfrac{1}{a-6}$     D) $a-6$

**Yechish:** Qavs ichidagi kasrlarni umumiy maxrajga keltiramiz:

$$\frac{3a}{a+6} - \frac{2a}{a^2+12a+36} = \frac{3a}{a+6} - \frac{2a}{(a+6)^2} =$$

$$= \frac{3a^2+18a-2a}{(a+6)^2} = \frac{3a^2+16a}{(a+6)^2}$$

Birinchi kasrning suratidan umumiy ko'paytuvchi $a$ ni qavs oldiga chiqarib, keyingi amallarni bajaramiz:

$$\frac{a(3a+16)}{(a+6)^2} \cdot \frac{(a-6)(a+6)}{3a+16} + \frac{6(a-6)}{a+6} =$$

$$= \frac{a(a-6)}{a+6} + \frac{6(a-6)}{a+6} = \frac{(a-6)(a+6)}{a+6} = a-6.$$

**Javob:** $a-6$ (D).

2. (96-7-19) Soddalashtiring:

$$\left(\frac{5m}{m+3} - \frac{14m}{m^2+6m+9}\right) : \frac{5m+1}{m^2-9} + \frac{3 \cdot (m-3)}{m+3}$$

A) $\dfrac{3}{m+3}$     B) 3     C) $m-3$     D) 1

3. (97-7-19) Soddalashtiring.

$$\left(\frac{2x}{x-5} + \frac{x}{x^2-10x+25}\right) : \frac{2x-9}{x^2-25} - \frac{5(x+5)}{x-5}$$

A) 5     B) $\dfrac{x+5}{x-5}$     C) $\dfrac{5}{x+5}$     D) $5+x$

4. (98-1-21) Soddalashtiring.

$$\left(\frac{4a}{4-a^2} - \frac{a-2}{4+2a}\right) \cdot \frac{4}{a+2} - \frac{a}{2-a}$$

A) $-1$     B) $\dfrac{2a}{2-a}$     C) $\dfrac{3+a}{2-a}$     D) 1

5. (98-2-8) Soddalashtiring.

$$\frac{x^3-8}{x^2+2x+4} - \frac{x^2-4}{x-2}$$

A) 4     B) $2x$     C) $-2x$     D) $-4$

6. (98-2-29) Ushbu

$$\frac{x^{-3}+8}{x^{-2}-2x^{-1}+4}$$

ifodaning $x = 0,5$ dagi qiymatini hisoblang.
A) $4,5$     B) 3     C) 4     D) 5

**Yechish:** $x^{-1} = y$ almashtirish olsak, berilgan ifoda quyidagi ko'rinishga kelad:

$$\frac{x^{-3}+8}{x^{-2}-2x^{-1}+4} = \frac{y^3+2^3}{y^2-2y+2^2} = y+2.$$

Agar $x = 0,5$ bo'lsa, u holda $y = 2$ bo'ladi va natijada biz $y + 2 = 2 + 2 = 4$ ni olamiz. **Javob:** 4 (C).

(01-11-6) Ushbu

$$\frac{a^3+b^3}{a^2-ab+b^2} \cdot (a-b) \frac{a^3-b^3}{a^2+ab+b^2} \cdot (a+b)$$

ifodaning $a = 3$ va $b = 2$ bo'lgandagi qiymatini hisoblang.
A) 24     B) 25     C) 30     D) 32

7. (98-10-12) Soddalashtiring.

$$\frac{x^3+y^3}{x^2-xy+y^2} - \frac{x^2-y^2}{x+y}$$

A) $2x$   B) $2y$   C) $-2y$   D) $-2x$

8. (99-4-26) Ifodani soddalashtiring.

$$\frac{5x+6}{x^2-4} - \frac{x}{x^2-4} : \frac{x}{x-2} - \frac{x+2}{x-2}$$

A) 1     B) $-1$     C) $\dfrac{x-2}{x+2}$     D) $\dfrac{x^2+4}{4-x^2}$

9. (99-9-19) Soddalashtiring.

$$\left(\frac{1}{a(a+1)} + \frac{1}{(a+1)(a+2)}\right) \cdot \frac{a^2+2a}{8}$$

A) $\dfrac{1}{6}$     B) $\dfrac{1}{8}$     C) $\dfrac{3}{4}$     D) $\dfrac{1}{4}$

10. (00-3-16) Soddalashtiring.

$$\left(\frac{a^2-4}{a^2+4}\right)^2 + \left(\frac{4a}{a^2+4}\right)^2$$

A) $a-4$     B) 2     C) $\dfrac{a^2-4}{a^2+4}$     D) 1

11. (00-7-13) Soddalashtiring.

$$(a^3 - 3a^2b + 3ab^2 - b^3) \cdot (a+b) : \left(\frac{a^3+b^3}{a+b} - ab\right)$$

A) $b^2-a^2$   B) $a^2-b^2$   C) $(a-b)^2$   D) $(a+b)^2$

12. (01-5-6) Soddalashtiring.

$$\frac{x}{x^2+y^2} - \frac{y \cdot (x-y)^2}{x^4-y^4}$$

A) $\dfrac{1}{x+y}$    B) $\dfrac{1}{x-y}$    C) $x+y$    D) $x-y$

13. (01-6-10) Soddalashtiring.

$$\left(2a + \frac{2ab}{a-b}\right)\left(\frac{ab}{a+b} - a\right) : \frac{4,5a^2}{a^2 - b^2}$$

A) $\dfrac{4a^2}{9}$    B) $-\dfrac{2a^2}{9}$    C) $\dfrac{2a^2}{9}$    D) $-\dfrac{4a^2}{9}$

14. (01-8-18) Soddalashtiring.

$$\frac{a^2}{a^2 - 1} + \frac{1}{a+1} : \left(\frac{1}{2-a} + \frac{2}{a^2 - 2a}\right)$$

A) $\dfrac{a}{a^2 - 1}$    B) $\dfrac{1}{a-1}$    C) $\dfrac{2a^2 - a}{a^2 - 1}$    D) 1

15. (02-9-14) Soddalashtiring.

$$\left(\frac{2}{1-x^2} - \frac{2}{(x-1)^2}\right) \cdot (1-x)^2 - \frac{4}{1+x}$$

A) 4    B) $-4$    C) 0    D) $\dfrac{1-x}{1+x}$

16. (03-4-10) Soddalashtiring.

$$\left(\frac{a+x}{a} - \frac{x-y}{x}\right) \cdot \frac{a^2}{x^2 + ay} : \frac{a}{8x}$$

A) 10    B) 6    C) 7    D) 8

17. (03-6-7) Soddalashtiring.

$$\frac{x^3 y + 2x^2 y - 3xy}{x^3 + 5x^2 + 6x} : \frac{1 - x^2}{x^2 + 3x + 2}$$

A) $\dfrac{y}{x}$    B) $-x$    C) $-y$    D) $x$

18. (03-7-10) Soddalashtiring.

$$\frac{x^3 y + 2x^2 y - 3xy}{x^3 + 5x^2 + 6x} : \frac{x^2 - 1}{x^2 + 3x + 2}$$

A) $\dfrac{y}{x}$    B) $-x$    C) $-y$    D) $y$

## 2.8   Aralash tipdagi masalalar

1. (97-9-80) Hisoblang:

$$\frac{1000^3 + 3 \cdot 1000 \cdot 995 \cdot 1995 + 995^3}{1000^2 + 2 \cdot 1000 \cdot 995 + 995^2}$$

A) 1995    B) 195    C) 995    D) 2195

**Yechish:** Kasr suratidagi 1995 ni $1000 + 995$ shaklda yozib, qisqa ko'paytirish formulasining 7-dan foydalansak,

$$1000^3 + 3 \cdot 1000^2 \cdot 995 + 3 \cdot 1000 \cdot 995^2 + 995^3 =$$
$$= (1000 + 995)^3$$

ni olamiz. Kasrning maxraji esa $(1000+995)^2$ ga teng. Shunday ekan

$$\frac{(1000 + 995)^3}{(1000 + 995)^2} = 1995.$$

**Javob:** 1995 (A).

2. (99-7-2) Hisoblang.

$$889^3 + 3000 \cdot 889 \cdot 111 + 111^3 + 889 + 111$$

A) 10001000       B) 1001000
C) 1001001000       D) 1000001000

3. (96-1-13) Hisoblang.

$$\frac{1^2 - 0,4^2}{2,8 \cdot 0,4 - 2,8}$$

A) $\dfrac{1}{2}$    B) $-\dfrac{1}{2}$    C) $-5$    D) 5

4. (00-6-5) Soddalashtiring.

$$\frac{1,6^2 - 1,6 \cdot 0,8 + 0,4^2}{1,4^2 - 0,2^2}$$

A) $1,6$    B) $0,375$    C) $1,2$    D) $0,75$

5. (96-10-13) Hisoblang.

$$\frac{4,5^2 - 1,5^2}{0,3 \cdot 0,7 - 0,3}$$

A) $-20$    B) 20    C) 200    D) $-200$

6. Hisoblang.

$$\frac{10^{45} + 10^{46} + 10^{50}}{10^{49} + 10^{45} + 10^{44}}$$

A) 10    B) 30    C) 100    D) 1000

7. (98-7-10) Hisoblang.

$$\frac{(3,7^2 - 6,3^2) \cdot (13^2 - 12,6^2)}{(4,2^2 - 5,8^2) \cdot (2,3^2 - 0,3^2)}$$

A) 32    B) $0,32$    C) $3,2$    D) $\dfrac{1}{32}$

8. (98-8-9) Hisoblang.

$$\frac{0,5^2 - 0,5}{0,4^2 + 2 \cdot 0,04 + 0,1^2}$$

A) 1    B) $-1$    C) $-0,1$    D) 10

9. (98-1-9) Hisoblang.

$$\frac{0,2^2 - 2 \cdot 0,06 + 0,3^2}{0,5 \cdot 0,9 - 0,5}$$

A) $0,2$    B) $-2$    C) $-0,2$    D) $0,25$

10. (99-6-6) Hisoblang.

$$(202^2 - 54^2 + 256 \cdot 352) : (4^4 \cdot 10^2)$$

A) 4    B) 1    C) 2    D) 5

**Yechish:** $202^2 - 54^2$ ayirmani ikki son kvadratlarining ayirmasi uchun 2.4 ning 3-formulasiga ko'ra $202^2 - 54^2 = 148 \cdot 256$. Endi birinchi qavs ichini hisoblaymiz $148 \cdot 256 + 256 \cdot 352 = 256(148 + 352) = 256 \cdot 500$. Endi bo'linmani hisoblaymiz $256 \cdot 500 : (256 \cdot 100) = 5$.    **Javob:** 5 (D).

11. (97-4-14) Agar $x = 4,5$ va $y = 3,5$ bo'lsa,

$$x^3 - x^2y - xy^2 + y^3$$

ni hisoblang.
A) 10     B) $9,5$     C) 8     D) $7,2$

12. (01-8-5) Hisoblang.

$$\frac{0,6 \cdot 0,8 + 0,6 \cdot 1,2}{0,2^2 - 0,4^2}$$

A) $-10$     B) 10     C) $-0,1$     D) $-100$

13. (02-5-6) Hisoblang.

$$\frac{2,7(1,7^3 - 1,5^3)}{5,1^2 + 5,1 \cdot 4,5 + 4,5^2}$$

A) $0,45$     B) $0,27$     C) $0,3$     D) $0,06$

14. (03-11-61) Hisoblang.

$$\frac{0,6^2 - 0,6 \cdot 0,2 + 0,1^2}{1,5 - 1,5^2}$$

A) $-0,5$     B) $-\dfrac{1}{3}$     C) $-3$     D) $-1\dfrac{2}{3}$

15. (00-10-12) Soddalashtiring.

$$\frac{5 \cdot 2^{k-2} + 10 \cdot 2^{k-1}}{10^{k+2}}$$

A) $4^{-1} \cdot 5^{-k}$     B) $4^{-2} \cdot 5^{-k}$
C) $4 \cdot 5^{-k}$     D) $2^{-1} \cdot 5^{-k}$

16. (00-10-74) Kasrni qisqartiring.

$$\frac{2^{m+1} + 2^{-m+1}}{(4^m + 1)(3^{m+2} + 3^{m+1})}$$

A) $0,5 \cdot 6^{-m}$     B) $\left(\dfrac{2}{3}\right)^m$     C) $6^{-m-1}$     D) $3^m$

17. (97-2-6) Ushbu $\dfrac{12 - 3n}{n}$ ifoda $n$ ning nechta natural qiymatida natural son bo'ladi?
A) 6     B) 3     C) 5     D) 4

**Yechish:** Ushbu $\dfrac{12 - 3n}{n} = \dfrac{12}{n} - \dfrac{3n}{n} = \dfrac{12}{n} - 3$ tenglik o'rinli. Bu ayirma natural bo'lishi uchun $\dfrac{12}{n}$ bo'linma 3 dan katta natural son bo'lishi kerak. Bulardan $n = 1, 2, 3$ ekanligini olamiz. **Javob:** 3 (B).

18. (97-8-6) $\dfrac{10n - 24}{n}$ ifoda natural son bo'ladigan $n$ ning natural qiymatlari nechta?
A) 4     B) 7     C) 6     D) 5

19. (97-12-5) Ifoda natural son bo'ladigan $n$ ning barcha natural qiymatlari nechta?

$$\frac{16n^2 - 128}{n^2}$$

A) 5     B) 3     C) 2     D) 6

20. (98-8-11) Ushbu $\dfrac{3n - 1}{n + 2}$ ifoda $n$ ning nechta butun qiymatida natural son bo'ladi?
A) 1     B) 3     C) 4     D) 2

21. (96-6-6) $\dfrac{6n - 12}{n}$ ifoda $n$ ning nechta natural qiymatida natural son bo'ladi?
A) 6     B) 5     C) 3     D) 4

22. (03-10-10) $a$ ning nechta butun qiymatida

$$\frac{a^4 - 9}{a^3 - 3a} : \frac{a^3 + 3a}{a - 5a^2}$$

ifodaning qiymati butun son bo'ladi?
A) 2     B) 3     C) 1     D) 4

23. (01-7-7) $n$ ning nechta butun qiymatida $\dfrac{n^2 - n + 3}{n + 1}$ kasr butun son bo'ladi?
A) 1     B) 2     C) 3     D) 4

24. (01-11-30) Ushbu

$$\frac{4^{a+1} - 2^{2a-1}}{2^{2a}}$$

ifodaning qiymati 9 dan qancha kam?
A) 4     B) $3,5$     C) 3     D) $5,5$

**Yechish:** $4^{a+1} = (2^2)^{a+1} = 2^{2a+2}$ desak, u holda berilgan kasr quyidagiga teng bo'ladi:

$$\frac{2^{2a+2} - 2^{2a-1}}{2^{2a}} = \frac{2^{2a}(2^2 - 2^{-1})}{2^{2a}} = 4 - \frac{1}{2} = 3,5.$$

Bu qiymatni 9 dan ayirib, ya'ni $9 - 3,5 = 5,5$ ni olamiz.     **Javob:** $5,5$ (D).

25. (02-1-39) Agar $\dfrac{x}{y} = 2$ bo'lsa, $x^2 - 4y^2$ nimaga teng?
A) 4     B) 8     C) 0     D) $-8$

26. (02-6-2) Agar $a + b + c = 0$ bo'lsa, $a^3 + a^2c - abc + b^2c + b^3$ ifodaning qiymatini toping.
A) 0     B) 1     C) 2     D) $-1$

27. Soddalashtiring.

$$\frac{a - b}{ab} + \frac{b - c}{bc} + \frac{c - d}{cd} + \frac{d - a}{ad}$$

A) 1     B) $abcd$     C) $(abcd)^{-1}$     D) 0

28. $m$ ning qanday butun qiymatida quyidagi ifodani qisqartirish mumkin:

$$\frac{x^2 + mx + 36}{x^2 + 8x + 7}$$

A) $-37$     B) $-36$     C) $-35$     D) 37

29. Kasr qisqarishi mumkin bo'lsa, $m$ ning eng katta va eng kichik qiymatlari farqini toping.

$$\frac{x^3 - x^2 - 4x + 4}{x^2 + mx + 6}$$

A) 12     B) 5     C) 7     D) 10

# 3 - bob. Ildizlar

## 3.1 Arifmetik ildiz. Kvadrat ildiz

Kvadrati $a$ ga teng bo'lgan $b$ son $a$ sonning kvadrat ildizi deyiladi. Kvadrat ildizdan $b$ sonni topish $a$ sondan kvadrat ildiz chiqarish deyiladi. $a$ sondan kvadrat ildiz chiqarish quyidagicha belgilanadi:

$$\sqrt{a} = b, \qquad (3.1).$$

Bu yerda $a$ ildiz ostidagi ifoda deyiladi. (3.1) tenglik $a$ va $b$ sonlari orasida quyidagi bog'lanish borligini bildiradi: $b^2 = a$. Ixtiyoriy $a > 0$ son uchun $b^2 = a$ tenglikni qanoatlantiruvchi ikki haqiqiy son mabjud. Bu sonlar absolyut qiymatlari teng bo'lgan o'zaro qarama-qarshi sonlardir. Bu sonlarning musbati $a > 0$ sonning arifmetik ildizi deyiladi. Masalan, $\sqrt{16} = \pm 4$, bu yerda 4 soni 16 ning arifmetik ildizi hisoblanadi. Ko'pincha *ildiz* atamasi o'rniga *radikal* termini ham ishlatiladi. Kvadrat ildiz quyidagi xossalarga ega.

1. $(\sqrt{a})^2 = a, \quad a \geq 0$.
2. $\sqrt{ab} = \sqrt{a}\sqrt{b}, \quad a, b \geq 0$.
3. $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}, \quad a \geq 0, \quad b > 0$.
4. $a\sqrt{b} = \sqrt{a^2 b}, \quad a \geq 0$.
5. $a\sqrt{b} = -\sqrt{a^2 b}, \quad a \leq 0$.
6. $\sqrt{a^2} = |a|$.

1. (97-11-8) Soddalashtiring.

$$2\sqrt{5\frac{1}{2}} + \frac{1}{3}\sqrt{99} - 2\sqrt{2\frac{3}{4}}$$

A) $3\sqrt{11}$    B) $2\sqrt{22}$    C) $\sqrt{22}$    D) 2

**Yechish:** 4-formuladan foydalanib

$$2\sqrt{5\frac{1}{2}} + \frac{1}{3}\sqrt{99} - 2\sqrt{2\frac{3}{4}} = 2\sqrt{\frac{11}{2}} +$$

$$+\frac{1}{3}\sqrt{9\cdot 11} - 2\sqrt{\frac{11}{4}} = \sqrt{4\cdot\frac{11}{2}} + \frac{1}{3}\cdot 3\sqrt{11} - \sqrt{4\cdot\frac{11}{4}} =$$

$$= \sqrt{22} + \sqrt{11} - \sqrt{11} = \sqrt{22}$$

ekanini hosil qilamiz. **Javob:** $\sqrt{22}$ (C).

2. (02-2-6) Hisoblang.

$$3\sqrt{\frac{1}{5}} + \frac{1}{2}\sqrt{20} + \sqrt{\frac{4}{5}}$$

A) $2\sqrt{5}$    B) $\sqrt{5}$    C) $3\sqrt{5}$    D) $\dfrac{6}{\sqrt{5}}$

3. (02-9-10) Hisoblang.

$$3\sqrt{3\frac{2}{3}} - \sqrt{132} + 4\sqrt{2\frac{1}{16}}$$

A) 0    B) $2\sqrt{33}$    C) $3\sqrt{3}$    D) $4\sqrt{11}$

4. (97-1-8) Hisoblang.

$$4\sqrt{3\frac{1}{2}} - 0,5\sqrt{56} - 3\sqrt{1\frac{5}{9}}$$

A) $2\sqrt{14}$    B) $2\sqrt{7}$    C) 0    D) 2

5. (97-6-8) Hisoblang.

$$15\sqrt{\frac{3}{5}} - 0,5\sqrt{60} + 2\sqrt{3\frac{3}{4}}$$

A) 0    B) $\sqrt{15}$    C) $5\sqrt{3}$    D) $3\sqrt{15}$

6. (98-6-9) Hisoblang.

$$\frac{\sqrt{32} + \sqrt{98} - \sqrt{50}}{\sqrt{72}}$$

A) 2    B) 1    C) $\sqrt{2}$    D) $2\sqrt{2}$

7. (99-6-36) Hisoblang.

$$\sqrt{192} - \sqrt{108} + \frac{\sqrt{243}}{3}$$

A) $5\sqrt{3}$    B) $5\sqrt{2}$    C) $3\sqrt{5}$    D) $3\sqrt{3}$

8. (01-1-6) Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{3\sqrt{5} - 2\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$$

A) $\dfrac{1}{2}(\sqrt{5} + 3\sqrt{2})$    B) $\dfrac{1}{2}(3\sqrt{5} - 2\sqrt{2})$
C) $9 + 2,5\sqrt{10}$    D) $2,5\sqrt{10} - 9$

**Yechish:** Kasr maxrajini irratsionallikdan qutqarish deganda shu narsa tushuniladiki, bu kasr o'ziga teng bo'lgan boshqa bir kasr bilan almashtiriladi, bu kasrning maxrajida ildiz amali ishtirok etmaydi. Bu kasr surat va maxrajini $2\sqrt{5} + 3\sqrt{2}$ ga ko'paytiramiz va amallarni bajaramiz

$$\frac{3\sqrt{5} - 2\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}} \cdot \frac{2\sqrt{5} + 3\sqrt{2}}{2\sqrt{5} + 3\sqrt{2}} = \frac{18 + 5\sqrt{10}}{4\cdot 5 - 9\cdot 2}.$$

Bu kasrning maxraji 2 ga teng. Bulardan $9 + 2,5\sqrt{10}$ ni olamiz. **Javob:** $9 + 2,5\sqrt{10}$ (C).

9. (96-7-24) Hisoblang.

$$\frac{3 - \sqrt{5}}{3 + \sqrt{5}} + \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$$

A) 2    B) $3\dfrac{\sqrt{5}}{2}$    C) $4,5$    D) 7

10. (97-7-24) Ifodaning qiymatini toping.

$$\frac{3 + \sqrt{7}}{3 - \sqrt{7}} - \frac{3 - \sqrt{7}}{3 + \sqrt{7}}$$

A) $4 + \sqrt{7}$    B) $-3\sqrt{7}$    C) $6\sqrt{7}$    D) 3

11. Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{4}{1 - \sqrt{5} + \sqrt{20}}$$

A) $\sqrt{5} + 1$   B) $\sqrt{5} - 1$   C) $\sqrt{10}$   D) $1 - \sqrt{5}$

12. (01-6-23) Hisoblang.

$$\left( \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} + \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \right)^2$$

A) 12   B) 14   C) 18   D) 16

13. (02-6-26) Hisoblang.

$$\frac{1}{\sqrt{7} - \sqrt{6}} - \frac{3}{\sqrt{6} - \sqrt{3}} - \frac{4}{\sqrt{7} + \sqrt{3}}$$

A) 0   B) 1   C) 2   D) 3

14. (03-8-24) Hisoblang.

$$\left( \frac{15}{\sqrt{6} + 1} + \frac{4}{\sqrt{6} - 2} - \frac{12}{3 - \sqrt{6}} \right) \cdot (\sqrt{6} + 11)$$

A) $-115$   B) 127   C) 100   D) $-116$

15. (03-10-17) Hisoblang.

$$\frac{\sqrt{2} + 1}{3 + 2\sqrt{2}} - \frac{\sqrt{2} - 1}{3 - 2\sqrt{2}}$$

A) 1   B) $-1$   C) 2   D) $-2$

16. (96-6-50) Hisoblang.

$$\frac{1}{3 - \sqrt{8}} - 2\sqrt{2} + 6$$

A) 8   B) 7   C) 9   D) 10

**Yechish:** Dastlab birinchi kasr maxrajini irratsionallikdan qutqaramiz. Buning uchun kasrning surat va maxrajini $3 + \sqrt{8}$ ga ko'paytiramiz

$$\frac{1}{3 - \sqrt{8}} \cdot \frac{3 + \sqrt{8}}{3 + \sqrt{8}} - 2\sqrt{2} + 6 = \frac{3 + \sqrt{8}}{9 - 8} - 2\sqrt{2} + 6.$$

Endi $9 - 8 = 1$ va $\sqrt{8} = 2\sqrt{2}$ ekanligini hisobga olsak, $3 + 2\sqrt{2} - 2\sqrt{2} + 6 = 9$ ni olamiz. **Javob:** 9 (C).

17. (97-2-50) Hisoblang.

$$2\sqrt{3} - 5 - \frac{11}{\sqrt{12} - 1}$$

A) $2\sqrt{3} - 4$   B) 4   C) $-4$   D) $-6$

18. (97-8-50) Hisoblang.

$$\frac{19}{\sqrt{20} - 1} - 2\sqrt{5} + 3$$

A) $4\sqrt{5} + 4$   B) $4\sqrt{5} - 4$
C) $2\sqrt{5} + 4$   D) 4

19. (97-12-49) Hisoblang.

$$\frac{19}{\sqrt{20} + 1} + 6 - 2\sqrt{5}$$

A) 6   B) 5   C) $4\sqrt{5} - 7$   D) $4\sqrt{5} - 6$

20. (98-2-20) Hisoblang.

$$\frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{3} - 1}$$

A) 2   B) 3   C) 4   D) $\sqrt{3}$

**Yechish:** Birinchi kasr surat va maxrajini $2 - \sqrt{3}$ ga ikkinchi kasr surat va maxrajini $\sqrt{3} + 1$ ko'paytiramiz hamda $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$ va $(\sqrt{3} + 1) \cdot (\sqrt{3} - 1) = 2$ ekanligini hisobga olsak,

$$\frac{1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} + \frac{2}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} =$$

$$= 2 - \sqrt{3} + \sqrt{3} + 1 = 3 \text{ ni olamiz. } \textbf{Javob: } 3 \text{ (D)}.$$

21. (99-6-38) Hisoblang.

$$\frac{\sqrt{5}}{\sqrt{5} - 2} - \frac{10}{\sqrt{5}}$$

A) 1   B) 4   C) 3   D) 5

22. (99-10-15) Hisoblang.

$$\left( \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{2 + \sqrt{3}} \right) \cdot (2 + \sqrt{2})$$

A) $2\sqrt{2}$   B) $2\sqrt{3}$   C) 2   D) $3\sqrt{2}$

23. (00-10-47) Amallarni bajaring.

$$\frac{9}{5 - \sqrt{7}} + \frac{22}{7 + \sqrt{5}} - \frac{1}{\sqrt{7} + \sqrt{5}}$$

A) 1   B) 6   C) $\frac{1}{5}$   D) 5

24. (98-7-17) O'zaro teskari sonlarni aniqlang.

1) $\frac{\sqrt{7}}{2}$ va $\frac{2\sqrt{7}}{7}$   2) $\sqrt{6} - \sqrt{5}$ va $\sqrt{6} + \sqrt{5}$

3) $\frac{2\sqrt{5}}{9}$ va $\frac{9\sqrt{5}}{10}$   4) $\sqrt{3} - 1$ va $\sqrt{3} + 1$.

A) hammasi   B) $2; 3; 4$
C) $1; 3; 4$   D) $1; 2; 3$

25. (01-7-4) Soddalashtiring.

$$4 + 5\sqrt{2} + \frac{\sqrt{75}}{\sqrt{3} - \sqrt{6}}$$

A) $2\sqrt{2} + 1$   B) 3   C) 2   D) $-1$

26. (01-11-8) Hisoblang.

$$\left( \sqrt{18} + \sqrt{72} - \sqrt{12} \right) \cdot \left( \sqrt{18} + \sqrt{72} + \sqrt{12} \right)$$

A) 148   B) 149   C) 147   D) 150

27. (99-2-13) Ushbu

$$\sqrt{9 + \sqrt{65}} \cdot \sqrt{9 - \sqrt{65}}$$

son 14 dan qanchaga kam?
A) 8      B) 9      C) 10      D) 11

**Yechish:** 2-qoidadan foydalanib

$$\sqrt{9 + \sqrt{65}} \cdot \sqrt{9 - \sqrt{65}} = \sqrt{(9 + \sqrt{65}) \cdot (9 - \sqrt{65})}$$

$= \sqrt{81 - 65} = \sqrt{16} = 4$ ni olamiz. Ifodaning qiymati 14 dan 10 ta kam ekan. **Javob:** 10 (C).

28. (99-7-4) Hisoblang.

$$\sqrt{9 + \sqrt{77}} \cdot \sqrt{9 - \sqrt{77}}$$

A) 3      B) 12      C) 2      D) 4

29. (00-1-48) Hisoblang.

$$\sqrt{7 + 2\sqrt{10}} \cdot \sqrt{7 - 2\sqrt{10}}$$

A) 2      B) 3,2      C) 3      D) 2,5

30. (03-5-2) Hisoblang.

$$\frac{\sqrt{196} \cdot \sqrt{19,6}}{\sqrt{0,196} \cdot \sqrt{1,96}}$$

A) 1000      B) 100      C) 196      D) 10

31. (96-7-14) Hisoblang.

$$\sqrt{\frac{65^3 + 35^3}{100} - 35 \cdot 65}$$

A) 100      B) 30      C) 10      D) 45

**Yechish:** Ikki son kublari yig'indisidan foydalansak, $65^3 + 35^3 = 100 \cdot (65^2 - 65 \cdot 35 + 35^2)$ ni olamiz. Buni ildiz ostidagi ifodaga qo'yib

$$\sqrt{\frac{100 \cdot (65^2 - 65 \cdot 35 + 35^2)}{100} - 35 \cdot 65} =$$

$= \sqrt{65^2 - 2 \cdot 65 \cdot 35 + 35^2} = \sqrt{(65 - 35)^2} = 30$

ni olamiz. **Javob:** 30 (B).

32. (97-3-14) Ifodaning qiymatini toping.

$$\sqrt{\frac{68^3 - 32^3}{36} + 68 \cdot 32}$$

A) $16\frac{2}{3}$      B) 85      C) 100      D) $25\frac{5}{6}$

33. (99-5-11) Agar $\sqrt{t^5 + 3} - \sqrt{t^5 - 2} = 1$ bo'lsa,

$$\sqrt{t^5 + 3} + \sqrt{t^5 - 2}$$

ning qiymati nechaga teng bo'ladi?
A) 2      B) 3      C) 4      D) 5

34. (98-7-20) Agar $a : b = -\sqrt{5}$ bo'lsa, $a^2 - 5b^2$ ni hisoblang.
A) 0      B) $\sqrt{5}$      C) 5      D) $-5$

35. (03-5-12) Agar $\sqrt{5} = m$, $\sqrt{7} = n$ bo'lsa, $\sqrt{560}$ ni $m$ va $n$ orqali ifodalang.
A) $4mn$      B) $2mn$      C) $6mn$      D) $8mn$

### 3.1.1   Hisoblashga oid misollar

Ildiz qatnashgan ifodalarni hisoblashda (soddalashtirish-da) quyidagi formulalardan foydalaniladi.

1. $\sqrt{a + b\sqrt{c}} = \sqrt{\frac{a+A}{2}} + \sqrt{\frac{a-A}{2}}$, $A = \sqrt{a^2 - b^2 c}$.

2. $\sqrt{a - b\sqrt{c}} = \sqrt{\frac{a+A}{2}} - \sqrt{\frac{a-A}{2}}$, $A = \sqrt{a^2 - b^2 c}$.

3. $\sqrt{a + b + 2\sqrt{a \cdot b}} = \sqrt{a} + \sqrt{b}$, $a > 0$; $b > 0$.

4. $\sqrt{a + b - 2\sqrt{a \cdot b}} = \sqrt{a} - \sqrt{b}$, $a > b > 0$.

1. (03-4-11) Hisoblang.

$$\sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$$

A) $\sqrt{3}$      B) $2\sqrt{3}$      C) $2\sqrt{2}$      D) $\sqrt{2}$

**Yechish:** Bu misolni yechishda 1 va 2-qoidalardan foydalanamiz. Demak, $a = 2$, $b = 1$, $c = 3$ bunga ko'ra ko'ra $A = \sqrt{2^2 - 1^2 \cdot 3} = \sqrt{4 - 3} = 1$. 1-qoidaga ko'ra

$$\sqrt{2 + \sqrt{3}} = \sqrt{\frac{2 + 1}{2}} + \sqrt{\frac{2 - 1}{2}} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}.$$

Ikkinchi ifoda $\sqrt{2 - \sqrt{3}}$ ga 2-qoidani qo'llab

$$\sqrt{2 - \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

ni olamiz. Ularni ayirib $2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$ ga ega bo'lamiz.
**Javob:** $\sqrt{2}$ (D).

2. (01-1-4) Hisoblang.

$$\sqrt{5 - 2\sqrt{6}} - \sqrt{5 + 2\sqrt{6}}$$

A) $2\sqrt{2}$   B) $-4\sqrt{6}$   C) $\sqrt{2}$   D) $-2\sqrt{2}$

3. (01-11-36) Hisoblang.

$$\sqrt{6 + 4\sqrt{2}} + \sqrt{6 - 4\sqrt{2}}$$

A) $3,8$      B) $3,6$      C) 4      D) $4,2$

4. (02-10-43) Hisoblang.

$$\sqrt{52 - 30\sqrt{3}} - \sqrt{52 + 30\sqrt{3}}$$

A) $-10$      B) 10      C) $-8$      D) 8

5. (03-11-74) Hisoblang.

$$\sqrt{17 - 12\sqrt{2}} \cdot (6 + 4\sqrt{2})$$

A) $\sqrt{2}$      B) $-\sqrt{2}$      C) $\sqrt{3 + \sqrt{8}}$      D) 2

6. Hisoblang.

$$\sqrt{15 - \sqrt{56}} - \sqrt{14}$$

A) $\sqrt{3}$      B) $-2\sqrt{14}$      C) $-1$      D) 1

7. (96-3-73) Ayirmaning qiymatini toping.

$$\sqrt{9 - 2\sqrt{20}} - \sqrt{9 + 2\sqrt{20}}$$

A) $-3$    B) $-6$    C) $-4$    D) $-5$

**Yechish:** *1-usul.* $x = \sqrt{9 - 2\sqrt{20}} - \sqrt{9 + 2\sqrt{20}}$ deb belgilaymiz. $x < 0$ ekani ravshan. U holda

$$x^2 = 9 - 2\sqrt{20} - 2\sqrt{(9 - 2\sqrt{20})(9 + 2\sqrt{20})} + 9 +$$

$$+ 2\sqrt{20} = 18 - 2\sqrt{81 - 80} = 18 - 2 = 16.$$

$x < 0$ va $x^2 = 16$ dan $x = -4$ ni olamiz.

*2-usul.* 3 va 4-qoidalardan foydalanamiz bu misolni yechamiz. Ildiz ostidagi $9 - 2\sqrt{20}$ ni $5 + 4 - 2\sqrt{5 \cdot 4}$ deb yozib olamiz. U holda 3-ga ko'ra u $\sqrt{5} - \sqrt{4} = \sqrt{5} - 2$ ga teng bo'ladi. Ikkinchi qo'shiluvchi $\sqrt{9 + 2\sqrt{20}}$ esa $\sqrt{5} + \sqrt{4} = \sqrt{5} + 2$ ga teng. Ularni ayirib $\sqrt{5} - 2 - \sqrt{5} - 2 = -4$ ni olamiz. **Javob:** $-4$ (C).

8. (96-6-28) Hisoblang.

$$\sqrt{23 - 8\sqrt{7}} + \sqrt{23 + 8\sqrt{7}}$$

A) 7    B) 6    C) 8    D) 9

9. (96-9-13) Yig'indining qiymatini toping.

$$\sqrt{11 + 6\sqrt{2}} + \sqrt{11 - 6\sqrt{2}}$$

A) 6    B) 4    C) 8    D) 5

10. (96-12-71) Ayirmaning qiymatini toping.

$$\sqrt{9 + 2\sqrt{20}} - \sqrt{9 - 2\sqrt{20}}$$

A) 4    B) 5    C) 6    D) 3

11. (97-2-28) Hisoblang.

$$\sqrt{19 + 8\sqrt{3}} + \sqrt{19 - 8\sqrt{3}}$$

A) 6    B) 7    C) 9    D) 8

12. (96-13-13) Yig'indining qiymatini toping.

$$\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}$$

A) 3    B) 5    C) 4    D) 6

13. (00-10-4) Hisoblang.

$$\sqrt{21 - 2\sqrt{21 + 2\sqrt{19 - 6\sqrt{2}}}}$$

A) $3\sqrt{2}+1$   B) $3\sqrt{2}+2$   C) $3\sqrt{2}-2$   D) $3\sqrt{2}-1$

14. (01-7-6) Ushbu $\sqrt{45 \cdot 10 \cdot 18}$ va $\sqrt{21 \cdot 56 \cdot 6}$ sonlarning eng katta umumiy bo'luvchisini toping.
    A) 9    B) 10    C) 18    D) 6

15. (97-5-21) Soddalashtiring.

$$\sqrt{7 - 4\sqrt{3}}$$

A) $2 + \sqrt{3}$    B) $\sqrt{3} - 2$
C) $3 + \sqrt{3}$    D) $2 - \sqrt{3}$

**Yechish:** Misolni yechishda 3 va 4-qoidalardan foydalanamiz. Ildiz ostidagi $7 - 4\sqrt{3}$ ni $4 + 3 - 2\sqrt{4 \cdot 3}$ deb yozib olamiz. U holda 3-ga ko'ra u $\sqrt{4} - \sqrt{3} = 2 - \sqrt{3}$ ga teng bo'ladi. **Javob:** $2 - \sqrt{3}$ (D).

16. (98-5-10) Hisoblang.

$$\sqrt{19 - 8\sqrt{3}}$$

A) $4 - \sqrt{3}$    B) $4 + \sqrt{3}$    C) $3 + \sqrt{3}$    D) $3\sqrt{3}$

17. (00-8-28) Hisoblang.

$$\sqrt{6 - 2\sqrt{5}}$$

A) $\sqrt{5} - 1$    B) $1 - \sqrt{5}$    C) $\sqrt{3}$    D) $1 + \sqrt{5}$

18. (98-10-41) Hisoblang.

$$\sqrt{19 - 8\sqrt{3}} + \sqrt{3}$$

A) $-4$    B) 4    C) $4 + 2\sqrt{3}$    D) $2\sqrt{3} - 4$

19. (03-7-50) Hisoblang.

$$\sqrt{11 - 4\sqrt{7}}$$

A) $\sqrt{7} + 2$    B) $\sqrt{7} - 2$    C) $\sqrt{7}$    D) $2 - \sqrt{7}$

20. (97-8-27) Hisoblang.

$$\sqrt{11 - 6\sqrt{2}} + \sqrt{11 + 6\sqrt{2}}$$

A) 8    B) 4    C) 3    D) 6

21. (97-12-27) Hisoblang.

$$\sqrt{9 - 4\sqrt{2}} - \sqrt{9 + 4\sqrt{2}}$$

A) 2    B) 3    C) $-3$    D) $-2$

22. (98-7-14) Hisoblang.

$$\left(\sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}\right)^2 \cdot 0,5^{-2}$$

A) 38    B) 30    C) 40    D) 44

23. (99-10-14) Hisoblang.

$$\frac{\sqrt{3 + 2\sqrt{2}} + \sqrt{3 - 2\sqrt{2}}}{4\sqrt{2}}$$

A) $\dfrac{\sqrt{2}}{4}$    B) $0,5$    C) $\dfrac{\sqrt{2}}{2}$    D) $0,75$

24. (03-1-1) Soddalashtiring.

$$\sqrt{\left(\frac{\pi}{2} - \sqrt{3}\right)^2} + \sqrt{\left(\frac{\pi}{3} - \sqrt{2}\right)^2} - \sqrt{5 + 2\sqrt{6}}$$

A) $\sqrt{3} + \sqrt{2}$    B) $\dfrac{5\pi}{6} - 2(\sqrt{2} + \sqrt{3})$
C) $\dfrac{5\pi}{6}$    D) $\dfrac{-5\pi}{6}$

### 3.1.2 Ildizli ifodalarni soddalashtirish

Biz bu bandda arifmetik ildizning xossalari hamda qisqa ko'paytirish formulalaridan foydalanib misollar yechamiz.

1. (01-1-62) Ifodani $a \geq 0,5$ da soddalashtiring.

$$\sqrt{a^2} - \sqrt{a^2 + a + 0,25} + \sqrt{a^2 - a + 0,25}$$

A) $a - 0,25$    B) $a - 0,5$    C) $a - 0,75$    D) $a - 1$

**Yechish:** $a \geq 0,5$ shartdan hamda $a^2 + a + 0,25 = (a + 0,5)^2$, $a^2 - a + 0,25 = (a - 0,5)^2$ tengliklardan, 3.1-mavzudagi 6-qoidaga ko'ra

$$\sqrt{a^2} - \sqrt{a^2 + a + 0,25} + \sqrt{a^2 - a + 0,25} =$$

$$= a - (a + 0,5) + (a - 0,5) = a - 1$$

tenglikni olamiz. **Javob:** $a - 1$ (D).

2. (97-6-56) Quyidagi ifodani $m = 15$ va $n = 3\sqrt{2}$ bo'lgandagi qiymatini hisoblang.

$$\frac{(\sqrt{m} + n)\sqrt{m - 2\sqrt{m} \cdot n + n^2}}{m - n^2}$$

A) $1$        B) $-1$        C) $-3$        D) $0$

3. (01-7-14) Agar $a = (2 + \sqrt{3})^{-1}$ va $b = (2 - \sqrt{3})^{-1}$ bo'lsa,

$$(a + 1)^{-1} + (b + 1)^{-1}$$

ning qiymatini hisoblang.
A) $2$        B) $0,5$        C) $2\sqrt{3}$        D) $1$

4. (01-10-1) Agar $a = \sqrt{2}$ va $b = \sqrt[3]{3}$ bo'lsa,

$$\sqrt{a^2 - 2ab + b^2} + \sqrt{a^2 + 2ab + b^2}$$

ning qiymatini hisoblang.
A) $\sqrt{8}$        B) $\sqrt[3]{12}$        C) $\sqrt{18}$        D) $\sqrt[3]{24}$

5. (02-6-8) $x = 5\sqrt{6}$ va $y = 6\sqrt{5}$ bo'lsa,

$$\sqrt{x^2 + 2xy + y^2} - \sqrt{x^2 - 2xy + y^2}$$

ning qiymatini hisoblang.
A) $\sqrt{720}$        B) $\sqrt{700}$        C) $\sqrt{640}$        D) $\sqrt{600}$

6. (97-1-57) Ushbu

$$\frac{(x + \sqrt{y}) \cdot \sqrt{y - 2 \cdot \sqrt{y} \cdot x + x^2}}{y - x^2}$$

ifodani $x = 2\sqrt{6}$ va $y = 23$ bo'lganda hisoblang.
A) $1$        B) $-1$        C) $\frac{1}{2}$        D) $-\frac{1}{2}$

7. (03-10-12) Agar $x = \frac{4}{5}m$ bo'lsa,

$$\frac{\sqrt{m + x} + \sqrt{m - x}}{\sqrt{m + x} - \sqrt{m - x}}$$

ning qiymatini toping.
A) $2$        B) $2m$        C) $4$        D) $-2$

8. (03-10-15) Agar $x < 0$ bo'lsa,

$$\sqrt{x^2 - 12x + 36} - \sqrt{x^2}$$

ni soddalashtiring.
A) $6$        B) $-6$        C) $6 - 2x$        D) $2x - 6$

9. (01-11-7) Soddalashtiring.

$$\frac{3}{a - \sqrt{a^2 - 3}} + \frac{3}{a + \sqrt{a^2 - 3}}$$

A) $1,5a$        B) $3a$        C) $2a$        D) $2,5a$

**Yechish:** Birinchi kasr surat va maxraji $a + \sqrt{a^2 - 3}$ ga, ikkinchisini esa $a - \sqrt{a^2 - 3}$ ga ko'paytiramiz, natijada kasrlarning maxraji irratsionallikdan qutiladi va biz

$$a + \sqrt{a^2 - 3} + a - \sqrt{a^2 - 3} = 2a$$

ni olamiz. **Javob:** $2a$ (C).

10. (02-12-25) Soddalashtiring.

$$\frac{3}{a - \sqrt{a^2 - 3}} - \frac{3}{a + \sqrt{a^2 - 3}}$$

A) $3a$    B) $3\sqrt{a^2 - 3}$    C) $6a$    D) $2\sqrt{a^2 - 3}$

11. (02-11-12) Soddalashtiring.

$$\left(\frac{1 + \sqrt{x} + x}{x\sqrt{x} - 1}\right)^{-1} - x^{\frac{1}{2}}$$

A) $\sqrt{x} + 1$        B) $1$        C) $\sqrt{x} - 1$        D) $-1$

12. (02-12-13) Soddalashtiring.

$$\frac{\sqrt{x} + 1}{x\sqrt{x} + x + \sqrt{x}} : \frac{1}{\sqrt{x} - x^2} + x$$

A) $2x$        B) $2$        C) $1$        D) $2x - 1$

13. (03-8-41) Kasrni qisqartiring.

$$\frac{c - 2\sqrt{c} + 1}{\sqrt{c} - 1}$$

A) $\sqrt{c} - 1$    B) $c - 1$    C) $c + 1$    D) $\sqrt{c} + 1$

14. (03-11-5) Soddalashtiring.

$$a \cdot \left(\frac{\sqrt{a} + \sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b \cdot \left(\frac{\sqrt{a} + \sqrt{b}}{2a\sqrt{b}}\right)^{-1}$$

A) $2ab$        B) $ab$        C) $4ab$        D) $0,5ab$

15. (99-1-15) Soddalashtiring.

$$\left(\frac{1}{\sqrt{a} + \sqrt{b}} - \frac{\sqrt{a} + \sqrt{b}}{a - b}\right) \cdot \frac{a - 2\sqrt{a}\sqrt{b} + b}{2\sqrt{b}}$$

A) $\frac{\sqrt{b} - \sqrt{a}}{\sqrt{a} + \sqrt{b}}$        B) $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$

C) $\frac{\sqrt{b} + \sqrt{a}}{\sqrt{a} - \sqrt{b}}$        D) $\frac{\sqrt{a} - \sqrt{b}}{a + b}$

16. (00-1-20) Soddalashtiring.

$$\left(\frac{1}{\sqrt{a + 1} + \sqrt{a}} + \frac{1}{\sqrt{a} - \sqrt{a - 1}}\right)(\sqrt{a + 1} - \sqrt{a - 1})$$

A) $1$        B) $2$        C) $2\sqrt{a}$        D) $2\sqrt{a - 1}$

## 3.2   n-darajali ildiz

$n$ natural son uchun $n-$ darajasi $a$ ga teng bo'lgan $b$ son $a$ sonning $n-$ darajali ildizi deyiladi. $n-$ darajali ildizdan $b$ sonni topish $a$ sondan $n-$ darajali ildiz chiqarish deyiladi. $a$ sondan $n-$ darajali ildiz chiqarish quyidagicha belgilanadi:

$$\sqrt[n]{a} = b. \qquad (3.2)$$

Bu yerda $a$ ildiz ostidagi son, $n-$ ildiz ko'rsatkichi deyiladi. (3.2) tenglik $a$ va $b$ sonlari orasida quyidagi bog'lanish borligini bildiradi: $b^n = a$. Agar $a \geq 0$ ixtiyoriy son va $n > 1$ natural son bo'lsa, $b^n = a$ tenglikni qanoatlantiruvchi manfiy bo'lmagan $b$ son mavjud bo'lib, u yagonadir. Bu $b$ son $a \geq 0$ sonning $n-$ darajali arifmetik ildizi deyiladi. $a$ sonning $n-$ darajali ildizi $\sqrt[n]{a}$ yoki $a^{\frac{1}{n}}$ ko'rinishida yoziladi. Kasr ko'rinishdagi sonlar ratsional sonlar bo'lganligi uchun $a^{\frac{m}{n}}$ ni ratsional ko'psatkichli daraja ham deyiladi. Ko'pincha *ildiz* atamasi o'rniga *radikal* termini ham ishlatiladi. Ildiz ko'rsatkichi $n = 2$ bo'lgan ildiz, kvadrat ildiz deyiladi. $\frac{1}{n} = \frac{k}{kn}$ tenglikdan $\sqrt[n]{a} = a^{\frac{1}{n}} = a^{\frac{k}{kn}} = \sqrt[kn]{a^k}$ tengliklar kelib chiqadi. Bu xossadan foydalanib har xil ildiz ko'rsatkichga ega bo'lgan ildizlarni bir xil ildiz ko'rsatkich orqali ifodalash mumkin. Masalan, $\sqrt[3]{2}$ va $\sqrt[4]{4}$ larni $\sqrt[12]{2^4} = \sqrt[12]{16}$ va $\sqrt[12]{4^3} = \sqrt[12]{64}$ shaklda yozish mumkin.

1. **Agar ildiz ko'rsatkichi $n-$ juft son va $a > 0$ bo'lsa, $a$ son uchun ikkita $n-$ darajali ildiz mavjud bo'ladi. Bu sonlar absolyut qiymatlari teng bo'lgan o'zaro qarama-qarshi sonlardir. Masalan, $\sqrt{16} = \pm 4$, $\sqrt[4]{81} = \pm 3$.**

2. **Agar ildiz ko'rsatkichi $n-$ toq son va $a > 0$ bo'lsa, $a$ ning ildizi yagona va musbat bo'ladi. Masalan, $\sqrt[5]{243} = 3$, $\sqrt[3]{1,728} = 1,2$.**

3. **Agar ildiz ko'rsatkichi $n-$ toq son va $a < 0$ bo'lsa, $a$ ning ildizi yagona va manfiy bo'ladi. Masalan, $\sqrt[3]{-8} = -2$, $\sqrt[5]{-243} = -3$.**

4. **Agar ildiz ko'rsatkichi $n-$ juft son va $a < 0$ bo'lsa, $a$ sonning haqiqiy ildizi mavjud emas, chunki manfiy sonning juft darajasi musbat son bo'ladi.**

5. **Agar ildiz ko'rsatkichi $n-$ natural son va $a = 0$ bo'lsa, $a$ sonning ildizi nolga teng bo'ladi, ya'ni $\sqrt[n]{0} = 0$, chunki $0^n = 0$.**

$n-$ darajali arifmetik ildiz quyidagi xossalarga ega. Bu yerda $a > 0$, $b > 0$ va $n, m \in N$ sonlar.

1. $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.

2. $\sqrt[m]{a \cdot b} = \sqrt[m]{a} \cdot \sqrt[m]{b}$.

3. $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$.

4. $a\sqrt[m]{b} = \sqrt[m]{a^m \cdot b}$.

5. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$.

6. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$.

7. $\sqrt[n]{a} = \sqrt[nm]{a^m}$

8. $\left(\sqrt[n]{a}\right)^n = a, \quad a \geq 0$.

9. $\sqrt[2n+1]{-a} = -\sqrt[2n+1]{a}$.

10. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$.

1. (00-3-17) Soddalashtiring.

$$\frac{a - a\sqrt{a}}{\sqrt[3]{a^2} + \sqrt[6]{a^5} + a} + \frac{\sqrt[3]{a^2} - a}{\sqrt[3]{a} + \sqrt{a}} + 2\sqrt{a}$$

A) $2\sqrt[3]{a}$     B) $2\sqrt{a}$     C) $\sqrt[3]{a} + 2\sqrt{a}$     D) 0

**Yechish:** $\sqrt[6]{a} = x$ deb belgilaymiz. U holda $a = x^6$, $\sqrt[3]{a^2} = x^4$, $\sqrt{a} = x^3$, $\sqrt[3]{a} = x^2$ bo'ladi. U holda berilgan ifoda

$$\frac{x^6 - x^6 \cdot x^3}{x^4 + x^5 + x^6} + \frac{x^4 - x^6}{x^2 + x^3} + 2x^3$$

ko'rinishga keladi. Uni soddalashtiramiz:

$$\frac{x^6(1 - x^3)}{x^4(1 + x + x^2)} + \frac{x^4(1 - x^2)}{x^2(1 + x)} + 2x^3 =$$

$$= \frac{x^2(1 - x)(1 + x + x^2)}{1 + x + x^2} + \frac{x^2(1 - x)(1 + x)}{1 + x} + 2x^3 =$$

$$= x^2(1 - x) + x^2(1 - x) + 2x^3 = x^2 - x^3 +$$

$$+ x^2 - x^3 + 2x^3 = 2x^2 = 2\sqrt[3]{a}.$$

**Javob:** $2\sqrt[3]{a}$     (A).

2. (99-5-5) Soddalashtiring.

$$\frac{27a + 1}{9a^{\frac{2}{3}} - 3\sqrt[3]{a} + 1} - \frac{27a - 1}{9\sqrt[3]{a^2} + 3a^{\frac{1}{3}} + 1}$$

A) $\sqrt[3]{a} - 1$     B) 1     C) 2     D) $a + 1$

3. (00-8-53) Soddalashtiring.

$$\frac{a^{\frac{3}{4}} - 36a^{\frac{1}{4}}}{a^{\frac{1}{2}} - 6a^{\frac{1}{4}}}$$

A) $\sqrt[4]{a} - 6$     B) $\sqrt[4]{a} + 6$     C) $\sqrt{a} - 6$     D) $\sqrt{a} + 6$

4. (00-9-14) Soddalashtiring.

$$\frac{729a + 1}{81\sqrt[3]{a^2} - 9a^{\frac{1}{3}} + 1} - \frac{729a - 1}{81a^{\frac{2}{3}} + 9\sqrt[3]{a} + 1}$$

A) 1     B) 2     C) 3     D) 9

5. (97-9-81) Soddalashtiring.

$$\frac{\sqrt[3]{x^2} + 2\sqrt[3]{x} + 1}{x + 3\sqrt[3]{x^2} + 3\sqrt[3]{x} + 1} - \frac{1}{\sqrt[3]{x} + 1}$$

A) 1     B) $\dfrac{1}{\sqrt[3]{x} + 1}$     C) $\sqrt[3]{x}$     D) 0

6. (98-5-17) Ifodani soddalashtiring,

$$(a^{\frac{1}{2}} - b^{\frac{1}{2}})(a + a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} + b)$$

so'ng $a$ va $b$ lar daraja ko'rsatkichlarining yig'indisini hisoblang.
A) 2        B) 1        C) 4        D) 3

**Yechish:** Agar $a^{\frac{1}{2}} = x$ va $b^{\frac{1}{2}} = y$ desak, u holda $a = x^2$, $b = y^2$ bo'ladi. U holda berilgan ifoda qisqa ko'paytirish formulasining 6-ga ko'ra

$$(x - y)(x^2 + xy + y^2) = x^3 - y^3 = a^{\frac{3}{2}} - b^{\frac{3}{2}}$$

ko'rinishga keladi. Endi $a$ va $b$ larning darajalarini qo'shamiz $\frac{3}{2} + \frac{3}{2} = 3$. **Javob:** 3 (D).

7. (99-7-19) Ifodani soddalashtirib,

$$(a^{\frac{1}{2}} + b^{\frac{1}{2}})(a - a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} + b)$$

$a$ va $b$ asosli darajalar ko'rsatkichlarining yig'indisini toping.
A) 1        B) 4        C) 2        D) 3

8. (98-5-18) Soddalashtiring.

$$\frac{(5b^{\frac{1}{4}} + 10)(b^{\frac{3}{4}} - 2b^{\frac{1}{2}})}{b - 4b^{\frac{1}{2}}}$$

A) $1\frac{1}{4}$        B) $\frac{1}{5}$        C) 1        D) 5

9. (02-10-7) Soddalashtiring.

$$\left(\frac{9}{a+8} - \frac{a^{\frac{1}{3}} + 2}{a^{\frac{2}{3}} - 2a^{\frac{1}{3}} + 4}\right) \cdot \frac{a^{\frac{4}{3}} + 8a^{\frac{1}{3}}}{1 - a^{\frac{2}{3}}} + \frac{5 - a^{\frac{2}{3}}}{1 + a^{\frac{1}{3}}}$$

A) 5        B) $\frac{1}{1-a}$        C) $\frac{2}{1-a^{\frac{2}{3}}}$        D) 4

10. (01-5-5) Soddalashtiring.

$$\frac{a-b}{a+b+2\sqrt{ab}} : \frac{a^{-\frac{1}{2}} - b^{-\frac{1}{2}}}{a^{-\frac{1}{2}} + b^{-\frac{1}{2}}}$$

A) $-1$    B) $a+b$    C) $\frac{1}{\sqrt{a}+\sqrt{b}}$    D) $\frac{ab}{a+b}$

11. (98-9-18) Agar $n = 81$ bo'lsa, $\sqrt[3]{n\sqrt{n}}$ qiymati qanchaga teng bo'ladi?
A) 3        B) 6        C) 9        D) 4

**Yechish:** 4 va 5-xossalardan foydalanib $\sqrt[3]{n\sqrt{n}} = \sqrt[3]{\sqrt{n^2 \cdot n}} = \sqrt[6]{n^3} = \sqrt{n}$ ni olamiz. Endi $n = 81$ desak, $\sqrt{81} = 9$. **Javob:** 9 (C).

12. (02-12-44) Agar $a = 729$ bo'lsa,

$$\frac{a^{\frac{4}{3}} - 8a^{\frac{1}{3}}}{a^{\frac{2}{3}} + 2a^{\frac{1}{3}} + 4} : (\sqrt[3]{a} - 2)$$

ning qiymatini toping.
A) 9        B) 6        C) 12        D) 15

13. (03-4-9) Agar $x = 256$ bo'lsa,

$$\frac{x-1}{x^{\frac{3}{4}} + x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}} + x^{\frac{1}{4}}}{x^{\frac{1}{2}} + 1} \cdot x^{\frac{1}{4}} + 1$$

ning qiymatini toping.
A) 14        B) 15        C) 16        D) 13

14. (03-4-28) $a = 64$ bo'lganda,

$$\frac{a^{\frac{4}{3}} - 8a^{\frac{1}{3}}b}{a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}}} : \left(1 - \frac{2b^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right) - 4a^{\frac{2}{3}}$$

ning qiymatini hisoblang.
A) $-46$        B) $-48$        C) $-44$        D) $-50$

15. (03-8-14) Quyida berilgan ifodaning $x = \sqrt{3} - \sqrt[3]{2}$ bo'lgandagi qiymatini toping.

$$\frac{x^2 - 2x\sqrt{3} - \sqrt[3]{4} + 3}{x - \sqrt{3}}$$

A) $\sqrt{3}$        B) $\sqrt[3]{2}$        C) 1        D) 0

**Yechish:** Dastlab ifodani soddalashtiramiz, keyin $x$ o'rniga berilgan qiymatini qo'yamiz. Berilgan kasr surati $x^2 - 2x\sqrt{3} - \sqrt[3]{4} + 3$ ni $x^2 - 2x\sqrt{3} + (\sqrt{3})^2 - \sqrt[3]{2^2} = (x - \sqrt{3})^2 - (\sqrt[3]{2})^2$ shaklda yozib olamiz. Bu ifodaga ikki son kvadratlarining ayirmasi uchun formulani qo'llab $(x - \sqrt{3} - \sqrt[3]{2})(x - \sqrt{3} + \sqrt[3]{2})$ ni olamiz. Agar $x$ o'rniga $\sqrt{3} - \sqrt[3]{2}$ ni qo'ysak ikkinchi qavs ichi nolga aylanadi. Shuning uchun ko'paytma nol bo'ladi. **Javob:** 0 (D).

16. (99-9-2) Agar $a = 27$ bo'lsa,

$$\left(\frac{a-b}{\sqrt[3]{a} - \sqrt[3]{b}} + \sqrt[3]{ab}\right) : \left(\sqrt[3]{a} + \sqrt[3]{b}\right) +$$

$$+ \left(\sqrt[3]{a^2} - \sqrt[3]{b^2}\right) : \left(\sqrt[3]{a} + \sqrt[3]{b}\right)$$

ning qiymatini hisoblang.
A) 4        B) 4,5        C) 5        D) 6

17. (99-2-11) Agar $a = 8$, $b = 2$ bo'lsa,

$$\frac{a^{\frac{3}{2}} - b^{\frac{3}{2}}}{a^{\frac{1}{2}} - b^{\frac{1}{2}}} - \frac{a^{\frac{3}{2}} + b^{\frac{3}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}}$$

ning qiymati nechaga teng?
A) 10        B) 6        C) 8        D) 12

18. (01-6-32) Ushbu

$$\left(\frac{x^{\frac{3}{2}} - y^{\frac{3}{2}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}} - x - y\right) \cdot x^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$$

ifodani soddalashtiring, keyin $x = 16^{\frac{1}{3}}$ va $y = 4^{\frac{1}{3}}$ bo'lgandagi qiymatini hisoblang.
A) 2        B) 4        C) $2\sqrt[3]{4}$        D) 3

### 3.2.1 Hisoblashga oid misollar

1. (99-8-16) 243 sonini 9 asosli daraja shaklida ifo-
   dalang.
   A) $9^{5/2}$     B) $9^{3/4}$     C) $9^{5/3}$     D) $9^{3/2}$

   **Yechish:** Ma'lumki, $243 = 3^5$ dir. 3 esa $\sqrt{9} = 9^{\frac{1}{2}}$ ga teng. Bulardan $243 = 9^{5/2}$ ekanligi kelib chiqadi. **Javob:** $9^{5/2}$ (A).

2. $25 \cdot 5^{n+2}$ sonini 25 asosli daraja shaklida ifodalang.
   A) $25^{1+n/2}$   B) $25^{2+n/4}$   C) $25^{2+n/2}$   D) $25^{n/2}$

3. $64^{0,5} \cdot 16^{3n+9}$ sonini 8 asosli daraja shaklida ifo-
   dalang.
   A) $8^{12+4n}$   B) $8^{13+4n}$   C) $8^{13+3n}$   D) $8^{4n}$

4. (02-1-1) $\sqrt[3]{2\sqrt{2\sqrt[3]{2}}}$ ni 2 asosli daraja shaklida
   ifodalang.
   A) $2^{5/9}$     B) $2^{4/3}$     C) $2^{2/3}$     D) $2^{3/2}$

5. (99-2-12) $\sqrt{3 \cdot \sqrt[3]{18}} \cdot \sqrt[6]{96}$ ni hisoblang.
   A) 6     B) 18     C) 9     D) 10

6. (98-5-7) Hisoblang.

   $$\frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}}$$

   A) 45     B) 15     C) 5     D) 3

   **Yechish:** 15 ni $3 \cdot 5$ shaklda yozamiz va $a^n = \dfrac{1}{a^{-n}}$
   xossadan foydalansak

   $$\frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}} = 3^{\frac{2}{3}} \cdot 5^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 3^1 \cdot 5^1 = 15.$$

   **Javob:** 15 (B).

7. (99-7-9) $30^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} : 10^{-\frac{2}{3}}$ ni hisoblang.
   A) 15     B) 20     C) 60     D) 30

8. (00-3-6) Hisoblang.

   $$0,027^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} + 256^{\frac{3}{4}} - 3^{-1} + 5,5^0$$

   A) 33     B) 32,97     C) 31     D) 32

9. Hisoblang.

   $$\frac{\sqrt[4]{0,0016}}{\sqrt[5]{0,00032}} - \frac{\sqrt[3]{0,027}}{\sqrt[6]{0,000064}}$$

   A) 1,5     B) −1,5     C) −0,5     D) 0,5

10. $\sqrt[5]{(15^{10} - 10^{10}) : (3^{10} - 2^{10})}$ ni hisoblang.
    A) 3     B) 5     C) 25     D) 9

11. (98-11-59) Hisoblang.

    $$\frac{\sqrt[3]{-24} + \sqrt[3]{81} + \sqrt[3]{192}}{\sqrt[3]{-375}}$$

    A) −1     B) 1     C) $-\dfrac{83}{125}$     D) $\dfrac{83}{125}$

**Yechish:** 4 va 9-xossalardan foydalanib

$$\frac{-\sqrt[3]{2^3 \cdot 3} + \sqrt[3]{3^3 \cdot 3} + \sqrt[3]{4^3 \cdot 3}}{-\sqrt[3]{5^3 \cdot 3}} = \frac{\sqrt[3]{3}(-2 + 3 + 4)}{-5\sqrt[3]{3}}$$

ni olamiz. Bu ifodani soddalashtirsak, uning qiy-
mati $-1$ chiqadi. **Javob:** $-1$ (A).

12. (98-5-2) Hisoblang.

    $$\sqrt[3]{9 + \sqrt{73}} \cdot \sqrt[3]{9 - \sqrt{73}}$$

    A) 2     B) 3     C) 4     D) 1

13. (98-7-18) Hisoblang.

    $$\sqrt{2\sqrt{2} - 1} \cdot \sqrt[4]{9 + 4\sqrt{2}}$$

    A) 7     B) $\sqrt[4]{7}$     C) $2\sqrt{2} + 1$     D) $\sqrt{7}$

14. (98-12-13) Hisoblang.

    $$\left(\sqrt[6]{9 + 4\sqrt{5}} + \sqrt[3]{\sqrt{5} + 2}\right) \cdot \sqrt[3]{\sqrt{5} - 2}$$

    A) 2     B) 1     C) 3     D) 4

15. (00-3-2) Hisoblang.

    $$\sqrt[3]{216 \cdot 512} + \sqrt[5]{32 \cdot 243}$$

    A) 45     B) 48     C) 49     D) 54

16. (00-7-18) Hisoblang.

    $$\frac{\sqrt[4]{3 + 2\sqrt{2}}}{\sqrt{\sqrt{2} + 1}}$$

    A) 2     B) 1,5     C) 0,5     D) 1

17. (00-8-55) Hisoblang.

    $$\sqrt[3]{2 - \sqrt{3}} \cdot \sqrt[6]{7 + 4\sqrt{3}}$$

    A) 1     B) −1     C) 0     D) 7

18. (99-10-3) Hisoblang.

    $$\sqrt[4]{\frac{4,1^3 - 2,15^3}{1,95} + 4,1 \cdot 2,15}$$

    A) 1,5     B) 1,75     C) 2,25     D) 2,5

19. (01-3-22) Hisoblang.

    $$\sqrt{3 + 2\sqrt{2}} \cdot \sqrt[4]{17 - 12\sqrt{2}}$$

    A) 2     B) 1     C) $\sqrt{2}$     D) $2\sqrt{2}$

20. (01-9-7) Hisoblang.

    $$\sqrt{4 - 2\sqrt{2}} \cdot \sqrt[4]{6 + 4\sqrt{2}}$$

    A) 2     B) 1     C) 3     D) 4

**Yechish:** Ikkinchi ildiz $\sqrt[4]{6 + 4\sqrt{2}}$ ni quyidagicha
yozib olamiz $\sqrt{\sqrt{4 + 2 + 2\sqrt{4 \cdot 2}}} = \sqrt{\sqrt{4} + \sqrt{2}}$
Birinchi ildizdan 2 ni qavs oldiga chiqarib, ularni
ko'paytiramiz $\sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} = \sqrt{2(4 - 2)} = 2$. **Javob:** 2 (A).

21. (02-3-6) Hisoblang.

$$\sqrt[4]{68 + 8\sqrt{72}} \cdot \sqrt[8]{4 - \sqrt{15}} \cdot \sqrt[8]{4 + \sqrt{15}} + 1$$

A) $3 + \sqrt{2}$    B) $1 + \sqrt{3}$    C) $\sqrt{2} + \sqrt{3}$    D) $2\sqrt{2}$

22. (02-7-44) Hisoblang.

$$\sqrt[3]{2000 \cdot 1998 - 1997 \cdot 2001 + 5}$$

A) 2    B) 3    C) $\sqrt[3]{17}$    D) 4

23. (03-4-18) Hisoblang.

$$\sqrt[3]{16 + 16\sqrt{2}} \cdot \sqrt[6]{48 - 32\sqrt{2}}$$

A) 2    B) 6    C) 4    D) 8

24. (03-6-46) Hisoblang.

$$\sqrt[3]{1 - \sqrt{3}} \cdot \sqrt[6]{4 + 2\sqrt{3}}$$

A) $-\sqrt{2}$    B) $\sqrt[3]{2}$    C) $-\sqrt[3]{2}$    D) $\sqrt{2}$

25. Hisoblang.

$$\sqrt[3]{4\sqrt{2 \cdot \sqrt[3]{4\sqrt{2 \cdots}}}}$$

A) $\sqrt{6}$    B) $\sqrt[6]{8}$    C) $\sqrt[5]{8}$    D) 2

26. Hisoblang.

$$\sqrt[4]{11 + 2\sqrt{18}} \cdot \sqrt[8]{9 - \sqrt{80}} \cdot \sqrt[8]{9 + \sqrt{80}}$$

A) $\sqrt{3} + 2$    B) $\sqrt{2} + 3$    C) $\sqrt{3} + \sqrt{2}$    D) $\sqrt{2}$

27. Hisoblang.

$$\sqrt[4]{4 - \sqrt{12}} \cdot \sqrt[6]{(1 + \sqrt{3})^5} \cdot \sqrt[3]{\sqrt{3} - 1}$$

A) 4    B) $2^{5/6}$    C) $3^{2/3}$    D) $1 + \sqrt{2}$

28. Hisoblang.

$$\sqrt[4]{0,0016} \cdot \sqrt[3]{0,125}$$

A) $0,4$    B) $0,01$    C) $0,1$    D) 1

29. Hisoblang.

$$\sqrt[6]{5 - 2\sqrt{6}} \cdot \sqrt[3]{5 + \sqrt{24}} \cdot \sqrt[3]{\sqrt{2} - \sqrt{3}}$$

A) 1    B) 2    C) $-1$    D) $-2$

**Yechish:** Birinchi ildiz $\sqrt[6]{5 - 2\sqrt{6}}$ ni 3.2-dagi 5-qoidadan foydalanib, quyidagicha yozib olamiz

$$\sqrt[3]{\sqrt{3 + 2 - 2\sqrt{3 \cdot 2}}} = \sqrt[3]{\sqrt{3} - \sqrt{2}}.$$

3−ildizni 9-qoidadan foydalanib, $-\sqrt[3]{\sqrt{3} - \sqrt{2}}$ shaklda yozib uni birinchi ko'paytuvchi bilan ko'paytirib $-\sqrt[3]{5 - \sqrt{24}}$ ni olamiz. Bu qiymatni $\sqrt[3]{5 + \sqrt{24}}$ bilan ko'paytirib $-\sqrt[3]{25 - 24} = -1$ ni olamiz. **Javob:** $-1$ (C).

30. Hisoblang.

$$\sqrt{7 + \sqrt{1 + \sqrt{7 + \sqrt[3]{8}}}}$$

A) 1    B) 2    C) 3    D) 4

31. Hisoblang.

$$\frac{\sqrt{72} - \sqrt{108}}{\sqrt{3}\sqrt{2}}$$

A) 6    B) 3    C) $-3$    D) $-6$

32. Quyidagi ifoda natural son bo'ladigan $n$ ning qiymatini toping.

$$\sqrt[n]{3^8 + 9^4 + 81^2}$$

A) 4    B) 6    C) 8    D) 9

33. Hisoblang.

$$\left( \sqrt{\frac{3}{14}} - \sqrt{\frac{2}{21}} \right) \cdot \frac{42}{\sqrt{7}}$$

A) 1    B) 2    C) $-1$    D) $-2$

34. (03-8-9) Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{2}{2 + \sqrt[3]{2} + \sqrt[3]{4}}$$

A) $2 - \sqrt[3]{4}$    B) $1 - \sqrt[3]{4}$    C) $1 + \sqrt[3]{4}$    D) $\sqrt[3]{2}$

35. (97-4-3) Eng katta son berilgan javobni toping.
A) $\sqrt{15}$    B) $\sqrt[3]{65}$    C) $\sqrt[4]{81}$    D) 4

**Yechish:** Ma'lumki, $0 < a < b$ bo'lsa, $\sqrt[n]{a} < \sqrt[n]{b}$ bo'ladi. $3 = \sqrt[4]{81} = \sqrt{9} < \sqrt{15} < \sqrt{16} = 4$. Berilgan $\sqrt{15}$; $\sqrt[4]{81}$ va 4 sonlari ichida kattasi 4 soni ekan. $4 = \sqrt[3]{64} < \sqrt[3]{65}$. Demak, eng katta son $\sqrt[3]{65}$ ekan. **Javob:** $\sqrt[3]{65}$ (B).

36. (97-9-63) Eng katta sonni toping.
A) 3    B) $\sqrt[3]{26}$    C) $\sqrt{10}$    D) $\sqrt[4]{82}$

37. (02-5-3) $a = \sqrt{3}$, $b = \sqrt[3]{5}$ va $c = \sqrt[4]{7}$ sonlarni o'sish tartibida joylashtiring.
A) $a < b < c$    B) $c < b < a$
C) $b < a < c$    D) $b < c < a$

38. (02-10-42) Sonlarni o'sish tartibida joylashtiring.

$$m = \left( \frac{4}{7} \right)^{-\frac{2}{3}}, \; n = \left( \frac{49}{16} \right)^{\frac{4}{3}}, \; k = \left( \frac{16}{49} \right)^{-\frac{1}{4}}$$

A) $k < m < n$    B) $m < k < n$
C) $m < n < k$    D) $k < n < m$

39. (02-12-34) Sonlarni o'sish tartibida joylashtiring.

$$a = \sqrt[3]{2}, \quad b = \sqrt[4]{3}, \quad c = \sqrt[6]{5}$$

A) $a < b < c$    B) $c < b < a$
C) $a < c < b$    D) $b < a < c$

## 3.3 O'rta qiymatlar

O'rta qiymatlardan eng ko'p ishlatiladiganlari o'rta arifmetik, o'rta geometrik, o'rta vaznli va o'rta garmonik qiymatlardir. Ularni misollarda tushuntiramiz.

*O'rta arifmetik qiymat.* Berilgan $a_1, a_2, a_3, \ldots, a_n$ sonlarning o'rta arifmetik qiymati deb

$$A = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n} \qquad (3.3)$$

songa aytiladi. Masalan, $10$, $-12$, $20$ sonlarining o'rta arifmetik qiymati $(10 - 12 + 20) : 3 = 18 : 3 = 6$ ga teng.

*O'rta geometrik qiymat.* Berilgan $b_1, b_2, b_3, \ldots, b_n$ sonlarning o'rta geometrik qiymati deb

$$B = \sqrt[n]{b_1 \cdot b_2 \cdot b_3 \cdots b_n} \qquad (3.4)$$

songa aytiladi. Masalan, $4$, $10$, $25$ sonlarining o'rta geometrik qiymati $\sqrt[3]{4 \cdot 10 \cdot 25} = \sqrt[3]{1000} = 10$ ga teng. Berilgan miqdorlarning qiymatlari bir-biriga teng bo'lgan holdan boshqa barcha hollarda o'rta geometrik qiymat o'rta arifmetik qiymatdan kichik bo'ladi. Berilgan sonlar teng bo'lganda o'rta geometrik qiymat o'rta arifmetik qiymatga teng bo'ladi. Xususan $n = 2$ da

$$\frac{a + b}{2} \geq \sqrt{ab}.$$

$\sqrt{ab}$ miqdor berilgan $a$ va $b$ sonlarining o'rta proporsionali ham deyiladi. Ma'lumki, $a : x = x : b$ proporsiyada, proporsiyaning o'rta hadi $x = \sqrt{ab}$ ham $a$ va $b$ sonlarining o'rta proporsionali deyiladi. Masalan, $4$ va $9$ sonlarining o'rta proporsionali $\sqrt{4 \cdot 9} = 6$ ga teng.

*O'rta vaznli qiymat.* Quyidagi masalani qaraymiz. Zargarlik buyumi tayyorlash maqsadida 8 gramm oltin bilan 32 gramm kumush aralashtirildi. Agar 1 gramm oltinning bahosi 2000 so'm, 1 gramm kumushning bahosi 500 so'm bo'lsa, 1 gramm aralashmaning bahosi necha so'm bo'ladi.

*Masalani yechish uchun quyidagilarni topamiz.*
1) Oltinning jami bahosi:     $8 \cdot 2000 = 16000$ so'm.
2) Kumushning jami bahosi: $32 \cdot 500 = 16000$ so'm.
3) Aralashmaning massasi     $8 + 32 = 40$ gramm.
4) 1 gramm aralashmaning bahosi

$$\frac{16000 + 16000}{40} = \frac{32000}{40} = 800.$$

**Javob:** 800 so'm.

Berilgan $a_1, a_2, a_3, \ldots, a_m$ sonlarning o'rta vaznli qiymati deb

$$C = \frac{a_1 \cdot n_1 + a_2 \cdot n_2 + a_3 \cdot n_3 + \cdots + a_m \cdot n_m}{n_1 + n_2 + n_3 + \cdots + n_m} \quad (3.5)$$

songa aytiladi. Agar $n_1 = n_2 = \cdots = n_m$ bo'lsa, o'rta vaznli qiymat o'rta arifmetik qiymatga teng bo'ladi.

*O'rta garmonik qiymat.* Quyidagi masalani qaraylik. $A$ va $B$ shaharlar orasidagi masofa $S$ km. Poyezd $A$ dan $B$ ga $v_1$ km/soat, $B$ dan $A$ ga esa $v_2$ km/soat tezlik bilan yuradi. Borish va kelishdagi yo'lni poyezd o'rtacha necha km/soat tezlik bilan o'tgan.

*Masalani yechish uchun quyidagilarni topamiz.*
1) Poyezd $A$ dan $B$ ga borish uchun $t_1 = S : v_1$ soat,
2) $B$ dan $A$ ga borish uchun $t_2 = S : v_2$ soat sarflagan.
3) Hammasi bo'lib borib-kelish uchun sarflangan vaqt:

$$t_1 + t_2 = \frac{S}{v_1} + \frac{S}{v_2} = \frac{Sv_1 + Sv_2}{v_1 v_2}.$$

4) Bosib o'tilgan yo'lning hammasi $2S$ ga teng bo'lganligi uchun poyezdning o'rtacha tezligi

$$\frac{2S}{\dfrac{Sv_1 + Sv_2}{v_1 v_2}} = 2S \cdot \frac{v_1 v_2}{Sv_1 + Sv_2} = \frac{2v_1 v_2}{v_1 + v_2} \quad km/soat.$$

**Javob:** $\dfrac{2v_1 v_2}{v_1 + v_2}$ km/soat.

Berilgan $a$ va $b$ sonlarning o'rta garmonik qiymati deb

$$D = \frac{2ab}{a + b} \qquad (3.6)$$

songa aytiladi.

*To'g'ri va teskari proporsionallik.* To'g'ri va teskari proporsional bog'lanishlarga to'xtalamiz. $x$ va $y$ miqdorlar o'rtasidagi $y = kx$, $k > 0$ bog'lanishga to'g'ri proporsional bog'lanish, $y = \dfrac{k}{x}$ ga teskari proporsional bog'lanish deyiladi. $y = kx + b$ tenglikdan noma'lum $x$ ni topish uchun, uning ikkala qismidan $b$ soni ayriladi, keyin tenglikning ikkala qismi $k > 0$ ga bo'linadi, natijada $x = \dfrac{y - b}{k}$ tenglik hosil bo'ladi. Quyidagi masalani qaraymiz.

*1-misol.* 300 sonini 3, 5, 7 sonlariga to'gri proporsional (proporsional yoki mutanosib) bo'laklarga bo'ling.

**Yechish:** Masala shartiga ko'ra, 300 soni $3x$, $5x$ va $7x$ qismlarga bo'linadi. Demak, $3x + 5x + 7x = 300$. Bu yerdan $15x = 300$ ni olamiz. Bu tenglikning har ikkala qismini 15 ga bo'lib, $x = 20$ ni topamiz. $x$ o'rniga 20 qo'ysak, $3x = 60$, $5x = 100$ va $7x = 140$ ni olamiz.
**Javob:** 60, 100 va 140.

*2-misol.* 240 sonini 5 va 7 sonlariga teskari proporsional bo'laklarga bo'ling.

**Yechish:** Masala shartiga ko'ra, 240 soni $x/5$ va $x/7$ qismlarga bo'linadi. Demak, $\dfrac{x}{5} + \dfrac{x}{7} = 240$. Bu tenglikning ikkala qismini 35 ga ko'paytirib,

$$7x + 5x = 240 \cdot 35 \Longleftrightarrow 12x = 240 \cdot 35$$

ni olamiz. Bu yerdan $x = 700$ kelib chiqadi. Demak, $700 : 5 = 140$ va $700 : 7 = 100$. **Javob:** 140, 100.

1. Agar $a$, $1,8$ va $-5,6$ sonlarining o'rta arifmetigi $1,2$ ga teng bo'lsa, $a$ ning qiymatini toping.
   A) $7,4$     B) $7$     C) $6,8$     D) $7,(6)$
   **Yechish:** Masala shartiga ko'ra
   $$\frac{a + 1,8 - 5,6}{3} = 1,2 \Longleftrightarrow a - 3,8 = 3,6.$$
   Bu yerdan $a = 7,4$ ni olamiz. **Javob:** $7,4$ (A).

2. $0,32$, $0,28$, $0,4$ va $7$ sonlarining o'rta arifmetik qiymatini toping.
   A) $0,7$     B) $2$     C) $1,8$     D) $2,(6)$

3. $a_1$, $a_2$, $a_3$ sonlarining o'rta arifmetigi 4 ga, $a_4$, $a_5$, $a_6$, $a_7$, $a_8$ sonlarining o'rta arifmetigi esa 5 ga teng bo'lsa, $a_1 + a_2 + a_3 + \cdots + a_8$ ning qiymatini toping.
A) $37,4$    B) $37$    C) $36,8$    D) $37,(6)$

4. Real futbol jamoasidagi 11 ta o'yinchining o'rtacha yoshi 21 ga teng. Bir o'yinchi safdan chiqdi. Qolgan 10 o'yinchining o'rtacha yoshi $20,8$ ga teng. Safdan chiqqan o'yinchining yoshini toping.
A) $22$    B) $23$    C) $19$    D) $18$

5. (96-1-10) $x$; $-2,1$ va $3,3$ sonlarining o'rta arifmetigi $0,2$ ga teng. $x$ ni toping.
A) $0{,}6$    B) $-0,6$    C) $0,8$    D) $2$

6. (96-9-60) $5,4$; $y$; $-2,2$ sonlarining o'rta arifmetigi $1,2$ ga teng. $y$ ni toping.
A) $1,2$    B) $-0,8$    C) $0,4$    D) $-0,4$

7. (98-1-12) Bir son ikkinchi sondan 6 ta ortiq. Ularning o'rta arifmetigi 20 ga teng. Shu sonlardan kattasini toping.
A) $23$    B) $27$    C) $33$    D) $26$

**Yechish:** Bu sonlar kattasini $x$ desak, u holda kichigi $x - 6$ bo'ladi. Masala shartiga ko'ra, ularning o'rta arifmetigi 20, ya'ni

$$\frac{x + x - 6}{2} = 20. \Longleftrightarrow 2x - 6 = 40 \Longleftrightarrow 2x = 46.$$

Bu yerdan $x = 23$ ni olamiz. **Javob:** 23 (A).

8. (98-6-6) Uchta sonning o'rta arifmetigi $17,4$ ga teng. Agar sonlarning ikkitasi $17,5$ va $21,6$ bo'lsa, uchinchi sonni toping.
A) $12,1$    B) $-0,2$    C) $-8,4$    D) $13,1$

9. (98-8-12) Bir son ikkinchisidan 15 ga kichik. Shu sonlarning o'rta arifmetigi $11,5$ ga teng. Shu sonlardan kichigini toping.
A) $3$    B) $3,5$    C) $4$    D) $7$

10. (02-6-11) Uchta sonning o'rta arifmetigi 20 ga, boshqa ikkita sonning o'rta arifmetigi esa 25 ga teng. Shu beshta sonning o'rta arifmetigini toping.
A) $22,5$    B) $22,6$    C) $24$    D) $22$

11. (02-8-6) 7 ta sonning o'rta arifmetigi 13 ga teng. Bu sonlarga qaysi son qo'shilsa, ularning o'rta arifmetigi 18 ga teng bo'ladi?
A) $53$    B) $50$    C) $45$    D) $56$

12. (00-7-5) Uchta sonning o'rta arifmetigi 30 ga, dastlabki ikkitasiniki esa 25 ga teng. uchinchi sonni toping.
A) $44$    B) $40$    C) $45$    D) $38$

13. (03-12-52) Oltita o'quvchining o'rtacha bo'yi 120 sm, shulardan bir o'quvchining bo'yi 105 sm. Qolgan besh o'quvchining o'rtacha bo'yi qanchaga teng?
A) $122$    B) $123$    C) $121$    D) $124$

14. $b_1$, $b_2$ sonlarining o'rta geometrigi 3 ga teng. $b_3$ soni qanday tanlansa $b_1$, $b_2$, $b_3$ sonlarining o'rta geometrigi 4 ga teng bo'ladi?
A) $7$    B) $6,9$    C) $7,(1)$    D) $7,1$

**Yechish:** Masala shartiga ko'ra $\sqrt{b_1 \cdot b_2} = 3$ va $\sqrt[3]{b_1 \cdot b_2 \cdot b_3} = 4$. Birinchi tenglikdan $b_1 \cdot b_2 = 9$ ni olamiz, uni ikkinchi tenglikka qo'yib $\sqrt[3]{9 \cdot b_3} = 4$. Bu yerdan $b_3 = 64 : 9$ ni olamiz. **Javob:** $7,(1)$ (C).

15. $8$, $64$, $0,027$ sonlarining o'rta geometrik qiymatini toping.
A) $2,7$    B) $2$    C) $2,8$    D) $2,4$

16. $2$, $9$, $12$ sonlarining o'rta geometrigini toping.
A) $7$    B) $6$    C) $6,(1)$    D) $7,1$

17. $x$, $25$ va $-5$ sonlarining o'rta geometrik qiymati $-5$ ga teng bo'lsa, $x$ ni toping.
A) $1,4$    B) $1$    C) $1,6$    D) $-1$

18. $b_1$, $b_2$ sonlarining o'rta geometrigi 2 ga, $b_3$, $b_4$ sonlarining o'rta geometrigi esa 3 ga teng bo'lsa, $b_1 \cdot b_2 \cdot b_3 \cdot b_4$ ning qiymatini toping.
A) $37$    B) $49$    C) $36$    D) $32$

19. (03-4-4) 4 va 64 sonlarining o'rta arifmetigi ularning o'rta geometrigidan necha marta katta?
A) $2\frac{1}{4}$    B) $2\frac{3}{4}$    C) $2,2$    D) $2\frac{1}{8}$

**Yechish:** 4 va 64 sonlarining o'rta arifmetigi $(4 + 64) : 2 = 34$, ularning o'rta geometrigi $\sqrt{4 \cdot 64} = 2 \cdot 8 = 16$. Ularning nisbati $34 : 16 = 2\frac{1}{8}$. **Javob:** $2\frac{1}{8}$ (D).

20. Ikki sonning o'rta arifmetigi 10 ga, o'rta geometrigi esa 6 ga teng. Shu sonlarni toping.
A) $2; 8$    B) $2; 18$    C) $5; 15$    D) $6; 14$

21. (98-11-56) Uchta sonning o'rta geometrigi 6 ga teng bo'lib, ulardan ikkitasi 8 va 9 bo'lsa, uchinchi son necha bo'ladi?
A) $3$    B) $7$    C) $-5$    D) $-3$

22. (01-6-4) $a > 0$ sonining va 4 ning o'rta arifmetigi hamda o'rta geometrigi $a$ ning qanday qiymatida o'zaro teng bo'ladi?
A) $3$    B) $7$    C) $5$    D) $4$

23. (01-9-31) Ikki musbat sonning o'rta geometrigi 8 ga, va boshqa ikkita musbat sonning o'rta geometrigi 32 ga teng. Shu to'rtta sonning o'rta geometrigini toping.
A) $12$    B) $16$    C) $15$    D) $14$

24. (01-12-16) Ikki son o'rta geometrigining o'rta arifmetigiga nisbati $3 : 5$ kabi. Shu sonlardan kichigining kattasiga nisbatini toping.
A) $1 : 9$    B) $9 : 25$    C) $3 : 5$    D) $4 : 15$

25. 3 va 48 sonlarining o'rta proporsionalini toping.
A) $7$    B) $12$    C) $11$    D) $16$

26. 180 sonini 3, 5, 7 sonlariga proporsional bo'lak-larga bo'lganda kattasi nechaga teng bo'ladi?
    A) 84    B) 62    C) 48    D) 92

    **Yechish:** Masala shartiga ko'ra, 180 soni $3x$, $5x$ va $7x$ qismlarga bo'linadi. Demak, $3x + 5x + 7x = 180$. Bu yerdan $15x = 180$ ni olamiz. Bu tenglikning har ikkala qismini 15 ga bo'lib, $x = 12$ ni topamiz. Bu sonlarning kattasi $7x$ demak, $7 \cdot 12 = 84$. **Javob:** 84 (A)

27. 120 sonini 5, 7 sonlariga teskari proporsional bo'-laklarga bo'ling.
    A) 60; 84    B) 70; 50    C) 48; 72    D) 56; 64

28. $\dfrac{1}{n-1}$ va $\dfrac{1}{n+1}$ sonlarining o'rta garmonik qiymatini toping.
    A) $\dfrac{1}{n}$    B) $2n$    C) $\dfrac{1}{2n}$    D) $\dfrac{2}{n}$

29. Daryo bo'yidagi $A$ va $B$ pristanlar orasidagi masofa 60 km. Daryo oqimining tezligi 5 km/s, motorli qayiqning tezligi 30 km/s. Motorli qayiq $A$ pristandan $B$ pristanga borib qaytib keldi. Motorli qayiqning bu yo'ldagi o'rtacha tezligini toping.
    A) 30    B) 31    C) 29,1(6)    D) 29,16

30. (98-4-4) Massasi 300 g va konsentrasiyasi 15% bo'lgan eritma massasi 500 g va konsentrasiyasi 9% bo'lgan eritma bilan aralashtirildi. Hosil bo'l-gan aralashmaning konsentrasiyasini toping.
    A) 12,75    B) 11,75    C) 12,25    D) 11,25

    **Yechish:** Berilgan masala o'rta vaznli qiymat haqidagi masala ekan. (3)-formulaga ko'ra

    $$\frac{300 \cdot 15 + 500 \cdot 9}{300 + 500} = \frac{100(3 \cdot 15 + 5 \cdot 9)}{100 \cdot 8} = 11,25.$$

    **Javob:** $11,25\%$ (D).

31. (98-12-65) Massasi 400 g va konsentrasiyasi 8% bo'lgan eritma massasi 600 g va konsentrasiyasi 13% bo'lgan eritma bilan aralashtirildi. Hosil bo'lgan aralashmaning konsentrasiyasini toping.
    A) 11    B) 12    C) 9    D) 10

32. (00-4-29) Yog'liligi 2% bo'lgan 80 l sut bilan yog'li-ligi 5% bo'lgan necha litr sut aralashtirilsa, yog'li-ligi 3% bo'lgan sut olish mumkin?
    A) 20    B) 30    C) 40    D) 50

33. (97-2-3) Qotishma mis va qo'rg'oshindan iborat. Qotishmaning 60% i mis bo'lib, mis qo'rg'oshindan 2 kg ko'p. Qotishmada qancha mis bor?
    A) 5    B) 7    C) 6    D) 5,5

34. (00-4-28) Yig'ilgan 1 t mevaning 82% i suvdan iborat. Ma'lum vaqtdan keyin bu mevadagi suvning miqdori 70% ga tushdi. Endi bu mevaning og'irligi necha kg chiqadi?
    A) 810    B) 820    C) 700    D) 600

35. Bir idishda 40% li, ikkinchi idishda 35% li eritma bor. Ularni aralashtirib, 37% li 1 litr eritma ol-ish uchun har bir eritmadan necha litrdan olish kerak.
    A) 0,3 va 0,7    B) 0,6 va 0,4
    C) 0,2 va 0,8    D) 0,4 va 0,6

36. (00-5-15) 140 g suvga 60 g tuz qo'shish natijasida hosil bo'lgan tuzli eritmada necha protsent tuz bor?
    A) 20    B) 30    C) 25    D) 35

37. (01-11-4) Kumush va misdan iborat qotishma-ning og'irligi 2 kg. Kumushning og'irligi mis og'ir-ligining $\dfrac{1}{7}$ qismini tashkil etadi. Qotishmada necha gramm kumush bor.
    A) 310 g    B) 300 g    C) 270 g    D) 250 g

    **Yechish:** Qotishmadagi kumushning massasini $x$ bilan belgilaymiz. Masala shartiga ko'ra, mis-ning massasi $7x$ bo'ladi va biz $x+7x = 2$ tenglikni olamiz. Bu yerdan $x = 0,25$ kg $= 250$ g. **Javob:** 250 g. (D).

38. (01-12-14) 15 kg eritmaning 35 foizi tuzdan ibo-rat. Tuzning miqdori 25 foiz bo'lishi uchun erit-maga necha kg chuchuk suv qo'shish kerak?
    A) 6    B) 5    C) 5,5    D) 5,25

39. (01-12-35) 800 kg mevaning tarkibida 80% suv bor. Bir necha kundan keyin mevaning og'irligi 500 kg ga tushdi. Endi uning tarkibida necha foiz suv bor?
    A) 62    B) 68    C) 66    D) 60

40. (02-7-52) 20 litr tuzli suvning tarkibida 12% tuz bor. Bu eritmada tuz miqdori 15% bo'lishi uchun necha litr suv bug'lantirilishi kerak?
    A) 4    B) 3    C) 5    D) 4,2

41. (03-8-25) Qotishma kumush va oltindan iborat bo'lib, o'zaro 3 : 5 nisbatda. Agar qotishmada $0,45$ kg oltin bo'lsa, qotishmaning og'irligini (kg) toping.
    A) 0,72    B) 0,21    C) 1,21    D) 0,8

42. (03-5-15) Massasi 36 kg bo'lgan mis va rux qo-tishmasining tarkibida 45% mis bor. Qotishman-ing tarkibida 60% mis bo'lishi uchun unga yana necha kg mis qo'shish kerak?
    A) 13,5    B) 14    C) 12    D) 15

43. (03-8-7) Siment va qumdan iborat 30 kg qorish-maning 60%ini siment tashkil etadi. Qorishman-ing 40%i simentdan iborat bo'lishi uchun qorish-maga qancha qum qo'shish kerak?
    A) 10    B) 12    C) 15    D) 18

44. (03-12-12) $A$ aralashmaning bir kilogrammi 1000 so'm, $B$ aralashmaning bir kilogrammi esa 2000 so'm turadi. $B$ va $A$ aralashmadan 3 : 1 nisbatda tayyorlangan 1 kg aralashma necha so'm turadi?
    A) 1500  B) 1750  C) 1650  D) 1800

# 4 - bob. Tenglamalar

## 4.1 Ayniyat va tenglama

Agar ifodada qatnashuvchi o'zgaruvchilarning barcha qiymatlarida, ifodaning son qiymati nolga teng bo'lsa, bunday ifoda aynan nol ifoda deyiladi. Agar ikki ifodaning ayirmasi aynan nol ifoda bo'lsa, ular aynan teng ifodalar deyiladi. Agar ikkita aynan teng ifodani tenglik belgisi bilan birlashtirsak, ular ayniyat hosil qiladi deyiladi. Masalan, $(x+1)^2 = x^2 + 2x + 1$, $a+b = b+a$. Bir ifodani aynan unga teng bo'lgan boshqa ifoda bilan almashtirish ayniy almashtirish deyiladi. Bir yoki bir nechta noma'lum qatnashgan tenglikka tenglama deyiladi. Tenglamada qatnashgan noma'lumlar va ularning darajalariga qarab tenglama har xil turlarga bo'linadi. Masalan,

$$3x + 15 = 0; \ 2 - 7x = 9; \ ax + b = 0; \qquad (4.1)$$

$$3x - y = 15; \ 0,2z - 7x = 14; \ ax + by = c; \quad (4.2)$$

$$5x^2 - 6x = 11; \ x^2 = 4; \ ax^2 + bx + c = 0. \quad (4.3)$$

(4.1) dagi tenglamalar birinchi darajali bir noma'lumli (chiziqli), (4.2) dagi tenglamalar birinchi darajali ikki noma'lumli, (4.3) dagi ikkinchi darajali bir noma'lumli (kvadrat) tenglamalardir. Tenglamalardagi noma'lumlarning tenglamaning to'g'ri tenglikka aylantiruvchi qiymatlari tenglamaning ildizlari yoki yechimlari deyiladi. Tenglamaning ildizlari bitta, ikkita, uchta va hokazo cheksiz ko'p yoki tenglama umuman ildizga ega bo'lmasligi mumkin. Masalan, $3x + 15 = 0$ tenglama yagona ildizga ega, $2(x + 3) = 2x + 6$ tenglama esa cheksiz ko'p ildizga ega, $x^2 + 1 = 0$ tenglama haqiqiy ildizlarga ega emas. Tenglamani yechish deganda, uning barcha ildizlarini topish yoki ildizlari mavjud emasligini ko'rsatishga aytiladi. Ikki tenglamaning barcha ildizlari mos tushsa yoki ikkalasi ham yechimga ega bo'lmasa, bunday tenglamalar teng kuchli (ekvivalent) tenglamalar deyiladi. Teng kuchli tenglamalar quyidagi xossalarga ega:

1. **Tenglamaning ikkala qismini nolmas songa ko'paytirish yoki bo'lishdan hosil bo'lgan tenglama berilgan tenglamaga teng kuchlidir.**

2. **Tenglamaning ikkala qismiga bir xil ifodani qo'shsak yoki ayirsak, berilgan tenglamaga teng kuchli tenglama hosil bo'ladi.**

2-xossadan quyidagilar kelib chiqadi:

1) Agar tenglamaning har ikki qismida bir xil hadlar bo'lsa, ularni tashlab yuborish mumkin. Masalan, $3x + 1 + 6x^2 = 7 + 6x^2$ tenglama $3x + 1 = 7$ tenglamaga teng kuchlidir.

2) Tenglamaning hadlarini tenglikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o'tkazish mumkin. Masalan, $3x + 1 = 6x$ va $1 = 6x - 3x$ hamda $1 = 3x$ tenglamalar teng kuchlidir.

1. Quyidagilardan qaysi biri ayniyat emas?
   A) $(a + b)^2 = a^2 + 2ab + b^2$
   B) $(a - b)^2 = a^2 - 2ab + b^2$
   C) $a^2 + b^2 = (a - b)(a + b)$
   D) $a^2 - b^2 = (a - b)(a + b)$

   **Yechish:** *1-usul.* C) javobning o'ng tomonidagi qavsni ochamiz $(a-b)(a+b) = a^2 + ab - ba - b^2 = a^2 - b^2$. Bu ifoda tenglikning chap tomoni $a^2 + b^2$ ga teng emas.

   *2-usul.* C) da keltirilgan tenglikning ayniyat emasligini ko'rsatamiz. Buning uchun $a = 1$, $b = 1$ deymiz. Natijada tenglikning chap tomoni $1^2 + 1^2 = 2$, o'ng tomoni esa $(1 - 1)(1 + 1) = 0$ bo'ladi. Demak, C) javobda keltirilgan tenglik ayniyat emas. **Javob:** (C).

2. (97-8-12) Quyidagilardan qaysi biri ayniyat?
   A) $\dfrac{m^3 - n^3}{m + n} = m^2 + mn + n^2$
   B) $2mn - n^2 - m^2 = (m + n)^2$
   C) $m - (m - n) - (m - n) = 2n - m$
   D) $-\dfrac{m - n}{n} = \dfrac{-m - n}{n}$

3. Quyidagilardan qaysi biri ayniyat emas?
   A) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   B) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   C) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
   D) $a^2 - b^2 = (a - b)(a - b)$

4. Quyidagilardan qaysi biri ayniyat emas?
   A) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
   B) $(a - b)^3 = a^3 + 3a^2b + 3ab^2 - b^3$
   C) $-(a - b^2) = b^2 - a$
   D) $x^2 = |x|^2$

5. Quyidagilardan qaysi biri ayniyat emas?
   A) $a^n \cdot a^m = a^{n+m}$ \qquad B) $(ab)^n = a^n \cdot b^n$
   C) $(a^n)^m = a^{n+m}$ \qquad D) $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

6. (98-9-9) Agar $9x^2 + kx + kx^2 - 9x = x^2 - 17x$ ayniyat bo'lsa, $k$ ning qiymati nechaga teng?
   A) $-6$ \quad B) $-8$ \quad C) $-7$ \quad D) $-9$

   **Yechish:** Berilgan ifoda ayniyat bo'lganligi uchun

   $$9x^2 + kx + kx^2 - 9x - x^2 + 17x = (8+k)x^2 + (k+8)x \equiv 0.$$

   Bu yerdan $8 + k = 0$ shart kelib chiqadi. Bu shartdan $k = -8$ ni olamiz. **Javob:** $-8$ (B).

7. (98-1-19) $a$ va $b$ ning qanday qiymatida quyidagi tenglik ayniyat bo'ladi?

   $$\frac{1}{x^2 - 5x - 6} = \frac{a}{x - 6} + \frac{b}{x + 1}$$

   A) $a = 7, \ b = -1$ \qquad B) $a = \dfrac{1}{7}, \ b = -\dfrac{1}{7}$
   C) $a = 1, \ b = 1$ \qquad D) $a = -\dfrac{1}{7}, \ b = \dfrac{1}{7}$

8. (02-3-9) Agar $a(x-1)^2 + b(x-1) + c = 2x^2 - 3x + 5$ ayniyat bo'lsa, $a + b + c$ yig'indi nechaga teng?
   A) 7 \quad B) 8 \quad C) 6 \quad D) 4

9. (00-3-14) $a$ va $b$ ning qanday qiymatlarida

$$\frac{1}{4x^2 - 1} = \frac{a}{2x - 1} - \frac{b}{2x + 1}$$

munosabat ayniyat bo'ladi?

A) $a = -\frac{1}{2}, \ b = \frac{1}{2}$     B) $a = 1, \ b = -1$

C) $a = -1, \ b = 1$     D) $a = \frac{1}{2}, \ b = \frac{1}{2}$

10. (02-10-6) $a, b$ va $c$ ning qanday qiymatida

$$\frac{1}{(x+1)^2 \cdot (x+2)} = \frac{a}{x+1} + \frac{b}{(x+1)^2} + \frac{c}{x+2}$$

tenglik ayniyat bo'ladi?

A) $-1; \ 1; \ 1$     B) $0; \ 1; \ 2$

C) $1; \ -1; \ \frac{1}{2}$     D) $2; \ -2; \ \frac{1}{2}$

11. (96-13-12) Ushbu

$$(-3x + \alpha y)(\beta x - 2y) = \gamma x^2 + 7xy + 2y^2$$

ayniyatdagi noma'lum koeffitsiyentlardan biri $\beta$ ni toping.

A) $1$     B) $-1$     C) $2$     D) $-2$

12. Teng kuchli bo'lmagan tenglamani toping?

A) $2x + 1 = 0$    va    $2x + 2 = 1$
B) $x + y = 5$    va    $x = 5 - y$
C) $x^2 + 7 = 0$    va    $|x| + 1 = 0$
D) $2x - x^2 = 1$    va    $x^2 - 2x - 1 = 0$

**Yechish:** 1-usul. D) javobning o'ng tomonida keltirilgan tenglamaga teng kuchli tenglamani topamiz. Tenglamaning ikkala qismini $-1$ ga ko'paytiramiz va birinchi va ikkinchi hadlarining o'rinlarini almashtirib $2x - x^2 + 1 = 0$ ni olamiz. Endi 1 ni tenglikning o'ng tomoniga qarama-qarshi ishora bilan o'tkazib $2x - x^2 = -1$ ga ega bo'lamiz. Bu esa D) ning chap tomoni bilan bir xil emas.

2-usul. $x = 1$ soni D) ning chap tomonidagi tenglamaning yechimidir. $x = 1$ o'ng tomondagi tenglamaning yechimi emas. Haqiqatan ham, $1^2 - 2 \cdot 1 - 1 = -2 \neq 0$. **Javob:** (D).

13. Teng kuchli tenglamani toping?

A) $2x + 1 = 0$    va    $2x = 1$
B) $x + y = 5$    va    $x = y + 5$
C) $x^2 - 6x = 7$    va    $x^2 - 6x - 7 = 0$
D) $-x^2 = 1$    va    $|x|^2 = 1$

14. Teng kuchli bo'lmagan tenglamani toping?

A) $x^2 + 1 = 0$    va    $x^2 + 5 = 1$
B) $x + 3 = 5$    va    $x(x + 3) = 5x$
C) $5x^2 - 6x = 7x$    va    $5x^2 - 13x = 0$
D) $-x^2 = -4$    va    $|x|^2 = 4$

15. Teng kuchli tenglamani toping?

A) $x(y^2 + 1) = 0$    va    $x(y + 1)(y - 1) = 0$
B) $(x + 7)(x + y) = 5(x + 7)$    va    $x + y = 5$
C) $7x - 6 = 6x + 7$    va    $x - 13 = 0$
D) $x^2 y^2 = xy$    va    $xy(xy + 1) = 0$

## 4.2 Chiziqli tenglamalar

Chiziqli yoki birinchi darajali bir noma'lumli tenglama deb

$$ax + b = 0 \qquad (4.4)$$

shakldagi yoki ayniy almashtirishlardan keyin (4.4) ko'rinishga keltirish mumkin bo'lgan tenglamalarga aytiladi.

1. **Agar $a \neq 0$ bo'lsa, $ax + b = 0$ tenglama yagona $x = -\dfrac{b}{a}$ yechimga ega.**

2. **Agar $a = 0$, $b = 0$ bo'lsa, $ax + b = 0$ tenglama cheksiz ko'p yechimga ega, ya'ni ixtiyoriy haqiqiy son bu tenglamaning yechimi bo'ladi.**

3. **Agar $a = 0$, $b \neq 0$ bo'lsa, $ax + b = 0$ tenglama yechimga ega emas.**

1. (97-1-6) Tenglamani yeching.

$$\frac{3x - 11}{4} - \frac{3 - 5x}{8} = \frac{x + 6}{2}$$

A) $5$    B) $-4, 5$    C) $6, 5$    D) $7$

**Yechish:** Tenglamaning ikkala qismini 8 ga ko'paytiramiz:

$$6x - 22 - 3 + 5x = 4x + 24.$$

$x$ qatnashgan hadlarni tenglamaning chap qismiga, sonlarni o'ng qismiga o'tkazamiz:

$$6x + 5x - 4x = 24 + 22 + 3 \quad \Longleftrightarrow \quad 7x = 49.$$

Bu tenglikning har ikkala qismini 7 ga bo'lib $x = 7$ ni olamiz. **Javob:** 7 (D).

2. (97-6-6) Tenglamani yeching.

$$6 - \frac{x - 1}{2} = \frac{3 - x}{2} + \frac{x - 2}{3}$$

A) $4, 5$    B) $8$    C) $17$    D) $11$

3. (97-7-3) Tenglamani yeching.

$$0, 9(4x - 2) = 0, 5(3x - 4) + 4, 4$$

A) $1, 2$    B) $2, 5$    C) $-3$    D) $2$

4. (98-1-4) Tenglamani yeching.

$$2, 8x - 3(2x - 1) = 2, 8 - 3, 19x$$

A) $-20$    B) $20$    C) $-2$    D) $200$

5. (98-8-4) Tenglamani yeching.

$$5, 6 - 7(0, 8x + 1) = 14 - 5, 32x$$

A) $5, 5$    B) $55$    C) $-55$    D) $-5, 5$

6. (98-7-1) $x$ ni toping.

$$420 : (160 - 1000 : x) = 12$$

A) $8$    B) $\frac{1}{8}$    C) $35$    D) $36$

7. (98-12-1) $x$ ni toping.

$$(360 + x) \cdot 1002 = 731460$$

A) 370     B) 270     C) 470     D) 730

8. (97-11-6) Tenglamani yeching.

$$\frac{x-3}{6} + x = \frac{2x-1}{3} - \frac{4-x}{2}$$

A) 3     B) 2     C) −2     D) ∅

**Yechish:** Tenglamani undagi kasrlarning umumiy maxraji bo'lgan 6 ga ko'paytiramiz:

$$x - 3 + 6x = 2(2x - 1) - 3(4 - x).$$

Qavslarni ochamiz:

$$x - 3 + 6x = 4x - 2 - 12 + 3x.$$

$x$ qatnashgan hadlarni tenglamaning chap qismiga, sonlarni esa o'ng qismiga o'tkazamiz:

$$7x - 7x = -14 + 3, \qquad 0 = -11.$$

Noto'g'ri sonli tenglik hosil bo'ldi. Bu esa berilgan tenglama ildizga ega emasligini bildiradi. **Javob:** ∅ (D).

9. Tenglamani yeching.

$$1,6 \cdot (2 + 3x) = 6 \cdot (0,8x - 1) + 6,8$$

A) 5     B) −0,5     C) ∅     D) −2

10. (99-4-12) Tenglamani yeching.

$$0,(3)x - 3 = x - 2(0,5 + 0,(3)x)$$

A) 20     B) ∅     C) 0,2     D) 0,5

11. (96-1-20) $m$ ning qanday qiymatlarida $my + 1 = m$ tenglama yechimga ega bo'lmaydi?
A) $m = 1$   B) $m = 0$   C) $m = -1$   D) $m = 2$

12. (97-10-22) $n$ ning qanday qiymatlarida $nx + 5 = n - 2x$ tenglamaning ildizi mavjud emas?
A) 5     B) −2     C) 1     D) −5

13. (99-7-21) $a$ ning

$$(a^2 - 4)x + 5 = 0$$

tenglama yechimga ega bo'lmaydigan barcha qiymatlari ko'paytmasini toping.
A) 4     B) −4     C) 0     D) 2

14. (99-8-21) Tenglama $a$ ning qanday qiymatida yechimga ega emas?

$$6x - a - 6 = (a + 2)(x + 2)$$

A) 4     B) 2     C) −2     D) 6

15. (00-3-11) $k$ ning qanday qiymatida

$$k(k + 6)x = k + 7(x + 1)$$

tenglama yechimga ega bo'lmaydi?
A) 1 va 7     B) 1     C) 7     D) 1 va −7

16. (02-7-7) $a$ ning qanday qiymatida

$$\frac{3x - a}{5} = \frac{ax - 4}{3}$$

tenglama ildizga ega emas?
A) 1, 8     B) 2     C) 2, 2     D) 1

17. (02-11-9) Tenglamani yeching.

$$\frac{2x + 3}{2} + \frac{2 - 3x}{3} = 2,1(6)$$

A) ∅     B) 2     C) −2     D) $x \in R$

**Yechish:** Tenglamaning ikkala qismini 6 ga ko'paytiramiz:

$$6x + 9 + 4 - 6x = 6 \cdot 2,1(6).$$

$2,1(6)$ davriy kasrni oddiy kasrga aylantirsak, u $2\frac{16-1}{90} = 2\frac{1}{6} = \frac{13}{6}$ ko'rinish oladi. Natijada $(6 - 6)x = 13 - 13$ tenglikka ega bo'lamiz. Bu tenglik barcha $x \in R$ lar uchun o'rinli. **Javob:** $x \in R$ (D).

18. (03-8-11) Tenglamani yeching.

$$\frac{6x + 2}{4} + \frac{2x + 3}{2} - 2,5x + 2 = 4$$

A) ∅     B) $x \in R$     C) 10     D) −10

19. Tenglamani yeching.

$$\frac{x + 2}{3} + \frac{7x - 1}{2} = 3,8(3)x + 0,1(6)$$

A) ∅     B) $x \in R$     C) 10     D) −10

20. (98-1-20) $m$ ning qanday qiymatlarida

$$m(mx - 1) = 9x + 3$$

tenglama cheksiz ko'p ildizga ega?
A) $m = 0$         B) $m = 3$
C) $m = -3$        D) $m = -1$

**Yechish:** Qavslarni ochamiz.

$$m^2 x - m = 9x + 3$$

Uni $(m^2 - 9)x - (m + 3) = 0$ ko'rinishga keltiramiz. Bu tenglama cheksiz ko'p yechimga ega bo'lishi uchun (2-qoidaga qarang)
$$\begin{cases} m^2 - 9 = 0 \\ m + 3 = 0 \end{cases}$$ bo'lishi kerak. Demak, $m = -3$.
**Javob:** −3 (C).

21. (96-7-22) $a$ ning qanday qiymatlarida $ax - a = x - 1$ tenglama cheksiz ko'p yechimga ega bo'ladi?
A) $a = 1$     B) $a = 2$     C) $a = -1$     D) $a \in R$

22. (96-10-21) $n$ ning qanday qiymatlarida $nx + 1 = n+x$ tenglama cheksiz ko'p yechimga ega bo'ladi?
A) $n = 0$      B) $n = 1$      C) $n = 2$      D) $n \neq 1$

23. (97-7-22) $m$ ning qanday qiymatlarida $m^2x - m = x + 1$ tenglama ildizlari cheksiz ko'p bo'ladi?
A) $m = 1$   B) $m = 0$   C) $m = -1$   D) $m = \pm 1$

24. (98-12-28) Tenglama $a$ ning qanday qiymatlarida cheksiz ko'p yechimga ega?

$$10(ax - 1) = 2a - 5x - 9$$

A) $-\dfrac{1}{2}$      B) $2$      C) $\dfrac{1}{2}$      D) $-2$

25. (01-1-10) $a$ ning qanday qiymatida

$$(a^2 + 2)x = a(x - a) + 2$$

tenglamaning ildizlari cheksiz ko'p bo'ladi?
A) $-\sqrt{2}$   B) $\sqrt{2}$   C) $\sqrt{2}; -\sqrt{2}$   D) $\emptyset$

26. (03-7-44) Tenglamani yeching.

$$\frac{3 + 25x}{3x + 7} = 5$$

A) $-3, 2$      B) $1, 5$      C) $-1\dfrac{1}{5}$      D) $3, 2$

27. (03-7-48) Tenglamani yeching.

$$\left(1,7 : \left(1\frac{2}{3} \cdot x - 3,75\right)\right) : \frac{8}{25} = 1\frac{5}{12}$$

A) $5, 2$      B) $5\dfrac{3}{4}$      C) $4$      D) $4, 5$

28. (99-8-11) Tenglamani yeching.

$$\frac{(x - 12) : \frac{3}{8}}{0,3 \cdot 3\frac{1}{3} + 7} = 1$$

A) $25$      B) $14$      C) $15$      D) $16$

29. (01-8-4) Tenglamani yeching.

$$\left(4\frac{3}{8}x + 5\frac{1}{16}\right) \cdot \frac{4}{15} = \frac{5}{12}x + 2\frac{2}{5}$$

A) $\dfrac{1}{15}$      B) $1\dfrac{2}{5}$      C) $\dfrac{3}{185}$      D) $2\dfrac{1}{5}$

30. (02-7-43) $986^2 - 319^2 = 2001 \cdot n$ bo'lsa, $n$ ning qiymatini toping.
A) $435$      B) $443$      C) $515$      D) $475$

31. (03-11-57) Tenglamani yeching.

$$12\left(1\frac{3}{4}x + \frac{5}{8}\right) = -6\frac{1}{2}$$

A) $-\dfrac{1}{3}$      B) $-\dfrac{2}{3}$      C) $\dfrac{2}{3}$      D) $-\dfrac{13}{21}$

32. (02-7-6) $m$ ning qanday qiymatida

$$\frac{6x - m}{2} = \frac{7mx + 1}{3}$$

tenglamaning ildizi nolga teng bo'ladi?
A) $-\dfrac{2}{3}$      B) $\dfrac{4}{5}$      C) $-\dfrac{3}{2}$      D) $\dfrac{1}{2}$

### 4.2.1 Proporsiya

Proporsiya xossalari 1.2.5-bandda keltirilgan.

1. (96-9-75) Proporsiyaning noma'lum hadini toping.

$$3\frac{3}{5} : 2\frac{7}{10} = 3\frac{3}{4} : x$$

A) $2\dfrac{13}{16}$      B) $2\dfrac{3}{10}$      C) $3\dfrac{1}{3}$      D) $1\dfrac{15}{16}$

**Yechish:** Proporsiya o'rta hadlarining ko'paytmasi uning chetki hadlari ko'paytmasiga tengligidan

$$2\frac{7}{10} \cdot 3\frac{3}{4} = x \cdot 3\frac{3}{5} \iff \frac{27}{10} \cdot \frac{15}{4} : \frac{18}{5} = x$$

ni olamiz. Ko'paytirish va bo'lishni bajarib $x = 2\dfrac{13}{16}$ ekanligini olamiz. **Javob:** $2\dfrac{13}{16}$ (A).

2. (00-5-10) Tenglamani yeching.

$$1\frac{1}{12}x : 2\frac{1}{12} = 2\frac{3}{5}$$

A) $5$      B) $3$      C) $1\dfrac{5}{12}$      D) $4$

3. (98-12-12) Tenglamani yeching.

$$(12,5 - x) : 5 = (3,6 + x) : 6$$

A) $5\dfrac{2}{11}$      B) $5\dfrac{3}{11}$      C) $5\dfrac{4}{11}$      D) $5\dfrac{1}{11}$

4. (96-7-12) Proporsiyaning noma'lum hadini toping.

$$6,9 : 4,6 = x : 5,4$$

A) $7, 1$      B) $7, 7$      C) $8, 1$      D) $8, 4$

5. (97-3-12) Proporsiyaning noma'lum hadini toping.

$$3,5 : x = 0,8 : 2,4$$

A) $10, 5$      B) $9, 2$      C) $13, 5$      D) $7, 8$

6. (97-7-12) Proporsiyaning noma'lum hadini toping.

$$5,4 : 2,4 = x : 1,6$$

A) $3, 6$      B) $4$      C) $2, 8$      D) $4, 6$

7. (97-10-12) Proporsiyaning noma'lum hadini toping.

$$0,25 : 1,4 = 0,75 : x$$

A) $3, 6$      B) $2, 4$      C) $4, 2$      D) $5, 2$

8. (98-7-13) Tenglamani yeching.

$$\left(\frac{1}{3} + x\right) : 7 = \left(\frac{3}{4} + x\right) : 9$$

A) $1\dfrac{3}{8}$      B) $1\dfrac{1}{8}$      C) $1\dfrac{5}{8}$      D) $1\dfrac{7}{8}$

9. (03-11-55) Proporsiyaning noma'lum hadini toping.

$$12\frac{1}{2} : 2\frac{1}{2} = 16\frac{2}{3} : y$$

A) $3\dfrac{1}{3}$      B) $3\dfrac{2}{3}$      C) $3\dfrac{1}{6}$      D) $3\dfrac{5}{6}$

## 4.3 Kvadrat tenglamalar

$$ax^2 + bx + c = 0 \qquad (4.5)$$

ko'rinishdagi tenglamaga yoki ayniy almashtirishlardan keyin (4.5) ko'rinishga keltirish mumkin bo'lgan tenglamaga bir noma'lumli ikkinchi darajali tenglama yoki kvadrat tenglama deyiladi. Bunda $x-$ noma'lum, $a\,(a \neq 0), b$ va $c-$ lar ixtiyoriy sonlar. (4.5) tenglamaga kvadrat tenglamaning normal shakli deyiladi. $a$ ni birinchi koeffitsiyent, $b$ ni ikkinchi koeffitsiyent, $c$ ni esa uchinchi koeffitsiyent yoki ozod had deyiladi. Kvadrat tenglamaning ildizlari quyidagi formula orqali topiladi

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (4.6)$$

Bunda $b^2 - 4ac$ ga kvadrat tenglamaning diskriminanti deyiladi va u $D$ harfi bilan belgilanadi: $D = b^2 - 4ac$.

Agar (4.5) tenglamaning birinchi koeffitsiyenti, ya'ni $a = 1$ bo'lsa bunday tenglama keltirilgan kvadrat tenglama deyiladi. Keltirilgan kvadrat tenglama odatda

$$x^2 + px + q = 0 \qquad (4.7)$$

ko'rinishda yoziladi.

Agar (4.5) tenglamada $b = 0$ yoki $c = 0$ yoki $b = c = 0$ bo'lsa, ular chala kvadrat tenglamalar deyiladi. Masalan, $ax^2 + bx = 0$ chala kvadrat tenglamaning ildizlari $x_1 = 0$, $x_2 = -b/a$ lardir. $ax^2 + c = 0$ chala kvadrat tenglama $ac < 0$ shartda yechimga ega va uning ildizlari $x_{1,2} = \pm\sqrt{\frac{-c}{a}}$, ko'rinishga ega. $ax^2 = 0$ chala kvadrat tenglamaning ildizi esa $x_1 = x_2 = 0$ dir.

1. **Viyet teoremasi. Agar** $x_1, x_2$ **sonlar (4.7) tenglamaning ildizlari bo'lsa, u holda** 
$$\begin{cases} x_1 + x_2 = -p \\ x_1 \cdot x_2 = q \end{cases}$$ **tengliklar o'rinli.**

2. **Agar** $D > 0$ **bo'lsa, u holda (4.5) kvadrat tenglama ikkita har xil haqiqiy ildizga ega.**

3. **Agar** $D = 0$ **bo'lsa, u holda (4.5) kvadrat tenglama bitta haqiqiy ildizga ega.**

4. **Agar** $D < 0$ **bo'lsa, u holda (4.5) kvadrat tenglama haqiqiy ildizlarga ega emas.**

5. $ax^2 + bx + c$ **kvadrat uchhad**

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

**ko'paytuvchilarga ajraydi. Bunda** $x_1$, $x_2$ **sonlar (4.5) kvadrat tenglamaning ildizlari.**

Keltirilgan (4.7) kvadrat tenglamaning ildizlari quyidagi xossalarga ega.

6. $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = p^2 - 2q$.

7. $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1 x_2 = p^2 - 4q$.

8. $x_1^3 + x_2^3 = 3pq - p^3$.

9. $x_1^4 + x_2^4 = p^4 - 4p^2 q + 2q^2$.

1. Kvadrat tenglamani yeching.

$$x^2 + 5x - 6 = 0$$

A) $-6; 1$    B) $-1; 6$    C) $1; 6$    D) $2; 3$

**Yechish:** Berilgan kvadrat tenglamada $a = 1$, $b = 5$, $c = -6$. Endi kvadrat tenglama yechimga egami yoki yo'qmi shuni aniqlaymiz. Buning uchun uning diskriminantini hisoblaymiz: $D = 5^2 - 4 \cdot 1 \cdot (-6) = 25 + 24 = 49$. Kvadrat tenglama diskriminanti musbat, shuning uchun u ikkita ildizga ega. Ularni (4.6) formula yordamida hisoblaymiz:

$$x_1 = \frac{-5 - \sqrt{49}}{2 \cdot 1} = \frac{-5 - 7}{2} = -6,$$

$$x_2 = \frac{-5 + \sqrt{49}}{2 \cdot 1} = \frac{-5 + 7}{2} = 1.$$

**Javob:** $-6; 1$ (A).

2. Kvadrat tenglamani yeching.

$$2x^2 + 3x - 14 = 0$$

A) $-7; 2$    B) $-2; 3\frac{1}{2}$    C) $-3\frac{1}{2}; 2$    D) $2; 3\frac{1}{2}$

3. Kvadrat tenglamani yeching.

$$4x^2 + 12x + 9 = 0$$

A) $-7; 2$    B) $-1,5$    C) $-3$    D) $\emptyset$

4. Kvadrat tenglamani yeching.

$$9x^2 + 6x + 3 = 0$$

A) $-1; \frac{1}{3}$    B) $-2; \frac{1}{6}$    C) $-3$    D) $\emptyset$

5. (00-8-64) Tenglamani yeching.

$$1998x^2 - 2000x + 2 = 0$$

A) $1; \frac{2}{1998}$    B) $-1; \frac{2}{1998}$
C) $1; -\frac{2}{1998}$    D) $-1; -\frac{2}{1998}$

6. Tenglamani yeching.

$$x^2 - 97x + 2010 = 0$$

A) $30; 67$    B) $-30; -67$    C) $15; 134$    D) $2; 1005$

7. Chala kvadrat tenglamani yeching.

$$2x^2 - 6x = 0$$

A) $0; 3$    B) $-2; 6$    C) $3$    D) $\emptyset$

**Yechish:** Berilgan kvadrat tenglamada $a = 2$, $b = -6$, $c = 0$. Bu tenglama diskriminantini $D = (-6)^2 - 4 \cdot 2 \cdot 0 = 36 > 0$. Kvadrat tenglama ikkita ildizga ega. (4.6) formuladan $x_1 = 0$, $x_2 = 3$.
**Javob:** $0; 3$ (A).

8. Chala kvadrat tenglamani yeching.

$$x^2 - 7x = 0$$

A) 0; 3     B) 7     C) 0; 7     D) 0

9. Chala kvadrat tenglamani yeching.

$$9x^2 - 1 = 0$$

A) 0; $\frac{1}{3}$     B) $-\frac{1}{3}$; $\frac{1}{3}$     C) 0; $-\frac{1}{3}$     D) $\emptyset$

10. Chala kvadrat tenglamani yeching.

$$3x^2 + \frac{1}{3} = 0$$

A) 0; $\frac{1}{3}$     B) $-\frac{1}{3}$; 1     C) 0; $-\frac{1}{3}$     D) $\emptyset$

11. (96-1-18) Tenglamaning nechta ildizi bor?

$$3 - x = -\frac{4}{x}$$

A) 1     B) 2     C) 3     D) ildizi yo'q

**Yechish:** $x \neq 0$ deb, tenglikning ikkala qismini $x$ ga ko'paytiramiz va $3x - x^2 + 4 = 0$ ni olamiz. Bu kvadrat tenglamaning diskriminanti $D = 3^2 - 4 \cdot (-1) \cdot 4 = 25 > 0$. Shuning uchun u ikkita ildizga ega. **Javob:** 2 (B).

12. (96-9-69) Tenglamaning nechta ildizi bor?

$$\frac{2}{x} = x + 2$$

A) 3     B) 2     C) 1     D) ildizi yo'q

13. Tenglamaning nechta ildizi bor?

$$x^2 - 6x + 9 = 0$$

A) 1     B) 2     C) 3     D) ildizi yo'q

14. Tenglamaning nechta haqiqiy ildizi bor?

$$4x^2 + 8x + 7 = 0$$

A) 1     B) 2     C) 3     D) ildizi yo'q

15. (96-7-13) Agar

$$(3x - 1) \cdot (x - 2) = 0$$

bo'lsa, $3x - 1$ qanday qiymatlarni qabul qilishi mumkin?

A) faqat $\frac{1}{3}$     B) faqat 0

C) $\frac{1}{3}$ yoki 0     D) 0 yoki 5

**Yechish:** $(3x-1) \cdot (x-2) = 0$ dan $x_1 = \frac{1}{3}$, $x_2 = 2$ ekanligi kelib chiqadi. Bu qiymatlarni $3x - 1$ ga qo'yib $3x_1 - 1 = 0$ va $3x_2 - 1 = 5$ ni olamiz. **Javob:** 0 yoki 5 (D).

16. (97-3-13) Agar

$$(2x + 1) \cdot (x - 1, 5) = 0$$

bo'lsa, $2x + 1$ qanday qiymatlar qabul qiladi?

A) faqat 0     B) faqat $-\frac{1}{2}$

C) 0 yoki $-\frac{1}{2}$     D) 4 yoki 0

17. (97-7-13) Agar $(x - 5) \cdot (\frac{1}{5}x + 4) = 0$ bo'lsa, $\frac{1}{5}x + 4$ qanday qiymatlar qabul qiladi?

A) faqat 0     B) faqat $-20$

C) 0 yoki 5     D) 0 yoki 8

18. (97-10-13) Agar $(4x + 1) \cdot (x - \frac{1}{4}) = 0$ bo'lsa, $4x + 1$ qanday qiymatlar qabul qilishi mumkin?

A) faqat $-\frac{1}{4}$     B) faqat $\frac{1}{4}$

C) faqat 0     D) 0 yoki 2

19. (96-3-18) Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$$x^2 - x - 2$$

A) $(x - 1)(x + 2)$     B) $(x - 1)(x - 2)$
C) $(x + 1)(x + 2)$     D) $(x + 1)(x - 2)$

**Yechish:** Berilgan kvadrat uchhadni chiziqli ko'-paytuvchilarga ajratish uchun 5-xossadan foydala-namiz. Shu maqsadda kvadrat tenglamaning ildiz-larini topamiz. Uning diskriminanti $D = (-1)^2 - 4 \cdot 1 \cdot (-2) = 1 + 8 = 9$ ga teng. $D > 0$ bo'lganligi uchun tenglamaning ikkita ildizi bor. Ularni (4.6) formula yordamida topamiz

$$x_1 = \frac{1 - 3}{2} = -1, \quad x_2 = \frac{1 + 3}{2} = 2.$$

5-xossadan $x^2 - x - 2 = (x + 1)(x - 2)$ ni olamiz. **Javob:** $(x + 1)(x - 2)$ (D).

20. (96-11-19) Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$$x^2 - 3x + 2$$

A) $(x - 1)(x + 2)$     B) $(x - 2)(x + 1)$
C) $(x - 1)(x - 2)$     D) $(x + 1)(x + 2)$

21. (97-2-27) Kasrni qisqartiring.

$$\frac{x^2 - 16}{x^2 - 5x + 4}$$

A) $\frac{4 + x}{1 - x}$     B) $\frac{4 - x}{x + 1}$     C) $\frac{x + 4}{x + 1}$     D) $\frac{x + 4}{x - 1}$

22. (97-8-26) Kasrni qisqartiring.

$$\frac{y^2 - 3y - 4}{y^2 - 1}$$

A) $\frac{y + 4}{y + 1}$     B) $\frac{4 - y}{y - 1}$     C) $\frac{y + 4}{y - 1}$     D) $\frac{y - 4}{y - 1}$

23. (00-8-37) Ko'paytuvchilarga ajrating.

$$3x^2 - 6xm - 9m^2$$

A) $3(x + m)(x - 3m)$  B) $(x - 3m)^2$
C) $3(x - m)(x + 3m)$  D) $(3x - m)^2$

24. (00-3-18) Agar

$$x^2 - 3x - 6 = 0$$

tenglamaning ildizlari $x_1$ va $x_2$ bo'lsa, $\dfrac{1}{x_1^3} + \dfrac{1}{x_2^3}$ ni toping.

A) $\dfrac{1}{3}$  B) $0,5$  C) $-0,5$  D) $-0,375$

**Yechish:** Berilgan tenglama uchun $p = -3$, $q = -6$. Qiymati izlanayotgan ifodani

$$\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{x_1^3 + x_2^3}{(x_1 x_2)^3}$$

shaklda yozib olamiz. Viyet teoremasi va 8-xossaga ko'ra bu kasrning qiymati

$$\frac{x_1^3 + x_2^3}{(x_1 x_2)^3} = \frac{3 \cdot (-3) \cdot (-6) - (-3)^3}{(-6)^3} = -\frac{81}{216}$$

ga teng. Buni o'nli kasrga aylantirib $-0,375$ ni olamiz. **Javob:** $-0,375$ (D).

25. (96-13-18) $x_1$ va $x_2$ sonlar

$$x^2 + x - 5 = 0$$

tenglamaning ildizlari ekanligi ma'lum. $x_1^2 + x_2^2$ ning qiymatini toping.
A) 10  B) 12  C) 11  D) 9

26. (97-4-24) $a$ va $b$ sonlari

$$3x^2 - 2x - 6 = 0$$

tenglamaning ildizlari bo'lsa, $a^2 + b^2$ ni hisoblang.
A) 6  B) 8  C) $4\dfrac{4}{9}$  D) $4\dfrac{2}{9}$

27. (98-4-25) Agar

$$x^2 + x - 1 = 0$$

tenglamaning ildizlari $x_1$ va $x_2$ bo'lsa, $x_1^3 + x_2^3$ ning qiymati qanchaga teng bo'ladi?
A) 1  B) 3  C) 2  D) $-4$

28. (98-5-21) Ushbu

$$x^2 + 4x - 5 = 0$$

tenglamaning ildizlari $x_1$ va $x_2$ bo'lsa, $x_1^3 \cdot x_2^3$ ni hisoblang.
A) 124  B) $-125$  C) 130  D) 5

29. (99-7-23) Agar

$$x^2 + 2x + 1 = 0$$

tenglamaning ildizlari $x_1$ va $x_2$ bo'lsa, $x_1^3 - x_2^3$ ni hisoblang.
A) 1  B) 3  C) 4  D) 0

30. (01-10-2) Agar $x_1$ va $x_2$ lar $x^2 + x - 5 = 0$ tenglamaning ildizlari bo'lsa, $x_1^2 x_2^4 + x_1^4 x_2^2$ ning qiymatini hisoblang.
A) 225  B) 145  C) 125  D) 275

31. (02-11-13) Agar $x_1$ va $x_2$

$$9x^2 + 3x - 1 = 0$$

tenglamaning ildizlari bo'lsa, $\dfrac{3x_1 x_2}{x_1 + x_2}$ ning qiymatini toping.

A) $-1$  B) 1  C) 2  D) $\dfrac{1}{3}$

32. (03-1-5) Agar $x_1$ va $x_2$

$$x^2 + 3x - 3 = 0$$

tenglamaning ildizlari bo'lsa, $x_1^4 + x_2^4$ ning qiymatini hisoblang.
A) 207  B) 192  C) 243  D) 168

33. (03-8-19) Tenglamaning katta va kichik ildizlari kublarining ayirmasini toping.

$$x^2 - \frac{\sqrt{85}}{4}x + 1\frac{5}{16} = 0$$

A) $-2$  B) $-1$  C) 2  D) 1

34. (01-2-23) Tenglama ildizlarining o'rta proporsionalini toping.

$$x^2 - 13x + 36 = 0$$

A) 4  B) 9  C) $6,5$  D) 6

**Yechish:** $a$ va $b$ sonlarining o'rta proporsionali deganda $\sqrt{a\,b}$ qiymat tushuniladi. Berilgan tenglama diskriminanti $D = (-13)^2 - 4 \cdot 36 = 25 > 0$. Demak, bu tenglamaning ikkita ildizi bor. Viyet teoremasiga ko'ra, $\sqrt{x_1 \cdot x_2} = \sqrt{36} = 6$. **Javob:** 6 (D).

35. (00-1-12) Tenglama ildizlarining o'rta proporsionalini toping.

$$2x^2 - 26x + 72 = 0$$

A) 4  B) 5  C) 7  D) 6

36. (99-10-5) Tenglama ildizlarining o'rta arifmetigi ularning ko'paytmasidan qancha kam?

$$\frac{x^2 + 16}{x} = 10$$

A) 13  B) 12  C) 14  D) 11

37. (02-11-14) Ildizlaridan biri $3 + \dfrac{\sqrt{2}}{2}$ ga teng bo'lgan ratsional koeffitsiyentli kvadrat tenglama tuzing.
A) $x^2 - 3x + 9 = 0$  B) $x^2 - 6x + 17 = 0$
C) $x^2 - 12x + 9 = 0$  D) $2x^2 - 12x + 17 = 0$

38. (02-1-49) 3 va $-2$ sonlari qaysi tenglamaning ildizlari ekanligini ko'rsating.
A) $x^2 - x = 6$  B) $x^2 + x = 6$
C) $x^2 + 6 = x$  D) $x^2 + 6 = -x$

### 4.3.1 Parametrli kvadrat tenglamalar

Kvadrat tenglamalarni qo'shimcha xossalarini keltiramiz.

1. **(1) kvadrat tenglama $D := b^2 - 4ac = 0$ yoki $a = 0$ da bitta ildizga ega.**

2. **(1) kvadrat tenglama $D > 0$ va $ac > 0$ da bir xil ishorali ikkita ildizga ega.**

3. **(1) kvadrat tenglama $D > 0$ va $ac < 0$ da turli ishorali ikkita ildizga ega.**

4. **Agar (1) kvadrat tenglamaning ildizlaridan biri nol bo'lsa, u holda $c = 0$ va aksincha.**

5. **$ax^2 + bx + c$ kvadrat uchhad $a > 0$ va $D = 0$ bo'lganda to'la kvadrat bo'ladi.**

1. (97-2-24) Tenlama ildizlaridan biri 0 bo'ladigan $m$ ning barcha qiymatlari ko'paytmasini toping.

$$x^2 - 9x + (m^2 - 4)(m^2 - 9) = 0$$

A) 36    B) $4\sqrt{3}$    C) $-6$    D) 6

**Yechish:** *1-usul.* 4-qoidaga ko'ra tenlamaning ildizi 0 bo'lishi uchun, uning ozod hadi $c = 0$ bo'lishi zarur va yetarli. Bu shart $(m^2 - 4)(m^2 - 9) = 0$ ga teng kuchli. Bu yerdan $m_{1,2} = \pm 2$ va $m_{3,4} = \pm 3$ ekanligini olamiz. Ularning ko'paytmasi $m_1 \cdot m_2 \cdot m_3 \cdot m_4 = 36$ ga teng.

*2-usul.* Nol berilgan tenglamaning ildizi. Tenglamada $x = 0$ deb $(m^2 - 4)(m^2 - 9) = 0$ ni olamiz. Bu tenglamaning ildizlari $\pm 2$ va $\pm 3$ lardir. Ularning ko'paytmasi esa 36 ga teng. **Javob:** 36 (A).

2. (00-10-21) $p$ ning qanday qiymatida

$$x^2 + px + 15 = 0$$

tenglamaning ildizlaridan biri 5 ga teng bo'ladi?
A) $-4$    B) 4    C) $-2$    D) $-8$

3. (01-6-13) $a$ ning qanday qiymatida

$$x^2 - (a - 1)x + 36 = 0$$

tenglamaning ildizlaridan biri 4 ga teng bo'ladi?
A) 13    B) 14    C) 11    D) 10

4. (98-10-43) Tenglama ildizlaridan biri 2 ga teng. Ikkinchi ildizni toping.

$$2x^2 + x - a = 0$$

A) $2,5$    B) $-2,5$    C) $1,5$    D) $-1,5$

5. (97-12-24) Ushbu

$$x^2 + px - 12 = 0$$

tenglamaning ildizlaridan biri 2 ga teng. $p : (-12)$ nimaga teng?
A) $\dfrac{1}{3}$    B) $-\dfrac{5}{12}$    C) $\dfrac{2}{3}$    D) $-\dfrac{1}{3}$

6. (00-8-31) $b$ ning qanday qiymatida

$$x^2 + \frac{2}{3}x + b$$

uchhad to'la kvadrat bo'ladi?
A) $\dfrac{1}{9}$    B) $\dfrac{1}{3}$    C) $\dfrac{2}{9}$    D) $\dfrac{2}{3}$

**Yechish:** Berilgan kvadrat uchhadda    $a = 1$. 5-qoidaga ko'ra u to'la kvadrat bo'lishi uchun $D = (\frac{2}{3})^2 - 4b = 0$ bo'lishi zarur va yetarli. Bu shartdan $b = \dfrac{1}{9}$ ni olamiz. **Javob:** $\dfrac{1}{9}$ (A).

7. (00-8-34) $k$ ning qanday qiymatlarida

$$x^2 + 2(k - 9)x + k^2 + 3k + 4$$

ifodani to'la kvadrat shaklida tasvirlab bo'ladi?
A) $\dfrac{11}{3}$    B) 3    C) 4    D) $\dfrac{5}{7}$

8. (03-3-14) $m$ ning qanday qiymatlarida

$$(m - 1)x^2 + 2mx + 3m - 2$$

kvadrat uchhadni to'la kvadrat shaklida tasvirlash mumkin?
A) $2; \dfrac{1}{2}$    B) $-2$    C) 2    D) $\dfrac{1}{2}$

9. (98-7-35) Ushbu

$$x^2 + px + 6 = 0$$

tenglama ildizlari ayirmasining kvadrati 40 ga teng bo'lsa, ildizlarining yig'indisi qancha bo'lishini toping?
A) $\sqrt{40}$    B) 8    C) $-8$    D) $-8$ va 8

10. (98-12-32) $m$ ning qanday qiymatlarida

$$3x^2 + (3m - 15)x - 27 = 0$$

tenglamaning ildizlari qarama-qarshi sonlar bo'ladi?
A) 5    B) 0    C) $-3; 3$    D) $-5$

11. (98-12-33) Ushbu

$$x^2 + px + 6 = 0$$

tenglama ildizlari ayirmasining kvadrati 40 ga teng. $p$ ning qiymatini toping.
A) $-8; 8$    B) 8    C) $-8$    D) $4 + \sqrt{10}$

12. (99-1-18) $x^2 + px - 35 = 0$ tenglamaning ildizlaridan biri 7 ga teng. Ikkinchi ildiz va $p$ ning qiymatini toping.
A) $-5; -2$    B) $-5; 2$    C) $5; 2$    D) $5; -2$

**Yechish:** $x = 7$ tenglamaning ildizi bo'lganligi uchun $7^2 + 7p - 35 = 0$ bo'ladi. Bu yerdan $p = -2$ ekanligi kelib chiqadi. $x_1 x_2 = 7x_2 = -35$ dan $x_2 = -5$ ni olamiz. **Javob:** $-5; -2$ (A).

13. (01-11-9) Ushbu

$$x^2 + 2ax + a = 0$$

tenglamaning ildizlaridan biri 1 ga teng. Tenglamaning ikkinchi ildizini toping.

A) $-\dfrac{4}{3}$     B) $-\dfrac{1}{2}$     C) $\dfrac{1}{3}$     D) $-\dfrac{1}{3}$

14. (01-1-9) $k$ ning qanday qiymatida

$$kx^2 + 12x - 3 = 0$$

tenglamaning ildizlaridan biri 0,2 ga teng bo'ladi.

A) 135     B) 60     C) $-135$     D) 15

15. (01-12-39) $k$ ning qanday qiymatlarida

$$(k - 2)x^2 + 7x - 2k^2 = 0$$

tenglama $x = 2$ yechimga ega?

A) $1;3$    B) $1;-3$    C) $-1;3$    D) $-2;3$

16. (02-5-17) Tenglamaning ildizlaridan biri 2 ga teng bo'ladigan $a$ ning barcha qiymatlarini toping.

$$x^2 - 4x - (a - 1)(a - 5) = 0$$

A) $(-\infty; -2) \cup (2; +\infty)$     B) $(-\infty; \infty)$
C) $(-\infty; -4) \cup (4; +\infty)$     D) $\{3\}$

17. (00-7-12) Ildizlari $x^2 + px + q = 0$ tenglamaning ildizlariga teskari bo'lgan tenglamani ko'rsating.

A) $px^2 + qx + 1 = 0$     B) $qx^2 + px - 1 = 0$
C) $qx^2 + px + 1 = 0$     D) $qx^2 - px + 1 = 0$

18. (00-7-47) $m$ ning qanday qiymatlarida

$$x^2 - 4mx + 48 = 0$$

tenglamaning ildizlaridan biri boshqasidan 3 marta katta bo'ladi?

A) 2     B) $\pm 4$     C) $\pm 3$     D) 4

**Yechish:** Masala shartidan $x_2 = 3x_1$ ni olamiz. Viyet teotemasiga ko'ra $x_1 + x_2 = 4m \Longleftrightarrow x_1 + 3x_1 = 4m$. Bu yerdan $x_1 = m$ kelib chiqadi. Yana Viyet teotemasidan foydalansak,

$$x_1 x_2 = 48 \Longleftrightarrow x_1 \cdot 3x_1 = 3m^2 = 48 \Longleftrightarrow m^2 = 16.$$

Demak, $m = \pm 4$. **Javob:** $\pm 4$ (B).

19. (00-4-9) Ushbu

$$x^2 - 5x + a = 0$$

tenglamaning ildizlaridan biri ikkinchisidan 9 marta katta bo'lsa $a$ ning qiymatini toping.

A) $2,5$     B) $2,4$     C) $2,25$     D) $3,5$

20. (02-11-15) $q$ ning qanday qiymatida

$$x^2 - 8x + q = 0$$

tenglamaning ildizlaridan biri boshqasidan uch marta katta bo'ladi?

A) 6     B) 8     C) 12     D) 16

21. (03-7-78) $m$ ning qanday qiymatlarida

$$4x^2 - (3 + 2m)x + 2 = 0$$

tenglamaning ildizlaridan biri ikkinchisidan sakkiz marta kichik bo'ladi?

A) 3     B) $-6$     C) $-6; 3$     D) $3; 5$

22. (00-8-9) $x_1$ va $x_2$ sonlari

$$3x^2 + 2x + b = 0$$

tenglamaning ildizlari bo'lib, $2x_1 = -3x_2$ ekanligi ma'lum bo'lsa, $b$ ning qiymatini toping.

A) $-8$     B) 6     C) 4     D) $-3$

23. (01-1-15) $a$ ning qanday qiymatida

$$5(a + 4)x^2 - 10x + a = 0$$

tenglamaning ildizlari turli ishorali bo'ladi?

A) $(-1; 5)$     B) $(-4; 0)$
C) $(-5; 1)$     D) $(-5; -4) \cup (0; 1)$

24. (01-2-62) $a$ ning qanday qiymatlarida

$$x^2 + ax + 12 = 0$$

tenglamaning ildizlari orasidagi masofa 1 ga teng bo'ladi?

A) $\pm 5$     B) $\pm 6$     C) $\pm 7$     D) $\pm 8$

**Yechish:** Masala shartiga ko'ra $|x_1 - x_2| = 1$ bo'ladi. Bu tenglikning ikkala qismini kvadratga ko'taramiz va $|x|^2 = x^2$ ayniyatdan foydalanib

$$x_1^2 - 2x_1 x_2 + x_2^2 = (x_1 + x_2)^2 - 4x_1 x_2 = p^2 - 4q = 1^2$$

ni olamiz. Berilgan tenglamada $p = a$, $q = 12$. Bularni yuqorida hosil qilingan tenglikka qo'yib $a^2 - 4 \cdot 12 = 1 \Longleftrightarrow a^2 = 49$ ni hosil qilamiz. Demak, $a = \pm 7$ ekan. **Javob:** $\pm 7$ (C).

25. (01-5-20) $a$ ning qanday musbat qiymatida

$$8x^2 - 30x + a^3 = 0$$

tenglamaning ildizlaridan biri ikkinchisining kvadratiga teng bo'ladi?

A) 3     B) 1     C) 2     D) 4

26. (02-1-50) $a$ ning qanday qiymatlarida

$$ax^2 - 2x + 3 = 0$$

tenglama bitta ildizga ega bo'ladi?

A) $\dfrac{1}{3}$    B) 0 va 1    C) 3 va $1,5$    D) $\dfrac{1}{3}$ va 0

27. (02-7-2) Tenglamani yeching.

$$x^2 - 3ax + 2a^2 - ab - b^2 = 0$$

A) $a - b; 2a + b$     B) $-a + b; -2a + b$
C) $-a - b; 2a - b$     D) $a + b; 2a + b$

28. (02-7-4) $n$ ning qanday qiymatlarida

$$x^2 - 12x + n = 0$$

tenglama ildizlaridan biri ikkinchisidan $2\sqrt{5}$ ga ortiq bo'ladi?
A) 31     B) 30     C) 3     D) 29

29. (02-8-21) $a$ ning qanday qiymatlarida

$$x^2 + (a+2)x + a = 0$$

tenglama ildizlari kvadratlarining yig'indisi 3 ga teng bo'ladi?
A) $-1$     B) 1     C) $-2$     D) 3

30. (97-2-25) Ushbu $x^2 - 6x + q = 0$ tenglamaning ildizlaridan biri 2 ga teng. Bu tenglamaning barcha koeffitsiyentlari yig'indisini toping.
A) 2     B) $-6$     C) 3     D) $-5$

**Yechish:** $x^2 - 6x + q = 0$ tenglamada $x = 2$ deb $4 - 12 + q = 0$ ni olamiz. Bu yerdan $q = 8$ bo'ladi. U holda koeffitsiyentlar yig'indisi $1 + (-6) + 8 = 3$ ga teng. **Javob:** 3 (C).

31. (02-9-13) Tenglama yechimga ega bo'lmaydigan $k$ ning butun qiymatlari o'rta arifmetigini toping.

$$kx^2 + 3kx + 2k - 1 = 0$$

A) $-3$     B) $-2$     C) $-1,5$     D) 3

32. (96-3-77) $x_1$ va $x_2$ lar

$$x^2 + |a|x + 6 = 0$$

tenglamaning ildizlari bo'lib, $x_1^2 + x_2^2 = 13$ tenglikni qanoatlantirsa, $x_1 + x_2$ nechaga teng?
A) 5     B) $-6$     C) 6     D) $-5$

33. (01-8-22) $a$ ning nechta qiymatida

$$\frac{3x-a}{3-x} + \frac{x+a}{x+1} = 2$$

tenglama bitta yechimga ega?
A) 4     B) 3     C) 2     D) 1

34. (99-4-19) $a$ ning qanday qiymatlarida

$$ax^2 - (a+1)x + 2a - 1 = 0$$

tenglama bitta ildizga ega bo'ladi?
A) $-1; \dfrac{1}{7}$     B) $0; -1$
C) $1; -\dfrac{1}{7};$     D) $1; 0; -\dfrac{1}{7}$

35. (03-1-58) Tenglamaning ildizlari bir-biriga teng bo'ladigan $k$ ning barcha qiymatlari ko'paytmasini toping.
$$9x^2 + kx = 2x - k + 6$$

A) 100     B) $-120$     C) 220     D) $-196$

36. (03-3-11) Agar

$$x^2 - 3x + m = 0$$

tenglamaning $x_1$ va $x_2$ ildizlari uchun $3x_1 - 2x_2 = 14$ munosabat o'rinli bo'lsa, $m$ ning qiymatini toping.
A) $-4$     B) 4     C) 6     D) $-6$

37. (03-3-12) $p$ ning qanday qiymatida

$$x^2 - px + 5 = 0$$

tenglamaning ildizlaridan biri boshqasidan 4 ga katta?
A) 6     B) 4     C) $-4$     D) $\pm 6$

38. (03-3-25) $a$ ning qanday qiymatlarida

$$x + 4 = \frac{a}{x}$$

tenglama ikkita turli haqiqiy ildizga ega?
A) $(-4; \infty)$     B) $(-4; 0) \cup (0; \infty)$
C) $[-4; \infty)$     D) $[-4; 0) \cup (0; \infty)$

39. (03-4-12) $a$ ning qanday qiymatlarida

$$x^2 + 3x + a + 0,75 = 0$$

tenglamaning ikkala ildizi ham manfiy bo'ladi?
A) $0,5 < a < 2$     B) $-0,75 < a < 1,5$
C) $0,6 < a < 1,8$     D) $0,8 < a < 1,2$

40. (03-5-16) $a$ ning qanday qiymatida

$$x^2 - (a-2)x - a - 1 = 0$$

tenglama ildizlari kvadratlarining yig'indisi eng kichik qiymatga ega bo'ladi?
A) 1     B) 2     C) $\dfrac{1}{2}$     D) 4

**Yechish:** Masala shartiga ko'ra $x_1^2 + x_2^2$ ifodaning qiymati eng kichik bo'ladigan $a$ ni topishimiz kerak. 4.3-dagi 6-tenglikka ko'ra $x_1^2 + x_2^2 = p^2 - 2q = (a-2)^2 - 2(-a-1) = a^2 - 2a + 6 = (a-1)^2 + 5$. Haqiqiy sonning kvadrati manfiymasligidan $x_1^2 + x_2^2 = (a-1)^2 + 5 \geq 5$. Bu ifodaning qiymati minimal bo'lishi uchun $a - 1 = 0$, ya'ni $a = 1$ bo'lishi kerak. **Javob:** 1 (A).

41. (99-2-16) $x_1$ va $x_2$

$$x^2 - px + p - 1 = 0$$

tenglamaning ildizlari, $p$ ning qanday qiymatida $x_1^2 + x_2^2$ yig'indi eng kichik qiymatni qabul qiladi?
A) 2     B) $-2$     C) 1     D) $-1$

42. (00-1-13) $y_1$ va $y_2$

$$y^2 - by + b - 1 = 0$$

tenglamaning ildizlari bo'lsa, $b$ ning qanday qiymatida $y_1^2 + y_2^2$ ifodaning qiymati eng kichik bo'ladi?
A) $1,2$     B) $0,85$     C) 1     D) $1,5$

43. (01-7-16) $m$ ning qanday qiymatida
$$x^2 + (m-1)x + m^2 - 1,5 = 0$$
tenglama ildizlari kvadratlarining yig'indisi eng katta bo'ladi?
A) $1,5$    B) $-1,5$    C) $1$    D) $-1$

44. (01-8-16) $m$ ning qanday qiymatida
$$x^2 + (2-m)x - m - 3 = 0$$
tenglama ildizlari kvadratlarining yig'indisi eng kichik bo'ladi?
A) $2$    B) $1$    C) $-1$    D) $-3$

45. (01-12-25) $a$ ning qanday qiymatida
$$x^2 + (a+2)x + a = 3$$
tenglama ildizlari kvadratlarining yig'indisi eng kichik bo'ladi?
A) $0$    B) $-1$    C) $1$    D) $3$

46. (03-7-62) $q$ ning qanday qiymatida
$$x^2 - 8x + q = 0$$
tenglama ildizlari kvadratlarining yig'indisi 34 ga teng bo'ladi?
A) $15$    B) $-12$    C) $12$    D) $-15$
**Yechish:** Masala shartiga ko'ra $x_1^2 + x_2^2 = 34$. 4.3-dagi 6-tenglikka ko'ra $p^2 - 2q = 34 \iff (-8)^2 - 2q = 34 \iff q = 15$. **Javob:** 15 (A).

47. (03-10-14) $q$ ning qanday qiymatida
$$x^2 - x - q = 0$$
tenglama ildizlari kublarining yig'indisi 19 ga teng bo'ladi?
A) $6$    B) $5$    C) $7$    D) $4$

48. (03-11-1) $a$ parametrning qanday butun qiymatida
$$2x^2 + 6ax + a = 0$$
tenglama ildizlari kvadratlarining yig'indisi 38 ga teng bo'ladi?
A) $-2$    B) $2$    C) $-3$    D) $-1$

49. (03-11-6) Agar $m$ va $n$ sonlar
$$x^2 + 3mx - 5n = 0 \quad (m \cdot n \neq 0)$$
tenglamaning ildizlari bo'lsa, $n - m$ ning qiymati nechaga teng bo'ladi?
A) $25$    B) $24$    C) $18$    D) $12$

50. (03-5-29) $x_1$ va $x_2$ sonlar
$$x^2 + 3x + k = 0$$
tenglamaning ildizlari va $\dfrac{x_1}{x_2} = -\dfrac{2}{5}$ bo'lsa, $k$ ning qiymatini toping.
A) $-10$    B) $-7$    C) $-12$    D) $-8$

51. (03-9-4) $m$ ning qanday qiymatida
$$3x^2 - 21x + m = 0$$
tenglama ildizlari kvadratlarining yig'indisi 25 ga teng bo'ladi?
A) $36$    B) $-36$    C) $24$    D) $42$

## 4.4   Ratsional tenglamalar

Agar $P(x)$ ifodada $x-$ o'zgaruvchining 3 va undan yuqori darajalari qatnashsa, $P(x) = 0$ tenglama yuqori tartibli tenglama deyiladi. Bunday tenglamani yechishda asosan ikki usul qo'llaniladi. Bular "yangi o'zgaruvchi kiritish" va "ko'paytuvchilarga ajratish" usullaridir.

$$ax^4 + bx^2 + c = 0, \ a \neq 0 \qquad (4.8)$$

ko'rinishdagi tenglamaga bikvadrat tenglama deyiladi. Bikvadrat tenglamada $x^2 = y \geq 0$ deb yangi o'zgaruvchi kiritsak, (4.8) tenglama

$$ay^2 + by + c = 0 \qquad (4.9)$$

ko'rinishdagi kvadrat tenglamaga keladi. Agar (4.9) tenglama yechimga ega bo'lmasa, u holda (4.8) tenglama ham yechimga ega emas. Agar (4.9) tenglama $y_1$ va $y_2$ yechimga ega bo'lib, ular manfiymas bo'lsa, u holda $x_{1,2} = \pm\sqrt{y_1}$ va $x_{3,4} = \pm\sqrt{y_2}$ lar (4.8) tenglamaning yechimlari bo'ladi. Bikvadrat tenglamaning barcha yechimlari yig'indisi doim nolga teng.

*2-usul*, ko'paytuvchilarga ajratishga doir quyidagi misolni qaraymiz:

$$x^4 - 4x^3 - x^2 + 4x = (x^2 - 1)(x^2 - 4x) = 0. \quad (4.10)$$

Bu tenglama yechimga ega bo'lishi uchun ko'paytuvchilarning kamida bittasi biror $x$ da nolga teng bo'lishi kerak, yani

$$x^2 - 1 = 0, \qquad (4.11)$$
$$x^2 - 4x = 0 \qquad (4.12)$$

tenglamalardan kamida bittasining yechimi mavjud bo'lishi kerak. Demak, (4.11) yoki (4.12) tenglamalardan ixtiyoriy birining yechimi (4.10) tenglamaning ildizi bo'ladi. (4.11) tenglama $x_1 = -1, x_2 = 1$ ildizlarga, (4.12) tenglama esa $x_3 = 0, x_4 = 4$ ildizlarga ega. Demak, (4.10) tenglama $x_1 = -1, x_2 = 1, x_3 = 0, x_4 = 4$ ildizlarga ega. Agar bizga $P(x) : Q(x) = 0$ ko'pinishdagi ratsional tenglama berilgan bo'lsa, uning ildizlari $P(x) = 0$ tenglama ildizlaridan $Q(x) = 0$ tenglama ildizlarini chiqarib tashlashdan hosil bo'ladi. Quyidagi misolni qaraymiz:

$$\frac{x^2 + 5x + 6}{x^2 + x - 2} = 0.$$

Ma'lumki, kasrning qiymati nol bo'lishi uchun uning surati nolga teng bo'lishi kerak. Shuning uchun

$$x^2 + 5x + 6 = 0$$

tenglamani yechamiz. Bu tenglamaning ildizlari $x_1 = -2, x_2 = -3$. Topilgan bu qiymatlarni kasr maxrajiga qo'yamiz: $(-2)^2 + (-2) - 2 = 4 - 2 - 2 = 0$; $(-3)^2 + (-3) - 2 = 9 - 3 - 2 = 4 \neq 0$. Shunday qilib, berilgan tenglamaning ildizi $x = -3$ ekan. $x = -2$ esa tenglamaning ildizi bo'lmaydi. Chunki bu qiymatda kasrning maxraji nolga aylandi.

1. (00-3-26) Tenglamaning haqiqiy ildizlari yig'indisini toping.
$$(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$$

A) 3    B) $-3$    C) 2    D) $-5$

**Yechish:** Tenglama $y = x^2 + 5x$ belgilash yordamida $(y + 4)(y + 6) = 120$ tenglamaga keladi. Qavslarni ochamiz

$$y^2 + 10y + 24 - 120 = 0 \iff y^2 + 10y - 96 = 0.$$

Bu kvadrat tenglamaning yechimlari $y_1 = 6$, $y_2 = -16$ lardir. Endi berilgan tenglama ikkita tenglamaaga ajraydi.

1) $x^2 + 5x = 6$, $x^2 + 5x - 6 = 0$, $x_1 = -6$, $x_2 = 1$,

2) $x^2 + 5x + 16 = 0$,     $D = 25 - 64 = -39 < 0$.

2) tenglama yechimga ega emas. Demak, berilgan tenglama $x_1 = -6$ va $x_2 = 1$ ildizlarga ega. Ildizlar yig'indisi $x_1 + x_2 = -5$. **Javob:** $-5$ (D).

2. (96-7-15) Tenglama ildizlari yig'indisini toping.
$$x^4 - 13x^2 + 36 = 0$$
A) 13    B) 5    C) 0    D) 36

3. (97-7-15) Tenglamaning eng katta va eng kichik ildizlari ayirmasini toping.
$$x^4 - 10x^2 + 9 = 0$$
A) 1    B) 8    C) 2    D) 6

4. (98-4-33) Tenglamaning ildizlari yig'indisini toping.
$$2x^4 - 7x^2 + 2 = 0$$
A) 7    B) $3,5$    C) 0    D) 2

5. (98-6-20) Tenglama ildizlari ko'paytmasini toping.
$$(x + \frac{1}{x})^2 - 2(x + \frac{1}{x}) - 3 = 0$$
A) 3    B) $-1$    C) 4    D) 1

6. (98-11-10) Tenglamaning haqiqiy ildizlari ko'paytmasini aniqlang.
$$y^4 - 2y^2 - 8 = 0$$
A) 4    B) $-16$    C) 16    D) $-4$

7. (00-1-16) Ifoda nechta ratsional koeffitsiyentli ko'paytuvchilarga ajraladi?
$$(x^4 + x^2 + 1) \cdot (x^4 + x^2 + 2) - 12$$
A) 4    B) 2    C) 3    D) 5

8. (98-6-22) Tenglamani yeching.
$$\frac{2x^2 - 5x + 3}{(10x - 5)(x - 1)} = 0$$
A) 1    B) $1; \frac{3}{2}$    C) $\frac{3}{2}$    D) 5

**Yechish:** Berilgan tenglamada kasr suratini nolga tenglashtiramiz. $2x^2 - 5x + 3 = 0$ kvadrat tenglama ildizlari $x_1 = 1$, $x_2 = 1,5$ lardir. Berilgan tenglamadagi kasr maxraji $x = 1$ da nolga aylanadi, $x = 1,5$ da esa noldan farqli. Demak, $x = 1,5$ tenglamaning ildizi bo'ladi. **Javob:** $\frac{3}{2}$ (C).

9. (98-11-18) Tenglamaning yechimlari quyidagi oraliqlarning qaysi birida joylashgan?
$$\frac{x^2 + 1}{x} + \frac{x}{x^2 + 1} = -2,5$$
A) $(-\infty; -1)$    B) $[-1; 8)$    C) $[2; 8)$    D) $[3; 8)$

10. (98-11-71) Tenglamani yeching.
$$\frac{1 - \frac{1}{x-1}}{1 + \frac{1}{x-1}} = 0$$
A) $-2$    B) 0    C) $-1$    D) 2

11. (98-12-63) Tenglama ildizlari yig'indisini toping.
$$\frac{2}{3 - x} + \frac{1}{2} = \frac{6}{x(3 - x)}$$
A) 4    B) 7    C) 3    D) 10

12. (00-5-36) Tenglamaning ildizlari nechta?
$$\frac{x^2 - x - 2}{x^2 + x} = 0$$
A) 2    B) 4    C) 1    D) 3

13. (01-1-8) Tenglamani yeching.
$$\frac{2}{x - 3} = \frac{x + 5}{x^2 - 9}$$
A) $-2$    B) 2    C) 1    D) $-1$

14. (02-3-25) Tenglama ildizlari ko'paytmasini toping.
$$\frac{26}{5(x + x^{-1})} = 1$$
A) 1    B) 5    C) 2    D) $2, 4$

15. (02-4-4) Tenglamaning ildizlari sonini toping.
$$x^4 - (\sqrt{5} + \sqrt{3})x^2 + \sqrt{15} = 0$$
A) 2    B) 4    C) 1    D) 0

16. (02-7-41) Tenglama ildizlari yig'indisini toping.
$$(x + 1)(x + 2)(x + 4)(x + 5) = 40$$
A) $-6$    B) 0    C) $-5$    D) 6

**Yechish:** Berilgan tenglamaning chap qismidagi ko'paytmani $(x + 1)(x + 5)$ va $(x + 2)(x + 4)$ larning ko'paytmasi shaklida yozamiz va bu qavslarni ochamiz. Natijada, berilgan tenglamaga teng kuchli bo'lgan

$$(x^2 + 6x + 5)(x^2 + 6x + 8) - 40 = 0$$

tenglamaga ega bo'lamiz. Bu tenglamada $x^2 + 6x + 5 = y$ deb belgilash olib

$$y(y + 3) - 40 = 0 \iff y^2 + 3y - 40 = 0$$

kvadrat tenglamani hosil qilamiz. Bu tenglamaning ildizlari $y_1 = -8$, $y_2 = 5$ lardir. Topilgan bu qiymatlarni belgilashga qo'yamiz:

1) $x^2 + 6x + 5 = -8$;    2) $x^2 + 6x + 5 = 5$.

1-kvadrat tenglamaning diskriminanti $D = 6^2 - 4 \cdot 13 = -14 < 0$ manfiy, shuning uchun u haqiqiy ildizlarga ega emas. 2-tenglama $x^2 + 6x = 0$ ko'rinishdagi chala kvadrat tenglama bo'lib, uning ildizlari $x_1 = -6$, $x_2 = 0$ lardir. Tenglama ildizlari yig'indisi $-6$ ekan. **Javob:** $-6$ (A).

17. (02-11-20) Tenglama ildizlari yig'indisini toping.

$$\frac{3x^2 + 8x - 3}{x + 3} = x^2 - x + 2$$

A) $-8$      B) $-6$      C) $-4$      D) 4

18. (03-3-28) Tenglama ildizlari ko'paytmasini toping.

$$\frac{3x^2 + 8x - 3}{x + 3} = x^2 - x + 2$$

A) 2      B) $-2$      C) 6      D) 3

19. (03-6-8) Agar

$$\frac{4x^2 - 4xy + 3y^2}{2y^2 + 2xy - 5x^2} = 1$$

bo'lsa, $\dfrac{x + y}{x - y}$ ning qiymatini toping.

A) 2      B) $-2$      C) $\dfrac{1}{2}$      D) $-\dfrac{1}{2}$

20. (03-7-56) Tenglama ildizlari ayirmasining modulini toping.

$$\frac{x + 8}{3} = x - \frac{x - 3}{x}$$

A) $5, 5$      B) 5      C) $3, 5$      D) 4

21. (03-8-17) Tenglama ildizlari yig'indisini toping.

$$\frac{3x^2 + 4x - 4}{x + 2} = x^2 - 4x + 4$$

A) 10      B) $-5$      C) $-4$      D) 7

22. (03-8-42) Tenglama butun ildizlarining yig'indisini toping.

$$x^2 + 3x + \frac{6}{2 - 3x - x^2} = 1$$

A) $-3$      B) 1      C) $-5$      D) 3

23. (03-11-63) Tenglama ildizlari yig'indisini toping.

$$\frac{x^3 - 8}{x - 2} = 6x + 1$$

A) 6      B) 4      C) $-4$      D) 3

24. (03-12-2) Tenglamaning eng kichik va eng katta ildizlari ayirmasini toping.

$$3x^4 - 5x^2 + 2 = 0$$

A) 2      B) $\dfrac{2\sqrt{6}}{3}$      C) $-\dfrac{2\sqrt{6}}{3}$      D) $-2$

## 4.5   Tenglamalar sistemasi

### 4.5.1   Chiziqli tenglamalar sistemasi

Ikki noma'lumli birinchi darajali (chiziqli) tenglamalar sistemasi deb

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases} \qquad (4.13)$$

ko'rinishdagi sistemaga aytiladi. Bu yerda $a_1$, $b_1$, $c_1$, $a_2$, $b_2$, $c_2$ ma'lum sonlar bo'lib, ular berilgan bo'ladi, $x$, $y$ lar esa noma'lumlar hisoblanadi. (4.13) sistemaning yechimi deb birinchi va ikkinchi tenglamalarni qanoatlantiruvchi $(x; y)$ sonlar juftiga aytiladi. Sistemani yechish uning hamma yechimlarini topish yoki yechimlari yo'qligini isbotlashdan iborat. (4.13) sistemani yechishning quyidagi ikki usulini ko'rib o'tamiz.

*O'rniga qo'yish usuli.* Tenglamalar sistemasini o'rniga qo'yish usuli bilan yechish uchun tenglamalarning birortasidan noma'lumlardan birini ikkinchisi orqali ifodalab, ikkinchi tenglamaga olib borib qo'yiladi. Natijada bir noma'lumli chiziqli tenglama hosil bo'ladi. Bu tenglamani yechib, yechimni tenglamalar sistemasining istalgan bir tenglamasiga qo'yib ikkinchi noma'lumning qiymati topiladi. Quyidagi misolni qaraymiz:

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5. \end{cases} \qquad (4.14)$$

Bu sistemaning ikkinchisidan $x$ ni topamiz (ya'ni $x = 5 - 2y$) va uni sistemaning birinchi tenglamasiga qo'yamiz: $2(5 - 2y) - 3y = 3 \iff 7 = 7y$. Bu tenglama yagona $y = 1$ ildizga ega. Bu yechimni sistemaning ikkinchisiga qo'yib, $x + 2 \cdot 1 = 5$ dan $x = 3$ ni olamiz. Demak, (4.14) sistemaning yechimi (3; 1) dan iborat.

*Qo'shish usuli.* Tenglamalar sistemasini qo'shish usuli bilan yechish uchun berilgan sistemaga teng kuchli bo'lgan shunday sistema olamizki, unda $y$ (yoki $x$) oldidagi koeffitsiyentlar qarama-qarshi sonlar bo'ladi. Sistemaning tenglamalari hadma-had qo'shilsa, $x$ ga nisbatan bir noma'lumli chiziqli tenglama hosil bo'ladi. Bu tenglamadan $x$ ni topib, uni tenglamalar sistemasining istalgan bir tenglamasiga qo'yib $y$ noma'lumning qiymati topiladi. Quyidagi misolni qaraymiz:

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1. \end{cases} \qquad (4.15)$$

Bu sistemaning ikkinchi tenglamasini 2 ga kopaytiramiz va tenglamalarni hadma-had qo'shib $5x = 5$ tenglamani olamiz. Bu yerdan $x = 1$ ni topamiz va uni sistemaning ikkinchi tenglamasiga qo'yib $1 + 2y = 1$ dan $y = 0$ ni olamiz. Demak, (4.15) sistemaning yechimi (1; 0) dan iborat.

1. **Agar (1) sistemada $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ bo'lsa, u yagona yechimga ega.**

2. **Agar (1) sistemada $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ bo'lsa, u cheksiz ko'p yechimlarga ega.**

**3. Agar (1) sistemada $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ bo'lsa, sistema yechimga ega emas.**

1. (96-6-17) Agar

$$\begin{cases} 3x + y = 45 \\ z + 3y = -15 \\ 3z + x = 6 \end{cases}$$

bo'lsa, $x + y + z$ nimaga teng?

A) 12    B) 10    C) 15    D) 9

**Yechish:** Sistema tenglamalarini qo'shamiz:

$$4x + 4y + 4z = 45 - 15 + 6.$$

Bu yerdan

$$4(x + y + z) = 36 \iff x + y + z = 9$$

ekanini hosil qilamiz. **Javob:** 9 (D).

2. (96-1-21) $(x; y)$ sonlar jufti

$$\begin{cases} 2x - y = 5 \\ 3x + 2y = 4 \end{cases}$$

sistemaning yechimi bo'lsa, $x - y$ ni toping.

A) 1    B) $-1$    C) 3    D) 0

3. (96-3-24) Tenglamalar sistemasini qanoatlantiruvchi sonlar juftligini aniqlang.

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases}$$

A) $(2; 3)$    B) $(-2; 3)$    C) $(3; 2)$    D) $(-2; -3)$

4. (96-3-76) $x$ ni toping.

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$$

A) 1    B) 2    C) 3    D) $-2$

5. (96-9-17) $x$ ni toping.

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1 \end{cases}$$

A) 1    B) 0    C) $-1$    D) 2

6. (96-9-72) $(x; y)$ sonlar jufti

$$\begin{cases} 3x - 2y = -8 \\ x + 3y = 1 \end{cases}$$

sistemaning yechimi bo'lsa, $y - x$ ni toping.

A) 0    B) $-1$    C) $-2, 5$    D) 3

7. (96-11-25) Quyidagi juftliklardan qaysi biri tenglamalar sistemasini qanoatlantiradi?

$$\begin{cases} x + y = 5 \\ x - y = -1 \end{cases}$$

A) $(2; 3)$    B) $(1; 4)$    C) $(4; 1)$    D) $(3; 2)$

8. (96-12-74) Sistemani yeching va $y$ ning qiymatini toping.

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$$

A) 2    B) 1    C) 3    D) $1, 5$

9. (96-13-17) Sistemadan $x$ ni toping.

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1 \end{cases}$$

A) $-1$    B) 3    C) 2    D) 1

10. (97-1-11) $(x; y)$ sonlar jufti tenglamalar sistemasining yechimi, $x \cdot y$ ni toping.

$$\begin{cases} 2x + y - 8 = 0 \\ 3x + 4y - 7 = 0 \end{cases}$$

A) $-90$    B) 12    C) $-10$    D) 80

**Yechish:** Sistemaning birinchi tenglamasidan $y$ ni topamiz $y = 8 - 2x$ va uni ikkinchi tenglamaga qo'yamiz: $3x + 4(8 - 2x) = 7 \iff 25 = 5x$. Bu yerdan $x = 5$ ni olamiz. $x$ ning bu qiymatini $y = 8 - 2x$ ga qo'yib $y = -2$ ni olamiz. Ularning ko'paytmasi $xy = 5 \cdot (-2) = -10$ ekan. **Javob:** $-10$ (C).

11. (97-3-21) Agar

$$\begin{cases} 5x + 2y = -3 \\ x - 3y = -4 \end{cases}$$

bo'lsa, $x^2 - y^2$ ning qiymatini toping.

A) 2    B) 1    C) 0    D) $2, 5$

12. (97-6-11) $(x; y)$ sonlar jufti

$$\begin{cases} x + 2y - 3 = 0 \\ 2x - 3y + 8 = 0 \end{cases}$$

tenglamalar sistemasining yechimi, $x + y$ ni hisoblang.

A) $-1$    B) 1    C) 3    D) $4, 5$

13. (97-10-21) Agar

$$\begin{cases} 3x - 2y = 1 \\ 4x - y = -2 \end{cases}$$

bo'lsa, $y^2 - x^2$ ning qiymatini toping.

A) $-1$    B) $-3$    C) 3    D) 5

14. (98-3-16) Sistemadan $x$ ni toping.

$$\begin{cases} 3x + 4y = 11 \\ 5x - 2y = 1 \end{cases}$$

A) 2    B) $\dfrac{3}{2}$    C) $\dfrac{5}{2}$    D) 1

15. (98-10-64) Sistemadan $y$ ni toping.

$$\begin{cases} 3x + 4y = 11 \\ 5x - 2y = 1 \end{cases}$$

A) 0    B) 1    C) 2    D) $-2$

16. Tenglamalar sistemasini yeching.

$$\begin{cases} x + y = 8 \\ x + 2y = 12 \end{cases}$$

   A) $(4; 4)$    B) $(-4; -4)$    C) $(-4; 4)$    D) $(4; 6)$

17. (97-4-7) $a = 4b$ va $c + 3b = 0$ ($b \neq 0$) bo'lsa, $\dfrac{a}{c}$ ni toping.

   A) $-\dfrac{1}{3}$    B) $1\dfrac{1}{3}$    C) $1\dfrac{2}{3}$    D) $-\dfrac{4}{3}$

   **Yechish:** Sistemadan $a = 4b$, $c = -3b$ ekanligi kelib chiqadi. Demak, $\dfrac{a}{c} = \dfrac{4b}{-3b} = -\dfrac{4}{3}$. **Javob:** $-\dfrac{4}{3}$ (D).

18. (97-8-17) Agar $2m + n = 2$, $2n + p = 6$ va $2p + m = 4$ bo'lsa, $m + n + p$ ni toping.

   A) 6    B) 4    C) 5    D) 3

19. (97-12-16) Agar $2q - 4p = -9$, $2t - 4q = -7$ va $2p - 4t = 2$ bo'lsa, $p + q + t$ ning qiymatini toping.

   A) $-7$    B) 8    C) 7    D) $-8$

20. (00-4-39) Agar $3a + 4b = 16$ va $2c - b = 1$ bo'lsa, $3a + 8c$ ning qiymatini toping.

   A) 18    B) 4    C) 20    D) 23

21. (00-1-11) Agar

$$a^2 - 4a + 5 + b^2 - 2b = 0$$

   bo'lsa, $(a + b)^3$ ning qiymatini toping.

   A) 26    B) 27    C) 28    D) 25

22. (02-12-2) Agar $x + y = 4$, $y + z = 8$ va $x + z = 6$ bo'lsa, $x - y + 2z$ ning qiymatini hisoblang.

   A) 8    B) 6    C) 7    D) 10

23. (02-12-19) Nechta natural sonlar jufti

$$x^2 - y^2 = 105$$

   tenglikni qanoatlantiradi?

   A) 3    B) 4    C) 2    D) 5

### 4.5.2   Parametrli tenglamalar sistemasi

1. (00-5-27) $k$ ning qanday qiymatida

$$\begin{cases} kx + 4y = 4 \\ 3x + y = 1 \end{cases}$$

   tenglamalar sistemasi yagona yechimga ega bo'ladi?

   A) $k \neq 12$    B) $k = 9$    C) $k \neq 19$    D) $k = 12$

   **Yechish:** 1-qoidaga ko'ra, sistema $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ shartda yagona yechimga ega. Bu shart berilgan sistema uchun $\dfrac{k}{3} \neq \dfrac{4}{1}$ shartdan iborat. Bu yerdan $k \neq 12$. **Javob:** $k \neq 12$ (A).

2. (01-7-19) $a$ ning qanday qiymatida

$$\begin{cases} x + y = a \\ xy = 9 \end{cases}$$

   tenglamalar sistemasi yagona yechimga ega?

   A) 3    B) 6    C) $-3$    D) $-6; 6$

3. (01-10-28) $a$ ning qanday qiymatida

$$\begin{cases} 2x + 3y = 5 \\ x - y = 2 \\ x + 4y = a \end{cases}$$

   tenglamalar sistemasi yechimga ega?

   A) 0    B) 1    C) 2    D) 3

4. (02-1-46) Agar

$$\begin{cases} ax + by = 3 \\ bx + ay = 2 \end{cases}$$

   tenglamalar sistemasi $x = 3$, $y = 2$ yechimga ega bo'lsa, $a$ ning qiymatini toping.

   A) 4    B) 5    C) 3    D) 1

5. (02-9-15) Agar $y - x = 2$ va $a > 0$ bo'lsa,

$$\begin{cases} y^2 - x^2 = 8a \\ y + x = a^2 \end{cases}$$

   tenglamalar sistemasini yeching.

   A) $(5; 7)$    B) $(7; 9)$    C) $(4; 6)$    D) $(-6; -4)$

6. (99-9-16) $k$ ning qanday qiymatida

$$\begin{cases} 3x + 6y = k \\ 9x + 18y = k + 1 \end{cases}$$

   tenglamalar sistemasi cheksiz ko'p yechimga ega?

   A) $\dfrac{1}{3}$    B) 1    C) $\dfrac{1}{2}$    D) $\dfrac{2}{3}$

   **Yechish:** Sistemaning cheksiz ko'p yechimga ega bo'lishlik sharti 2-dan $\dfrac{3}{9} = \dfrac{6}{18} = \dfrac{k}{k+1}$ ni olamiz. Bu yerdan $k = \dfrac{1}{2}$. **Javob:** $\dfrac{1}{2}$ (C).

7. (01-2-15) $k$ ning qanday qiymatida

$$\begin{cases} 3x + (k-1)y = k + 1 \\ (k+1)x + y = 3 \end{cases}$$

   tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi?

   A) $-1$    B) $-2$    C) 0    D) 2

8. (98-3-24) $k$ ning qanday qiymatlarida

$$\begin{cases} (k^2 - k - 1)x + 2,5y = 5 \\ 2x + y = -k \end{cases}$$

   sistemaning birorta ham yechimi bo'lmaydi?

   A) $-2$    B) $-2$ va 3    C) 3    D) 4 va 3

   **Yechish:** Sistemaning yechimga ega bo'lmaslik sharti 3 − dan

$$\begin{cases} k^2 - k - 1 = 5 \\ k \neq -2 \end{cases}$$

ni olamiz. Sistema 1− tenglamasining yechimlari $k_1 = -2$ va $k_2 = 3$ lardir. Ikkinchi munosabat $k \neq -2$ dan yechim faqat $k = 3$ bo'ladi. **Javob:** $k = 3$ (C).

9. (98-5-20) $a$ ning qanday qiymatlarida

$$\begin{cases} ax - y = 0 \\ x + y = 10 \end{cases}$$

tenglamalar sistemasi yechimga ega bo'lmaydi?
A) $-1$    B) 2    C) 1    D) $-2$

10. (98-10-71) $k$ ning qanday qiymatlarida

$$\begin{cases} (k^2 + k + 1)x + 3y - 6 = 0 \\ x + y + k = 0 \end{cases}$$

sistema birorta ham yechimga ega bo'lmaydi?
A) $-2$    B) 1    C) $-2$ va 1    D) 2 va 3

11. (02-5-10) $m$ ning qanday qiymatlarida

$$\begin{cases} mx + 2y + 4 = 0 \\ 2x + my - 8 = 0 \end{cases}$$

tenglamalar sistemasi yechimga ega emas?
A) 4    B) $-4$    C) 2    D) $-2; 2$

12. (99-2-17) $a$ ning qanday qiymatida

$$\begin{cases} 2x + ay = 2 \\ ax + 2y = 3 \end{cases}$$

tenglamalar sistemasi yechimga ega bo'lmaydi?
A) 3    B) $\pm 3$    C) 4    D) $\pm 2$

13. (99-1-17) Tenglamalar sistemasi $a$ ning qanday qiymatlarida yechimga ega emas?

$$\begin{cases} a^2 x + 3y = 3 \\ 3x + y = 4 \end{cases}$$

A) $\pm 3$    B) $\pm 1$    C) $\pm\sqrt{3}$    D) 0

14. (03-10-30) $a$ ning nechta qiymatida

$$\begin{cases} (a - 2)x + 3y = 5 \\ 7x - 18y = 1 \end{cases}$$

tenglamalar sistemasi yechimga ega emas?
A) 1    B) 2    C) 4    D) $\emptyset$

15. (97-4-23) Agar

$$\begin{cases} x + 2y = 2 \\ 2x + y = k \end{cases}$$

bo'lsa, $k$ ning qanday qiymatida $x + y = 2$ tenglik o'rinli bo'ladi?
A) 2    B) 4    C) 1    D) 5

**Yechish:** Sistema tenglamalarini qo'shamiz: $3x + 3y = 2 + k \iff 3(x + y) = 2 + k$. Bu yerdan hamda $x + y = 2$ ekanligini hisobga olsak, $3 \cdot 2 = 2 + k$ ni olamiz. Demak, $k = 4$ ekan. **Javob:** 4 (B).

16. (97-9-83) Agar

$$\begin{cases} x + 3y = 6 \\ 2x + ky = 8 \end{cases}$$

bo'lsa, $k$ ning qanday qiymatida $x + y = 2$ tenglik o'rinli bo'ladi?
A) 0    B) 1    C) 2    D) 4

### 4.5.3   Ikkinchi darajali tenglamalar sistemasi

1. (97-8-20) Tenglamalar sistemasini yeching.

$$\begin{cases} y + 4 = 2 \\ x^2 y = -2 \end{cases}$$

A) $(1; -2)$    B) $(-1; -2)$
C) $(1; 2)$    D) $(-1; -2)$ va $(1; -2)$

**Yechish:** Sistemaning 1-tenglamasidan $y = -2$ ekanligini olamiz va uni 2-tenglamaga qo'yamiz

$$x^2 \cdot (-2) = -2 \iff x^2 = 1.$$

Bu chala kvadrat tenglamaning yechimlari $x_1 = -1$, $x_2 = 1$ lardir. Shunday qilib, sistema $(x_1; y) = (-1; -2)$ va $(x_2; y) = (1; -2)$ yechimlarga ega.
**Javob:** $(-1; -2)$ va $(1; -2)$ (D).

2. (96-9-70) Sistemaning yechimini toping.

$$\begin{cases} x^2 + y^2 - 2xy = 16 \\ x + y = -2 \end{cases}$$

A) $(1; -3)$    B) $(-3; 1)$
C) $(0; -2)$    D) $(1; -3)$ va $(-3; 1)$

3. (96-10-20) Sistemaning yechimini toping.

$$\begin{cases} x - y = 4 \\ x^2 + y^2 + 2xy = 4 \end{cases}$$

A) $(3; 1)$    B) $(3; -1)$
C) $(3; -1)$ va $(1; -3)$    D) $(2; -2)$

4. (97-12-19) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - y^2 + 2x + 4 = 0 \\ x - y = 0 \end{cases}$$

A) $(2; 2)$    B) $(-2; -2)$    C) $(-1; -1)$    D) $(1; 1)$

5. (97-2-20) Tenglamalar sistemasini yeching.

$$\begin{cases} x + 2 = 0 \\ xy^2 = -8 \end{cases}$$

A) $(-2; -2)$    B) $(-2; 2)$
C) $(-2; 2)$ va $(-2; -2)$    D) $(2; 2)$

6. (96-1-19) Sistemaning yechimini toping.

$$\begin{cases} x^2 + y^2 - 2xy = 1 \\ x + y = 3 \end{cases}$$

A) $(2; 1)$    B) $(1; 2)$
C) $(1,5; 1,5)$    D) $(2; 1)$ va $(1; 2)$

7. (96-6-20) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - y^2 - 3x = 12 \\ x - y = 0 \end{cases}$$

A) $(-4; 4)$    B) $(4; -4)$    C) $(4; 4)$    D) $(-4; -4)$

8. (01-3-34) Tenglamalar sistemasini yeching.

$$\begin{cases} x + 3 = 0 \\ xy^2 = -12 \end{cases}$$

A) $(-3; 2)$             B) $(-3; -2)$
C) $(-3; -2), (-3; 2)$         D) $\emptyset$

9. (96-7-23) Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 9 \\ y - x = -3 \end{cases}$$

A) 1       B) 2       C) 3       D) 4

**Yechish:** Sistemaning 2-tenglamasidan $y = x - 3$ ni topamiz va uni 1-tenglamaga qo'yamiz

$$x^2 + (x - 3)^2 = 9 \iff 2x^2 - 6x = 0.$$

Bu chala kvadrat tenglamaning yechimlari $x_1 = 0$, $x_2 = 3$ lardir. Bu qiymatlarni $y = x - 3$ ga qo'yib $y_1 = -3$, $y_2 = 0$ ni olamiz. Demak, sistema ikkita $(x_1, y_1) = (0, -3)$ va $(x_2, y_2) = (3, 0)$ yechimlarga ega. **Javob:** 2 (B).

10. (97-10-23) Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 4 \\ x - y = -2 \end{cases}$$

A) 4       B) 3       C) 2       D) 1

11. (97-3-23) Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 25 \\ x - y = 5 \end{cases}$$

A) 4       B) 3       C) 2       D) 1

12. (98-12-19) Agar $a - b = 12$ va $-ab + a^2 = 144$ bo'lsa, $a$ ning qiymati qanchaga teng bo'ladi?
A) 12       B) $-12$       C) 36       D) 6

**Yechish:** 2-tenglikni $a(a - b) = 144$ shaklda yozamiz va $a - b = 12$ ekanligidan foydalansak $12a = 144$ ni olamiz. Bu tenglikning ikkala qismini 12 bo'lib $a = 12$ ni olamiz. **Javob:** 12 (A).

13. (98-11-60) Agar $x^2 + y^2 = 281$ va $x - y = 11$ bo'lsa, $xy$ qanchaga teng bo'ladi?
A) 80       B) 160       C) $-80$       D) 40

14. (02-12-30) Agar $x^2 - 4xy + y^2 = 4 - 2xy$ va $x + y = 12$ bo'lsa, $xy$ ning qiymatini toping.
A) 32       B) 35       C) 30       D) 34

15. (97-9-67) Agar $ab = 9$ va $3b = 8c$ ($b \neq 0$) bo'lsa, $ac$ ni hisoblang.
A) $3\frac{1}{3}$     B) $3\frac{5}{8}$     C) $3\frac{4}{9}$     D) $3\frac{3}{8}$

16. (01-3-33) Sistemadan $3xy$ ni toping.

$$\begin{cases} x^2 + y^2 - xy = 1 \\ x + y = -2 \end{cases}$$

A) 1       B) $-1$       C) 3       D) $-3$

**Yechish:** Berilgan sistema

$$\begin{cases} x^2 + y^2 = 1 + xy \\ x + y = -2 \end{cases}$$

sistemaga teng kuchli. Bu sistemaning 2-tenglamasini (har ikkala qismini) kvadratga ko'taramiz

$$(x + y)^2 = (-2)^2 \iff x^2 + y^2 = 4 - 2xy.$$

Endi sistemaning 1-tenglamasidan foydalansak $1 + xy = 4 - 2xy$ ni olamiz. Bu tenglikdan $3xy = 3$ ni olamiz. **Javob:** 3 (C).

17. (01-3-32) Sistemadan $x$ ni toping.

$$\begin{cases} x + y = 6 \\ x^2 - y^2 = 12 \end{cases}$$

A) 4       B) 2       C) 1       D) 3

18. (96-3-75) Sistemadan $x$ ni toping.

$$\begin{cases} x + y = 3 \\ x^2 - y^2 = 6 \end{cases}$$

A) $1, 5$     B) $2, 5$     C) 3     D) 1

19. (96-12-73) Sistemani yeching va $x \cdot y$ ning qiymatini toping?

$$\begin{cases} x^2 + y^2 = 3 \\ x - y = 1 \end{cases}$$

A) 2       B) 3       C) $1, 5$       D) 1

20. (98-12-64) Agar

$$\begin{cases} x + y = 3 \\ x \cdot y = 1 \end{cases}$$

bo'lsa, $x^5 \cdot y + x \cdot y^5$ ni hisoblang.
A) 47       B) 29       C) 51       D) 24

**Yechish:** $x + y = 3$, $xy = 1$ tengliklardan

$$x^2 + y^2 = \underbrace{(x + y)^2}_{3} - 2\underbrace{xy}_{1} = 9 - 2 = 7$$

ekani kelib chiqadi. Shuning uchun

$$x^5 y + xy^5 = xy(x^4 + y^4) = x^4 + y^4 =$$
$$= (x^2 + y^2)^2 - 2(xy)^2 = 7^2 - 2 = 47$$

**Javob:** 47 (A).

21. (01-4-23) Agar

$$\begin{cases} x^2 - y^2 = 6 \\ x + y = 1 \end{cases}$$

bo'lsa, $x - y$ ning qiymatini toping.
A) 1       B) $-1$       C) 6       D) $-6$

22. (03-12-3) Sistemadan $ab$ ni toping

$$\begin{cases} b + a = 18 \\ a^2 + b^2 = 170 \end{cases}$$

A) 45       B) 72       C) 77       D) 80

### 4.5.4 Ikkinchi va undan yuqori darajali tenglamalar sistemasi

1. (98-10-17) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - 1 = 0 \\ xy^2 = -4 \end{cases}$$

A) $(-1; 2)$      B) $(2; -1)$
C) $(2; 1)$      D) $(-1; -2)$ va $(-1; 2)$

**Yechish:** Sistemaning 1-tenglamasidan $x_1 = -1$, $x_2 = 1$ ni olamiz. Ikki holni alohida qaraymiz:

$$1) \ x_1 y^2 = -4 \Longleftrightarrow -y^2 = -4 \Longleftrightarrow y^2 = 4.$$

Bu tenglama $y_1 = -2$ va $y_2 = 2$ yechimlarga ega. Demak, $(x_1; y_1) = (-1; -2)$ va $(x_1; y_2) = (-1; 2)$ juftliklar berilgan sistemaning yechimlari bo'ladi.

$$2) \ x_2 y^2 = -4 \Longleftrightarrow y^2 = -4.$$

Bu tenglama haqiqiy ildizlarga ega emas. Shunday qilib sistemaning yechimlari $(-1; -2)$ va $(-1; 2)$ juftliklar ekan. **Javob:** $(-1; -2)$ va $(-1; 2)$ (D).

2. (02-9-11) Sistema nechta yechimga ega?

$$\begin{cases} y = \dfrac{4}{x} \\ y = -x^2 + 6x - 5 \end{cases}$$

A) $\emptyset$    B) 1    C) 2    D) 3

3. (03-9-6) Sistemaning yechimlaridan iborat barcha $x$ va $y$ larning yig'indisini toping.

$$\begin{cases} x^3 + y^3 = 35 \\ x^2 y + xy^2 = 30 \end{cases}$$

A) 0    B) 2    C) 6    D) 10

4. (02-11-27) $x$ ning

$$\begin{cases} x^5 \cdot y^7 = 32 \\ x^7 \cdot y^5 = 128 \end{cases}$$

tenglamalar sistemasining yechimidan iborat barcha qiymatlari yig'indisini toping.
A) 0    B) 4    C) 8    D) 12

5. (97-4-25) Agar

$$\begin{cases} x^3 - y^3 - 3x^2 y = 5 \\ xy^2 = 1 \end{cases}$$

bo'lsa, $\dfrac{x - y}{2}$ ni hisoblang.
A) 2    B) 1    C) 3    D) $4, 5$

**Yechish:** Berilgan tengliklardan foydalanib

$$(x - y)^3 = \underbrace{x^3 - y^3 - 3x^2 y}_{5} + 3\underbrace{xy^2}_{1} = 5 + 3 \cdot 1 = 8$$

ni hosil qilamiz. U holda $x - y = 2$ va

$$\frac{x - y}{2} = \frac{2}{2} = 1$$

bo'ladi. **Javob:** 1 (B).

6. (98-2-16) Agar $m^2 - mn = 48$ va $n^2 - mn = 52$ bo'lsa, $m - n$ nechaga teng?
A) 10    B) 8    C) $\pm 10$    D) $\pm 8$

7. (98-5-22) Agar

$$\begin{cases} x^2 - 2xy + y^2 = 9 \\ xy = 10 \end{cases}$$

bo'lsa, $|x + y|$ ni hisoblang.
A) 7    B) 6    C) 5    D) 8

8. (98-10-65) $(x + y)^2$ ni toping.

$$\begin{cases} x^2 + y^2 = 10 \\ xy = 3 \end{cases}$$

A) 13    B) 7    C) 16    D) 19

9. (99-6-37) Agar $ab = 18$, $bc = 25$ va $ac = 8$ bo'lsa, $\sqrt{abc}$ nimaga teng.
A) $2\sqrt{15}$    B) $15\sqrt{2}$    C) $6\sqrt{5}$    D) $8\sqrt{3}$

10. (99-10-11) Agar $x^2 \cdot y = 50$, $x \cdot y^2 = 20$ bo'lsa, $xy$ ning qiymatini hisoblang.
A) 8    B) 10    C) 6    D) 12

11. (00-1-23) Agar $a - b = 1$ va $(a^2 - b^2) \cdot (a - b) = 9$ bo'lsa, $ab$ ning qiymatini toping.
A) 19    B) 22    C) 21    D) 20

12. (97-8-11) Agar $(x - 4)^2 + (x - y^2)^2 = 0$ bo'lsa, $x + 2y$ nechaga teng?
A) 0    B) 4    C) 6    D) 0 yoki 8

13. (00-8-14) Agar

$$\begin{cases} xy = 6 \\ yz = 12 \\ zx = 8 \end{cases}$$

bo'lsa, $x + y + z$ ning qiymatini toping.
A) $-9$ yoki 9    B) 18    C) 0    D) 36

**Yechish:** Tengliklarni ko'paytirib, $x^2 y^2 z^2 = 576$ ekanini hosil qilamiz. Ikkita hol bo'lishi mumkin:
1) $xyz = 24$. Bu tenglikni sistemaning birinchi, ikkinchi va uchinchi tengliklariga ketma-ket bo'lib, $z = 4$; $x = 2$; $y = 3$ ni hosil qilamiz. Bu holda $x + y + z = 2 + 3 + 4 = 9$.
2) $xyz = -24$. Bu tenglikni ham sistemaning birinchi, ikkinchi va uchinchi tengliklariga bo'lib, $z = -4$; $x = -2$; $y = -3$ ni hosil qilamiz. Bu holda $x + y + z = -9$. **Javob:** $-9$ yoki 9 (A).

14. (03-8-40) Sistemadan $x$ ni toping.

$$\begin{cases} \dfrac{xy}{x + y} = \dfrac{10}{7} \\ \dfrac{yz}{y + z} = \dfrac{40}{13} \\ \dfrac{zx}{x + z} = \dfrac{5}{8} \end{cases}$$

A) $\dfrac{80}{79}$    B) $\dfrac{5}{7}$    C) $\dfrac{7}{13}$    D) $\dfrac{79}{80}$

15. (98-6-2) Agar $xy = 6$, $yz = 2$ va $xz = 3$ ($x > 0$) bo'lsa, $xyz$ ni toping.
    A) $-6$    B) $6$    C) $5$    D) $12$

16. (98-11-52) Agar $xy = 4$, $yz = 7$ va $xz = 28$ ($y > 0$) bo'lsa, $xyz$ ni toping.
    A) $-28$    B) $28$    C) $27$    D) $56$

17. (98-6-11) Agar $m$ va $n$ natural sonlar

$$\sqrt{2}(n - 5) + n^2 - 6mn + 5m = 0$$

tenglikni qanoatlantirsa. $n - m$ ni toping.
    A) $2$    B) $5$    C) $6$    D) $4$

**Yechish:** Agar $\sqrt{2}(n-5)$ noldan farqli bo'lsa, u holda $\sqrt{2}(n-5)+n^2-6mn+5m$ ifoda ham noldan farqli bo'ladi. Chunki irratsional son plyus butun son bu irratsionaldir (nol irratsional emas). Shuning uchun $\sqrt{2}(n-5) + n^2 - 6mn + 5m = 0$ tenglikdan $\sqrt{2}(n-5) = 0$ va $n^2 - 6mn + 5m = 0$ tengliklar kelib chiqadi. 1-tenglikdan $n = 5$ ni olamiz. Uni 2-tenglikka qo'yib $25-30m+5m = 0$ chiziqli tenglamaga ega bo'lamiz. Uning yechim $m = 1$ dir. Demak, $n - m = 5 - 1 = 4$. **Javob:** 4 (D).

18. (01-10-8) Nechta butun $x$ va $y$ sonlar jufti $x^2 - y^2 = 31$ tenglikni qanoatlantiradi?
    A) $\emptyset$    B) $1$    C) $2$    D) $4$

19. (99-10-22) Agar $x^2 \cdot y + x \cdot y^2 = 48$ va $x^2 \cdot y - x \cdot y^2 = 16$ bo'lsa, $\dfrac{x}{y}$ ning qiymatini hisoblang.
    A) $\dfrac{1}{4}$    B) $-2$    C) $2$    D) $-\dfrac{1}{2}$

20. (02-6-32) Sistemadan $x \cdot y$ ni toping

$$\begin{cases} x^3 + y^3 = 35 \\ x + y = 5 \end{cases}$$

    A) $3$    B) $4$    C) $5$    D) $6$

21. (02-7-54) Agar $8a^3 - b^3 = 37$ va $ab^2 - 2a^2b = -6$ bo'lsa, $2a - b$ ning qiymatini toping.
    A) $1$    B) $-1$    C) $2$    D) $-2$

22. (02-8-11)

$$\begin{cases} xy + x + y = 11 \\ x^2y + y^2x = 30 \end{cases}$$

tenglamalar sistemasi uchun $x + y$ ning eng katta qiymatini toping.
    A) $6$    B) $5$    C) $7$    D) $4$

**Yechish:** Sistemaning 1-tenglamasidan $xy = 11 - (x+y)$ ekanligini topamiz. Agar $x+y$ ni $t$ desak, u holda 2-tenglama

$$xy(x + y) = 30 \iff (11 - t)t = 30$$

ko'rinishga keladi. Bu kvadrat tenglamaning ildizlari $t_1 = 5$, $t_2 = 6$ lardir. $x + y = t$ bo'lganligi uchun, uning katta qiymati 6. **Javob:** 6 (A).

23. (02-12-26) Agar $\dfrac{1}{n} + \dfrac{1}{m} = \dfrac{1}{7}$ va $m + n = -4$ bo'lsa, $mn$ ning qiymatini toping.
    A) $20,5$    B) $-20,5$    C) $21$    D) $-28$

24. (02-12-29) Agar $x^3 + 3xy^2 = 185$ va $y^3 + 3x^2y = 158$ bo'lsa, $x - y$ ning qiymatini toping.
    A) $4$    B) $3,5$    C) $2$    D) $3$

25. (03-11-3) Agar

$$\begin{cases} \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{y}} = \dfrac{4}{3} \\ xy = 9 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.
    A) $10$    B) $9$    C) $8$    D) $12$

**Yechish:** Sistemaning 2-tenglamasi $xy = 9$ dan foydalanib uning 1-tenglamasini

$$\frac{\sqrt{x} + \sqrt{y}}{\sqrt{9}} = \frac{4}{3} \iff \sqrt{x} + \sqrt{y} = 4$$

ko'rinishda yozib olamiz. Bu tenglikning ikkala qismini kvadratga ko'tarib va yana bir marta $xy = 9$ dan foydalanib

$$x + y + 2\sqrt{xy} = 16 \iff x + y = 10$$

ni olamiz. **Javob:** 10 (A).

26. (03-1-65) Agar

$$\begin{cases} x^2y + xy^2 = 120 \\ x^2y - xy^2 = 30 \end{cases}$$

bo'lsa, $x^2 - y^2$ ning qiymatini hisoblang.
    A) $16$    B) $20$    C) $25$    D) $34$

27. (03-11-65) Agar

$$\begin{cases} \dfrac{x + 3y + 1}{y} - \dfrac{y - x + 3}{2(x - 2)} = 2 \\ y - x = 1 \end{cases}$$

bo'lsa, $x \cdot y$ ning qiymatini toping.
    A) $15$    B) $-6$    C) $-8$    D) $12$

28. (07-112-29) $\begin{cases} x^3 + y^3 = 126 \\ x^2y + xy^2 = 30. \end{cases}$ tenglamalar sistemasining haqiqiy yechimlaridan iborat barcha $x$ va $y$ larning yig'indisini toping.
    A) $2$    B) $12$    C) $10$    D) $6$

29. (98-6-10) Agar $x^2 + y^2 = 225$ va $x^2 - y^2 = 63$ bo'lsa, $|x| - |y|$ ni toping.
    A) $3$    B) $4$    C) $5$    D) $6$

30. (98-9-16) Agar $p^2 + pq = 96$ va $q^2 + pq = 48$ bo'lsa, $p + q$ ning qiymati qanchaga teng bo'ladi?
    A) $12$    B) $14$    C) $\pm 12\sqrt{2}$    D) $\pm 12$

31. (99-1-19) Tenglamalar sistemasini yeching.

$$\begin{cases} y - x^3 = 0 \\ y = 16x \end{cases}$$

    A) $(0;0)$, $(4;64)$ va $(-4;-64)$
    B) $(0;0)$, $(8;2)$ va $(27;3)$
    C) $(0;0)$, $(2;8)$ va $(64;4)$
    D) $\emptyset$

# 5 - bob. Tengsizliklar

Agar ikkita ifoda > yoki < ($\geq$ yoki $\leq$) belgisi bilan bog'langan bo'lsa, ular tengsizlik hosil qiladi deyiladi. Masalan, $3x + 5 > 0$; $13x + 1 < 5$; $x^2 + 7 < 0$. Agar tengsizlik > yoki < belgisidan iborat bo'lsa, uni qat'iy tengsizlik deymiz. Masalan, $x - 5 < 7$; $x^2 - 3 > 0$. Agar tengsizlik $\geq$ yoki $\leq$ belgisidan iborat bo'lsa, uni qat'iymas tengsizlik deymiz. Masalan, $x - 9 \geq 7$; $x^2 - 5 \leq 0$. Tengsizlik belgisining chap tomonidagi ifodaga tengsizlikning chap qismi, o'ng tomonidagi ifodaga tengsizlikning o'ng qismi deyiladi. Tengsizlikning ikkala qismi ham sonlardan iborat bo'lsa, unga *sonli tengsizlik* deyiladi. $a < b$, $c < d$ tengsizliklar bir xil ma'noli, $a < b$, $c > d$ tengsizliklar esa har xil ma'noli tengsizliklar deyiladi. Sonli tengsizliklar quyidagi xossalarga ega.

1. **Agar $a > b$ bo'lsa, u holda $b < a$ bo'ladi.**

2. **Agar $a > b$ va $b > c$ bo'lsa, u holda $a > c$ bo'ladi.**

3. **Agar $a > b$ va $c-$ ixtiyoriy son bo'lsa, u holda $a + c > b + c$ bo'ladi. Xuddi shunday $a > b$ dan $a - c > b - c$ kelib chiqadi.**

4. **Agar $a > b$ bo'lib, $c > 0$ ixtiyoriy son bo'lsa, u holda $a \cdot c > b \cdot c$ hamda $a : c > b : c$ bo'ladi. Ya'ni tengsizlikning ikkala qismini musbat songa ko'paytirsak, yoki bo'lsak tengsizlik belgisi saqlanadi.**

5. **Agar $a > b$ bo'lib, $c < 0$ ixtiyoriy son bo'lsa, u holda $a \cdot c < b \cdot c$ hamda $a : c < b : c$ bo'ladi. Ya'ni tengsizlikning ikkala qismini manfiy songa ko'paytirsak, yoki bo'lsak tengsizlik belgisi qarama-qarshisiga o'zgaradi.**

6. **Bir xil ma'noli tengsizliklarni hadlab qo'shish mumkin, ya'ni $a < b$ va $c < d$ bo'lsa, $a + c < b + d$ bo'ladi.**

7. **Har xil ma'noli tengsizliklarni hadlab ayirish mumkin, ya'ni $a > b$ va $c < d$ bo'lsa, $a - c > b - d$ bo'ladi. Ayirmada kamayuchi tengsizlikning belgisi saqlanadi.**

8. **Nomanfiy hadli bir xil ma'noli tengsizliklarni hadlab ko'paytirish mumkin, ya'ni $0 \leq a < b$ va $0 \leq c < d$ bo'lsa, $a \cdot c < b \cdot d$ bo'ladi.**

9. **Nomanfiy hadli tengsizliklarning har ikkala qismini bir xil natural darajaga ko'tarish mumkin, ya'ni $0 \leq a < b$ va $n \in N$ bo'lsa, $a^n < b^n$ bo'ladi.**

Agar bir tengsizlikning har qanday yechimi shu o'zgaruvchilar qatnashgan ikkinchi tengsizlikning ham yechimi bo'lsa, va aksincha, ikkinchi tengsizlikning ham qanday yechimi birinchi tengsizlikning yechimi bo'lsa, bu tengsizliklar teng kuchli tengsizliklar deyiladi. Yechimga ega bo'lmagan tengsizliklar ham teng kuchli tengsizliklar hisoblanadi.

## 5.1 Chiziqli tengsizliklar

$$ax + b > 0, \quad ax + b \geq 0 \qquad (5.1)$$
$$ax + b < 0, \quad ax + b \leq 0 \qquad (5.2)$$

ko'rinishdagi yoki soddalashtirishdan so'ng shunday ko'rinishga keladigan tengsizlik chiziqli tengsizlik deyiladi. (5.1) va (5.2) da $x$ noma'lum, $a$ va $b$ lar haqiqiy sonlar, $a$ noma'lum oldidagi koeffitsiyent, $b$ ozod had deyiladi. Eng sodda $x > a$, $x \leq b$ yoki $a < x \leq b$, $a \leq x \leq b$ ko'rinishdagi tengsizliklarning yechimlari $(a; \infty)$, $(-\infty; b]$ yoki $(a; b]$, $[a; b]$ shaklda yozish qabul qilingan.

1. $3x + 1 < 10$ tengsizlikni yeching.
   A) $(3; \infty)$  B) $(1; \infty)$  C) $(-\infty; 3)$  D) $\emptyset$

   **Yechish:** Berilgan tengsizlikda 1 ni tengsizlikning o'ng qismiga o'tkazib, keyin tengsizlikning ikkala qismiga 3 ga bo'lib $x < 3$ ni olamiz. **Javob:** $(-\infty; 3)$ (C) .

2. $7x + 5 > 19$ tengsizlikni yeching.
   A) $(2; \infty)$  B) $(7; \infty)$  C) $(-\infty; 2)$  D) $\emptyset$

3. $2x + 7 \leq 11$ tengsizlikni yeching.
   A) $[2; \infty)$  B) $(7; \infty)$  C) $(-\infty; 2]$  D) $\emptyset$

4. $5x + 9 \geq 14$ tengsizlikni yeching.
   A) $[1; \infty)$  B) $[5; \infty)$  C) $(-\infty; 1]$  D) $\emptyset$

5. (00-6-10) Tengsizlikni yeching.

   $$1 - \frac{17 - 3x}{2} > 1,5x$$

   A) $(-2, 5; 0)$    B) $(-\infty; -2, 5)$
   C) $(-\infty; 0)$    D) $\emptyset$

   **Yechish:** Berilgan tengsizlikning har ikkala qismini 2 ga ko'paytirsak, tengsizlik saqlanadi

   $$2 - 17 + 3x > 3x \iff -15 > 0 \cdot x.$$

   Bu tengsizlik yechimga ega emas. **Javob:** $\emptyset$ (D).

6. Tengsizlikni yeching.

   $$\frac{1 - x}{2} < \frac{2x + 1}{4}$$

   A) $(\frac{3}{4}; \infty)$  B) $(1; \infty)$  C) $(-\infty; \frac{3}{4})$  D) $\emptyset$

7. (01-8-10) Tengsizlikni yeching.

   $$\frac{1 - x}{2} + 3 < 3x - \frac{2x + 1}{4}$$

   A) $(1\frac{1}{3}; \infty)$  B) $(1\frac{1}{13}; \infty)$  C) $(1\frac{1}{4}; \infty)$  D) $\emptyset$

8. (98-2-17) Quyida keltirilgan tengsizliklardan qaysi biri

   $$3x - a > b - 2x$$

   tengsizlikka teng kuchli emas?
   A) $5x - a > b$        B) $6x - 2a > 2b - 4x$
   C) $3x > a + b - 2x$    D) $a - 3x > 2x - b$

9. $2x + 5 > a$ ga teng kuchli tengsizlikni toping.
A) $2x + 5 - a \geq 0$    B) $2x + 5 - a > 0$
C) $2x < a - 5$    D) $-2x < -5 - a$

10. (00-8-33) $k$ ning $4y^2 - 3y + k = 0$ tenglama haqiqiy ildizlarga ega bo'lmaydigan eng kichik butun qiymatini toping.
A) 1    B) 3    C) 4    D) 5

**Yechish:** $4y^2 - 3y + k = 0$ kvadrat tenglama haqiqiy ildizlarga ega bo'lmasligi uchun $D < 0$ bo'lishi kerak:

$$D = 9 - 16k < 0 \iff k > \frac{9}{16}.$$

Bu tengsizlikni qanoatlantiruvchi eng kichik butun son $k = 1$. **Javob:** 1 (A) .

11. (02-2-10) Tengsizlikni qanoatlantiruvchi eng katta butun sonni toping.

$$(x + 1)^2 > (x + 2)^2$$

A) $-2$    B) $-1$    C) $-3$    D) $-4$

12. (02-3-18) Tengsizlikni qanoatlantiruvchi eng kichik butun manfiy son nechaga teng?

$$8 + \frac{6x - 8}{10} > \frac{x - 2}{6} + \frac{1 - 5x}{8} + \frac{1}{4}$$

A) $-6$    B) $-7$    C) $-5$    D) $-4$

13. (03-11-64) Tengsizlikning eng katta butun yechimini ko'rsating.

$$\frac{2x - 7}{6} + \frac{7x - 2}{3} < 3 - \frac{1 - x}{2}$$

A) 2    B) $-1$    C) 1    D) 0

## 5.2  Chiziqli tengsizliklar sistemasi

1. (97-7-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 3x + 7 \geq 5(x + 1) + 6 \\ (x - 2)^2 - 8 < x(x - 2) + 10 \end{cases}$$

A) $(-11; 2)$    B) $[-2; 7)$    C) $(-7; -2]$    D) $[2; 11)$

**Yechish:** Qavsrlarni ochamiz:

$$\begin{cases} 3x + 7 \geq 5x + 5 + 6 \\ x^2 - 4x + 4 - 8 < x^2 - 2x + 10 \end{cases}$$

Endi $x$ qatnashgan hadlarni tengsizlikning chap qismiga, sonlarni tengsizlikning o'ng qismiga o'tkazib quyidagi sistemani olamiz:

$$\begin{cases} -2x \geq 4 \\ -2x < 14 \end{cases} \iff \begin{cases} x \leq -2 \\ x > -7 \end{cases} \iff -7 < x \leq -2.$$

Demak, $x \in (-7; -2)$. **Javob:** $(-7; -2]$ (C).

2. (97-3-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 2x - 3(x - 5) > 10 - 3x \\ x(x + 2) - 4 \leq (x - 1)^2 + 7 \end{cases}$$

A) $[2; 12, 5)$    B) $[2, 5; \infty)$    C) $[-3; 2)$    D) $(-2, 5; 3]$

3. (96-7-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} x(x + 1) + 10 > (x + 1)^2 + 3 \\ 3x - 4(x - 7) \geq 16 - 3x \end{cases}$$

A) $[-3; 5)$    B) $(2; 4]$    C) $[-6; 6)$    D) $[6, \infty)$

4. (97-10-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 4(x - 3) - 3 > 8x + 1 \\ 2 + x(x + 3) \leq (x + 2)^2 + 5 \end{cases}$$

A) $(4; 7]$    B) $(-\infty; -7)$    C) $(-4; \infty)$    D) $[-7; -4)$

5. (01-8-14) Tengsizliklar sistemasini yeching.

$$\begin{cases} \dfrac{3x - 2}{4} > \dfrac{1 - 5x}{6} \\ 3x - 1 \leq 3 - 2x \end{cases}$$

A) $(\dfrac{8}{19}; \infty)$    B) $(\dfrac{8}{19}; \dfrac{4}{5}]$    C) $(-\infty; \dfrac{4}{5}]$    D) $x \in R$

6. (97-6-14) Tengsizliklar

$$\begin{cases} 7x + 3 \leq 9x - 1 \\ 20 - 3x \geq 4x - 15 \end{cases}$$

sistemasi butun yechimlarining o'rta arifmetigini toping.
A) $3, 5$    B) 7    C) 4    D) 3

**Yechish:** Sistemada $x$ qatnashgan hadlarni tengsizlikning chap qismiga, ozod hadlarni tengsizlikning o'ng qismiga o'tkazib, berilgan sistemaga teng kuchli bo'lgan quyidagi sistemani olamiz:

$$\begin{cases} -2x \leq -4 \\ -7x \geq -35 \end{cases} \iff \begin{cases} x \geq 2 \\ x \leq 5 \end{cases} \iff 2 \leq x \leq 5.$$

So'nggi tengsizlikni butun yechimlari 2, 3, 4, 5 lardir. Ularning o'rta arifmetigi $(2 + 3 + 4 + 5) : 4 = 14 : 4 = 3, 5$. **Javob:** $3, 5$ (A).

7. (97-1-14) Tengsizliklar sistemasi butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 5x - 2 \geq 2x + 1 \\ 2x + 3 \leq 18 - 3x \end{cases}$$

A) 3    B) $2, 5$    C) 2    D) $1, 5$

8. (97-11-14) Tengsizliklar sistemasi butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 2x - 1 \geq 3x - 4 \\ 8x + 7 > 5x + 4 \end{cases}$$

A) 2    B) $2, 5$    C) $1, 5$    D) $0, 75$

9. (02-2-11) Tengsizliklar sistemasi butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 2x - 10 > 0 \\ 27 - x > 0 \end{cases}$$

A) 16    B) 18    C) 17    D) 15

10. (96-1-22) Tengsizliklar sistemasi nechta butun yechimga ega?

$$\begin{cases} 3 + 4x \geq 5 \\ 2x - 3(x-1) > -1 \end{cases}$$

A) 5     B) 3     C) 4     D) 2

11. (96-6-16) Tengsizliklar sistemasining eng katta butun yechimini toping.

$$\begin{cases} -2x < 22 \\ x + 4 < 8 \end{cases}$$

A) 4     B) 3     C) −11     D) −12

12. (96-9-73) Tengsizliklar sistemasi nechta butun yechimga ega?

$$\begin{cases} 3 - 4x > 5 \\ 2 + 3(x-1) \leq 4x + 3 \end{cases}$$

A) 1     B) 2     C) 4     D) 6

13. (97-2-16) Tengsizliklar sistemasining eng kichik butun yechimimini toping?

$$\begin{cases} x + 8 < 12 \\ -3x < 15 \end{cases}$$

A) −5     B) −3     C) −6     D) −4

14. (97-8-16) Tengsizliklar sistemasining barcha butun yechimlari ko'paytmasini toping.

$$\begin{cases} -4y < 12 \\ y + 6 < 6 \end{cases}$$

A) 2     B) 6     C) −6     D) −2

15. (97-12-15) Tengsizliklar sistemasining eng katta va eng kichik butun yechimlari yig'indisini toping.

$$\begin{cases} -2x > -26 \\ x - 3 > 1 \end{cases}$$

A) 17     B) 16     C) 18     D) 19

16. (98-3-15) Tengsizliklar sistemasining butun yechimlari yig'indisini toping?

$$\begin{cases} x + 1 < 2x - 4 \\ 3x + 1 < 2x + 10 \end{cases}$$

A) 9     B) 5     C) 20     D) 21

17. (98-1-6) Qo'sh tengsizlikni yeching.

$$-3 < 2 - 5x < 1$$

A) $(-1; 0, 2)$      B) $(-1; -0, 2)$
C) $(-0, 2; 1)$      D) $(0, 2; 1)$

**Yechish:** Sistemaning har bir qismidan 2 ni ayirib $-5 < -5x < -1$ ni olamiz. Bu tengsizlikni $-5$ ga bo'lamiz (bu holda tengsizliklar qarama-qarshisiga o'zgaradi, chunki $-5$ manfiy son) va natijada $1 > x > 0, 2$ yoki $0, 2 < x < 1$ ga ega bo'lamiz. **Javob:** $(0, 2; 1)$ (D).

18. (98-8-6) Qo'sh tengsizlikni yeching.

$$-4 < 2 - 4x < -2$$

A) $(-1, 5; -1)$    B) $(1; 2)$    C) $(0; 1)$    D) $(1; 1, 5)$

19. (99-8-9) $5 < x < 98$ tengsizlikni qanoatlantiruvchi va bo'luvchisi 12 ga teng bo'lgan nechta natural son mavjud?
A) 8     B) 10     C) 12     D) 6

20. (99-8-79) Tengsizlik nechta natural yechimga ega?

$$17, 556 : 5, 7 \leq y < 31, 465 : 3, 5$$

A) 1     B) 2     C) 4     D) 5

21. (98-8-1) Tengsizlikning barcha natural yechimlarini toping.

$$1256 : 314 < 9x - 32 \leq 2976 : 96$$

A) $4; 5; 6$    B) $5; 6; 7$    C) $6; 7; 8$    D) $7; 8$

22. (98-1-1) Tengsizlikning barcha natural yechimlarini toping.

$$6798 : 103 < 54 + 6x < 9156 : 109$$

A) $2; 3; 4$    B) $4; 5; 6$    C) $3; 4$    D) $4; 5$

23. (99-9-24) Tengsizlikning eng katta butun yechimi, eng kichik butun yechimidan qanchaga katta?

$$\begin{cases} 2x - 3 \leq 17 \\ 14 + 3x > -13 \end{cases}$$

A) 17     B) 19     C) 16     D) 18

**Yechish:** Sistemada ozod hadlarni tengsizlikning o'ng qismiga o'tkazib, berilgan sistemaga teng kuchli bo'lgan quyidagi sistemani olamiz:

$$\begin{cases} 2x \leq 20 \\ 3x > -27 \end{cases} \iff \begin{cases} x \leq 10 \\ x > -9 \end{cases} \iff -9 < x \leq 10.$$

Bu tengsizlikning eng katta butun yechimi 10, eng kichik butun yechimi $-8$ dir. Ularning farqi $10 - (-8) = 18$. **Javob:** 18 (D).

24. (98-10-40) Sistemaning eng katta butun va eng kichik butun yechimlari yig'indisini toping.

$$\begin{cases} 2x - 3 < 17 \\ 4x + 6 > 8 \end{cases}$$

A) 8     B) 11     C) 12     D) 10

25. (98-10-63) Tengsizliklar sistemasining butun yechimlari yig'indisini toping.

$$\begin{cases} -x - 5 < -2x - 2 \\ -2x + 2 > 3 - 3x \end{cases}$$

A) 0     B) 1     C) 2     D) 3

26. (98-11-25) Tengsizliklar sistemasining butun yechimlari yig'indisini toping.

$$\begin{cases} 0, 4(2x - 3) > x - 2 \\ 3x - 7 \geq x - 6 \end{cases}$$

A) 10     B) 5     C) 6     D) 8

## 5.3 Oraliqlar usuli

Kasr ratsional tengsizliklarni yechishning eng qo'lay usullaridan biri – oraliqlar usulidir. Bizdan

$$\frac{x^2 + 2x - 3}{x^2 + 2x - 8} > 0 \qquad (5.3)$$

kasr tengsizlikni yechish talab qilinsin. Ma'lumki, kasr musbat bo'lishi uchun uning maxraji va surati bir xil ishorali bo'lishi kerak. Shunday ekan (5.3) tengsizlik quyidagi ikki tengsizliklar sistemasiga teng kuchlidir:

$$\begin{cases} x^2 + 2x - 3 > 0 \\ x^2 + 2x - 8 > 0 \end{cases} \text{ yoki } \begin{cases} x^2 + 2x - 3 < 0 \\ x^2 + 2x - 8 < 0 \end{cases}$$

Bu sistemani yechish yana qismlarga ajratiladi. Agar

$$\frac{(x-1)(x-3)(x-5)}{(x+2)(x+4)(x+6)} > 0$$

ko'rinishdagi tengsizliklarni yechish talab qilinsa, ish yanada qiyinlashadi. Shuning uchun bunday ko'rinishdagi tengsizliklar odatda oraliqlar usuli bilan yechiladi. Oraliqlar usulining mohiyati quyidagicha: Agar $P(x)$ ko'phad bo'lib, uni

$$P(x) = p(x)(x - x_1)^{r_1}(x - x_2)^{r_2} \cdots (x - x_n)^{r_n} \quad (5.4)$$

ko'rinishda tasvirlash mumkin bo'lsin. Bunda $p(x)$ haqiqiy ildizlarga ega bo'lmagan ko'phad bo'lib, $x$ ning barcha qiymatlarida faqat yo musbat yo manfiy qiymatlar qabul qiladi. Aniqlik uchun $p(x) > 0$ va $x_1 < x_2 < \cdots < x_n$ bo'lsin. Agar $x > x_n$ bo'lsa, (5.4) dagi hamma ko'paytuvchilar musbat bo'lib, $P(x) > 0$ bo'ladi. Agar $(x - x_n)$ chiziqli ko'paytuvchi (5.4) da toq ($r_n = 2m - 1$ toq son) daraja bilan qatnashsa, u holda $x_{n-1} < x < x_n$ bo'lganda (5.4) dagi so'nggi ko'paytuvchi manfiy, boshqa hamma ko'paytuvchilar musbat bo'ladi. Bu holda $P(x) < 0$ bo'ladi. Bunda $P(x)$ ko'phad $x_n$ ildizdan o'tganda ishorasini o'zgartiradi deyiladi. Agar $(x - x_n)$ chiziqli ko'paytuvchi (5.4) da juft ($r_n = 2m$ juft son) daraja bilan qatnashsa, u holda $x_{n-1} < x < x_n$ bo'lganda (5.4) dagi barcha ko'paytuvchilar musbat bo'lib, $P(x) > 0$ bo'ladi. Bunda $P(x)$ ko'phad $x_n$ ildizdan o'tganda ishorasini o'zgartirmaydi deyiladi.

Xuddi shunday muhokama qilish usuli bilan quyidagi xulosaga kelish mumkin: $P(x)$ ko'phad, $x$ o'zgaruvchi $x_k$ ildizdan o'tganda $(x - x_k)$ chiziqli ko'paytuvchi toq darajada bo'lsa, ishorasini o'zgartiradi va juft darajada bo'lsa, ishorasini o'zgartirmaydi. Ko'phadning bu xossasi yuqori darajali tengsizliklarni yechishda foydalaniladi. Quyidagi ikki misolni qaraymiz:

*1-misol.* Tengsizlikni yeching.

$$P(x) = (x^2 + 2x + 3)(x-1)^2(x-3)^5(x-7)^9 \le 0. \quad (5.5)$$

**Yechish:** Bu yerda $p(x) = x^2 + 2x + 3 = (x+1)^2 + 2 > 0$. Ko'phad ildizlari 1; 3 va 7 larni sonlar o'qida belgilab chiqamiz. Natijada sonlar o'qi $(-\infty; 1)$, $(1; 3)$, $(3; 7)$ va $(7; \infty)$ oraliqlarga ajraydi (5.1-chizmaga qarang). Agar $x > 7$ bo'lsa, (5.5) dagi barcha ko'paytuvchilar musbat bo'lib $P(x) > 0$ bo'ladi. Endi $3 \le x \le 7$ bo'lsin, u holda $(x-7)^9 \le 0$ bo'lib, qolgan ko'paytuvchilar

musbat bo'ladi va $P(x) \le 0$. Endi $1 < x < 3$ bo'lsin, u holda $(x-3)$ toq daraja bilan qatnashgani uchun $P(x)$ ishorasini o'zgartiradi, ya'ni $P(x) > 0$ bo'ladi. Agar $x < 1$ bo'lsa, $P(x)$ ko'phad ishorasini o'zgartirmaydi, chunki $x - 1$ chiziqli ikki had (5.5) da juft daraja bilan qatnashyapti, ya'ni $P(x) > 0$ bo'ladi. $P(1) = 0$ bo'lganligi uchun 1 nuqta ham (5.5) tengsizlikning yechimi bo'ladi. Demak, (5.5) tengsizlikning yechimi $[3; 7] \cup \{1\}$ to'plamdan iborat.

*2-misol.* Tengsizlikni yeching.

$$\frac{(x-1)^2(x-3)^4(x-5)^3}{(x+2)^5(x+4)^7(x+6)^8} > 0. \qquad (4)$$

**Yechish:** Kasr surati va maxrajidagi ko'phadning ildizlari 1, 3, 5 va $-2$, $-4$, $-6$ larni sonlar o'qida belgilab chiqamiz va oraliqlarga $+$ va $-$ ishoralarni qo'yib chiqamiz (5.2-chizmaga qarang). Demak, (4) tengsizlikning yechimi $(-4; -2) \cup (5; \infty)$ to'plamdan iborat ekan.

### 1. Oraliqlar usuli.

1. (96-3-25) Tengsizlikni yeching.

$$(x - 2)(x + 3) > 0$$

A) $(-\infty; 2) \cup (3; \infty)$     B) $(-\infty; -3) \cup (2; \infty)$
C) $(-\infty; -2) \cup (3; \infty)$     D) $(-\infty; \infty)$

**Yechish:** Berilgan ifoda nolga aylanadigan $-3$, 2 nuqtalarni sonlar o'qida belgilab olamiz, natijada sonlar o'qi $(-\infty; -3)$, $(-3; 2)$ va $(2; \infty)$ oraliqlarga bo'linadi (5.3-chizmaga qarang). Agar $x > 2$ bo'lsa, ifodadagi barcha ko'paytuvchilar musbat bo'ladi. Ikkinchi $(-3; 2)$ oraliqda ifoda ishorasini o'zgartiradi, ya'ni manfiy qiymatlar qabul qiladi. Uchinchi $(-\infty; -3)$ oraliqda ifoda yana ishorasini o'zgartiradi, ya'ni ifoda musbat qiymatlar qabul qiladi. Shunday qilib tengsizlikning yechimi $(-\infty; -3) \cup (2; \infty)$ to'plamdan iborat bo'ladi. **Javob:** $(-\infty; -3) \cup (2; \infty)$ (B).

2. (96-11-26) Tengsizlikni yeching.

$$(x - 1)(x + 2) > 0$$

A) $(-\infty; 1) \cup (2; \infty)$    B) $(0; \infty)$
C) $(-\infty; -2) \cup (1; \infty)$    D) $(-\infty; \infty)$

3. (96-12-26) Tengsizlikni yeching.

$$(x + 2)(x - 3) > 0$$

A) $(-\infty; \infty)$    B) $(-\infty; -3) \cup (2; \infty)$
C) $(0; \infty)$    D) $(-\infty; -2) \cup (3; \infty)$

4. (97-5-23) Tengsizlikni yeching.

$$\frac{x - 1}{x - 2} \geq 0$$

A) $[1; 2)$    B) $(-\infty; 1) \cup (2; \infty)$
C) $(1; 2)$    D) $(-\infty; 1] \cup (2; \infty)$

5. (97-9-23) Tengsizlikni yeching.

$$\frac{x - 2}{x - 1} \leq 0$$

A) $(1; 2]$    B) $[1; 2)$    C) $[1; 2]$    D) $(-\infty; 1)$

6. (97-9-24) Tengsizlikni yeching.

$$\frac{(x + 3)(x - 5)}{x + 1} \geq 0$$

A) $(-3; -1] \cup [5; \infty)$    B) $(-3; -1) \cup [5; \infty)$
C) $[-3; -1) \cup [5; \infty)$    D) $[-3; -1)$

7. (01-3-35) Tengsizlikni yeching.

$$\frac{x + 1}{x - 2} \leq 0$$

A) $(-\infty; 1]$    B) $[-1; 2)$    C) $(-1; 2]$    D) $(2; \infty)$

8. Tengsizlikni yeching.

$$(x - 2)(x - 4)(x - 5)^2 \leq 0$$

A) $(-\infty; 2]$    B) $[2; 4] \cup \{5\}$    C) $[2; 4]$    D) $\emptyset$

9. (97-12-22) Tengsizlikning eng katta va eng kichik butun yechimlari yig'indisini toping.

$$\frac{(x + 4)(3 - x)}{(x - 2)^2} > 0$$

A) 1    B) $-1$    C) $-2$    D) 2

**Yechish:** Berilgan kasrning surati nolga aylanadigan $-4$ va 3, maxraji nolga aylanadigan 2 larga mos keluvchi nuqtalarni sonlar o'qida belgilab olamiz va oraliqlar usulini qo'llaymiz (5.4-chizmaga qarang). Chizmadan ma'lum bo'ldiki, bu tengsizlikning yechimi $(-4; 2) \cup (2; 3)$ to'plamdan iborat. Bu to'plamdagi eng katta butun son 1, eng kichik butun son esa $-3$ dir. Ularning yig'indisi $1 + (-3) = -2$. **Javob:** $-2$ (C).

10. (96-6-23) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(y + 6)(y + 2) < 0$$

A) 12    B) 20    C) $-12$    D) $-20$

11. (99-5-13) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(x - 1)(x + 1)^2 (x - 3)^3 (x - 4)^4 \leq 0$$

A) 6    B) 7    C) 8    D) 9

12. (00-9-21) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(x + 3)(x - 2)^2 (x + 1)^3 (x - 5)^4 \leq 0$$

A) 1    B) 2    C) 3    D) 4

13. (01-6-15) Tengsizlikning barcha butun sonlardagi yechimlari yig'indisini toping.

$$\frac{x - 4}{2x + 6} \leq 0$$

A) 7    B) 6    C) 8    D) 5

14. (02-4-12) Tengsizlikni qanoatlantiruvchi eng katta butun sonni toping.

$$\frac{x + 5}{(x + 6)^2} < 0$$

A) 6    B) $-6$    C) 5    D) $-7$

**2. Oraliqlar usuliga keltirib yeching.**

15. (98-6-23) Tengsizlikni yeching.

$$\frac{x^2 - 2x + 3}{x - 1} \geq 0$$

A) $(1; \infty)$    B) $[1; \infty)$    C) $(-\infty; 1)$    D) $(-\infty; 1]$

**Yechish:** Berilgan kasrning surati $x^2 - 2x + 3 = (x - 1)^2 + 2 > 0$ doim musbat. Shuning uchun uning maxraji ham musbat bo'lishi kerak, ya'ni $x - 1 > 0 \iff x > 1$. **Javob:** $(1; \infty)$ (A).

16. (99-3-13) Tengsizlikni yeching.

$$\frac{x + 2 - x^2}{x^3 + 1} \geq 0$$

A) $(-\infty; 2]$    B) $[2; \infty)$
C) $(-\infty; -1) \cup (-1; 2]$    D) $(-1; 2)$

17. (01-3-36) Tengsizlikni yeching.

$$\frac{x^2 - 4x + 5}{x - 2} \geq 0$$

A) $[2; \infty)$   B) $(-\infty; 2)$   C) $(-\infty; 2]$   D) $(2; \infty)$

18. (02-10-48) Tengsizlikni yeching.

$$(9x^2 + 12x + 4)(x - 2) < 0$$

A) $(-\infty; -\frac{2}{3}) \cup (-\frac{2}{3}; 2)$     B) $(-\infty; -2)$

C) $(2; \infty)$                D) $(-\frac{2}{3}; 2)$

19. (98-10-60) Tengsizlikning butun yechimlari nechta?

$$\frac{(x^2 + x + 1)(x^2 + 2x - 3)}{x^2 + 3x + 2} \leq 0$$

A) 5     B) 4     C) 3     D) cheksiz ko'p

20. (99-9-7) Tengsizlikning eng katta va eng kichik butun yechimlari ayirmasini toping.

$$\frac{(x + 3)(x - 7)}{2x^2 - x + 4} < 0$$

A) 10     B) 9     C) 8     D) 7

21. (00-4-33) Tengsizlikning eng katta butun manfiy va eng kichik butun musbat yechimlari ko'paytmasini toping.

$$\frac{x^4 - 3x^3 + 2x^2}{30 - x^2 - x} < 0$$

A) $-30$     B) $-35$     C) $-36$     D) $-42$

**Yechish:** Berilgan kasr surat va maxrajini ko'paytuvchilarga ajratamiz

$x^4 - 3x^3 + 2x^2 = x^2(x^2 - 3x + 2) = x^2(x - 1)(x - 2),$

$$30 - x^2 - x = -(x - 5)(x + 6).$$

Endi

$$\frac{x^4 - 3x^3 + 2x^2}{30 - x^2 - x} = \frac{x^2(x - 1)(x - 2)}{-(x - 5)(x + 6)} < 0$$

tengsizlikka oraliqlar usulini qo'llab $(-\infty; -6) \cup (1; 2) \cup (5; \infty)$ yechimni olamiz. Bu to'plamdagi eng katta butun manfiy son $-7$, eng kichik butun musbat son esa 6 dir. Ularning ko'paytmasi $-7 \cdot 6 = -42$. **Javob:** $-42$ (D).

22. (96-7-20) Tengsizlikning butun yechimlari ko'paytmasini toping.

$$2x^2 - 9x + 4 < 0$$

A) 0     B) 4     C) 24     D) 6

23. (97-3-20) Tengsizlikning butun yechimlari yig'indisini toping.

$$2x^2 \leq 5x + 12$$

A) 4     B) 9     C) 7     D) 5

24. (97-7-20) Tengsizlikning butun yechimlari ko'paytmasini toping.

$$3x^2 \leq 13x - 4$$

A) 12     B) 6     C) 30     D) 24

25. (97-10-20) Tengsizlikning butun yechimlari yig'indisini toping.

$$2x^2 - 3x \leq 9$$

A) 3     B) 4     C) 5     D) 6

26. (00-7-46) Tengsizliklar sistemasining eng katta va eng kichik yechimlari yig'indisini toping.

$$\begin{cases} x^2 - 3x - 4 \leq 0 \\ x^2 - 6x + 8 \leq 0 \end{cases}$$

A) 3     B) 4     C) 5     D) 6

27. (01-10-18) Kasrning qiymati manfiy bo'ladigan $x$ ning barcha qiymatlarini toping.

$$\frac{x^2 - 4x - 5}{2x - 5}$$

A) $(2, 5; 5)$          B) $(-\infty; -1] \cup (2, 5; 5]$
C) $(-\infty; -1)$       D) $(-\infty; -1) \cup (2, 5; 5)$

28. (02-2-2) Tengsizlikni qanoatlantiruvchi $n$ ning nechta butun qiymati bor?

$$(n^2 - 3)(n^2 - 21) < 0$$

A) 6     B) 5     C) 3     D) 4

**Yechish:** Berilgan tengsizlikni

$$(n - \sqrt{3})(n + \sqrt{3})(n - \sqrt{21})(n + \sqrt{21}) < 0$$

shaklda yozib olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(-\sqrt{21}; -\sqrt{3}) \cup (\sqrt{3}; \sqrt{21})$ yechimni olamiz. Agar biz $\sqrt{3} \approx 1,732$ va $\sqrt{21} \approx 4,582$ ekanligini hisobga olsak, $(\sqrt{3}; \sqrt{21})$ oraliqda $2, 3$ va 4 butun sonlari yotishiga ishonch hosil qilamiz. Xuddi shunday $(-\sqrt{21}; -\sqrt{3})$ oraliqda $-4, -3, -2$ butun sonlari yotadi. Demak, $n$ ning 6 ta butun qiymati berilgan tengsizlikni qanoatlantiradi. **Javob:** 6 (A).

29. (01-7-22) Nechta tub son

$$x^2 - 50 < 0$$

tengsizlikning yechimi bo'ladi?
A) 2     B) 3     C) 4     D) 5

30. (00-7-19) Tengsizlik o'rinli bo'ladigan $n$ ning barcha natural qiymatlari yig'indisini toping.

$$n^2(n^2 - n - 6) \leq 0$$

A) 4     B) 2     C) 5     D) 6

31. (03-4-14) Tengsizlikning eng katta va eng kichik yechimlari ayirmasini toping.

$$\frac{x^2 - 13x + 36}{x^4 + 25} \leq 0$$

A) 6     B) 4     C) 5     D) 7

32. (03-7-63) Tengsizlikning butun sonlardan iborat yechimlaridan eng kattasidan eng kichigining ayirmasini toping.

$$\frac{x^2 - x - 12}{x^2 - 2x - 35} \leq 0$$

A) 10     B) 12     C) 11     D) 9

33. (03-8-56) Tengsizlikni qanoatlantiruvchi natural sonlar nechta?

$$\frac{x^2 - 5x - 14}{x + 4} \leq 0$$

A) 7     B) 8     C) 9     D) 5

34. (03-3-24) Ushbu

$$\frac{(x-1)^2 + 2x - 2}{(x-5)^3} \geq 0$$

tengsizlikning $[-3; 8]$ kesmadagi butun sonlardan iborat yechimlari sonini aniqlang.

A) 3     B) 4     C) 5     D) 6

35. (03-11-75) Tengsizlikning butun sonlardan iborat yechimlaridan nechtasi $[-5; 6]$ kesmada joylashgan?

$$\frac{(x+4)^2 - 8x - 25}{(x-6)^2} > 0$$

A) 2     B) 3     C) 4     D) 5

**3. Kvadrat tengsizliklar.**

Dastlab kvadrat uchhad chiziqli ko'paytuvchilarga ajratiladi, keyin unga oraliqlar usuli qo'llaniladi.

36. (97-1-10) Tengsizlikni yeching.

$$(x-2)^2 + 3(x-2) \geq 7 - x$$

A) $[-2; 1]$     B) $[0; 1] \cup [3; \infty)$
C) $[-3; 3]$     D) $(-\infty; -3] \cup [3; \infty)$

**Yechish:** Berilgan tengsizlikdagi qavslarni ochamiz, tengsizlikning o'ng qismidagi $7-x$ ni tengsizlikning chap qismiga o'tkazamiz va o'xshash hadlarni soddalashtirib, natijada unga teng kuchli bo'lgan

$$x^2 - 9 \geq 0 \Longleftrightarrow (x-3)(x+3) \geq 0$$

tengsizlikni olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(-\infty; -3] \cup [3; \infty)$ yechimni olamiz. **Javob:** $(-\infty; -3] \cup [3; \infty)$ (D).

37. (97-6-10) Tengsizlikni yeching.

$$(x+2)(x-2) - 2(x-1) \leq 23 - 2x$$

A) $(-\infty; 5]$     B) $(0; 25]$
C) $[-5; 5]$     D) $[-\sqrt{21}; \sqrt{21}]$

38. (97-11-10) Tengsizlikni yeching.

$$2 \cdot (x-1) \cdot (x+1) - x(x+3) < 2 - 3x$$

A) $(-\infty; 2)$     B) $(-2; 2)$     C) $(0; 4)$     D) $(1; \infty)$

39. (01-2-26) $x^2 + 2x - 15 < 0$ tengsizlikning natural yechimlari ko'paytmasini toping.
A) 0     B) 2     C) 4     D) 6

**4. Tengsizlikning bir qismiga o'tkazing.**

40. (99-1-20) $\frac{1}{x} > x$ tengsizlikni yeching.
A) $(-\infty; -1) \cup (0; 1)$     B) $[0; 1)$
C) $(0; 1)$     D) $\emptyset$

**Yechish:** Tengsizlikning o'ng qismidagi $x$ ni tengsizlikning chap qismiga o'tkazamiz va umumiy maxraj beramiz va

$$\frac{1-x}{x} > 0$$

tengsizlikni olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(0; 1)$ yechimni olamiz. **Javob:** $(0; 1)$ (C).

41. (99-6-30) Tengsizlikni yeching.

$$\frac{x^2}{x+3} < x - 3$$

A) $(-\infty; -3)$     B) $(-3; 3)$     C) $(0; 3)$     D) $\emptyset$

42. (99-6-45) Tengsizlikni yeching.

$$\frac{5x + 8}{4 - x} < 2$$

A) $(-\infty; 0) \cup (4; \infty)$     B) $(-\infty; -4) \cup (0; 4)$
C) $[-4; 4]$     D) $\emptyset$

43. (00-4-32) Tengsizlikni yeching.

$$1 - \frac{6}{x} > \frac{2}{1-x}$$

A) $(0; 1) \cup (2; 3)$     B) $(-\infty; 0) \cup (1; 2) \cup (3; \infty)$
C) $(0; 1) \cup (3; \infty)$     D) $(-\infty; 1) \cup (2; 3) \cup (5; \infty)$

44. (01-5-22) Tengsizlikni yeching.

$$\frac{1}{x-1} \leq 2$$

A) $(-\infty; 1) \cup [1,5; \infty)$     B) $(1; 2]$
C) $(1; 2)$     D) $(1; 1,5]$

45. (02-12-12) Tengsizlikni yeching.

$$\frac{x^2 - 5x + 2}{x - 3} > x$$

A) $(-3; 1)$     B) $(1; 3)$     C) $(-1; 3)$     D) $(-\infty; 1)$

46. (03-1-66) Tengsizlikni yeching.

$$\frac{2}{x^2-9} < \frac{3}{x^2-16}$$

A) $(-\infty; \infty)$
B) $(-4; -3) \cup (3; 4)$
C) $(-\infty; -4) \cup (-3; 3) \cup (4; \infty)$
D) $(-\infty; -4) \cup (4; \infty)$

47. (03-5-17) Tengsizlikni yeching.

$$\frac{1}{x-2002} \le \frac{x}{x-2002}$$

A) $(-\infty; 1] \cup (2002; \infty)$     B) $(-\infty; 1]$
C) $(2002; \infty)$             D) $[1; 2002)$

48. (01-1-72) Ushbu $x \ge \dfrac{6}{x-5}$ tengsizlikni qanoatlantiruvchi eng kichik butun musbat yechimning eng kichik butun manfiy yechimga nisbatini toping.
A) $-1$     B) $-2$     C) $-0,5$     D) $-4$

49. (01-2-68) Tengsizlikning eng katta va eng kichik butun ildizlari ayirmasini toping.

$$(x^2 - x - 1)(x^2 - x - 7) \le -5$$

A) 2     B) 3     C) 4     D) 5

**Yechish:** Tengsizlikda $x^2 - x - 4 = t$ belgilash olsak, berilgan tengsizlik

$$(t+3)(t-3) + 5 \le 0 \iff (t-2)(t+2) \le 0$$

ko'rinish oladi. Yana eski o'zgaruvchiga qaytib

$$(x^2 - x - 4 - 2)(x^2 - x - 4 + 2) \le 0$$

ga ega bo'lamiz. Ko'paytmadagi kvadrat uchhadlarni ko'paytuvchilarga ajratamiz

$$x^2 - x - 6 = (x+2)(x-3), \; x^2 - x - 2 = (x+1)(x-2)$$

va berilgan tengsizlikka teng kuchli bo'lgan

$$(x+2)(x-3)(x+1)(x-2) \le 0$$

tengsizlikka ega bo'lamiz. Bu tengsizlikka oraliqlar usulini qo'llab $[-2; -1] \cup [2; 3]$ yechimni olamiz. Bu to'plamda eng katta butun son 3, eng kichik butun son esa $-2$ dir. Ularning ayirmasi $3 - (-2) = 5$. **Javob:** 5 (D).

50. (97-1-58) $x^4 < 9x$ tengsizlikning butun sonlardagi yechimi nechta?
A) 1     B) 2     C) 3     D) 4

51. (01-1-12) Nechta tub son

$$3 < \frac{5x-1}{2x-3} < 5$$

tengsizlikning yechimi bo'ladi?
A) 0     B) 1     C) 2     D) 3

52. (01-10-17) Nechta butun son

$$x^4 - 8x^2 + 7 \le 0$$

tengsizlikni qanoatlantiradi?
A) 0     B) 1     C) 2     D) 4

53. (02-1-21) $x > 1$ va $x^2 > x$ tengsizliklar teng kuchli bo'ladigan sonli oraliqni ko'rsating.
A) $(0; \infty)$    B) $(-\infty; 0)$    C) $(-\infty; \infty)$    D) $\emptyset$

**Yechish:** 1-tengsizlikni $x > 0$ (musbat songa ko'paytirilganda tengsizlik saqlanadi) ga ko'paytirsak 2-tengsizlik hosil bo'ladi. Demak, $x > 0$ da tengsizliklar teng kuchli bo'ladi. **Javob:** $(0; \infty)$ (A).

54. (02-1-63) Nechta tub son

$$2 < \frac{x+7}{2x-19} < 4$$

tengsizlikning yechimi bo'ladi?
A) 1     B) 13     C) 7     D) 3

55. (02-8-7) Tengsizlikning eng kichik butun yechimini toping.

$$\frac{x-10}{2-x} > 1$$

A) 3     B) 4     C) 1     D) $-2$

56. (02-10-13) Tengsizlikning manfiy bo'lmagan butun yechimlarini toping.

$$\frac{x+3}{x^2-4} - \frac{1}{x+2} < \frac{2x}{2x-x^2}$$

A) 1     B) $0; 1; 2$     C) $1; 2; 3$     D) $1; 2$

57. (03-1-14) Tengsizlikning butun yechimlari nechta?

$$x^4 - 10x^2 + 9 \le 0$$

A) 2     B) 3     C) 4     D) 6

58. (03-3-19) Tengsizlikning butun sonlardan iborat yechimlari nechta?

$$\frac{x^2 - 12x + 23}{(x+1)(x-4)} \le -\frac{2}{x-4}$$

A) 2     B) 3     C) 4     D) 5

59. (03-6-42) Tengsizlikni qanoatlantiruvchi $x$ ning barcha qiymatlarini toping.

$$\frac{x+1}{x} \le 1$$

A) $-1 \le x < 0$       B) $x < 0$
C) $-1 < x < 0$       D) $x > 0$

**5. Tengsizliklarning qo'llanilishi.**

60. (97-1-16) $k$ ning qanday qiymatlarida $k(x+1) = 5$ tenglamaning ildizi musbat bo'ladi?
A) $(0;\infty)$   B) $(0;5)$   C) $(-5;0)$   D) $(5;\infty)$

**Yechish:** Berilgan tenglama $x = 5 : k - 1$ yechimga ega. Masala shartiga ko'ra u musbat bo'lishi kerak, yani

$$\frac{5}{k} - 1 > 0 \iff \frac{5-k}{k} > 0.$$

Oraliqlar usulini qo'llasak, bu tengsizlikning yechimi $(0; 5)$ ekanligini olamiz. **Javob:** $(0; 5)$ (B).

61. (97-11-16) $b$ ning qanday qiymatlarida $b(2-x) = 6$ tenglamaning ildizi manfiy?
A) $b \in (-\infty;0)$      B) $b \in (0;3)$
C) $b \in (-3;0)$      D) $b \in [3;\infty)$

62. (99-10-4) Tenglamaning ildizlari manfiy bo'ladigan $k$ ning barcha natural qiymatlari yig'indisini toping.
$$(k-2)^2 \cdot y = k^2 - 25$$
A) 10      B) 13      C) 11      D) 8

63. (00-3-13) $k$ ning qanday qiymatlarida
$$\frac{4x-1}{x-1} = k+3$$
tenglama manfiy yechimga ega bo'ladi?
A) $(-\infty;-2)$      B) $(-\infty;-2) \cup (1;\infty)$
C) $(1;\infty)$      D) $(-2;1)$

64. (02-12-6) $m$ ning qanday qiymatlarida
$$4 - m = \frac{2}{x-1}$$
tenglamaning ildizlari musbat bo'ladi?
A) $(4;6)$      B) $(-\infty;1) \cup (1;4)$
C) $(-\infty;4) \cup (6;\infty)$      D) $(-\infty;2) \cup (4;\infty)$

65. (02-9-20) $t$ ning qanday qiymatlarida
$$3x - 4 = 2(x-t)$$
tenglama musbat ildizga ega?
A) $t > -2$   B) $t < 2$   C) $t \leq 1$   D) $t \geq 2$

66. (03-3-6) $k$ ning qanday qiymatlarida
$$\frac{3x+1}{x+1} = k-2$$
tenglama manfiy ildizga ega?
A) $(3;5)$      B) $(-\infty;3) \cup (5;\infty)$
C) $(2;4)$      D) $(1;3)$

67. (99-2-18) Tenglama yechimga ega bo'lmaydigan $k$ ning eng katta butun qiymatini toping.
$$kz^2 + 2(k-12)z + 2 = 0$$
A) 16      B) 18      C) 20      D) 17

**Yechish:** Agar $k = 0$ bo'lsa, bu tenglama $z = 1 : 12$ yechimga ega. Shuning uchun $k \neq 0$

ni qaraymiz. Kvadrat tenglama yechimga ega bo'lmasligi uchun uning diskriminanti $D = (2(k-12))^2 - 4 \cdot 2k < 0$ bo'lishi kerak. Bu kvadrat tengsizlikni yechib, $k \in (8; 18)$ ekanligini olamiz. $k$ ning eng katta butun qiymati 17. **Javob:** 17 (D).

68. (00-9-12) Tenglama ildizga ega bo'lmaydigan $m$ ning barcha natural qiymatlari yig'indisini hisoblang.
$$\frac{t-6}{m-8} = \frac{m}{t}$$
A) 20      B) 25      C) 28      D) 30

69. (00-3-19) $k$ ning qanday eng kichik butun qiymatida
$$x^2 - 2(k+2)x + 6 + k^2 = 0$$
tenglama ikkita turli haqiqiy ildizlarga ega bo'ladi?
A) $-2$      B) $-1$      C) 2      D) 1

### 5.3.1   Parametrli tengsizliklar

1. (96-3-78) Tengsizliklar sistemasi $a$ ning qanday qiymatlarida yechimga ega emas?
$$\begin{cases} ax > 5a - 1 \\ ax < 3a + 1 \end{cases}$$
A) $\{1\}$      B) $(-\infty;0) \cup [1;\infty)$
C) $(-\infty;0)$      D) $[1;\infty)$

**Yechish:** Berilgan sistema
$$5a - 1 < ax < 3a + 1 \tag{1}$$
qo'sh tengsizlikka teng kuchli. Quyidagi ikki
1) $a = 0$ va 2) $a \neq 0$ holni qaraymiz.
1-holda $-1 < 0 \cdot x < 1$ tengsizlikni hosil qilamiz va u $x$ ning barcha qiymatlarida o'rinli. Demak, bu holda yechim mavjud.

2-hol, $a \neq 0$ bo'lsin. Ma'lumki, $a < x < b$ oraliq bo'sh to'plam bo'lishi uchun $b \leq a$ bo'lishi kerak. Demak, (1) qo'sh tengsizlik yechimga ega bo'lmasligi uchun $5a - 1 \geq 3a + 1$ bo'lishi kerak. Bu yerdan
$$5a - 3a \geq 2 \iff 2a \geq 2 \iff a \geq 1$$
ekani kelib chiqadi. Demak, $[1;\infty)$. **Javob:** (D).

2. (98-10-61) $kx^2 + 2x + k + 2 > 0$ tengsizlik yechimga ega bo'lmaydigan $k$ ning butun qiymatlari orasidan eng kattasini toping.
A) $-1$   B) $-2$   C) eng kattasi yo'q   D) $-3$

3. (00-6-20) $a$ ning qanday qiymatlarida
$$\begin{cases} 3 - 7x < 3x - 7 \\ 1 + 2x < a + x \end{cases}$$
tengsizliklar sistemasi yechimga ega emas?
A) $a < 4$   B) $a \leq 1$   C) $a < 2$   D) $a \leq 2$

4. (96-9-19) Tengsizliklar sistemasi $a$ ning qanday qiymatlarida yechimga ega bo'lmaydi?

$$\begin{cases} ax > 7a - 3 \\ ax \leq 3a + 3 \end{cases}$$

A) $(1, 5; \infty)$      B) $[1, 5; \infty)$
C) $(-\infty; 0)$      D) $(-\infty; 0) \cup (1, 5; \infty)$

5. (96-13-19) Tengsizliklar sistemasi $b$ ning qanday qiymatlarida yechimga ega bo'lmaydi?

$$\begin{cases} bx \geq 5b - 3 \\ bx \leq 4b + 3 \end{cases}$$

A) $(6; \infty)$      B) $[6; \infty)$
C) $(-\infty; 0)$      D) $(-\infty; 0) \cup (6; \infty)$

6. (98-3-13) Tengsizlik yechimga ega bo'lmaydigan $k$ ning eng katta butun qiymatini toping.

$$kx^2 + 4x + k + 1 > 0$$

A) $-1$      B) eng kattasi yo'q
C) $-3$      D) $-2$

7. (00-5-33) $a$ ning qanday qiymatida

$$a(x - 1) > x - 2$$

tengsizlik $x$ ning barcha qiymatlarida o'rinli bo'ladi?
A) 0      B) 1      C) 2      D) 3

**Yechish:** Berilgan tengsizlik $(a - 1)x > a - 2$ tengsizlikka teng kuchli. Quyidagi ikki
1) $a = 1$ va 2) $a \neq 1$ holni qaraymiz.
1-holda $0 \cdot x > -1$ tengsizlikni hosil qilamiz va u $x$ ning barcha qiymatlarida bajariladi. Demak, $a = 1$ masala shartini qanoatlantiradi.

2-hol $a \neq 1$ bo'lsin. Dastlab $a > 1$ holni qaraymiz. Bu holda $a - 1 > 0$ bo'lganligi uchun $(a - 1)x > a - 2$ tengsizlikning yechimi $x > (a - 2) : (a - 1)$ dan iborat. Endi $a < 1$ holni qaraymiz. Bu holda $a - 1 < 0$ bo'lganligi uchun $(a - 1)x > a - 2$ tengsizlikning yechimi $x < (a - 2) : (a - 1)$ dan iborat. Demak, $a \neq 1$ holda tengsizlik $x$ ning barcha qiymatlarida o'rinli emas. **Javob:** $a = 1$ (B).

8. (99-9-17) $a$ ning qanday qiymatlarida

$$ax^2 + 8x + a < 0$$

tengsizlik $x$ ning barcha qiymatlarida o'rinli bo'ladi?
A) $(0; 4)$    B) $(-4; 0)$    C) $(-4; 4)$    D) $(-\infty; -4)$

9. (01-2-78) $n$ ning 10 dan oshmaydigan nechta natural qiymatida $nx^2 + 4x > 1 - 3n$ tengsizlik $x$ ning ixtiyoriy qiymatida o'rinli bo'ladi?
A) 10      B) 9      C) 8      D) 7

10. (03-8-12) $m$ ning qanday qiymatida

$$\frac{mx + 9}{x} \geq -10$$

tengsizlikning eng katta manfiy yechimi $-3$ ga teng bo'ladi?
A) $-9$      B) $-8$      C) $-7$      D) $-6$

### 5.3.2 Shartli tengsizliklar

1. (97-9-68) Agar $a < 0 < b$ va $|a| > |b|$ bo'lsa,

$$\frac{1}{a^3 + b^3}, \qquad \frac{1}{a^4 + b^3}, \qquad \frac{1}{a^3}$$

larni taqqoslang.
A) $\dfrac{1}{a^3} > \dfrac{1}{a^3 + b^3} > \dfrac{1}{a^4 + b^3}$
B) $\dfrac{1}{a^4 + b^3} > \dfrac{1}{a^3} > \dfrac{1}{a^3 + b^3}$
C) $\dfrac{1}{a^4 + b^3} > \dfrac{1}{a^3 + b^3} > \dfrac{1}{a^3}$
D) $\dfrac{1}{a^3 + b^3} > \dfrac{1}{a^3} > \dfrac{1}{a^4 + b^3}$

**Yechish:** $a < 0 < b$, $|a| > |b|$ ekanidan $a^3 < a^3 + b^3 < 0 < a^4 + b^3$ tengsizliklarni hosil qilamiz. Shuning uchun

$$\frac{1}{a^4 + b^3} > \frac{1}{a^3} > \frac{1}{a^3 + b^3}. \qquad \textbf{Javob:} \text{ (B).}$$

2. (96-6-11) Quyidagi tengsizliklardan qaysilari $a$ ning barcha qiymatlarida o'rinli?
1) $a^2 > 0$,      2) $a^2 - 10 < 0$
3) $(a - 5)^2 \geq 0$,      4) $\dfrac{1}{a^2} + a^2 > 2$
A) 1      B) 2      C) 1; 3 va 4      D) 3

3. (98-12-34) $a > b > 0$ shartni qanoatlantiruvchi $a$ va $b$ sonlar uchun quyidagi munosabatlardan qaysilari o'rinli?
1) $a^3 > ab^2$;      2) $a^4 \geq a^2 b^2$
3) $a^2 b^2 < b^4$;      4) $\dfrac{2}{a} > \dfrac{2}{b}$
A) 1      B) 1; 2      C) 3      D) 4

4. (99-5-24) Agar $x$ va $y$ sonlari uchun $x \cdot y = 20$ va $0 < x < 0, 8$ munosabat o'rinli bo'lsa, quyidagi tengsizliklardan qaysi biri doimo o'rinli bo'ladi?
A) $\dfrac{x}{y} < 20$      B) $x + y < 20$
C) $y < 16$      D) $y > 25$

5. (99-5-34) Agar $2 < a < 3$ va $-3 < b < -2$ bo'lsa, quyidagilarning qaysi biri doim o'rinli bo'ladi?
A) $a^2 b^2 - 50 < 0$      B) $\dfrac{(a + b)^2 - 2ab}{a - b} < 0$
C) $b^3 a^2 - 5 < 0$      D) $a^3 b^2 - 2 < 0$

6. (01-6-16) Agar

$$\begin{cases} p^2 + q^2 < 20 \\ pq < 22 \end{cases}$$

bo'lsa, $|p + q|$ ning butun qiymatlari nechta?
A) 5      B) 6      C) 9      D) 8

**Yechish:** Berilgan sistemaning 2-tengsizligini 2 ga ko'paytirib 1-tengsizlikka qo'shsak $p^2 + q^2 + 2pq < 64$ ni olamiz. Agar $|p + q| = t \geq 0$ desak, so'nggi tengsizlik $t^2 < 64$ ko'rinishni oladi. $t \geq 0$ shartdan $0 \leq t < 8$ ni olamiz. $[0; 8)$ oraliqda 8 ta butun son bor. **Javob:** 8 (D).

7. (99-8-15) Agar $-2 < a < -1$ va $-3 < b < -2,5$ bo'lsa, $a - b$ ayirma qaysi sonlar orasida bo'ladi?
A) $(0,5;2)$  B) $(1;1,5)$  C) $(-1,5;-1)$  D) $(-1,5;1)$

8. (00-4-31) Agar $a < -1$ bo'lsa, quyidagi keltirilgan ifodalardan qaysi birining qiymati eng katta bo'ladi?
A) $a^{-1}$      B) $a^{-3}$      C) $a^{-5}$      D) $a^3$

# 6 - bob. Modulli ifodalar

## 6.1   Modulli tenglamalar

Haqiqiy sonning moduli (absolyut qiymati) xossalari 1.4-bandda keltirilgan. Modul qatnashgan tenglamalarni yechishda qo'llaniladigan ba'zi qoidalarni keltiramiz.

**1.** $|f(x)| = f(x) \iff f(x) \geq 0$.

**2.** $|f(x)| = -f(x) \iff f(x) \leq 0$.

**3.** $|f(x)| = |g(x)| \iff \left[ \begin{array}{l} f(x) = g(x) \\ f(x) = -g(x). \end{array} \right.$

**4.** $|f(x)| = a \ (a \geq 0) \iff \left[ \begin{array}{l} f(x) = a \\ f(x) = -a. \end{array} \right.$

Noma'lumi modul belgisi ostida bo'lgan tenglamalarni yechishning quyidagi uch usulini bayon qilamiz.
*1-usul.* Modulning ta'rifidan foydalanib yechish.
*2-usul.* Tenglamaning ikkala tomonini kvadratga ko'tarish.
*3-usul.* Geometrik usul.

Bu usullarni misollarda ko'rib chiqamiz.

*1-misol.* $x^2 - 3|x| - 40 = 0$ tenglamani 1-usul yordamida yeching.

**Yechish:** Absolyut qiymat ta'rifiga asosan berilgan tenglama quyidagi sistemalarga teng kuchli:

$\left\{ \begin{array}{l} x \leq 0 \\ x^2 + 3x - 40 = 0 \end{array} \right.$ va $\left\{ \begin{array}{l} x \geq 0 \\ x^2 - 3x - 40 = 0. \end{array} \right.$

Birinchi sistemadagi kvadrat tenglamaning yechimlari $x_1 = -8$ va $x_2 = 5$ lardir. Ammo $x_2 = 5$ yechim $x \leq 0$ shartni qanoatlantirmaydi, shunung uchun 1-sistemaning yechimi $x_1 = -8$ dir. Ikkinchi sistemadagi kvadrat tenglamaning yechimlari $x_1 = -5$ va $x_2 = 8$ lardir. Bu yerda $x_2 = -5$ yechim $x \geq 0$ shartni qanoatlantirmaydi, shunung uchun 2-sistemaning yechimi $x_2 = 8$ dir. **Javob:** $x_1 = -8$, $x_2 = 8$.

*2-misol.* $|x| = |2x-5|$ tenglamani 2-usul yordamida yeching.

**Yechish:** $|x|^2 = x^2$ ayniyatga ko'ra, berilgan tenglama

$$x^2 = (2x - 5)^2 \iff 3x^2 - 20x + 25 = 0$$

tenglamaga teng kuchli. Bu kvadrat tenglamaning yechimlari $x_1 = 5/3$, $x_2 = 5$ dir. **Javob:** $x_1 = 5/3$, $x_2 = 5$.

Ba'zi misollarni yechishda "geometrik usul" tez natija beradi.

*3-misol.* $|x - 3| = 5$ tenglamani geometrik usul yordamida yeching.

**Yechish:** $|x-3|$ miqdor sonlar o'qida $x$ va 3 nuqtalar orasidagi masofani ifodalaydi. Demak, sonlar o'qida

koordinatasi 3 bo'lgan nuqtadan 5 birlik masofada yotuvchi nuqtalarga mos sonlarni topishimiz kerak. Sonlar o'qida koordinatasi 3 bo'lgan nuqtadan 5 birlik chapda $-2$ ni, 5 birlik o'ngda yotuvchi 8 ni (6.1-chizmaga qarang), topamiz. **Javob:** $x_1 = -2$, $x_2 = 8$.

$|x| = ax^2 + bx + c$ ko'rinishdagi tenglamalarni yechishda 1-usul (modul ta'rifidan) foydalanishni tavsiya qilamiz.

$|ax+b| = |cx+d|$ ko'rinishdagi tenglamalarni yechishda 2-usul (kvadratga ko'trish usuli) yaxshi natija beradi.

$|x-a| + |x-b| + |x-c| = d$ ko'rinishdagi tenglamalarni yechishda "geometrik usul" tez natija beradi.

1. (99-6-48) Tenglamani yeching.

$$|2 - 3x| - |5 - 2x| = 0$$

A) $-3; \dfrac{7}{5}$   B) $3; \dfrac{7}{5}$    C) $3; -1$    D) $-3; 0$

**Yechish:** Berilgan tenglamani

$$|2 - 3x| = |5 - 2x|$$

ko'rinishda yozamiz. Bu tenglama 3-qoidaga ko'ra 2 ta tenglamaga ajraladi:
1)  $2 - 3x = 5 - 2x$    uning yechimi $x = -3$.
2)  $2 - 3x = -(5 - 2x)$,    bu tenglamaning yechimi esa $x = \dfrac{7}{5}$.   **Javob:** $-3; \dfrac{7}{5}$ (A).

2. (97-1-75) Tenglamaning nechta ildizi bor?

$$|x + 1| = |2x - 1|$$

A) 4      B) 3      C) 2      D) 1

3. (97-6-71) Tenglamaning nechta ildizi bor?

$$|x| = |2x - 5|$$

A) 1      B) 2      C) 3      D) cheksiz ko'p

4. (02-10-10) Tenglamani yeching.

$$|x - 2| = 3 \cdot |3 - x|$$

A) $2,75; 3,5$      B) $2,75$      C) 2      D) $2,5$

5. (02-11-26) Tenglamaning ildizlari yig'indisini toping.

$$|x + 1| = 2|x - 2|$$

A) 7      B) 5      C) 4      D) 6

6. (02-12-11) Tenglamaning butun sonlardan iborat ildizi nechta.

$$|x - 1| \cdot |x + 2| = 4$$

A) 2      B) 3      C) 4      D) 1

7. (98-4-24) Tenglamaning barcha natural yechim-lari yig'indisini toping.

$$|x^2 - 8x + 7| = -7 + 8x - x^2$$

A) 8      B) 40      C) 25      D) 28

**Yechish:** Berilgan tenglamani yechish 2-qoidaga ko'ra quyidagi

$$x^2 - 8x + 7 \le 0 \iff (x - 1)(x - 7) \le 0$$

tengsizlikni yechishga teng kuchli. Bu tengsizlik oraliqlar usuli yordamida yechiladi, uning yechimi $[1; 7]$ kesmadan iborat. Bu kesmada 7 ta ( 1, 2, 3, 4, 5, 6, 7) natural son bor. Ularning yig'indisi $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$.   **Javob:** 28 (D).

8. (00-5-22) Tenglamani yeching.

$$|2x - 3| = 3 - 2x$$

A) $\dfrac{3}{2}$   B) $(-\infty; \dfrac{3}{2}]$   C) $(-\infty; \dfrac{3}{2})$   D) $(-\infty; \infty)$

9. (99-4-24) Tenglamaning nechta butun ildizi bor?

$$|x^2 - 2x| = 2x - x^2$$

A) 1     B) 2     C) 3     D) birorta ham ildizi yo'q

10. (98-1-8) $m$ ning qanday qiymatlarida $|m + 1| = m + 1$ tenglik o'rinli bo'ladi?
A) $m = -1$   B) $m \in R$   C) $m = 0$   D) $m \ge -1$

11. (98-8-8) $a$ ning qanday qiymatlarida

$$|a + 2| = -a - 2$$

tenglik o'rinli bo'ladi?
A) $a = -2$   B) $a \in \emptyset$   C) $a < -2$   D) $a \le -2$

12. (99-6-47) Tenglamaning ildizlari yig'indisini to-ping?
$$|x^2 + 5x| = 6$$

A) 10       B) $-6$       C) $-3$       D) $-10$

**Yechish:** Berilgan tenglamani yechish 4-qoidaga ko'ra quyidagi ikki

$$\left[ \begin{array}{l} x^2 + 5x = 6 \\ x^2 + 5x = -6. \end{array} \right. \iff \left[ \begin{array}{l} x^2 + 5x - 6 = 0 \\ x^2 + 5x + 6 = 0. \end{array} \right.$$

tenglama yechimlari bilan ustma-ust tushadi. Bi-rinchi $x^2 + 5x - 6 = 0$ kvadrat tenglamaning ildizlari $x_1 = -6$, $x_2 = 1$ lardir. Ikkinchi $x^2 + 5x + 6 = 0$ kvadrat tenglamaning ildizlari esa $x_1 = -3$, $x_2 = -2$. Ularning yig'indisi $-6 + 1 + (-3) + (-2) = -10$.   **Javob:** $-10$ (D).

13. (99-10-9) Tenglamaning manfiy ildizlari nechta?

$$\left( \frac{y}{6} + \frac{y}{3} + \frac{y}{2} \right)(y^2 - 3|y| + 2) = 0$$

A) 1       B) 2       C) 3       D) 4

14. (00-4-11) Tenglama ildizlari ko'paytmasini toping.

$$x^2 - 3|x| - 40 = 0$$

A) $-40$       B) 40       C) $-32$       D) $-64$

15. (99-2-14) Tenglama ildizlari ko'paytmasini toping.

$$(x - 2)^2 - 4|x - 2| + 3 = 0$$

A) 3       B) 15       C) $-3$       D) $-15$

16. (00-6-12) Tenglama ildizlari yig'indisini toping?

$$|1 - |1 - x|| = 0,5$$

A) 0       B) 4       C) 3       D) 1

**Yechish:** Berilgan tenglamani yechish 4-qoidaga ko'ra quyidagi ikki

$$\left[ \begin{array}{l} 1 - |1 - x| = 0,5 \\ 1 - |1 - x| = -0,5 \end{array} \right.$$

tenglama yechimlari bilan ustma-ust tushadi. Bi-rinchi $1 - |1 - x| = 0,5 \iff |1 - x| = 0,5$ tenglamaning ildizlari 4-qoidaga ko'ra $1 - x = 0,5$ hamda $1 - x = -0,5$ tenglamaning ildizlari bi-lan bir xil. Oxirgi ikki tenglamaning yechimlari $x_1 = 0,5$  hamda  $x_2 = 1,5$ dir. Xuddi shunday $1 - |1 - x| = -0,5 \iff |1 - x| = 1,5$ tenglama-ning ildizlari $1 - x = 1,5$ hamda $1 - x = -1,5$ tenglamaning ildizlari bilan bir xil. Bu tenglamalar ildizlar esa $x_3 = -0,5$ hamda $x_4 = 2,5$ dir. Ularning yig'indisi $0,5 + 1,5 + (-0,5) + 2,5 = 4$.
**Javob:** 4 (B).

17. (01-8-13) Tenglama ildizlari ko'paytmasini toping.

$$|3 - |2 + x|| = 1$$

A) 24       B) 48       C) $-12$       D) 0

18. (96-1-11) Ushbu $|y| : (-0,5) = -2,5$ tenglamani qanoatlantiradigan $y$ ning barcha qiymatlarini to-ping.
A) $0,5$   B) 5 va $-5$   C) $\dfrac{5}{4}$ va $-\dfrac{5}{4}$   D) 5

19. (96-9-61) $-4,8 : |x| = -0,5$ tenglikni qanoat-lantiruvchi $x$ ning barcha qiymatlarini toping.
A) $2,4$                B) $2,4$ va $-2,4$
C) $9,6$ va $-9,6$       D) $9,6$

20. (96-9-20) Tenglama nechta ildizga ega?

$$|x + 2| + |x| + |x - 2| = 4$$

A) 2     B) cheksiz ko'p     C) 1     D) 0

**Yechish:** Ma'lumki, $|a - b|$ miqdor $a$ va $b$ nuq-talar orasidagi masofani ifodalaydi. Barcha $x \in [-2; 2]$ larda $|x + 2| + |x| + |x - 2| = x + 2 + |x| - (x - 2) = 4 + |x|$ tenglik o'rinli. Bu yerdan $x = 0$ berilgan tenglamaning yechimi ekanligi ke-lib chiqadi. Agar $x \notin [-2; 2]$ bo'lsa, u holda yo $|x + 2|$ yo $|x - 2|$ to'rtdan katta bo'ladi. Shuning

uchun $|x+2|$, $|x|$, $|x-2|$ larning manfiymasligidan $|x+2|+|x|+|x-2| > 4$ tengsizlik bajariladi. Demak, $(-\infty; -2) \cup (2; \infty)$ to'plamda berilgan tenglama yechimga ega emas. Yuqoridagilardan tenglama yagona $x = 0$ yechimga ega ekanligi kelib chiqadi. **Javob:** 1 (C).

21. (96-12-77) Tenglama yechimlarining yig'indisini toping?

$$|x+4| + |x-2| + |x-3| = 7$$

A) 2   B) ildizi yo'q   C) 0   D) $-2$

22. (96-13-20) Tenglamaning ildizlari nechta?

$$|x-4| + |x-1| + |x+2| = 6$$

A) ildizi yo'q      B) 2      C) 3      D) 1

23. (98-3-19) Tenglamaning nechta ildizi bor?

$$x^2 + |x| - 2 = 0$$

A) 0      B) 1      C) 2      D) 3

24. (98-12-97) Tenglamaning ildizlari ko'paytmasini toping?

$$|x-1|^2 - 8 = 2|x-1|$$

A) 15      B) $-3$      C) 5      D) $-15$

25. (98-9-17) Agar $y^2 > x > 0$ bo'lsa,

$$|x - y^2| + |x + 9| - 25 = 0$$

tenglik $y$ ning qanday qiymatlarida o'rinli bo'ladi?
A) 4      B) $\pm 3$      C) $\pm 4$      D) 3

26. (01-3-5) Tenglama ildizlari yig'indisini toping?

$$|x| = x^2 + x - 4$$

A) $2 - \sqrt{5}$   B) $1 - 2\sqrt{5}$   C) $-1 - \sqrt{5}$   D) $1 - \sqrt{5}$

27. (98-2-15) Tenglamani yeching.

$$|z|z^4 - 27|z^2| = 0$$

A) $0; 3$   B) $3; -3$   C) $0; \pm 9$   D) $-3; 0; 3$

**Yechish:** $|a|^2 = a^2$ ayniyatga asosan $z^4 = |z|^4$ ham ayniyat bo'ladi. Shuning uchun berilgan tenglama

$$|z|^5 - 27|z|^2 = 0 \iff |z|^2(|z|^3 - 27) = 0$$

tenglamaga teng kuchli. Bu tenglamaning yechimlari $|z|^2 = 0$ hamda $|z|^3 - 27 = 0$ tenglama yechimlaridan iborat. So'nggi ikki tenglamaning yechimlari esa $z_1 = 0$ hamda $z_2 = -3$, $z_3 = 3$ dan iborat. **Javob:** $-3; 0; 3$ (D).

28. (01-9-42) Tenglamani yeching.

$$2|x| = \frac{1}{2}x - 1$$

A) 1      B) $\frac{2}{5}$      C) $-\frac{2}{3}$      D) $\emptyset$

29. (03-1-21) Tenglamani yeching.

$$|x| = x^2 - 6$$

A) 2; 3        B) $\pm 2$        C) $-3$        D) $\pm 3$

30. (03-3-16) Tenglamani yeching.

$$x|x| + 2x + 1 = 0$$

A) 1      B) $-1$      C) $1 - \sqrt{2}$      D) $1 + \sqrt{2}$

31. (02-2-16) Agar $|x - 2| + 3x = -6$ bo'lsa, $|x|$ ni toping.
A) 4        B) 3        C) 2        D) 6

32. (02-5-9) Tenglama ildizlari ko'paytmasini toping.

$$(2|x| - 1)^2 = |x|$$

A) $\frac{1}{16}$      B) $-\frac{1}{16}$      C) $\frac{1}{4}$      D) $-\frac{1}{4}$

33. (02-8-8) $|5 - x| = 2(2x - 5)$ bo'lsa, $5 + x$ ning qiymati nechaga teng?
A) 8        B) 7        C) 9        D) 11

## 6.2   Modulli tengsizliklar

Modul qatnashgan tengsizliklarni yechishda quyidagi teng kuchliliklardan foydalaniladi.

**1.** $|f(x)| < a$, $(a > 0) \iff -a < f(x) < a$.

**2.** $|f(x)| > a$, $(a > 0) \iff \left[ \begin{array}{l} f(x) > a \\ f(x) < -a. \end{array} \right.$

**3.** $|f(x)| < |g(x)| \iff f^2(x) < g^2(x) \iff$

$$\iff (f(x) - g(x))(f(x) + g(x)) < 0.$$

Noma'lumi modul belgisi ostida bo'lgan tengsizliklarni yechishning bir nechta usullari bor. Bu usullarni misollarda ko'rib chiqamiz.

*1-misol.* $|x - 1| \le 3$ tengsizlikni yeching.

**Yechish:** *1-usul – modul ta'rifi yordamida.* Absolyut qiymat ta'rifiga asosan berilgan tengsizlik quyidagi sistemalarga teng kuchli:

$\left\{ \begin{array}{l} x - 1 \le 0 \\ -(x - 1) \le 3 \end{array} \right.$   va   $\left\{ \begin{array}{l} x - 1 \ge 0 \\ x - 1 \le 3. \end{array} \right.$

Birinchi sistemaning 2-tengsizligini $-1$ ga ko'paytiramiz, natijada $-3 \le x - 1 \le 0$ qo'sh tengsizlikni olamiz. Bu tengsizlikning barcha qismlariga 1 ni qo'shib $-2 \le x \le 1$ ni olamiz. Demak, birinchi sistemaning yechimi $[-2; 1]$ kesmadan iborat. Ikkinchi sistema $0 \le x - 1 \le 3$ qo'sh tengsizlikka teng kuchli. Bu tengsizlikning barcha qismlariga 1 ni qo'shish orqali $1 \le x \le 4$ ni olamiz. Bu yechimlarni birlashtirib, berilgan tengsizlikning yechimi bo'lgan $[-2; 1] \cup [1; 4] = [-2; 4]$ ni olamiz. **Javob:** $[-2; 4]$.

*2-misol.* $|x - 1| \le 3$ tengsizlikni yeching.

**Yechish:** *2-usul – kvadratga ko'tarish.* Berilgan tengsizlikning ikkila tomoni ham $x$ ning barcha qiymatlarida nomanfiy bo'lganligi uchun, tengsizlikni kvadratga ko'tarib, o'ziga teng kuchli $|x - 1|^2 \le 3^2$ tengsizlikni hosil qilamiz. $|a|^2 = a^2$ ayniyatga ko'ra bu tengsizlik

$$x^2 - 2x - 8 \le 0 \iff (x + 2)(x - 4) \le 0 \qquad (1)$$

tengsizlikka teng kuchli. (1) tengsizlikka oraliqlar usulini qo'llab $[-2; 4]$ yechimni olamiz. **Javob:** $[-2; 4]$

*3-misol.* $|x - 1| \leq 2$ tengsizlikni yeching.

**Yechish:** *3-usul – "geometrik usul".* $|x-1|$ miqdor sonlar o'qida $x$ va 1 nuqtalar orasidagi masofani ifodalaydi. Demak, berilgan tengsizlikning yechimi sonlar o'qida koordinatasi 1 bo'lgan nuqtadan masofasi 2 birlik va undan kichik $x$ ning barcha qiymatlaridan iborat (6.2-chizmaga qarang). Sonlar o'qida koordinatasi 1 bo'lgan nuqtadan 2 birlik chapda $-1$ ni, 2 birlik o'ngda yotuvchi 3 ni, topamiz. Demak, yechim $[-1; 3]$ kesmadan iborat. **Javob:** $[-1; 3]$

1. (99-4-5) Tengsizlik nechta butun yechimga ega?

$$4 \leq |x| \leq 8$$

A) 12    B) 10    C) 8    D) 6

**Yechish:** Tengsizlikni yechishda 1-usuldan foydalanamiz. Berilgan tengsizlik yechimi
$$\begin{cases} x \leq 0 \\ 4 \leq -x \leq 8 \end{cases} \quad \text{va} \quad \begin{cases} x \geq 0 \\ 4 \leq x \leq 8 \end{cases}$$
sistema yechimlari birlashmasidan iborat. 1-sistemaning ikkinchi tengsizligini $-1$ ga ko'paytirib $-8 \leq x \leq -4$ ni olamiz. 2-sistemaning yechimi tayyor shaklda yozilgan, ularni birlashtirib $[-8; -4] \cup [4; 8]$ yechimni olamiz. Bu to'plamda $-8, -7, -6, -5, -4, 4, 5, 6, 7, 8$ butun sonlari yotadi, ular 10 ta. **Javob:** 10 (B).

2. (99-1-7) Tengsizlikni yeching.

$$1 < |x| < 4$$

A) $(-\infty; -4) \cup (4; \infty)$    B) $(-4; -1) \cup (1; 4)$
C) $(-\infty; -1) \cup (1; \infty)$    D) $(-1; 1)$

3. (03-5-20) Tengsizlikni yeching.

$$1 < |x - 2| < 3$$

A) $(-1; 1) \cup (3; 5)$    B) $(-1; 1)$
C) $(3; 5)$    D) $(-1; 5)$

4. (00-3-24) Tengsizlik nechta natural yechimga ega?

$$|x - 3| \leq 6 - x$$

A) 0    B) 1    C) 2    D) 4

5. (00-6-6) Tengsizlikni yeching.

$$|x^2 - 5| < 4$$

A) $(-3; 0) \cup (0; 3)$    B) $(-3; 3)$
C) $(-3; -1) \cup (1; 3)$    D) $(-3; -1)$

6. (00-10-70) Tengsizlikni yeching.

$$\frac{|x - 3|}{x^2 - 5x + 6} \geq 2$$

A) $[\frac{3}{2}; 2)$    B) $[\frac{5}{2}; 4)$    C) $\emptyset$    D) $[-10; 10]$

7. (01-3-7) Tengsizlikni qanoatlantiruvchi butun sonlarning yig'indisini aniqlang.

$$x^2 - 3|x| - 4 \leq 0$$

A) 0    B) 2    C) 3    D) 1

8. (02-11-23) Tengsizlikning butun yechimlari nechta?

$$|x^2 - 3| < 2$$

A) 2    B) 3    C) 4    D) 5

9. (03-12-69) Tengsizlikning butun yechimlari nechta?

$$x^2 - 2|x| < 3$$

A) 7    B) 6    C) 5    D) 4

10. (98-10-66) Tengsizlikning butun yechimlari nechta?

$$2 \cdot |x - 1| \leq |x + 3|$$

A) 6    B) 5    C) cheksiz ko'p    D) 0

**Yechish:** Tengsizlikni yechishda 2-usuldan foydalanamiz. Tengsizlikning har ikkala qismini kvadratga ko'taramiz va hadlarni tengsizlikning chap qismiga o'tkazamiz:

$$(2x - 2)^2 - (x + 3)^2 \leq 0.$$

Qisqa ko'paytirish formulalarining 3-dan foydalanib, ifodani ko'paytuvchilarga ajratamiz:

$$(2x - 2 + x + 3)(2x - 2 - x - 3) \leq 0$$

$$\iff (3x + 1)(x - 5) \leq 0.$$

Bu tengsizlikni oraliqlar usuli bilan yechib, $[-\frac{1}{3}; 5]$ yechimni olamiz. Bu kesmada $0, 1, 2, 3, 4, 5$ butun sonlari yotadi, ular 6 ta. **Javob:** 6 (A).

11. (01-12-17) Tengsizlikni yeching.

$$|x + 1| > 2|x + 2|$$

A) $(-2; -1)$    B) $[-3; -1]$
C) $(-3; -\frac{5}{3})$    D) $(-3; 0)$

12. (03-7-22) Tengsizlikni yeching.

$$|x - 4| < |x + 4|$$

A) $(-4; 4)$    B) $(0; 4) \cup (4; \infty)$
C) $(0; \infty)$    D) $(-\infty; -4) \cup (-4; 0)$

13. (01-5-24) Tengsizlikni butun yechimlari yig'indisini toping.

$$|5 - 2x| \leq 3$$

A) 10    B) 15    C) 6    D) 3

14. (02-1-47) Tengsizlikni qanoatlantiruvchi butun son-
lar nechta?
$$|3x + 8| \leq 2$$
A) 1      B) 2      C) 3      D) 4

15. (96-3-26) $|x - 1| \geq 2$ tengsizlikni yeching.
A) $(-\infty; -1]$          B) $[-1; 3]$
C) $(-\infty; -1] \cup [3; \infty)$     D) $[1; 3]$

**Yechish:** Misolni 3-usul, ya'ni geometrik usul-
dan foydalanib yechamiz. $|x - 1|$ miqdor son-
lar o'qida $x$ va 1 nuqtalar orasidagi masofani ifo-
dalaydi. Demak, berilgan tengsizlikning yechimi
sonlar o'qida koordinatasi 1 bo'lgan nuqtadan ma-
sofasi 2 birlik va undan katta $x$ ning barcha qiy-
matlaridan iborat (6.3-chizmaga qarang). Son-
lar o'qida koordinatasi 1 bo'lgan nuqtadan 2 bir-
lik chapda $-1$ ni, 2 birlik o'ngda yotuvchi 3 ni,
topamiz. Shunday qilib tengsizlikning yechimi
$(-1; 3)$ intervalning tashqarisi bo'ladi. Demak,
yechim $(-\infty; -1] \cup [3; \infty)$ dan iborat. **Javob:**
$(-\infty; -1] \cup [3; \infty)$. (C)

16. (96-7-8) Tengsizlik nechta butun yechimga ega?
$$|x - 2| \leq 5$$
A) 11      B) 10      C) 8      D) 7

17. (96-11-27) Tengsizlikni yeching.
$$|x - 1| \geq 1$$
A) $[0; 2]$          B) $(-\infty; 0] \cup [2; \infty)$
C) $[-2; 0]$        D) $[2; \infty)$

18. (96-12-27) Tengsizlikni yeching.
$$|x - 1| \leq 2$$
A) $(-\infty; 3]$        B) $(-\infty; -1] \cup [3; \infty)$
C) $[-1; 3]$        D) $[1; 3]$

19. (97-7-8) Tengsizlik nechta butun yechimga ega.
$$|x + 2| \leq 3$$
A) 5      B) 6      C) 7      D) 4

20. (97-1-73) Tengsizlikning eng katta natural yechi-
mini toping.
$$|3x - 7| < 5$$
A) 4      B) 3      C) 2      D) 1

21. (97-3-8) Tengsizlik nechta butun yechimga ega.
$$|3 - x| < 4$$
A) 4      B) 5      C) 6      D) 7

22. (97-10-8) Tengsizlik nechta butun yechimga ega?
$$|4 - x| < 6$$
A) 3      B) 5      C) 8      D) 11

23. (98-5-23) Tengsizlikning eng kichik natural yechi-
mini toping.
$$|x - 7| \leq 1$$
A) 5      B) 7      C) 8      D) 6

24. (99-7-24) Tengsizlikning eng kichik natural yechi-
mini toping.
$$|x - 6| \leq 8$$
A) 2      B) 7      C) 3      D) 1

25. (99-9-18) Ushbu $|x - 4| \leq 12$ tengsizlikning eng
kichik va eng katta butun yechimlari yig'indisini
toping.
A) 6      B) 8      C) $-6$      D) $-8$

## 6.3 Modulli tenglamalar va tengsizlik-
lar sistemasi

1. (98-8-25) $b$ ning qanday qiymatlarida tenglamalar
sistemasi yagona yechimga ega?
$$\begin{cases} x = 3 - |y| \\ 2x - |y| = b \end{cases}$$
A) $b = 0$      B) $b > 0$      C) $b < 1$      D) $b = 6$

**Yechish:** Agar $(x_0; y_0)$ berilgan sistemaning ye-
chimi bo'lsa, u holda $|y| = |-y|$ tenglikka ko'ra,
$(x_0; -y_0)$ ham uning yechimi bo'ladi. Demak,
sistema yagona yechimga ega bo'lishi uchun $y_0 =
0$ bo'lishi kerak ekan. Uni sistemaning birinchi
tenglamasiga qo'yib $x = 3$ ekanini, ikkinchi tengla-
madan esa $b = 6$ ekanini hosil qilamiz. **Javob:**
$b = 6$ (D).

2. (98-1-25) $a$ ning qanday qiymatlarida
$$\begin{cases} 3|x| + y = 2 \\ |x| + 2y = a \end{cases}$$
sistema yagona yechimga ega?
A) $a = 0$   B) $a > 0$   C) $a = 2$   D) $a = 4$

3. (98-8-23) Agar
$$\begin{cases} x + 2|y| = 3 \\ x - 3y = 5 \end{cases}$$
bo'lsa, $x - y$ ning qiymatini toping.
A) 3      B) 2      C) 1      D) $-1$

4. (98-1-23) Agar
$$\begin{cases} |x| + y = 2 \\ 3x + y = 4 \end{cases}$$
bo'lsa, $x + y$ ning qiymatini toping.
A) 3      B) 1      C) $2,5$      D) 2

5. (00-1-18) Tengsizliklar sistemasini yeching.

$$\begin{cases} x \geq 3 \\ |x - 3| \leq 1 \end{cases}$$

A) $2 \leq x \leq 3$      B) $-2 \leq x \leq 4$
C) $3 \leq x \leq 4$      D) $x \leq 4$

**Yechish:** 6.2-dagi 1-qoidaga ko'ra berilgan sistemaning 2-tengsizligi $-1 \leq x - 3 \leq 1$ qo'sh tengsizlikka teng kichli. Bu tengsizlikning har bir qismiga 3 ni qo'shib $2 \leq x \leq 4$ ni olamiz. Sistemaning 1-tengsizligi $x \geq 3$ ni hisobga olsak, $3 \leq x \leq 4$ javobni olamiz. **Javob:** $3 \leq x \leq 4$ (C).

6. (02-10-55) Tengsizliklar sistemasini yeching.

$$\begin{cases} |2x - 3| \leq 1 \\ 5 - 0,4x > 0 \end{cases}$$

A) $[1; 2]$            B) $(-\infty; 2]$
C) $(-\infty; 1] \cup (2; \infty)$      D) $(-0, 4; 2)$

7. (01-7-20) Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} |x| + |y| = 1 \\ x^2 + y^2 = 4 \end{cases}$$

A) 1      B) 2      C) 4      D) $\emptyset$

8. (02-12-17) Agar

$$\begin{cases} |x - 1| + |y - 5| = 1 \\ y = 5 + |x - 1| \end{cases}$$

bo'lsa, $x + y$ qanday qiymatlar qabul qilishi mumkin?
A) 6 yoki 8    B) 7    C) 8 yoki 10    D) 6 yoki 7

9. (00-7-20) Agar

$$\begin{cases} (x - 2)^2 + |y| = 4 \\ |x - 2| + |y| = 2 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.
A) 4 yoki 2 yoki 0      B) 0 yoki 3
C) 2 yoki 4            D) 0 yoki 4

10. (03-10-31) Agar

$$\begin{cases} |x + y| = 5 \\ xy = 4, 75 \end{cases}$$

bo'lsa, son o'qida $x$ va $y$ sonlari orasidagi masofani toping.
A) $\sqrt{6}$     B) $\sqrt{3}$     C) $\sqrt{5}$     D) $\sqrt{7}$

# 7 -bob. Irratsional tenglama va tengsizliklar

## 7.1 Irratsional tenglamalar

Noma'lumi ildiz belgisi ostida bo'lgan tenglamalar *irratsional tenglamalar* deyiladi. Masalan,

$$\sqrt[4]{x - 3} = 3 - \sqrt{x}; \quad (5 - x)\sqrt{x - 3} = 3 - \sqrt[3]{2x}$$

tenglamalar irratsional tenglamalardir. Irratsional tenglamalarni yechish ma'lum bir shakl almashtirishlar yordamida ularni ratsional tenglamalarga keltirishga asoslangan. Radikallardan (ildiz belgisidan) qutilish maqsadida tenglamaning ikkala qismini bir xil darajaga ko'tariladi. Lekin darajaga ko'targanda chet ildizlar hosil bo'lishi mumkin. Shuning uchun oxirgi tenglamani yechishda topilgan ildizlarni, berilgan irratsional tenglamaning o'ziga qo'yib tekshirib ko'rish lozim.

Irratsional tenglamalarni yechishda ko'p qo'llaniladigan quyidagi tengkuchliliklarni keltiramiz.

1. $\sqrt[2k]{f(x)} = \varphi(x) \iff \begin{cases} f(x) = [\varphi(x)]^{2k} \\ \varphi(x) \geq 0 \end{cases}$

2. $\sqrt[2k+1]{f(x)} = \varphi(x) \iff f(x) = [\varphi(x)]^{2k+1}$

Irratsional tenglamalarni yechish usullarini misollarda namoyish qilamiz.

*1-misol.* $\sqrt{x + 2} + x = 0$ tenglamani yeching.

**Yechish:** Tenglamada $x$ ni tenglikning o'ng qismiga o'tkazamiz va ikkala qismini kvadratga ko'taramiz, natijada

$$x + 2 = (-x)^2 \iff x^2 - x - 2 = 0$$

kvadrat tenglamaga kelamiz. Bu kvadrat tenglamaning ildizlari $x_1 = -1$ va $x_2 = 2$ lardir. Topilgan yechimlarni berilgan tenglamaga qo'yamiz. Dastlab $x_1 = -1$ ni qo'yamiz: $\sqrt{-1 + 2} + (-1) = 1 - 1 = 0$. Endi $x_2 = 2$ ni $\sqrt{2 + 2} + 2 = 2 + 2 = 4 \neq 0$. Demak, $x_2 = 2$ chet ildiz ekan, $x_1 = -1$ esa tenglamani qanoatlantiradi. **Javob:** $x = -1$.

*2-misol.* $\sqrt{x + 1} = -7$ tenglamani yeching.

**Yechish:** Arifmetik ildizning manfiymasligidan $\sqrt{x + 1} \geq 0$ ekanligi kelib chiqadi. Tenglamaning o'ng qismi $-7$ esa manfiy son. Shuning uchun tenglama ildizga ega emas. **Javob:** $\emptyset$.

Xuddi shunday ko'rsatish mumkinki,

$$|x^2 - 1| + \sqrt{2x + 1} = -1$$

irratsional tenglama ham yechimga ega emas.

*3-misol.* Quyidagi tenglamani yeching:

$$x^2 - 3x + \sqrt{x^2 - 3x + 5} = 7. \tag{1}$$

**Yechish:** Tenglamada $x^2 - 3x = t$ belgilash olamiz, natijada berilgan tenglama $t + \sqrt{t + 5} = 7$ ko'rinishga ega bo'ladi. Bu tenglamada $t$ ni tenglikning o'ng qismiga o'tkazib, keyin tenglikning har ikkala qismini kvadratga ko'tarib, $t + 5 = (7 - t)^2$ kvadrat tenglamaga kelamiz. Bu kvadrat tenglamani yechib $t_1 = 4$, $t_2 = 11$ ni olamiz. $t_2 = 11$ ildiz $t + \sqrt{t + 5} = 7$ irratsional tenglamani qanoatlantirmaydi. Shuning uchun faqat $t_1 = 4$ qiymatni $x^2 - 3x = t$ belgilashga qo'yib,

$$x^2 - 3x = t_1 \iff x^2 - 3x = 4$$

kvadrat tenglamaga ega bo'lamiz. Bu kvadrat tenglamaning ildizlari $x_1 = -1$, $x_2 = 4$ lardir. Tekshirish mumkinki, ular (1) tenglamani ham qanoatlantiradi. **Javob:** $x_1 = -1$, $x_2 = 4$.

1. (98-9-19) Tenglama ildizlari yig'indisini toping.

$$\sqrt{x^4 + 5x^2} = -3x$$

A) 0     B) $-2$     C) $-4$     D) 2

**Yechish:** 1-qoidaga ko'ra berilgan tenglama

$$\begin{cases} x^4 + 5x^2 = 9x^2 \\ -3x \geq 0 \end{cases}$$

sistemaga teng kuchli. Uning birinchi tenglamasini

$$x^4 - 4x^2 = 0 \iff x^2(x^2 - 4) = 0$$

shaklda yozib olamiz. Bu tenglamaning yechimlari $x_1 = 0$, $x_2 = -2$, $x_3 = 2$ lardir. Endi $-3x \geq 0$ shartni tekshiramiz. Uni faqat $x_1 = 0$ va $x_2 = -2$ sonlar qanoatlantiradi. Shunday qilib, berilgan tenglamaning ildizlari $x_1 = 0$ va $x_2 = -2$ ekan. Ular yig'indisi $0 + (-2) = -2$. **Javob:** $-2$ (B).

2. (97-1-72) Tenglamani yeching.

$$\sqrt{x + 2} + x = 0$$

A) $-1$     B) $-2$     C) 2     D) 0

3. (97-5-39) Tenglamalar sistemasini yeching.

$$\begin{cases} \sqrt{(x + 2)^2} = x + 2 \\ \sqrt{(x - 2)^2} = 2 - x \end{cases}$$

A) $x \geq -2$     B) $x < 2$
C) $x \leq 2$     D) $-2 \leq x \leq 2$

4. (97-7-61) Ushbu $\sqrt{3 + 2x} = -x$ tenglik $x$ ning qanday qiymatlarida o'rinli?
A) $-1$     B) 1     C) $-3$     D) 3

5. (98-2-21) Agar $\sqrt{x^4 - 9x^2} = -4x$ tenglamaning katta ildizi $x_0$ bo'lsa, $x_0 + 10$ nechaga teng?
A) 10     B) 12     C) 20     D) 15

**Yechish:** Berilgan tenglamaning har ikkala tomonini kvadratga ko'tarib, o'ng tomondagi hadni chap tomonga o'tkazib,

$$x^4 - 25x^2 = 0 \iff x^2(x^2 - 25) = 0$$

tenglamani olamiz. Bu tenglamaning ildizlari $x_1 = -5$, $x_2 = 0$, $x_3 = 5$ lardir. Lekin $x_3 = 5$ dastlabki tenglamani qanoatlantirmaydi. Shuning uchun tenglamaning katta ildizi $x_2 = 0$ bo'ladi. $x_0 + 10 = 0 + 10 = 10$. **Javob:** 10 (A).

6. (99-2-19) Tenglamaning ildizlari yig'indisini toping.
$$\sqrt{x^2 - 3x + 5} + x^2 - 3x = 7$$

A) 4     B) $-3$     C) 3     D) $-4$

7. (99-6-41) $\sqrt{a} - \sqrt{b} = 4$ va $a - b = 24$ bo'lsa, $\sqrt{a} + \sqrt{b}$ nimaga teng.
A) 6     B) 4     C) 5     D) 3

8. (99-5-15) Tenglamaning natural ildizlari nechta?

$$\sqrt{(3x - 13)^2} = 13 - 3x$$

A) 0     B) 1     C) 2     D) 4

9. (99-8-3) Tenglamani yeching.

$$\sqrt{x + 1} + \sqrt{2x + 3} = 1$$

A) $-1$     B) 3     C) $-1; 3$     D) 1

10. (99-9-11) Tenglama ildizlari o'rta arifmetigini toping.

$$(x^2 - 25)(x - 3)(x - 6)\sqrt{4 - x} = 0$$

A) $4\dfrac{1}{3}$     B) $1\dfrac{1}{3}$     C) $\dfrac{2}{3}$     D) $4\dfrac{1}{2}$

11. (99-9-12) Tenglama ildizlari ko'paytmasini toping.

$$\sqrt{x^2 + 77} - 2\sqrt[4]{x^2 + 77} - 3 = 0$$

A) $-3$     B) 3     C) 4     D) $-4$

**Yechish:** Tenglamada $\sqrt[4]{x^2 + 77} = t$ belgilash olamiz, natijada berilgan tenglama $t^2 - 2t - 3 = 0$ ko'rinishga ega bo'ladi. Bu kvadrat tenglama yechimlari $t_1 = -1$, $t_2 = 3$ lardir. Yechimlarni $\sqrt[4]{x^2 + 77} = t$ belgilashga qo'yib, $\sqrt[4]{x^2 + 77} = -1$ va $\sqrt[4]{x^2 + 77} = 3$ tenglamalarni hosil qilamiz. Ammo $\sqrt[4]{x^2 + 77} = -1$ tenglama yechimga ega emas (chunki tenglamaning chap qismi musbat, o'ng qismi esa manfiy). Ikkinchi tenglamaning har ikkala qismini 4-darajaga ko'tarib,

$$x^2 + 77 = 3^4 \iff x^2 = 4$$

tenglamaga ega bo'lamiz. Bu tenglamaning yechimlari $x_1 = -2$, $x_2 = 2$ lardir. Tekshirish natijasi ko'rsatadiki, $x_1 = -2$, $x_2 = 2$ lar dastlabki tenglamani qanoatlantiradi. Ularning ko'paytmasi $(-2) \cdot 2 = -4$. **Javob:** $-4$ (D).

12. (00-1-19) Agar

$$\sqrt{1 - \frac{1}{x}} = \frac{x - 1}{x} - 6$$

bo'lsa, $6\dfrac{1}{8} + x$ ning qiymatini hisoblang.
A) $-7$     B) 6     C) 7     D) $-6$

13. (00-2-19) Tenglamani yeching.

$$\sqrt{(2x - 1)^2 (3 - x)} = (2x - 1)\sqrt{3 - x}$$

A) $[0, 5; 3]$    B) $[0; 3]$    C) $[1; 3]$    D) $(-\infty; 0, 5]$

**Yechish:** Tenglama $x \leq 3$ da ma'noga ega. Demak, berilgan tenglama

$$\begin{cases} \sqrt{(2x - 1)^2(3 - x)} = (2x - 1)\sqrt{3 - x} \\ x \leq 3 \end{cases}$$

sistemaga teng kuchli. Uning birinchi tenglamasi $x \leq 3$ shartda

$$|2x - 1|\sqrt{(3-x)} - (2x-1)\sqrt{3-x} = 0$$

tenglamaga teng kuchli. Bu tenglama esa

$$\sqrt{3-x} \cdot (|2x-1| - (2x-1)) = 0$$

ga teng kuchli. Bu tenglama yechimlari esa $\sqrt{3-x} = 0$ hamda $|2x-1| = 2x-1$ tenglama yechimlari bilan ustma-ust tushadi. 1-tenglamadan $x = 3$ ni olamiz. 2-tenglik modul ta'rufiga ko'ra (6.1-ning 1-qoidasiga qarang) $2x - 1 \geq 0$ shartda o'rinli. Bu tengsizlikning yechimi $x \geq 0,5$ dan iborat. Agar $x \leq 3$ shartni hisobga olsak, $x \in [0,5\,;3]$ ni olamiz. **Javob:** $[0,5\,;3]$ (A).

14. (00-3-10) Tenglamani yeching.

$$3\sqrt{2x} - 5\sqrt{8x} + 7\sqrt{18x} = 28$$

A) 1    B) 2    C) 3    D) 4

15. (00-3-22) Tenglama ildizlari yig'indisini toping.

$$\sqrt{x+1} + \sqrt{2x+3} = 1$$

A) 2    B) 3    C) 4    D) $-1$

16. (00-4-7) Tenglamani yeching.

$$\frac{2\sqrt{x} - \sqrt{2x}}{2} + 3 = \sqrt{x} + 1$$

A) 8    B) 4    C) 9    D) 1

17. (00-5-29) Tenglamani yeching.

$$\sqrt{x^2 - x - 2} = x - 3$$

A) 5    B) 2,2    C) 4    D) $\emptyset$

18. (00-6-33) Agar

$$\sqrt{3x^2 - 6x + 16} = 2x - 1$$

bo'lsa, $x^2 \cdot (x+2)$ ning qiymatini toping.
A) $-75$    B) $-45$    C) 15    D) 45

19. (00-8-5) Tenglamani yeching.

$$(x^2 - 9)\sqrt{x+1} = 0$$

A) $-1;3$    B) $\pm 3$    C) $\pm 3;1$    D) 2

20. (00-8-25) Agar

$$\sqrt{8-a} + \sqrt{5+a} = 5$$

bo'lsa, $\sqrt{(8-a)(5+a)}$ ning qiymatini toping.
A) 6    B) 20    C) 12    D) 10

**Yechish:** Tenglamaning ikkala qismini kvadratga ko'taramiz, natijada

$$8 - a + 2\sqrt{8-a}\sqrt{5+a} + a + 5 = 25$$

$$\iff 2\sqrt{(8-a)(5+a)} = 12$$

tenglikni olamiz. Bu yerdan $\sqrt{(8-a)(5+a)} = 6$ ni olamiz. **Javob:** 6 (A).

21. (00-8-26) Agar

$$\sqrt{25 - x^2} + \sqrt{15 - x^2} = 5$$

bo'lsa, $\sqrt{25 - x^2} - \sqrt{15 - x^2}$ ifodaning qiymatini toping.
A) 2    B) 3    C) 5    D) 6

22. (00-9-31) Agar $\sqrt[4]{ab} = 2\sqrt{3}$ va $a, b \in N$ bo'lsa, $a - b$ quyidagi keltirilgan qiymatlardan qaysi birini qabul qila olmaydi?
A) $-32$    B) 10    C) 0    D) 25

23. (97-5-26) Agar

$$\begin{cases} \sqrt{x} + \sqrt{y} = 3 \\ \sqrt{xy} = 2 \end{cases}$$

bo'lsa, $x + y$ ni toping.
A) 2    B) 3    C) 4    D) 5

24. (01-1-19) Agar

$$\begin{cases} x - y = 21 \\ \sqrt{x} - \sqrt{y} = 3 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.
A) 7    B) 12    C) 23    D) 29

25. (01-2-24) Tenglama ildizlari o'rta arifmetigini toping.

$$x - 5\sqrt{x} + 4 = 0$$

A) 16    B) $8,5$    C) 3    D) 2

26. (01-5-9) Tenglama ildizlari yig'indisini toping.

$$(x^2 - 4)\sqrt{x+1} = 0$$

A) 1    B) $-1$    C) 3    D) 2

**Yechish:** Tenglamaning chap qismi $x \geq -1$ da ma'noga ega. Ko'paytma nolga aylanishi uchun, ko'paytuvchilardan biri nolga aylanishi yetarli. Shuning uchun $x^2 - 4 = 0$ va $x + 1 = 0$ tenglama yecimlari ichidan $x \geq -1$ shartni qanoatlantiruvchilarini olishimiz kerak. Ularning yechimlari $x_1 = -2$, $x_2 = 2$ va $x_3 = -1$ lardir. $x_1 = -2$ yechim $x \geq -1$ shartni qanoatlantirmaydi. Demak, $x_2 = 2$ va $x_3 = -1$ lar berilgan tenglamaning yechimi bo'ladi. Ularning yig'indisi $2 + (-1) = 1$. **Javob:** 1 (A).

27. (01-6-25) Agar $\sqrt{x+1} + x - 11 = 0$ bo'lsa, $x + 12$ ning qiymatini toping.
A) 15    B) 16    C) 20    D) 19

28. (01-9-12) Ushbu

$$\begin{cases} y = \sqrt{16 - x^2} \\ y - x = 4 \end{cases}$$

tenglamalar sistemasining nechta yechimi bor?
A) 2    B) 1    C) 0    D) 3

29. (01-10-20) Tenglama nechta ildizga ega?

$$\frac{x-9}{\sqrt{x}+3} = x - 15$$

A) 0    B) 1    C) 2    D) 3

30. (01-12-43) Tenglamani yeching.

$$\sqrt{3x-7} - \sqrt{7-3x} = 0$$

A) $2,3$    B) $\frac{3}{7}$    C) $\frac{7}{3}$    D) $\emptyset$

31. (02-1-8) Tenglama nechta ildizga ega?

$$\sqrt{x^2+1} - \sqrt{x^2-1} = 1$$

A) $\emptyset$    B) 1    C) 2    D) 3

32. (03-7-20) Tenglamani yeching.

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x...}}} = 8$$

A) 56    B) 48    C) 60    D) 64

33. (03-8-38) Tenglamani yeching.

$$\sqrt{x} + \sqrt[4]{x} - 12 = 0$$

A) 81    B) 16    C) 25    D) 9

## 7.2  Irratsional tengsizliklar

Irratsional tengsizliklarni yechishda quyidagi teng kuch-liliklardan foydalaniladi.

**1.** $\sqrt[2k]{f(x)} > \varphi(x) \iff \begin{cases} f(x) > [\varphi(x)]^{2k} \\ \varphi(x) \geq 0; \end{cases}$
$\cup \begin{cases} f(x) \geq 0, \\ \varphi(x) < 0. \end{cases}$

**2.** $\sqrt[2k+1]{f(x)} > \varphi(x) \iff f(x) > [\varphi(x)]^{2k+1};$

**3.** $\sqrt[2k]{f(x)} < \varphi(x) \iff \begin{cases} f(x) \geq 0, \\ \varphi(x) \geq 0, \\ f(x) < [\varphi(x)]^{2k} \end{cases}$

**4.** $\sqrt[2k+1]{f(x)} < \varphi(x) \iff f(x) < [\varphi(x)]^{2k+1};$

1. (97-10-34) Tengsizlikning yechimini ko'rsating.

$$(x-1)\sqrt{6+x-x^2} \leq 0$$

A) $(-\infty; 1]$    B) $[-2; 3]$
C) $[-2; 1] \cup \{3\}$    D) $[3; \infty)$

**Yechish:** Berilgan tengsizlikni yechish uchun ikki holni qaraymiz. 1) $6 + x - x^2 = 0$. Uning ildizlari $x_1 = -2$, $x_2 = 3$. Bu sonlar berilgan tengsizlik-ning ham yechimi bo'ladi.

2) $\begin{cases} 6 + x - x^2 > 0 \\ x - 1 \leq 0. \end{cases}$

Birinchi tengsizliklarni $-1$ ga ko'paytirib, uning chap qismini ko'paytuvchilarga ajratamiz.

$$\begin{cases} (x+2)(x-3) < 0 \\ x - 1 \leq 0. \end{cases}$$

Bu holda $(-2; 1]$ yechimni hosil qilamiz. Endi yuqorida topilgan $x_1 = -2$, $x_2 = 3$ sonlarni hisobga olib berilgan tengsizlikning $[-2; 1] \cup \{3\}$ yechimini hosil qilamiz. **Javob:** $[-2; 1] \cup \{3\}$ (C).

2. (96-7-34) Tengsizlikning yechimini ko'rsating.

$$(x+3)\sqrt{x^2-x-2} \geq 0$$

A) $[-3; \infty)$    B) $[-1; 2]$
C) $[-3; -1] \cup [2; \infty)$    D) $[2; \infty)$

3. (97-3-34) Quyidagilardan qaysi biri

$$(x-3)\sqrt{x^2+x-2} \leq 0$$

tengsizlikning yechimi?
A) $(-\infty; 3]$    B) $(-\infty; -2] \cup [1; 3]$
C) $[-2; 3]$    D) $[-1; 2] \cup [3; \infty)$

4. (97-7-34) Tengsizlikning yechimini ko'rsating.

$$(x-2)\sqrt{3+2x-x^2} \geq 0$$

A) $[2; \infty)$  B) $[-1; 3]$  C) $[3; \infty)$  D) $[2; 3] \cup \{-1\}$

5. (00-3-21) Tengsizlikni yeching.

$$\sqrt{3x+10} > \sqrt{6-x}$$

A) $[-1; 6]$    B) $[-\frac{10}{3}; 6]$

C) $(-1; 6]$    D) $[-\frac{10}{3}; -1) \cup (-1; 6]$

6. (01-10-19) Tengsizlikni yeching.

$$\sqrt{3x-8} > \sqrt{5-x}$$

A) $(3,25; \infty)$    B) $(\frac{8}{3}; 5)$
C) $(3,25; 5]$    D) $(3,25; 5)$

7. (01-12-46) Tengsizlikni yeching.

$$\sqrt{5x - 2x^2 - 42} > 3$$

A) $\{-2\}$    B) $\{1\}$    C) $\{2\}$    D) $\emptyset$

8. (02-1-48) Tengsizlikni yeching.

$$\sqrt{x+1} < 4$$

A) $(-\infty; 15)$  B) $[0; 15]$  C) $[0; 15)$  D) $[-1; 15)$

9. (02-1-68) Tengsizlikni yeching.

$$(x+3)\sqrt{10-3x-x^2} \geq 0$$

A) $[-3; \infty)$    B) $[2; \infty)$
C) $[-3; 2]$    D) $\{-5\} \cup [-3; 2]$

10. (02-10-12) Tengsizlikni yeching.

$$\frac{\sqrt{6 + x - x^2}}{2x + 5} \geq \frac{\sqrt{6 + x - x^2}}{x + 4}$$

A) $[-2; -1] \cup \{3\}$  B) $[-2; 1]$  C) $[1; 3]$  D) $[-2; 3]$

11. (98-4-23) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x + 2} > x$$

A) 3    B) 2    C) 4    D) 1

**Yechish:** Berilgan tengsizlikni yechishda 1-qoidadan foydalanamiz:

$$\begin{cases} x + 2 > x^2 \\ x \geq 0; \end{cases} \cup \begin{cases} x + 2 \geq 0, \\ x < 0. \end{cases}$$

1-sistemani yechamiz. Uning 1-tengsizligi

$$x^2 - x - 2 < 0 \iff (x + 1)(x - 2) < 0$$

kvadrat tengsizlikka teng kuchli bo'lib, uning yechimi $(-1; 2)$ dan iborat. Agar sistemaning ikkinchi tengsizligi $x \geq 0$ ni hisobga olsak, 1-sistemaning yechimi $[0; 2)$ ekanligini olamiz.

2-sistemani quyidagicha yozib olamiz:

$$\begin{cases} x + 2 \geq 0, \\ x < 0. \end{cases} \iff \begin{cases} x \geq -2, \\ x < 0. \end{cases} \iff -2 \leq x < 0.$$

Bu qo'sh tengsizlik 2-sistemaning yechimi $[-2; 0)$ ekanligini bildiradi. Bu yechimlarni birlashtirib, berilgan tengsizlikning yechimi $[-2; 2)$ ekanligini olamiz. Bu yarim intervalda $-2, -1, 0, 1$ butun sonlar bor. Ularning soni 4 ta. **Javob:** 4 (C).

12. (98-12-82) Tengsizlikni qanoatlantiruvchi butun sonlar nechta?

$$\sqrt{5 - x^2} > x - 1$$

A) 5    B) 3    C) 4    D) 2

13. (99-2-20) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x^2 - 6x + 9} < 3$$

A) 4    B) 6    C) 7    D) 5

14. (00-2-15) Tengsizlikning eng kichik butun musbat yechimini toping?

$$\frac{\sqrt{x + 5}}{1 - x} < 1$$

A) 6    B) 3    C) 5    D) 2

15. (00-7-23) Tengsizlikning eng katta butun va eng kichik butun yechimlari ayirmasini toping.

$$\sqrt{x^2 - 16} < \sqrt{4x + 16}$$

A) 4    B) 5    C) 2    D) 3

16. (01-5-23) Tengsizlikning nechta butun yechimi bor?

$$\sqrt{\frac{3x - 4}{8 - x}} > 1$$

A) 4    B) 1    C) 2    D) 3

17. (02-4-26) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x - 50} \cdot \sqrt{100 - x} > 0$$

A) 43    B) 54    C) 49    D) 51

**Yechish:** Arifmetik ildizning manfiymasligidan berilgan tenglama

$$\begin{cases} x - 50 > 0 \\ 100 - x > 0 \end{cases} \iff \begin{cases} x > 50 \\ x < 100 \end{cases}$$

sistemaga teng kuchli bo'ladi. Uning yechimi $(50; 100)$ oraliqdan iborat. Bu oraliqda 49 ta butun son bor. **Javob:** 49 (C).

18. (01-6-26) Tengsizlikning butun yechimlari yig'indisini toping.

$$\frac{\sqrt{2x + 7}}{6 - 3x} \geq 0$$

A) $-4$    B) $-3$    C) 4    D) $-5$

19. (02-9-26) Tengsizlikning eng kichik va eng katta butun yechimlari ayirmasini toping.

$$x - 4\sqrt{x} - 5 \leq 0$$

A) $-25$    B) $-24$    C) $-27$    D) $-5$

20. (02-9-28) Tengsizlikning butun yechimlari nechta?

$$\frac{\sqrt{3 + 2x - x^2}}{x - 2} \leq 0$$

A) 3    B) 4    C) 5    D) 2

21. (02-12-14) Tengsizlikning butun yechimlari nechta?

$$\frac{5 - \sqrt{x}}{\sqrt{x} - 2} > 0$$

A) 20    B) 19    C) 21    D) 2

22. (02-12-35) Qanday eng kichik butun son

$$\sqrt{12 - x} < 2$$

tengsizlikni qanoatlantiradi?
A) 8    B) 9    C) 6    D) 10

23. (03-1-8) Tengsizlikning eng kichik butun yechimini toping.

$$\sqrt{\frac{2 - 3x}{x + 4}} > -2$$

A) 0    B) $-1$    C) $-2$    D) $-3$

**Yechish:** Arifmetik ildizning manfiymasligini hisobga olsak,

$$\frac{2 - 3x}{x + 4} \geq 0$$

tengsizlikni yechish kifoya. Bu tengsizlikni oraliqlar usuli yordamida yechib, $(-4; 2/3]$ ni olamiz. Bu yarim intervaldagi eng kichik butun son $-3$ dir. **Javob:** $-3$ (D).

24. (03-1-30) Tengsizlikni qanoatlantiruvchi butun son-larning yig'indisini toping.

$$\sqrt{x} \geq x - 6$$

A) 6     B) 15     C) 28     D) 45

25. (03-3-20) Tengsizlikning butun yechimlari nechta?

$$\sqrt{x-4} - \sqrt{x-7} \geq 1$$

A) 0     B) 1     C) 2     D) 4

26. (03-3-30) Tengsizlikning butun yechimlari nechta?

$$\sqrt{5 - |2x-1|} < 2$$

A) 2     B) 3     C) 4     D) 6

27. (03-8-37) Tengsizlikning eng kichik natural yechi-mini toping.

$$\sqrt{x^2 - 3x + 2} \geq 0$$

A) 1     B) 2     C) 3     D) 5

28. (03-9-9) Tengsizlikning butun yechimlari nechta?

$$\sqrt{\frac{x^2 - 2}{x}} \leq 1$$

A) $\emptyset$     B) 1     C) 2     D) 3

29. (03-11-73) Tengsizlikning butun yechimlari nechta?

$$\sqrt{8 + 2x - x^2} > 6 - 3x$$

A) 2     B) 3     C) 4     D) 5

# 8 -bob. Progressiyalar

## 8.1 Arifmetik progressiya

Arifmetik progressiya deb shunday sonlar ketma-ketligi-ga aytiladiki, unda ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadga shu ketma-ketlik uchun o'zgar-mas bo'lgan biror $d$ sonni qo'shish natijasida hosil bo'ladi. Masalan, 1) $1, 2, 3, 4, \ldots$;    2) $10, 12, 14, 16, \ldots$ ketma-ketliklar arifmetik progressiya tashkil qiladi. Chunki har bir son, ikkinchisidan boshlab, mos ravishda 1 va 2 sonlarini oldingisiga qo'shish natijasida hosil bo'ladi.

Arifmetik progressiyani tashkil qiluvchi sonlar un-ing hadlari deyiladi va umumiy ko'rinishda

$$a_1, a_2, a_3, \ldots, a_{n-1}, a_n, \ldots \qquad (1)$$

yoziladi. Arifmetik progressiyaning keyingi hadini hosil qilish uchun oldingi hadiga qo'shiladigan $d$ son arifmetik progressiya ayirmasi deyiladi. Agar $d > 0$ bo'lsa, pro-gressiya o'suvchi, $d < 0$ bo'lsa, progressiya kamayuvchi deyiladi. Agar $d = 0$ bo'lsa, arifmetik progressiyaning barcha hadlari o'zaro teng bo'ladi. $d = 0$ hol odatda qaralmaydi.

Arifmetik progressiyaning $n -$ hadi $a_n$ quyidagi for-mula yordamoda topiladi: $a_n = a_1 + (n-1)d$.

Arifmetik progressiya hadlarining xossalari.

*1-xossa.* Arifmetik progressiyaning ikkinchi hadi-dan boshlab istalgan hadi o'ziga qo'shni bo'lgan ikki hadning o'rta arifmetik qiymatiga teng, ya'ni

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}.$$

*2-xossa.* Chekli arifmetik progressiyada boshidan va oxiridan teng uzoqlikda to'rgan hadlar yig'indisi chetki hadlar yig'indisiga teng, ya'ni

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots = a_k + a_{n-k+1}.$$

*3-xossa.* Arifmetik progressiyaning dastlabki $n$ ta yig'indisi, ya'ni $a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$ ni $S_n$ bilan belgilaymiz. Arifmetik progressiyaning dastlabki $n$ ta yig'indisi $S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n$ chetki hadlar yig'indisining yarmi bilan hadlar soni ko'paytmasiga teng, ya'ni

$$S_n = \frac{a_1 + a_n}{2} n.$$

Arifmetik progressiya xossalarini jamlab, ularni quyidagi tartibda keltiramiz.

**1.** $a_n = a_1 + (n-1)d; \quad a_n = a_{n-1} + d.$

**2.** $a_n - a_m = (n - m)d, \quad n > m.$

**3.** $a_n = \dfrac{a_{n-1} + a_{n+1}}{2} = \dfrac{a_{n-k} + a_{n+k}}{2}, \ k < n.$

**4.** $a_k + a_m = a_p + a_q, \quad k + m = p + q.$

**5.** $S_n = \dfrac{a_1 + a_n}{2} n, \quad S_n = \dfrac{2a_1 + d(n-1)}{2} n.$

**6.** $S_n - S_{n-1} = a_n.$

1. Arifmetik progressiyada $a_1 = 5$, $d = 2$ bo'lsa, $a_7$ ni toping.
   A) 12    B) 18    C) 17    D) 10

   **Yechish:** 1-xossadan $a_7 = a_1 + 6d$ ni olamiz. $a_1$ va $d$ lar o'rniga berilganlarni qo'yib, $a_7 = 5 + 6 \cdot 2 = 5 + 12 = 17$. **Javob:** 17 (C) .

2. Arifmetik progressiyada $a_1 = 3$, $d = 4$ bo'lsa, $a_9$ ni toping.
   A) 36    B) 35    C) 39    D) 34

3. Arifmetik progressiyada $a_2 = 5$, $a_3 = 8$ bo'lsa, shu progressiyaning ayirmasini toping.
   A) 2    B) 3    C) 5    D) 1,6

4. Agar arifmetik progressiyada $a_5 = 16$, $d = 5$ bo'lsa, $a_3$ ni toping.
   A) 36    B) 35    C) 39    D) 34

5. Agar arifmetik progressiyada $a_3 = 5$, $a_9 = 25$ bo'lsa, $a_6$ ni toping.
   A) 16    B) 15    C) 19    D) 14

6. Agar arifmetik progressiyada $a_1 + a_9 = 20$ bo'lsa, $a_7 + a_3$ ni toping.
   A) 16    B) 15    C) 20    D) 25

7. Agar arifmetik progressiyada $a_9 - a_1 = 32$ bo'lsa, $d$ ni toping.
   A) 6     B) 5     C) 2     D) 4

8. (96-9-78) Arifmetik progressiyada $a_4 - a_2 = 4$ va $a_7 = 14$. Shu progressiyaning beshinchi hadini toping.
   A) 12     B) 8     C) 7     D) 10

   **Yechish:** 2-xossadan hamda birinchi shartdan $2d = 4$ ni olamiz. Yana 2-xossadan foydalanamiz:
   $$a_7 - a_5 = 2d \iff a_5 = a_7 - 2d = 14 - 4 = 10.$$
   **Javob:** $a_5 = 10$ (D).

9. (96-1-27) Arifmetik progressiyada $a_2 = 12$ va $a_5 = 3$. Shu progressiyaning o'ninchi hadini toping.
   A) $-6$     B) 0     C) $-12$     D) $-30$

10. (98-12-36) Arifmetik progressiya uchun quyidagi formulalardan qaysilari to'g'ri?
    1) $a_1 - 2a_2 + a_3 = 0$     2) $a_1 = a_3 - a_2$
    3) $n = \dfrac{a_n - a_1 + d}{d}$
    A) 1; 3     B) 1     C) 2     D) 1; 2

11. (99-1-22) Arifmetik progressiyada $a_{20} = 0$ va $a_{21} = -41$ bo'lsa, $a_1$ ni toping.
    A) 779     B) $-779$     C) 41     D) $-41$

12. (99-9-26) Arifmetik progressiyada $a_2 - a_1 = 6$ bo'lsa, $a_8 - a_6$ ni toping.
    A) 10     B) 12     C) 9     D) 18

13. (00-5-32) Arifmetik progressiyada $a_2 = 9$ va $a_{26} = 105$ bo'lsa, shu progressiya birinchi hadi va ayirmasining o'rta proporsional qiymatini toping.
    A) 20     B) $4,5$     C) $2\sqrt{5}$     D) 9

14. (00-10-22) 4; 9; 14; ... arifmetik progressiyaning sakkizinchi hadi to'rtinchi hadidan nechtaga ortiq?
    A) 16     B) 18     C) 20     D) 22

15. (02-4-16) Arifmetik progressiyada $a_1 = 3$ va $d = 2$ bo'lsa, $a_1 - a_2 + a_3 - a_4 + ... + a_{25} - a_{26} + a_{27}$ ning qiymatini hisoblang.
    A) 31     B) 30     C) 29     D) 28

16. (02-9-18)   $-\dfrac{1}{4}; -\dfrac{1}{5}, ...$ arifmetik progressiyaning nechta hadi manfiy?
    A) 10     B) 6     C) 5     D) 7

    **Yechish:** Demak, $a_1 = -\dfrac{1}{4}$, $a_2 = -\dfrac{1}{5}$. U holda progressiyaning ayirmasi $d = -\dfrac{1}{5} + \dfrac{1}{4} = \dfrac{1}{20}$. 1-qoidadan $a_n = -\dfrac{1}{4} + (n-1)\dfrac{1}{20}$ ekanligi kelib chiqadi. Masala shartiga ko'ra $a_n = -\dfrac{1}{4} + (n-1)\dfrac{1}{20} < 0$ tengsizlikning yechimi bo'lgan eng katta natural sonni topishimiz kerak. Bu chiziqli tengsizlikning yechimi $n < 6$ dir. Demak, arifmetik progressiyaning 5 ta hadi manfiy ekan. **Javob:** 5 (C).

17. (02-11-38) Arifmetik progressiyaning to'rtinchi hadi va o'n birinchi hadlari mos ravishda 2 va 30 ga teng. Shu progressiyaning uchinchi va o'ninchi hadlari yig'indisini toping.
    A) 16     B) 18     C) 24     D) 28

18. (03-2-67) Kinoteatrning birinchi qatorida 21 ta o'rin bor. Har bir keyingi qatorda o'rinlar soni oldingi qatordagidan 2 tadan ko'p. 40 - qatorda nechta o'rin bor?
    A) 42     B) 80     C) 99     D) 100

19. (03-3-36) Arifmetik progressiyada $a_2 + a_5 - a_3 = 10$ va $a_1 + a_6 = 17$ bo'lsa, uning o'ninchi hadini toping.
    A) 24     B) 26     C) 28     D) 29

**3-4 xossalariga oid misollar**

20. (97-4-27) Arifmetik progressiyaning dastlabki 6 ta hadlari $7, a_2, a_3, a_4, a_5$ va 22 bo'lsa, $a_2 + a_3 + a_4 + a_5$ ni hisoblang.
    A) 65     B) 60     C) 82     D) 58

    **Yechish.** Shartga ko'ra $a_1 = 7$, $a_6 = 22$ ekan. Arifmetik progressiyaning 4-xossasiga ko'ra
    $$a_2 + a_5 = a_3 + a_4 = a_1 + a_6 = 7 + 22 = 29.$$
    U holda $a_2 + a_3 + a_4 + a_5 = 29 + 29 = 58$.
    **Javob:** 58 (D).

21. (97-12-36) Ikkinchi, to'rtinchi va oltinchi hadlarining yig'indisi $-18$ ga teng arifmetik progressiyaning to'rtinchi hadini toping.
    A) 6     B) $-5$     C) $-6$     D) $-4$

22. (98-3-20) Birinchi hadi 1 ga, o'n birinchi hadi 13 ga teng bo'lgan arifmetik progressiyaning oltinchi hadini toping.
    A) 4     B) 5     C) 6     D) 7

23. (98-10-67) Ikkinchi hadi 5 ga sakkizinchi hadi 15 ga teng bo'lgan arifmetik progressiyaning beshinchi hadini toping.
    A) $7,5$     B) $12,5$     C) 10     D) $8,5$

    **Yechish.** Shartga ko'ra $a_2 = 5$, $a_8 = 15$. Arifmetik progressiyaning 3-xossasiga ko'ra
    $$\frac{a_2 + a_8}{2} = a_5 \iff \frac{5 + 15}{2} = 10 = a_5.$$
    **Javob:** 10 (C).

24. (02-1-40) Uchta sonning o'rta arifmetigi 2,6 ga, birinchi son esa 2,4 ga teng. Agar keyingi har bir son avvalgisidan ayni bir songa farq qilsa, keyingi sondan oldingisining ayirmasini toping.
    A) $\dfrac{1}{3}$     B) $0,1$     C) $\dfrac{1}{4}$     D) $0,2$

25. (02-5-29) Arifmetrik progressiyaning birinchi va to'rtinchi hadi yig'indisi 26 ga teng, ikkinchi hadi esa beshinchi hadidan 6 ga ko'p. Shu progressiyaning uchinchi va beshinchi hadi yig'indisini toping.
    A) 20     B) 21     C) 22     D) 23

26. (08-103-27) Arifmetrik progressiyaning to'rtinchi va o'n birinchi hadlari mos ravishda 15 va 43 ga teng. Shu progressiyaning uchinchi va o'ninchi hadi yig'indisini toping.
A) 68     B) 60     C) 50     D) 24

**Dastlabki $n$ ta hadi yig'indisiga oid misollar**

27. (96-3-27) Arifmetik progressiya uchinchi va to'qqizinchi hadlarining yig'indisi 8 ga teng. Shu progressiyaning dastlabki 11 ta hadi yig'indisini toping.
A) 22     B) 33     C) 44     D) 55

**Yechish.** Shartga ko'ra $a_3 + a_9 = 8$. 4-xossaga ko'ra $a_1 + a_{11} = a_3 + a_9 = 8$. 5-xossaga ko'ra

$$S_{11} = \frac{a_1 + a_{11}}{2} \cdot 11 = \frac{8}{2} \cdot 11 = 44.$$

**Javob:** 44 (C).

28. (96-11-28) Arifmetik progressiyada $a_3 + a_5 = 12$. $S_7$ ni toping.
A) 18     B) 36     C) 42     D) 48

29. (96-12-28) Arifmetik progressiyada $a_4 + a_6 = 10$. $S_9$ ni toping.
A) 25     B) 30     C) 35     D) 45

30. (98-10-18) Arifmetik progressiyada $a_2 + a_{19} = 40$. Shu progressiyaning dastlabki 20 ta hadlari yig'indisini toping.
A) 300     B) 360     C) 400     D) 420

31. (96-6-36) Ikkinchi va o'n to'qqizinchi hadlarining yig'indisi 12 ga teng bo'lgan arifmetik progressiyaning dastlabki yigirmata hadining yig'indisini toping.
A) 110     B) 120     C) 130     D) 115

32. (96-10-29) Arifmetik progressiyada $a_2 = 10$ va $a_5 = 22$. Shu progressiyaning dastlabki sakkizta hadining yig'indisini toping.
A) 162     B) 170     C) 115     D) 160

33. (98-11-26) Arifmetik progressiyaning uchinchi hadi va beshinchi hadi, mos ravishda 11 va 19 ga teng bo'lsa, $S_{10}$ ni toping.
A) 210     B) 190     C) 230     D) 220

34. (98-11-75) $(a_n)$ arifmetik progressiyada $a_1 = 3$, $a_{60} = 57$ bo'lsa, progressiyaning dastlabki 60 ta hadi yig'indisi qanchaga teng bo'ladi?
A) 1500     B) $\frac{3423}{2}$     C) 1600     D) 1800

35. (00-4-22) Arifmetik progressiyaning beshinchi hadi 6 ga teng. Uning dastlabki to'qqizta hadi yig'indisini toping.
A) 36     B) 48     C) 54     D) 45

**Yechish.** 3-xossaga ko'ra $\frac{a_1 + a_9}{2} = a_5 = 6$. 5-xossaga ko'ra

$$S_9 = \frac{a_1 + a_9}{2} \cdot 9 = 6 \cdot 9 = 54. \text{ **Javob:** } 54 \ (C).$$

36. (99-4-28) Arifmetik progressiyaning o'n uchinchi hadi 5 ga teng. Uning dastlabki 25 ta hadlarining yig'indisini toping.
A) 125     B) 100     C) 75     D) 225

37. (03-6-56) Arifmetik progressiyada $a_{10} = 56$ bo'lsa, uning dastlabki 19 ta hadlari yig'indisini toping.
A) 1024     B) 1032     C) 1056     D) 1064

38. (00-7-25) Arifmetik progressiyaning birinchi va to'qqizinchi hadlari yig'indisi 64 ga teng. Shu progressiyaning dastlabki 9 ta hadlari yig'indisi va beshinchi hadi ayirmasini toping.
A) 256     B) 260     C) 270     D) 208

39. (03-4-19) 15 ta haddan iborat arifmetik progressiyaning sakkizinchi hadi 18 ga teng. Shu progressiyaning hadlari yig'indisini toping.
A) 280     B) 270     C) 250     D) 300

40. (08-106-27, 08-123-27) Arifmetik progressiyning dastlabki 16 ta hadi yig'indisi 528 ga va $a_{16} = 63$. Shu progressiyaning ayirmasini toping.
A) 7     B) 4     C) 5     D) 6

41. (08-122-27) Arifmetik progressiyning oltinchi hadi 17 ga, dastlabki 16 ta hadi yig'indisi 392 ga teng. Shu progressiyaning 9-hadini toping.
A) 24     B) 26     C) 13     D) 18

42. (97-2-36) $(a_n)$ arifmetik progressiyaning dastlabki $n$ ta hadi yig'indisi 120 ga teng. Agar $a_3 + a_{n-2} = 30$ bo'lsa, yig'indida nechta had qatnashgan?
A) 6     B) 10     C) 8     D) 12

**Yechish:** 4-xossaga ko'ra $a_1 + a_n = a_3 + a_{n-2} = 30$. Shartga ko'ra

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{30}{2} \cdot n = 15n = 120.$$

Bu yerdan $n = 8$ ni olamiz. **Javob:** 8 (C).

43. (08-126-27) 25 va 4 sonlari orasiga shu sonlar bilan arifmetrik progressiya tashkil etadigan bir nechta son joylashtirilgan. Agar joylashtirilgan sonlarning yig'indisi 87 ga teng bo'lsa, nechta had joylashtirilgan.
A) 6     B) 11     C) 12     D) 9

44. (98-8-27) Arifmetik progressiyaning uchinchi hadi 8 ga, to'rtinchi hadi 5 ga va dastlabki bir nechta hadlari yig'indisi 28 ga teng. Yig'indida nechta had qatnashgan?
A) 10     B) 7     C) 11     D) 8

45. (99-6-54) Arifmetik progressiyaning dastlabki $n$ ta hadining yig'indisi 91 ga teng. Agar $a_3 = 9$ va $a_7 - a_2 = 20$ ekanligi ma'lum bo'lsa, $n$ ni toping.
A) 7     B) 5     C) 3     D) 9

46. (03-12-63) 10; 15; 20; ... arifmetik progressiyaning dastlabki nechta hadining yig'indisi 2475 ga teng bo'ladi?
A) 40     B) 25     C) 30     D) 35

47. (97-7-27) 100 dan katta bo'lmagan 4 ga karrali barcha natural sonlarning yig'indisini toping.
    A) 1250     B) 1300     C) 1120     D) 1000

    **Yechish:** Ma'limki, 4 ga karrali sonlar $4n$, $n = 1, 2, 3, \ldots$ shaklda bo'lib, ular arifmetik progressiya tashkil qiladi. Bu progressiya uchun $a_n = 4n$ dir. Bundan tashqari $a_{25} = 100$. Demak,

$$S_{25} = \frac{4 + 100}{2} \cdot 25 = 52 \cdot 25 = 1300.$$

    **Javob:** 1300 (B).

48. (96-7-27) 100 dan katta bo'lmagan 3 ga karrali barcha natural sonlarning yig'indisini toping.
    A) 1683     B) 2010     C) 1500     D) 1080

49. (97-6-17) $a_n = 4n - 2$ formula bilan berilgan ketma-ketlikning dastlabki 50 ta hadi yig'indisini toping.
    A) 4500     B) 5050     C) 3480     D) 5000

50. (97-11-17) Hadlari $b_n = 3n - 1$ formula bilan berilgan ketma-ketlikning dastlabki 60 ta hadining yig'indisini toping.
    A) 4860     B) 4980     C) 5140     D) 5430

51. (02-11-37) 9 ga bo'lganda, qoldig'i 4 ga teng bo'ladigan barcha ikki xonali sonlarning yig'indisini toping.
    A) 527     B) 535     C) 536     D) 542

52. (03-9-26) 7 ga bo'lganda, qoldig'i 2 ga teng bo'ladigan barcha ikki xonali sonlarning yig'indisini toping.
    A) 640     B) 647     C) 650     D) 654

53. (03-1-70) Dastlabki mingta natural sonlarning o'rta arifmetigini toping.
    A) 500     B) 501     C) 501,5     D) 500,5

54. (00-2-11) 25 ta ketma-ket natural sonning yig'indisi 1000 ga teng. Bu sonlarning kichigi nechaga teng bo'ladi?
    A) 30     B) 28     C) 26     D) 27

55. (98-1-27) Arifmetik progressiyaning dastlabki 16 ta hadlari yig'indisi 840 ga va $a_{16} = 105$ bo'lsa shu progressiyaning ayirmasini toping.
    A) 9     B) 7     C) 15     D) 5

56. (98-2-18) Arifmetik progressiyada $S_{20} - S_{19} = -30$ va $d = -4$ bo'lsa, $a_{25}$ ning qiymatini toping.
    A) $-40$     B) $-50$     C) $-48$     D) $-56$

    **Yechish:** 6-qoidaga ko'ra, $a_{20} = S_{20} - S_{19} = -30$. 2-qoidaga ko'ra, $a_{25} - a_{20} = 5d$. Bu ifodaga $a_{20} = -30$ va $d = -4$ larni qo'yib, $a_{25} = -50$ ni olamiz.  **Javob:** $-50$ (B).

57. (00-3-44) Arifmetik progressiyaning dastlabki 13 ta hadi yig'indisi 104 ga teng. Yettinchi hadining kvadratini toping.
    A) 25     B) 36     C) 49     D) 64

58. (00-5-1) 1 dan 75 gacha bo'lgan toq sonlar yig'indisi qanday raqam bilan tugaydi?
    A) 0     B) 2     C) 3     D) 4

59. (00-9-13) $y; 3y+5; 5y+10; \ldots$ arifmetik progressiyaning dastlabki 8 ta hadi yig'indisi 396 ga teng. $y$ ning qiymatini toping.
    A) 2     B) 3     C) 4     D) 5

60. (01-1-26) Agar soat 1 da bir marta, 2 da ikki marta va hokazo 12 da o'n ikki marta zang ursa, bir sutkada necha marta zang uradi?
    A) 72     B) 78     C) 108     D) 156

61. (01-5-28) Arifmetik progressiya uchun $a_{17} = 2$ ga teng bo'lsa, $S_{21} - S_{12}$ ni toping
    A) 18     B) 15     C) 16     D) 17

62. (02-1-55) Arifmetik progressiya birinchi o'nta hadining yig'indisi 140 ga teng bo'lsa, $a_2 + a_9$ ni aniqlang.
    A) 24     B) 26     C) 30     D) 28

63. (02-4-22) Agar arifmetik progressiya hadlari uchun $a_1 + a_3 + \cdots + a_{19} = a_2 + a_4 + \cdots + a_{20} + 10$ tenglik o'rinli bo'lsa, arifmetik progressiyaning ayirmasini toping.
    A) 1     B) $-1$     C) 0     D) $-2$

    **Yechish:** Ma'lumki, barcha $n \geq 2$ lar uchun $a_n - a_{n-1} = d$ tenglik o'rinli. Masala shartida berilgan tenglikni

$$-10 = (a_2 - a_1) + (a_4 - a_3) + \cdots + (a_{20} - a_{19})$$

    shaklda yozib olamiz. Har bir qavs ichidagi ayirma $d$ ga teng va qavslar soni 10 ta. Demak, $-10 = 10d$ tenglik o'rinli. Bu yerdan $d = -1$ ni olamiz.
    **Javob:** $-1$ (B).

64. (02-4-18) Arifmetik progressiya hadlari uchun $a_1 + a_3 + \cdots + a_{21} = a_2 + a_4 + \cdots + a_{20} + 15$ tenglik o'rinli bo'lsa, $a_{11}$ ni toping.
    A) 11     B) 13     C) 15     D) 17

65. (03-5-27) Arifmetik progressiyaning oltinchi hadi 10 ga, dastlabki 16 ta hadining yig'indisi 200 ga teng. Bu progressiyaning 12-hadini toping.
    A) 16     B) 14     C) 18     D) 20

66. (03-8-50) Agar arifmetik progressiyada $a_1 + a_2 + \cdots + a_{16} + a_{17} = 136$ bo'lsa, $a_6 + a_{12}$ ni hisoblang.
    A) 16     B) 10     C) 12     D) 14

## 8.2   Geometrik progressiya

Birinchi hadi noldan farqli bo'lib, ikkinchi hadidan boshlab bir hadi o'zidan oldingi hadni shu ketma-ketlik uchun o'zgarmas va noldan farqli bo'lgan biror $q$ songa ko'paytirishdan hosil bo'lgan sonlar ketma-ketligi geometrik progressiya deyiladi. Masalan, 1) $1, 3, 9, \ldots$;  2) $20, 10, 5, \ldots$ ketma-ketliklar geometrik progressiya tashkil qiladi. Birinchi misolda $q = 3$, ikkinchisida $q = 0, 5$.

Geometrik progressiyani tashkil qiluvchi sonlar uning hadlari deyiladi va umumiy ko'rinishda

$$b_1, b_2, b_3, \ldots, b_{n-1}, b_n, \ldots \qquad (1)$$

yoziladi. Geometrik progressiyaning keyingi hadini hosil qilish uchun oldingi hadiga ko'paytiriladigan $q$ son geometrik progressiya maxraji deyiladi. Agar $b_1 > 0$ va $q > 1$ bo'lsa, progressiya o'suvchi deyiladi. Agar $|q| < 1$ bo'lsa, progressiya kamayuvchi, $q < 0$ bo'lsa, progressiya ishorasi o'zgaruvchi deyiladi, $q = 1$ hol odatda qaralmaydi. Geometrik progressiyaning $n-$ hadi $b_n$ quyidagi formula yordamida topiladi: $b_n = b_1 q^{(n-1)}$.

Geometrik progressiya hadlarining xossalari.

*1-xossa.* Agar geometrik progressiyaning barcha hadlari musbat bo'lsa, u holda uning ikkinchi hadidan boshlab istalgan hadi o'ziga qo'shni bo'lgan ikki hadning o'rta geometrik qiymatiga teng, ya'ni

$$b_n = \sqrt{b_{n-1}\,b_{n+1}}.$$

*2-xossa.* Chekli geometrik progressiyada boshidan va oxiridan teng uzoqlikda to'rgan hadlar ko'paytmasi chetki hadlar ko'paytmasiga teng, ya'ni

$$b_1\,b_n = b_2\,b_{n-1} = b_3\,b_{n-2} = \cdots = b_k\,b_{n-k+1}.$$

*3-xossa.* Geometrik progressiyaning dastlabki $n$ ta hadi yig'indisi $S_n = b_1 + b_2 + b_3 + \cdots + b_{n-1} + b_n$ bo'lsin. Geometrik progressiyaning dastlabki $n$ ta hadi yig'indisi $S_n$ uchun quyidagi formulalar o'rinli:

$$S_n = \frac{b_1 - b_n\,q}{1-q}, \quad S_n = \frac{b_n\,q - b_1}{q-1} \quad S_n = \frac{b_1\,(q^n - 1)}{q-1}.$$

*4-xossa.* Cheksiz kamayuvchi geometrik progressiya barcha hadlarining yig'indisi $S$ uchun quyidagi formula o'rinli. $S = b_1 + b_2 + \cdots + b_n + \cdots = \dfrac{b_1}{1-q}$.

**1.** $b_n = b_1 \cdot q^{n-1}, \quad b_n = q\,b_{n-1}.$

**2.** $b_n : b_m = q^{n-m}, \quad n > m.$

**3.** $b_n^2 = b_{n-1}b_{n+1}, \quad n \geq 2.$

**4.** $b_k b_m = b_p b_q, \quad k + m = p + q.$

**5.** $S_n = \dfrac{b_1(1-q^n)}{1-q}, \quad S_n = \dfrac{b_n\,q - b_1}{q-1}, \quad (q \neq 1).$

**6.** $S_n - S_{n-1} = b_n.$

**7.** $S = \dfrac{b_1}{1-q}.$

1. Agar geometrik progressiyada $b_1 = 2$, $q = 3$ bo'lsa, $b_5$ ni toping.
   A) 162        B) 158        C) 120        D) 254

   **Yechish:** 1-xossaga ko'ra $b_5 = b_1 q^4$. Endi $b_1$ va $q$ larning qiymatlari qo'yib $b_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$ ni olamiz. **Javob:** 162 (A).

2. Agar geometrik progressiyada $b_2 = 1$, $q = 2$ bo'lsa, $b_5 - b_4$ ni hisoblang.
   A) 2        B) 5        C) 4        D) 8

3. Agar geometrik progressiyada $b_3 = 10$, $q = 3$ bo'lsa, $b_5$ ni toping.
   A) 2        B) 5        C) 4        D) 8

4. Geometrik progressiyada $b_5 = 64$, $b_7 = 16$ bo'lsa, geometrik progressiyaning maxrajini toping.
   A) $0,2$        B) $0,5$        C) 4        D) 2

5. Geometrik progressiyada $b_3 b_5 = 64$ bo'lsa, $b_4$ ni toping.
   A) 8        B) $-8$        C) $\pm 8$        D) 4

6. Agar ishorasi almashinuvchi geometrik progressiyada $b_3 = 4$, $b_7 = 9$ bo'lsa, $b_5$ ni toping.
   A) 2        B) $-6$        C) 6        D) 5

7. (98-4-21) Nolga teng bo'lmagan $x, y, z$ sonlar ko'rsatilgan tartibda ishorasi o'zgaruvchi geometrik progressiyani, $x + y; y + z; z + x$ sonlar esa arifmetik progressiyani tashkil etadi. Geometrik progressiyaning maxrajini toping.
   A) $-2$        B) $-1$        C) $-3$        D) $-4$

   **Yechish:** Geometrik progressiyaning maxraji $q$ bo'lsin. U holda $y = qx$, $z = q^2 x$. Endi $x + y$, $y + z$, $z + x$ sonlar arifmetik progressiya tashkil etganligi uchun $2(y+z) = x + y + z + x$, ya'ni $y + z = 2x$ bo'ladi. $y, z$ larning o'rniga ularning ifodalarini qo'yib $qx + q^2 x - 2x = 0$ tenglamani, bu yerdan esa $x(q^2 + q - 2) = 0$ ekanini topamiz. Masala shartiga ko'ra $x \neq 0$, shuning uchun $q^2 + q - 2 = 0$ bo'ladi. Uning ildizlari $q_1 = 1$, $q_2 = -2$. Geometrik progressiyaning ishorasi o'zgaruvchiligidan $q = -2$ ekani kelib chiqadi. **Javob:** $-2$ (A).

8. (08-120-27) O'suvchi geometrik progressiyaning birinchi hadi 2 ga, yettinchi va to'rtinchi hadlarining ayirmasi 1404 ga teng. Shu progressiyaning maxrajini toping.
   A) 2        B) 3        C) $2\sqrt{2}$        D) 4

9. (97-9-87) Geometrik progressiyaning dastlabki 6 ta hadi $2, b_2, b_3, b_4, b_5$ va 486 bo'lsa, $b_2 + b_3 + b_4 + b_5$ ni hisoblang.
   A) 200        B) 260        C) 230        D) 240

10. (98-7-38) Quyidagi ketma-ketliklardan qaysilari geometrik progressiyani tashkil etadi?

    1) $a_n = 2x^n$    2) $c_n = ax^n + 1$    3) $b_n = (\dfrac{3}{5})^n$

    A) 1;3     B) 2;3     C) hech biri     D) 1;2;3

11. (00-2-21) Nechanchi hadidan boshlab $-8; 4; -2; \ldots$ geometrik progressiya hadlarining absolyut qiymati $0,001$ dan kichik bo'ladi?
    A) 16        B) 12        C) 15        D) 14

12. (00-10-23) $64; 32; 16; \ldots$ geometrik progressiyaning to'qqizinchi hadi oltinchi hadidan nechtaga kam?
    A) $1,025$        B) $1,5$        C) $1,25$        D) $1,75$

**Yechish:** Berilgan geometrik progressiyada $b_1 = 64$, $b_2 = 32$. Bu yerdan $q = 1/2$ ni olamiz. U holda 1-xossadan

$$b_6 = b_1 q^5 = \frac{64}{32} = 2, \quad b_9 = b_1 q^8 = \frac{64}{256} = \frac{1}{4}.$$

Ularning farqi $b_6 - b_9 = 2 - 0,25 = 1,75$. **Javob:** $1,75$ (D).

13. (02-4-23) Agar geometrik progressiya hadlari uchun $b_1 b_3 \cdots b_{13} = b_2 b_4 \cdots b_{14}/128$ tenglik o'rinli bo'lsa, progressiyaning maxrajini toping.
    A) 1    B) 2    C) 3    D) 4

14. (03-2-5) $(b_n)$ geometrik progressiyada $b_4 - b_2 = 24$ va $b_2 + b_3 = 6$ bo'lsa, $b_1$ ning qiymatini toping.
    A) 0,4    B) 1    C) $1\frac{1}{5}$    D) $\frac{1}{5}$

15. (03-6-57) Ikkinchi hadi 6 ga, birinchi uchta hadining yig'indisi 26 ga teng o'suvchi geometrik progressiyaning uchinchi va birinchi hadlari ayirmasini toping.
    A) 15    B) 16    C) 14    D) 13

16. (03-12-66) O'suvchi geometrik progressiyaning birinchi hadi 3 ga, yettinchi va to'rtinchi hadlarining ayirmasi 168 ga teng. Shu progressiyaning maxrajini toping.
    A) 3    B) $\frac{3}{2}$    C) $\sqrt{7}$    D) 2

    **3-4-xossalariga oid misollar**

17. (00-9-40) $x$ ning qanday qiymatlarida $0,(16)$; $x$ va $0,(25)$ sonlar ishoralari almashinuvchi geometrik progressiyaning ketma-ket keluvchi hadlari bo'ladi?
    A) $0,(20)$    B) $\pm 0,(20)$    C) $-0,(20)$    D) $-0,(21)$

    **Yechish:** Geometrik progressiyaning ishora almashinuvchiligidan $x < 0$ ekani, 3-xossasidan esa

    $$x^2 = 0,(16) \cdot 0,(25) = \frac{16}{99} \cdot \frac{25}{99}$$

    ekani kelib chiqadi. Shuning uchun

    $$x = -\frac{4 \cdot 5}{99} = -\frac{20}{99} = -0,(20).$$

    **Javob:** $-0,(20)$ (C).

18. (96-6-37) Geometrik progressiyada $b_2 \cdot b_3 \cdot b_4 = 216$ bo'lsa, uning uchinchi hadini toping.
    A) 12    B) 8    C) 4    D) 6

19. (97-8-36) $b_3 \cdot b_4 \cdot b_5 = 64$ ga teng bo'lgan progressiyaning to'rtinchi hadini toping.
    A) 10    B) 12    C) 4    D) 8

20. (00-3-46) Geometrik progressiyada uchinchi va yettinchi hadlarining ko'paytmasi 144 ga teng. Uning beshinchi hadini toping.
    A) 6    B) $\pm 12$    C) $-8$    D) $-12$

21. (00-3-47) $\frac{1}{3}$ va $\frac{1}{48}$ sonlar orasiga shunday uchta musbat sonni joylashtiringki, natijada geometrik progressiya hosil bo'lsin. O'sha qo'yilgan uchta sonning yig'indisini toping.
    A) 0,5    B) $\frac{7}{12}$    C) 0,375    D) $\frac{7}{24}$

22. (00-7-26) Barcha hadlari musbat bo'lgan geometrik progressiyaning birinchi hadi 2 ga, beshinchi hadi 18 ga teng. Shu progressiyaning beshinchi va uchinchi hadlari ayirmasini toping.
    A) 10    B) 12    C) 8    D) 11

23. (02-8-9) 3 va 19683 sonlari o'rtasiga 7 ta shunday musbat sonlar joylashtirilganki, hosil bo'lgan to'qqizta son geometrik progressiya tashkil etsin. 5-o'rinda turgan son nechaga teng?
    A) 243    B) 343    C) 286    D) 729

    **Dastlabki $n$ ta hadi yig'indisiga oid misollar**

24. Geometrik progrogressiyada $b_1 = 3$, $q = 2$. Shu progrogressiyaning daslabki oltita hadining yig'indisini toping.
    A) 63    B) 189    C) 126    D) 184

    **Yechish:** 5-xossaga ko'ra

    $$S_6 = \frac{b_1(1 - q^6)}{1 - q} = \frac{3(1 - 2^6)}{1 - 2} = 3 \cdot 63 = 189.$$

    **Javob:** 189 (B).

25. Geometrik progrogressiyada $b_1 = 3$, $q = 2$. Shu progrogressiyaning daslabki oltita hadining yig'indisini toping.
    A) 63    B) 189    C) 126    D) 184

26. $81, 27, 9, \ldots$ geometrik progressiyaning nechta hadining yig'indisi 121 ga teng bo'ladi.
    A) 3    B) 4    C) 5    D) 6

27. Geometrik progressiyada $b_6 - b_1 = 84$, $q = 3$. Shu progressiyaning daslabki beshta hadining yig'indisini toping.
    A) 63    B) 89    C) 42    D) 21

28. (97-12-35) Daslabki beshta hadining yig'indisi $-62$ ga, dastlabki oltita hadining yig'indisi $-126$ ga va maxraji 2 ga teng geometrik progressiyaning birinchi hadini toping.
    A) $-1$    B) $-3$    C) $-4$    D) $-2$

    **Yechish:** $S_5 = -62$, $S_6 = -126$ ekanidan 7-xossaga ko'ra $b_6 = S_6 - S_5 = -126 + 62 = -64$ ni topamiz. $q = 2$ va $b_6 = b_1 \cdot q^5$ ekanidan $b_1 \cdot 2^5 = -64$, bundan esa $b_1 = -2$ ni hosil qilamiz.
    **Javob:** $-2$ (D).

29. (98-2-19) Geometrik progressiyada $S_6 - S_5 = -128$ va $q = -2$. $b_8$ ning qiymatini toping.
    A) 512    B) 256    C) $-512$    D) $-256$

30. (98-3-21) Geometrik progressiyaning maxraji 3 ga, dastlabki to'rta hadlari yig'indisi 80 ga teng. Uning to'rtinchi hadini toping.
    A) 24        B) 32        C) 54        D) 27

31. (98-8-26) Geometrik progressiyaning birinchi hadi 486 ga, maxraji $\frac{1}{3}$ ga teng. Shu progressiyaning dastlabki to'rtta hadi yig'indisini toping.
    A) 680        B) 840        C) 720        D) 760

32. (98-1-26) Geometrik progressiyaning maxraji $-2$ ga, dastlabki beshta hadining yig'indisi 5,5 ga teng. Progressiyaning beshinchi hadini toping.
    A) 4        B) $-8$        C) 8        D) $-16$

33. (98-11-27) Agar olti hadli geometrik progressiyaning dastlabki uchta hadining yig'indisi 112 ga va oxiridagi uchta hadining yig'indusi 14 ga teng bo'lsa, birinchi hadi nechaga teng bo'ladi?
    A) 72        B) 64        C) 56        D) 63

34. (99-4-29) Geometrik progressiyada $q = 2$ va $S_4 = 5$. $b_2$ ni toping.
    A) 0,4        B) 0,8        C) $1\frac{1}{3}$        D) $\frac{2}{3}$

35. (00-6-25) O'suvchi geometrik progressiyaning dastlabki to'rtta hadi yig'indisi 15 ga, undan keyingi to'rttasiniki esa 240 ga teng. Shu progressiyaning dastlabki oltita hadi yig'indisini toping.
    A) 31        B) 48        C) 63        D) 127

    **Yechish:** Masala sharti va 1-xossadan

    $$\begin{cases} b_1 + b_2 + b_3 + b_4 = b_1(1 + q + q^2 + q^3) = 15 \\ b_5 + b_6 + b_7 + b_8 = q^4 b_1(1 + q + q^2 + q^3) = 240 \end{cases}$$

    ni olamiz. $b_1(1+q+q^2+q^3)$ ning qiymatini sistemaning 2-tenglamasiga qo'yib $q^4 \cdot 15 = 240$ ni, bu yerdan esa $q = 2$ ni olamiz. $q$ ning qiymatini sistemaning 1-tenglamasiga qo'yib $b_1 = 1$ ekanligini topamiz. 5-xossadan $S_6 = 2^6 - 1 = 63$. **Javob: 63 (C).**

36. (01-1-28) Oltita haddan iborat geometrik progressiyaning dastlabki uchta hadining yig'indisi 168 ga, keyingi uchtasiniki esa 21 ga teng. Shu progressiyaning maxrajini toping.
    A) $\frac{1}{4}$        B) $\frac{1}{3}$        C) $\frac{1}{2}$        D) 2

37. (02-1-56) Agar hadlari haqiqiy sondan iborat bo'lgan o'suvchi geometrik progressiyaning birinchi uchta hadi yig'indisi 7 ga, ko'paytmasi 8 ga teng bo'lsa, shu progressiyaning beshinchi hadini toping.
    A) 6        B) 32        C) 12        D) 16

38. (02-4-21) Geometrik progressiyada $b_1 = 1$ va $q = \sqrt{2}$ bo'lsa, $b_1 + b_3 + b_5 + ... + b_{15}$ ning qiymatini hisoblang.
    A) 253        B) 254        C) 255        D) 256

39. (02-5-28) Geometrik progressiyaning dastlabki oltita hadi yig'indisi 1820 ga, maxraji esa 3 ga teng.

Shu progressiyaning birinchi va beshinchi hadlari yig'indisini toping.
    A) 164        B) 246        C) 328        D) 410

40. (02-11-39) Geometrik progressiyaning ikkinchi hadi 2 ga, beshinchi hadi 16 ga teng. Shu progressiyaning dastlabki oltita hadi yig'indisini toping.
    A) 81        B) 72        C) 65        D) 63

41. (03-9-25) Geometrik progressiyaning birinchi hadi va maxraji 2 ga teng. Shu progressiyaning dastlabki nechta hadlari yig'indisi 1022 ga teng bo'ladi?
    A) 5        B) 8        C) 9        D) 10

42. (03-4-20) Geometrik progressiyaning oltinchi va birinchi hadi ayirmasi 1210 ga, maxraji 3 ga teng. Shu progressiyaning dastlabki beshta hadi yig'indisini toping.
    A) 610        B) 615        C) 600        D) 605

**Cheksiz kamayuvchi geometrik progressiya**

43. (99-1-23) Geometrik progressiyaning yig'indisini toping:

    $$\sqrt{5},\ 1,\ \frac{1}{\sqrt{5}},\ \ldots$$

    A) $\frac{5}{\sqrt{5}-1}$     B) $\frac{6\sqrt{5}}{5}$     C) $\frac{\sqrt{5}-1}{\sqrt{5}}$     D) 4,16

    **Yechish:** Berilgan ketma–ketlik cheksiz kamayuvchi geometrik progressiya bo'lib, bunda $b_1 = \sqrt{5}$ va $q = \frac{1}{\sqrt{5}}$. U holda 7-formulaga ko'ra

    $$S = \frac{\sqrt{5}}{1 - \frac{1}{\sqrt{5}}} = \frac{5}{\sqrt{5}-1}. \quad \textbf{Javob}: \frac{5}{\sqrt{5}-1}\ (A).$$

44. (99-10-25) Cheksiz kamayuvchi geometrik progressiyaning hadlari yig'indisi 8 ga, dastlabki to'rttasiniki esa $\frac{15}{2}$ ga teng. Agar uning barcha hadlari musbat bo'lsa, $b_1$ ni toping.
    A) 2        B) 4,5        C) 4        D) 3

45. (00-3-48) Cheksiz kamayuvchi geometrik progressiyaning hadlari yig'indisi 9 ga, maxraji esa $\frac{1}{3}$ ga teng. Uning birinchi hamda uchinchi hadlari ayirmasini toping.
    A) $5\frac{1}{3}$     B) $4\frac{2}{3}$     C) $5\frac{2}{3}$     D) $2\frac{1}{3}$

46. (01-8-25) Cheksiz kamayuvchi geometrik progressiyaning birinchi hadi ikkinchisidan 8 ga ortiq, hadlarining yig'indisi esa 18 ga teng. Progressiyaning uchinchi hadini toping.
    A) $1\frac{1}{3}$     B) $-33\frac{1}{3}$     C) $-1\frac{1}{3}$     D) $2\frac{2}{3}$

47. (02-1-16) Cheksiz kamayuvchi geometrik progressiyaning yig'indisi 56 ga, hadlari kvadratlarining yig'indisi esa 448 ga teng. Progressiyaning maxrajini toping.
    A) 0,75        B) 0,8        C) 0,25        D) 0,5

**Yechish:** Masala shartiga ko'ra $S = \dfrac{b_1}{1-q} = 56$ va $b_1^2 + b_2^2 + \cdots + b_n^2 + \cdots = b_1^2(1 + q^2 + \cdots + q^{2(n-1)} + \cdots) = 448$. Qavs ichidagi ifoda maxraji $q^2$ va $b_1 = 1$ bo'lgan cheksiz kamayuvchi geometrik progressiyaning yig'indisini ifodalaydi. Uning yig'indisi $\dfrac{1}{1-q^2}$ dir. Natijada biz

$$\begin{cases} b_1 : (1-q) = 56 \\ b_1^2 : (1-q^2) = 448 \end{cases}$$

sistemaga ega bo'lamiz. Sistemaning 1-tenglamasidan $b_1 = 56(1-q)$ ni, 2-tenglamaga qo'yib, $q$ ga nisbatan $7(1-q) = 1+q$ tenglamani olamiz. Bu tenglamaning yechimi $q = 6/8 = 0,75$. **Javob:** $0,75$ (A).

48. (02-5-30) $a$ ning qanday qiymatida

$$2a + a\sqrt{2} + a + \frac{a}{\sqrt{2}} + \ldots$$

cheksiz kamayuvchi geometrik progressiya yig'indisi 8 ga teng bo'ladi?

A) 1    B) $\dfrac{4}{\sqrt{2}}$    C) $2-\sqrt{2}$    D) $2(2-\sqrt{2})$

49. (02-12-32) Cheksiz kamayuvchi geometrik progressiyaning yig'indisi 243 ga, dastlabki beshta hadiniki esa 275 ga teng. Bu progressiyaning maxraji $\dfrac{1}{5}$ dan qanchaga kam?

A) $\dfrac{7}{15}$    B) $\dfrac{8}{15}$    C) $\dfrac{3}{5}$    D) $\dfrac{13}{15}$

50. (03-4-21) Cheksiz kamayuvchi geometrik progressiyaning birinchi hadi 2 ga, hadlarining yig'indisi esa 5 ga teng. Shu progressiyaning hadlari kvadratlaridan tuzilgan progressiyaning hadlari yig'indisini toping.

A) 6,25    B) 6,5    C) 5,75    D) 6,75

# 9 -bob. Matnli masalalar

## 9.1 Sonlarga oid masalalar

1. **Hech bir uchtasi bitta to'g'ri chiziqda yotmaydigan $n$ ta nuqtadan $\dfrac{n(n-1)}{2}$ ta to'g'ri chiziq o'tkazish mumkin.**

2. **$N(A) -$ $A$ to'plamning elementlari soni bo'lsa, $N(A \cup B) = N(A) + N(B) - N(A \cap B)$ o'rinli.**

3. **Ikki xonali $\overline{xy}$ soni $10x+y$ ko'rinishda yoziladi, bunda $x, y$ lar raqamlar.**

1. (96-10-39) Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqta berilgan. Shu 6 ta nuqtalar orqali nechta turlicha to'g'ri chiziq o'tkazish mumkin?

A) 6    B) 12    C) 10    D) 15

**Yechish:** 1-qoidaga ko'ra, hech qaysi uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqtadan

$$\frac{6(6-1)}{2} = 15$$

to'g'ri chiziq o'tkazish mumkin. **Javob:** 15 (D).

2. (96-1-36) Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 7 ta nuqta berilgan. Shu 7 ta nuqtalar orqali nechta turlicha to'g'ri chiziq o'tkazish mumkin?

A) 28    B) 21    C) 42    D) 35

3. (01-2-43) Istalgan uchtasi bir to'g'ri chiziqda yetmaydigan to'rtta nuqtani juft-juft ravishda tutashtirish natijasida nechta kesma hosil bo'ladi?

A) 4    B) 5    C) 6    D) 7

4. (98-12-101) 13 kishi bir-biri bilan salomlashganda, qo'l berib ko'rishishlar soni qancha bo'ladi?

A) 169    B) 156    C) 78    D) 130

5. (96-6-4) 2 soat 30 minut 3 sekund necha sekund bo'ladi?

A) 10203    B) 8203    C) 9003    D) 9803

6. (97-2-4) $3\,m^2\,1\,dm^2\,5sm^2$ necha $sm^2$ ga teng?

A) 3015    B) 3105    C) 30015    D) 30105

7. (97-8-4) 1 soat 160 minut 2 sekund necha sekunddan iborat?

A) 106002    B) 12202    C) 14202    D) 13202

8. (97-12-4) $2m^2\,3dm^2\,4sm^2$ necha $sm^2$ ga teng?

A) 2034    B) 20244    C) 21034    D) 20304

9. (97-5-8) Chumoli 5 minutda $15\dfrac{5}{6}$ m yuradi. U 1 minutda necha metr yuradi?

A) $3\dfrac{5}{6}$    B) $15\dfrac{1}{6}$    C) $3\dfrac{1}{6}$    D) 3

**Yechish:** Chumoli 1 minutda $x$ metr yursin. U holda $15\dfrac{5}{6} : 5 = x : 1$ bo'ladi. Bu yerdan $x = 3\dfrac{1}{6}$. **Javob:** $3\dfrac{1}{6}$ (C).

10. (03-2-64) Toshbaqa 1 minutda 50 sm yo'l bosadi. U 0,1 km masofani qancha soatda o'tadi?

A) $2\dfrac{2}{3}$    B) $2\dfrac{1}{2}$    C) $3\dfrac{1}{3}$    D) $3\dfrac{1}{2}$

11. (97-9-8) G'ildirak 7 minutda $12\dfrac{3}{5}$ marta aylanadi. U 1 minutda necha marta aylanadi?

A) $1\dfrac{4}{5}$    B) 1    C) $1\dfrac{3}{5}$    D) $1\dfrac{2}{5}$

12. (98-7-12) Piyoda kishi 1 km yo'lni $\dfrac{2}{9}$ soatda o'tadi. U $\dfrac{3}{4}$ km yo'lni qancha soatda o'tadi?

A) $\dfrac{1}{5}$    B) $\dfrac{1}{6}$    C) $\dfrac{8}{27}$    D) $\dfrac{1}{4}$

13. (00-5-24) 1 $l$ dengiz suvida o'rtacha $0,00001$ mg oltin bor. $1km^3$ dengiz suvida necha kg oltin bor?

A) 0,1    B) 0,01    C) 1    D) 10

14. (01-2-6) Tiko avtomashinasida 100 km yo'lni o'tish uchun $5,8\ l$ yonilg'i sarflanadi. $8,7\ l$ yonilg'i bilan bu avtomashinada necha km yul yurish mumkin?
    A) 160    B) 154,8    C) 150    D) 145,4

15. (98-11-21) Xaritada ikki shahar orasidagi masofa 4,5 sm ga teng. Xaritadagi masshtab 1 : 2000000 bo'lsa, shaharlar orasidagi haqiqiy masofa necha km bo'ladi?
    A) 0,9    B) 9    C) 90    D) 900

16. (00-5-11) Xaritada 3,6 sm uzunlikdagi kesmaga 72 km masofa mos keladi.Agar xaritada ikki shahar orasidagi masofa 12,6 sm bo'sa, ular orasidagi masofa necha km?
    A) 240    B) 244    C) 246    D) 252

17. Boks tushadigan maydoncha (ring) - tomoni 6 m ga teng kvadrat. Ring uch qator yo'g'on arqon bilan o'raladi. Buning uchun necha metr arqon kerak?
    A) 80    B) 72    C) 76    D) 88

18. (97-9-11) Avtomashina bakiga $70\ l$ benzin quyildi. Gulistonga borish uchun benzinning $\frac{2}{5}$ qismi, Chimyonga borish uchun esa $\frac{3}{7}$ qismi sarflandi. Bakda necha litr benzin qolgan?
    A) 13    B) 15    C) 18    D) 12

19. (98-2-6) Agar kamayuvchini 16 ta va ayriluvchini 20 ta orttirilsa, ayirma qanday o'zgaradi?
    A) 4 ta kamayadi       B) 36 ta ortadi
    C) 36 ta kamayadi      D) 4 ta ortadi

20. (98-3-7) Agar $4\frac{3}{5}$ son $2\frac{1}{2}$ marta oshirilgan bo'lsa, u qanchaga ko'paygan?
    A) 6,6    B) 6    C) 7    D) 6,9

    **Yechish:** Agar $4\frac{3}{5}$ soni $2\frac{1}{2}$ marta oshirilsa, $4\frac{3}{5} \cdot 2\frac{1}{2} = \frac{23}{2}$ son hosil bo'ladi. Ularning farqi $\frac{23}{2} - 4\frac{3}{5} = \frac{69}{10} = 6,9$. **Javob:** 6,9 (D).

21. (99-6-59) $\frac{65}{6}$ va $\frac{39}{8}$ kasrlar butun qismlarining o'rta arifmetigini toping.
    A) 7    B) 6    C) 8    D) 5

22. (99-9-21) [1; 3] oraliqdagi maxraji 3 ga teng bo'lgan barcha qisqarmas kasrlarning yig'indisini toping.
    A) $8\frac{1}{3}$    B) $8\frac{2}{3}$    C) $7\frac{1}{3}$    D) 8

23. (00-2-2) $32 < a < 92$ shartni qanoatlantiruvchi ikki xonali $a$ sonning birinchi raqami o'chirilganda u 31 marta kamaydi. O'chirilgan raqam nechaga teng?
    A) 5    B) 4    C) 6    D) 7

24. "$9^A$ sinfdagi barcha o'quvchilar a'lochi" tasdig'ining inkorini toping.
    A) "$9^A$ sinfdagi barcha o'quvchilar ikkichi"

25. "Tenglama yagona yechimga ega" tasdig'ining inkorini toping.
    B) "$9^A$ sinfda birorta ham a'lochi o'quvchi yo'q"
    C) "$9^A$ sinfda kamida bir o'quvchi a'lochi emas"
    D) "$9^A$ sinfda birgina a'lochi o'quvchi bor"

25. "Tenglama yagona yechimga ega" tasdig'ining inkorini toping.
    A) "Tenglama yechimga ega emas"
    B) "Tenglama cheksiz ko'p yechimga ega"
    C) "Tenglama ikkitadan ko'p yechimga ega"
    D) "Tenglama yechimga ega emas yoki tenglama bittadan ko'p yechimga ega"

26. "$x$ haqiqiy soni 1 dan katta" tasdig'ining inkorini toping.
    A) "$x$ haqiqiy soni 1 dan kichik"
    B) "$x$ haqiqiy soni 1 dan kichik yoki teng"
    C) "$x$ manfiy bo'lgan haqiqiy son"
    D) "$x$ haqiqiy soni 0 yoki undan kichik"

27. (00-2-14) Ikkita toq sonning yig'indisi 5 ga bo'linadi. Bu sonlar kublarining yig'indisi qanday raqam bilan tugaydi?
    A) 6    B) 5    C) 4    D) 0

    **Yechish:** 1-xulosa: ikkita toq sonning yig'indisi juft sondir. 2-xulosa: 5 ga bo'linadigan juft son faqat 0 raqami bilan tugaydi. 3-xulosa: bu toq sonlarning oxirgi raqamlari 1 va 9 yoki 3 va 7 bo'lishi mumkin. Har ikkala holda ham

    $$\ldots 1^3 + \ldots 9^3 = \ldots 1 + \ldots 9 = \ldots 0,$$

    $$\ldots 3^3 + \ldots 7^3 = \ldots 7 + \ldots 3 = \ldots 0$$

    raqami bilan tugaydi. **Javob:** 0 (D).

28. (02-1-28) 1, 2 va 3 raqamlari yordamida yozilgan turli raqamli barcha uch xonali sonlar yig'indisi toping.
    A) 1233    B) 2133    C 1332    D) 2331

29. 2 va 0 raqamlari yordamida yoziladigan barcha to'rt xonali sonlarni yozing. Ulardan eng kattasi bilan eng kichigining ko'paytmasini toping.
    A) 2222    B) 2000    C) 4222    D) 4444000

30. Fermada 43 ta sigir va shuncha buzoq bor. Har bir sigirga bir kunda 8 kg dan, har bir buzoqqa esa 5 kg dan ozuqa beriladi. 30 kunda mollar uchun qancha ozuqa kerak?
    A) 28770    B) 12560    C) 16770    D) 3000

31. (02-1-30) Agar $a$ va $b$ ixtiyoriy natural sonlar bo'lsa, u xolda $2a + 8b$ ufoda quyidagi sonlarning qaysi biriga qoldiqsiz bo'linadi?
    A) 2    B) 3    C) 4    D) 12

32. (01-10-11) Sayohatchilar guruhidagi erkaklarning ayollar soniga nisbati 3 : 4 kabi. Quyida keltirilganlardan qaysi biri guruhdagi sayohatchilar soniga teng bo'lsa olmaydi?
    A) 28    B) 21    C) 23    D) 35

33. (99-5-8) Qishloqda bolalar kattalardan 2 marta ko'p, nafaqaxo'rlar esa qolgiz aholidan 3 marta kam. Agar 15 sonining o'ng va chap tomoniga bir xil raqam yozilsa, qishloq aholisining soni hosil bo'ladi. Bu qanday raqam?
A) 2   B) 3   C) 4   D 6

34. (96-3-62) 1 dan 100 gacha bo'lgan sonlar orasida 2 ga ham, 3 ga ham bo'linmaydiganlari nechta?
A) 33   B) 30   C) 32   D) 21

**Yechish:** Dastlab 1 dan 100 gacha bo'lgan sonlar orasidan 2 ga ham, 3 ga ham bo'linadiganlari sonini topamiz. $A$ bilan 1 dan 100 gacha bo'lgan sonlar orasida 2 ga bo'linadigan sonlar to'plamini, $B$ bilan 1 dan 100 gacha bo'lgan sonlar orasida 3 ga bo'linadigan sonlar to'plamini belgilaymiz. U holda $A \cap B$ to'plam 6 ga bo'linadigan sonlar to'plami bo'ladi. Demak, $N(A) = 50$, $N(B) = 33$ va $N(A \cap B) = 16$ bo'ladi. 1-tenglikka ko'ra $N(A \cup B) = 50 + 33 - 16 = 67$. Shunday qilib 2 ga ham, 3 ga ham bo'linmaydiganlari soni $100 - 67 = 33$ ta ekan. **Javob:** 33 (A).

35. (96-12-60) 1 dan 100 gacha bo'lgan sonlar orasida 2 ga ham, 5 ga ham bo'linmaydiganlari nechta?
A) 35   B) 40   C) 41   D) 32

36. (98-9-3) Sinfdagi 35 ta o'quvchidan 28 tasi suzish sektsiyasiga, 14 tasi voleybol sektsiyasiga qatnashadi. Agar har bir o'quvchi hech bo'lmaganda bitta sektsiyaga qatnashsa, ikkala sektsiyaga qatnashadigan o'quvchilar necha foizni tashkil etadi?
A) 20   B) 18   C) 25   D) 15

37. (03-10-34) 30 kishidan 22 tasi o'yin to'garagiga, 17 tasi esa xorda ashula aytadi. Necha kishi faqat o'yin to'garagiga qatnashadi?
A) 8   B) 10   C) 12   D) 13

38. (03-12-54) 30 ta turistdan 20 tasi ingliz tilini, 15 tasi fransuz tilini bilishadi. Shu turistlarlardan nechtasi ikkala tilni ham bilishadi?
A) 5   B) 10   C) 15   D) 8

**Tenglama yoki tenglamalar sistemasi yordamida yechiladigan masalalar.**

39. (98-12-61) Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son ayirmasi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?
A) 5   B) 11   C) 9   D) 4

**Yechish:** Raqamlari $a$ va $b$ bo'lgan kiki xonali sonni $10a + b$ ko'rinishda yozish mumkin. Uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan ikki xonali son $10b + a$ dir. Ularning ayirmasi

$$10a + b - (10b + a) = 9a - 9b = 9(a - b).$$

Demak, ayirma 9 ga qoldiqsiz bo'linadi. **Javob:** 9 (C).

40. (00-1-5) Ikki xonali sonning o'ng tomoniga 0 raqami yozilsa, berilgan sonning yarmi bilan 323 ning yig'indisiga teng bo'ldan son hosil bo'ladi. Berilgan sonni toping.
A) 54   B) 14   C) 24   D) 34

41. (98-4-2) Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son yig'indisi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?
A) 3   B) 11   C) 9   D) 4

42. (98-12-67) $A$, $B$- raqamlar; $AB$ va $5A$ esa ikki xonali sonlar. Agar $AB \cdot 3 = 5A$ bo'lsa, $A^2 + B^2$ ning qiymati qanchaga teng bo'ladi?
A) 65   B) 13   C) 50   D) 37

43. (99-7-13) Ikki xonali son o'zining raqamlari yig'indisidan 4 marta katta. Raqamlari kvadratlarining yig'indisi 5 ga teng. Shu ikki xonali sonning kvadratini hisoblang.
A) 441   B) 169   C) 121   D) 144

44. (01-2-5) Raqamlari yig'indisining uchlanganiga teng ikki xonali sonni toping.
A) 17   B) 21   C) 13   D) 27

45. (03-1-63) Raqamlari yig'indisiga bo'lganda, bo'linmasi 4 ga va qoldig'i nolga teng bo'ladigan ikki xonali sonlar nechta?
A) 2   B) 3   C) 4   D) 5

46. (96-3-22) Onasi 50, qizi 28 yoshda. Necha yil oldin qizi onasidan 2 marta yosh bo'lgan.
A) 5 yil   B) 6 yil   C) 8 yil   D) 4 yil

**Yechish:** Faraz qilaylik, $x$ yil oldin qizi onasidan 2 marta yosh bo'lgan. $x$ yil oldin onasi $50 - x$ yoshda, qizi esa $28 - x$ yoshda bo'lgan. Masala shartiga ko'ra, $2(28 - x) = 50 - x$. Bu yerdan $x = 6$ ni olamiz. **Javob:** 6 yil (B).

47. (96-12-23) Buvisi 100, nabirasi 28 yoshda. Necha yil oldin nabirasi buvisidan 4 marta yosh bo'lgan.
A) 8 yil   B) 5 yil   C) 4 yil   D) 6 yil

48. (02-1-41) Olim otasidan 32 yosh kichik. Otasi esa bobosidan shuncha yosh kichik. Uch yil avval ularning yoshlari yig'indisi 111 ga teng bo'lgan bo'lsa, hozir Oliming bobosi necha yoshda?
A) 69   B) 72   C) 75   D) 80

49. (02-7-50) 36 yoshdagi onaning yoshi 4 ta bolalari yoshlari yig'indisidan 3 marta ortiq. Necha yildan keyin onaning yoshi bolalari yoshlarining yig'indisiga teng bo'ladi?
A) 8   B) 9   C) 10   D) 7

50. (03-1-61) Egizaklar yoshining yig'indisi 10 yilda ikki marta ortdi. Yana 10 yildan keyin ulardan har birining yoshi nechaga teng bo'ladi?
A) 20   B) 30   C) 40   D) 25

51. (96-1-2) Bir nechta natural sonlarning yig'indisi 75 ga teng. Agar shu sonlarning har biridan 2 ni ayirib yig'indi hisoblansa, u 61 ga teng bo'ladi.

Yig'indida nechta son qatnashgan?
A) 5     B) 7     C) 14     D) 8

**Yechish:** Faraz qilaylik, yig'indida $n$ ta son qatnashgan bo'lsin. Masala shartiga ko'ra, $a_1 + a_2 + \cdots + a_n = 75$ va $a_1 - 2 + a_2 - 2 + \cdots + a_n - 2 = 61$. Bulardan

$$a_1 + a_2 + \cdots + a_n - 2n = 61 \iff 75 - 2n = 61.$$

Bu yerdan $n = 7$ ni olamiz. **Javob:** 7 (B).

52. (98-9-57) Ko'paytmaning har bir hadi 2 ga ko'paytirildi, natijada ko'paytma 1024 marta ortdi. Ko'paytmada nechta had qatnashgan.
A) 8     B) 9     C) 10     D) 11

53. (98-10-49) Berilgan beshta sonning har biri 3 ga ko'paytirilib, so'ngra hosil bo'lgan sonlarning har biriga 2 qo'shildi. Hosil bo'lgan sonlar yig'indisi 70 ga teng bo'lsa, berilgan sonlar yig'indisi nechaga teng bo'lgan?
A) 20     B) 22     C) 15     D) 25

54. (96-3-5) Ikki sonning yig'indisi 51 ga, ayirmasi esa 21 ga teng. Shu sonlarni toping.
A) 36; 15    B) 35; 16    C) 37; 14    D) 33; 18

**Yechish:** Faraz qilaylik, bu sonlarning biri $x$, ikkinchisi $y$ bo'lsin. Masala shartiga ko'ra,

$$\begin{cases} x + y = 51 \\ x - y = 21. \end{cases}$$

Bu sistemani qo'shish usuli yordamida yechamiz. Sistema tenglamalarini qo'shib $2x = 72$ ni olamiz. Bu yerdan $x = 36$ kelib chiqai. Topilgan $x$ ning bu qiymatini sistemaning 1-tenglamasiga qo'yib, $y = 51 - 36 = 15$ ni olamiz. **Javob:** 36; 15 (A).

55. Katakda quyonlar va tovuqlar bor. Agar oyoqlar 314 ta va boshlar 100 ta bo'lsa, quyonlar nechta?
A) 46     B) 37     C) 57     D) 43

56. Bir qutida 100 so'mlik va 500 so'mlik puldan 90 dona bor. 100 so'mliklarning yig'indisi 500 so'mliklarning yig'indisiga teng bo'lsa, qutidagi necha ming so'm pul bor?
A) 21     B) 25     C) 15     D) 18

57. 60 kishilik harbiy qo'shin 25 kunga yetadigan oziq-ovqat bilan ta'minlangan edi. 5 kundan keyin do'shman bilan tuqnashuvda 10 kishi halok bo'ldi. Qolgan oziq-ovqat harbiy qo'shinga necha kunga yetadi?
A) 28     B) 25     C) 24     D) 20

58. (98-12-30) Ikki sonning yig'indisi $6,5$ ga teng. Ulardan biri ikkinchisidan 4 marta kichik. Shu sonlarning kattasini toping.
A) 5,2     B) 5     C) 4     D) 5,3

59. (96-11-5) Ikki sonning yig'indisi 7 ga teng. Ulardan biri ikkinchisidan 4 marta kichik bo'lsa, Shu sonlarning kattasini toping.
A) 5,2     B) 6,2     C) 5,6     D) 5,4

60. (03-3-1) Ikki sonning yig'indisi 6 ga, kvadratlarining ayirmasi esa 48 ga teng. Shu sonlarning ko'paytmasini toping.
A) 8     B) $-8$     C) 7     D) $-7$

61. (03-12-5) Ikki sonning ko'paytmasi ularning yig'indisidan 29 ga, ayirmasidan 41 ga ortiq. Shu ikki sondan birini toping.
A) 7     B) 8     C) 9     D) 10

62. (00-2-12) 100 shunday ikki musbat songa ajratilganki, ulardan biri 7 ga, ikkinchisi 11 ga bo'linadi. Bu sonlar ayirmasining moduli nimaga teng.
A) 8     B) 14     C) 10     D) 12

63. (00-4-15) Biror sonni 2 ga bo'lsak, bo'linma berilgan sondan 4 taga katta chiqadi. Berilgan sonni toping.
A) 4     B) 6     C) 8     D) $-8$

64. (00-8-30) Kasr surati va maxrajining yig'indisi 23 ga teng. Surati maxrajidan 9 ta kam. Kasrni toping.
A) $\dfrac{7}{16}$     B) $\dfrac{8}{15}$     C) $\dfrac{16}{7}$     D) $\dfrac{10}{13}$

65. (00-10-81) Kasr suratiga 2 qo'shilsa, kasr 1 ga, maxrajiga uch qo'shilsa, u $\dfrac{1}{2}$ ga teng bo'ladi. Shu kasrning $\dfrac{3}{5}$ qismini toping.
A) $\dfrac{3}{7}$     B) $\dfrac{4}{7}$     C) $\dfrac{3}{5}$     D) $\dfrac{3}{4}$

66. (01-3-27) Kasrning maxraji suratidan 4 birlik ortiq. Agar kasrning surati va maxraji 1 birlik ortirilsa, $\dfrac{1}{2}$ soni hosil bo'ladi. Berilgan kasrning kvadratini toping.
A) $\dfrac{25}{81}$     B) $\dfrac{49}{121}$     C) $\dfrac{9}{49}$     D) $\dfrac{121}{225}$

67. (01-7-11) Ikki sonning yig'indisi 64 ga teng. Shu sonlardan kattasini uning kichigiga bo'lganda, bo'linma 3 ga, qoldiq 4 ga teng chiqdi. Berilgan sonlardan kattasini toping.
A) 54     B) 42     C) 56     D) 49

68. (02-1-37) $a$ ning qanday qiymatida $9 - a$ va $15 - a$ lar qarama-qarshi sonlar bo'ladi?
A) 9     B) 10     C) 12     D) 15

**Yechish:** Masala shartiga ko'ra,

$$\begin{cases} 15 - a = x \\ 9 - a = -x. \end{cases}$$

Sistema tenglamalarini qo'shib $24 - 2a = 0$ ni olamiz. Bu yerdan $a = 12$ kelib chiqadi. **Javob:** 12 (C).

69. (03-10-7) Ikki sonning yig'indisi 6 ga, ko'paytmasi 7 ga teng bo'lsa, bu sonlar kublarining yig'indisini toping.
A) 90     B) 48     C) 64     D) 72

70. (03-11-60) Birinchi son ikkinchi sondan $2, 5$ ga ortiq. Birinchi sonning $\dfrac{1}{5}$ qismi ikkinchi sonining $\dfrac{4}{5}$ qismiga teng. Shu sonlarning yig'indisini toping.
A) 4      B) 6      C) $6\dfrac{1}{3}$      D) $4\dfrac{1}{6}$

71. (02-7-47) $a$ sonining $b$ songa nisbati $\dfrac{2}{3}$ ga, $c$ sonining $b$ songa nisbati $\dfrac{1}{2}$ ga teng. $c$ sonining $a$ soniga nisbati nechaga teng?
A) $\dfrac{3}{4}$      B) $\dfrac{5}{7}$      C) $\dfrac{5}{6}$      D) $\dfrac{2}{3}$

72. (02-9-9) $a$ sonini 3 ga bo'lgandagi qoldiq 1 ga, 4 ga bo'lgandagi qoldiq esa 3 ga teng. $a$ sonni 12 ga bo'lgandagi qoldiqni toping.
A) 1      B) 3      C) 5      D) 7

73. (03-5-8) $a$ soni $b^2 - 3$ bilan to'g'ri proporsional. $b = 5$ bo'lganda , $a = 88$ bo'lsa, $b = -3$ bo'lganda, $a$ soni nechaga teng bo'ladi?
A) 24      B) 6      C) 18      D) 12

74. (03-8-21) $25\dfrac{1}{2}$ sonini 7; 8; 2 sonlariga mutanosib bo'laklarga bo'lgandagi eng kichik sonni toping.
A) 3      B) 4      C) 5      D) 3,5

75. (03-6-30) $\dfrac{5}{7}$ qismi 4 ga teng bo'lgan sonni toping.
A) $5\dfrac{6}{7}$      B) $5\dfrac{1}{5}$      C) $5\dfrac{2}{5}$      D) $5\dfrac{3}{5}$

**Yechish:** Izlanayotgan son $x$ bo'lsin. U holda masala shartiga ko'ra, $x \cdot \dfrac{5}{7} = 4$ bo'ladi. Bu yerdan $x = 5\dfrac{3}{5}$ kelib chiqadi. **Javob:** $5\dfrac{3}{5}$ (D).

76. (03-6-33) $0, 23$ qismi 690 ga teng sonni toping.
A) 3000      B) 2500      C) 2800      D) 3500

77. (03-8-26) $0, 4(6)$ qismi 360 sonining $0, 6(4)$ qismiga teng sonni toping.
A) $497\dfrac{1}{7}$      B) $506\dfrac{2}{7}$      C) $400\dfrac{3}{7}$      D) $497\dfrac{5}{7}$

78. (97-10-10) Turist butun yo'lning $0, 85$ qismini o'tganda, ko'zlangan manziligacha $6, 6$ km qolgani ma'lum bo'ldi. Butun yo'lning uzunligi necha km?
A) 52 km   B) 44 km   C) 36, 6 km   D) 64, 4 km

**Yechish:** Butun yo'lning uzunligi $x$ km bo'lsin. U holda masala shartiga ko'ra, $x \cdot 0, 85 + 6, 6 = x$ bo'ladi. Bu yerdan $6, 6 = 0, 15x \Longleftrightarrow x = 44$ kelib chiqadi. **Javob:** 44 (B).

79. (96-7-10) Turist butun yo'lning $0, 35$ qismini o'tganda unga, yo'lning yarmigacha $18, 3$ km qolgani ma'lum bo'ldi. Butun yo'lning uzunligini toping.
A) 110 km   B) 102 km   C) 122 km   D) 98 km

80. (97-3-10) Velosipedchi bir soatda butun yo'lning $0, 65$ qismini o'tdi, bu esa yo'lning yarmidan $7, 5$ km ko'p. Butun yo'lning uzunligini toping.
A) 47, 5 km   B) 62, 5 km   C) 50 km   D) 65 km

81. (00-1-6) Sayohat uchun ma'lum miqdorda pul yig'ish kerak edi. Agar har bir sayohatchi 750 so'mdan to'lasa, to'lovga 1200 so'm yetmaydi, agar har bir sayohatchi 800 so'mdan to'lasa, keragidan 1200 so'm ortib qoladi. Sayohatga necha kishi qatnashishi kerak edi?
A) 38      B) 48      C) 45      D) 46

82. (01-2-10) Fermada tovuq va qo'ylar bor. Ularning jami soni 170 ta, oyoqlari soni 440 ta. Qo'ylar tovuqlardan nechta kam?
A) 50      B) 60      C) 70      D) 80

83. (01-8-2) Kostyum paltodan 5950 so'm arzon. Agar palto kostyumdan $1, 7$ marta qimmat bo'lsa, kostyum necha so'm turadi?
A) 8750      B) 7550      C) 3500      D) 8500

84. (01-10-10) 30 so'mlik va 35 so'mlik daftarlardan jami 490 so'mlik xarid qilindi. Quyida keltirilganlardan qaysi biri 30 so'mlik daftarlar soniga teng bo'lishi mumkin?
A) 5      B) 6      C) 7      D) 8

**Yechish:** 30 so'mlik daftardan $x$ dona, 35 so'mlik daftardan $y$ dona xarid qilingan bo'lsin. U holda masala shartiga ko'ra, $30x + 35y = 490$ bo'ladi. Bu yerda $x, y$ lar natural son. Shuning uchun 7   30 so'mlik daftarlar soni bo'lishi mumkin. **Javob:** 7 (B).

85. (02-6-16) 20 va 25 so'mlik daftarlardan hammasi bo'lib 350 so'mlik xarid qilindi. Quyida keltirilgan sonlardan qaysi biri 25 so'mlik daftarlarning soniga teng bo'lishi mumkin?
A) 4      B) 5      C) 6      D) 7

86. (02-1-34) Do'konga birinchi kuni $5, 42$ t ikkinchi kuni birinchi kundagiga qaraganda $2, 43$ t kam, uchinchi kuni esa dastlabki 2 kundagidan $3, 21$ t kam un keltirildi. Uchinchi kuni qancha un keltirilgan?
A) 13,61      B) 2,99      C) 7,85      D) 5,2

87. (02-3-4) Sinfda o'qiydigan o'g'il bolalar sonining barcha o'quvchilar soniga nisbati $\dfrac{4}{7}$ ga teng bo'lsa, qiz bolalar sonining o'g'il bolalar soniga nisbati nechaga teng bo'ladi?
A) $\dfrac{3}{4}$      B) $\dfrac{3}{5}$      C) $\dfrac{1}{2}$      D) $\dfrac{2}{5}$

88. (00-5-12) Sexda 120 ta samovar va 20 ta patnis yasalgan. Sarf qilingan hamma materialning $0, 96$ qismi samovarga ketgan. Agar har bir samovarning og'irligi $3, 2$ kg dan bo'lsa, har bir patnis necha kg bo'lgan?
A) 0,8      B) 0,04      C) 7,68      D) 0,768

89. (03-6-29) 2 o'ram bir xil sim xarid qilindi. Birinchi o'ram 3060 so'm, ikkinchi esa 1904 so'm turadi. Agar birinchi o'ram ikkinchi o'ramdan 17 m uzun bo'lsa, birinchi o'ramda necha m sim bor?
A) 40      B) 45      C) 47      D) 28

**Yechish:** 1 m simning bahosi $x$ so'm, birinchi o'ramda $y$ m uzunlikdagi sim bo'lsin. U holda ikkinchi o'ramda $y - 17$ m sim bo'ladi. Masala shartiga ko'ra,

$$\begin{cases} xy = 3060 \\ x(y-17) = 1904. \end{cases}$$

Sistema 2-tenglamasini $xy - 17x = 1904$ shaklda yozib, 1-tenglikdan foydalansak, $3060 - 17x = 1904$ ni olamiz. Bu yerdan $x = 68$ kelib chiqadi. $x$ ning bu qiymatini sistemaning 1-tenglamasiga qo'yib $y = 45$ hosil qilamiz. **Javob:** 45 (B).

90. (03-6-32) Binoni 3 ta bo'yoqchi birgalikda bo'yadi. Birinchisi binoning $\frac{5}{13}$ qism yuzasini bo'yadi. Ikkinchi esa, uchinchisiga nisbatan 3 marta ko'p yuzani bo'yadi. Uchinchi bo'yoqchi qancha qism yuzani bo'yagan?
A) $\frac{1}{18}$      B) $\frac{1}{13}$      C) $\frac{1}{9}$      D) $\frac{2}{13}$

91. (03-10-19) 7 ta kitob va 4 ta jurnalning birgalikdagi bahosi, 4 ta kitob va 7 ta jurnalning birgalikdagi bahosidan 525 so'm ortiq. Kitob jurnalga qaraganda qancha so'm qimmat turishini aniqlang.
A) 150      B) 175      C) 200      D) 125

92. (00-8-2) Kitob betlarini sahifalab chiqish uchun 1012 ta raqam ishlatildi. Agar sahifalash 3-betdan boshlangan bo'lsa, kitob necha betlik?
A) 374      B) 400      C) 506      D) 421

93. (03-11-59) Traktorchilar maydonni uch kunda haydab bo'lishdi. Birinchi kuni ular maydonning $\frac{3}{7}$ qismini, ikkinchi kuni butun yer maydonining 40% ini, uchinchi kuni qolgan 72 ga maydonni haydashgan bo'lsa, maydonning yuzi necha gektar bo'lgan?
A) 420      B) 450      C) 500      D) 350

## 9.2   Foizga oid masalalar

**1.** $a$ **soni** $t\%$ **ga oshirilsa,** $(1 + \frac{t}{100}) \cdot a$ **soni hosil bo'ladi. Masalan,** $a$ **soni 30% ga oshirilsa,** $1,3a$; **40% ga oshirilsa,** $1,4a$ **hosil bo'ladi.**

**2.** $a$ **soni t% ga kamaytirilsa,** $(1 - \frac{t}{100}) \cdot a$ **soni hosil bo'ladi. Masalan,** $a$ **soni 30% ga kamaytirilsa,** $(1-0,3)a=0,7a$; **40% ga kamaytirilsa,** $(1-0,4)a=0,6a$ **hosil bo'ladi.**

1. (98-3-6) Ishchining maoshi dastlab 20% ga, so'ngra yana 20% oshirilgan bo'lsa, uning maoshi necha

foizga oshgan?
A) 40      B) 50      C) 42      D) 44

**Yechish:** Ishchining dastlab maoshi $x$ ga teng bo'lsin. Uning maoshi 20% ga oshgandan keyin ishchi $x + 0,2x = 1,2x$ miqdorda maosh oladi. Ikkinshi marta 20% ga oshgandan keyin u

$$1,2x + 0,2 \cdot 1,2x = (1,2 + 0,24)x = 1,44x$$

miqdorda maosh oladi. Demak, uning maoshi jami $1,44x - x = 0,44x$ ga ortadi ekan. **Javob:** 44% (D).

2. (96-1-4) Do'konga 96 t karam keltirildi. Agar karamning 80% i sotilgan bo'lsa, do'konda qancha karam qolgan.
A) 16      B) 19,2      C) 24      D) 20,2

3. (96-1-9) Ikki sonning ayirmasi 33 ga teng. Agar shu sonlardan kattasining 30% i kichigining $\frac{2}{3}$ qismiga teng bo'lsa, shu sonlarni toping.
A) 56; 23    B) 27; 60    C) 17; 50    D) 37; 70

4. (98-3-4) Ishchining oylik maoshi 350 so'm. Agar uning maoshi 30% ortsa, qancha maosh oladi?
A) 405      B) 380      C) 1050      D) 455

5. (96-6-3) Magazinga keltirilgan tarvuzlarning 56% i birinchi kuni, qolgan 132 tasi ikkinchi kuni sotildi. Birinchi kuni qancha tarvuz sotilgan?
A) 168      B) 148      C) 178      D) 138

6. (96-10-9) Ikki sonning ayirmasi 5 ga teng. Agar shu sonlardan kattasining 20% i kichigining $\frac{2}{9}$ qismiga teng bo'lsa, shu sonlarni toping.
A) 30; 35    B) 36; 41    C) 45; 50    D) 63; 68

7. (96-9-59) Ikki sonning yig'indisi 24 ga teng. Agar shu sonlardan birining 85% ikkinchisining $\frac{7}{20}$ qismiga teng bo'lsa, shu sonlarni toping.
A) 18; 6    B) 20; 4    C) 7; 17    D) 8; 16

8. (96-11-4) Nafaqaxo'rning oylik nafaqasi 450 so'm. Agar uning nafaqasi 20% ortsa, qancha nafaqa oladi?
A) 540      B) 470      C) 900      D) 490

9. Talabaning stipendiyasi 120 ming so'm. Agar uning stipendiyasi 20% ortsa, u necha ming so'm stipendiya oladi?
A) 140      B) 144      C) 142      D) 148

10. (96-12-63) Go'sht qaynatilganda o'z vaznining 40% ini yuqotadi. 6 kg go'sht qaynatilganda vazni necha kg kamayadi?
A) 2,4      B) 2,2      C) 1,9      D) 2

**Yechish:** 6 ning 40% ini hisoblaymiz:

$$\frac{6}{100} \cdot 40 = 2,4.$$

Demak, 2,4 kg kamayadi. **Javob:** 2,4 (A).

11. (96-9-55) Olxo'ri quritilganda 35% olxo'ri qoqisi hosil bo'ladi. 64 kg olxo'ri quritilsa, gancha olxo'ri qoqisi olinadi?
    A) 20      B) 18,2      C) 22,4      D) 25

12. (07-118-26) 15 kg eritmaning 40 foizi tuzdan iborat. Tuzning miqdori 25 foiz bo'lishi uchun eritmaga necha kg chuchuk suv qo'shish kerak.
    A) 6      B) 9      C) 8      D) 10

13. (97-4-5) 30 ta talabadan 25 tasi qishki sinovlarning hammasini topshirdi. Ba'zi sinovlarni topshira olmagan talabalar barcha sinovlarni topshirgan talabalarning necha foizini tashkil etadi?
    A) 10%      B) 15%      C) 20%      D) 25%

14. (97-5-5) Qutiga 25 kg massali yuk joylandi. Agar qutining massasi yuk massasining 12% ini tashkil etsa, qutining massasini toping.
    A) 3 kg      B) 3,5 kg      C) 4 kg      D) 4,5 kg

15. (97-6-5) Noma'lum sonning 14% i 80 ning 35% iga teng. Noma'lum sonni toping.
    A) 120      B) 168      C) 200      D) 280

16. (97-8-3) Kutubxonadagi kitoblarning 55% i o'zbek tilida, qolgan kitoblar rus tilida Rus tilidagi kitoblar 270 ta. Kutubxonada o'zbek tilida nechta kitob bor?
    A) 325      B) 310      C) 320      D) 330

17. (98-1-2) Ushbu $2\frac{3}{5}$; $\frac{1}{2}$ sonlar ayirmasining 10% ini toping.
    A) 0,22      B) 0,3      C) 0,021      D) 0,21

18. (98-5-3) Paxtadan 30% foiz tola olinsa, 60 tonna tola olish uchun qancha paxta kerak?
    A) 100      B) 400      C) 200      D) 300

19. (98-10-5) 21 kg shakar va 129 kg boshqa mahsulotlardan muzqaymoq tayyorlandi. Shakar muzqaymoqning necha foizini tashkil etadi?
    A) 13      B) 15      C) 16      D) 14

20. (99-1-5) Institutdagi talabalarning 35% ini qizlar tashkil qiladi. Yigitlar qizlardan 252 taga ko'p. Talabalarning umymiy sonini toping.
    A) 840      B) 640      C) 546      D) 740

    **Yechish:** Masala shartidan talabalarning 65% i yigitlar ekanligi kelib chiqadi. Yigitlar qizlardan 30% ga ko'p. Agar jami alabalar sonini $x$ desak, u holda $\frac{x}{100} \cdot 30 = 252$ bo'ladi. Bu yerdan $x = 840$.
    **Javob:** 840 (A).

21. (99-1-6) 520 sonini shunday ikki bo'lakka bo'lingki, ulardan birining 80% i ikkinchisining 24% ini tashkil qilsin. Bo'laklarning kattasini toping.
    A) 400      B) 120      C) 420      D) 460

22. (99-3-3) 200 soni 30 foizga orttirildi, hosil bo'lgan son 20 foizga kamaytirildi. Natijada qanday son hosil bo'ladi?
    A) 206      B) 210      C) 208      D) 212

23. (99-7-5) Bug'doydan 90% un olinadi. 3 t bug'doydan qancha un olish mumkin.
    A) 2,5      B) 2,6      C) 2,1      D) 2,7

24. (00-1-3) Inflyasiya natijasida mahsulotning narxi 25% ga oshirildi. Lekin mahsulotga talabning kamligi tufayli uning narxi 10% ga kamaytirildi. Mahsulotning oxirgi narxi dastlabkisiga qaraganda necha foiz ortdi?
    A) 12,8      B) 11,5      C) 12      D) 12,5

25. (00-7-6) Mahsulotning narxi birinchi marta 25% ga, ikkinchi marta yangi bahosi yana 20% oshirildi. Mahsulotning oxirgi bahosi necha foizga kamaytirilsa, uning narxi dastlabki narxiga teng bo'ladi?
    A) 45      B) 48      C) 50      D) $33\frac{1}{3}$

26. (00-9-57) Agar ikkita sondan biri 20% ga, ikkinchisi 12,5% ga kamaytirilsa, ularning ko'paytmasi necha foizga kamayadi?
    A) 40      B) 50      C) 45      D) 30

27. (01-1-63) Agar sonning 40% ini 5 ga, ko'paytirganda 8 chiqsa, shu sonning o'zini toping.
    A) 2      B) 4      C) 6      D) 8

28. (01-2-71) Uy bekasi kilosi 150 so'mdan yong'oq sotib oldi. Yong'oqlar qobig'idan tozalangach, umumiy og'irligining 60% i qoldi. Uy bekasi bir kilogramm tozalangan yong'oq uchun necha so'm sarflagan?
    A) 190      B) 180      C) 220      D) 250

29. (01-2-72) Xo'jalikda paxta ishlab chiqarish har yili 10% ga ortsa, 3 yilda paxta ishlab chiqarish necha foizga ortadi?
    A) 30      B) 32      C) 33      D) 33,1

30. (01-5-7) Xodimning oylik maoshi ketma-ket ikki marta bir xil foizga oshirilgandan so'ng, u dastlabki maoshdan 69% ga oshgan bo'lsa, maosh har gal necha foizga oshirilgan?
    A) 30      B) 34,5      C) 40      D) 35

31. (01-10-5) 17 ning 17% ini toping.
    A) 1      B) 3,24      C) 2,89      D) 10

32. (02-6-12) 19 ning 19% ini toping.
    A) 1      B) 2,89      C) 3,69      D) 3,61

33. (01-10-7) Ishlab chiqarish samaradorligi birinchi yili 15% ga, ikkinchi yili 12% ga ortdi. Shu ikki yil ichida samaradorlik necha foizga ortgan?
    A) 27      B) 28      C) 28,6      D) 28,8

34. (01-12-26) Firma mahsulotni 380 so'mga sotib, 4 foiz zarar qildi. Shu mahsulotning tannarxini toping.
    A) 400      B) 495      C) $395\frac{5}{6}$      D) $395\frac{1}{6}$

    **Yechish:** Mahsulotning tannarxi $x$ bo'lsin, u holda $x - \frac{x}{100} \cdot 4 = 380$ bo'ladi. Bu yerdan $x = 395\frac{5}{6}$. **Javob:** $395\frac{5}{6}$ (C).

35. (02-1-36) Kitob 200 so'm turadi. Uning narxi 2 marta 5% dan arzonlashtirildi. Kitobning narxi necha so'm bo'ldi?
A) 180    B) 180,2    C) 180,3    D) 180,5

36. (02-2-7) Mehnat unumdorligi 40% oshgach, korxona kuniga 560 ta buyum ishlab chiqaradigan bo'ldi. Korxonada oldin kuniga nechta buyum ishlab chiqarilgan?
A) 400    B) 420    C) 380    D) 440

37. (02-10-44) 6 foizi 30 ning 22 foiziga teng bo'lgan sonni toping.
A) 110    B) 108    C) 96    D) 90

38. (02-12-1) Ikki sonning yig'indisi 24 ga teng. Shu sonlardan birining 35% i ikkinchisining 85% iga teng. Shu sonlardan kichigini toping.
A) 3,5    B) 7    C) 6    D) 9

39. (03-1-64) Kilosi 600 so'mdan baliq sotib olindi. Tozalangandan keyin baliqning og'irligi dastlabki og'irligining 80% ini tashkil etdi. 1 kg tozalangan baliq necha so'mga tushgan?
A) 480    B) 500    C) 640    D) 750

40. (03-2-36) 12% ga arzonlashtirilgandan keyin mahsulotning bahosi 1100 so'm bo'ldi. Mahsulotning dastlabki bahosini aniqlang.
A) 1200    B) 1240    C) 1280    D) 1250

41. (01-6-4) $x$ $(x > 0)$ ga teskari bo'lgan son $x$ ning 36% ini tashkil etadi. $x$ ning qiymatini toping.
A) $2\frac{1}{3}$    B) $1\frac{2}{3}$    C) $1\frac{1}{3}$    D) $3\frac{1}{3}$

**Yechish:** Masala shartidan $\frac{1}{x} = \frac{36\,x}{100}$ tenglikni olamiz. Bu yerdan

$$x^2 = \frac{100}{36} \Longleftrightarrow x = \frac{10}{6} = 1\frac{2}{3}. \quad \textbf{Javob: } 1\frac{2}{3} \ (B).$$

42. (03-10-23) Biznesmen o'z pulining 50% ini yo'qotdi. Qolgan puliga aksiya sotib olgach, u 40% daromad (foyda) oldi. Uning oxirgi puli dastlabki pulining necha foizini tashkil etadi?
A) 60    B) 70    C) 80    D) 100

## 9.3   Harakatga oid masalalar

1. **$S$ masofa $v$ tezlik bilan $t$ vaqtda bosib o'tilsa, $S = vt$ bo'ladi.**

2. **$A$ va $B$ punktlar orasidagi masofa $S$ bo'lsin.**

   **a) $A$ va $B$ lardan bir-biriga qarab ikki yo'lovchi $v_1$ va $v_2$ tezliklar bilan yo'lga chiqib, $t$ vaqtdan keyin uchrashsa, $(v_1 + v_2)t = S$ bo'ladi.**

   **b) $A$ va $B$ lardan bir tomonga ikki yo'lovchi $v_1$ va $v_2$ tezliklar bilan yo'lga chiqib, $t$ vaqtdan keyin 1 - yo'lovchi 2- siga yetib olsa, $v_1 t - v_2 t = S$ bo'ladi.**

3. **Qayiqning turg'un suvdagi tezligi $v$ ga, daryo oqimining tezligi $u$ ga teng bo'lsin. U holda qayiq oqim bo'ylab $v+u$, oqimga qarshi esa $v - u$ tezlik bilan suzadi.**

1. (97-12-6) Motosiklchi va velosipedchi bir tomonga qarab harakat qilishmoqda. Velosipedchining tezligi 12 km/soat, motosiklchiniki 30 km/soat va ular orasidagi masofa 72 km bo'lsa, necha soatdan keyin motosiklchi velosipedchini quvib yetadi?
A) 3    B) 4    C) 3,5    D) 2,5

**Yechish:** Motosiklchi velosipedchini $t$ soatdan keyin quvib yetsin. $t$ soatda motosiklchi $30t$ km, velosipedchi esa $12t$ km masofa bosib o'tadi. Bu yerdan $30t - 12t = 72$ tenglamani hosil qilamiz. Uni yechib $t = 4$ ekanini topamiz. **Javob:** 4 (B).

2. (96-3-3) Passajir va yuk poyezdi bir-biriga tomon harakatlanmoqda. Ular orasidagi masofa 275 km. Yuk poyezdining tezligi 50 km/soat. Passajir poyezdining tezligi yuk poyezdining tezligidan 20% ortiq. Ular necha soatdan keyin uchrashadi?
A) 3    B) 2    C) 2,5    D) 4

3. (96-3-69) Uzunligi 400 m bo'lgan poyezd uzunligi 500 m bo'lgan tunneldan 30 s da o'tib ketdi. Poyezdning tezligini toping.
A) 35 m/s    B) 30 m/s    C) 40 m/s    D) 45 m/s

4. (96-6-7) Ikki shahardan bir-biriga qarab ikki turist yo'lga chiqdi. Birinchisi avtomashinada, tezligi 62 km/soat. Ikkinchisi avtobusda tezligi 48 km/soat. Agar ular $0,6$ soatdan keyin uchrashgan bo'lsa, shaharlar orasidagi masofani toping.
A) 70 km    B) 64 km    C) 62 km    D) 66 km

5. (96-9-9) Poyezdning uzunligi 800 m. Poyezdning ustun yonidan 40 s da o'tib ketgani ma'lum bo'lsa, uning tezligini toping.
A) 30 m/s    B) 15 m/s    C) 25 m/s    D) 20 m/s

6. (99-3-9) Yo'lovchilar poyezdining 3 soatda yurgan masofasi yuk poyezdining 4 soatda yurgan masofasidan 10 km ortiq. Yuk poyezdining tezligi yo'lovchilar poyezdining tezligidan 20 km/soat ga kam. Yuk poyezdining tezligini toping.
A) 40    B) 45    C) 48    D) 50

**Yechish:** Yuk poyezdining tezligi $v$ bo'lsin. U holda yo'lovchilar poyezdining tezligi $v+20$ bo'ladi. Yo'lovchilar poyezdi 3 soatda $3(v + 20)$ km masofani bosib o'tadi, yuk poyezdi 4 soatda $4v$ km masofani bosib o'tadi. Masala shartiga ko'ra

$$3(v + 20) = 4v + 10.$$

Bu yerdan $v = 50$ ni olamiz. **Javob:** 50 (D).

7. (97-8-7) Oralaridagi masofa 200 km bo'lgan $A$ va $B$ punktlardan bir vaqtning o'zida ikki turist bir-biriga qarab yo'lga chiqdi. Birinchisi avtobusda tezligi 40 km/soat, ikkinchisi avtomobilda. Agar ular 2 soatdan keyin uchrashgan bo'lishsa, avtomobilning tezligini toping.

A) 58 km/soat     B) 55 km/soat
C) 65 km/soat     D) 60 km/soat

8. (97-10-4) Muayyan masofani bosib o'tish uchun ketadigan vaqtni 25% ga kamaytirish uchun tezlikni necha foiz orttirish kerak?
A) 25     B) 20     C) $33\frac{1}{3}$     D) 30

9. (99-9-4) $A$ va $B$ shaharlar orasidagi masofa 188 km. Bir vaqtning o'zida bir-biriga qarab $A$ shahardan velosipedchi, $B$ shahardan motosiklchi yo'lga tushdi va ular $A$ shahardan 48 km masofada uchrashdi. Agar velosipedchining tezligi 12 km/soat bo'lsa, motosiklchining tezligini toping.
A) 45     B) 42     C) 30     D) 35

10. (01-10-15) Uzunligi 200 m bo'lgan poyezd balandligi 40 m bo'lgan ustun yonidan 50 sekundda o'tib ketdi. Uzunligi 520 m bo'lgan ko'prikdan shu poyezd o'sha tezlik bilan necha minutda o'tib ketadi?
A) 2     B) 2,5     C) 3     D) 4

11. (96-11-3) Kater va teploxod bir-biriga tomon harakatlanmoqda. Ular orasidagi masofa 120 km. Teploxodning tezligi 50 km/ soat. Katerning tezligi teploxodning tezligidan 60% kam. Ular necha soatdan keyin uchrashadi?
A) $1\frac{5}{7}$     B) 2     C) $2\frac{1}{4}$     D) $2\frac{1}{3}$

12. (98-2-7) $A$ va $B$ pristanlar orasidagi masofa 96 km. $A$ pristandan oqim bo'ylab sol jo'natildi. Xuddi shu paytda $B$ pristandan oqimga qarshi motorli qayiq jo'nadi va 4 soatdan keyin sol bilan uchrashdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligini toping.
A) 20 km/soat     B) 19 km/soat
C) 17 km/soat     D) 24 km/soat

**Yechish:** Solning tezligi daryo oqimining tezligi bilan bir xil, ya'ni 3 km/s. Qayiqning turg'un suvdagi tezligi $v$ km/s bo'lsin, u holda uning oqimga qarshi tezligi $v - 3$ km/s bo'ladi. 2-qoidaning a) bandiga ko'ra $(3 + v - 3) \cdot 4 = 96$ ni olamiz. Bu yerdan $v = 24$. **Javob:** 24 (D).

13. (98-9-6) Ikki pristan orasidagi masofa 63 km. Bir vaqtning o'zida oqim bo'ylab birinchi pristandan sol, ikkinchisidan motorli qayiq jo'natildi va motorli qayiq solni 3 soatda quvib yetdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligi qanchaga teng bo'ladi?
A) 21     B) 20     C) 22     D) 19

14. (01-9-34) Matorli qayiqning daryo oqimi bo'yicha tezligi 21 km/soat dan ortiq va 23 km/soat dan kam. Oqimga qarshi tezligi esa 19 km/soat dan ortiq va 21 km/soatdan kam. Qayiqning turg'un suvdagi tezligi qanday oraliqda bo'ladi?
A) (18;20)   B) (19;21)   C) (18;19)   D) (20;22)

15. (02-1-2) Katerning daryo oqimi bo'ylab va oqimga qarshi tezliklari yig'indisi 30 km/soat. Katerning turg'un suvdagi tezligi (km/soat)ni toping.
A) 15     B) 16     C) 10     D) 18

16. (03-3-10) Paroxod daryo oqimi bo'ylab 48 km va oqimga qarshi shuncha masofani 5 soatda bosib o'tdi. Agar daryo oqimining tezligi soatiga 4 km bo'lsa, Paroxodning turg'un suvdagi tezligini toping.
A) 12     B) 16     C) 20     D) 24

17. (03-6-10) Avtomobil butun yo'lning $\frac{3}{7}$ qismini 1 soatda qolgan qismini $1,5$ soatda bosib o'tdi. Uning birinchi tezligi ikkinchi tezligidan necha marta katta?
A) $\frac{2}{3}$     B) $\frac{3}{2}$     C) $\frac{9}{8}$     D) $\frac{8}{9}$

18. (03-7-15) Avtomobil butun yo'lning $\frac{3}{7}$ qismini 1 soatda, qolgan qismini 2 soatda bosib o'tdi. Uning birinchi tezligi ikkinchi tezligidan necha marta katta?
A) $\frac{2}{3}$     B) $\frac{3}{2}$     C) $\frac{9}{8}$     D) $\frac{8}{9}$

## 9.4 Ishga oid masalalar

1. **Agar** 1-kombayn hosilni $x$ soatda, 2-kombayn $y$ soatda, ikkala kombayn birgalikda hosilni $z$ soatda yig'ib olsa, u holda

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}. \tag{1}$$

1. (98-10-11) Bir kombayn daladagi hosilni 15 soatda, boshqasi esa shu hosilni 10 soatda yig'ib olishi mumkin. Ikkala kombayn birgalikda hosilni qancha soatda yig'ib olishi mumkin?
A) 7     B) 8     C) 5,5     D) 6

**Yechish:** (1) tenglikda $x = 15$, $y = 10$ deb $z$ ni topamiz:

$$\frac{1}{z} = \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}, \qquad z = 6.$$

**Javob:** $z = 6$ (D).

2. (96-3-67) Meshdagi suv Anvarning o'ziga 20 kunga, ukasiga esa 60 kunga yetadi. Meshdagi suv ikkalasiga necha kunga yetadi?
A) 15     B) 14     C) 12     D) 16

3. (96-9-7) Meshdagi suv Anvarning o'ziga 14 kunga, ukasi ikkalasiga esa 10 kunga yetadi. Meshdagi suv Anvarning ukasiga necha kunga yetadi?
A) 35     B) 39     C) 28     D) 26

4. (96-12-8) Birinchi kuni ish normasining $\frac{1}{3}$ qismi bajarildi. Ikkinchi kuni birinchi kunda bajarilgan ishning $\frac{1}{6}$ qismicha ko'p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi?
A) $0,5$     B) $\frac{2}{9}$     C) $\frac{13}{18}$     D) $\frac{5}{6}$

5. (98-12-73) Birinchi quvur hovuzni 3 soatda to'ldiradi, ikkinchisi esa 5 soatda. Ikkala quvur birgalikda hovuzni qancha vaqtda to'ldiradi?

   A) $1\dfrac{7}{8}$    B) $2\dfrac{1}{2}$    C) $2\dfrac{1}{5}$    D) $1\dfrac{4}{5}$

6. (99-2-7) Hovuzga 2 ta quvur o'tkazilgan. Birinchi quvur bo'sh hovuzni 10 soatda to'ldiradi, ikkinchisi esa 15 soatda bo'shatadi. Hovuz bo'sh bo'lgan vaqtda ikkala quvur birdaniga ochilsa, hovuz necha soatdan keyin to'ladi?

   A) 25    B) 28    C) 30    D) 32

7. Ali o'rtog'i bilan ishning 20% bajardi. Keyin bir o'zi 4 kun ishlab qolgan ishning 25% ni bajardi. Ali bu ishning hammasini necha kunda qila oladi?

   A) 20    B) 25    C) 30    D) 16

   **Yechish:** Ali 4 kunda qolgan 80% ishning 25% ni bajardi. Demak, u 4 kunda jami ishning

   $$\frac{0,8 \cdot 25}{100} = \frac{20}{100} = 20\%$$

   ni bajardi. Bu yerdan Ali 20 kunda ishning 100% ni bajarishi kelib chiqadi. **Javob:** 20 (A).

8. (00-4-19) Usta muayyan ishni 12 kunda, uning shogirdi esa 30 kunda bajaradi. Agar 3 ta usta va 5 ta shogird birga ishlasalar, o'sha ishni necha kunda bajarishadi?

   A) 2,4    B) 3,6    C) 2,5    D) 1,2

9. (00-7-9) Muayyan ishni bajarishga bir ishchi 3 soat, ikkinchi ishchi esa 6 soat vaqt sarflaydi. Birinchi ishchi 1 soat ishlagandan keyin, unga ikkinchi ishchi qo'shildi. Ikkala ishchi birgalikda qolgan ishni necha soatda tugatishadi?

   A) 2 soat 30 min   B) 1 soat 40 min
   C) 1 soat 20 min   D) 2 soat

10. (03-9-7) Ikkita ishchi birgalikda ishlab, ma'lum ishni 12 kunda tamomlaydi. Agar ishchilarning bittasi shu ishning yarmini bajargandan keyin, ikkinchi ishchi qolgan yarmini bajarsa, shu ishni 25 kunda tamomlashi mumkin. Ishchilardan biri boshqasiga qaraganda necha marta tez ishlaydi?

    A) 1,2    B) 1,5    C) 1,6    D) 1,8

11. (03-10-24) Eski traktor maydonni 6 soatda, yangisi esa 4 soatda haydaydi. Shu maydonni 3 ta eski va 2 ta yangi traktor qancha vaqtda haydaydi?

    A) 1 soatda    B) 1,5 soatda
    C) 2 soatda    D) 2,5 soatda

# 10  -bob. Funksiyalar

Tabiatda ikki xil miqdorlar, o'zgaruvchi va oz'garmas miqdorlar uchraydi. Masalan, bizga bir nechta aylana (har xil radiusli) berilgan bo'lsin. Barcha aylanalar uchun aylana uzunligining o'z radiusiga nisbati o'zgarmas bo'lib u $2\pi$ ga teng, lekin ularning diametrlari, aylana uzunliklari radius o'zgarishi bilan, o'zgarib turadi. Agar bizga har xil (katta, kichik) kvadratlar berilgan bo'lsa, bu kvadratlarning yuzalari tomoni o'zgarishi bilan o'zgarib turadi, lekin ularning burchaklari $90^0$ ligicha o'zgarmasdan qolaveradi. Odatda o'zgarmas miqdorlar $a$, $b$, $c$, $d$, ...; o'zgaruvchi miqdorlar $x$, $y$, $z$, $u$, $v$, ... harflari bilan belgilanadi. Matematikada ko'pincha o'zaro bir-biriga bog'liq ravishda o'zgaradigan miqdorlar qaraladi. Yuqoridagi misolimizda aylananing uzunligi $\ell$ bilan uning radiusi $R$ orasida $\ell = 2\pi R$ bog'lanish bor. Ma'lumki, kvadratning yuzasi uning tomoni kvadratiga teng, ya'ni kvadratning yuzasi $S$, uning tomoni uzunligini $a$ desak, u holda ular orasida $S = a^2$ bog'lanish mavjud.

Agar $x$ miqdorning har bir qiymatiga $y$ miqdorning yagona qiymati mos kelsa, $y$ miqdor $x$ miqdorning funksiyasi deyiladi. Bu holda $x-$ argument yoki erkli o'zgaruvchi, $y-$ esa funksiya yoki erksiz o'zgaruvchi deyiladi. $x$ va $y$ miqdorlar o'rtasidagi bog'lanishni o'rnatuvchi moslik $f$ orqali belgilanadi va quyidagicha yoziladi: $y = f(x)$. Argumentning qabul qilishi mumkin bo'lgan qiymatmatlari to'plami funksiyaning aniqlanish sohasi, funksiyaning o'zi qabul qilishi mumkin bo'lgan qiymatmatlari to'plami funksiyaning o'zgarish sohasi yoki qiymatlar to'plami deyiladi. $f$ funksiyaning aniqlanish sohasi $D(f)$ bilan, uning qiymatlar to'plami $E(f)$ bilan belgilanadi. Funksiya analitik, jadval va grafik usullar bilan berilishi mumkin. Agar moslik biror formula yordamida berilgan bo'lsa, funksiya analitik usulda berilgan deyiladi. Masalan,

$$1)\, y = 3x;\ 2)\, y = x^2;\ 3)\, y = \sqrt{5-x};\ 4)\, y = \frac{x^3 + 8}{x - 2}$$

funksiyalar analitik usulda berilgan. Agar analitik usulda berilgan funksiyaning aniqlanish sohasi to'g'risida alohida shart qo'yilmagan bo'lsa, u holda $y = f(x)$ da o'ng tomonda turuvchi ifoda ma'noga ega bo'ladigan $x$ ning barcha qiymatlari olinadi. Yuqorida keltirilgan 1- va 2-funksiyalarning aniqlanish sohasi haqiqiy sonlar to'plami, ya'ni $D(f) = R$ dir. 3-funksiyaning aniqlanish sohasi $D(f) = (-\infty; 5]$ dan, 4-funksiyaning aniqlanish sohasi $D(f) = (-\infty; 2) \cup (2; \infty)$ dan iborat.

   *Juft va toq funksiyalar.*
Agar $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ dan $-x \in D(f)$ ekanligi kelib chiqsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(-x) = f(x)$ bo'lsa, $f$ ga *juft funksiya* deyiladi.
Agar $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ dan $-x \in D(f)$ ekanligi kelib chiqsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(-x) = -f(x)$ bo'lsa, $f$ ga *toq funksiya deyiladi.* $y = 2x-$ funksiya toq, $y = 3x^2-$ funksiya juft funksiyaga misol bo'ladi.

Agar $f$ juft funksiya bo'lsa, uni $f_+$ bilan, agar $f$ toq funksiya bo'lsa, uni $f_-$ bilan belgilaymiz. Juft va toq funksiyalar quyidagi xossalarga ega:

1. $\alpha \cdot f_+ = \varphi_+$, $(\alpha \cdot f_- = \varphi_-)$ **juft (toq) funksiyaning songa ko'paytmasi juft (toq) funksiya bo'ladi.**

2. $f_+ + g_+ = \varphi_+$ **juft funksiyalar yig'indisi va** $f_+ - g_+ = \psi_+$ **juft funksiyalar ayirmasi yana juft funksiya bo'ladi.**

3. $f_+ \cdot g_+ = \varphi_+$ **juft funksiyalar ko'paytmasi va** $f_+ : g_+ = \psi_+$ **juft funksiyalar nisbati yana juft funksiya bo'ladi.**

4. $f_+ \cdot g_- = \varphi_-$ **juft va toq funksiyalar ko'paytmasi va** $f_+ : g_- = \psi_-$ **juft va toq funksiyalar nisbati toq funksiya bo'ladi.**

5. $f_- \cdot g_+ = \varphi_-$ **toq va juft funksiyalar ko'paytmasi va** $f_- : g_+ = \psi_-$ **toq va juft funksiyalar nisbati toq funksiya bo'ladi.**

6. $f_+^n = \varphi_+, n \in N$ **juft funksiyaning ixtiyoriy natural darajasi juft funksiya bo'ladi.**

7. $f_- + g_- = \varphi_-$ **toq funksiyalar yig'indisi va** $f_- - g_- = \psi_-$ **toq funksiyalar ayirmasi toq funksiya bo'ladi.**

8. $f_- \cdot g_- = \varphi_+$ **toq funksiyalar ko'paytmasi va** $f_- : g_- = \psi_+$ **toq funksiyalar nisbati juft funksiya bo'ladi.**

9. $f_-^{2n} = \varphi_+, n \in N$ **toq funksiyaning ixtiyoriy juft natural darajasi juft funksiya bo'ladi.**

10. $f_-^{2n-1} = \varphi_-, n \in N$ **toq funksiyaning ixtiyoriy toq natural darajasi toq funksiyadir.**

11. $f_+ + g_-$ **juft va toq funksiyalar yig'indisi na toq na juft funksiya bo'ladi.**

12. $f_+ - g_-$ **juft va toq funksiyalar ayirmasi na toq na juft funksiya bo'ladi.**

*Davriy funksiyalar.*

Shunday $T > 0$ soni mavjud bo'lib, $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ uchun $T + x \in D(f)$ bo'lsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(T + x) = f(x)$ bo'lsa, $f$ ga *davriy funksiya* deyiladi. Bu holda $T$ soni $y = f(x)$ funksiyaning davri deyiladi. Davriy funksiyaga misol sifatida $x$ ning kasr qismi, ya'ni $f(x) = \{x\}$ ni olish mumkin. Bu funksiyaning davri $T = 1$. Haqiqatan ham $f(x+1) = \{x+1\} = \{x\} = f(x)$. Agar $f$ va $g$ lar $T$ davrli funksiyalar bo'lsa, u holda ularning yig'indisi, ayirmasi, ko'paytmasi va nisbati ham $T$ davrli funksiyalar bo'ladi. Keyinchalik biz ko'rsatamizki, trigonometrik funksiyalardan $\sin x$ va $\cos x$ funksiyalar $2\pi$ davrli, $tgx$ va $ctgx$ funksiyalar $\pi$ davrli funksiyalar bo'ladi. Agar $f$ va $g$ larning davrlari o'lchovdosh bo'lmasa, ularning yig'indisi (ayirmasi) davriy funksiya bo'lmaydi. Masalan, $\varphi(x) = \{x\}+\sin x$ davriy funksiya emas. Ammo $f(x) = \{x\}$ davriy funksiya bo'lib, davri $T = 1, g(x) = \sin x$ esa $2\pi$ davrli funksiyadir.

*Monoton funksiyalar.*

$[a;b]$ kesmada aniqlangan $y = f(x)$ funksiya, shu kesmadan olingan har qanday $x_1$, $x_2$ lar uchun $x_1 < x_2$ bo'lganda $f(x_1) < f(x_2)$ bo'lsa, $f$ ga $[a;b]$ kesmada *o'suvchi funksiya* deyiladi. $[a;b]$ kesmada aniqlangan $y = f(x)$ funksiya, shu kesmadan olingan har qanday $x_1$, $x_2$ lar uchun $x_1 < x_2$ bo'lganda $f(x_1) > f(x_2)$ bo'lsa, $f$ ga $[a;b]$ kesmada *kamayuvchi funksiya* deyiladi. Berilgan kesmada faqat o'suvchi yoki kamayuvchi bo'lgan

funksiyalar *monoton funksiyalar* deyiladi. $y = 2x$ funksiya $[-1;0]$ kesmada o'suvchi, $y = x^2$ funksiya esa $[-1;0]$ da kamayuvchi funksiyaga misol bo'ladi. Agar $f$ va $g$ funksiyalar $[a;b]$ kesmada o'suvchi bo'lsa, ularning yig'indisi ham $[a;b]$ da o'suvchi bo'ladi. Agar $f$ va $g$ funksiyalar $[a;b]$ kesmada kamayuvchi bo'lsa, ularning yig'indisi ham $[a;b]$ da kamayuvchi funksiya bo'ladi. Ammo o'suvchi funksiyalarning ayirmasi o'suvchi bo'lmasligi mumkin. Masalan, $f(x) = x^2 + 1$ funksiya $[0;2]$ kesmada o'suvchi, $g(x) = 2x$ ham $[0;2]$ kesmada o'suvchi. Ularning ayirmasi bo'lgan $\varphi(x) = x^2 + 1 - 2x = (x - 1)^2$ funksiya $[0;1]$ da kamayuvchi, $[1;2]$ kesmada o'suvchi. Demak, $\varphi(x)$ funksiya $[0;2]$ kesmada monoton emas.

1. (96-9-10) Funksiyaning aniqlanish sohasini toping.
$$y = \sqrt{\frac{x(x+1)}{(x-2)(4-x)}}$$
A) $[-1;0] \cup (2;4)$     B) $(-1;0) \cup [2;4]$
C) $(-1;0] \cup [2;4)$     D) $(-\infty;-1) \cup (0;2) \cup (4;\infty)$

**Yechish:** Berilgan funksiya aniqlangan bo'lishi uchun ildiz ostidagi ifoda manfiy bo'lmasligi kerak, ya'ni
$$\frac{x(x+1)}{(x-2)(4-x)} \geq 0.$$
Uni oraliqlar usuli bilan yechib $x \in [-1;0] \cup (2;4)$ ekanini hosil qilamiz. **Javob:** $[-1;0] \cup (2;4)$ (A).

2. (96-3-16) $f(x) = \dfrac{x-2}{x^2-1}$ funksiyaning aniqlanish sohasini toping.
A) $(0;\infty)$     B) $(-\infty;1) \cup (1;\infty)$
C) $(-\infty;\infty)$     D) $(-\infty;-1) \cup (-1;1) \cup (1;\infty)$

3. (96-12-17) $f(x) = \dfrac{x+2}{x^2-1}$ funksiyaning aniqlanish sohasini toping.
A) $(-\infty;\infty)$     B) $(-\infty;-1) \cup (-1;1) \cup (1;\infty)$
C) $(-\infty;0)$     D) $(0;\infty)$

4. (99-1-12) $y = \dfrac{2x-3}{x(x+2)}$ funksiyaning aniqlanish sohasini toping.
A) $(-\infty;-2) \cup (-2;0) \cup (0;\infty)$
B) $(-\infty;0) \cup (2;\infty)$
C) $(-\infty;-2) \cup (0;\infty)$
D) $(-\infty;1,5) \cup (1,5;\infty)$

5. (00-6-21) Funksiyaning aniqlanish sohasini toping.
$$y = \sqrt{\frac{x^2 - 4x + 4}{1 - x^2}}$$
A) $(-1;1)$     B) $(-1;1) \cup \{2\}$
C) $(-1;2)$     D) $(-\infty;-1) \cup \{2\}$

6. (96-3-70) Funksiyaning aniqlanish sohasini toping.
$$y = \sqrt{\frac{(x-1)(3-x)}{x(4-x)}}$$
A) $[0;1) \cup [3;4)$     B) $(0;1] \cup [3;4)$
C) $(0;1] \cup (3;4)$     D) $(-\infty;0) \cup [1;3] \cup (4;\infty)$

7. (96-13-10) Funksiyaning aniqlanish sohasini to-
ping.
$$y = \sqrt{\frac{(x-2)(4-x)}{x(x+1)}}$$
A) $(-1;0) \cup [2;4]$    B) $[-1;0] \cup (2;4)$
C) $(-1;0] \cup [2;4)$    D) $(-\infty;-1) \cup (0;2] \cup [4;\infty)$

8. (99-4-23) Funksiyaning aniqlanish sohasini toping.
$$y = \sqrt{x^2 - 9} + \frac{2}{\sqrt{-x}}$$
A) $(0;3)$    B) $[-3;0)$    C) $(-\infty;0)$    D) $(-\infty;-3]$

9. (99-6-46) $y = \sqrt{3x - x^3}$ funksiyaning aniqlanish
sohasini toping.
A) $(-\infty;-\sqrt{3}] \cup [0;\sqrt{3}]$    B) $(-\infty;-\sqrt{3}) \cup (0;\sqrt{3})$
C) $[0;\sqrt{3})$    D) $(-\infty;\sqrt{3}) \cup (\sqrt{3};\infty)$

10. $y = [x]$ funksiyaning qiymatlar sohasini toping.
A) $N$    B) $(-\infty;\infty)$    C) $Z$    D) $0, 1$

**Yechish:** Berilgan funksiyaning aniqlanish so-
hasi $R = (-\infty;\infty)$ dan iborat. Agar $x$ barcha
haqiqiy sonlarni qabul qilib chiqsa, u holda un-
ing butun qismi $[x]$ barcha butun sonlarni qabul
qiladi, ya'ni $E(f) = Z$. **Javob:** $Z$ (C).

11. $y = \{x\}$ funksiyaning qiymatlar sohasini toping.
A) $N$    B) $[0;1)$    C) $[0;1]$    D) $0, 1$

12. $y = x^2$ funksiyaning qiymatlar sohasini toping.
A) $(0;\infty)$    B) $(-\infty;\infty)$    C) $[0;\infty)$    D) $(2;\infty)$

13. $y = \sqrt{x^2 + 4}$ funksiyaning qiymatlar sohasini to-
ping.
A) $(0;\infty)$    B) $[2;\infty)$    C) $(2;\infty)$    D) $(-\infty;2)$

14. $y = 7 - x^2$ funksiyaning qiymatlar sohasini to-
ping.
A) $(7;\infty)$    B) $(-\infty;7)$    C) $(-\infty;7]$    D) $(0;\infty)$

15. (98-11-66) Ushbu $y = \sqrt{9 - x^2}$ funksiyaning qiy-
matlar sohasini ko'rsating.
A) $(-\infty;\infty)$    B) $[-3;3]$    C) $[0;\infty)$    D) $[0;3]$

16. (96-7-26) Quyidagi funksiyalardan qaysi biri juft?
A) $g(x) = \dfrac{5x^3}{(x-3)^2}$    B) $g(x) = \dfrac{x(x-2)(x-4)}{x^2 - 6x + 8}$

C) $g(x) = \dfrac{9x^2}{x^2 - 25}$    D) $g(x) = x^2 + |x+1|$

**Yechish:** $9x^2$ juft funksiya, $x^2 - 25$ ham juft
funksiya, ularning nisbati 5-qoidaga ko'ra juft funk-
siya bo'ladi. **Javob:** (C).

17. (97-3-26) Quyidagi funksiyalardan qaysi biri toq?
A) $y = \dfrac{5x^3}{(x-3)^2}$    B) $y = \dfrac{x(x-4)(x-2)}{x^2 - 6x + 8}$

C) $y = \dfrac{9x^2}{x^2 - 25}$    D) $y = \dfrac{x^4 - 2x^2}{3x}$

18. Quyidagi funksiyalardan qaysi juft emas.
A) $y = 5x^4 - 7x^8$    B) $y = 2|x|$
C) $y = \dfrac{9x^3}{x^3 - x}$    D) $y = \dfrac{x^4 - 8x^2}{7x}$

19. Quyidagi funksiyalardan qaysi toq emas.
A) $y = x + 1$    B) $y = 4x^9 - 8x^7$
C) $y = x|x|$    D) $y = x(\sqrt{1+x} + \sqrt{1-x})$

20. Juft ham, toq ham bo'lmagan funksiyani toping.
A) $y = x^8 - 7x^{100}$    B) $y = 2x - 5$
C) $y = |x| + 5x^{64}$    D) $y = x$

21. Juft ham, toq ham bo'lmagan funksiyani toping.
A) $y = x$    B) $y = [x]$
C) $y = |x|$    D) $y = x^2$

22. (99-1-13) $y = x|x|$, $x \in R$ funksiya uchun qaysi
tasdiq to'g'ri?
A) toq funksiya
B) juft funksiya
C) kamayuvchi funksiya
D) juft funksiya ham, toq funksiya ham emas

23. $[0;\infty)$ da o'suvchi funksiyani toping.
A) $y = 1 - x$    B) $y = (x-5)^2$
C) $y = x|x|$    D) $y = 7 - x^3$

**Yechish:** Berilgan funksiyalar $[0;\infty)$ da aniqlan-
gan, ya'ni $x \geq 0$. Bu shartda $y = x|x| = x^2$. Bu
funksiyani ta'rif yordamida, $[0;\infty)$ da o'suvchi
ekanligini ko'rsatamiz. Faraz qilaylik, $x_1, x_2 \in$
$[0;\infty)$ lar $x_1 < x_2$ shartni qanoatlantiruvchi ix-
tiyoriy nuqtalar bo'lsin. Nomanfiy hadli tengsiz-
likni kvadratga ko'tarish mumkin. Bundan $f(x_1)$
$= x_1^2 < x_2^2 = f(x_2)$ kelib chiqadi. Demak, $y =$
$x|x|$ funksiya $[0;\infty)$ da o'suvchi ekan. **Javob:**
$y = x|x|$ (C).

24. $(-\infty;0)$ da kamayuvchi funksiyani toping.
A) $y = 5 - x^2$    B) $y = x^2$
C) $y = x|x|$    D) $y = x^3$

25. $R = (-\infty;\infty)$ da monoton funksiyani ko'rsating.
A) $y = 1 - 7x$    B) $y = (x-3)^2$
C) $y = 1 + |x|$    D) $y = x^2 - 5x - 6$

26. $[0;2]$ da monoton bo'lmagan funksiyani toping.
A) $y = (x-1)^2$    B) $y = x^2$
C) $y = (x-2)^2$    D) $y = (x-3)^2$

27. $[0;2]$ da monoton bo'lmagan funksiyani toping.
A) $y = \{0,2 \cdot x\}$    B) $y = \{0,3 \cdot x\}$
C) $y = \{0,4 \cdot x\}$    D) $y = \{0,5 \cdot x\}$

28. (00-2-8) Agar $f(x) = x^2 - 8x + 7$ bo'lsa, $f(4 - \sqrt{11})$ ni hisoblang.
A) $2$    B) $2 - \sqrt{2}$    C) $2 + \sqrt{11}$    D) $3$

**Yechish:** Berilgan funksiyani $f(x) = (x-4)^2 - 9$
shaklda yozib olamiz va $x = 4 - \sqrt{11}$ deymiz,
natijada
$$f(4 - \sqrt{11}) = (4 - \sqrt{11} - 4)^2 - 9 = 11 - 9 = 2$$
bo'ladi. **Javob:** 2 (A).

29. (96-1-16) Agar $f(x) = (1 + \dfrac{1}{x})(7 + 4x)$ bo'lsa,
$f(-\dfrac{1}{2})$ ni toping.
A) $9$    B) $-3$    C) $15$    D) $-5$

30. (02-8-17) Agar

$$f(x) = \sqrt{x^3 - 1} \quad \text{bo'lsa,} \quad f(\sqrt[3]{x^2 + 1}) = ?$$

A) $|x|$     B) $x$     C) $-x$     D) $0$

31. (03-1-15) Agar

$$f(x) = \begin{cases} |x+1|, & x > -2 \\ 3 - 4|x|, & x \le -2 \end{cases}$$

bo'lsa, $f(-1) - f(-3)$ ni hisoblang.

A) $0$     B) $3$     C) $6$     D) $9$

32. (03-6-13) Agar

$$f\left(\frac{3x-2}{2}\right) = x^2 - x - 1$$

bo'lsa, $f(0)$ ni toping.

A) $-\dfrac{5}{9}$     B) $-\dfrac{13}{9}$     C) $-\dfrac{7}{9}$     D) $-\dfrac{11}{9}$

33. (03-11-17) Agar $f(x + 2) = x^3 + 6x^2 + 12x + 8$ bo'lsa, $f(\sqrt{3})$ ni toping.

A) $3\sqrt{3}$     B) $2\sqrt{3}$     C) $4\sqrt{3}$     D) $12$

## 10.1 Tekislikda koordinatalar sistemasi

Tekislikda perpendikulyar ikki to'g'ri chiziq berilgan bo'lsin. Qo'laylik uchun ularning birini gorizontal, ikkinchisini vertikal qilib olamiz. Bu to'g'ri chiziqlarning kesishish nuqtasini $O$ bilan belgilaymiz va uni sanoq boshi deb ataymiz. Gorizontal to'g'ri chiziqni abssissalar o'qi yoki $Ox$ o'qi, vertikal to'g'ri chiziqni ordinatalar o'qi yoki $Oy$ o'qi deb ataymiz. Bularning hammasi birgalikda tekislikda to'g'ri burchakli koordinatalar sistemasi deyiladi. Bundan tashqari bu o'qlardagi nuqtalar bilan, haqiqiy sonlar o'rtasida moslik o'rnatilgan bo'lsin. Abssissalar o'qida $O$ nuqtadan o'ngdagi nuqtalarga musbat sonlar, chapdagi nuqtalarga manfiy sonlar mos qo'yilgan bo'lsin. Xuddi shunday ordinatalar o'qida $O$ nuqtadan yuqoridagi nuqtalarga musbat sonlar, pastdagi nuqtalarga manfiy sonlar mos qo'yilgan bo'lsin. Abssissalar o'qida $O$ nuqtadan o'ng tomomga yo'nalish, musbat yo'nalish hisoblanadi. $Ox$ o'qining musbat yo'nalishidan soat strelkasining aylanishiga teskari yo'nalish to'g'ri yo'nalish hisoblanadi. Bu o'qlar tekislikni to'rtta qismga, 4 chorakka ajratadi. Ularni to'g'ri yo'nalish bo'yicha joylashtirib I chorak, II chorak, III chorak va IV chorak (10.1-chizma) ni olamiz.

To'g'ri burchakli koordinatalar sistemasida nuqtaning vaziyati quyidagicha aniqlanadi. Faraz qilaylik, tekislikda $M$ nuqta berilgan bo'lsin. $M$ nuqtadan koordinata o'qlariga perpendikulyarlar tushiramiz. Perpendikulyarning abssissalar o'qi bilan kesishgan nuqtasini $x$ bilan, ordinatalar o'qi bilan kesishgan nuqtasini $y$ bilan belgilaymiz (10.1-chizma) va ularni $M$ nuqtaning koordinatalari deb ataymiz. Odatda $M$ nuqtaning koordinatalarini ko'rsatish maqsadida $M(x; y)$ shaklda yozamiz. Shunday qilib tekislikdagi $M$ nuqtalar bilan tartiblangan $(x; y)$ $(x, y \in R)$ sonlar jufti o'rtasida o'zaro bir qiymatli moslik o'rnatiladi. $y = f(x)$ funksiyaning grafigi deb, tekislikda $\{(x; f(x)) : x \in D(f)\} = Gr(f)$ ko'rinishdagi nuqtalar to'plamiga aytiladi.

1. **Agar $x > 0$, $y > 0$ bo'lsa, $M(x; y)$ nuqta I chorakda yotadi va aksincha.**

2. **Agar $x < 0$, $y > 0$ bo'lsa, $M(x; y)$ nuqta II chorakda yotadi va aksincha.**

3. **Agar $x < 0$, $y < 0$ bo'lsa, $M(x; y)$ nuqta III chorakda yotadi va aksincha.**

4. **Agar $x > 0$, $y < 0$ bo'lsa, $M(x; y)$ nuqta IV chorakda yotadi va aksincha.**

5. **Berilgan $M(x_0; y_0)$ nuqta $y = f(x)$ funksiyaning grafigiga tegishli bo'lishi uchun $y_0 = f(x_0)$ tenglik o'rinli bo'lishi zarur va yetarli.**

6. $\begin{cases} y = f(x) \\ y = g(x) \end{cases}$ **sistemaning yechimlari $f$ va $g$ funksiya grafiklarining kesishish nuqtalari bo'ladi va aksincha.**

7. $f(x) = g(x)$ **tenglamaning yechimlari $f$ va $g$ funksiyalar grafiklarining kesishish nuqtalarining abssissasi bo'ladi va aksincha.**

1. $M(3; 2)$ nuqta qaysi chorakda yotadi.

A) I     B) II     C) III     D) IV

**Yechish:** Berilgan $M$ nuqtaning abssissasi ($3 > 0$) ham ordinatasi ($2 > 0$) ham musbat. 1-qoidaga ko'ra $M(3; 2)$ nuqta I chorakda yotadi (10.2-chizmaga qarang). **Javob:** I (A).

2. $M(-1; 2)$ nuqta qaysi chorakda yotadi.

A) I     B) II     C) III     D) IV

3. $M(-3; -2)$ nuqta qaysi chorakda yotadi.
   A) I     B) II     C) III     D) IV

4. $M(5; -4)$ nuqta qaysi chorakda yotadi.
   A) I     B) II     C) III     D) IV

5. $y = 2x$ funksiya grafigiga tegishli nuqtani toping.
   A) $(2; 4)$   B) $(1; -3)$   C) $(-1; 2)$   D) $(0; 1)$

   **Yechish:** Qaysi nuqtaning koordinatalari $y = 2x$ tenglikni qanoatlantirsa, shu nuqta funksiya grafigiga tegishli bo'ladi. A javobdagi $(2; 4)$ nuqta $4 = 2 \cdot 2$ tenglikni qanoatlantiradi. **Javob:** $(2; 4)$ (A).

6. $y = x^3 - 5$ funksiya grafigiga tegishli nuqtani toping.
   A) $(1; -3)$    B) $(2; 3)$    C) $(1; 4)$    D) $(0; 0)$

7. $y = [x] + 3$ funksiya grafigiga tegishli nuqtani toping.
   A) $(1,5; 3)$   B) $(2,2; 3)$   C) $(1,7; 4)$   D) $(0; 0)$

8. $y = \{x\}$ funksiya grafigiga tegishli nuqtani toping.
   A) $(1; 0)$    B) $(2; 1)$    C) $(1,7; 1)$    D) $(0; 1)$

9. $y = |x + 1|$ funksiya grafigiga tegishli nuqtani toping.
   A) $(1; 1)$    B) $(2; 1)$    C) $(1; 2)$    D) $(1; 0)$

10. (96-3-15) Quyidagi nuqtalarning qaysi biri $f(x) = -3x + 4$ funksiyaning grafigiga tegishli?
    A) $(3; -5)$   B) $(-3; 5)$   C) $(5; -3)$   D) $(2; 4)$

11. (96-12-16) Quyidagi nuqtalarning qaysi biri $f(x) = -4x + 3$ funksiyaning grafigiga tegishli?
    A) $(-1; 1)$   B) $(2; 5)$   C) $(-5; 2)$   D) $(1; -1)$

12. (07-112-3) Quyidagilardan qaysi biri $f(x) = -2x + 7$ funksiya grafigiga tegishli.
    A) $(2; 1)$    B) $(1; 2)$    C) $(2; 4)$    D) $(3; 1)$

13. Grafigi $M(1; 0)$ nuqtadan o'tuvchi funksiyani toping.
    A) $y = |x - 3|$      B) $y = x^2 - 1$
    C) $y = 3x$         D) $y = [x]$

14. $M(1; 5)$ qaysi funksiyaning grafigiga tegishli?
    A) $y = |x + 3|$      B) $y = x^2 + 1$
    C) $y = 3x + 1$     D) $y = [x + 4, 5]$

15. $y = x^2 - 2x + 1$ funksiya grafigining $Ox$ o'qi bilan kesishish nuqtasi abssissasini toping.
    A) $-3$     B) $3$     C) $-1$     D) $1$

    **Yechish:** $Ox$ o'qida yotuvchi nuqtalar $(x; 0)$ ko'rinishda bo'ladi. Demak, $0 = x^2 - 2x + 1$ tenglamaning ildizlari masalaning yechimi bo'ladi. Bu yerdan $x = 1$ ni olamiz. **Javob:** $1$ (D).

16. (02-4-5) $y = (x + 3)(x^2 + x + 1)$ funksiya grafigining $Oy$ o'qi bilan kesishish nuqtasi ordinatasini toping.
    A) $-3$     B) $3$     C) $-1$     D) $1$

17. $y = |x - 1|$ va $y = 1 - x^2$ funksiya grafiklarining kesishish nuqtasini toping.
    A) $(1; 0)$    B) $(0; 1)$    C) $(-1; 0)$     D) $(1; 1)$

18. $y = |x|$ va $y = x^2$ funksiya grafiklarining kesishish nuqtalarini toping.
    A) $(-1; 1)$         B) $(-1; 1)$   va   $(1; 1)$
    C) $(-1; -1)$      D) $(1; 1)$   va   $(-1; 1)$

## 10.2   Chiziqli funksiya

$y = kx + b$ funksiyaga chiziqli funksiya deyiladi. Bu yerda $k \neq 0$, $b \in R$ haqiqiy sonlar. Bu funksiyaning aniqlanish sohasi $D(f) = R$. Bu funksiyaning qiymatlar sohasi $E(f) = R$. $y = kx + b$ funksiyaning grafigi to'g'ri chiziq bo'ladi (10.3-chizma), $k$ esa to'g'ri chiziqning burchak koeffitsiyenti deyiladi.

Chiziqli funksiyalar quyidagi xossalarga ega.

1. $y = kx + b$ **funksiyaning grafigi** $Oy$ **o'qini** $(0; b)$ **nuqtada kesib o'tuvchi to'g'ri chiziqdir. Funksiya**
   a) $k > 0$ **da o'suvchi;**
   b) $k < 0$ **da kamayuvchi;**
   c) $k = 0$ **da o'zgarmas.**

2. $y = kx + b$ **funksiyaning grafigi** $Ox$ **o'qining musbat yo'nalishi bilan** $\alpha$ **burchak tashkil etsa, u holda** $tg\alpha = k$ **bo'ladi.**

3. $y = k_1 x + b_1$ **va** $y = k_2 x + b_2$ **to'g'ri chiziqlar:**
   a) $k_1 \neq k_2$ **da kesishadi;**
   b) $k_1 \cdot k_2 = -1$ **da perpendikulyar bo'ladi.**
   c) $k_1 = k_2$ **da parallel bo'ladi;**
   **Xususan,** $k_1 = k_2$, $b_1 = b_2$ **da ustma-ust tushadi;** $k_1 = k_2$, $b_1 \neq b_2$ **da kesishmaydi.**

4. **Agar** $y = k_1 x + b_1$ **va** $y = k_2 x + b_2$ **to'g'ri chiziqlar orasidagi burchak** $\varphi$ **bo'lsa, u holda** $tg\varphi = \dfrac{k_2 - k_1}{1 + k_1 \cdot k_2}$ **tenglik o'rinli.**

5. $(x_0; y_0)$ **nuqtadan** $ax + by + c = 0$ **to'g'ri chiziqqacha bo'lgan masofa:**

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$

1. (99-1-47) $2y = 2x + 3$ to'g'ri chiziqning $Ox$ o'qi bilan hosil qilgan burchagini toping.
   A) $45^0$     B) $30^0$     C) $60^0$     D) $75^0$

   **Yechish:** Tenglikdan $y$ ni topamiz:

   $$y = x + \frac{3}{2}.$$

   Agar $y = kx + b$ to'g'ri chiziqning $Ox$ o'qi bilan hosil qilgan burchagi $\alpha$ bo'lsa, 2-qoidaga ko'ra $tg\alpha = k$ bo'ladi. Bu holda $k = 1$ bo'lgani uchun $tg\alpha = 1$, ya'ni $\alpha = 45^0$. **Javob:** $\alpha = 45^0$ (A).

2. $2y = x + 7$ to'g'ri chiziqning $Ox$ o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
   A) $45^0$     B) $30^0$     C) $60^0$     D) $75^0$

3. $2y = \sqrt{3}x - 9$ to'g'ri chiziqning $Ox$ o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
   A) $45^0$     B) $30^0$     C) $60^0$     D) $75^0$

4. $2y = \sqrt{2}x - 9$ to'g'ri chiziqning $Ox$ o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
   A) $45^0$     B) $30^0$     C) $60^0$     D) $75^0$

5. $y = 7 - x$ to'g'ri chiziqning $Ox$ o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
   A) $145^0$     B) $135^0$     C) $120^0$     D) $75^0$

6. (98-3-44) $k$ ning qanday qiymatlarida $kx + 3y + 5 = 0$ va $(k + 1) \cdot x - 2y - 1 = 0$ to'g'ri chiziqlar parallel bo'ladi?

   A) $-3$ va $5$     B) $\frac{3}{5}$     C) $-5$ va $3$     D) $-\frac{3}{5}$

   **Yechish:** $kx+3y+5 = 0$ tenglikdan $y = -\frac{k}{3}x - \frac{5}{3}$ ni, $(k+1)x-2y-1 = 0$ dan esa $y = \frac{k+1}{2}x - \frac{1}{2}$ ni topamiz. Ma'lumki, $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar parallel bo'lishi uchun $k_1 = k_2$ bo'lishi kerak. Bu yerdan $\frac{k+1}{2} = -\frac{k}{3}$ ni olamiz. Uning yechimi $k = -\frac{3}{5}$. **Javob:** $-\frac{3}{5}$ (D).

7. $k$ ning qanday qiymatlarida $kx + 4y + 7 = 0$ va $x - 2y - 1 = 0$ to'g'ri chiziqlar parallel bo'ladi?
   A) $-3$     B) $-1\frac{1}{4}$     C) $-2$     D) $-\frac{3}{4}$

8. (01-12-3) $y_1 = \sqrt{3}x + \frac{1}{\sqrt{3}}$ va $y_2 = -\frac{1}{\sqrt{3}}x - \sqrt{3}$ to'g'ri chiziqlar orasidagi burchakni toping.
   A) $90^0$     B) $60^0$     C) $80^0$     D) $95^0$

9. $a$ ning qanday qiymatlarida $ax + 2y = 3$ va $2x - y = -1$ to'g'ri chiziqlar perpendikulyar bo'ladi?
   A) $a = -1$     B) $a = 2$     C) $a = 0$     D) $a = 1$

10. (96-1-26) $a$ ning qanday qiymatlarida $ax+2y = 3$ va $2x - y = -1$ to'g'ri chiziqlar kesishadi?
    A) $a \neq 4$     B) $a \neq 2$     C) $a \in R$     D) $a \neq -4$

11. (99-9-13) $y = \sqrt{3}x + 2$ va $y = -x + 2$ to'g'ri chiziqlarning kesishishidan hosil bo'lgan o'tkir burchakni toping.
    A) $65^0$     B) $75^0$     C) $60^0$     D) $85^0$

12. (96-7-16) $k$ ning qanday qiymatida $y = kx + 6$ funksiyaning grafigi $M(0,5; 4,5)$ nuqtadan o'tadi?
    A) $3$     B) $-3$     C) $-2$     D) $4$

13. (99-6-4) $k$ ning qanday qiymatida $y = kx - 10$ funksiyaning grafigi $A(-4; 14)$ nuqtadan o'tadi?
    A) $-2$     B) $-1$     C) $-6$     D) $-3$

14. (99-3-10) $a$ ning qanday qiymatlarida $2ax+3y = 3$ va $4x + 3y = 7$ to'g'ri chiziqlar kesishish nuqtasining abssissasi manfiy bo'ladi.
    A) $a < 3$     B) $a > 3$     C) $a < 2$     D) $a > 2$

    **Yechish:** Ma'lumki, $\begin{cases} 2ax + 3y = 3 \\ 4x + 3y = 7 \end{cases}$ sistemaning yechimi berilgan to'g'ri chiziqlarning kesishish nuqtasi bo'ladi. Bu sistemaning 2-tenglamasini $-1$ ga ko'paytirib, 1-tenglamaga qo'shamiz va $(2a - 4)x = -4$ ni olamiz. Bu tenglamaning yechimi $x = \frac{2}{2 - a}$ dir. Hozir biz berilgan to'g'ri chiziqlar kesishish nuqtasining abssissasini topdik. Uni manfiy deymiz, ya'ni $\frac{2}{2 - a} < 0$. Bu tengsizlik o'rinli bo'lishi uchun $2 - a < 0$ bo'lishi kerak, ya'ni $2 < a$. **Javob:** $a > 2$ (D).

15. (98-9-15) Koordinata o'qlari $\frac{x}{8} - \frac{y}{6} = 1$ to'g'ri chiziqdan qanday uzunlikdagi kesma ajratadi?
    A) $12$     B) $14$     C) $9$     D) $10$

16. (98-10-42) $n$ ning qanday qiymatida $2y = 8+n-(3n+4)x$ va $3y = 5-2n-(4n-3)x$ tenglamalar bilan berilgan to'g'ri chiziqlarning kesishish nuqtasi $Oy$ o'qida yotadi?
    A) $2$     B) $1,5$     C) $-1,5$     D) $-2$

17. (99-8-33) $f(-2) = 3$ va $f(2) = 5$ shartni qanoatlantiruvchi chiziqli funksiyani aniqlang.
    A) $f(x) = \frac{1}{2}x + 4$     B) $f(x) = 2x - 1$
    C) $f(x) = 2x + 1$     D) $f(x) = 3x + 9$

18. (96-6-13) Agar $k < 0$ va $l > 0$ bo'lsa, $y = kx + l$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
    A) I; II     B) I; II; III
    C) II; I; IV     D) I; III; IV

    **Yechish:** $y = kx + l$ to'g'ri chiziqning koordinata o'qlari bilan kesishish nuqtalarini topamiz. Agar $x = 0$ desak, $y = l$ bo'ladi, $y = 0$ desak, $x = -l/k$. Demak, $y = kx + l$ funksiyaning grafigi koordinata o'qlarini $(0; l)$ va $(-l/k; 0)$ nuqtalarda kesib o'tadi. Bu nuqtalar koordinata o'qlarining musbat yo'nalishlarida turibdi (10.4-chizmaga qarang). Shunday qilib, funksiya grafigi koordinatalar tekisligining II; I va IV choraklari orqali o'tadi. **Javob:** II; I; IV (C).

19. Agar $k > 0$ va $l > 0$ bo'lsa, $y = kx + l$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashadi?
    A) I; II va III      B) I va II
    C) I; III va IV      D) I; II va IV

20. (97-8-13) Ushbu $y = kx + l$ ($k < 0$ va $l < 0$) funksiyaning grafigi qaysi choraklarda joylashgan?
    A) I; II va III      B) I; III va IV
    C) II va IV      D) II;III va IV

21. (96-12-24) Grafigi rasmda tasvirlangan funksiyaning qiymatlari $x$ ning qanday qiymatlarida $-2$ dan kichik bo'ladi?
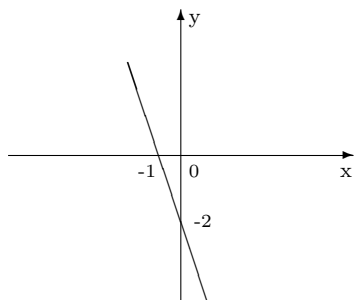


    A) $x \geq 0$    B) $x > 0$    C) $x < 0$    D) $x \leq 0$

22. (97-8-60) Rasmda $a = 4$; $b = 3$ va $c = 5$ bo'lsa, $OC$ to'g'ri chiziqning burchak koeffitsiyentini toping.
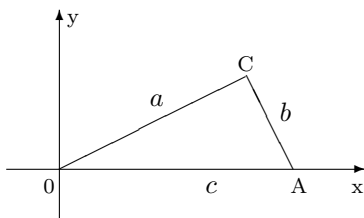


    A) $\dfrac{4}{3}$     B) $\dfrac{3}{5}$     C) $\dfrac{4}{5}$     D) $\dfrac{3}{4}$

23. (98-10-91) $k$ ning qanday qiymatlarida $kx + 3y + 1 = 0$ va $2x + (k+1)y + 2 = 0$ to'g'ri chiziqlar parallel bo'ladi?
    A) 2     B) $-2$     C) $-3$     D) $-3$ va 2

24. (98-11-14) $y = -\dfrac{41}{5}x$ funksiyaning grafigi $y = kx + \dfrac{41}{5}$ funksiyaning grafigiga $k$ ning qaysi qiymatida parallel bo'ladi?
    A) $\left(\dfrac{5}{41}\right)^{-1}$   B) $\dfrac{5}{41}$   C) $-\left(\dfrac{5}{41}\right)^{-1}$   D) $-\dfrac{5}{41}$

25. (99-1-46) $x + y = 1$ tenglama bilan berilgan to'g'ri chiziqqa parallel to'g'ri chiziqni toping.
    A) $2x + 2y + 3 = 0$      B) $y = x - 1$
    C) $x - y = 2$      D) $y = x + 1$

26. (98-3-41) $y = 1$ ga nisbatan $y = 2x + 1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
    A) $y = 2x - 1$      B) $y = 2x + 1$
    C) $y = 1 - 2x$      D) $y = 2x$

    **Yechish:** $y = 2x + 1$ to'g'ri chiziqning ikkita nuqtasini olamiz. Masalan, $(0; 1), (1; 3)$. Endi ularga $y = 1$ ga nisbatan simmetrik bo'lgan $(0; 1), (1; -1)$ nuqtalar orqali o'tuvchi to'g'ri chiziq tenglamasini topamiz: **Javob:** $y = 1 - 2x$ (C).

27. (98-10-88) $y = x$ ga nisbatan $y = 2x + 1$ ga simmetrik bo'gan to'g'ri chiziqning tenglamasini toping.
    A) $y = 2x - 1$      B) $y = \dfrac{x}{2} - 1$
    C) $y = \dfrac{x}{2} + 1$      D) $y = \dfrac{x-1}{2}$

28. (98-12-29) $Ox$ o'qqa nisbatan $y = 2x + 3$ to'g'ri chiziqqa simmetrik bo'gan to'g'ri chiziqning tenglamasini ko'rsating.
    A) $y = -2x - 3$      B) $y = 2x - 3$
    C) $y = -2x + 3$      D) $y = 3x - 2$

29. (01-3-12) Ushbu
    $$(a + 3)x + (a^2 - 16)y + 2 = 0$$
    to'g'ri chiziq $a$ ning qanday qiymatida abssissa o'qiga parallel bo'ladi?
    A) $-3$     B) 2     C) $-2$     D) 3

30. (01-12-40) $m$ va $n$ ning qanday qiymatlarida $2xm - 3ny = 12$ va $3xm + 2ny = 44$ to'g'ri chiziqlar $(1; 2)$ nuqtada kesishadi?
    A) $m = 10$, $n = 4$      B) $m = 8$, $n = 6$
    C) $m = 4$, $n = 10$      D) $m = 12$, $n = 2$

31. (02-1-45) Agar barcha $x$ lar uchun $f(x) = 6x - 3$ bo'lsa, $y = f(x-1)$ tenglama bilan aniqlanadigan to'g'ri chiziqning burchak koeffitsiyentini toping.
    A) 6     B) 5     C) 7     D) $-6$

32. (02-12-5) $y = 2x + 1$ va $y = -2 - x$ funksiyalarning grafiklari qaysi koordinatalar choragida kesishadi?
    A) I     B) II     C) III     D) IV

33. (96-11-31) $M(2; 1)$ nuqtadan $y = x + 2$ to'g'ri chiziqqacha bo'lgan masofani toping.
    A) $2{,}25$     B) $1{,}5\sqrt{2}$     C) $\dfrac{1}{4}$     D) $\dfrac{1}{2}$

**Yechish:** 5-qoidadan foydalanib, $M(2;1)$ nuqtadan $x - y + 2 = 0$ to'g'ri chiziqqacha bo'lgan masofani hisoblaymiz

$$d = \frac{|2 - 1 + 2|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 1,5\sqrt{2}.$$

**Javob:** $1,5\sqrt{2}$ (B).

34. (96-12-31) $M(2;2)$ nuqtadan $y = x + 1$ to'g'ri chiziqqacha bo'lgan masofani toping.

    A) $1,5$    B) $\dfrac{\sqrt{2}}{2}$    C) $\dfrac{1}{2}$    D) $2,25$

35. (03-11-30) Koordinatalar boshidan $5x + 12y = 60$ to'g'ri chiziqqacha bo'lgan masofani aniqlang.

    A) $4\dfrac{8}{13}$    B) $5$    C) $5\dfrac{3}{13}$    D) $4\dfrac{7}{13}$

## 10.3 Kvadratik funksiya

$y = ax^2 + bx + c$ ko'rinishdagi funksiyaga kvadratik funksiya deyiladi. Bu yerda $a$, $b$, $c$ lar berilgan sonlar bo'lib, $a \neq 0$. Kvadratik funksiyaning aniqlanish sohasi $D(y) = R$. Kvadratik funksiya quyidagi xossalarga ega.

1. $y = ax^2 + bx + c$ $(a \neq 0)$ **kvadrat uchhadning grafigi paraboladan iborat (10.5-chizma):**
   **a)** $a > 0$ **da parabola shoxlari yuqoriga yo'nalgan;**
   **b)** $a < 0$ **da parabola shoxlari pastga yo'nalgan;**
   **c)** $D > 0$ **da parabola** $Ox$ **o'qi bilan 2 ta umumiy nuqtaga ega;**
   **d)** $D = 0$ **da parabola** $Ox$ **o'qiga urinadi, ya'ni 1 ta umumiy nuqtaga ega;**
   **e)** $D < 0$ **da parabola** $Ox$ **o'qi bilan umumiy nuqtaga ega emas.**

2. **Parabola uchining koordinatalari** $(x_0; y_0)$ **quyidagi formula yordamida hisonlanadi:**

$$x_0 = -\frac{b}{2a}, \quad y_0 = ax_0^2 + bx_0 + c = -\frac{b^2 - 4ac}{4a}.$$

3. **Agar parabola uchi** $(x_0; y_0)$ **da bo'lsa,** $y = ax^2 + bx + c$ **kvadrat uchhad** $y = a(x - x_0)^2 + y_0$ **ko'rinishda tasvirlanadi.**

4. $y = ax^2 + bx + c$ **funksiyaning qiymatlar sohasi:**
   **a)** $a > 0$ **da** $E(y) = [y_0; \infty)$,
   **b)** $a < 0$ **da** $E(y) = (-\infty; y_0]$.

5. **Parabola simmetriya o'qining tenglamasi** $x = x_0$. **Bu yerda** $x_0$ **parabola uchining abssissasi.**

6. $ax^2 + bx + c = 0$ **tenglamaning** $x_1$, $x_2$ **ildizlari** $y = ax^2 + bx + c$ **funksiyaning nollari deyiladi va** $\dfrac{x_1 + x_2}{2} = x_0$ **bo'ladi. Bu yerda** $x_0$ **parabola uchining abssissasi.**

7. $y = f(x)$ **funksiyani** $(a; b)$ **vektorga parallel ko'chirsak** $y = f(x - a) + b$ **funksiya hosil bo'ladi.**

1. (98-8-24) Agar $B(-2; -7)$ nuqta $y = kx^2 + 8x + m$ parabolaning uchi bo'lsa, $k$ va $m$ ning qiymatlarini toping.
   A) $k = 1$, $m = -9$    B) $k = 2$, $m = -1$
   C) $k = -1$, $m = -16$    D) $k = 2$, $m = 1$

   **Yechish:** Ma'lumki $y = ax^2 + bx + c$ parabola uchining abssissasi $x_0 = -\dfrac{b}{2a}$ formuladan topiladi. Shuning uchun $-2 = -\dfrac{8}{2k}$, yani $k = 2$ bo'ladi. Endi $y = 2x^2 + 8x + m$ tenglikka $B$ nuqtaning koordinatalarini qo'yib $m$ ning qiymatini topamiz. $-7 = 8 - 16 + m$, $m = 1$. **Javob:** $k = 2, m = 1$. (D).

2. (96-6-21) $y = x^2 - 4x + 3$ parabolaning uchi koordinatalar tekisligining qayerida joylashgan.
   A) IV chorakda    B) $Ox$ o'qida
   C) III chorakda    D) II chorakda

3. (97-2-21) $y = x^2 + 4x - 2$ parabolaning uchi koordinatalar tekisligining qayerida joylashgan.
   A) I chorakda    B) II chorakda
   C) $Oy$ o'qida    D) III chorakda

4. (97-8-21) $y = x^2 - 6x + 10$ parabolaning uchi koordinatalar tekisligini qayerida joylashgan.
   A) II chorakda    B) III chorakda
   C) $Oy$ o'qida    D) I chorakda

5. (97-3-16) $k$ ning qanday qiymatida $y = kx^2 - 2$ funksiyaning grafigi $A(-1; 1)$ nuqtadan o'tadi?
   A) $4$    B) $-3$    C) $3$    D) $2$

6. (98-4-45) $y = kx^2 - 2kx + 3$ va $y = 2 - kx$ funksiyalarning grafiklari $k$ ning nechta butun qiymatlarida kesishmaydi?
   A) $3$    B) $2$    C) cheksiz ko'p    D) $4$

   **Yechish:** Ma'lumki, (10.1-ning 6-qoidasiga qarang) funksiyalarning grafiklari kesishmasa

$$\begin{cases} y = kx^2 - 2kx + 3 \\ y = 2 - kx \end{cases}$$

sistema yechimga ega emas. Bu yerdan $kx^2 - 2kx + 3 = 2 - kx$ tenglamaning yechimga ega emasligi kelib chiqadi. Agar $k = 0$ bo'lsa, $3 = 2$ tenglik hosil bo'ladi. Bu tenglik to'g'ri emas. Demak, $k = 0$ da funksiyalarning grafiklari kesishmaydi. Endi $k \neq 0$ holni qaraymiz. Bu holda $kx^2 - kx + 1 = 0$ kvadrat tenglama yechimga ega emas. Ma'lumki, kvadrat tenglamaning diskriminanti $D < 0$ bo'lsa, u yechimga ega bo'lmaydi. $D < 0$ shart $k^2 - 4k < 0 \iff k(k-4) < 0$ shartga teng kuchli. Bu tengsizlik oraliqlar usuli yordamida oson yechiladi, uning yechimi $(0; 4)$ dan iborat. Bu intervalda 1, 2, 3 butun sonlari bor. Yuqorida ko'rsatildiki $k = 0$ da ham sistema yechimga ega emas edi. Shunday qilib $k$ ning 4 ta butun qiymatida funksiyalarning grafiklari kesishmaydi. **Javob:** 4 (D).

7. (98-12-94) $y = (k-2)x^2 - 3kx + 2$ va $y = kx^2 + kx + 4$ funksiyalarning grafiklari kesishmaydigan $k$ ning barcha butun qiymatlari yig'indisini toping.
   A) 0     B) 1     C) $-2$     D) 3

8. (01-12-18) $a$ ning qanday qiymatlarida $y = 2ax + 1$ va $y = (a-6)x^2 - 2$ funksiyalarning grafiklari kesishmaydi?
   A) $(-3; 6)$     B) $(-\infty; 6) \cup (3; \infty)$
   C) $\emptyset$         D) $(-6; 3)$

9. (99-3-11) $a$ ning qanday qiymatlarida $y = 9x^2 - 12x + 35a$ parabola abssissalar o'qi bilan ikkita umumiy nuqtaga ega bo'ladi?
   A) $a = \dfrac{4}{35}$   B) $a < \dfrac{4}{35}$   C) $a > \dfrac{4}{35}$   D) $a < \dfrac{18}{35}$

10. (98-8-17) Agar $f(x) = 2 - ax^2$ va $g(x) = 2b + x$ funksiyalarning qiymatlari $x = -1$ va $x = 0$ da teng bo'lsa, $a$ va $b$ ning qiymatini toping.
    A) $a = -1$, $b = 1$      B) $a = 1$, $b = 1$
    C) $a = 1$, $b = -1$     D) $a = 5$, $b = -1$

11. (98-10-59) $A(1; 1)$, $B(0; 3)$ va $C(2; 3)$ nuqtalardan o'tuvchi parabola qaysi funksiyaning grafigi hisoblanadi?
    A) $y = 2x^2 + 2x - 3$      B) $y = 2x^2 - 2x - 3$
    C) $y = 2x^2 - 4x + 3$      D) $y = 2x^2 - 3x + 2$

12. (98-11-79) $m$ ning qanday qiymatida $y = 1$ to'g'ri chiziq, $y = x^2 - 2x + m$ parabolaga urinadi?
    A) 4     B) 1     C) 3     D) 2

13. (00-6-11) $a$ ning qanday qiymatlarida $y = ax^2 + 4x + c$ parabola koordinata o'qlarini $A(1; 0)$ va $B(0; 4)$ nuqtalarda kesib o'tadi?
    A) $-8$     B) 4     C) $-4$     D) 1

14. (00-7-22) $a$ ning qanday qiymatida $y = x^2 - 4x + 12 - a$ parabolaning uchi $M(2; 4)$ nuqtada yotadi?
    A) 3     B) 2     C) 4     D) 5

    **Yechish:** 3-xossaga ko'ra, parabola tenglamasi $y = (x-2)^2 + 4$ ko'rinishda bo'ladi. Uni masala

berilishidagi $y = x^2 - 4x + 12 - a$ bilan tenglashtirib, $4 + 4 = 12 - a$ ni hosil qilamiz. Bu yerdan $a = 4$. **Javob:** 4 (C).

15. (98-6-31) $y = 2x^2 + bx + c$ parabolaning uchi $(-3; -5)$ nuqtada joylashgan. Bu funksiya nollarining o'rta arifmetigini toping.
    A) $-1$     B) $-2$     C) $-3$     D) 1

16. (00-2-26) $A(1; 9)$ nuqta $y = -x^2 + ax + 4$ parabolaga tegishli. Parabola uchining ordinatasini toping.
    A) 13     B) 6     C) 4     D) 2

17. (02-11-18) $y = -3x^2 + 12x - 16$ parabola uchining koordinatalari yig'indisini toping.
    A) $-1$     B) 1     C) 0     D) $-2$

18. (02-11-19) $a$ ning nechta butun qiymatida $y = (x - 4a)^2 + a^2 + 10a + 21$ parabola uchining abssissasi musbat, ordinatasi esa manfiy bo'ladi?
    A) 0     B) 1     C) 2     D) 3

19. (03-8-18) $a$ ning nechta butun qiymatida $y = (x - 2a)^2 + a^2 - 9a + 14$ parabola uchining abssissasi musbat, ordinatasi esa manfiy bo'ladi?
    A) 1     B) 2     C) 4     D) 5

20. (97-12-21) Agar $a < 0$ va $b^2 - 4ac < 0$ bo'lsa, $y = ax^2 + bx + c$ funksiya grafigi koordinatalar tekisligining qaysi choraklarida joylashadi?
    A) I, II    B) III, IV    C) II, III    D) I, II va IV

    **Yechish:** $a < 0$ shartdan parabola shoxlari pastga qaragan ekanligi kelib chiqadi. $b^2 - 4ac < 0$ shartdan parabola $Ox$ o'qini kesmasligi kelib chiqadi. Demak, parabola $Ox$ o'qidan pastga, ya'ni III va IV choraklarda joylashgan. **Javob:** III, IV (B).

21. Agar $a > 0$ va $b^2 - 4ac < 0$ bo'lsa, $y = ax^2 + bx + c$ funksiya grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
    A) I, IV         B) I, II va IV
    C) faqat IV     D) III, IV

22. (98-11-13) Ushbu $y = -3x^2 + 8x - 8$ funksiyaning grafigi qaysi choraklarda joylashgan?
    A) II, III, IV     B) barcha choraklarda
    C) III, IV         D) I, II, III

23. (00-8-11) Ushbu $f(x) = -4x^2 + 2x - 1$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
    A) III; IV    B) I; II; III    C) I; III    D) II; IV

24. (98-1-16) Rasmda qaysi funksiyaning grafigi tasvirlangan?
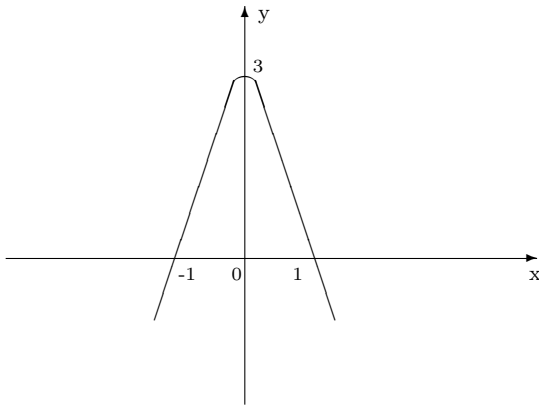
A) $y = 3x - x^2$          B) $y = 3x^2 - 3$

C) $y = 3(1 - x^2)$          D) $y = x^2 + 3x$

25. (01-9-38) $y = x^2 + px + q$ parabola $x = 5$ nuqtada $Ox$ o'qiga urinadi. $\dfrac{q}{p}$ ni toping.

A) 1        B) $-2$        C) 2,5        D) $-2,5$

**Yechish:** Masala shartidan parabolaning uchi $(5; 0)$ nuqtada ekanligi kelib chiqadi. 3-ga ko'ra $y = (x-5)^2 = x^2 - 10x + 25$ ni olamiz. Bu yerdan $p = -10$, $q = 25$ ekanligi kelib chiqadi. Demak, $p/q = -2,5$. **Javob:** $-2,5$ (D).

26. (01-12-41) $t$ ning qanday qiymatlarida $f(x) = 3x^2 + 2tx - (t-1)^2$ funksiya $f(-1) = -2$ shartni qanoatlantiradi?
A) $\pm 3$        B) $\pm 1$        C) 3        D) $\pm 2$

27. (01-2-25) Ushbu

$$y = 4x^2 + 4x + 1 \quad \text{va} \quad y = 2x + 1$$

funksiyalar grafiklari kesishish nuqtalari koordinatalarining yig'indisini toping.
A) $-0,5$        B) 1        C) 0,5        D) 1,5

28. (02-5-12) $m$ ning qanday qiymatlarida $y = (m+4)x^2 - 2(m+2)x + 1$ kvadrat uchhadning grafigi abssissalar o'qidan pastda joylashadi?
A) $(-\dfrac{1}{4}; 1)$     B) $(-2; 1)$     C) $\emptyset$     D) $(-\infty; \infty)$

29. (03-5-34) $y = ax^2 + c$ funksiya grafigi $A(-1; -3)$ va $B(3; 0)$ nuqtalardan o'tishi ma'lum bo'lsa, $\dfrac{c}{a}$ ning qiymati nechaga teng.
A) $-9$        B) 9        C) $-8$        D) $-10$

30. (03-6-50) $x$ ning qanday qiymatlarida $y = x^2$ funksiyaning qiymati 9 dan katta bo'ladi?
A) $-3 < x < 3$          B) $x < -3$
C) $x > 3$          D) $x < -3, x > 3$

31. (03-7-57) $m$ ning qanday qiymatida $y = mx + 2$ to'g'ri chiziq va $y = -5x^2$ parabola abssissasi $x = -1$ bo'lgan nuqtada kesishadi?
A) 3        B) $-3$        C) $-7$        D) 7

32. (03-8-46) $y = -2x^2 + 5x - 3$ funksiyaning eng katta qiymatini toping.
A) $\dfrac{1}{8}$        B) $\dfrac{1}{4}$        C) 5        D) $-3$

## 10.4    Teskari funksiya

Bizga $X$ to'plamni $Y$ to'plamga akslantiruvchi $y = f(x)$ funksiya berilgan bo'lsin. Faraz qilaylik, $D(f) = X$ va $E(f) = Y$ bo'lsin. Agar har bir $y \in Y$ uchun

$$f(x) = y \tag{1}$$

tenglama yagona $x \in D(f)$ yechimga ega bo'lsa, $f$ funksiya teskarilanuvchan deyiladi. Agar $f$ teskarilanuvchan funksiya bo'lsa, u holda har bir $y \in E(f)$ ga (1) tenglamaning yagona yechimi bo'lgan $x \in D(f)$ ni mos qo'yuvchi akslantirish $f$ ga teskari funksiya deyiladi va u $f^{-1}$ shaklda belgilanadi, ya'ni $x = f^{-1}(y)$. Teskari funksiya ta'rifidan quyidagilar kelib chiqadi.

1. $D(f) = E(f^{-1})$ **va** $D(f^{-1}) = E(f)$ **tengliklar o'rinli.**

2. **Barcha** $x \in D(f)$ **uchun** $f^{-1}(f(x)) = x$ **o'rinli.**

3. **Barcha** $x \in D(f^{-1})$ **uchun** $f(f^{-1}(x)) = x$ **o'rinli.**

4. **Agar** $(x_0; y_0)$ **nuqta** $f$ **funksiyaning grafigiga tegishli bo'lsa, u holda** $(y_0; x_0)$ **nuqta** $f^{-1}$ **funksiyaning grafigiga tegishli bo'ladi.**

Agar $f : D(f) \to R$ funksiya uchun, biror $y \in E(f)$ da (1) tenglama ikki yoki undan ko'p yechimga ega bo'lsa, u holda $f$ funksiya teskarilanuvchan emas, ya'ni $f$ ga teskari funksiya mavjud emas. Bu holda berilgan funksiyaning aniqlanish sohasi $D(f)$ ni "kichraytirish" hisobiga (1) tenglamani barcha $y \in E(f)$ larda yagona yechimga ega bo'ladigan qilish mumkin. Buni quyidagi misolda tushuntiramiz:

$$f : R \longrightarrow R_+ = [0; \infty), \quad f(x) = x^2$$

funksiyani qaraymiz. $f(x) = 4 \iff x^2 = 4$ tenglama ikkita $x_1 = -2$, $x_2 = 2$ yechimlarga ega. Agar bu funksiyaning aniqlanish sohasini $R_+$ desak, u holda istalgan $y \in R_+$ uchun $x^2 = y$ tenglama $D(f) = R_+$ da yagona $x = \sqrt{y}$ yechimga ega bo'ladi. Demak, $f : R_+ \to R_+$, $f(x) = x^2$ funksiyaga teskari funksiya $f^{-1}(y) = \sqrt{y}$ ekan.

1. (97-1-9) Quyidagilardan qaysi biri $y = \dfrac{3}{x+1} - 2$ funksiyaga teskari funksiya?

A) $y = \dfrac{3}{x-2}$          B) $y = \dfrac{x+1}{3} - 2$

C) $y = \dfrac{x+1}{3} - \dfrac{1}{2}$          D) $y = \dfrac{3}{x+2} - 1$

**Yechish:** Berilgan funksiyaning aniqlanish sohasi $D(f) = (-\infty; -1) \cup (-1; \infty)$ dan qiymatlar sohasi esa $E(f) = (-\infty; -2) \cup (-2; \infty)$ dan iborat. Istalgan $y \in E(f)$ uchun $\dfrac{3}{x+1} - 2 = y$ tenglama yagona $x = \dfrac{3}{y+2} - 1$ yechimga ega. Demak, $y = f^{-1}(x) = \dfrac{3}{x+2} - 1$. **Javob:** (D).

2. (97-11-9) Quyidagilardan qaysi biri $y = \dfrac{3}{2-x} - 1$ funksiyaga teskari funksiya?

   A) $y = x - 2$          B) $y = \dfrac{3}{x-2} + 1$

   C) $y = \dfrac{x-2}{3} + 1$       D) $y = 2 - \dfrac{3}{x+1}$

3. (00-3-61) Ushbu $y = x^2 - 4x + 7$ funksiyaga $(-\infty; 2]$ oraliqda teskari funksiyani toping.

   A) $2 \pm \sqrt{x-3}$       B) $2 - \sqrt{x-3}$

   C) $2 + \sqrt{x-3}$       D) $2 + \sqrt{3-x}$

4. $y = \sqrt{x} + 1$, $x \geq 0$ funksiyaga teskari funksiyani toping.

   A) $y = (x-1)^2$         B) $y = (x+1)^2$

   C) $y = x^2 + 1$          D) $y = x^2 - 1$

5. (01-8-19) Ushbu $y = \dfrac{4}{2-x} - 3$ funksiyaga teskari bo'lgan funksiyani ko'rsating.

   A) $y = \dfrac{4}{x-3} - 2$     B) $y = \dfrac{4}{3-x} - 2$

   C) $y = \dfrac{4}{x+3} + 2$     D) $y = -\dfrac{4}{x+3} + 2$

6. (97-6-9) Quyidagilardan qaysi biri $y = \dfrac{2}{x-1} - 1$ funksiyaga teskari funksiya?

   A) $y = 1 - \dfrac{2}{x+1}$      B) $y = 2 - \dfrac{3}{x}$

   C) $y = -\dfrac{2}{x+1}$       D) $y = \dfrac{2}{x+1} + 1$

7. (99-3-29) Funksiyaga teskari funksiyani toping.

   $$y = \frac{x-1}{2-3x}$$

   A) $y = \dfrac{2-3x}{x-1}$       B) $y = -\dfrac{2-3x}{x-1}$

   C) $y = \dfrac{2-3x}{1-x}$       D) $y = \dfrac{2x+1}{3x+1}$

8. (01-1-66) Ushbu $y = x^2 - 8$ $(x \geq 0)$ funksiyaga teskari bo'lgan funksiyaning aniqlanish sohasini toping.

   A) $(-8; \infty)$    B) $[-8; \infty)$    C) $(-8; 8)$    D) $[-8; 8]$

   **Yechish:** Berilgan funksiyaning aniqlanish sohasi $D(f) = [0; \infty)$ dan qiymatlar sohasi esa $E(f) = [-8; \infty)$ dan iborat. 1-qoidaga ko'ra $D(f^{-1}) = [-8; \infty)$. **Javob:** $[-8; \infty)$ (B).

9. (98-11-15) Ushbu $y = 2x^2 - \dfrac{1}{2}$ $(x \geq 0)$ funksiyaga teskari bo'lgan funksiyani toping.

   A) $\sqrt{2x+1} \cdot 2^{-1}$       B) $\sqrt{2x+1} \cdot 4^{-1}$

   C) $\sqrt{2x+1} \cdot 2^{-1} - \dfrac{1}{2}$     D) $\sqrt{2x+1} \cdot 4^{-1} - \dfrac{1}{2}$

10. $D(f)$ qanday tanlansa $f(x) = x^2 - 2x + 3$ funksiyaga teskari funksiya mavjud bo'ladi.

    A) $[0; \infty)$    B) $[1; \infty)$    C) $[-2; \infty)$    D) $[-2; 2)$

11. $D(f)$ qanday tanlansa $f(x) = |x-3|$ funksiyaga teskari funksiya mavjud bo'ladi.

    A) $[3; \infty)$    B) $[1; \infty)$    C) $[-2; \infty)$    D) $[-3; 5)$

12. $D(f)$ qanday tanlansa $f(x) = \{x\}$ funksiyaga teskari funksiya mavjud bo'ladi.

    A) $[1; \infty)$    B) $[1; 5)$    C) $[-2; 0)$    D) $[0; 1)$

13. (98-6-14) Qaysi nuqta $y = x^3 + 5x - 2$ funksiyaga teskari funksiyaning grafigiga tegishli?

    A) $(-2; 1)$    B) $(0; -2)$    C) $(4; 1)$    D) $(-8; 1)$

    **Yechish:** 4-qoidaga ko'ra, agar $(x_0; y_0) \in Gr(f)$ bo'lsa, u holda $(y_0; x_0) \in Gr(f^{-1})$ bo'ladi. Shu sababli javoblarda keltirilgan nuqtalarning koordinatalari o'rinlarini almashtirib berilgan funksiyaga qo'yib tekshiramiz. $y(1) = 4$ bo'lgani uchun $(1; 4)$ nuqta berilgan funksiyaning grafigiga tegishli bo'ladi. Demak, $(4; 1)$ nuqta unga teskari funksiyaning grafigiga tegishli. **Javob:** $(4; 1)$ (C).

## 10.5    Aralash tipdagi masalalar

1. **$y = f(x)$ va $y = g(x)$ funksiyalar grafiklari kesishgan nuqtalarning absssissalari $f(x) = g(x)$ tenglamaning ildizlari bo'ladi.**

2. **Tekislikdagi $M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalar orasidagi masofa**

   $$|M_1 M_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

3. **$y = |x-a| + |x-b|$ $(a < b)$ funksiyaning qiymatlar sohasi $E(y) = [b-a; \infty)$ dan iborat.**

1. (00-3-59) Agar $f(x+1) = x^2 - 3x + 2$ bo'lsa, $f(x)$ ni toping.

   A) $x^2 - 3x - 1$       B) $x^2 - 5x + 1$

   C) $x^2 - 5x + 6$       D) $x^2 - 4$

   **Yechish:** $x + 1 = t$ deb olamiz, u holda $x = t - 1$ bo'ladi. Uni $f(x+1) = x^2 - 3x + 2$ tenglikka qo'yamiz:

   $$f(t) = (t-1)^2 - 3(t-1) + 2 = t^2 - 5t + 6.$$

   **Javob:** $f(x) = x^2 - 5x + 6$ (C).

2. (00-9-60) Agar $f(x-1) = x^2 + 3x - 2$ bo'lsa, $f(x)$ ni aniqlang.

   A) $x^2 + 2x - 3$       B) $x^2 + 5x - 4$

   C) $x^2 + 5x + 2$       D) $x^2 - x - 2$

3. (97-7-16) $k$ ning qanday qiymatlarida $y = \dfrac{k}{x} - 1$ funksiyaning grafigi $C(-\dfrac{1}{2}; -3)$ nuqtadan o'tadi?

   A) $1$     B) $-2$     C) $-1$     D) $\dfrac{1}{2}$

4. (97-10-16) $k$ ning qanday qiymatida $y = kx^3 + 2$ funksiyaning grafigi $B(-2; 10)$ nuqtadan o'tadi?

   A) $2$     B) $1$     C) $-0,5$     D) $-1$

5. (99-4-15) $y = \dfrac{k}{x+2}$, $(k > 0)$ funksiyaning grafigi qaysi choraklar orqali o'tadi?

   A) I va III       B) II va IV

   C) I, III, IV      D) I, II va III

6. (03-6-40) $y = x|x|^{-1}$ funksiya grafigi koordinata-lar tekisligining qaysi choraklarida joylashgan?
A) III      B) IV      C) II, III      D) I, III

7. (99-5-40) $y(x) = |x - 1| + |x - 3|$ funksiyaning qiymatlar sohasini toping.
A) $[0; \infty)$   B) $[1; \infty)$   C) $[2; \infty)$   D) $[3; \infty)$

**Yechish:** 3-qoidaga ko'ra, $E(y) = [2; \infty)$ ni olamiz.
**Javob:** $[2; \infty)$ (C).

8. (03-12-26) $y = |x - 1| + |x - 3|$ funksiyaning eng kichik qiymatini toping.
A) 3      B) 4      C) 2      D) 1

9. (00-9-45) $f(x) = |x + 2| + |x + 8|$ funksiyaning qiymatlar sohasini toping.
A) $[0; \infty)$    B) $[3; \infty)$    C) $[4; \infty)$    D) $[6; \infty)$

10. (02-7-13) $f(x) = \dfrac{3}{x - 4}$ funksiyaning qiymatlar to'plamini toping.
A) $(-\infty; 0) \cup (0; \infty)$      B) $(-\infty; 4) \cup (4; \infty)$
C) $(-\infty; 3) \cup (3; \infty)$      D) $(-\infty; -1) \cup (-1; \infty)$

11. (00-3-60) Argumentning qanday qiymatida

$$y = \frac{5x}{2|x + 1| - 5}$$

funksiya 2 ga teng?
A) $-\dfrac{4}{3}$      B) $-\dfrac{5}{3}$      C) $-2$      D) $-\dfrac{14}{9}$

12. (00-7-17) Koordinata boshidan $y = x^2$ va $y = \dfrac{1}{x}$ funksiyalarning grafiklari kesishgan nuqtagacha bo'lgan masofani aniqlang.
A) 2      B) 1,5      C) $\sqrt{2}$      D) $\dfrac{1}{2}\sqrt{2}$

**Yechish:** 1-qoidaga ko'ra, funksiyalar grafiklari kesishgan nuqtalar abssissalari $x^2 = \dfrac{1}{x} \iff x^3 = 1$ tenglama yechimlari bo'ladi. Bu tenglamaning yechimi $x = 1$. Bundan $y = 1$ ni olamiz. Koordinata boshi $O(0; 0)$ dan $N(1; 1)$ nuqtagacha masofa (2-ga qarang)

$$|ON| = \sqrt{(1 - 0)^2 + (1 - 0)^2} = \sqrt{2}.$$

**Javob:** $\sqrt{2}$ (C).

13. (03-12-32) $3x + 4y + 7 = 0$ va $3x + y - 5 = 0$ to'g'ri chiziqlarning kesishish nuqtasi koordinata boshidan qanday masofada joylashgan?
A) 5      B) 6      C) 8      D) $8\sqrt{2}$

14. (01-4-12) $M(x, y)$ nuqtaning koordinatalari yig'in-disi 3 ga teng. Bu nuqta va koordinata boshi orasidagi eng qisqa masofa qanchaga teng bo'ladi?
A) $3\sqrt{2}$   B) $2\sqrt{3}$   C) $1,5\sqrt{2}$   D) $4,5\sqrt{2}$

15. (00-10-8) $y = |x - 1| - 5$ va $y = 0$ funksiyalar grafiklari kesishgan nuqtalar abssissalarining kvad-ratlari yig'indisini toping.
A) 36      B) 48      C) 24      D) 52

**Yechish:** 1-qoidaga ko'ra, funksiyalar grafiklari kesishgan nuqtalar abssissalari $|x - 1| - 5 = 0 \iff |x - 1| = 5$ tenglama yechimlari bo'ladi. Bu tenglamani geometrik usulda yechsak, sonlar o'qida koordinatasi 1 bo'lgan nuqtadan 5 birlik chapda $-4$ ni, 5 birlik o'ngda 6 ni olamiz. Ular kvadrat-lari yig'indisi $(-4)^2 + 6^2 = 16 + 36 = 52$. **Javob:** 52 (D).

16. (98-11-12) $y = |x - 2| + 1$ va $y = 5$ funksiyalar grafiklari kesishgan nuqtalar abssissalari kvadrat-larining yig'indisini toping.
A) 52      B) 32      C) 40      D) 36

17. (01-11-14) Ushbu $y = x^4$ va $y = 2x^2 - 1$ funksiya-larning grafiklari nechta umumiy nuqtaga ega?
A) 4      B) 3      C) 1      D) 2

18. (02-1-53) Agar $y = x^3 + 1$ va $-1 < x < 2$ bo'lsa, $y$ qanday oraliqda o'zgaradi?
A) $(-1; \infty)$    B) $(0; 9)$    C) $(1; 8)$    D) $(-1; 9)$

19. (02-12-47) $y = ax^3 + b$ kubik parabolaning grafigi $A(1; 18)$ va $B(-1; 14)$ nuqtalardan o'tadi. Qaysi nuqtada bu funksiyaning grafigi $Ox$ o'qini kesib o'tadi?
A) $(0; 2)$    B) $(-3; 0)$    C) $(3; 0)$    D) $(-2; 0)$

20. (98-4-41) $f(x) = 1 - 2x$ funksiya berilgan. Agar $f(\varphi(x)) = x$ bo'lsa, $\varphi(x)$ funksiyani toping.
A) $\dfrac{1 - x}{2}$   B) $\dfrac{x + 1}{2}$   C) $\dfrac{x - 1}{2}$   D) $\dfrac{2x - 1}{4}$

**Yechish:** Masala shartiga ko'ra $f(\varphi(x)) = 1 - 2\varphi(x) = x$. Bu yerdan $1 - x = 2\varphi(x)$, ya'ni $\varphi(x) = \dfrac{1 - x}{2}$. **Javob:** (A).

21. (00-9-43) $f(x) = 2x^2$ va $\varphi(x) = x + 1$ bo'lsa, $x$ ning nechta qiymatida $f(\varphi(x)) = \varphi(f(x))$ bo'ladi?
A) 0      B) 1      C) 2      D) 3

22. Agar $f(x) = x^2 - 1$ va $\varphi(x) = x + 1$ bo'lsa, $f(\varphi(x))$ ning qiymatini toping.
A) $x^2$      B) $x^2 + 2x$      C) $2x$      D) $x^2 - 2x$

23. (01-7-46) Agar $f(x) = \dfrac{1 - x}{1 + x}$ bo'lsa, $f(\dfrac{1}{x}) + \dfrac{1}{f(x)}$ ning qiymatini toping.
A) $\dfrac{4x}{1 - x^2}$   B) $\dfrac{4x}{x^2 - 1}$   C) $\dfrac{x^2 + 1}{x^2 - 1}$   D) $\dfrac{2(x^2 + 1)}{x^2 - 1}$

# 11 -bob. Ko'rsatkichli tenglama va tengsizliklar

## 11.1 Ko'rsatkichli funksiya

$y = a^x$, $a > 0$, $a \neq 1$ ko'rinishdagi funksiyaga ko'rsatkich-li fuksiya deyiladi. Uning aniqlanish $D(y) = (-\infty; \infty)$ va qiymatlar sohasi $E(y) = (0; \infty)$. $y = a^x$ funksiya $a > 1$ da o'suvchi, $0 < a < 1$ da kamayuvchi. $y = a^x$ funksiyaning grafigi I va II choraklarda yotadi (11.1-chizmaga qarang).

Ixtiyoriy $a > 0$, $b > 0$ sonlar uchun quyidagi tengliklar o'rinli:

**1.** $a^{x+y} = a^x \cdot a^y$.

**2.** $(a^x)^y = a^{xy}$.

**3.** $\dfrac{a^x}{a^y} = a^{x-y}$.

**4.** $\left(a \cdot b\right)^x = a^x \cdot b^x$.

**5.** $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$.

**6.** $\left(\dfrac{a}{b}\right)^{-x} = \left(\dfrac{b}{a}\right)^x$.

**7.** $a^{-x} = \dfrac{1}{a^x}$.

**8.** $a^{-m/n} = \dfrac{1}{a^{m/n}}$.

1. (98-7-23) Quyidagilardan qaysilari kamayuvchi funksiyalar?

   1) $y = 0,37^x$;    2) $y = (\sqrt[3]{11})^x$;    3) $y = 3 \cdot (\frac{1}{2})^x$;

   4) $y = (\frac{3}{4})^x$;    5) $y = \frac{1}{2} \cdot 3^x$

   A) 1; 3; 5    B) 2; 3; 4    C) 1; 4    D) 1; 3; 4

   **Yechish:** $y = a^x$ funksiya $0 < a < 1$ da kamayuvchi bo'lgani uchun 1; 3; 4 funksiyalar kamayuvchi bo'ladi. **Javob:** 1; 3; 4 (D).

2. Kamayuvchi funksiyalarni ajrating.

   $y_1 = 2^x$;    $y_2 = e^x$;    $y_3 = \pi^{-x}$;    $y_4 = (\frac{e}{\pi})^x$

   A) $y_1$; $y_2$    B) $y_2$; $y_3$    C) $y_3$; $y_4$    D) $y_4$; $y_1$

3. O'suvchi funksiyalarni ajrating.

   $y_1 = 3^x$;    $y_2 = e^x$;    $y_3 = \pi^x$

   A) $y_1$    B) $y_2$    C) $y_3$    D) barchasi

4. $f(x) = e^x - 1$ funksiyaning qiymatlar sohasini toping.

   A) $(0; \infty)$    B) $(1; \infty)$    C) $(-1; \infty)$    D) $[0; \infty)$

5. $f(x) = 2^x - 2^{-x}$ funksiyaning qiymatlar sohasini toping.

   A) $(0; \infty)$    B) $(1; \infty)$    C) $(-\infty; \infty)$    D) $[0; \infty)$

6. $f(x) = 2^x - 2^{-x}$ funksiyaning $x = 2$ nuqtadagi qiymatini toping.

   A) 0    B) 2    C) 3,75    D) 3,25

   **Yechish:** $f(2) = 2^2 - 2^{-2} = 4 - \dfrac{1}{2^2} = 4 - \dfrac{1}{4}$.
   Demak, $f(2) = 3,75$ ekan. **Javob:** $3,75$ (C).

7. $y = 2^x - 1$ funksiya grafigi qaysi choraklar orqali o'tadi?

   A) I; III    B) II; IV    C) I; II    D) II; III

8. Grafigi koordinata boshidan o'tuvchi funksiyani toping.

   A) $y = 2^x - e^x$    B) $y = e^x + 1$
   C) $y = 3^x - 3$    D) $y = 2^x - 2$

9. $y = 7^x - 1$ funksiya grafigi qaysi nuqta orqali o'tadi?

   A) $(0; 1)$    B) $(1; 6)$    C) $(2; 13)$    D) $(-1; 0)$

10. (97-7-16) $k$ ning qanday qiymatida $y = 2^{kx} - 1$ funksiyaning grafigi $C(2; 15)$ nuqtadan o'tadi?

    A) 1    B) $-2$    C) 4    D) 2

    **Yechish:** Masala shartidan $15 = 2^{2k} - 1$ tenglikni olamiz. Bu yerdan

    $$16 = 2^{2k} \iff 2^4 = 2^{2k} \iff 4 = 2k$$

    kelib chiqadi. Demak, $k = 2$. **Javob:** 2 (D).

11. (98-2-30) Quyidagi sonlardan qaysi biri 1 dan katta?

    $a = 0,7^{2,3} \cdot 0,3^{0,8}$,    $b = 3,2^{-4,2} \cdot 1,2^{-0,8}$,
    $c = 0,7^{-1,2} \cdot 0,6^{-0,4}$,    $d = 0,6^{0,4} \cdot 0,3^{0,6}$,

    A) $a, d$    B) $b, c$    C) $c$    D) $d, c$

12. (98-5-31) Ushbu $y = a^x$ funksiya uchun qaysi mulahaza noto'g'ri?

    A) aniqlanish sohasi barcha haqiqiy sonlar to'plami
    B) qiymatlari to'plami barcha musbat haqiqiy sonlar to'plami
    C) garifigi (0;1) nuqtadan o'tadi
    D) aniqlanish sohasida har doim o'suvchi

13. Toq funksiyalarni ajrating.

    $y_1 = a^x - a^{-x}$;    $y_2 = x(a^x + a^{-x})$;    $y_3 = x \cdot e|x|$.

    A) $y_1$    B) $y_2$    C) $y_2$; $y_3$    D) barchasi

14. (99-3-27) Funksiyalardan qaysilari juft funksiya?

    $y_1 = \dfrac{a^x + a^{-x}}{2}$;    $y_2 = \dfrac{a^x + 1}{a^x - 1}$;

    $y_3 = \dfrac{x}{a^x - 1}$;    $y_4 = x\dfrac{a^x - 1}{a^x + 1}$;

    A) $y_1$    B) $y_2$    C) $y_2$; $y_3$    D) $y_1$; $y_4$

## 11.2  Ko'rsatkichli tenglamalar

### A. Bir xil asosga keltirib yechiladigan tenglamalar

Bu yerda quyidagi tenglikdan foydalaniladi:

**1.** $a^{f(x)} = a^{g(x)} \iff f(x) = g(x)$,    $a > 0$, $a \neq 1$.

1. (97-9-94) Tenglamani yeching.

$$\left(\frac{25}{64}\right)^{7x^2-6} = \left(\frac{64}{25}\right)^{2+3x-6x^2}$$

A) $-4; 1$    B) $-1; 4$    C) $1; 4$    D) $-4; -1$

**Yechish:** Tenglamaning har ikkala qismini bir xil asosga keltiramiz (2.1-ning 6-ga qarang).

$$\left(\frac{64}{25}\right)^{-(7x^2-6)} = \left(\frac{64}{25}\right)^{2+3x-6x^2}$$

1-ga ko'ra $-7x^2 + 6 = 2 + 3x - 6x^2$. Bu yerdan $x^2 + 3x - 4 = 0$ kvadrat tenglamani hosil qilamiz va uni yechib $x_1 = -4$, $x_2 = 1$ ildizlarni olamiz.
**Javob:** $-4; 1$ (A).

2. (96-1-34) Tenglamani yeching.

$$3^1 \cdot 3^2 \cdot 3^3 \cdot \dots \cdot 3^x = \frac{1}{9^{-33}}$$

A) 12 va $-11$    B) 11    C) 12    D) 33

3. (96-6-51) Ushbu

$$\left(\frac{4}{3}\right)^x \cdot \left(\frac{3}{8}\right)^x = 2$$

tenglamaning ildizi $x_0$ bo'lsa, quyidagi munosabatlardan qaysi biri o'rinli?
A) $x_0 > -1$    B) $x_0 < -1$
C) $x_0 = -1$    D) $\frac{x_0}{2} = -1$

4. (97-1-76) Tenglamani yeching.

$$(0,75)^{x-1} = \left(1\frac{1}{3}\right)^3$$

A) 1    B) $-1$    C) 2    D) $-2$

5. (97-6-57) Tenglamani yeching.

$$(0,8)^{3-2x} = (1,25)^3$$

A) 0    B) 1    C) 2    D) 3

6. (99-1-29) Tenglamani yeching.

$$4^{x-4} = 0,5$$

A) 3,5    B) 4,5    C) $-4,5$    D) $-3,5$

7. (99-6-8) Tenglamani yeching.

$$(3,5)^{x-5} = \left(\frac{4}{49}\right)^2$$

A) 3    B) 2    C) 1    D) 4

8. (99-6-27) Tenglamani yeching.

$$\frac{1}{27} \cdot \sqrt[4]{9^{3x-1}} = 27^{-\frac{2}{3}}$$

A) $-1$    B) 2    C) 1    D) $-2$

**Yechish:** Tenglamaning har ikkala qismini 27 ga ko'paytirib, 27 va 9 larni $3^3$ va $3^2$ shaklga keltirib, natijada

$$\sqrt[4]{(3^2)^{3x-1}} = 3^3 \cdot (3^3)^{-\frac{2}{3}} \iff 3^{(3x-1):2} = 3^1$$

ni olamiz. 1-ga ko'ra $(3x-1):2 = 1$ bo'ladi. Bu tenglamaning yechimi $x = 1$ dir. **Javob:** 1 (C).

9. (99-6-58) Tenglamani yeching.

$$(0,1(6))^{3x-5} = 1296$$

A) $\frac{1}{3}$    B) 3    C) $-3$    D) $-\frac{1}{3}$

10. (99-10-39) Tenglamaning ildizi 10 dan qancha kam?

$$3^{x+1} \cdot 27^{x-1} = 9^7$$

A) 5    B) 4    C) 8    D) 6

11. (00-3-32) Tenglamani yeching.

$$0,125 \cdot 4^{2x-3} = \left(\frac{\sqrt{2}}{8}\right)^{-x}$$

A) 2    B) $-2$    C) 4    D) 6

12. (96-10-37) Tenglamani yeching.

$$5^2 \cdot 5^4 \cdot 5^6 \cdot \dots \cdot 5^{2x} = 0,04^{-28}$$

A) 5    B) 10    C) 14    D) 7

13. (01-5-13) Tenglama ildizlarining ko'paytmasini toping.

$$2^{x^2 - 6x - 5/2} = 16\sqrt{2}$$

A) $-7$    B) $-2$    C) 3    D) 2

14. (01-6-35) Agar

$$\frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{24}{81}$$

bo'lsa, $x - y$ ning qiymatini toping.
A) 6    B) 5    C) 4    D) 3

15. (01-7-30) Tenglamani yeching.

$$(0,25)^{2-x} = \frac{1}{2^{x+3}}$$

A) 2    B) 3    C) $\frac{1}{2}$    D) $\frac{1}{3}$

16. (02-2-23) Tenglama ildizlarining ko'paytmasini toping.

$$\left(\frac{1}{4}\right)^{(4-x^2)/2} = 8^x$$

A) $-4$    B) 6    C) 4    D) $-6$

17. (02-3-17) Agar $3^{\alpha-3} = 11$ bo'lsa, $3^{5-\alpha}$ ning qiymatini toping.
A) $\frac{9}{11}$    B) 99    C) $\frac{3}{16}$    D) $\frac{11}{9}$

**Yechish:** $3^{5-\alpha}$ ni quyidagicha yozib olamiz:

$$3^{5-\alpha} = \frac{1}{3^{\alpha-5}} = \frac{3^2}{3^{\alpha-3}} = \frac{9}{11}.$$

**Javob:** $\frac{9}{11}$ (A).

18. (02-7-53) Agar

$$\sqrt[4]{9^{\frac{n-3}{5}}} = 243$$

bo'lsa, $n$ nechaga teng?
A) 53     B) 38     C) 47     D) 43

19. (03-3-31) Tenglama ildizlari yig'indisini toping.

$$\left(\frac{\sqrt{5}}{3}\right)^{2x^2-5x} = 1,8$$

A) 5     B) $-5$     C) 2,5     D) $-2,5$

20. (03-4-29) Tenglamaning ildizi 12 dan qancha kam?

$$\frac{2^{2x-1} \cdot 4^{x+1}}{8^{x-1}} = 64$$

A) 8     B) 9     C) 6     D) 10

21. (03-6-45) Tenglamani yeching.

$$\sqrt{5^2 - 4^2} = \sqrt[x]{81}$$

A) 2     B) 4     C) 3     D) 6

### B. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish usuli

22. (98-2-31) Tenglamaning kichik ildizini toping.

$$2^{-4x^2+2} - 3 \cdot 2^{-4x^2} = 2^{-16}$$

A) 2     B) $-3$     C) $-2$     D) $-1$

**Yechish:** Tenglamaning chap qismida umumiy ko'paytuvchi $2^{-4x^2}$ ni qavsdan tashqariga chiqaramiz:

$$2^{-4x^2}(2^2 - 3) = 2^{-16} \iff 2^{-4x^2} = 2^{-16}.$$

1-ga ko'ra $-4x^2 = -16 \iff x^2 = 4$. Bu tenglamaning ildizlari $x_1 = -2$, $x_2 = 2$ lardir. Ularning kichigi $x_1 = -2$. **Javob:** $-2$ (C).

23. (98-8-34) Tenglamani yeching.

$$\left(\frac{1}{7}\right)^{-2x+3} + 49^{x-1} + 7^{2x-1} = 399$$

A) 5     B) 4     C) 3     D) 2

24. (98-9-31) 18 va $2^{x-4} + 2^{x+1} = 132$ tenglama ildizi ayirmasini toping.
A) 9     B) 10     C) 8     D) 12

25. (99-3-18) Agar

$$3^{5x+1} + 3^{5x-1} = 30$$

bo'lsa, $\dfrac{x}{x+1}$ ning qiymatini hisoblang.

A) $\dfrac{2}{5}$     B) $\dfrac{1}{3}$     C) $\dfrac{2}{7}$     D) $\dfrac{4}{9}$

26. (01-7-31) Tenglamani yeching.

$$6 \cdot 9^{0,5x-2} + 2 \cdot 3^{x-6} = 56$$

A) 1     B) 2     C) 6     D) 3

27. (02-12-43) Agar

$$4^{x-1} - \frac{1}{2} \cdot 2^{2x} = -64$$

bo'lsa, $x + 13$ ning qiymatini toping.
A) 19     B) 15     C) 17     D) 13

### C. Yangi noma'lum kiritish usuli

28. (99-6-49) Tenglamani yeching.

$$3^{\sqrt{x}} - 3^{1-\sqrt{x}} = \frac{26}{3}$$

A) $\emptyset$     B) 9     C) 2     D) 4

**Yechish:** Tenglamada $3^{\sqrt{x}} = y \geq 0$ almashtirish olsak, u

$$y - \frac{3}{y} = \frac{26}{3} \iff 3y^2 - 26y - 9 = 0$$

ko'rinishni oladi. Bu kvadrat tenglamaning ildizlari $y_1 = -3^{-1}$, $y_2 = 9$ lardir. $y_1$ yechim $y \geq 0$ shartni qanoatlantirmaydi. $3^{\sqrt{x}} = y_2 \iff 3^{\sqrt{x}} = 9$ dan $x = 4$ ni olamiz. **Javob:** 4 (D).

29. (99-8-2) $5^x - 5^{3-x} = 20$ tenglamani yeching.
A) $-5$     B) 1     C) $-5; 1$     D) 2

30. (01-1-20) $5^x - 24 = 5^{2-x}$ tenglamani yeching.
A) $-2$     B) 0     C) $-1$     D) 2

31. (02-9-37) Tenglama ildizlari yig'indisini toping.

$$25^{x^2+0,5} - 5^{x^2} = 5^{x^2+3} - 25$$

A) 0     B) 1     C) $2\sqrt{2}$     D) 2

32. (02-11-28) Tenglama ildizlari ko'paytmasini toping.

$$8 \cdot 4^{|x|} - 33 \cdot 2^{|x|} + 4 = 0$$

A) 4     B) $\dfrac{1}{4}$     C) $-4$     D) $-\dfrac{1}{4}$

33. (03-7-19) Tenglamani yeching.

$$4^{x+1} - 2^{x+4} + 3 \cdot 2^{x+2} = 48$$

A) 1     B) 2     C) 3     D) 4

### D. Guruhlash usuli

34. (97-6-26) Tenglamani yeching.

$$2^{3x+7} + 5^{3x+4} + 2^{3x+5} - 5^{3x+5} = 0$$

A) 1     B) 0     C) $-1$     D) 2

**Yechish:** 2 asosli darajalarni tenglamaning chap qismida qoldirib, 5 asosli darajalarni tenglamaning o'ng qismiga o'tkazib, umumiy ko'paytuvchini qavsdan tashqariga chiqarsak,

$$2^{3x+5}(2^2 + 1) = 5^{3x+4}(5 - 1) \iff 2^{3x+3} = 5^{3x+3}$$

ni olamiz. Tenglikning har ikkala qismini $5^{3x+3}$ ga bo'lib, $0,4^{3x+3} = 1 = 0,4^0$ ni hosil qilamiz. Bu yerdan $3x + 3 = 0$ yoki $x = -1$ ekanligi kelib chiqadi. **Javob:** $-1$ (C).

35. Tenglamani yeching.

$$5^{3x} - 7^x - 35 \cdot 5^{3x} + 35 \cdot 7^x = 0$$

A) 1      B) 0      C) −1      D) 2

36. $2^x = 5^x$ tenglamani yeching.
A) 1      B) 0      C) −1      D) 2

37. $3 \cdot 2^x = 2 \cdot 3^x$ tenglamani yeching.
A) 1      B) 0      C) −1      D) 2

38. Tenglamani yeching.

$$9 \cdot 16^x - 7 \cdot 12^x = 16 \cdot 9^x$$

A) 2      B) −2      C) 3      D) −1

**Yechish:** $m_1 \cdot a^x + m_2 \cdot b^x = m_3 \cdot c^x$ ko'rinishdagi tenglamalarni $ac = b^2$ $(a < b < c)$ shartda yechish mumkin. Bizning holimizda $16 \cdot 9 = 12^2$ tenglik bajariladi. Berilgan tenglamaning har ikkala qismini $16^x$ ga bo'lib,

$$9 - 7 \cdot \left(\frac{12}{16}\right)^x = 16 \cdot \left(\frac{9}{16}\right)^x \iff 9 - 7 \cdot \left(\frac{3}{4}\right)^x = 16 \cdot \left(\frac{3}{4}\right)^{2x}$$

ni olamiz. Tenglamada $\left(\frac{3}{4}\right)^x = y > 0$ belgilash kiritib, uni $16y^2 + 7y - 9 = 0$ shaklga keltiramiz. Bu kvadrat tenglamaning ildizlari $y_1 = -1$, $y_2 = 9/16$. $y_1 = -1$ yechim $y > 0$ shartni qanoatlantirmaydi. $\left(\frac{3}{4}\right)^x = y_2 \iff \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^2$ dan $x = 2$ ni olamiz. **Javob:** 2 (A).

39. $4^x + 6^x = 2 \cdot 9^x$ tenglamani yeching.
A) 0      B) 0; −1      C) −1      D) 1

**Tenglamalar sistemasi**

40. (96-7-17) Agar $\begin{cases} 3^x = 9^{y+1} \\ 4y = 5 - x \end{cases}$ ekanligi ma'lum bo'lsa, $x + y$ ning qiymatini toping.
A) 3,5      B) 5      C) 2      D) −4

**Yechish:** Sistemaning 2-tenglamasidan $x = 5 - 4y$ ni olamiz. Uni sistemaning 1-tenglamasiga qo'yib $3^{5-4y} = 9^{y+1} = 3^{2y+2}$ ni olamiz. 1-xossaga ko'ra, $5 - 4y = 2y + 2$. Bu yerdan $y = 0,5$ ni olamiz. Uni sistemaning 2-tenglamasi qo'yib $x = 3$ ni olamiz. Ularning yig'indisi $x + y = 3,5$. **Javob:** 3,5 (A).

41. (97-3-17) Agar $3^{x-1} = 9^y$ va $2x - y = 5$ bo'lsa, $x - y$ ni toping.
A) 2      B) 3      C) −1      D) −0,5

42. (97-7-17) Agar $2^{x+1} = 4^y$ va $x + y = -4$ bo'lsa, $y - x$ ni toping.
A) 4      B) −2      C) 2      D) −3

43. (00-3-30) Agar

$$\begin{cases} 9^{x+y} = 729 \\ 3^{x-y-1} = 1 \end{cases}$$

bo'lsa, $x^2 - y^2$ ni toping.
A) 1      B) 4      C) 3      D) 2

44. (02-1-58) Agar

$$\begin{cases} 2^x + 2^y = 5 \\ 2^{x+y} = 4 \end{cases}$$

bo'lsa, $x \cdot y$ ni toping.
A) 0      B) 1      C) 2      D) 3

45. (03-4-31) Agar $2^{x^2} \cdot 2^{y^2} = 64$ va $2^{xy} = \sqrt{8}$ bo'lsa, $|x + y|$ ning qiymatini toping.
A) 4,5      B) 3,5      C) 2,5      D) 3

## 11.3   Ko'rsatkichli tengsizliklar

$a^{f(x)} > a^{g(x)}$ yoki $a^{f(x)} < a^{g(x)}$ ko'rinishdagi tengsizliklar sodda ko'rsatkichli tengsizliklar deyiladi. Bunday tengsizliklarni yechish ko'rsatkichli $y = a^x$ funksiyaning $a > 1$ bo'lganda o'suvchi $0 < a < 1$ bo'lganda kamayuvchi ekanligiga asoslanadi, yani:

1. **Agar** $0 < a < 1$ **bo'lsa,**

$$a^{f(x)} > a^{g(x)} \iff f(x) < g(x).$$

2. **Agar** $a > 1$ **bo'lsa,**

$$a^{f(x)} > a^{g(x)} \iff f(x) > g(x).$$

1. (98-2-32) Tengsizlikning eng katta butun yechimini toping.

$$\left(\frac{4}{9}\right)^x \cdot \left(\frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{-2x}$$

A) 2      B) 3      C) 4      D) 1

**Yechish:** Tengsizlikning chap qismiga $a^x \cdot b^x = (ab)^x$ formulani, o'ng qismiga esa $a^x \cdot a^y = a^{x+y}$ formulani qo'llab

$$\left(\frac{4}{9} \cdot \frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^{6-2x} \iff \left(\frac{2}{3}\right)^x > \left(\frac{2}{3}\right)^{6-2x}$$

tengsizlikni hosil qilamiz. Asos $\frac{2}{3} < 1$ bo'lgani uchun 1-ga ko'ra

$$x < 6 - 2x \iff 3x < 6 \iff x < 2.$$

Bu shartni qanoatlantiruvchi eng katta butun son 1 dir. **Javob:** 1 (D).

2. (96-6-54) Tengsizlikni yeching.

$$0,25^x \geq 0,5^{4x-8}$$

A) $(-\infty; 4)$   B) $(-\infty; 2]$   C) $[2; \infty)$   D) $[4; \infty)$

3. (97-6-55) Tengsizlikni yeching.

$$2^{\sqrt{x}-1} \cdot (4x^2 - 4x + 1) > 0$$

A) $(1; \infty)$          B) $[1; \infty)$
C) $[\frac{1}{2}; \infty)$          D) $[0; \frac{1}{2}) \cup (\frac{1}{2}; \infty)$

4. (97-9-76) $x$ ning qanday qiymatlarida $y = 5^x - 5$ funksiya musbat qiymatlar qabul qiladi?
A) $x < 1$    B) $x > 1$    C) $x \geq 1$    D) $x \leq 2$

5. (99-1-30) Tengsizlikni yeching.
$$(\sqrt{6})^x \leq \frac{1}{36}$$
A) $(-\infty; -4]$        B) $[-4; \infty)$
C) $[-4; 4]$        D) $(-\infty; 6]$

**Yechish:** Tengsizlikning quyidagicha yozib olamiz:
$$(6)^{x/2} \leq 6^{-2} \Longleftrightarrow \frac{x}{2} \leq -2 \Longleftrightarrow x \leq -4$$
tengsizlikni hosil qilamiz. **Javob:** $(-\infty; -4]$ (A).

6. (99-2-35) Ushbu $\left(\frac{1}{2}\right)^{20-2x} > 1$ tengsizlikning eng kichik butun yechimini toping.
A) 6      B) 11      C) 10      D) 9

7. (99-6-16) Tengsizlikning eng katta butun yechimini toping.
$$2^{3-6x} > 1$$
A) 0      B) 1      C) $-1$        D) $-2$

8. (00-8-10) Tengsizlikni yeching.
$$\left(\frac{1}{2}\right)^{2x-1} > \frac{1}{16}$$
A) $(-\infty; 2,5)$        B) $(2,5; \infty)$
C) $(-2,5; \infty)$        D) $(-\infty; 0) \cup (0; 2,5)$

9. (03-4-30) Tengsizlikning eng kichik butun yechimini toping.
$$\frac{1}{8} \cdot 2^{4x-2} > (\sqrt{2})^{10}$$
A) 2      B) 1      C) 3      D) 4

10. (03-5-31) $f(x) = \sqrt{3^x - 4^x}$ funksiyaning aniqlanish sohasini toping.
A) $(-\infty; 0]$    B) $(0; 1)$    C) $[0; 1)$    D) $[0; \infty)$

11. (00-6-31) Tengsizlikning butun yechimlari yig'indisini toping.
$$3^{8x} - 4 \cdot 3^{4x} \leq -3$$
A) 8      B) 7      C) 4      D) 0

**Yechish:** Tengsizlikda $3^{4x} = y > 0$ belgilash olib, uni
$$y^2 - 4y + 3 \leq 0 \Longleftrightarrow (y-1)(y-3) \leq 0$$
shaklda yozib olamiz. Bu tengsizlikni oraliqlar usuli bilan yechib $1 \leq y \leq 3$ ni olamiz. Belgilashga qaytib $1 \leq 3^{4x} \leq 3 \Longleftrightarrow 3^0 \leq 3^{4x} \leq 3^1$ ni olamiz. Bu yerdan $0 \leq 4x \leq 1 \Longleftrightarrow 0 \leq x \leq 0,25$. Tengsizlikning birgina butun yechimi 0 dir. **Javob:** 0 (D).

12. (02-5-20) Tengsizlikni yeching.
$$4^x - 5 \cdot 2^{x+1} + 16 \leq 0$$
A) $(1; 3)$        B) $(0; 1) \cup (3; \infty)$
C) $[1; 3]$        D) $[0; 1] \cup [3; \infty)$

13. (01-4-30) Tengsizlikni yeching.
$$9^{-x} - 28 \cdot 3^{-x-1} + 3 < 0$$
A) $(-2; 1)$    B) $(-\infty; 2]$    C) $[1; \infty)$    D) $(-2; 0)$

14. (01-1-21) Tengsizlikni yeching.
$$3^{\frac{1}{x+1}} > 9$$
A) $(-1; 1)$        B) $\left(-1; -\frac{1}{2}\right)$
C) $\left(-\frac{1}{2}; 1\right)$        D) $(0; 1)$

15. (01-2-70) Nechta natural son
$$(0,7)^{2+4+\cdots+2n} > (0,7)^{72}$$
tengsizlikni qanoatlantiradi?
A) 7      B) 8      C) 9      D) 10

**Yechish:** Arifmetik progressiya dastlabki $n$ ta hadi yig'indisi formulasidan foydalansak,
$$S_n = 2 + 4 + \cdots + 2n = n(n+1).$$
Asos $0,7 < 1$ bo'lganligi uchun, 1-ga ko'ra
$$n(n+1) < 72 \Longleftrightarrow (n-8)(n+9) < 0$$
tengsizlik o'rinli. Bu tengsizlikni oraliqlar usuli bilan yechib $-9 < n < 8$ ni olamiz. Bu shartni qanoatlantiruvchi natural sonlar 1, 2, 3, 4, 5, 6, 7 lardir. Ular 7 ta **Javob:** 7 (A).

16. (98-5-16) Ushbu $14 \leq 2^n < 64$ qo'sh tengsizlikni qanoatlantiruvchi natural sonlar nechta?
A) 2      B) 3      C) 1      D) 4

17. (99-7-18) $n$ ning nechta natural qiymati
$$9 \leq 3^n \leq 79$$
qo'sh tengsizlikni qanoatlantiradi?
A) 1      B) 3      C) 4      D) 2

18. (01-8-32) Ushbu $3^{|x|+2} \leq 81$ tengsizlikning butun yechimlari yig'indisini toping.
A) $-1$      B) 3      C) 4      D) 0

19. (01-9-18) Ushbu $0,5^{x^2-4} > 0,5^{3x}$ tengsizlikning butun yechimlari o'rta arifmetigini toping.
A) $1,5$      B) 2      C) 1      D) 3

20. (02-2-25) Tengsizlikni yeching.
$$5^{\frac{1}{x}} + 5^{\frac{1}{x}+2} > 130$$
A) $(0; 1)$    B) $(0; 3)$    C) $\left(0; \frac{3}{4}\right)$    D) $(1; 2)$

**Yechish:** Umumiy ko'paytuvchi $5^{\frac{1}{x}}$ ni qavs oldiga chiqaramiz:

$$5^{\frac{1}{x}}(1 + 25) > 130 \iff 5^{\frac{1}{x}} > 5^1.$$

Asos $5 > 1$ bo'lganligi uchun, 2-ga ko'ra

$$\frac{1}{x} > 1 \iff \frac{1}{x} - 1 > 0 \iff \frac{1-x}{x} > 0.$$

Bu tengsizlikni oraliqlar usuli bilan yechib $0 < x < 1$ ni olamiz. **Javob:** $(0; 1)$ (A).

21. (06-121-34) Tengsizlikni yeching.

$$3^{\frac{1}{x}} + 3^{\frac{1}{x}+3} > 84$$

A) $(0; 1)$  B) $(-\infty; 0)$
C) $(1; \infty)$  D) $(0; 1) \cup (1; \infty)$

22. (03-6-58) Tengsizlikni yeching.

$$3^{3x-2} + 3^{3x+1} - 3^{3x} < 57$$

A) $x > 1$  B) $x < 1\frac{1}{2}$  C) $x < 1$  D) $x > \frac{2}{3}$

23. (03-7-79) Tengsizlikning natural yechimlari yig'indisini toping.

$$3^{x+2} + 3^{x+3} \leq 972$$

A) 1  B) 3  C) 6  D) 10

24. (02-5-22) Tengsizlik yechimlari orasida nechta tub son bor?

$$(1,25)^{1-x} > (0,64)^{2(1+\sqrt{x})}$$

A) 5  B) 7  C) 9  D) 12

25. (97-2-54) Tengsizlikni yeching.

$$0,2^{x^2+1} + 0,2^{x^2-1} < 1,04$$

A) $(-\infty; -1)$  B) $(1; \infty)$
C) $(-\infty; -1] \cup [1; \infty)$  D) $(-\infty; -1) \cup (1; \infty)$

# 12  -bob. Logarifmik funksiya

Ko'rsatkichli $f(x) = a^x$, $a \neq 1$, $a > 0$ funksiyani qaraymiz. Har bir tayinlangan $y \in (0; \infty)$ uchun

$$a^x = y \tag{1}$$

tenglama yagona yechimga ega. Bu yechim $x = \log_a y$ shaklda yoziladi. $y$ sonining $a$ asosga ko'ra logarifmi deb $y$ sonini hosil qilish uchun $a$ ni ko'tarish kerak bo'lgan darajaga aytiladi. Demak, (1) tenglikda $x$ soni $y$ ning $a$ asosga ko'ra logarifmi ekan. Yuqoridagilardan kelib chiqadiki $y = \log_a x$, $a > 0$, $a \neq 1$ funksiya, ko'rsatkichli $f(x) = a^x$ funksiyaga teskari funksiya ekan. $y = \log_a x$, $a > 0$, $a \neq 1$ funksiyaga logarifmik fuksiya deyiladi. Uning aniqlanish sohasi $D(y) = (0; \infty) = E(f)$ va qiymatlar sohasi $E(y) = (-\infty; \infty) = D(f)$. Teskari funksiya ta'rifidan quyidagilar kelib chiqadi:

barcha $x \in R$ lar uchun $\log_a a^x = x$, barcha $x > 0$ lar uchun $a^{\log_a x} = x$. Bu tenglik asosiy logarifmik ayniyat deyiladi. $y = \log_a x$ funksiya $a > 1$ da o'suvchi, $0 < a < 1$ da kamayuvchi. Agar logarifm asosida $e = 2,71\ldots$ bo'lsa, u $\ln x = \log_e x$ shaklda yoziladi. 10 asosga ko'ra logarifm $\lg x = \log_{10} x$ shaklda yoziladi. Logarifmik funksiyaning grafigi I va IV choraklarda yotadi (12.1-chizmaga qarang).

Logarifmik funksiya quyidagi xossalarga ega.
**Ixtiyoriy** $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ **uchun**

1. $a^{\log_a x} = x$, $\quad x > 0$.

2. $\log_a a = 1$, $\quad \log_a 1 = 0$.

3. $\log_a(xy) = \log_a x + \log_a y$, $\quad x, y > 0$.

4. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$ $\quad x, y > 0$.

5. $\log_a x^p = p \log_a x$, $\quad x > 0$.

6. $\log_{a^p} x = \frac{1}{p} \log_a x$, $\quad x > 0$.

7. $\log_a x = \dfrac{\log_b x}{\log_b a}$.

8. $\log_a b \cdot \log_b a = 1 \iff \log_a b = \dfrac{1}{\log_b a}$.

9. $\log_a b = \log_{a^p} b^p$, $\quad p \neq 0$.

10. $\log_{a^x} b^y = \dfrac{y}{x} \log_a b$.

11. $a^{\log_b c} = c^{\log_b a}$.

12. $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$.

## 12.1  Aniqlanish sohasi va xossalari

1. (96-6-52) Ushbu $y = \log_3(2 - x)$ funksiyaning aniqlanish sohasini toping.
   A) $(-\infty; 2)$  B) $(2; \infty)$  C) $(0; 2)$  D) $(0; 2]$

   **Yechish:** Logarifmik funksiyaning aniqlanish sohasi $(0; \infty)$ dan iborat. Shuning uchun $2 - x > 0 \iff x < 2$. Demak, funksiya $(-\infty; 2)$ to'plamda aniqlangan ekan. **Javob:** $(-\infty; 2)$ (A).

2. Ushbu $y = \log_7(4 - x^2)$ funksiyaning aniqlanish sohasini toping.
   A) $(0; 2)$     B) $(2; 4)$     C) $(-2; 2)$     D) $(0; 2]$

3. (98-12-42) $y = \log_3 x$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida yotadi?
   A) I, IV     B) I, II     C) II, III     D) III, IV

4. (99-2-36) $f(x) = \dfrac{\sqrt{8 - x}}{\lg(x - 1)}$ funksiyaning aniqlanish sohasiga tegishli butun sonlar nechta ?
   A) 4     B) 8     C) 7     D) 6

5. (99-3-26) Funksiyaning aniqlanish sohasini toping.

$$y = \frac{1}{\ln(1 - x)} + \sqrt{x + 2}$$

   A) $[-2; \infty)$         B) $[-2; 1]$
   C) $(-\infty; 1)$         D) $[-2; 0) \cup (0; 1)$

6. (99-5-39) Ushbu $f(x) = \log_2(64^{-x} - 8^{1-x})$ funksiyaning aniqlanish sohasini toping.
   A) $(-\infty; 0)$         B) $(-\infty; -1)$
   C) $(-\infty; -2)$         D) $(1; \infty)$

7. (99-6-29) Ushbu $y = \log_3(x(x-3)) - \log_3 x$ funksiyaning aniqlanish sohasini toping.
   A) $(3; \infty)$     B) $(-\infty; 3)$     C) $[3; \infty)$     D) $(-\infty; 3]$

8. (97-2-52) Ushbu $y = \log_{x^2}(4 - x)$ funksiyaning aniqlanish sohasini toping.
   A) $(-\infty; 4)$
   B) $(-\infty; -1) \cup (-1; 0) \cup (0; 1) \cup (1; 4)$
   C) $(-\infty; -1) \cup [-1; 1] \cup (1; 4)$
   D) $(-\infty; 1) \cup (4; \infty)$

   **Yechish:** $\log_a b$ son $b > 0,\quad a > 0,\quad a \neq 1$ da aniqlangani uchun

$$\begin{cases} 4 - x > 0 \\ x^2 > 0 \\ x^2 \neq 1. \end{cases}$$

   sistemani hosil qilamiz. Uni yechimi

$$\begin{cases} x < 4 \\ x \neq -1, 0, 1. \end{cases}$$

   Demak, berilgan funksiya $(-\infty; -1) \cup (-1; 0) \cup (0; 1) \cup (1; 4)$ to'plamda aniqlangan ekan. **Javob:** (B).

9. (97-1-63) Ushbu $y = \log_x(3 - x)$ funksiyaning aniqlanish sohasini toping.
   A) $(-\infty; 3)$         B) $(0; \infty)$
   C) $(0; 1) \cup (1; 3)$       D) $(0; 3)$

10. (97-6-64) Ushbu $f(x) = \log_x(6 - x)$ funksiyaning aniqlanish sohasini toping.
    A) $(-\infty; 6)$         B) $(1; 6)$
    C) $(0; 1)$         D) $(0; 1) \cup (1; 6)$

11. (97-8-52) Ushbu $y = \log_{x-1}(x - \dfrac{1}{4})$ funksiyaning aniqlanish sohasini toping.
    A) $(\dfrac{1}{4}; \infty)$         B) $(1; 2) \cup (2; \infty)$
    C) $(-0, 25; 2) \cup (2; \infty)$     D) $[-0, 25; 2) \cup [2; \infty)$

12. (97-9-75) $n$ ning qanday butun qiymatlarida $y = \lg(nx^2 - 5x + 1)$ funksiyaning aniqlanish sohasi $(-\infty; \dfrac{1}{4}) \cup (1; \infty)$ bo'ladi?
    A) 1         B) 4         C) 3         D) 5

    **Yechish:** Masalada $nx^2 - 5x + 1 > 0$ tengsizlikning yechimi $(-\infty; \dfrac{1}{4}) \cup (1; \infty)$ to'plam bo'ladigan $n \in Z$ sonini topish talab qilingan. Demak, parabolaning shoxlari yuqoriga yo'nalgan va $Ox$ o'qini $\dfrac{1}{4}$ va 1 nuqtalarda kesib o'tadi. Bu yerdan $nx^2 - 5x + 1$ kvadrat uchhadning nollari $x_1 = \dfrac{1}{4}$ va $x_2 = 1$ ekanligi kelib chiqadi.

$$nx^2 - 5x + 1 = n(x - \frac{1}{4})(x - 1)$$

    tenglikdan $n = 4$ ni olamiz. **Javob:** 4 (B).

13. (99-7-15) $k$ ning qanday qiymatlarida $y = \lg(kx^2 - 2x + 1)$ funksiya faqat $x = 1$ nuqtada aniqlanmagan?
    A) $k < 2$     B) $k < 3$     C) $k \leq 1$     D) $k = 1$

14. (99-8-34) Quyidagi funksiyalardan qaysi birining aniqlanish sohasi $(0, 1)$ oraliqdan iborat?
    A) $y = \sqrt{1/(1 - x)} + \log_2 x$         B) $y = 1/\sqrt{1 - x^2}$
    C) $y = \sqrt{1 - x} - \sqrt{x}$         D) $y = \dfrac{1}{1 - x}$

15. (99-8-36) Ushbu $f(x) = \log_3(x^2 - 6x + 36)$ funksiyaning eng kichik qiymatini toping.
    A) 1         B) 9         C) 2         D) 3

16. (96-12-90) Agar $a = \log_{\frac{1}{6}} 4$, $b = \log_{\frac{1}{5}} 6$ va $c = \log_{\frac{1}{5}} 4$ bo'lsa, $a, b$ va $c$ sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?
    A) $c < b < a$         B) $b < c < a$
    C) $c < a < b$         D) $a < b < c$

    **Yechish:** Asos $a \in (0; 1)$ da $y = \log_a x$ kamayuvchi, shunga ko'ra $b = \log_{\frac{1}{5}} 6 < \log_{\frac{1}{5}} 4 = c$. Endi $a$ bilan $c$ ni taqqoslaymiz. Ularni 4 asosli logarifm shaklida tasvirlab olamiz:

$$a = \frac{\log_4 4}{\log_4 \frac{1}{6}} = -\frac{1}{\log_4 6}, \quad c = \frac{\log_4 4}{\log_4 \frac{1}{5}} = -\frac{1}{\log_4 5}.$$

    Quyidagilardan $5 < 6 \Longrightarrow \log_4 5 < \log_4 6$ yana

$$\frac{1}{\log_4 5} > \frac{1}{\log_4 6} \Longrightarrow c = -\frac{1}{\log_4 5} < -\frac{1}{\log_4 6} = a.$$

    Demak, $b < c < a$ ekan. **Javob:** $b < c < a$ (B).

17. (96-13-31) Agar $a = \log_{\frac{1}{5}} 4$, $b = \log_{\frac{1}{5}} 6$, $c = \log_{\frac{1}{6}} 4$ bo'lsa, $a, b$ va $c$ sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?
    A) $b < c < a$         B) $c < a < b$
    C) $a < c < b$         D) $b < a < c$

18. (03-5-63) Sonlarni o'sish tartibida joylashtiring.

$$a = 2\log_2 5, \quad b = 3\log_{\frac{1}{8}} \frac{1}{23}, \quad c = 4\log_{\frac{1}{4}} \frac{5}{26}$$

    A) $b < a < c$         B) $a < b < c$
    C) $b < c < a$         D) $c < b < a$

19. (02-2-20) Qaysi javobda manfiy son ko'psatilgan?
    A) $\log_{\frac{1}{2}} 2$      B) $\log_{\sqrt{2}} \sqrt{3}$
    C) $\log_{\frac{1}{7}} \dfrac{1}{\sqrt{45}}$      D) $\log_2 1,2$

20. (99-9-47) Agar $0 < p < 1$ va $1 < n < m$ bo'lsa, quyidagi ko'paytmalardan qaysi biri musbat?
    A) $\log_p m \cdot \log_m 1$      B) $\log_p n \cdot \log_p m$
    C) $\log_m p \cdot \log_n m$      D) $\log_p m \cdot \log_m 1$

21. (01-3-21) Ushbu
    $$y = \log_{\sqrt{10}}(6 + x - x^2)$$
    funksiyaning aniqlanish sohasidagi butun sonlar yig'indisini toping.
    A) 0     B) 3     C) 2     D) 5

22. (01-9-46) Ushbu
    $$y = \log_\pi \frac{x^2 - 13x - 30}{25 - 9x^2}$$
    funksiyaning aniqlanish sohasiga nechta natural son tegishli?
    A) 13     B) 15     C) 0     D) 8

23. (01-9-47) Ushbu
    $$y = \log_{15} \frac{x^2 - 2x - 15}{2x + 3}$$
    funksiyaning aniqlanish sohasiga tegishli eng katta manfiy butun sonni va funksiyaning shu nuqtadagi qiymatini toping.
    A) $y(-1) = \log_{15} 2$      B) $y(-5) = \log_{15} 20$
    C) $y(-3) = 4$      D) $y(-2) = \log_{15} 7$

    **Yechish:** Berilgan funksiyaning aniqlanish sohasi
    $$\frac{x^2 - 2x - 15}{2x + 3} > 0 \iff \frac{(x+3)(x-5)}{2(x+1,5)} > 0$$
    tengsizlik yechimidan iborat. Bu tengsizlik oraliqlar isuli bilan yechiladi, uning yechimi $(-3; -1,5) \cup (5; \infty)$ to'plamdir. Bu to'plamga tegishli eng katta manfiy butun son $-2$ dir. $y(-2) = \log_{15} 7$.
    **Javob:** $y(-2) = \log_{15} 7$ (D).

24. (98-7-21) Tenglamada $x$ ning qabul qilishi mumkin bo'lgan qiymatlar to'plamini ko'rsating.
    $$\lg(x - 3) - \lg(x + 9) = \lg(x - 2)$$
    A) $(2; 3)$   B) $(9; \infty)$   C) $(-9; \infty)$   D) $(3; \infty)$

25. (02-7-20) $y = \lg\left(\dfrac{3x + 1}{x + 2} - 1\right)$ funksiyaning aniqlanish sohasini toping.
    A) $(-\infty; -2) \cup (\frac{1}{2}; \infty)$      B) $(-2; \frac{1}{2})$
    C) $(-\infty; -2)$         D) $(\frac{1}{2}; \infty)$

26. (02-9-29) $y = \sqrt{2 + \log_{\frac{1}{2}}(3 - x)}$ funksiyaning aniqlanish sohasini toping.
    A) $(-1; 3)$   B) $[-1; 3)$   C) $(-\infty; 3)$   D) $(-\infty; -1]$

27. (02-12-51) Funksiyaning aniqlanish sohasini toping.
    $$f(x) = \sqrt{x + 4} + \log_2(x^2 - 4)$$
    A) $[-2; 2]$      B) $(-4; 2)$
    C) $(-2; 2)$      D) $[-4; -2) \cup (2; \infty)$

28. (03-6-43) Funksiyaning aniqlanish sohasini toping.
    $$y = \sqrt{\frac{8}{|x|} - 1} + \lg(x^2 - 1)$$
    A) $-8 < x < -1$     B) $1 < x < 8$
    C) $-1 < x < 1$     D) $-8 \le x < -1,\ 1 < x \le 8$

29. (03-10-38) $y = \dfrac{\ln(7 - x^2)}{x + 1}$ funksiyaning aniqlanish sohasiga tegishli butun sonlarning yig'indisini toping.
    A) 0     B) 1     C) $-1$     D) 2

### 12.1.1   Logarifmik ifodalarda shakl almashtirish

1. $\log_2 8$ ni hisoblang.
   A) 4     B) 3     C) 1     D) 2

   **Yechish:** $8 = 2^3$ dan va 5-xossadan foydalanib $\log_2 2^3 = 3 \cdot \log_2 2 = 3$ ni olamiz. **Javob:** 3 (B).

2. $\log_4 8 + \log_4 32$ ni hisoblang.
   A) 4     B) 3     C) 5     D) 2

3. (02-4-38) Hisoblang.
   $$\log_{\frac{1}{6}} 2 + \log_{\frac{1}{6}} 3$$
   A) $-3$     B) $-1$     C) 0     D) 1

4. $\log_2 18 - \log_2 9$ ni hisoblang.
   A) 4     B) 3     C) 1     D) 2

5. (08-121-28)   $\log_2 \log_3 81$ ni hisoblang.
   A) 4     B) 3     C) 1     D) 2

6. (97-5-37)   $\log_2 \lg 100$ ni hisoblang.
   A) 1     B) 4     C) 3     D) 2

7. (08-120-28) Hisoblang:
   $$\frac{\log_9 12}{\log_{36} 3} - \frac{\log_9 4}{\log_{108} 3}$$
   A) 2     B) 3     C) 6     D) 1

8. (96-9-31) Hisoblang.
   $$\left(\sqrt[3]{7}\right)^{\frac{3}{\log_9 7}}$$
   A) 10     B) 9     C) 3     D) 7

   **Yechish:** 8-xossadan foydalanib $\dfrac{1}{\log_9 7} = \log_7 9$ ni olamiz. Endi asosiy ayniyatdan
   $$\left(7^{\frac{1}{3}}\right)^{3 \cdot \log_7 9} = 7^{\log_7 9} = 9.$$
   **Javob:** 9 (B).

9. (96-3-89) $\left(2^{\frac{1}{\log_3 16}}\right)^4$ ni hisoblang.

 A) $\sqrt{3}$    B) 4    C) 2    D) 3

10. (98-4-15) Hisoblang.

$$\frac{5^{\lg 20}}{20^{\lg 5+1}}$$

 A) 0,25    B) 0,1    C) 0,2    D) 0,05

11. (99-2-31) Hisoblang.

$$100^{\frac{1}{2}\lg 27-\lg 3} \cdot 10$$

 A) 20    B) 40    C) 30    D) 10

12. (00-3-34) Hisoblang.

$$343^{\log_{49} 4}$$

 A) 8    B) 4    C) 7    D) 6

13. (00-10-42) Hisoblang.

$$\log_{2\sqrt{2}} 512$$

 A) 8    B) 6    C) 4    D) 10

14. (01-3-14) Hisoblang.

$$4^{\log_2(\sqrt[3]{2\sqrt{2}})^2}$$

 A) 16    B) 2    C) 4    D) 64

15. (01-5-16) Ifodaning qiymatini toping.

$$49^{1-\log_7 2} + 5^{-\log_5 4}$$

 A) 12,5    B) 13    C) 14    D) 23

16. (96-9-84) Hisoblang.

$\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$

 A) 1    B) 3    C) 6    D) 2

**Yechish:** 10 asosli logarifmga o'tamiz

$$\frac{\lg 4}{\lg 3} \cdot \frac{\lg 5}{\lg 4} \cdot \frac{\lg 6}{\lg 5} \cdot \frac{\lg 7}{\lg 6} \cdot \frac{\lg 8}{\lg 7} \cdot \frac{\lg 9}{\lg 8} = \frac{\lg 3^2}{\lg 3} = 2.$$

**Javob:** 2 (D).

17. (00-5-66) Hisoblang.

$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7$

 A) $\frac{1}{2}$    B) $\frac{1}{3}$    C) $\frac{1}{4}$    D) $\frac{1}{5}$

18. (99-6-13) Hisoblang.

$$\log_9 17 \cdot \log_{17} 7 \cdot \log_7 3$$

 A) $\frac{1}{2}$    B) $\frac{1}{7}$    C) 1    D) 2

19. (96-6-53) Sonlardan qaysi biri 2 dan kichik?
 $M = \log_5 100 - \log_5 4$,    $N = 4\log_2 3 - \log_2 9$
 $P = \log_6 72 - \log_6 2$,    $Q = \log_4 16 + \log_4 \frac{1}{8}$
 A) $N$    B) P    C) M    D) Q

 **Yechish:** Yig'indini ko'paytmaga keltirish (3-ga qarang) formulasidan

$$Q = \log_4 16 + \log_4 \frac{1}{8} = \log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}.$$

 **Javob:** $Q$ (D).

20. (97-8-53) Sonlardan qaysi biri 2 dan kichik?
 A) $\log_4 2 + \log_4 8$        B) $\log_2 36 - \log_2 3$
 C) $2\log_2 5 - \log_2 25$       D) $\log_2 6 + \frac{1}{2}\log_2 9$

21. (97-12-52) Sonlardan qaysi biri 1 ga teng emas?
 A) $\log_3 12 - \log_3 4$        B) $\frac{1}{2}\log_4 36 + \log_4 \frac{2}{3}$
 C) $\log_5 125 - \frac{1}{2}\log_5 625$     D) $2\log_2 5 - \log_2 30$

22. (00-1-39) Eng katta sonni toping.
 A) $\log_2 18 - \log_2 9$       B) $3^{\log_3 6}$
 C) $\lg 25 + \lg 4$             D) $\log_{13} 169^2$

23. (03-1-20) Agar $x = \log_5 2 + \log_{11} 3$ bo'lsa, quyidagi sonlarning qaysi biri eng katta bo'ladi?
 A) $x$    B) $x^2$    C) $x^3$    D) $\sqrt[3]{x}$

24. (98-1-33) Soddalashtiring.

$$\frac{\log_2^2 14 + \log_2 14 \log_2 7 - 2\log_2^2 7}{\log_2 14 + 2\log_2 7}$$

 A) 2    B) $\log_2 7$    C) $-\log_2 7$    D) 1

 **Yechish:** $\log_2 7 = x$ belgilash olsak, $\log_2 14 = \log_2 2 + \log_2 7 = 1 + x$ bo'ladi. Endi kasr suratini hisoblaymiz: $(1+x)^2 + (1+x)x - 2x^2 = 1 + 2x + x^2 + x + x^2 - 2x^2 = 1 + 3x$. Kasr maxraji $1 + x + 2x = 1 + 3x$. Ularning nisbati 1 ga teng.
 **Javob:** 1 (D).

25. (98-8-33) Soddalashtiring.

$$\frac{2\log_3^2 2 - \log_3^2 18 - \log_3 2 \cdot \log_3 18}{2\log_3 2 + \log_3 18}$$

 A) 1    B) $\frac{1}{2}$    C) $-2$    D) $-\frac{1}{2}$

26. (01-6-36) Hisoblang.

$$2\log_2 12 + \log_2 20 - \log_2 15 - \log_2 3$$

 A) 4    B) 5    C) 7    D) 6

27. (01-9-17) Soddalashtiring

$$\frac{\lg^2(x^3)}{\lg^3(x^2)} \cdot \lg\sqrt{x}$$

 A) $\frac{9}{16}$    B) $\frac{3}{4}$    C) $1\frac{7}{9}$    D) $\frac{3}{2}$

28. (01-11-25) Hisoblang.

$$\log_5 2 \cdot \log_4 243 \cdot \log_2 5 \cdot \log_3 4$$

A) 4     B) 3     C) 5     D) 6

**Yechish:** Logarifmning 8-xossasiga ko'ra, $\log_5 2 \cdot \log_2 5 = 1$. Endi 5 va 8-xossalardan foydalansak

$\log_4 243 \cdot \log_3 4 = \log_4 3^5 \cdot \log_3 4 = 5\log_4 3 \cdot \log_3 4 = 5$

bo'ladi. **Javob:** 5 (C).

29. (01-11-26) Hisoblang.

$$\frac{3\lg 2 + 3\lg 5}{\lg 1300 - \lg 13}$$

A) $1,8$     B) $1,6$     C) $2,3$     D) $1,5$

30. (02-2-53) Hisoblang.

$$\frac{\log_5 30}{\log_{30} 5} - \frac{\log_5 150}{\log_6 5}$$

A) $1$     B) $-1$     C) $\dfrac{1}{2}$     D) $-\dfrac{1}{2}$

31. (02-3-32) Agar $a > 0$ va $a \neq 1$ bo'lsa, $\log_{\sqrt{a}} \sqrt[3]{a}$ ifodaning qiymatini toping.

A) $\dfrac{2}{3}$     B) $\dfrac{3}{2}$     C) $3$     D) $6$

32. (02-3-33) Hisoblang.

$$\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \frac{1}{\log_{16} 4} +$$

$$+ \frac{1}{\log_{32} 4} + \frac{1}{\log_{64} 4} + \frac{1}{\log_{128} 4}$$

A) 14     B) 16     C) 7     D) 32

33. (02-5-24) Hisoblang.

$$\log_3^{-1} \sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}$$

A) 27     B) $-27$     C) $\dfrac{1}{27}$     D) 3

**Yechish:** Logarifmning 5-xossasi va ildizning xossasidan foydalansak

$$\log_3^{-1} \sqrt[3]{\sqrt[3]{\sqrt[3]{3}}} = \frac{1}{\log_3 3^{\frac{1}{27}}} = \frac{1}{\frac{1}{27}\log_3 3} = 27$$

bo'ladi. **Javob:** 27 (A).

34. (03-5-39) Hisoblang.

$$y = \log_5 \log_5 \sqrt[5]{\sqrt[5]{\sqrt[5]{\sqrt[5]{5}}}}$$

A) $-4$     B) $\dfrac{1}{5}$     C) $-\dfrac{1}{4}$     D) 4

35. (02-10-73) Hisoblang.

$$\left( \frac{\log_6 27 + 2\log_6 2}{\log_6 \sqrt[3]{0,25} + \log_6 \frac{1}{3}} \right)^3$$

A) $-27$     B) 27     C) $-8$     D) $8\log_6 27$

36. (02-12-48) Hisoblang.

$$\frac{3\lg 2 + 3\lg 5}{\lg 1300 - \lg 0,13}$$

A) $0,8$     B) $0,6$     C) $0,7$     D) $0,75$

37. (03-2-20) Hisoblang.

$$\frac{1 + 2\log_3 2}{(1 + \log_3 2)^2} + \log_6^2 2$$

A) 2     B) $0,5$     C) 1     D) $\dfrac{1}{4}$

38. (03-3-33) Hisoblang.

$$\log_8 5^{2\log_{25} 32}$$

A) $\dfrac{1}{2}$     B) $\dfrac{1}{3}$     C) $\dfrac{5}{3}$     D) 2

39. (03-4-32) $\ln(3^{\log_3 0,64} + 8^{\log_8 0,36})$ ning qiymati $-11$ dan qancha ko'p?

A) 10     B) 9     C) 11     D) 12

40. (03-4-33) Hisoblang.

$$2\log_4 8 - 3\log_8 4 + \log_2 32 + 18$$

A) 22     B) 24     C) 26     D) 20

41. (98-5-29) Agar $\log_3 5 = a$, $\log_3 2 = b$ bo'lsa, $\log_6 45$ ni $a$ va $b$ orqali ifodalang.

A) $\dfrac{b+2}{a+2}$   B) $\dfrac{2+a}{1+b}$   C) $\dfrac{a}{1+b}$   D) $\dfrac{b}{1+a}$

**Yechish:** $\log_3 5 = a$, $\log_3 2 = b$ berilgan. Bir asosdan boshqa asosga o'tish formulasi 7-ga ko'ra

$$\log_6 45 = \frac{\log_3(9 \cdot 5)}{\log_3(2 \cdot 3)} = \frac{\log_3 9 + \log_3 5}{\log_3 2 + \log_3 3} = \frac{2+a}{b+1}$$

ekanini hosil qilamiz. **Javob:** $\dfrac{2+a}{b+1}$ (B).

42. (96-10-36) Agar $\log_4 125 = a$ bo'lsa, $\lg 64$ ni $a$ orqali ifodalang.

A) $\dfrac{3}{2}a + 4$   B) $\dfrac{2}{3}a + 6$   C) $\dfrac{18}{2a+3}$   D) $\dfrac{6}{3a+2}$

43. (96-9-28) Agar $a = \log_{50} 40$ bo'lsa, $\log_5 2$ ni $a$ orqali ifodalang.

A) $\dfrac{3a-1}{2-a}$   B) $\dfrac{a-3}{1-2a}$   C) $\dfrac{a-3}{2a-1}$   D) $\dfrac{1-2a}{a-3}$

44. (96-3-86) Agar $a = \log_{98} 56$ bo'lsa, $\log_7 2$ ni $a$ orqali ifodalang.

A) $\dfrac{3-a}{2a-1}$   B) $\dfrac{2a-1}{3-a}$   C) $\dfrac{a-3}{2a-1}$   D) $\dfrac{1-2a}{3-a}$

45. (00-1-38) Agar $a = \log_{12} 2$ bo'lsa, $\log_6 16$ ning qiymatini toping.

A) $\dfrac{4a}{1+a}$   B) $\dfrac{2a}{1-a}$   C) $\dfrac{4a}{1-a}$   D) $\dfrac{3a}{1+a}$

46. (00-6-32) Agar $\log_{0,5} 27 = a$ bo'lsa, $\log_{\sqrt{3}} \sqrt[6]{1,5}$ ning qiymatini toping.

A) $\dfrac{1}{3} + a^{-1}$   B) $a^2 - 1$   C) $3 + a^{-1}$   D) $1 + a^{-3}$

47. (00-10-34) Agar $a = \log_2 3$ bo'lsa, $\log_8 0,75$ ni $a$ orqali ifodalang.

A) $\dfrac{1}{3}(a-1)$     B) $\dfrac{1}{3}(a+1)$

C) $\dfrac{1}{3}(a-2)$     D) $\dfrac{1}{3}(a+2)$

48. (00-10-66) Agar $\log_a 27 = b$ bo'lsa, $\log_{\sqrt{3}} \sqrt[6]{a}$ ni toping.

A) $\dfrac{1}{b}$     B) $\dfrac{2}{b}$     C) $-\dfrac{b}{2}$     D) $2b$

49. (99-10-35) Agar $\log_2 a = 2$ va $\log_3 b = 2$ bo'lsa, $\log_6 ab$ ning qiymatini toping.

A) $-2$     B) $3$     C) $-3$     D) $2$

50. (00-8-38) Agar $\lg 5 = a$ va $\lg 3 = b$ bo'lsa, $\log_{30} 8$ ni $a$ va $b$ orqali ifodalang.

A) $\dfrac{3-3a}{1+b}$     B) $\dfrac{3(1-b)}{1+a}$

C) $\dfrac{3(a-b)}{a+b}$     D) $\dfrac{b-1}{a+1}$

51. (01-8-31) Agar $\log_{0,2} 27 = a$ bo'lsa, $\log_{\sqrt{3}} \sqrt[6]{1,8}$ ni $a$ orqali ifodalang.

A) $a^2 - \dfrac{2}{3}$     B) $a^{-1} + 1,5$

C) $a^{-3} + 2$     D) $a^{-1} + \dfrac{2}{3}$

52. (02-8-12) Agar   $7^{\log_5 b} = 4$   bo'lsa, $b^{\log_5 \sqrt{7}}$ ni hisoblang.

A) $2$     B) $3$     C) $1$     D) $4$

53. (02-8-13) Agar $\lg 2 = a$ va $\lg 3 = b$ bo'lsa, $\log_9 20$ ni $a$ va $b$ orqali ifodalang.

A) $\dfrac{1+a}{2b}$   B) $\dfrac{1-a}{2b}$   C) $\dfrac{b}{1+2a}$   D) $\dfrac{b}{1-2a}$

**Yechish:** Boshqa asosga o'tish formulasi 7-ga ko'ra

$$\log_9 20 = \frac{\lg 20}{\lg 9} = \frac{\lg 10 + \lg 2}{\lg 3^2} = \frac{1+a}{2b}$$

ekanini hosil qilamiz. **Javob:**  $\dfrac{1+a}{2b}$  (A).

54. (02-9-38) Agar $\log_{\frac{b}{a}} \left(\dfrac{a^2}{b}\right) = -\dfrac{1}{2}$ bo'lsa, $\log_{a^2 b}(ab)$ ni hisoblang.

A) $-\dfrac{1}{4}$     B) $-1$     C) $1$     D) $0,8$

55. (02-10-27) $\lg 2 = a$ va $\log_2 7 = b$ bo'lsa, $\lg 56$ ni $a$ va $b$ orqali ifodalang.

A) $3a + ab$   B) $2a + 3b$   C) $3a + 2b$   D) $\dfrac{2a + 5b}{3}$

56. (03-4-37) Agar   $\log_a 8 = 3$ va $\log_b 243 = 5$ bo'lsa, $ab$ ning qiymatini toping.

A) $4$     B) $5$     C) $6$     D) $8$

57. (03-7-67) Agar $\lg 5 = a$ va $\lg 3 = b$ bo'lsa, $\log_{30} 8$ ni $a$ va $b$ orqali ifodalang.

A) $\dfrac{a}{2a + 3b}$   B) $\dfrac{b-3}{1-2a}$   C) $\dfrac{3a-3}{b+2}$   D) $\dfrac{3(1-a)}{1+b}$

58. (03-8-43) Agar $a = \log_5 4$ va $b = \log_5 3$ bo'lsa, $\log_{25} 12$ ni $a$ va $b$ orqali ifodalang.

A) $\dfrac{a+b}{2}$     B) $\dfrac{a-b}{4}$     C) $\dfrac{ab}{2}$     D) $\dfrac{a^2 + b}{4}$

## 12.2   Logarifmik tenglamalar

O'zgaruvchisi logarifm belgisi ostida kelgan tenglamalar logarifmik tenglamalar deyiladi. Masalan,

$$\log_2 x = 3, \quad \log_x 2 = 1, \quad \log_3(x^2 - 5x + 3) = 0.$$

Eng sodda logarifmik tenglamaning ko'rinishi $\log_a x = b$ bo'lib, uning yechimi $x = a^b$. Agar

$$\log_a f(x) = \log_a g(x) \qquad (1)$$

tenglamada $f(x) > 0$, $g(x) > 0$ shart bajarilganda, u

$$f(x) = g(x) \qquad (2)$$

tenglamaga teng kuchli bo'ladi. (1) tenglamadan (2) tenglamaga o'tganda chet ildizlar paydo bo'lishi mumkin. Chet ildizlarni aniqlash uchun, ildizlarni dastlabki tenglamaga qo'yib tekshirib ko'riladi. Quyidagi teng kuchliliklarni keltiramiz.

**1.** $\log_a f(x) = b \iff f(x) = a^b$.

**2.** $\log_a f(x) = \log_a g(x) \iff \begin{cases} f(x) = g(x), \\ f(x) > 0, \\ g(x) > 0. \end{cases}$

**3.** $\log_{f(x)} g(x) = b \iff \begin{cases} [f(x)]^b = g(x), \\ f(x) > 0, \quad f(x) \neq 1. \end{cases}$

**A.** $\log_a f(x) = b, \quad \log_a f(x) = \log_a g(x)$
**ko'rinishdagi tenglamalar**

1. $\log_5 x = 2$ tenglamani yeching.
A) $10$     B) $25$     C) $\sqrt{5}$     D) $32$

**Yechish:** 1-xossaga ko'ra $x = 5^2 = 25$.
**Javob:** $25$ (B).

2. $\lg x = -1$ tenglamani yeching.
A) $10$     B) $0,1$     C) $\sqrt{10}$     D) $-1$

3. $\ln x = \ln(8 - x)$ tenglamani yeching.
A) $e^2$     B) $0,8$     C) $\sqrt{8}$     D) $4$

4. $\log_2 x^2 = 4$ tenglamani yeching.
A) $4$     B) $\pm 2$     C) $\pm 4$     D) $16$

5. $\log_2 \log_3 x = 0$ tenglamani yeching.
A) $8$     B) $\pm 3$     C) $3$     D) $9$

6. $\lg(x - 4) = \lg(4 - x)$ tenglama nechta ildizga ega.
A) $1$     B) $2$     C) $0$     D) $4$

7. (00-7-33) $a$ ning qanday qiymatlarida

$$\lg x + \lg(x-6) = \lg(-a)$$

tenglama bitta ildizga ega bo'ladi?
A) 9      B) $a \in (-\infty; 0)$      C) 7      D) 6

**Yechish:** Berilgan tenglamaning aniqlanish sohasi $x > 6$ to'plamdan iborat. $\log_a b + \log_a c = \log_a bc$ formula yordamida tenglamaning chap qismini almashtiramiz. $\lg x(x-6) = \lg(-a)$. Uni potenserlaymiz. $x(x-6) = -a$ $(a < 0)$. Hosil bo'lgan tenglamani yechamiz.

$$x^2 - 6x + a = 0; \quad D = 36 - 4a = 4(9-a).$$

Bu tenglama $a \le 9$ da yechimga ega va uning ildizlari quyidagilar

$$x_{1,2} = \frac{6 \pm 2\sqrt{9-a}}{2} = 3 \pm \sqrt{9-a}$$

$x_1 = 3 - \sqrt{9-a} \le 3$ bo'lgani uchun u berilgan tenglamaning aniqlanish sohasiga kirmaydi. Demak, u chet ildiz. $x_2 = 3 + \sqrt{9-a}$ berilgan tenglamaning ildizi bo'lishi uchun $x_2 > 6$ bo'lishi kerak. Bu tengsizlikdan $a$ ni topamiz.

$$\sqrt{9-a} > 3 \Longleftrightarrow 9 - a > 9 \Longleftrightarrow a < 0.$$

Shunday qilib, $a \in (-\infty; 0)$ da berilgan tenglama bitta ildizga ega. **Javob:** $a \in (-\infty; 0)$ (B).

8. (98-9-34) Tenglamani yeching.

$$\lg(x^2 + 2x - 3) = \lg(x - 3)$$

A) 0      B) $-1$      C) $0; -1$      D) $\emptyset$

9. (99-6-26) Tenglamani yeching.

$$\log_{18} \log_2 \log_2 \left(-\frac{1}{x}\right) = 0$$

A) $-\frac{1}{16}$      B) $-\frac{1}{8}$      C) $\frac{1}{8}$      D) $-\frac{1}{4}$

10. (99-6-50) Tenglamani yeching.

$$\log_{\frac{1}{5}} \log_5 \sqrt{5x} = 0$$

A) $-5$      B) 1      C) 0      D) 5

11. (00-2-22) Agar

$$\begin{cases} 3^x \cdot 2^y = 972, \\ \log_{\sqrt{3}}(x-y) = 2 \end{cases}$$

bo'lsa, $xy$ ning qiymatini toping.
A) 14      B) 12      C) 10      D) 8

12. (01-3-26) Tenglama ildizlari yig'indisini toping.

$$\lg\left(3^{\sqrt{\frac{x^2-4x}{x-3}}} + 1\right) = 1$$

A) 10      B) 2      C) 8      D) 25

13. (01-7-25) Tenglamani yeching

$$\lg(3 + 2\lg(1+x)) = 0$$

A) 0      B) 1      C) $-15$      D) $-0,9$

**Yechish:** Berilgan tenglamani

$$\lg(3 + 2\lg(1+x)) = \lg 1$$

shaklda yozamiz. Bu yerdan $3 + 2\lg(1+x) = 1$ ni, undan esa $\lg(1+x) = -1$ ni olamiz. Ta'rifga ko'ra $1 + x = 10^{-1} \Longleftrightarrow x = -0,9$. Bevosita tekshirish ko'rsatadiki, $x = -0,9$ tenglamani qanoatlantiradi. **Javob:** $-0,9$ (D).

14. (01-7-26) Tenglamani yeching

$$\log_2 |x - 1| = 1$$

A) 3      B) 2      C) $-1$      D) $3; -1$

15. (01-9-41) Ushbu

$$\lg(5x - 2) = \lg(2 - 5x)$$

tenglamaning aniqlanish sohasini toping.
A) $(0,4; \infty)$    B) $\emptyset$    C) $(-\infty; 0,4)$    D) $\{2,5\}$

16. (02-3-35) Tenglama ildizlari ayirmasining moduli nechaga teng?

$$\log_3(4 \cdot 3^x - 1) = 2x + 1$$

A) 1      B) 2      C) 3      D) 0

17. (02-10-69) Tenglamani yeching.

$$\log_2(2^{2x} + 16^x) = 2\log_4 12$$

A) $\log_4 3$      B) $\log_2 3$      C) 2      D) $\log_4 6$

18. (02-10-71) Agar

$$\begin{cases} \log_2(x-y) = 1 \\ 2^x \cdot 3^{y+1} = 72 \end{cases}$$

bo'lsa, $x$ va $y$ ning o'rta proporsional qiymatini toping.
A) $\sqrt{3}$      B) 2      C) $\sqrt{2}$      D) $2\sqrt{2}$

**B.** $a^{\log_a f(x)} = g(x)$ **ko'rinishdagi tenglamalar**

19. (96-6-55) Tenglamaning ildizini toping.

$$3^{2\log_3 x} = 16$$

A) 3      B) $-4$      C) 4      D) $\pm 4$

**Yechish:** Tenglama $x > 0$ da aniqlanish. Logarifmning 5-xossasidan foydalanib, uni

$$3^{\log_3 x^2} = 16$$

shaklda yozamiz. Asosiy logarifmik ayniyatdan $x^2 = 16$ ni olamiz. Bu yerdan $x_1 = -4$, $x_2 = 4$ kelib chiqadi. $x_1 = -4$ tenglamaning aniqlanish sohasida yotmaydi. $x = 4$ tenglamani qanoatlantiradi. **Javob:** 4 (C).

20. (97-2-55) Tenglamaning ildizi 20 dan qancha katta?

$$4^{\log_4(x-5)} = 19$$

A) 6     B) 2     C) 4     D) 3

21. (97-8-40) Tenglamani yeching.

$$4^{2\log_4 x} = 25$$

A) 5     B) $\pm 5$     C) $-5$     D) 10

22. Tenglamani yeching.

$$2^{\log_4 x} = \frac{1}{4}$$

A) 1     B) 2     C) 0,5     D) 0,25

23. (01-5-12) Tenglamani yeching

$$x^{\log_x(x^2-1)} = 3$$

A) 2     B) 1     C) 3     D) 4

**C. 3-4 va 5-xossalariga oid tenglamalar**

24. (97-12-54) Tenglamaning ildizi 8 dan qanchaga kam?

$$\log_2(x+2) + \log_2(x+3) = 1$$

A) 7     B) 9     C) 10     D) 6

**Yechish:** Tenglamaning aniqlanish sohasi $x > -2$ to'plam. 3-xossadan foydalansak, tenglamani $\log_2(x+2)(x+3) = \log_2 2$ shaklda yozish mumkin. Bu yerdan

$$(x+2)(x+3) = 2 \Longleftrightarrow x^2 + 5x + 4 = 0$$

kelib chiqadi. Bu kvadrat tenglamaning ildizlari $x_1 = -4$, $x_2 = -1$ dir. $x_1 = -4$ tenglamaning aniqlanish sohasida yotmaydi. $x = -1$ tenglamani qanoatlantiradi. $8 - (-1) = 9$. **Javob:** 9 (B).

25. (00-3-38) Tenglamani yeching

$$\lg\left(\frac{1}{2} + x\right) = \lg\frac{1}{2} - \lg x$$

A) 2     B) $\frac{1}{2}$     C) 1     D) $-1$

26. (99-3-20) Tenglamani yeching

$$\lg\sqrt{x-5} + \lg\sqrt{2x-3} + 1 = \lg 30$$

A) $\frac{1}{2}$     B) 6     C) $\frac{1}{2}; 6$     D) $\frac{1}{2}; 8$

27. (02-12-50) Agar $\lg(x^2 + y^2) = 2$, $\lg 2 + \lg xy = \lg 96$ va $x > 0$ bo'lsa, $x+y$ yig'indining qiymatini toping.

A) 12     B) 14     C) 16     D) 18

28. (03-7-21) Tenglamani yeching.

$$\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$$

A) $\frac{3}{2}$     B) $\frac{5}{4}$     C) 2     D) 2,5

29. (99-6-28) Tenglamani yeching.

$$\log_2(54 - x^3) = 3\log_2 x$$

A) $-3$     B) 2     C) 1     D) 3

**Yechish:** 5-xossadan foydalansak, tenglamani $\log_2(54 - x^3) = \log_2 x^3$ shaklda yozish mumkin. Bu yerdan

$$54 - x^3 = x^3 \Longleftrightarrow 27 = x^3 \Longleftrightarrow x = 3$$

kelib chiqadi. $x = 3$ tenglamani qanoatlantiradi. **Javob:** 3 (D).

30. (00-2-24) $\log_5 x = 2\log_5 3 + 4\log_{25} 7$ bo'lsa, $x$ ni toping.

A) 441     B) 125     C) 256     D) 400

31. (00-3-28) Tenglamani yeching.

$$\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{\lg 4}{\lg 8}$$

A) 3     B) 4     C) 2     D) 1

32. (00-8-15) Tenglamani yeching.

$$\log_2(9^{x-1} + 7) = 2\log_2(3^{x-1} + 1)$$

A) 2     B) 1     C) 3     D) 4

33. (01-5-11) Tenglamani yeching

$$\log_a x - \log_{a^2} x + \log_{a^4} x = \frac{3}{4}$$

A) $a$     B) $a^2$     C) $a^4$     D) 2

34. (03-4-34) Agar

$$\log_4\frac{(2-x)^2}{(3-x)^3} = -3\log_4|3 - x|$$

bo'lsa, $x - 27$ ni hisoblang.

A) $-25$     B) $-29$     C) $-26$     D) $-24$

35. (03-11-13) Tenglamani yeching.

$$7^{(2x^2-5x-9)/2} = (\sqrt{2})^{3\log_2 7}$$

A) $-1,5; 1$     B) $1,5$     C) $-2,5; 4$     D) $-1,5; 4$

**D. 6-7 va 8-xossalarga oid tenglamalar**

36. (99-6-55) Tenglamani yeching.

$$\log_{\sqrt{2}} x + \frac{2}{\log_x 2} = 4$$

A) 2     B) 1     C) 3     D) 4

**Yechish:** 6-8-xossalardan foydalanib, tenglamani

$$\frac{1}{\frac{1}{2}}\log_2 x + 2\log_2 x = 4 \Longleftrightarrow 4\log_2 x = 4$$

shaklda yozamiz. Bu yerdan $\log_2 x = 1$ kelib chiqadi. Logarifm ta'rifidan $x = 2^1$ ni olamiz. **Javob:** 2 (A).

37. (98-11-45) Tenglama ildizlari ko'paytmasini toping.

$$\log_x 2 \log_{2x} 2 = \log_{4x} 2$$

A) 1    B) $\dfrac{1}{\sqrt{2}}$    C) $-\dfrac{1}{\sqrt{2}}$    D) $\dfrac{1}{2}$

38. (99-3-21) Tenglamani yeching.

$$\log_4(x + 12) \cdot \log_x 2 = 1$$

A) 4    B) $-3$    C) 2    D) 4;2

39. (02-3-36) Tenglama ildizlari ko'paytmasini toping.

$$\log_x 2 + \log_{4x} 4 = 1$$

A) 2    B) 4    C) 1    D) 8

### E. 11-xossa yoki logarifmlash yordamida yechiladigan tenglamalar

40. (97-6-59) Tenglamani yeching.

$$x^{\lg 9} + 9^{\lg x} = 6$$

A) 1    B) 10    C) $\sqrt{10}$    D) 2

**Yechish:** 11-xossadan foydalanib, tenglamani

$$9^{\lg x} + 9^{\lg x} = 6 \iff 2 \cdot 9^{\lg x} = 6 \iff 9^{\lg x} = 3$$

shaklda yozamiz. Bu tenglikning ikkala qismini 3 asosga ko'ra logarifmlaymiz

$$\log_3 9^{\lg x} = \log_3 3 \iff \lg x \cdot 2 = 1 \iff \lg x = \dfrac{1}{2}.$$

Logarifm ta'rifidan $x = 10^{1/2}$ ni olamiz. **Javob:** $\sqrt{10}$ (C).

41. (00-3-39) Tenglama ildizlari ko'paytmasini toping.

$$x^{\lg x - 1} = 100$$

A) 10    B) 20    C) 100    D) 1

42. (01-2-73) Tenglama ildizlari ko'paytmasini toping.

$$x^{(\lg x + 5)/3} = 10^{5 + \lg x}$$

A) 100    B) 10    C) 1    D) 0,01

43. (01-9-9) Tenglama ildizlarining o'rta proporsional qiymatini toping.

$$x^{3 - \log_3 x} = 9$$

A) $3\sqrt{3}$    B) $\sqrt{2}$    C) $2\sqrt{3}$    D) $\sqrt{3}$

44. (02-6-34) Tenglama ildizlari ko'paytmasini toping.

$$x^{2\lg x} = 10x^2$$

A) 1    B) 10    C) 100    D) 0,1

45. (02-7-9) Tenglamani yeching.

$$2 \cdot 3^{\log_7 x} + 3x^{\log_7 3} = 45$$

A) 49    B) 4    C) 7    D) 8

46. (03-4-36) Agar

$$\begin{cases} x^{\lg y} = 1000, \\ \log_y x = 3 \end{cases}$$

bo'lsa, $y$ ning qiymatini toping.

A) 10    B) 0,01    C) 10 yoki $0,1$    D) 30

### F. Yangi o'zgaruvchi kiritish yordamida yechiladigan tenglamalar

47. (96-10-38) Tenglama ildizlari ko'paytmasini toping.

$$\log_2^2 x - 5 \cdot \log_2 x + 6 = 0$$

A) 5    B) 6    C) 32    D) $\dfrac{3}{2}$

**Yechish:** Tenglamada $\log_2 x = y$ o'zgaruvchi kiritib, uni $y^2 - 5y + 6 = 0$ shaklda yozamiz. Bu kvadrat tenglamaning ildizlari $y_1 = 2$, $y_2 = 3$ lardir. Ularni belgilashga qo'yib, $\log_2 x = y_1 = 2$ va $\log_2 x = y_2 = 3$ larni olamiz. Bu yerdan $x_1 = 2^{y_1} = 4$, $x_2 = 2^{y_2} = 8$. Ularning ko'paytmasi $x_1 \cdot x_2 = 2^{y_1 + y_2} = 2^5 = 32$. **Javob:** 32 (C).

*Xulosa:* Agar $\log_a^2 x + b \cdot \log_a x + c = 0$ ko'rinishdagi tenglamaning ikkita ildizi borligi ma'lum bo'lsa, u holda $x_1 \cdot x_2 = a^{-b}$ bo'ladi. Bu xulosadan foydalanib 2-5-misollarni og'zaki yechib ko'ring.

48. (96-1-35) Tenglama ildizlari ko'paytmasini toping.

$$\lg^2 x - \lg x - 2 = 0$$

A) 1    B) $-2$    C) 10    D) 100

49. (96-9-86) Tenglama ildizlari ko'paytmasini toping.

$$\log_3^2 x - 4 \log_3 x + 3 = 0$$

A) 4    B) 81    C) 24    D) $9\dfrac{1}{3}$

50. (98-3-33) Tenglama ildizlari ko'paytmasini toping.

$$\log_3^2 x - 3 \log_3 x + 2 = 0$$

A) 6    B) 3    C) 27    D) 15

51. (98-6-24) Tenglama ildizlari ko'paytmasini toping.

$$\log_2^2 x - 4 \log_2 x - 1 = 0$$

A) 8    B) 4    C) 16    D) $\dfrac{1}{8}$

52. (00-1-47) Tenglama ildizlari yig'indisini toping.

$$\log_2^2 x - 2 \log_2 x^2 + 3 = 0$$

A) 4    B) $-4$    C) $-10$    D) 10

53. (02-11-33) Tenglama ildizlari ko'paytmasini to-ping.
$$\log_2^2 \frac{x}{2} - \log_2 4x = 3$$
A) 2     B) 4     C) 6     D) 8

54. (03-3-34) Tenglama ildizlari ko'paytmasini toping.
$$\log_{0,2}^2 \frac{x}{25} + \log_{0,2}^2 \frac{x}{5} = 1$$
A) $\dfrac{1}{125}$     B) 125     C) 25     D) $\dfrac{1}{25}$

## 12.3 Logarifmik tengsizliklar

$\log_a f(x) > \log_a g(x)$ yoki $\log_a f(x) < \log_a g(x)$ ko'rinishdagi tengsizliklar sodda logarifmik tengsizliklar deyiladi. Bunday tengsizliklarni yechish $y = \log_a x$ logarifmik funksiyaning $a > 1$ bo'lganda o'suvchi $0 < a < 1$ bo'lganda kamayuvchi ekanligiga asoslanadi, yani:

1. **Agar $a > 1$ bo'lsa,**

$$\log_a g(x) < \log_a f(x) \Longleftrightarrow 0 < g(x) < f(x).$$

2. **Agar $0 < a < 1$ bo'lsa,**

$$\log_a g(x) < \log_a f(x) \Longleftrightarrow 0 < f(x) < g(x).$$

1. (97-1-56) Tengsizlikni yeching.
$$\log_5(5 - 2x) \le 1$$
A) $(-\infty; 2,5)$     B) $(0; 2,5)$
C) $(-\infty; 2,5]$     D) $[0; 2,5)$

**Yechish:** Berilgan tengsizlikni quyidagicha $\log_5(5 - 2x) \le \log_5 5$ yozib olamiz. 1-xossaga ko'ra $0 < 5 - 2x < 5$. Bu yerdan $-5 < -2x < 0$ ni, bundan esa $2,5 > x > 0$ ni hosil qilamiz.
**Javob:** $(0; 2,5)$ (B).

2. Tengsizlikni yeching.
$$\log_2(5 - x) \le 1$$
A) $(-\infty; 5)$   B) $(0; 2,5)$   C) $(-\infty; 3)$   D) $(-\infty; 3]$

3. (97-3-33) Tengsizlikni yeching
$$\log_{\sqrt{3}}\Big(\frac{3x}{3x - 1,5}\Big) > 0$$
A) $(0,5; \infty)$   B) $(0; 0,5)$   C) $(-\infty; 0)$   D) $(0; \infty)$

4. (08-101-10) Tengsizlikni yeching:
$$3^{\log_3(4-x)} > 9$$
A) $-5 < x < 4$     B) $x < 4$
C) $x \le -5$     D) $x < -5$

5. (08-104-10) Tengsizlikni yeching:
$$9^{\log_9(x-4)} > 3$$
A) $4 < x < 7$     B) $x \ge 8$
C) $x \ge 9$     D) $x > 7$

6. (08-110-10) Tengsizlikni yeching:
$$5^{\log_5(x-7)} \le 4$$
A) $x \ge 11$     B) $7 \le x \le 11$
C) $x > 11$     D) $7 < x \le 11$

7. (99-5-14) Tengsizlikni yeching.
$$\log_{0,5}(x + 5)^4 > \log_{0,5}(3x - 1)^4$$
A) $(3; \infty)$    B) $(-\infty; 1)$    C) $(-\infty; 1) \cup (3; \infty)$
D) $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$

**Yechish:** 2-ga ko'ra $0 < (x + 5)^4 < (3x - 1)^4$. Bu yerdan $0 < (x + 5)^2 < (3x - 1)^2$ ekanini hosil qilamiz. Bu qo'sh tengsizlik

$$(x + 5)^2 - (3x - 1)^2 < 0, \quad x \ne -5$$

ga teng kuchli. Tengsizlikning chap qismini ko'paytuvchiga ajratib $(x + 5 - 3x + 1)(x + 5 + 3x - 1) < 0$ ni, bundan $(6 - 2x)(4x + 4) < 0$ ni, bu yerdan esa

$$2 \cdot 4(3 - x)(x + 1) < 0 \Longleftrightarrow (3 - x)(x + 1) < 0$$

ni hosil qilamiz. Bu tengsizlik oraliqlar usuli bilan oson yechiladi, uning yechimi $(-\infty; -1) \cup (3; \infty)$ to'plamdan iborat. $x \ne -5$ bo'lganligi uchun $-5$ ni yechimdan chiqarib, $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$ yechimni hosil qilamiz.
**Javob:** $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$ (D).

8. (96-3-87) Ushbu $\quad y = \log_2 \log_3 \sqrt{4x - x^2 - 2}$ funksiyaning aniqlanish sohasini toping.
A) $(1,5; 2,5)$     B) $(1; 3)$
C) $\{2\}$     D) $(-\infty; 1) \cup (3; \infty)$

9. (96-7-33) Tengsizlikni yeching.
$$\log_{\frac{1}{\sqrt{2}}} \frac{4x - 1}{4x + 8} < 0$$
A) $(\frac{1}{4}; \infty)$   B) $(2; \infty)$   C) $(-2; \infty)$   D) $(-\infty; -2)$

10. (97-1-24) Tengsizlikni yeching.
$$\log_{\frac{1}{\sqrt{3}}}(x - 5) + 2\log_{\sqrt{3}}(x - 5) < 4$$
A) $(6; 15)$    B) $(5; 14)$    C) $(5; 81)$    D) $(10; 20)$

**Yechish:** $\frac{1}{\sqrt{3}} = (\sqrt{3})^{-1}$ tenglikdan hamda logarifmning 6-xossasidan foydalansak, berilgan tengsizlikni quyidagicha

$$-\log_{\sqrt{3}}(x - 5) + 2\log_{\sqrt{3}}(x - 5) < 4$$

yozish mumkin. Tengsizlikning chap qismini soddalashtirib, $4 = \log_{\sqrt{3}} 9$ dan va 1-xossadan

$$\log_{\sqrt{3}}(x - 5) < \log_{\sqrt{3}} 9 \Longleftrightarrow 0 < x - 5 < 9$$

ni olamiz. Bu tengsizlikning barcha qismlariga 5 ni qo'shib $5 < x < 14$ ni olamiz. **Javob:** $(5; 14)$ (B).

11. (97-6-24) Tengsizlikni yeching.

$$\log_2(3 - 2x) - \log_{\frac{1}{8}}(3 - 2x) > \frac{4}{3}$$

A) $(-\infty; 0, 5)$　　B) $(-\infty; 1, 5)$
C) $(-4; -1)$　　　　D) $(0; 1)$

12. (97-11-24) Tengsizlikni yeching.

$$\log_{\frac{1}{3}}(x + 2) - \log_9(x + 2) > -\frac{3}{2}$$

A) $(0; 1)$　　B) $(1; \infty)$　　C) $(2; 3)$　　D) $(-2; 1)$

13. (98-2-37) Tengsizlikning barcha manfiy yechimlari to'plamini ko'rsating

$$\log_{0,2}(x^4 + 2x^2 + 1) > \log_{0,2}(6x^2 + 1)$$

A) $(-2; 2)$　　　　　B) $(-2; 0)$
C) $(-\infty; -2) \cup (0; 2)$　　D) $(-\infty; -2)$

14. (97-4-16) $x$ ning qanday qiymatlarida $y = 2 - \lg x$ funksiya manfiy qiymatlar qabul qiladi?
A) $x > 100$　B) $x > 10$　C) $x \leq 100$　D) $x < 10$

15. (98-3-32) $\log_5(3 - x) - \log_5 12 < 0$ tengsizlikni qanoatlantiradigan butun sonlar nechta?
A) cheksiz ko'p　　　B) 5　　　C) 10　　　D) 11

16. (99-2-33) $\log_{3x^2+5}(9x^4 + 27x^2 + 28) > 2$ tengsizlikning butun yechimini toping.
A) 1　　B) 2　　　C) $-1$　　D) 0

17. (99-3-17) Tengsizlikni yeching.

$$\log_2 \log_{\frac{1}{3}} \log_5 x > 0$$

A) $(0; \infty)$　　　　　B) $(-\infty; \sqrt[3]{5})$
C) $(-\infty; 0) \cup (\sqrt[3]{5}; \infty)$　　D) $(1 ; \sqrt[3]{5})$

**Yechish:** $0 = \log_2 1$ tenglikdan hamda 1-xossadan foydalanib $\log_{\frac{1}{3}} \log_5 x > 1 = \log_{\frac{1}{3}} \frac{1}{3}$ ni, 2-xossadan

$$0 < \log_5 x < \frac{1}{3} \Longleftrightarrow \log_5 1 < \log_5 x < \log_5 5^{\frac{1}{3}}$$

ni olamiz. Bu tengsizlikning yechimi $1 < x < \sqrt[3]{5}$ dir. **Javob:** $(1 ; \sqrt[3]{5})$ (D).

18. (99-6-9) Tengsizlikning eng katta butun yechimini toping.

$$\log_2(2x - 1) < 3$$

A) 2　　B) 5　　C) 1　　D) 4

19. (00-9-22) Tengsizlikni yeching.

$$\log_{\frac{1}{5}}(x + 17)^8 \leq \log_{\frac{1}{5}}(x + 13)^8$$

A) $(-15; -13) \cup (-13; \infty)$
B) $[-15; -13) \cup (-13; \infty)$
C) $(-13; \infty)$
D) $(-\infty; -17) \cup (-17; -13) \cup (-13; \infty)$

20. (96-12-87) Funksiyaning aniqlanish sohasini toping.

$$y = \log_2 \log_{\frac{1}{2}} \sqrt{4x - 4x^2}$$

A) $\{\frac{1}{2}\}$　　B) $(0; \frac{1}{2})$　　C) $(\frac{1}{2}; 1)$　　D) $(0; \frac{1}{2}) \cup (\frac{1}{2}; 1)$

21. (96-13-28) Ushbu　$y = \log_2(\log_3 \sqrt{4x - 4x^2})$ funksiyaning aniqlanish sohasini toping.
A) $\{\frac{1}{2}\}$　　　　　　　B) $\emptyset$
C) $(0; \frac{1}{2}) \cup (\frac{1}{2}; 1)$　　　D) $(-\infty; 0) \cup (1; \infty)$

22. (98-11-39) Tengsizlikni yeching.

$$\log_x 6 > \log_x 12$$

A) $(0; \frac{1}{2})$　　B) $(\frac{1}{2}; 1)$　　C) $(0; 1)$　　D) $(0; 2)$

23. (98-11-49) Tengsizlikni yeching.

$$x^{\log_2 x + 4} < 32$$

A) $(2^{-1}; 2)$　B) $(2^{-2}; 2)$　C) $(2^{-3}; 2)$　D) $(2^{-5}; 2)$

24. (98-4-39) $x$ ning nechta natural qiymatida

$$\frac{\sqrt{6 - x}}{\log_{\frac{1}{3}}(x - 3)} \geq 0$$

tengsizlik o'rinli bo'ladi?
A) bunday qiymatlar yo'q
B) 1　　　C) 2　　　D) 3

25. (01-1-24) Tengsizlikni yeching.

$$\log_2 x \leq \frac{2}{\log_2 x - 1}$$

A) $(0; 1)$　B) $(0; 4]$　C) $(0; 2)$　D) $(0; \frac{1}{2}] \cup (2; 4]$

26. (01-2-28) Ushbu　$\log_{x^2}(3 - 2x) > 1$　tengsizlikning butun yechimlari nechta?
A) 4　　B) 3　　C) 2　　D) 1

27. (00-4-41) Tengsizlikni yeching.

$$\log_{x^2}(x + 2) \leq 1$$

A) $(-\infty; -1] \cup [2; \infty)$
B) $(-\infty; -1) \cup [2; \infty)$
C) $(-2; -1) \cup (-1; 0) \cup (0; 1) \cup [2; \infty)$
D) $(-1; 2]$

28. (01-4-28) Tengsizlikni yeching.

$$\log_{1/3}(5 - 2x) > -2$$

A) $(-2; -1)$　　　B) $(-2; 2, 5)$
C) $(0; 2, 5)$　　　D) $(0; 2)$

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29. (01-6-38) Tengsizlikning butun yechimini toping.

$$\log_{\frac{1}{2}}(2^x - 128) \geq -7$$

A) 5    B) 6    C) 9    D) 8

**Yechish:** $\frac{1}{2} = 2^{-1}$ tenglikdan va logarifmning 6-xossasidan

$$-\log_2(2^x - 128) \geq -7\log_2 2$$

ni olamiz. Tengsizlikning har ikkala qismini $-1$ ga ko'paytirib, logarifmning 5-xossasidan hamda logarifm tengsizlikning 1-xossasidan foydalanib

$$\log_2(2^x - 128) \leq \log_2 2^7 \Longleftrightarrow 0 < 2^x - 128 \leq 2^7$$

ni hosil qilamiz. Bu tengsizlikning barcha qismlariga $128 = 2^7$ ni qo'shib

$$2^7 < 2^x \leq 2^8 \Longleftrightarrow 7 < x \leq 8$$

ga kelamiz. Yechimlar ichida faqat 8 butun soni bor. **Javob:** 8 (D).

30. (01-6-39) Nechta butun son

$$\begin{cases} \log_2 x^2 \geq 2 \\ \log_5 x^2 \leq 2 \end{cases}$$

tengsizliklar sistemasini qanoatlantiradi?
A) 6    B) 7    C) 9    D) 8

31. (01-7-28) Tengsizlikni yeching.

$$\log_{\frac{1}{3}}(x-1) - 2\log_{\frac{1}{9}}(2x-3) < 0$$

A) $(\frac{3}{2}; 2)$    B) $(-\infty; 2)$    C) $(2; \infty)$    D) $(\frac{3}{2}; \infty)$

32. (97-12-53) Tengsizlikning eng kichik butun musbat yechimini toping

$$\left(\frac{1}{2}\right)^{\log_{0,5} x(x-4)} > 0$$

A) 4    B) 6    C) 5    D) 5,5

33. (01-7-35) Tengsizlikning eng katta butun yechimini toping.

$$0,5^{\log_3(x^2+6x-7)} \geq \frac{1}{4}$$

A) 1    B) 2    C) 4    D) 1,5

34. (02-4-42) Tengsizlikni qanoatlantiruvchi eng kichik butun sonni toping.

$$-\lg x < 1$$

A) $-2$    B) $-1$    C) 10    D) 1

35. (02-4-43) Tengsizlikning eng kichik butun yechimini toping.

$$\log_{16}(3x+1) > \frac{1}{2}$$

A) $-2$    B) $-1$    C) 0    D) 2

36. (01-9-45) Tengsizlikni yeching.

$$\sqrt{4x^2 - 5x - 9} < \ln\frac{1}{2}$$

A) $(-5; 4)$    B) $(2; 3)$    C) $(-5; 2)$    D) $\emptyset$

**Yechish:** $\ln\frac{1}{2}$ logarifm asosida $e = 2,71\ldots$ soni bor. Logarifmning 4-xossasidan hamda $\ln 1 = 0$ tenglikdan

$$\sqrt{4x^2 - 5x - 9} < -\ln 2$$

ni olamiz. Tengsizlikning o'ng qismi $-\ln 2 < 0$, ya'ni manfiy, chap qismi esa manfiymas. Shuning uchun berilgan tengsizlik yechimga ega emas. **Javob:** $\emptyset$ (D).

37. (01-11-32) Tengsizlikning butun yechimlari yig'indisini toping.

$$\frac{x-5}{\log_x^2 3} < 0$$

A) 7    B) 8    C) 9    D) 10

38. (01-9-3) Tengsizlikni yeching.

$$\frac{2\log_2(3-2x)}{\log_2 0,1} < 0$$

A) $(-\infty; 1)$    B) $(-\infty; 1]$    C) $(1; \infty)$    D) $(-1; 2)$

39. (02-5-26) Tengsizlikni yeching.

$$2\log_8(x-2) - \log_8(x-3) > \frac{2}{3}$$

A) $(-\infty; 4)$    B) $\{2\} \cup (4; \infty)$
C) $(-\infty; 4) \cup (4; \infty)$    D) $(3; 4) \cup (4; \infty)$

40. (02-6-38) Tengsizlikni yeching.

$$(x^2 - 8x + 7) \cdot \sqrt{\log_5(x^2 - 3)} \leq 0$$

A) $[-2; 1] \cup [2; 7]$    B) $[2; 7] \cup \{-2\}$
C) $[1; 7]$    D) $[3; 7]$

**Yechish:** Arifmetik ildiz manfiymas, shuning uchun berilgan tengsizlik

$$\begin{cases} x^2 - 8x + 7 \leq 0 \\ \log_5(x^2 - 3) \geq 0 \end{cases} \Longleftrightarrow \begin{cases} (x-1)(x-7) \leq 0 \\ x^2 - 3 \geq 1 \end{cases}$$

sistemaga teng kuchli. Bu tengsizliklarning ikkalasini ham oraliqlar usuli bilan yechish qo'lay. 1-tengsizlikning yechimi $[1; 7]$ dan iborat. 2-tengsizlik

$$x^2 - 3 \geq 1 \Longleftrightarrow x^2 - 4 \geq 0 \Longleftrightarrow (x-2)(x+2) \geq 0$$

ko'rinishda bo'lib, uning yechimi $(-\infty; -2] \cup [2; \infty)$ dan iborat. Ularning umumiy qismi (kesishmasi) $[2; 7]$ to'plamdir. $x = -2$ da tengsizlikning ikkala qismi ham nolga aylanadi. Shuning uchun uni yechimlar to'plamiga kiritib $[2; 7] \cup \{-2\}$ ni olamiz. **Javob:** $[2; 7] \cup \{-2\}$ (B).

41. (02-9-35) Tengsizlikning yechimlaridan nechtasi butun sondan iborat?

$$\lg(x - 2) < 2 - \lg(27 - x)$$

A) 9     B) 8     C) 7     D) 6

42. (02-11-35) Tengsizlikning yechimlaridan iborat tub sonlarning yig'indisini toping.

$$\frac{2\log_3 x}{2 + \log_3 x} \le 1$$

A) 5     B) 6     C) 16     D) 17

43. (03-1-29) Tengsizlikni yeching.

$$\log_x 3 < 2$$

A) $(\sqrt{3}; \infty)$        B) $(3; \infty)$
C) $(0; 1) \cup (\sqrt{3}; \infty)$     D) $(0; 1)$

44. Tengsizlikning butun sonlardan iborat nechta yechimi bor?

$$\log_4(2 - \sqrt{x + 3}) < 1$$

A) 6     B) 4     C) 5     D) 3

# 13 - bob. Trigonometriya

## 13.1 Burchak va yoy, ularning o'lchovi

Tekislikda umumiy boshlang'ich nuqtaga ega bo'lgan ikki turli yarim to'g'ri chiziqdan iborat figura burchak deyiladi. Burchakning bu ta'rifi trigonometrik funksiyalarni o'rganish uchun qo'lay emas. Har qanday burchakni $OA$ nurni o'zining boshlang'ich $O$ nuqtasi atrofida burish (13.1-chizma) yordamida hosil qilish mumkin. To'la bir aylanish $360^0$ deb qabul qilingan.

$OA$ nur o'zining boshlang'ich $O$ nuqtasi atrofida aylanib burchak yasaganda uning $O$ nuqtadan boshqa istalgan nuqtasi aylana yoyini chizadi. Nurni boshlang'ich nuqtasi atrofida ikki yo'nalish bo'yicha aylantirish mumkin. Nurni soat strelkasiga qarama-qarshi yo'nalish bo'yicha aylantirishdan hosil bo'lgan burchak va yoyni musbat, soat strelkasi yo'nalishi bo'yicha aylantirishdan hosil bo'lgan burchak va yoyni manfiy deb atash qabul qilingan (13.3-chizma).

Burchak va yoylarning o'lchov birligi qilib 1 gradusli burchak ($1^0$) va 1 gradusli yoy qabul qilingan. 1 gradusli burchak bu nurni o'z boshlang'ich nuqtasi atrofida soat strelkasiga qarama-qarshi yo'nalishda to'la aylanishni $1/360$ ga burishdan hosil bo'lgan burchakdir. 1 gradusli yoy bu 1 gradusli burchakni hosil qilishda nurning nuqtasi chizadigan yoydir. Gradusning $1/60$ qismi bu minut ($1'$), minutning $1/60$ qismi bu sekund ($1''$) hisoblanadi. Uchi aylana markazida, tomonlari raduslardan iborat bo'lgan burchak markaziy burchak deyiladi. 13.4-chizmada $AOB$ markaziy burchak, $AB$ yoy markaziy burchakka mos yoydir. $\alpha$ markaziy burchakka mos $AB$ yoy uzunligining radusga nisbati $\alpha$ burchakning radian o'lchovi deyiladi. Agar $AB$ yoy uzunligi aylana radusiga teng bo'lsa, u holda $AOB$ burchak 1 radianli burchak deyiladi. Agar $\alpha$ burchakning radian o'lchovi $a$, unga mos yoy uzunligi $\ell$, radus $r$ bo'lsa, u holda $a = \ell/r$ bo'ladi. Agar yoy uzunligi ikki radusga teng bo'lsa, u holda burchak 2 radianga, agar yoy uzunligi yarim radusga teng bo'lsa, u holda burchak $0,5$ radianga teng bo'ladi.

Agar $OA$ nur $O$ nuqta atrofida chorak aylanish qilsa, to'g'ri ($90^0$) burchak hosil bo'ladi (13.2 $a-$ chizma), agar $OA$ nur $O$ nuqta atrofida yarim aylanish qilsa yoyiq ($180^0$) burchak hosil bo'ladi (13.2 $b-$ chizma).

Ma'lumki, aylana uzunligining radusga nisbati $2\pi$ ga teng. Demak, $360^0$ li burchakka $2\pi$ radian mos keladi. Xuddi shunday yoyiq burchakka $\pi$ burchak radiani mos keladi. To'g'ri burchakka esa $\pi/2$ burchak radiani mos keladi. Endi radiandan gradusga, gradusdan radianga o'tish formulalarini beramiz.

1. **$\alpha$ radiandan gradusga o'tish:** $\dfrac{180^0}{\pi} \cdot \alpha = \alpha^0.$

2. **$n^0$ gradusdan radianga o'tish:** $\dfrac{\pi}{180} \cdot n^0 = n.$

1. (97-2-31) $\dfrac{5\pi}{4}$ radian necha gradus bo'ladi?

   A) $220^0$    B) $230^0$    C) $225^0$    D) $240^0$

   **Yechish:** 1-formuladan foydalansak,

   $$\frac{5\pi}{4} = \frac{180^0}{\pi} \cdot \frac{5\pi}{4} = \frac{5 \cdot 180^0}{4} = 225^0.$$

   **Javob:** $225^0$ (C).

2. $\dfrac{\pi}{4}$ radian necha gradus bo'ladi?

   A) $20^0$    B) $30^0$    C) $45^0$    D) $60^0$

3. 1 radianga mos burchakni minut aniqligida toping.

   A) $57^020'$    B) $57^018'$    C) $57^017'$    D) $57^019'$

4. (97-12-30) $\dfrac{4\pi}{3}$ radian necha gradusga teng?

   A) $230^0$    B) $220^0$    C) $250^0$    D) $240^0$

5. $\dfrac{5\pi}{6}$ radian necha gradusga teng?

   A) $130^0$    B) $120^0$    C) $150^0$    D) $140^0$

6. (96-6-31) $240^0$ ning radian o'lchovini toping.

   A) $\dfrac{5\pi}{4}$    B) $\dfrac{2\pi}{3}$    C) $\dfrac{4\pi}{3}$    D) $\dfrac{6\pi}{3}$

   **Yechish:** 2-formuladan foydalansak,

   $$240^0 = \frac{\pi}{180} \cdot 240 = \frac{24 \cdot \pi}{18} = \frac{4\pi}{3}.$$

   **Javob:** $\dfrac{4\pi}{3}$ (C).

7. $15^0$ ning radian o'lchovini toping.

   A) $\dfrac{\pi}{15}$    B) $\dfrac{\pi}{6}$    C) $\dfrac{\pi}{12}$    D) $\dfrac{\pi}{9}$

8. (97-8-30) $216^0$ ning radian o'lchovini toping.

   A) $\dfrac{4\pi}{3}$    B) $\dfrac{5\pi}{4}$    C) $\dfrac{3\pi}{2}$    D) $\dfrac{6\pi}{5}$

9. $30^0$ ning radian o'lchovini toping.

   A) $\dfrac{\pi}{3}$    B) $\dfrac{\pi}{4}$    C) $\dfrac{\pi}{2}$    D) $\dfrac{\pi}{6}$

10. $45^0$ ning radian o'lchovini toping.

    A) $\dfrac{\pi}{3}$    B) $\dfrac{\pi}{4}$    C) $\dfrac{\pi}{2}$    D) $\dfrac{\pi}{6}$

11. (00-8-58) $72^0$ ning radian o'lchovini toping.

    A) $72$    B) $1$    C) $0,3$    D) $\dfrac{2\pi}{5}$

12. Radusi 3 ga teng bo'lgan aylananing $120^0$ li yoyining uzunligini toping.

    A) $\pi$    B) $2\pi$    C) $1,5\pi$    D) $1,6\pi$

## 13.2    Trigonometrik funksiyalar

Tekislikda to'g'ri burchakli $Oxy$ koordinatalar sistemasini qaraymiz. Markazi koordinatalar boshida va radusi birga teng bo'lgan aylana birlik aylana deyiladi. Uning tenglamasi $x^2 + y^2 = 1$ ko'rinishda bo'ladi. Koordinatalar boshi $O$ ni istalgan burchakning uchi qilib, yarim musbat abssissa o'qini $OA$ nur deb qabul qilamiz. $OA$ nur birlik aylanani koordinatalari $(1; 0)$ bo'lgan $P_0$ nuqtada kesib o'tadi. $OA$ nurni $\alpha$ burchakka buramiz, natijada $OB$ nurga ega bo'lamiz. $OB$ nur birlik aylanani $P_\alpha(x; y)$ nuqtada kesib o'tsin (13.5-chizma).

$\alpha$ burchakning sinusi deb $P_\alpha(x; y)$ nuqtaning ordinatasiga aytiladi va $\sin \alpha = y$ shaklda yoziladi. $\alpha$ burchakning kosinusi deb $P_\alpha(x; y)$ nuqtaning abssissasiga aytiladi va $\cos \alpha = x$ shaklda yoziladi. $\alpha$ burchakning tangensi deb $P_\alpha(x; y)$ nuqta ordinatasining abssissasiga nisbatiga aytiladi va quyidagicha yoziladi:

$$tg\alpha = \frac{y}{x} = \frac{\sin \alpha}{\cos \alpha}.$$

$\alpha$ burchakning kotangensi deb $P_\alpha(x; y)$ nuqta abssissasining ordinatasiga nisbatiga aytiladi, bu quyidagicha yoziladi:

$$ctg\alpha = \frac{x}{y} = \frac{\cos \alpha}{\sin \alpha}.$$

Sinus, kosinus, tangens va kotangenslar $\alpha$ burchakning funksiyalaridir. Bu funksiyalar trigonometrik funksiyalar deyiladi.

Tangens va kotangenslarning quyidagi ta'rifi qo'lay. $x = 1$ to'g'ri chiziq tangenslar o'qi, $y = 1$ to'g'ri chiziq esa kotangenslar o'qi deyiladi. $OA$ nurni $\alpha$ ($\alpha \neq 90^0 + 180^0 n$, $n \in Z$) burchakka buramiz, natijada bu nur yoki uning davomi tangenslar o'qini $P_\alpha(1; y)$ nuqtada kesib o'tadi (13.6-chizma). Bu nuqtaning ordinatasi $\alpha$ burchakning tangensi deyiladi, ya'ni $tg\alpha = y$.

| $\alpha$ | $0^0$ | $30^0$ | $45^0$ | $60^0$ | $90^0$ | $180^0$ | $270^0$ |
|---|---|---|---|---|---|---|---|
| $\sin\alpha$ | 0 | $\dfrac{1}{2}$ | $\dfrac{\sqrt{2}}{2}$ | $\dfrac{\sqrt{3}}{2}$ | 1 | 0 | $-1$ |
| $\cos\alpha$ | 1 | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{\sqrt{2}}{2}$ | $\dfrac{1}{2}$ | 0 | $-1$ | 0 |
| $tg\alpha$ | 0 | $\dfrac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | $-$ | 0 | $-$ |
| $ctg\alpha$ | $-$ | $\sqrt{3}$ | 1 | $\dfrac{1}{\sqrt{3}}$ | 0 | $-$ | 0 |

13.1-jadval.

Xuddi shunday kotangens funksiyaga ta'rif berish mumkin. $OA$ nurni $\alpha$ ($\alpha \neq n\pi$, $n \in Z$) burchakka buramiz, natijada bu nur yoki uning davomi kotangenslar o'qini $P_\alpha(x; 1)$ nuqtada kesib o'tadi (13.7-chizma). Bu nuqtaning abssissasi $\alpha$ burchakning kotangensi deyiladi, ya'ni $ctg\alpha = x$.

Ma'lumki, $M(x; y)$ nuqta tekislikning I choragida yotsa, uning koordinatalari $x > 0$, $y > 0$ shartni, II choragida yotsa, $x < 0$, $y > 0$ shartni, III choragida yotsa, $x < 0$, $y < 0$ shartni va IV chorakda yotsa, uning koordinatalari $x > 0$, $y < 0$ shartni qanoatlantiradi. Bulardan foydalanib trigonometrik funksiyalarning ishoralari uchun 13.2-jadvalni tuzish mumkin.

| Funksiya | I chor $(0;\frac{\pi}{2})$ | II chor $(\frac{\pi}{2};\pi)$ | III chor $(\pi;\frac{3\pi}{2})$ | IV chor $(\frac{3\pi}{2};2\pi)$ |
|---|---|---|---|---|
| $\sin\alpha$ | $+$ | $+$ | $-$ | $-$ |
| $\cos\alpha$ | $+$ | $-$ | $-$ | $+$ |
| $tg\alpha$ | $+$ | $-$ | $+$ | $-$ |
| $ctg\alpha$ | $+$ | $-$ | $+$ | $-$ |

13.2-jadval.

Trigonometrik funksiyalarning ta'riflaridan foydalanib, to'g'ridan-to'g'ri ba'zi burchaklarda trigonometrik funksiyalarning son qiymatlarini hisoblash mumkin. Masalan, $\alpha = 0^0$ bo'lsa, $P_0$ nuqtaning koordinatasi (1; 0) bo'ladi. Shuning uchun $\sin 0^0 = 0$, $\cos 0^0 = 1$, $tg0^0 = 0$. $ctg0^0$ esa mavjud emas. Agar $\alpha = 90^0$($\alpha = \dfrac{\pi}{2}$) bo'lsa, $P_\alpha$ nuqtaning koordinatasi (0; 1) bo'ladi. Shuning uchun $\sin 90^0 = 1$, $\cos 90^0 = 0$, $ctg0^0 = 0$. $tg0^0$ esa mavjud emas. Endi $\alpha = 45^0$ bo'lsin, u holda $P_\alpha$ nuqtaning koordinatasi $(x, x)$ ko'rinishda bo'ladi. Bu nuqta birlik aylanada yotgani uchun uning tenglamasini qanoatlantiradi, ya'ni $x^2 + x^2 = 1$. Bu yerdan $x = \dfrac{\sqrt{2}}{2}$ ekanligi kelib chiqadi. Demak,

$$\sin 45^0 = \frac{\sqrt{2}}{2},\ \cos 45^0 = \frac{\sqrt{2}}{2},\ tg45^0 = 1,\ ctg45^0 = 1.$$

Bundan tashqari $30^0$, $60^0$ va $180^0$ da ham trigonometrik funksiyalarning qiymatlarini hisoblash mumkin. Bu qiymatlarni quyidagi 13.1-jadval shaklida beramiz.

Sinus va kosinus funksiyalar grafiklari 13.8-chizmada, tangens va kotangensning grafiklari 13.9-chizmada tasvirlangan.

1. (00-2-32) Quyidagilardan qaysi biri musbat?
   A) $\cos 3$     B) $\sin 4$     C) $\sin 2$     D) $tg2$

   **Yechish:** $\dfrac{\pi}{2} < 2 < \pi$ bo'lgani uchun 2 radianga mos burchak II chorakda yotadi. Shuning uchun $\sin 2$ musbat bo'ladi. **Javob:** $\sin 2$ (C).

2. (97-12-32) Quyidagi sonlardan qaysi biri manfiy?
   A) $\sin 122^0 \cdot \cos 322^0$     B) $\cos 148^0 \cdot \cos 289^0$
   C) $tg196^0 \cdot ctg189^0$     D) $tg220^0 \cdot \sin 100^0$

3. (96-6-33) Quyidagi sonlardan qaysi biri musbat?

   $$M = \frac{\cos 320^0}{\sin 217^0}, \quad N = \frac{ctg187^0}{tg340^0}$$

   $$P = \frac{tg185^0}{\sin 140^0}, \quad Q = \frac{\sin 135^0}{ctg140^0}$$

   A) M     B) N     C) P     D) Q

4. (97-2-33) Quyidagi sonlardan qaysi biri manfiy?
   A) $tg247^0 \cdot \sin 125^0$     B) $ctg215^0 \cdot \cos 300^0$
   C) $tg135^0 \cdot ctg340^0$     D) $\sin 247^0 \cdot \cos 276^0$

5. Quyidagi sonlardan qaysi biri manfiy?

   1) $\cos 3,78;$   2) $ctg2,91;$   3) $tg4,45$

   A) 1; 2     B) 1     C) 2; 3     D) 1; 3

6. (96-3-56) Hisoblang.

   $$5\sin 90^0 + 2\cos 0^0 - 2\sin 270^0 + 10\cos 180^0$$

   A) $-3$     B) $-6$     C) $-1$     D) 9

   **Yechish:** 13.1-jadvaldan ko'rish mumkinki,

   $\sin 90^0 = \cos 0^0 = 1,\ \sin 270^0 = \cos 180^0 = -1.$

   Shu sababli berilgan ifoda

   $5\cdot1+2\cdot1-2\cdot(-1)+10\cdot(-1) = 5+2+2-10 = -1$

   ga teng bo'ladi. **Javob:** $-1$ (C).

7. Hisoblang.

   $$5\sin 30^0 + 7\cos 60^0 - 11\sin 90^0 + 10\cos 0^0$$

   A) 3     B) 6     C) 5     D) 9

8. Hisoblang.

   $$\sqrt{2}\sin 45^0 + 8\sqrt{3}\cos 30^0 - \sqrt{27}\,tg30^0 + ctg45^0$$

   A) 10     B) 11     C) 15     D) 13

9. (96-11-58) Hisoblang.

   $$\sin 180^0 + \sin 270^0 - ctg90^0 + tg180^0 - \cos 90^0$$

   A) $-1$     B) 0     C) 1     D) $-2$

10. (96-12-11) Hisoblang.

    $$3tg0^0 + 2\cos 90^0 + 3\sin 270^0 - 3\cos 180^0$$

    A) 6     B) 0     C) $-6$     D) 9

11. (01-2-58) Hisoblang.

    $$\cos\left(\frac{12\pi}{5}(\log_2 0,25 + \log_{0,25} 2)\right)$$

    A) 0     B) 1     C) $-1$     D) $\frac{1}{2}$

12. (96-12-58) Agar $\sin\alpha\cdot\cos\alpha < 0$ bo'lsa, $\alpha$ burchak qaysi chorakka tegishli?
    A) I yoki II          B) I yoki III
    C) I yoki IV          D) II yoki IV

13. (96-3-42) $P(3;0)$ nuqtani koordinata boshi atrofida $90^0$ ga burganda u qaysi nuqtaga o'tadi?
    A) $(-3;0)$   B) $(0;-3)$   C) $(3;3)$   D) $(0;3)$

    **Yechish:** Berilgan $P(3;0)$ nuqta abssissa o'qida yotadi. $OA$ nurni koordinata boshi atrofida $90^0$ ga burganda u ordinata o'qining musbat yo'nalishi bilan ustma-ust tushadi. Demak, $P$ nuqtaning koordinatasi $(0;3)$ bo'ladi. **Javob:** $(0;3)$ (D).

14. $P(3;0)$ nuqtani koordinata boshi atrofida $180^0$ ga burganda u qaysi nuqtaga o'tadi?
    A) $(-3;0)$   B) $(0;-3)$   C) $(3;3)$   D) $(0;3)$

15. (96-11-43) $P(-3;0)$ nuqtani koordinata boshi atrofida $90^0$ ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.
    A) $(3;0)$   B) $(0;-3)$   C) $(3;3)$   D) $(0;3)$

16. $P(0;2)$ nuqtani koordinata boshi atrofida $270^0$ ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.
    A) $(-2;0)$   B) $(0;-2)$   C) $(2;2)$   D) $(2;0)$

17. $P(1;1)$ nuqtani koordinata boshi atrofida $135^0$ ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.
    A) $(\sqrt{2};0)$   B) $(0;\sqrt{2})$   C) $(-\sqrt{2};0)$   D) $(-1;1)$

Sinus, kosinus, tangens va kotangens funksiyalarning asosiy xossalarini keltitamiz.

**1. Aniqlanish sohasi:**

2. $D(\sin) = R = (-\infty; \infty)$.

3. $D(\cos) = R$.

4. $D(tg) = R \setminus \{\frac{\pi}{2} + n\pi, \quad n \in Z\}$.

5. $D(ctg) = R \setminus \{n\pi, \quad n \in Z\}$.

**6. Qiymatlar sohasi:**

7. $E(\sin) = [-1; 1]$.

8. $E(\cos) = [-1; 1]$.

9. $E(tg) = R$.

10. $E(ctg) = R$.

**11. Davriyligi:**

12. $\sin(x + 2\pi) = \sin x, \quad T = 2\pi$.

13. $\cos(x + 2\pi) = \cos x, \quad T = 2\pi$.

14. $tg(x + \pi) = tgx, \quad T = \pi$.

15. $ctg(x + \pi) = ctgx, \quad T = \pi$.

**16. Juft-toqligi:**

17. $\sin(-x) = -\sin x$,     toq funksiya.

18. $\cos(-x) = \cos x$,       juft funksiya.

19. $tg(-x) = -tgx$,      toq funksiya.

20. $ctg(-x) = -ctgx$,     toq funksiya.

**21. Monotonligini jadval yordamida beramiz:**
↗ **o'suvchilik,** ↘ **kamayuvchilik belgisi.**

| Funksiya | I-chorak | II-chorak | III-chorak | IV-chorak |
|---|---|---|---|---|
| $\sin \alpha$ | ↗ | ↘ | ↘ | ↗ |
| $\cos \alpha$ | ↘ | ↘ | ↗ | ↗ |
| $tg\alpha$ | ↗ | ↗ | ↗ | ↗ |
| $ctg\alpha$ | ↘ | ↘ | ↘ | ↘ |

13.3-jadval.

### 13.2.1 Trigonometriyaning asosiy ayniyatlari

**1.** $sin^2x + cos^2x = 1 -$ **asosiy trigonometrik ayniyat.**

**2.** $1 + tg^2x = \frac{1}{cos^2x}$.

**3.** $1 + ctg^2x = \frac{1}{sin^2x}$.

**4.** $tgx \cdot ctgx = 1$.

1-4 ayniyatlardan yana bir nechta hosilaviy formulalar kelib chiqadi. Ularni 13.4-jadval ko'rinishida beramiz:

| Funks | $\sin \alpha$ | $\cos \alpha$ | $tg\alpha$ |
|---|---|---|---|
| $\sin \alpha$ | $\sin \alpha$ | $\pm\sqrt{1 - \cos^2 \alpha}$ | $\dfrac{\pm tg\alpha}{\sqrt{1 + tg^2\alpha}}$ |
| $\cos \alpha$ | $\pm\sqrt{1 - \sin^2 \alpha}$ | $\cos \alpha$ | $\dfrac{\pm 1}{\sqrt{1 + tg^2\alpha}}$ |
| $tg\alpha$ | $\dfrac{\pm \sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$ | $\dfrac{\sqrt{1 - \cos^2 \alpha}}{\pm \cos \alpha}$ | $tg\alpha$ |
| $ctg\alpha$ | $\dfrac{\sqrt{1 - \sin^2 \alpha}}{\pm \sin \alpha}$ | $\dfrac{\pm \cos \alpha}{\sqrt{1 - \cos^2 \alpha}}$ | $\dfrac{1}{tg\alpha}$ |

13.4-jadval.

Xususan $ctg\alpha$ ning qiymati berilgan bo'lsa, $\sin \alpha$, $\cos \alpha$ va $tg\alpha$ lar quyidagicha topiladi:

$$\sin \alpha = \frac{\pm 1}{\sqrt{1 + ctg^2\alpha}}; \quad \cos \alpha = \frac{\pm ctg\alpha}{\sqrt{1 + ctg^2\alpha}}; \quad tg\alpha = \frac{1}{ctg\alpha}.$$

Asosiy trigonometrik ayniyatlar va ulardan kelib chiqadigan hosilaviy formulalar yordamida trigonometrik funksiyalardan birining qiymatiga ko'ra boshqalarining qiymatini topish mumkin. Yuqorida keltirilgan formulalarda + yoki − ishora $\alpha$ burchak qaysi chorakda bo'lsa, trigonometrik funksiyaning shu chorakdagi ishorasi olinadi.

1. (98-5-48) Agar $\frac{\pi}{2} < \alpha < \pi$ va $\sin\alpha = \frac{3}{5}$ bo'lsa, $tg\alpha$ ni toping.
   A) $-\frac{4}{5}$     B) $-\frac{3}{4}$     C) $\frac{3}{4}$     D) $-\frac{3}{5}$

   **Yechish:** 13.4-jadvaldan

   $$tg\alpha = \frac{\pm \sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$$

   ekanligini olamiz. Bu tenglikda $\sin\alpha = \frac{3}{5}$ desak

   $$tg\alpha = \pm\frac{3}{5} : \sqrt{1 - \frac{9}{25}} = \pm\frac{3}{5} : \frac{4}{5} = \pm\frac{3}{4}$$

   bo'ladi. Berilgan $\alpha$ burchak II chorakda, tangens funksiyaning II chorakdagi qiymati manfiy. Shuning uchun $tg\alpha = -\frac{3}{4}$. **Javob:** $-\frac{3}{4}$ (B).

2. (99-7-47) Agar $\alpha \in (\frac{\pi}{2}; \pi)$ va $\sin\alpha = \frac{1}{4}$ bo'lsa, $ctg\alpha$ ni hisoblang.
   A) $-4$    B) $-\sqrt{17}$    C) $-\frac{1}{\sqrt{15}}$    D) $-\sqrt{15}$

3. (00-8-61) Agar $0 < \alpha < \frac{\pi}{2}$ va $tg\alpha = 2$ bo'lsa, $\cos \alpha$ ni toping.
   A) $\frac{5}{\sqrt{5}}$     B) $\frac{2}{\sqrt{5}}$     C) $\frac{\sqrt{5}}{5}$     D) $\sqrt{5}$

4. Agar $\alpha \in (\frac{3\pi}{2}; 2\pi)$ va $\cos\alpha = \frac{1}{4}$ bo'lsa, $tg\alpha$ ni hisoblang.
   A) $-\sqrt{15}$    B) $-\sqrt{17}$    C) $-\frac{1}{\sqrt{15}}$    D) $\sqrt{15}$

5. Agar $\pi < \alpha < \dfrac{3\pi}{2}$ va $ctg\alpha = -\sqrt{15}$ bo'lsa, $\sin\alpha$ ni toping.

A) $-\dfrac{5}{\sqrt{5}}$    B) $-\dfrac{2}{5}$    C) $-\dfrac{1}{4}$    D) $-\sqrt{5}$

6. (01-7-37) Agar $\dfrac{\pi}{2} < \alpha < \pi$ va $tg\alpha = -\dfrac{3}{4}$ bo'lsa, $\sin\alpha - \cos\alpha$ ning qiymatini toping.

A) $-\dfrac{1}{5}$    B) $\dfrac{1}{5}$    C) $\dfrac{7}{5}$    D) $-\dfrac{7}{5}$

7. Agar $tg\alpha = 2$ bo'lsa,

$$\frac{3\sin\alpha}{5\sin^3\alpha + 10\cos^3\alpha}$$

ning qiymati qanchaga teng bo'ladi?

A) $\dfrac{4}{5}$    B) $\dfrac{3}{5}$    C) $\dfrac{8}{15}$    D) $\dfrac{7}{15}$

**Yechish:** Qiymati izlanayotgan kasrning surat va maxrajini $\sin\alpha$ ga bo'lib

$$\frac{3}{5\sin^2\alpha + 10\cos^2\alpha\; ctg\alpha}$$

olamiz. 4-ayniyatdan $ctg\alpha = 2^{-1}$ kelib chiqadi. Bu qiymatni yuqoridagi tenglikka qo'yib

$$\frac{3}{5sin^2\alpha + 10cos^2\alpha\; ctg\alpha} = \frac{3}{5\sin^2\alpha + 5\cos^2\alpha} = \frac{3}{5}$$

ni olamiz. **Javob:** $\dfrac{3}{5}$ (B).

8. (98-4-17) Agar $tg\alpha = 3$ bo'lsa,

$$\frac{3\sin\alpha}{5\sin^3\alpha + 10\cos^3\alpha}$$

ning qiymati qanchaga teng bo'ladi?

A) $\dfrac{16}{39}$    B) $\dfrac{4}{9}$    C) $\dfrac{8}{15}$    D) $\dfrac{18}{29}$

9. (02-8-41) Agar $ctg\alpha = 2$ bo'lsa,

$$\frac{\sin^2\alpha - 2\cos^2\alpha}{3\sin\alpha \cdot \cos\alpha + \cos^2\alpha}$$

ifodaning qiymatini hisoblang.

A) $-0,7$    B) $-0,5$    C) $\dfrac{\sqrt{3}}{2}$    D) $-\dfrac{\sqrt{3}}{2}$

10. (01-9-23) Agar $\cos\alpha = \dfrac{\sqrt{3}}{2}$ bo'lsa,

$$\frac{1 - \sin^2\alpha + \cos^2\alpha \cdot sin\alpha}{1 + \sin\alpha}$$

ifodaning qiymatini toping.

A) $\dfrac{3}{4}$    B) $1,5$    C) $1\dfrac{1}{3}$    D) $1$

11. (98-11-97) Agar $tg\alpha + ctg\alpha = a$, $a > 0$ bo'lsa, $\sqrt{tg\alpha} + \sqrt{ctg\alpha}$ ning qiymati qanchaga teng bo'ladi?

A) $\sqrt{a+2}$   B) $a - 2$   C) $\sqrt{2} + \sqrt{a}$   D) $a + 2$

**Yechish:** $\sqrt{tg\alpha} + \sqrt{ctg\alpha} = x \geq 0$ belgilash olib, bu tenglikni kvadratga ko'taramiz:

$$tg\alpha + ctg\alpha + 2 = x^2 \iff x^2 = a + 2.$$

Bu yerdan $x = \pm\sqrt{a+2}$. $x$ ning nomanfiyligidan $x = \sqrt{a+2}$. **Javob:** $\sqrt{a+2}$ (A).

12. (98-8-62) Agar $tg\alpha + ctg\alpha = p$ bo'lsa, $tg^3\alpha + ctg^3\alpha$ ni $p$ orqali ifodalang.

A) $-p^3 - 3p$   B) $p^3 - 3p$   C) $p^3 + 3p$   D) $3p - p^3$

13. (99-6-33) Agar

$$\frac{2\sin x - \cos x}{2\cos x + \sin x} = 3$$

bo'lsa, $tgx$ ni hisoblang.

A) $7$    B) $-3$    C) $3$    D) $-7$

14. (99-6-51) Soddalashtiring.

$$\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha \cdot \cos^2\alpha$$

A) $-1$    B) $0$    C) $1$    D) $2$

**Yechish:** Ikki son yig'indisining kubi formulasidan

$$\sin^6\alpha + \cos^6\alpha = (\sin^2\alpha)^3 + (\cos^2\alpha)^3 =$$

$$= (\sin^2\alpha + \cos^2\alpha)(\sin^4\alpha - \sin^2\alpha \cdot \cos^2\alpha + \cos^4\alpha)$$

ni olamiz. Endi asosiy trigonometrik ayniyatga ko'ra, berilgan ifodani

$$\sin^4\alpha + 2\sin^2\alpha \cdot \cos^2\alpha + \cos^4\alpha = (\sin^2\alpha + \cos^2\alpha)^2$$

shaklda yozish mumkin. Yana bir marta asosiy ayniyatdan foydalansak, berilgan ifodaning qiymati 1 ga tengligini olamiz. **Javob:** 1 (C).

15. (99-8-80) $\sin^2 x + \cos^2 x + tg^2 x$ ni soddalashtiring.

A) $-\dfrac{1}{\cos^2 x}$   B) $-\dfrac{1}{\sin^2 x}$   C) $\dfrac{1}{\sin^2 x}$   D) $\dfrac{1}{\cos^2 x}$

16. (97-1-47) Soddalashtiring.

$$(ctg\alpha - \cos\alpha) \cdot \left(\frac{\sin^2\alpha}{\cos\alpha} + tg\alpha\right)$$

A) $\cos^2\alpha$   B) $tg\alpha$   C) $\dfrac{1}{\cos\alpha}$   D) $ctg^2\alpha$

17. (98-1-55) Soddalashtiring:

$$\frac{3\sin^2\alpha + \cos^4\alpha}{1 + \sin^2\alpha + \sin^4\alpha}$$

A) $2\sin\alpha$    B) $2$    C) $ctg^2\alpha$    D) $1$

18. (98-8-55) Soddalashtiring.

$$\frac{1 + \cos^2\alpha + \cos^4\alpha}{3\cos^2\alpha + \sin^4\alpha}$$

A) $3$    B) $2$    C) $1\dfrac{1}{2}$    D) $1$

19. (02-4-30) Soddalashtiring.

$$(tgx + ctgx)^2 - (tgx - ctgx)^2$$

A) $0$    B) $-4$    C) $-2$    D) $4$

20. (02-7-39) Soddalashtiring.

$$(\sin\alpha + \cos\alpha)^2 + (\sin\alpha - \cos\alpha)^2 - 2$$

A) 0     B) 4     C) $2\sin 2\alpha$     D) 1

21. (01-1-69) Agar $\sin x + \cos x = 0,5$ bo'lsa, $16(\sin^3 x + \cos^3 x)$ ni toping.

A) 8     B) 14     C) 11     D) 16

**Yechish:** Ikki son kublarining yig'indisi formulasidan va asosiy ayniyatdan foydalansak

$$\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x \cdot \cos x)$$

ni olamiz. Masala sharti $\sin x + \cos x = 0,5$ va

$$\sin x \cdot \cos x = \frac{1}{2}\left((\cos x + \sin x)^2 - 1\right)$$

tenglikdan foydalansak, berilgan ifoda ucun

$$16 \cdot 0,5(1 - \cos x \sin x) = 16 \cdot \frac{1}{2}\left(1 + \frac{1}{2} \cdot \frac{3}{4}\right) = 11$$

ni olamiz. **Javob:** 11 (C).

22. (00-2-45) Agar $ctg\alpha = \frac{13}{4}$ bo'lsa,

$$\frac{2\cos\alpha + \sin\alpha}{\cos\alpha - 2\sin\alpha}$$

kasrning qiymatini toping.

A) 6     B) 5     C) 6,2     D) 4,8

23. (03-8-55) Agar $\cos x = \frac{1}{\sqrt{10}}$ bo'lsa,

$$(1 + tg^2 x)(1 - \sin^2 x) - \sin^2 x$$

ifodaning qiymatini toping.

A) 0,1     B) 0,2     C) 0,3     D) $\frac{2}{\sqrt{10}}$

### 13.2.2 Trigonometrik funksiyalarning xossalari

1. $y = sinx$    va    $y = cosx$ **funksiyalarning eng kichik musbat davri $2\pi$ ga teng.**

2. $y = tgx$    va    $y = ctgx$ **funksiyalarning eng kichik musbat davri $\pi$ ga teng.**

3. **Agar $f(x)$ funksiyaning davri $T$ bo'lsa, u holda $af(x)+b$ funksiyaning davri $T$ bo'ladi.**

4. **Agar $f(x)$ funksiyaning davri $T$ bo'lsa, u holda $f(ax+b)$ funksiyaning davri $T/a$ bo'ladi.**

5. **Agar $f(x)$ va $g(x)$ funksiyalarning davrlari mos ravishda $T_1$ va $T_2$ bo'lsa, u holda $f(x)\pm g(x)$ funksiyaning davri $T = EKUK(T_1; T_2)$ bo'ladi.**

6. **Agar $g(x)$ davriy funksiya bo'lsa, u holda $f(g(x))$ funksiya ham davriy bo'ladi.**

7. $y = \cos x$ **funksiya juft.**

8. $y = \sin x,\ y = tgx,\ y = ctgx$ **funksiyalar toq.**

1. (03-1-17) $y = 2 + 3\cos(8x - 7)$ funksiyaning eng kichik musbat davrini toping.

A) $2\pi$     B) $\frac{\pi}{2}$     C) $\frac{\pi}{3}$     D) $\frac{\pi}{4}$

**Yechish:** 1 va 4-xossalarga ko'ra $y = \cos(8x - 7)$ funksiyaning eng kichik musbat davri

$$T = \frac{2\pi}{8} = \frac{\pi}{4}$$

ga teng. 3-xossaga ko'ra $y = 2 + 3\cos(8x - 7)$ funksiyaning eng kichik musbat davri ham $\frac{\pi}{4}$ ga teng bo'ladi. **Javob:** $\frac{\pi}{4}$ (D).

2. $y = 3 + \sin 2x$ funksiyaning davrini toping.

A) $\frac{2\pi}{3}$     B) $\pi$     C) $\frac{\pi}{3}$     D) $2\pi$

3. (98-10-102) $y = \sin(3x + 1)$ funksiyaning davrini toping.

A) $\frac{2\pi}{3}$     B) $\pi$     C) $\frac{\pi}{3}$     D) $2\pi$

4. (98-12-56) $y = \cos(\frac{5x}{2} - \frac{5}{2})$ funksiyaning eng kichik musbat davrini aniqlang.

A) $\frac{4\pi}{5}$     B) $2\pi$     C) $\pi$     D) $\frac{2\pi}{5}$

5. (01-11-35) Funksiyaning davrini toping.

$$f(x) = 2^{\sin x} + 3^{tgx}$$

A) $\frac{\pi}{2}$     B) $2\pi$     C) $3\pi$     D) $4\pi$

6. (96-12-105) Funksiyaning eng kichik musbat davrini toping.

$$y = tg\frac{x}{3} - 2\sin x + 3\cos 2x$$

A) $6\pi$     B) $3\pi$     C) $4\pi$     D) $9\pi$

**Yechish:** 4-xossaga ko'ra $y = tg\frac{x}{3}$ funksiyaning eng kichik musbat davri $T_1 = \pi : \frac{1}{3} = 3\pi$ ga, $y = \sin x$ funksiyaning davri $T_2 = 2\pi$ ga, $y = \cos 2x$ funksiyaning davri $T_3 = \frac{2\pi}{2} = \pi$ ga teng. Ularning eng kichik umumiy karralisi $K(T_1; T_2; T_3) = 6\pi$ bo'lganligi uchun berilgan funksiyaning eng kichik musbat davri $6\pi$ ga teng. **Javob:** $6\pi$ (A).

7. (96-9-48) Funksiyaning eng kichik davrini toping.

$$y = tg\frac{x}{3} - 2\sin\frac{x}{2} + 3\cos\frac{2}{3}x$$

A) $4\pi$     B) $6\pi$     C) $3\pi$     D) $12\pi$

8. (96-13-14) Funksiyaning eng kichik davrini toping.

$$y = ctg\frac{x}{3} + tg\frac{x}{2}$$

A) $6\pi$     B) $2\pi$     C) $3\pi$     D) $12\pi$

9. Qaysi funksiyaning davri eng kichik va u nechaga teng. $y_1 = tg3x$, $\qquad y_2 = ctg6x$, $y_3 = \cos(3x + 1)$, $\qquad y_4 = \sin(6x + 4)$

A) $y_1$; $\dfrac{2\pi}{3}$    B) $y_2$; $\dfrac{\pi}{6}$    C) $y_3$; $\dfrac{2\pi}{3}$    D) $y_4$; $\dfrac{\pi}{3}$

10. (02-3-41) Quyidagi funksiyalardan qaysi biri davriy emas?

A) $y = \sin\sqrt{x}$      B) $y = \sqrt{\sin x}$
C) $y = |\sin|x||$      D) $y = \sin^2 x$

11. (02-3-45) Quyidagi funksiyalardan qaysi birlari davriy emas?

1) $y = \sin\sqrt{x}$;    2) $y = \lg|\cos x|$
3) $y = x\cos x$;    4) $y = \sin^2 x + 1$

A) 1;3      B) 1;2      C) 2;3      D) 1;4

12. (03-4-38) Eng kichik musbat davrga ega bo'lgan funksiyani ko'rsating.

A) $y = \sin\dfrac{4}{3}x$      B) $y = \cos\dfrac{5}{3}x$

C) $y = ctg\dfrac{3}{2}x$      D) $y = \sin x\cos x$

13. (97-2-41) Quyidagi funksiyalardan qaysi biri juft?

A) $f(x) = \sin x + x^3$      B) $f(x) = \cos x\, tgx$

C) $f(x) = x^2 \cdot ctgx$      D) $f(x) = \dfrac{x^4 + x^2}{\cos x}$

**Yechish:** $f(x) = \sin x + x^3$ funksiya toq funksiyalar yig'indisi sifatida toq (10-bob, 7-xossa), $f(x) = \cos x\, tgx$ va $f(x) = x^2 \cdot ctgx$ funksiyalar juft va toq funksiyalar ko'paytmasi sifatida toq (10-bob, 4-xossa) funksiya bo'ladi. D javobdagi $f$ funksiya juft funksiyalar nisbati sifatida juft (10-bob, 3-xossa) funksiya bo'lagi. **Javob:** (D).

14. (97-4-17) $k$ ning qanday natural qiymatlarida $y = (\sin x)^{5k+4}$ juft funksiya bo'ladi?

A) toq qiymatlarida
B) juft qiymatlarida
C) 5 ga karrali qiymatlarida
D) barcha qiymatlarida

15. (00-10-72) Quyidagilardan qaysi biri toq funksiya?

A) $y = \lg\dfrac{1+x}{1-x}$      B) $y = \lg x^3$

C) $y = \cos(x - a)$      D) $y = \dfrac{a^x + a^{-x}}{2}$

16. (97-12-40) Quyidagi funksiyalardan qaysi biri toq?

A) $f(x) = \dfrac{\cos 5x + 1}{|x|}$      B) $f(x) = \dfrac{\sin^2 x}{x^2 - 1}$

C) $f(x) = \dfrac{\cos^2 x}{x(x^2 - 1)}$      D) $f(x) = \dfrac{\sin\frac{x}{2}}{x^3}$

17. (99-2-38) Quyidagi funksiyalardan qaysi biri toq funksiya?

A) $f(x) = \sin x \cdot tgx$      B) $f(x) = \cos x \cdot ctgx$
C) $f(x) = \sin|x|$      D) $f(x) = e^{|x|}$

18. (99-7-17) Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?

$y_1 = 2^x + 2^{-x}$;      $y_2 = 5^x - 5^{-x}$;
$y_3 = \sqrt{\sin x} + \sqrt{\cos x}$;    $y_4 = \cos x + x^3$.

A) $y_1$; $y_2$    B) $y_1$; $y_3$    C) $y_3$; $y_4$    D) $y_3$

**Yechish:** Ta'rif yordamida ko'rsatish mumkinki $y_1$ juft, $y_2$ toq funksiyadir. $y_4$ funksiya juft va toq funksiyalar yig'indisi sifatida juft ham, toq ham bo'lmagan funksiyadir (10-bob, 11-xossa). $y_3$ funksiyaning juft yoki toqligini aniqlamasdan ham testning to'g'ri javobini topishimiz mumkin. Buning uchun quyidagicha yo'l tutamiz. Qaysi javoblarda $y_1$ yoki $y_2$ funksiyalar qatnashgan bo'lsa, ular to'g'ri javob bo'la olmaydi. Chunki $y_1$ va $y_2$ funksiyalar juft yoki toqlik xossasini namayon qiladi. To'g'ri javobda $y_4$ albatta qatnashishi kerak. Demak, to'g'ri javob C ekan. **Javob:** $y_3$; $y_4$ (C).

19. Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?

$y_1 = \sin x + \cos(-x)$;      $y_2 = tgx - ctg^2x$;
$y_3 = \sin^2 x + \cos^2 x$;      $y_4 = \cos^5 x + \sin^3 x$.

A) $y_1$; $y_2$; $y_4$    B) $y_1$; $y_3$    C) $y_3$; $y_4$    D) $y_2$; $y_3$

20. Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?

$y_1 = \sin^3 x : \cos x$;      $y_2 = \cos(x - 2)$;
$y_3 = \sin^7 x + 7$;      $y_4 = \cos^5 x \cdot \sin^3 x$.

A) $y_1$; $y_2$; $y_4$    B) $y_1$; $y_3$    C) $y_3$; $y_4$    D) $y_2$; $y_3$

21. Juft ham, toq ham bo'lmagan funksiyani toping.

A) $1 + \sin x$      B) $1 - \cos^5 3x$
C) $1 + tg^2x$      D) $1 - ctg^2x$

22. (01-2-16) Quyidagi funksiyalardan qaysi biri toq funksiya?

A) $x^3 + x + 4$      B) $\cos x + tgx$
C) $\sin x + tgx - 1$      D) $\sin 2x \cdot \cos x / tg^2x$

23. (01-11-34)   $y_1 = \sin(\dfrac{\pi}{2} - x)$;   $y_2 = ctg^2x \sin^2 x$ va $y_3 = \lg(|x| + 1)$ funksiyalardan qaysi biri toq?

A) $y_1$      B) $y_2$      C) $y_3$
D) Berilgan funksiyalar ichida toq funksiya yo'q.

### 13.2.3 Qo'shish va keltirish formulalari

Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalari uchun formulalar keltiramiz. Quyida keltirilgan formulalar *"qo'shish teoremalari"* deb ham yuritiladi.

**1.** $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

**2.** $\sin(x - y) = \sin x \cos y - \cos x \sin y$.

**3.** $\cos(x + y) = \cos x \cos y - \sin x \sin y$.

**4.** $\cos(x - y) = \cos x \cos y + \sin x \sin y$.

**5.** $tg(x + y) = \dfrac{tgx + tgy}{1 - tgx\, tgy}$.

**6.** $tg(x - y) = \dfrac{tgx - tgy}{1 + tgx\, tgy}$.

Argumentlari $\dfrac{n\pi}{2} \pm \alpha$, $n \in Z$ ko'rinishidagi trigonometrik funksiyalarni $\alpha$ burchakning trigonometrik

funksiyalariga keltiruvchi formulalar *keltirish formulalari* deyiladi. Qo'shish teoremalaridan bevosita kelib chiqadigan va *"keltirish formulalari"* teb ataluvchi 13.4-jadvalni keltiramiz.

| **x** | $\sin x$ | $\cos x$ | **tgx** | **ctgx** |
|---|---|---|---|---|
| $\frac{\pi}{2} - \alpha$ | $\cos\alpha$ | $\sin\alpha$ | $ctg\alpha$ | $tg\alpha$ |
| $\frac{\pi}{2} + \alpha$ | $\cos\alpha$ | $-\sin\alpha$ | $-ctg\alpha$ | $-tg\alpha$ |
| $\pi - \alpha$ | $\sin\alpha$ | $-\cos\alpha$ | $-tg\alpha$ | $-ctg\alpha$ |
| $\pi + \alpha$ | $-\sin\alpha$ | $-\cos\alpha$ | $tg\alpha$ | $ctg\alpha$ |
| $\frac{3\pi}{2} - \alpha$ | $-\cos\alpha$ | $-\sin\alpha$ | $ctg\alpha$ | $tg\alpha$ |
| $\frac{3\pi}{2} + \alpha$ | $-\cos\alpha$ | $\sin\alpha$ | $-ctg\alpha$ | $-tg\alpha$ |
| $2\pi - \alpha$ | $-\sin\alpha$ | $\cos\alpha$ | $-tg\alpha$ | $-ctg\alpha$ |

13.5-jadval.

Keltirish formulalaridan foydalanib, trigonometrik funksiyalarning ixtoyoriy burchakdagi qiymatini o'tkir burchakning trigonometrik funksiyalari orqali ifodalash mumkin. Bu yerdan kelib chiqadiki, trigonometrik funksiyalarning qiymatlarini hisoblash uchun, ularning faqat o'tkir burchakdagi qiymatlarini bilish kifoya ekan.

Quyidagi qoidalar yodda saqlansa, keltirish formulalarini eslab qolish oson bo'ladi.

1) $\pi \pm \alpha$, $2\pi \pm \alpha$ (butun $\pi$ lar) burchak funksiyalaridan $\alpha$ burchak funksiyalariga o'tilayotganda funksiyaning nomi o'zgarmaydi.

2) $\frac{\pi}{2} \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$ (yarimtalik $\pi$ lar) burchak funksiyalaridan $\alpha$ burchak funksiyalariga o'tilayotganda funksiyaning nomi o'zgaradi.

3) $\alpha$ ni o'tkir burchak deb hisoblab $\alpha$ burchak funksiyasi oldiga, keltirilayotgan funksiyaning $\beta = \frac{\pi}{2} \pm \alpha$ ($\pi \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$ $2\pi \pm \alpha$) burchak yotgan chorakdagi ishorasi qo'yiladi.

Buni quyidagi misolda ko'rib chiqamiz.

*1-misol.* $\sin 300^0$ ni o'tkir burchakning trigonometrik funksiyasi orqali ifodalang.

**Yechish:** *1-usul.* $\sin 300^0$ ni $\sin(360^0 - 60^0)$ shaklda yozib olamiz. $360^0$ – bu $2\pi$ radianga mos burchak, shuning uchun $60^0$ o'tsak, sinus funksiyaning ismi o'zgarmaydi. Endi oldiga qo'yiladigan ishorani aniqlaymiz. $360^0 - 60^0 = 300^0$ li burchak, IV chorakka tegishli, IV chorakda sinus funksiya manfiy qiymatlar qabul qiladi, demak " $-$ " ishorasi qo'yiladi. Shunday qilib, $\sin(360^0 - 60^0) = -\sin 60^0$.

*2-usul.* $\sin 300^0 = \sin\frac{10\pi}{6}$ ni $\sin(\frac{3\pi}{2} + \frac{\pi}{6})$ shaklda yozib olamiz. $\frac{3\pi}{2}$ (bir yarim $\pi$) bo'lganligi uchun $\frac{3\pi}{2}$ tashlanganda sinus funksiyaning ismi o'zgarib kosinusga almashadi. Endi oldiga qo'yiladigan ishorani aniqlaymiz. $\frac{3\pi}{2} < \frac{3\pi}{2} + \frac{\pi}{6} < 2\pi$ burchak IV chorakka tegishli, IV chorakda sinus funksiyaning qiymati manfiy, demak " $-$ " ishorasi qo'yiladi. Shunday qilib, $\sin(\frac{3\pi}{2} + \frac{\pi}{6}) = -\cos\frac{\pi}{6}$. **Javob:** $\sin 300^0 = -\sin 60^0 = -\cos\frac{\pi}{6}$.

1. $\sin 15^0$ ni hisoblang.

A) $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$      B) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

C) $\frac{\sqrt{2}}{2}(\sqrt{3} - 1)$      D) $\frac{\sqrt{2}}{4}(\sqrt{6} - \sqrt{2})$

**Yechish:** $\sin 15^0$ ni $\sin(45^0 - 30^0)$ shaklda yozib, unga 2-formulani qo'llasak

$$\sin(45^0 - 30^0) = \sin 45^0 \cos 30^0 - \cos 45^0 \sin 30^0$$

ni olamiz. 13.1-jadvaldan sinus va kosinuslarning $30^0$ va $45^0$ dagi qiymatlarini topib, ayirmani hisoblaymiz:

$$\sin 15^0 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1).$$

**Javob:** $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$ (A).

2. $\cos 15^0$ ni hisoblang.

A) $\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$      B) $\frac{1}{4}(\sqrt{6} - \sqrt{2})$

C) $\frac{\sqrt{2}}{2}(\sqrt{3} + 1)$      D) $\frac{\sqrt{2}}{4}(\sqrt{3} + \sqrt{2})$

3. $\sin 75^0$ ni hisoblang.

A) $\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$      B) $\frac{1}{4}(\sqrt{6} - \sqrt{2})$

C) $\frac{\sqrt{2}}{2}(\sqrt{3} + 1)$      D) $\frac{\sqrt{2}}{4}(\sqrt{3} + \sqrt{2})$

4. $\cos 75^0$ ni hisoblang.

A) $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$      B) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

C) $\frac{\sqrt{2}}{2}(\sqrt{3} - 1)$      D) $\frac{\sqrt{2}}{4}(\sqrt{6} - \sqrt{2})$

5. $\cos 75^0 + \cos 105^0$ ni hisoblang.
A) 0    B) $\sqrt{3}$    C) 1    D) $\sqrt{2}$

6. (98-6-54) Hisoblang.

$$\cos 45^0 \cdot \cos 15^0 + \sin 45^0 \cdot \sin 15^0$$

A) $\frac{1}{2}$     B) $\frac{\sqrt{2}}{2}$     C) $\frac{\sqrt{3}}{2}$     D) 0

7. (01-6-27) Hisoblang.

$$\cos 15^0 + \sqrt{3}\sin 15^0$$

A) $\sqrt{3}$    B) $\sqrt{2}$    C) $\frac{\sqrt{2}}{2}$    D) $\frac{\sqrt{3}}{2}$

8. (00-5-31) $\sin 2010^0$ ni hisoblang.

A) $-\frac{1}{2}$     B) $-\frac{\sqrt{3}}{2}$     C) $\frac{\sqrt{3}}{2}$     D) 1

**Yechish:** $\sin 2010^0 = \sin(6 \cdot 360^0 - 150^0)$ tenglikdan hamda sinusning $2\pi$ davriy va toq funksiya ekanligidan foydalansak

$$\sin 2010^0 = -\sin 150^0 = -\sin(180^0 - 30^0)$$

ni olamiz. Bunga keltirish formulasini qo'llab, $\sin 30^0 = 0,5$ ekanligini hisobga olsak, $\sin 2010^0 = -0,5$ ni olamiz. **Javob:** $-\frac{1}{2}$ (A).

9. (01-2-17) $\cos 870^0$ ni hisoblang.

   A) $\dfrac{\sqrt{3}}{2}$    B) $-\dfrac{1}{2}$    C) $-\dfrac{\sqrt{3}}{2}$    D) $\dfrac{1}{2}$

10. (02-4-29) $\sin^2\left(3570^0\right)$ ni hisoblang.

   A) $0,2$    B) $0,3$    C) $0,25$    D) $0,35$

11. (03-2-41) $tg 1395^0$ ni hisoblang.

   A) $\sqrt{3}$    B) $-\dfrac{1}{\sqrt{3}}$    C) $-1$    D) $1$

12. $tg 105^0$ ni hisoblang.
   A) $-2-\sqrt{3}$         B) $2-\sqrt{3}$
   C) $-2+\sqrt{3}$         D) $2+\sqrt{3}$

   **Yechish:** $tg 105^0$ ni $tg(45^0 + 60^0)$ shaklda yozib, unga 5-formulani qo'llasak

   $$tg 105^0 = \frac{tg 45^0 + tg 60^0}{1 - tg 45^0 tg 60^0}$$

   ni olamiz. 13.1-jadvaldan tangensning $45^0$ va $60^0$ dagi qiymatlarini topib, nisbatni hisoblaymiz:

   $$tg 105^0 = \frac{1+\sqrt{3}}{1-\sqrt{3}} = -2-\sqrt{3}.$$

   **Javob:** $-2-\sqrt{3}$ (A).

13. $ctg 105^0$ ni hisoblang.
   A) $\dfrac{-1}{2+\sqrt{3}}$         B) $\dfrac{1}{2-\sqrt{3}}$
   C) $\dfrac{1}{\sqrt{3}-2}$         D) $\dfrac{1}{2+\sqrt{3}}$

14. $ctg 15^0$ ni hisoblang.
   A) $-2-\sqrt{3}$         B) $2-\sqrt{3}$
   C) $-2+\sqrt{3}$         D) $2+\sqrt{3}$

15. $tg 75^0$ ni hisoblang.
   A) $-2-\sqrt{3}$         B) $2-\sqrt{3}$
   C) $-2+\sqrt{3}$         D) $2+\sqrt{3}$

16. $ctg 75^0 + tg 15^0$ ni hisoblang.
   A) $-4-2\sqrt{3}$         B) $4-2\sqrt{3}$
   C) $-4+2\sqrt{3}$         D) $4+2\sqrt{3}$

17. (03-4-23) Hisoblang.

   $$(tg 60^0 \cos 15^0 - \sin 15^0) \cdot 7\sqrt{2}$$

   A) $16$    B) $12$    C) $18$    D) $14$

18. (96-1-54) $2tg(-765^0)$ ning qiymatini toping.

   A) $-\sqrt{2}$    B) $\dfrac{2}{\sqrt{3}}$    C) $-2$    D) $4$

   **Yechish:** $765^0 = 2 \cdot 360^0 + 45^0$ tenglikni va tangensning toq funksiya hamda $\pi$ davriy ekanligini hisobga olsak

   $$2tg(-765^0) = -2tg(2 \cdot 360^0 + 45^0) = -2tg 45^0$$

   ni olamiz. $tg 45^0 = 1$ dan $2tg(-765^0) = -2$ kelib chiqadi. **Javob:** $-2$ (C).

19. (97-11-43) Hisoblang.

   $$cos(-45^0) + \sin(315^0) + tg(-855^0)$$

   A) $0$    B) $\sqrt{2}-1$    C) $1+\sqrt{3}$    D) $1$

20. (98-5-49) $tg 1^0 \cdot tg 2^0 \cdots tg 88^0 \cdot tg 89^0$ ni hisoblang.

   A) $0$    B) $\dfrac{1}{2}$    C) $1$    D) $\sqrt{3}$

21. (98-10-36) Hisoblang.

   $$tg\frac{\pi}{6} \cdot \sin\frac{\pi}{3} \cdot ctg\frac{5\pi}{4}$$

   A) $1,5$    B) $0,5$    C) $-\dfrac{1}{2}$    D) $\dfrac{\sqrt{3}}{4}$

22. (02-3-76) Hisoblang.

   $$\sin\frac{\pi}{9} - \cos\frac{7\pi}{18}$$

   A) $0$    B) $\dfrac{1}{2}$    C) $\dfrac{\sqrt{2}}{2}$    D) $\dfrac{\sqrt{3}}{2}$

23. (03-12-77) Hisoblang.

   $$\left(\left(tg^2\frac{7\pi}{24} - tg^2\frac{\pi}{24}\right) : \left(1 - tg^2\frac{7\pi}{24} \cdot tg^2\frac{\pi}{24}\right)\right)^2$$

   A) $\dfrac{1}{9}$    B) $9$    C) $\dfrac{1}{3}$    D) $3$

24. (96-3-111) Agar $tg(\frac{\pi}{4} - \alpha) = 2$ bo'lsa, $tg\alpha$ ni toping.

   A) $3$    B) $-3$    C) $\dfrac{1}{3}$    D) $-\dfrac{1}{3}$

   **Yechish:** $tg\alpha = tg\left(\frac{\pi}{4} - (\frac{\pi}{4} - \alpha)\right)$ tenglikning o'ng tomoniga 6-formulani qo'llasak

   $$tg\alpha = \frac{tg\frac{\pi}{4} - tg\left(\frac{\pi}{4} - \alpha\right)}{1 + tg\frac{\pi}{4}tg\left(\frac{\pi}{4} - \alpha\right)} = \frac{1-2}{1+2} = -\frac{1}{3}.$$

   **Javob:** $-\dfrac{1}{3}$ (D).

25. (96-9-46) Agar $tg(\frac{\pi}{4} - \alpha) = 2$ bo'lsa, $ctg\alpha$ ni toping.
   A) $3$    B) $\dfrac{1}{3}$    C) $-\dfrac{1}{3}$    D) $-3$

26. (01-1-42) Agar

   $$tg\alpha = \frac{1}{2}, \quad tg\beta = \frac{1}{3}, \quad \pi < \alpha + \beta < 2\pi$$

   bo'lsa, $\alpha + \beta$ ning qiymatini toping.
   A) $\dfrac{7\pi}{3}$    B) $\dfrac{5\pi}{3}$    C) $\dfrac{5\pi}{4}$    D) $\dfrac{7\pi}{4}$

27. (97-1-60) Agar

   $$\begin{cases} tg(x+y) = 3 \\ tg(x-y) = 2 \end{cases}$$

bo'lsa, $tg2x$ ni hisoblang.

A) 5     B) 2,5     C) 1     D) $-1$

**Yechish:** Agar $tg2x = tg\big((x+y)+(x-y)\big)$ tenglikning o'ng tomoniga 5-formulani qo'llasak

$$tg2x = \frac{tg(x+y)+tg(x-y)}{1-tg(x+y)\cdot tg(x-y)} = \frac{3+2}{1-3\cdot 2} = -1.$$

**Javob:** $-1$ (D).

28. (97-6-60) Agar

$$\begin{cases} tg(\alpha+\beta) = 5 \\ tg(\alpha-\beta) = 3 \end{cases}$$

bo'lsa, $tg2\beta$ ni hisoblang.

A) 15     B) 8     C) $\dfrac{1}{8}$     D) 1

29. (97-6-68) Agar

$$\begin{cases} 2tg\alpha = 3+\sqrt{x}, \\ 2tg\beta = 3-\sqrt{x} \\ 4(\alpha+\beta) = \pi \end{cases}$$

bo'lsa, $x$ ni toping.

A) $\dfrac{\pi}{3}$     B) $-17$     C) $-\dfrac{\pi}{6}+\pi k$     D) 17

30. (98-6-48) Agar $tg(x+y) = 5$ va $tgx = 3$ bo'lsa, $tgy$ ni toping.

A) 2     B) $\dfrac{1}{8}$     C) 8     D) $\dfrac{1}{2}$

**Yechish:** Agar $tgy = tg\big((x+y)-x\big)$ tenglikning o'ng tomoniga 6-formulani qo'llasak

$$tgy = \frac{tg(x+y)-tgx}{1+tg(x+y)\cdot tgx} = \frac{5-3}{1+5\cdot 3} = \frac{1}{8}.$$

**Javob:** $\dfrac{1}{8}$ (B).

31. (00-1-29) Agar $\alpha = -45^0$ va $\beta = 15^0$ bo'lsa, $\cos(\alpha+\beta)+2\sin\alpha\cdot\sin\beta$ ning qiymatini toping.

A) $-\dfrac{1}{2}$     B) $\dfrac{\sqrt{3}}{2}$     C) $-\dfrac{\sqrt{3}}{2}$     D) $\dfrac{1}{2}$

32. (02-6-46) Agar

$$\begin{cases} \cos x\cdot\cos y = \dfrac{1}{6} \\ tgx\cdot tgy = 2 \end{cases}$$

bo'lsa, $\cos(x+y)$ ni toping.

A) $\dfrac{1}{2}$     B) $\dfrac{1}{3}$     C) $-\dfrac{1}{2}$     D) $-\dfrac{1}{6}$

33. (01-1-49) Agar $\sin\alpha = -\dfrac{1}{3}$ va $\cos\beta = -\dfrac{1}{2}$ bo'lsa, $\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)$ ning qiymatini toping.

A) $-\dfrac{23}{36}$     B) $\dfrac{23}{36}$     C) $\dfrac{3}{4}$     D) $-\dfrac{3}{4}$

34. (03-1-25) Agar

$$\begin{cases} 3\sin x\cdot\cos y = -1 \\ 3\cos x\cdot\sin y = 2 \end{cases}$$

bo'lsa, $ctg(x-y)$ ni hisoblang.

A) 0     B) 1     C) $-\dfrac{1}{2}$     D) $\dfrac{1}{2}$

35. (00-1-26) Soddalashtiring.

$$\frac{\sin(\frac{\pi}{2}-\alpha)\cdot\cos(\pi+\alpha)}{ctg(\pi+\alpha)\cdot tg(\frac{3\pi}{2}-\alpha)}$$

A) $-\sin^2\alpha$     B) $-\sin^2\alpha\cdot tg^2\alpha$

C) $-\cos^2\alpha$     D) $\cos^2\alpha\cdot ctg^2\alpha$

**Yechish:** Berilgan kasr suratiga keltirish formulasini qo'llab $\cos\alpha(-\cos\alpha) = -\cos^2\alpha$ ni olamiz. Kasr maxrajiga ham keltirish formulasini qo'llab $ctg\alpha\cdot ctg\alpha = ctg^2\alpha$ ni olamiz. U holda berilgan kasr

$$-\cos^2\alpha\cdot tg^2\alpha = -\cos^2\alpha\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = -\sin^2\alpha$$

ga teng bo'ladi. **Javob:** $-\sin^2\alpha$ (A).

36. (96-1-57) Soddalashtiring.

$$\frac{\cos(\alpha+\beta)+2\sin\alpha\cdot\sin\beta}{\sin(\alpha+\beta)-2\cos\beta\cdot\sin\alpha}$$

A) $ctg(\beta-\alpha)$     B) $tg(\alpha-\beta)$

C) $2tg(\alpha+\beta)$     D) $2ctg(\alpha-\beta)$

37. (01-11-24) Soddalashtiring.

$$\frac{\sin\alpha+\cos\alpha}{\sqrt{2}\cos(\frac{\pi}{4}-\alpha)}$$

A) 1,6     B) $ctg(\dfrac{\pi}{4}+\alpha)$     C) 1,5     D) 1

38. (99-1-41) Soddalashtiring.

$$tg\alpha\cdot ctg(\pi+\alpha)+ctg^2\alpha$$

A) $\dfrac{1}{\sin^2\alpha}$     B) $\dfrac{1}{\cos^2\alpha}$     C) $tg\alpha$     D) $tg^2\alpha$

39. (00-8-60) Soddalashtiring.

$$\frac{tg(\pi-\alpha)}{\cos(\pi+\alpha)}\cdot\frac{\sin(\frac{3\pi}{2}+\alpha)}{tg(\frac{3\pi}{2}+\alpha)}$$

A) $tg^2\alpha$     B) $ctg^2\alpha$     C) $-tg^2\alpha$     D) $\dfrac{1}{tg\alpha}$

### 13.2.4   Ikkilangan va yarim burchak formulalari

Qo'shish teoremalaridan ikkilangan burchakning trigonometrik funksiyalari uchun quyidagilar kelib chiqadi.

**1.** $\sin 2x = 2\sin x\cos x$.

**2.** $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = $
$= 1 - 2\sin^2 x$.

**3.** $\sin 2x = \dfrac{2tgx}{1+tg^2x}$.

**4.** $\cos 2x = \dfrac{1-tg^2x}{1+tg^2x}$.

**5.** $tg2x = \dfrac{2tgx}{1 - tg^2x}$.

Ikkilangan burchak formulalaridan yarim burchak uchun formulalar kelib chiqadi.

**6.** $\sin^2\dfrac{x}{2} = \dfrac{1 - \cos x}{2}$, $\quad \sin\dfrac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{2}}$.

**7.** $\cos^2\dfrac{x}{2} = \dfrac{1 + \cos x}{2}$, $\quad \cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1 + \cos x}{2}}$.

**8.** $tg\dfrac{x}{2} = \dfrac{\sin x}{1 + \cos x} = \dfrac{1 - \cos x}{\sin x}$.

**9.** $ctg\dfrac{x}{2} = \dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$.

**10.** $tg^2\dfrac{\alpha}{2} = \dfrac{1 - \cos\alpha}{1 + \cos\alpha}$, $\quad ctg^2\dfrac{\alpha}{2} = \dfrac{1 + \cos\alpha}{1 - \cos\alpha}$.

1. (97-6-51) Hisoblang.

$$\sin\frac{\pi}{8} \cdot \cos^3\frac{\pi}{8} - \sin^3\frac{\pi}{8} \cdot \cos\frac{\pi}{8}$$

A) 0 $\qquad$ B) 1 $\qquad$ C) 2 $\qquad$ D) $\dfrac{1}{4}$

**Yechish:** 1 va 2-dan, ya'ni $\sin\alpha \cdot \cos\alpha = \dfrac{1}{2}\sin 2\alpha$ va $\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$ ekanligidan

$$\sin\frac{\pi}{8} \cdot \cos^3\frac{\pi}{8} - \sin^3\frac{\pi}{8} \cdot \cos\frac{\pi}{8} = \sin\frac{\pi}{8} \cdot \cos\frac{\pi}{8}$$

$$\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right) = \frac{1}{2}\sin\frac{\pi}{4}\cos\frac{2\pi}{8} =$$

$$= \frac{1}{2}\sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4} = \frac{1}{4}\sin\frac{\pi}{2} = \frac{1}{4}$$

ni hosil qilamiz. **Javob:** $\dfrac{1}{4}$ (D).

2. (97-1-52) Hisoblang.

$$\sin\frac{\pi}{16} \cdot \cos^3\frac{\pi}{16} - \sin^3\frac{\pi}{16} \cdot \cos\frac{\pi}{16}$$

A) $\dfrac{\sqrt{2}}{2}$ $\qquad$ B) $\dfrac{\sqrt{2}}{3}$ $\qquad$ C) $\dfrac{\sqrt{2}}{4}$ $\qquad$ D) $\dfrac{\sqrt{2}}{8}$

3. (99-6-53) Hisoblang.

$$\cos\frac{\pi}{7} \cdot \cos\frac{4\pi}{7} \cdot \cos\frac{5\pi}{7}$$

A) $-\dfrac{1}{8}$ $\qquad$ B) $\dfrac{1}{4}$ $\qquad$ C) $\dfrac{1}{2}$ $\qquad$ D) $\dfrac{1}{8}$

**Yechish:** Keltirish formulasi $\cos\alpha = -\cos(\pi - \alpha)$ dan foydalanib berilgan ifodada quyidagicha shakl almashtirish qilamiz:

$$A = \cos\frac{\pi}{7} \cdot \cos\frac{4\pi}{7} \cdot \cos\frac{5\pi}{7} = -\cos\frac{\pi}{7} \cdot$$

$$\cdot \cos\frac{4\pi}{7} \cdot \cos(\pi - \frac{5\pi}{7}) = -\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7}\cos\frac{4\pi}{7}.$$

Tenglikni $8\sin\dfrac{\pi}{7}$ ga ko'paytirib va 1-formulani bir necha marta qo'llab

$$8A\sin\frac{\pi}{7} = -8\cos\frac{4\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\sin\frac{\pi}{7} =$$

$$= -4\cos\frac{4\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\sin\frac{2\pi}{7} = -2\cos\frac{4\pi}{7}\sin\frac{4\pi}{7} =$$

$$= -\sin\frac{8\pi}{7}$$ ni hosil qilamiz. Demak,

$$A = -\frac{\sin\dfrac{8\pi}{7}}{8\sin\dfrac{\pi}{7}} = -\frac{\sin(\pi + \dfrac{\pi}{7})}{8\sin\dfrac{\pi}{7}} = \frac{1}{8}.$$

**Javob:** $\dfrac{1}{8}$ (D).

4. (98-1-54) Agar $tg\alpha = -\dfrac{1}{4}$ bo'lsa,

$$\frac{2\cos^2\alpha - \sin 2\alpha}{2\sin^2\alpha - \sin 2\alpha}$$

ni hisoblang.

A) $-4$ $\qquad$ B) 4 $\qquad$ C) $\dfrac{1}{4}$ $\qquad$ D) $-\dfrac{1}{2}$

5. (98-10-101) Agar $tg\alpha = \dfrac{1}{2}$ bo'lsa, $tg2\alpha$ ni toping.

A) $\dfrac{5}{3}$ $\qquad$ B) $\dfrac{4}{3}$ $\qquad$ C) $\dfrac{3}{4}$ $\qquad$ D) $\dfrac{3}{5}$

6. (98-8-54) Agar $ctg\alpha = \dfrac{1}{8}$ bo'lsa,

$$\frac{\sin 2\alpha + 2\sin^2\alpha}{\sin 2\alpha + 2cos^2\alpha}$$

ni hisoblang.

A) $\dfrac{1}{8}$ $\qquad$ B) 8 $\qquad$ C) $\dfrac{1}{4}$ $\qquad$ D) 4

7. (96-10-35) Agar $\cos\alpha = \dfrac{1}{5}$ bo'lsa,

$$\frac{2\sin\alpha + \sin 2\alpha}{2\sin\alpha - \sin 2\alpha}$$

ni hisoblang.

A) 0,5 $\qquad$ B) 1,5 $\qquad$ C) 3 $\qquad$ D) $\dfrac{2}{3}$

8. (98-11-17) $tg22,5^0 + tg^{-1}22,5^0$ ni hisoblang.

A) $\sqrt{2}$ $\qquad$ B) $\sqrt{2^{-1}}$ $\qquad$ C) $4\sqrt{2}$ $\qquad$ D) $2\sqrt{2}$

**Yechish:** 8-formuladan foydalansak

$$tg22,5^0 + tg^{-1}22,5^0 = \frac{1 - \cos 45^0}{\sin 45^0} + \frac{\sin 45^0}{1 - \cos 45^0}.$$

ni olamiz. Endi $\cos 45^0 = \sin 45^0 = \dfrac{\sqrt{2}}{2}$ ni yuqoridagi ifodaga qo'yib, uni soddalashtirsak $2\sqrt{2}$ ni olamiz. **Javob:** $2\sqrt{2}$ (D).

9. (98-10-32) $tg15^0 - ctg15^0$ ni hisoblang:

A) $2\sqrt{3}$ $\quad$ B) $-2\sqrt{3}$ $\quad$ C) $-\dfrac{2\sqrt{3}}{3}$ $\quad$ D) $\dfrac{2\sqrt{3}}{3}$

10. (98-4-29) Hisoblang:

$$cos92^0 \cdot cos2^0 + 0,5 \cdot sin4^0 + 1$$

A) $\dfrac{1}{2}$ $\qquad$ B) 1 $\qquad$ C) 0 $\qquad$ D) 2

11. (99-6-12) Hisoblang:

$$\frac{2tg(240^0)}{1 - tg^2(240^0)}$$

A) $-\sqrt{3}$    B) $\sqrt{3}$    C) $\dfrac{\sqrt{3}}{3}$    D) $\dfrac{2}{\sqrt{3}}$

12. (00-10-13) Hisoblang.

$$\cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}$$

A) $\dfrac{1}{2}$    B) $\dfrac{1}{8}$    C) $\dfrac{1}{4}$    D) $\dfrac{1}{12}$

13. (96-9-47) Soddalashtiring.

$$\frac{1 + \sin 2\alpha}{\sin\alpha + \cos\alpha} - \sin\alpha$$

A) $\cos\alpha$    B) $\sin\alpha$    C) $-\cos\alpha$    D) $-2\sin\alpha$

**Yechish:** Asosiy trigonometrik ayniyat va 1-formuladan foydalanib berilgan kasr suratini quyidagicha yozib olamiz:

$$1 + \sin 2\alpha = 1 + 2\sin\alpha\cos\alpha = (\sin\alpha + \cos\alpha)^2.$$

Endi soddalashtiramiz

$$\frac{(\sin\alpha + \cos\alpha)^2}{\sin\alpha + \cos\alpha} - \sin\alpha = \sin\alpha + \cos\alpha - \sin\alpha.$$

O'xshash hadlarni ixchamlab $\cos\alpha$ ga ega bo'lamiz.
**Javob:** $\cos\alpha$ (A).

14. (97-7-56) Soddalashtiring:

$$\frac{\sin(\pi - 2\alpha)}{1 - \sin(\frac{\pi}{2} - 2\alpha)}$$

A) $-tg\alpha$    B) $2\sin\alpha$    C) $ctg\alpha$    D) $tg\alpha$

15. (96-3-112) Soddalashtiring.

$$\frac{\sin 3\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\cos\alpha}$$

A) $2\cos\alpha$    B) $2$    C) $2\sin\alpha$    D) $1$

16. (96-12-85) Soddalashtiring.

$$\frac{2}{tg\alpha + ctg\alpha}$$

A) $\cos 2\alpha$    B) $\dfrac{1}{\cos 2\alpha}$    C) $\dfrac{1}{\sin 2\alpha}$    D) $\sin 2\alpha$

17. (96-13-38) Soddalashtiring.

$$\frac{2}{ctg\alpha - tg\alpha}$$

A) $ctg2\alpha$    B) $\sin 2\alpha$    C) $tg2\alpha$    D) $\cos 2\alpha$

18. (00-1-27) Soddalashtiring.

$$\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} + 1$$

A) $\cos^{-2}\alpha$    B) $\sin^{-2}\alpha$    C) $\sin^2\alpha$    D) $\cos^2\alpha$

19. (00-2-48) Soddalashtiring.

$$(\cos 3x + \cos x)^2 + (\sin 3x + \sin x)^2$$

A) $4\cos^2 x$    B) $2\cos^2 x$
C) $3\sin^2 x$    D) $4\sin^2 x$

20. (96-7-56) Soddalashtiring.

$$\frac{\sin 2\alpha + \cos(\pi - \alpha) \cdot \sin\alpha}{\sin(\frac{\pi}{2} - \alpha)}$$

A) $\cos\alpha$    B) $\sin\alpha$    C) $-2\sin\alpha$    D) $-\cos\alpha$

21. (97-3-56) Soddalashtiring.

$$\frac{\cos 2\alpha + \cos(\frac{\pi}{2} - \alpha) \cdot \sin\alpha}{\sin(\frac{\pi}{2} + \alpha)}$$

A) $\cos\alpha$    B) $2\sin\alpha$    C) $-\cos\alpha$    D) $tg\alpha$

22. (97-10-56) Soddalashtiring.

$$\frac{\sin(2\alpha - \pi)}{1 - \sin(\frac{3\pi}{2} + 2\alpha)}$$

A) $tg\alpha$    B) $-tg\alpha$    C) $-2ctg\alpha$    D) $\sin\alpha$

23. (99-6-23) Soddalashtiring.

$$1 + \frac{tg^2(-\alpha) - 1}{\sin(0,5\pi + 2\alpha)}$$

A) $-tg^2\alpha$    B) $tg^2\alpha$    C) $ctg^2\alpha$    D) $-ctg^2\alpha$

24. (98-8-57) Hisoblang.

$$\sin^4\left(\frac{23\pi}{12}\right) - \cos^4\left(\frac{13\pi}{12}\right)$$

A) $\dfrac{\sqrt{3}}{2}$    B) $\dfrac{1}{2}$    C) $-\dfrac{\sqrt{3}}{2}$    D) $-\dfrac{\sqrt{2}}{2}$

**Yechish:** Argumentlarni $\dfrac{23\pi}{12} = 2\pi - \dfrac{\pi}{12}$ va $\dfrac{13\pi}{12} = \pi + \dfrac{\pi}{12}$ shaklda yozib, keyin keltirish formulalarini qo'llab, berilgan ifodani

$$\sin^4\left(\frac{\pi}{12}\right) - \cos^4\left(\frac{\pi}{12}\right) = \left(\sin^2\left(\frac{\pi}{12}\right)\right)^2 - \left(\cos^2\left(\frac{\pi}{12}\right)\right)^2$$

shaklda yozib olamiz. Bu ifodaga ikki son kvadratlari ayirmasi formulasini, keyin asosiy trigonometrik ayniyatni qo'llab

$$\sin^2\frac{\pi}{12} - \cos^2\frac{\pi}{12} = -(\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}) = -\cos\frac{2\pi}{12}$$

ni olamiz. $\cos\dfrac{2\pi}{12} = \cos\dfrac{\pi}{6}$ ning qiymatini 13.1-jadvaldan qarab, berilgan ifodaning qiymati $-\dfrac{\sqrt{3}}{2}$ ekanligini olamiz. **Javob:** $-\dfrac{\sqrt{3}}{2}$ (C).

25. (98-12-90) Hisoblang.

$$\frac{\sqrt{3}}{\sin 100^0} + \frac{1}{\cos 260^0}$$

A) $2$    B) $-4$    C) $-3$    D) $-1$

26. (01-11-18) Hisoblang.

$$\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$$

A) 3,5    B) 2,5    C) 3    D) 4

27. (02-7-11) Hisoblang.

$$\sin^4 105^0 \cdot \cos^4 75^0$$

A) $\frac{1}{256}$    B) $\frac{\sqrt{2}}{2}$    C) $\frac{1}{128}$    D) $\frac{\sqrt{6}}{4}$

28. (00-8-41) Hisoblang.

$\log_2 \cos 20^0 + \log_2 \cos 40^0 + \log_2 \cos 60^0 + \log_2 \cos 80^0$

A) $-4$    B) $-3$    C) $\frac{1}{2}$    D) 1

29. (03-3-39) Hisoblang.

$$ctg35^0 - tg35^0 - 2tg20^0$$

A) $\frac{1}{2}$    B) 0    C) 1    D) $\frac{\sqrt{3}}{2}$

30. (03-8-53) Hisoblang.

$$\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \cdot tg \frac{\pi}{8} \cdot ctg \frac{9\pi}{8}$$

A) $\frac{1}{2\sqrt{2}}$    B) $\sqrt{2}$    C) $\frac{1}{2}$    D) $\frac{\sqrt{3}}{2}$

31. (00-8-46) Hisoblang.

$$\cos 50^0 \cdot \cos 40^0 - 2 \cos 20^0 \cdot \sin 50^0 \cdot \sin 20^0$$

A) 0    B) 1    C) $-1$    D) $\cos 20^0$

32. (01-3-1) Ifodaning qiymatini hisoblang.

$$\sin 50^0 + \sin 40^0 \cdot tg20^0$$

A) $\sin^2 20^0$    B) 0,5    C) 1    D) $\cos^2 20^0$

33. (01-1-43) Agar $tg\alpha = -\frac{4}{3}$ bo'lsa, $\sin 2\alpha$ ning qiymatini toping.
A) 0,96    B) $-0,96$    C) 0,25    D) $-0,5$
**Yechish:** 3-formuladan foydalansak

$$\sin 2\alpha = \frac{2tg\alpha}{1 + tg^2\alpha} = \frac{2 \cdot (-\frac{3}{4})}{1 + (-\frac{3}{4})^2} = -\frac{3}{2} : \frac{25}{16}.$$

Uni soddalashtirib $\sin 2\alpha = -0,96$ ni olamiz. **Javob:** $-0,96$ (B).

34. (99-9-31) Agar $tg\alpha + ctg\alpha = 4$ bo'lsa, $\sin 2\alpha$ ni hisoblang.
A) $\frac{1}{2}$    B) $\frac{1}{4}$    C) $\frac{1}{3}$    D) $\frac{2}{3}$

35. (01-7-38) Agar $tg\alpha = 2 - \sqrt{3}$ bo'lsa, $\alpha$ o'tkir burchakning qiymatini toping.
A) $\frac{\pi}{8}$    B) $\frac{\pi}{12}$    C) $\frac{5}{12}\pi$    D) $\frac{3}{8}\pi$

36. (01-9-21) Soddalashtiring.

$$\sin^2 \alpha tg\alpha + \cos^2 \alpha ctg\alpha + \sin 2\alpha$$

A) $\frac{2}{\sin 2\alpha}$    B) $\frac{2}{\sin \alpha \cos \alpha}$    C) 1    D) $\sin^2 \alpha$

37. (02-8-40) Soddalashtiring.

$$\cos(\pi + 2\alpha) + \sin(\pi + 2\alpha) \cdot tg(\frac{\pi}{2} + \alpha)$$

A) 1    B) 2    C) $\sin \alpha$    D) $\cos \alpha$

38. (03-4-24) Soddalashtiring.

$$\frac{1 - \cos 2\alpha}{1 + tg^2\alpha}$$

A) $\sin^2 2\alpha$    B) $\frac{1}{2} \sin^2 2\alpha$    C) $\cos^2 2\alpha$    D) $\frac{1}{2} \cos^2 2\alpha$

39. (02-11-41) Soddalashtiring.

$$\frac{1 + \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}{1 - \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}$$

A) $tg \frac{\alpha}{4}$    B) $\cos \frac{\alpha}{2}$    C) $-ctg \frac{\alpha}{4}$    D) $\sin \frac{\alpha}{4}$

40. (03-2-26) Agar $ctg\alpha = \sqrt{2} - 1$ bo'lsa, $\cos 2\alpha$ ning qiymatini toping.
A) $\sqrt{2}$    B) $\frac{\sqrt{2} + 1}{2}$    C) $-\frac{1}{\sqrt{2}}$    D) $-\frac{1}{2}$

**Yechish:** 4-ayniyatdan va $tgx = \frac{1}{ctgx}$ dan foydalansak

$$\cos 2\alpha = \frac{1 - tg^2 x}{1 + tg^2 x} = \frac{ctg^2 x - 1}{ctg^2 x + 1}.$$

Endi $ctg\alpha = \sqrt{2} - 1$ ning qiymatini oxirgi ifodaning o'ng tomoniga qo'ysak

$$\cos 2\alpha = \frac{(\sqrt{2} - 1)^2 - 1}{(\sqrt{2} - 1)^2 + 1} = \frac{1 - \sqrt{2}}{2 - \sqrt{2}} =$$

$$\frac{1 - \sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{-\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}.$$

**Javob:** $-\frac{1}{\sqrt{2}}$ (C).

41. (02-10-59) Agar $90^0 < \alpha < 180^0$ va $\sin \alpha = \frac{1}{3}$ bo'lsa, $tg2\alpha$ ni hisoblang.
A) $-\frac{4\sqrt{2}}{7}$    B) $-\frac{4\sqrt{3}}{7}$    C) $\frac{2}{3}$    D) $-\frac{\sqrt{2}}{4}$

42. (03-6-26) Agar $\sin 37^0 = a$ bo'lsa, $\sin 16^0$ ni $a$ orqali ifodalang.
A) $a^2 - 1$    B) $a - 1$    C) $2a^2 - 1$    D) $1 - 2a^2$

43. (03-9-31) Agar $tg\frac{\alpha}{2} = -2$ bo'lsa, $\sin \alpha + 2 \cos \alpha$ ning qiymatini hisoblang.
A) $\frac{1}{2}$    B) $-\frac{1}{2}$    C) $-2$    D) $\frac{4}{5}$

44. (03-10-40) Agar $tg\alpha = \dfrac{1}{2}$ bo'lsa

$$\sin\left(2\alpha + \dfrac{\pi}{4}\right)$$

ning qiymatini toping.

A) $\dfrac{\sqrt{2}}{5}$    B) $\dfrac{2\sqrt{2}}{3}$    C) $\dfrac{2\sqrt{2}}{5}$    D) $\dfrac{7\sqrt{2}}{10}$

45. (03-11-22) $\alpha$ o'tkir burchak va

$$\sin^4\alpha \cdot \cos^4\alpha = \dfrac{1}{64}$$

bo'lsa, $\alpha$ quyidagilarning qaysi biriga teng?

A) $\dfrac{\pi}{8}; \dfrac{3\pi}{8}$   B) $\dfrac{\pi}{8}; \dfrac{\pi}{4}$   C) $\dfrac{\pi}{16}$   D) $\dfrac{\pi}{6}; \dfrac{3\pi}{8}$

46. (98-10-100) $\sin 105^0 + \sin 75^0$ ni hisoblang.

A) $\dfrac{\sqrt{2+\sqrt{3}}}{2}$     B) $\dfrac{\sqrt{2-\sqrt{3}}}{2}$

C) $\sqrt{\sqrt{3}-\sqrt{2}}$     D) $\sqrt{2+\sqrt{3}}$

**Yechish:** $105^0 = 90^0 + 15$, $75^0 = 90^0 - 15^0$ teng-liklardan va keltirish formulasidan

$\sin 105^0 + \sin 75^0 = \cos 15^0 + \cos 15^0 = 2\cos 15^0$.

Endi 7-ayniyat va $15^0$ burchakni birinchi cho-rakda yotishini hisobga olsak

$$2\cos 15^0 = 2\sqrt{\dfrac{1+\cos 30^0}{2}} = \sqrt{2+\sqrt{3}}$$

ni olamiz. **Javob:** $\sqrt{2+\sqrt{3}}$ (D).

47. (96-7-55) $\sin\dfrac{\pi}{12}$ ni hisoblang.

A) $\sqrt{2-\sqrt{3}}$     B) $\dfrac{\sqrt{2+\sqrt{3}}}{2}$

C) $\dfrac{\sqrt{2-\sqrt{3}}}{2}$     D) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

48. (00-3-50) $\sin 112,5^0$ ni hisoblang.

A) $\dfrac{1}{2}\sqrt{2-\sqrt{2}}$     B) $\dfrac{1}{2}\sqrt{1+\sqrt{2}}$

C) $\dfrac{1}{2}\sqrt{2+\sqrt{2}}$     D) $\dfrac{1}{2}\sqrt{\sqrt{2}-1}$

49. (01-2-85) $\cos 2227^0 30'$ ni hisoblang.

A) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$     B) $\dfrac{\sqrt{2-\sqrt{2}}}{4}$

C) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$     D) $\dfrac{\sqrt{2+\sqrt{2}}}{4}$

50. (98-1-57) Hisoblang.

$$8\sin^2\dfrac{15\pi}{16} \cdot \cos^2\dfrac{17\pi}{16} - 1$$

A) $-\dfrac{\sqrt{2}}{2}$    B) $\dfrac{\sqrt{2}}{2}$    C) $-\dfrac{1}{2}$    D) $\dfrac{1}{2}$

51. (00-3-53) Qaysi $\alpha$ o'tkir burchak uchun

$$\cos\alpha = \dfrac{1}{2}\sqrt{2+\sqrt{3}}$$

tenglik to'g'ri?

A) $7,5^0$    B) $22,5^0$    C) $75^0$    D) $15^0$

52. (97-5-28) $8\cos 30^0 + tg^2 15^0$ ni hisoblang.

A) 5    B) 6    C) 7    D) 8

53. (01-3-3) Hisoblang.

$$\sin^4 15^0 + \cos^4 15^0$$

A) $\dfrac{5}{6}$    B) $\dfrac{2}{3}$    C) $\dfrac{7}{8}$    D) $\dfrac{5}{7}$

54. (02-3-73) Hisoblang.

$$8\sin^2\dfrac{7\pi}{8} \cdot \cos^2\dfrac{9\pi}{8}$$

A) 0    B) $\dfrac{\sqrt{2}}{2}$    C) 1    D) $\dfrac{1}{2}$

55. (97-7-55) $\cos\dfrac{5\pi}{12}$ ni hisoblang.

A) $\dfrac{\sqrt{2+\sqrt{3}}}{3}$     B) $\dfrac{\sqrt{3}}{4}$

C) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$     D) $\dfrac{\sqrt{2-\sqrt{3}}}{2}$

56. (97-9-28) $4ctg 30^0 + tg^2 15^0$ ni hisoblang.

A) 5    B) 7    C) 9    D) 8

57. (97-6-44) Agar

$$\cos\alpha = \dfrac{1}{2}, \quad \dfrac{3\pi}{2} < \alpha < 2\pi$$

bo'lsa, $\sin(\pi - \dfrac{\alpha}{2})$ ni toping.

A) $-\dfrac{1}{2}$    B) $-\dfrac{\sqrt{3}}{2}$    C) $\dfrac{1}{4}$    D) $\dfrac{1}{2}$

**Yechish:** Keltirish formulasiga ko'ra $\sin(\pi - \dfrac{\alpha}{2}) = \sin\dfrac{\alpha}{2}$. 6-ayniyat va $\dfrac{\alpha}{2}$ burchak ikkinchi chorakda yotganligi sababli

$$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{2}} = \sqrt{\dfrac{1-\frac{1}{2}}{2}} = \dfrac{1}{2}.$$

**Javob:** $\dfrac{1}{2}$ (D).

58. (96-1-55) Agar $\cos 2\alpha = \dfrac{1}{2}$ bo'lsa, $\cos^2\alpha$ ni hisob-lang.

A) $\dfrac{1}{4}$    B) $\dfrac{\sqrt{3}}{2}$    C) $\dfrac{3}{4}$    D) $\dfrac{3}{8}$

59. (97-1-45) Agar

$$\cos\alpha = -\dfrac{1}{2}, \quad \pi < \alpha < \dfrac{3\pi}{2}$$

bo'lsa, $\sin(\dfrac{\pi}{2} + \dfrac{\alpha}{2})$ ni toping.

A) $\dfrac{1}{2}$    B) $-\dfrac{\sqrt{3}}{2}$    C) $\dfrac{\sqrt{3}}{2}$    D) $-\dfrac{1}{2}$

60. (98-11-20) Agar

$$\cos\alpha = \dfrac{7}{18}, \quad 0 < \alpha < \dfrac{\pi}{2}$$

bo'lsa, $6\cos\dfrac{\alpha}{2}$ ni toping.

A) 3     B) 5     C) 6     D) 4

**Yechish:** 4-ayniyatga asosan va $\dfrac{\alpha}{2}$ birinchi chorakda joylashganligi sababli

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}.$$

Bundan

$$6\cos\frac{\alpha}{2} = 6\sqrt{\frac{1+\frac{7}{18}}{2}} = 6\sqrt{\frac{25}{36}} = 5.$$

**Javob:** 5 (B).

61. (01-1-68) Agar

$$\sin\alpha = -0,8, \quad \alpha \in (\pi; \frac{3\pi}{2})$$

bo'lsa, $tg\dfrac{\alpha}{2}$ ni hisoblang.

A) 1     B) $-1$     C) 2     D) $-2$

62. (02-3-74) Agar $\cos(\pi - 4\alpha) = -\dfrac{1}{3}$ bo'lsa, $\cos^4(\dfrac{3\pi}{2} - 2\alpha)$ ni hisoblang.

A) $\dfrac{1}{9}$     B) $\dfrac{1}{3}$     C) $\dfrac{3}{4}$     D) $\dfrac{8}{9}$

63. (02-7-16) Soddalashtiring.

$$\frac{2\cos^2(45^0 - \frac{\alpha}{2})}{\cos\alpha}$$

A) $ctg(45^0 - \dfrac{\alpha}{2})$     B) $\sin\dfrac{\alpha}{2}$

C) $2\sin(45^0 - \dfrac{\alpha}{2})$     D) $\cos\dfrac{\alpha}{2}$

64. (02-9-39) Hisoblang.

$$\frac{2\sin^2 70^0 - 1}{2ctg115^0 \cdot \cos^2 155^0}$$

A) $-1$     B) 1     C) $\dfrac{1}{2}$     D) $\dfrac{\sqrt{3}}{2}$

65. (02-11-42) Agar

$$ctg\alpha = \frac{5}{12}, \quad \alpha \in (540^0; 630^0)$$

bo'lsa, $\sin\dfrac{\alpha}{2}$ ning qiymatini hisoblang.

A) $\dfrac{3}{4}$     B) $-\dfrac{3}{4}$     C) $-\dfrac{1}{2}$     D) $-\dfrac{3}{\sqrt{13}}$

66. (02-12-38) Soddalashtiring.

$$\frac{tg\alpha + \sin\alpha}{2\cos^2\frac{\alpha}{2}}$$

A) $ctg\alpha$     B) $tg\alpha$     C) $tg\dfrac{\alpha}{2}$     D) $ctg\dfrac{\alpha}{2}$

67. (99-8-76) Soddalashtiring.

$$\frac{\sin^2 2,5\alpha - \sin^2 1,5\alpha}{\sin 4\alpha \cdot \sin\alpha + \cos 3\alpha \cdot \cos 2\alpha}$$

A) $2tg2\alpha$     B) $tg2\alpha \cdot tg\alpha$     C) $2\sin 2\alpha$     D) $4\sin^2\alpha$

68. (03-5-40) Agar $\sin\alpha\left(1 - 2\sin^2\dfrac{\alpha}{2}\right) = \dfrac{1}{3}$ bo'lsa, $\cos(\dfrac{\pi}{4} - \alpha) \cdot \sin(\dfrac{3\pi}{4} - \alpha)$ ni hisoblang.

A) $\dfrac{5}{6}$     B) $\dfrac{3}{4}$     C) $\dfrac{4}{5}$     D) $\dfrac{\sqrt{3}}{4}$

69. (03-7-35) Agar

$$\cos 15^0 + \sin 15^0 = \frac{a}{4\cos 15^0}$$

bo'lsa $a$ ning qiymatini toping.

A) $\sqrt{3}$     B) $\sqrt{3}+1$     C) $\sqrt{3}+2$     D) $\sqrt{3}+3$

### 13.2.5 Yig'indi va ayirma uchun formulalar

1. $\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$.

2. $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$.

3. $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$.

4. $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$.

   1, 2 va 3-formulalardan mos ravishda 5, 6 va 7-*ko'paytma uchun* formulalar kelib chiqadi:

5. $\sin x \cdot \sin y = \dfrac{1}{2}(\cos(x-y) - \cos(x+y))$.

6. $\cos x \cdot \cos y = \dfrac{1}{2}(\cos(x-y) + \cos(x+y))$.

7. $\sin x \cdot \cos y = \dfrac{1}{2}(\sin(x-y) + \sin(x+y))$.

1. (98-11-103) $\sin 75^0 - \sin 15^0$ ni hisoblang.

   A) $\dfrac{\sqrt{2}}{2}$     B) $\dfrac{\sqrt{3}}{2}$     C) $\sqrt{2}$     D) $-\sqrt{2}$

   **Yechish:** 4-ayniyatga asosan

   $$\sin 75^0 - \sin 15^0 = 2\cos 45^0 \sin 30^0 = \frac{\sqrt{2}}{2}.$$

   **Javob:** $\dfrac{\sqrt{2}}{2}$ (A).

2. (00-1-28) Hisoblang.

   $$\frac{\sin 35^0 + \cos 65^0}{2\cos 5^0}$$

   A) 0,25     B) 0,75     C) 0,5     D) 0,6

3. (00-8-59) Hisoblang.

   $$\sin 10^0 + \sin 50^0 - \cos 20^0$$

   A) 0     B) $-1$     C) 1     D) $\cos 20^0$

4. (99-5-54) Hisoblang.

$$\sqrt[3]{8 + \left(\cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5}\right)^3}$$

A) 1        B) 2        C) 3        D) 4

5. (96-6-35) Soddalashtiring.

$$\frac{\cos\alpha - \cos 3\alpha}{\sin\alpha}$$

A) $-2\cos 2\alpha$   B) $2\cos 2\alpha$   C) $\sin 2\alpha$   D) $2\sin 2\alpha$

**Yechish:** 4-formuladan foydalansak ifoda quyidagiga teng bo'ladi

$$\frac{-2\sin\frac{\alpha+3\alpha}{2}\sin\frac{\alpha-3\alpha}{2}}{\sin\alpha} = \frac{-2\sin 2\alpha(-\sin\alpha)}{\sin\alpha}.$$

Bundan esa ifodani $2\sin 2\alpha$ ekanligiga kelamiz.
**Javob:** $2\sin 2\alpha$ (D).

6. (97-12-34) Soddalashtiring.

$$\frac{\cos 6\alpha - \cos 4\alpha}{\sin 5\alpha}$$

A) $2\sin\alpha$   B) $2\cos\alpha$   C) $-2\cos\alpha$   D) $-2\sin\alpha$

7. (98-10-35) Soddalashtiring.

$$\frac{\sin 4\alpha - \sin 6\alpha}{\cos 5\alpha}$$

A) $\sin 2\alpha$   B) $2\sin 2\alpha$   C) $-2\cos\alpha$   D) $-2\sin\alpha$

8. (98-8-58) Soddalashtiring.

$$\frac{1 - \sin\alpha - \cos 2\alpha + \sin 3\alpha}{\sin 2\alpha + 2\cos\alpha \cdot \cos 2\alpha}$$

A) $2ctg\alpha$   B) $tg\alpha$   C) $2\sin\alpha$   D) $ctg\alpha$

9. (01-7-40) Soddalashtiring.

$$\frac{\sin\alpha + \sin 2\alpha - \sin(\pi + 3\alpha)}{2\cos\alpha + 1}$$

A) $\sin\alpha$      B) $\cos\alpha$        C) $\sin 2\alpha$        D) $\cos 2\alpha$

10. (00-8-48) Hisoblang.

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

A) $-\frac{1}{2}$        B) $\frac{1}{4}$        C) $\frac{1}{3}$        D) $\frac{\sqrt{2}}{3}$

**Yechish:** Berilgan ifodani A bilan belgilaymiz.

$$A = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

Bu tenglikni $2\sin\frac{\pi}{7}$ ga ko'paytirib, har bir qo'shiluvchiga
$2\sin\alpha\cos\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$ formulani qo'llaymiz:

$$2A\sin\frac{\pi}{7} = 2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{4\pi}{7} +$$

$$+2\sin\frac{\pi}{7}\cos\frac{6\pi}{7} = -\sin\frac{\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{3\pi}{7} +$$

$$+\sin\frac{5\pi}{7} - \sin\frac{5\pi}{7} + \sin\frac{7\pi}{7} = -\sin\frac{\pi}{7}.$$

U holda $A = -\frac{1}{2}$. **Javob:** $-\frac{1}{2}$ (A).

11. (96-3-57) Hisoblang.

$$\sin 20^0 \cdot \sin 40^0 \cdot \sin 80^0$$

A) $\frac{1}{2}$        B) $\frac{1}{3}$        C) $\frac{1}{4}$        D) $\frac{\sqrt{3}}{8}$

12. (01-1-45) $5^0, 10^0, 15^0, \ldots$ burchaklarning qiymatlari arifmetik progressiya tashkil qiladi. Shu progressiyaning birinchi hadidan boshlab eng kamida nechtasini olganda, ularning kosinuslari yig'indisi nolga teng bo'ladi?
A) 18        B) 17        C) 19        D) 35

13. (03-9-30) Hisoblang.

$$\cos 55^0 \cdot \cos 65^0 \cdot \cos 175^0$$

A) $-\frac{1}{8}$    B) $-\frac{\sqrt{3}}{8}$    C) $\frac{\sqrt{3}}{8}$    D) $-\frac{1}{8}\sqrt{2 + \sqrt{3}}$

### 13.2.6   Qiymatlar sohasi va monotonligi

1. $y = \sin x$ **va**   $y = \cos x$ **funksiyalarning qiymatlari sohasi** $[-1; 1]$ **kesmadan iborat.**

2. $y = tgx$ **va**   $y = ctgx$ **funksiyalarning qiymatlar sohasi** $(-\infty; \infty)$ **oraliqdan iborat.**

3. $y = a\sin x + c$ **va** $y = a\cos x + c$ **funksiyalarning qiymatlari sohasi** $[c - |a|; c + |a|]$ **kesmadan iborat.**

4. $y = a\sin x + b\cos x + c$ **funksiyaning qiymatlari sohasi** $[-\sqrt{a^2 + b^2} + c; \sqrt{a^2 + b^2} + c]$ **kesmadan iborat.**

5. $y = a\sin^2 x + b\cos^2 x$ $(a < b)$ **funksiyaning qiymatlari sohasi** $[a; b]$ **kesmadan iborat.**

6. $y = \sin x$ **funksiya** $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ **kesmada o'suvchi.**

7. $y = \cos x$ **funksiya** $[0; \pi]$ **kesmada kamayuvchi.**

8. $y = tgx$ **funksiya** $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ **oraliqda o'suvchi.**

9. $y = ctgx$ **funksiya** $(0; \pi)$ **oraliqda kamayuvchi.**

1. $y = 3\sin x$ funksiyaning qiymatlar sohasini toping.
A) $[0; 3]$   B) $(-3; 3)$    C) $[-3; 0]$   D) $[-3; 3]$

**Yechish:** 3-xossaga ko'ra $y = 3\sin x$   ($a = 3, c = 0$) funksiyaning qiymatlar to'plami $[-3; 3]$ kesmadan iborat. **Javob:** $[-3; 3]$ (D).

2. $y = 2\cos x$ funksiyaning qiymatlar sohasini toping.
A) $[0; 2]$    B) $(-2; 2)$    C) $[-2; 2]$    D) $[-2; 0]$

3. $y = 2 + 5tg3x$ funksiyaning qiymatlar sohasini toping.
A) $(-\infty; 2]$      B) $(-\infty; \infty)$
C) $[2; \infty)$      D) $[-2; 2]$

4. $y = 5 - 7ctg(3x + 2)$ funksiyaning qiymatlar sohasini toping.
A) $(-\infty; 5]$      B) $(-\infty; \infty)$
C) $[2; \infty)$      D) $[12; \infty)$

5. $y = 2\cos x - 3$ funksiyaning qiymatlar sohasini toping.
A) $[-5; 2]$   B) $[-5; 1)$   C) $[-5; -1]$   D) $[-3; 2]$

6. (01-10-51) Funksiyaning qiymatlar sohasini toping.

$$y = (\sin x + \cos x)^2 - \frac{1 - \cos 4x}{2\sin 2x} - \cos x$$

A) $[0; 2]$      B) $(0; 2)$
C) $(0; 1) \cup (1; 2)$      D) $[0; 1) \cup (1; 2]$.

**Yechish:** $\sin 2x \neq 0$ deb funksiyaning ko'rinishini quyidagicha o'zgartiramiz:

$$y = (\sin x + \cos x)^2 - \frac{1 - \cos 4x}{2\sin 2x} - \cos x = 1 - \cos x.$$

3-xossadan $y = 1 - cosx$   $(a = -1, \ c = 1)$ funksiyaning qiymatlar to'plami $[0; 2]$ kesmadan iborat ekanligi kelib chiqadi. Ammo $x \neq \frac{\pi}{2}k$ shartni e'tiborga olsak, berilgan funksiyaning qiymatlar to'plami $(0; 1) \cup (1; 2)$ oraliqdan iborat ekanligiga kelamiz.
**Javob:** $(0; 1) \cup (1; 2)$ (C).

7. (01-11-23) Ushbu $f(x) = 2\cos\frac{x}{2} + 3$ funksiyaning qiymatlar sohasini toping.
A) $[3; 5]$    B) $[4; 5]$    C) $[2; 5]$    D) $[1; 5]$

8. (02-1-23) $y = 1 + \cos x$ funksiya grafigining $Ox$ o'qi bilan urinish nuqtalarining koordinatlarini toping.
A) $\pi + 2\pi n, \quad n \in Z$      B) $2\pi n, \quad n \in Z$
C) $\pi + \pi n, \quad n \in Z$      D) $\pi n, \quad n \in Z$

9. (02-2-60) $y = \cos^4 x - 2\sin^2 x + 7$ funksiyaning eng kichik qiymatini toping.
A) 5     B) 3     C) 2     D) 1

10. (02-12-53) Funksiyaning qiymatlar sohasini toping.
$$f(x) = 3^{\log_2(3\sin^2 x + 1)}$$
A) $[1; 9]$    B) $[0; 9]$    C) $[0; 9)$    D) $(1; 9)$

11. (00-1-25) Sonlarning eng kattasini toping.
A) $\sin 1$    B) $\cos(\frac{\pi}{2} - \frac{1}{2})$    C) $\sin 4$    D) 1

**Yechish:** $\cos(\frac{\pi}{2} - \frac{1}{2}) = \sin\frac{1}{2}$ va $\sin 4 = \sin(\pi - (\pi - 4)) = \sin(\pi - 4)$. Endi $-\frac{\pi}{2} < \pi - 4 < \frac{1}{2} < 1 < \frac{\pi}{2}$ va sinus funksiyaning $[-\frac{\pi}{2}, \frac{\pi}{2}]$ kesmada o'suvchiligini hiobga olsak $\sin(\pi - 4) < \sin\frac{1}{2} < \sin 1 < \sin\frac{\pi}{2} = 1$. Demak, berilgan sonlarning eng kattasi 1 ekan. **Javob:** 1 (D).

12. (03-4-22) $tg240^0$, $\sin 120^0$, $\cos 150^0$ va $ctg225^0$ sonlardan eng kattasining eng kichigiga ko'paytmasini toping.
A) $-1,4$    B) $-1,5$    C) $-\frac{\sqrt{6}}{2}$    D) $1,5$

13. (03-2-11) $y = \cos^2 x - \frac{\sqrt{3}}{2}\sin 2x$ funksiyaning eng katta va eng kichik qiymatlari yig'indisini toping.
A) 1,5    B) 0,5     C) 1      D) 2

14. (03-2-29) Funksiyaning qiymatlar sohasini toping.

$$y = (1 + tg^2 x)\cos^2 x - \frac{\sin 2x}{2\cos x}$$

A) $[0; 2]$    B) $(0; 2)$    C) $[-1; 1]$    D) $(-2; 0)$

15. (01-3-15) Ushbu

$$f(x) = \sin\frac{x}{2} \cdot \cos^3\frac{x}{2} - \sin^3\frac{x}{2} \cdot \cos\frac{x}{2}$$

funksiyaning eng katta qiymatini toping.
A) 1     B) $\frac{1}{2}$     C) 2     D) $\frac{1}{4}$

**Yechish:** Bu ifodani ko'paytuvchilarga ajratib

$$f(x) = \sin\frac{x}{2} \cdot \cos\frac{x}{2}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)$$

ga kelamiz. 1.2.4-bandning 1 va 2- formulalaridan foydalanib

$$f(x) = \frac{1}{2}\sin x \cdot \cos x = \frac{1}{4}\sin 2x$$

tenglikga kelamiz. $g(x) = \sin 2x$ funksiyaning eng katta qiymati 1 bo'lganligi sababli, $f(x) = \frac{1}{4}\sin 2x$ ning eng katta qiymati $\frac{1}{4}$ ga teng bo'ladi.
**Javob:** $\frac{1}{4}$ (D).

16. (03-7-36) $y = (\cos x + 5) \cdot (3 - \cos x)$ funksiyaning eng katta qiymatini toping.
A) 8     B) 12     C) 15     D) 16

17. (03-7-49) $y = \frac{3}{4} \cdot \cos^2(x - \frac{\pi}{4}) - 1$ funksiyaning qiymatlar sohasini toping.
A) $\left[-\frac{3}{4}; \frac{3}{4}\right]$      B) $[-1; 0]$
C) $[-1; -0,25]$      D) $[-0,25; 0]$

18. (96-10-15) $y = 2\sin x + \cos x$ funksiyaning eng katta qiymatini toping.
A) 3     B) $\sqrt{5}$     C) 2     D) $-1$

**Yechish:** 4-qoidaga ko'ra $y = 2\sin x + \cos x$ funksiyaning qiymatlar sohasi $[-\sqrt{5}; \sqrt{5}]$ kesmadan iborat bo'ladi. Shuning uchun uning eng katta qiymati $\sqrt{5}$ dir. **Javob:** $\sqrt{5}$ (B).

19. (00-1-30) Agar $\alpha$ - o'zgaruvchi miqdor bo'lsa; $4(\sqrt{3}\cos\alpha + \sin\alpha)$ ning eng katta qiymati qanchaga teng bo'ladi?
    A) 9,5      B) 7      C) 8      D) 6,5

20. (00-3-57) $y = 6\sin 2x + 8\cos 2x$ funksiyaning qiymatlar to'plamini toping.
    A) $[-10;10]$      B) $[-14;14]$
    C) $(-\infty;\infty)$      D) $[0;6]$

21. (01-6-43) Ushbu $f(x) = \sin x + \cos x$ funksiyaning eng katta qiymatini toping.
    A) 1,4      B) $\sqrt{2}$      C) $\sqrt{3}$      D) 1,6

22. (01-7-45) Ushbu $f(x) = (\sin x + \cos x)^2$ funksiyaning qiymatlar sohasini toping.
    A) $[-1;1]$    B) $[-2;2]$    C) $[0;2]$    D) $[-\frac{1}{2};\frac{1}{2}]$

23. (02-1-18) $y = (2,(1)+1,(8))\sin x + (1,(2)+1,(7))\cos x$ funksiyaning qiymatlar to'plamini toping.
    A) $[-5;5]$    B) $[-4;4]$    C) $[-3;3]$    D) $(-4;4)$

24. (02-4-34) $y = 3\sin x - 4\cos x$ funksiyaning eng katta qiymatini toping.
    A) 3      B) 4      C) 5      D) 6

25. (03-12-23) $y = 1 - 6\sin 2x + 8\cos 2x$ funksiyaning eng katta qiymatini toping.
    A) 15      B) 14      C) 13      D) 11

26. (03-12-27) Funksiyaning eng katta qiymatini toping.
    $$y = \frac{8\sin x - 15\cos x + 3}{4}$$
    A) 6,5      B) 7,5      C) 5      D) 6

27. (96-7-57) Ushbu
    $$x = \cos\frac{11\pi}{12}, \quad y = \cos\left(-\frac{\pi}{3}\right), \quad z = \sin\frac{11\pi}{12}$$
    sonlar uchun qo'yidagi munosabatlardan qaysi biri o'rinli?
    A) $x < y < z$      B) $x < z < y$
    C) $y < z < x$      D) $z < y < x$

    **Yechish:** $\cos(-\alpha) = \cos\alpha$ ekanidan
    $$y = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3},$$
    tenglikni, $\sin\alpha = \cos\left(\frac{\pi}{2}-\alpha\right)$ ekanidan esa
    $$z = \sin\frac{11\pi}{12} = \cos\left(\frac{\pi}{2}-\frac{11\pi}{12}\right) = \cos\left(-\frac{5\pi}{12}\right) = \cos\frac{5\pi}{12}$$
    ni hosil qilamiz. Endi $\cos\frac{11\pi}{12}$, $\cos\frac{\pi}{3}$, $\cos\frac{5\pi}{12}$ sonlarni taqoslaymiz.
    $$\frac{\pi}{3} < \frac{5\pi}{12} < \frac{11\pi}{12}$$

va $y = \cos x$ funksiya $[0;\pi]$ oraliqda kamayuvchi bo'lgani uchun
$$\cos\frac{\pi}{3} > \cos\frac{5\pi}{12} > \cos\frac{11\pi}{12}$$
munosabatlarni hosil qilamiz. **Javob:** $x < z < y$ (B).

28. (97-3-57) Ushbu $x = tg\frac{5\pi}{7}; \quad y = \sin\frac{\pi}{6}; \quad z = tg\frac{3\pi}{7}$ sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?
    A) $z > y > x$      B) $x > z > y$
    C) $y > x > z$      D) $x > y > z$

29. (97-7-57) Ushbu $x = tg\left(\frac{5\pi}{6}\right), \quad y = \cos\left(\frac{2\pi}{5}\right),$ $z = tg\left(-\frac{\pi}{8}\right)$ sonlarni kamayish tartibida yozing.
    A) $x > y > z$      B) $y > x > z$
    C) $x > z > y$      D) $y > z > x$

30. (98-9-21) Quyidagi ayirmalardan qaysi birining qiymati manfiy?
    A) $\sin 140^0 - \sin 150^0$      B) $\cos 10^0 - \cos 50^0$
    C) $tg 87^0 - tg 85^0$      D) $ctg 45^0 - ctg 40^0$

31. (98-11-98) Eng katta sonni toping.
    A) $\sin 170^0$      B) $\sin 20^0$
    C) $\sin(-30^0)$      D) $\sin 100^0$

32. (99-1-50) $x = \sin 60^0, \quad y = \cos(-600^0),$ $z = ctg\frac{31\pi}{6}$ sonlarni kamayish tartibida yozing.
    A) $z > x > y$      B) $x > y > z$
    C) $y > z > x$      D) $z > y > x$

33. (99-6-32) Sonlarni o'sish tartibida joylashtiring.
    $$a = \cos(-13^0), \quad b = -\sin(-75^0), \quad c = \sin 100^0$$
    A) $b < a < c$      B) $a < b < c$
    C) $a < c < b$      D) $b < c < a$

34. (99-9-27) $M = \sin 72^0, \quad N = \cos 220^0$ va $Q = ctg 184^0 \cdot \sin 4^0$ sonlarni kamayish tartibida yozing.
    A) $N > Q > M$      B) $N > M > Q$
    C) $Q > M > N$      D) $Q > N > M$

35. (99-10-26) $k = tg 248^0, \quad t = \cos 32^0$ va $q = \sin 112^0$ sonlarni o'sish tartibida joylashtiring.
    A) $q < t < k$      B) $k < t < q$
    C) $t < k < q$      D) $t < q < k$

36. (96-6-32) $y = 2\sin^2 x + \cos^2 x$ funksiyaning eng katta qiymatini toping.
    A) 1      B) 1,5      C) 2,6      D) 2

    **Yechish:** 5-xossadan $y = 2\sin^2 x + \cos^2 x$ funksiyaning qiymatlar sohasi $[1;2]$ kesmadan iborat ekanligi kelib chiqadi. Demak, funksiyaning eng katta qiymati 2 ekan. **Javob:** 2 (D).

37. (96-1-56) $y = 2\sin 3x + \cos 3x$ funksiyaning eng katta qiymatini toping.
    A) 3      B) 2      C) $\sqrt{5}$      D) 4

38. (96-7-30) $y = 5^{1-\sin x} - e^{\ln 2}$ funksiyaning eng kichik qiymatini toping.

    A) $1 - e^2$     B) 3     C) $-1$     D) $-2,29$

**Yechish:** Asosiy logarifmik ayniyatga ko'ra $e^{\ln 2} = 2$ va u o'zgarmas. 3-qoidaga ko'ra $g(x) = 1 - \sin x$ funksiyaning qiymatlar sohasi [0; 2] kesmadan iborat. $f(t) = 5^t$ o'suvchi funksiya ekanligidan $y = 5^{1-\sin x} - e^{\ln 2}$ funksiyaning eng kichik qiymati $y_0 = 5^0 - 2 = 1 - 2 = -1$ ekanligi kelib chiqadi. **Javob:** $-1$ (C).

39. (98-3-55) Ushbu $y = tg3x + ctg2x$ funksiya $x$ ning qanday qiymatida aniqlanmagan?

    A) $\dfrac{\pi k}{2}, \dfrac{\pi}{6} + \dfrac{\pi k}{3}, \quad k \in Z$     B) $\dfrac{\pi}{6} + \dfrac{\pi k}{3}, \quad k \in Z$

    C) $\dfrac{\pi}{2} + \dfrac{k\pi}{3}, \quad k \in Z$     D) $\dfrac{k\pi}{4}, \quad k \in Z$

40. (97-1-21) $y = 1 + \cos x$ funksiyaning $[\dfrac{\pi}{3}; \dfrac{\pi}{2}]$ kesmadagi eng kichik qiymatini toping.

    A) 0    B) 1    C) $1\dfrac{1}{2}$    D) $1 + \dfrac{\sqrt{3}}{2}$

41. (97-8-31) $\sin^2 \alpha + 2\cos^2 \alpha$ ning eng katta qiymatini toping.

    A) 1,2     B) 1,4     C) 1,6     D) 2

42. (97-10-30) Quyidagilardan qaysi biri

$$y = \frac{10}{5^{|\cos x|}} + 2\ln e^3$$

funksiyaning eng katta qiymati?

    A) 8     B) 16     C) $2 + 2e^3$     D) 18

43. (97-11-21) $y = 2 - 2\sin x$ funksiyaning $[0; \dfrac{\pi}{6}]$ kesmadagi eng kichik qiymatini hisoblang.

    A) 0     B) $\dfrac{1}{2}$     C) $2 - \sqrt{3}$     D) 1

**Yechish:** Agar $y = f(x)$ o'suvchi funksiya bo'lsa, u holda ixtiyoriy $c < 0$ uchun $y = c\,f(x) + b$ funksiya kamayuvchi bo'ladi. Bu yerdan va sin funksiyaning $[0; \dfrac{\pi}{6}]$ kesmada o'suvchiligidan $y = 2 - 2\sin x$ funksiyaning $[0; \dfrac{\pi}{6}]$ kesmada kamayuvchi ekanligi kelib chiqadi. Demak, $y_0 = 2 - 2\sin \dfrac{\pi}{6} = 2 - 1 = 1$ berilgan funksiyaning $[0; \dfrac{\pi}{6}]$ kesmadagi eng kichik qiymati bo'ladi. **Javob:** 1 (D).

44. (97-3-30) Funksiyaning eng katta qiymatini toping.

$$y = \frac{1}{2^{\cos x}} + \ln e^2$$

    A) 2,5     B) 3     C) $1 + e^2$     D) 4

45. (98-8-30) Ushbu $y = 2 - \sin x$ funksiyaning $[0; \dfrac{7\pi}{6}]$ oraliqdagi eng katta qiymatini toping.

    A) 3     B) 2     C) 2,5     D) 1

46. (98-5-14) $f(x) = 5\sin x + 6$ funksiyaning eng katta qiymatini toping.

    A) $-1$     B) 11     C) 1     D) 6

47. (98-1-30) $y = 0,5\cos x$ funksiyaning $[-\dfrac{\pi}{4}; \dfrac{3\pi}{4}]$ kesmadagi eng kichik qiymatini toping.

    A) $-\dfrac{1}{2}$     B) $-1$     C) 0     D) $-\dfrac{\sqrt{2}}{4}$

48. (00-7-24) Ifodaning eng kichik qiymatini toping.

$$2\sin^2 x + \sqrt{3}\cos 2x$$

    A) $-1$     B) 1     C) $2 - \sqrt{3}$     D) $3 - 2\sqrt{2}$

49. (00-2-30) Qaysi funksiya $x \in (-\dfrac{\pi}{6}; \dfrac{5\pi}{6})$ oraliqda faqat musbat qiymatlarni qabul qiladi?

    A) $y = \sin(x + \dfrac{\pi}{6})$     B) $y = \sin(x + \dfrac{5\pi}{6})$

    C) $y = \sin(x - \dfrac{5\pi}{6})$     D) $y = \sin(x - \dfrac{\pi}{6})$

50. (01-2-18) Ushbu $y = \dfrac{x}{2} + \sin^2 x$ funksiyaning $[-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$ kesmadagi eng katta qiymatini toping.

    A) $-\dfrac{\pi}{2} + 1$    B) $-\dfrac{\pi}{4} + 1$    C) $\dfrac{\pi}{6} + 1$    D) $\dfrac{\pi}{4} + 1$

## 13.3    Teskari trigonometrik funksiyalar

Ma'lumki, tayinlangan $y \in [-1; 1]$ da $\sin x = y$ tenglama yechimi yagona emas. Masalan, $x_0 = 0$ va $x_1 = \pi$ sonlari $\sin x = 0$ tenglamaning yechimlari bo'ladi. Bu esa $y = \sin x$ funksiyaga teskari funksiya mavjud emasligini bildiradi. Xuddi shunday mulohaza yuritib $y = \cos x$ $(y = tgx, \ y = ctgx)$ funksiya uchun ham teskari funksiya mavjud emasligiga kelamiz.

Agar $y = \sin x$ funksiyaning aniqlanish sohasini qisqartirib, uni $D(y) = [-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$ deb olsak, u holda har bir tayinlangan $y \in [-1; 1]$ da $\sin x = y$ tenglama yagona yechimga ega bo'ladi, ya'ni teskari funksiya mavjud. Demak, aniqlanish sohasi $D(\sin) = [-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$ bo'lgan $y = \sin x$ funksiyaga teskari funksiya mavjud bo'lib, u *arksinus* deyiladi va $x = \arcsin y$ shaklda yoziladi.

Xuddi shunday mulohaza yuritib $y = \cos x$, $D(\cos) = [0; \pi]$; $y = tgx$, $D(tg) = (-\dfrac{\pi}{2}; \dfrac{\pi}{2})$ $y = ctgx$, $D(ctg) = (0; \pi)$ funksiyalar uchun ham teskari funksiyalar mavjudligiga kelamiz. Ularga teskari bo'lgan funksiyalar mos ravishda *arkkosinus, arktangens va arkkotangens* deyiladi va $y = \arccos x$, $y = arctgx$ va $y = arcctgx$ shaklda yoziladi. Teskari funksiya ta'rifidan foydalanib, teskari trigonometrik funksiyalar qiymatlarini quyidagi $13.6a - 13.6b$ jadvallar shaklida berish mumkin:

| $a$ | 0 | $\dfrac{1}{2}$ | $\dfrac{\sqrt{2}}{2}$ | $\dfrac{\sqrt{3}}{2}$ | 1 |
|---|---|---|---|---|---|
| $arcsina$ | 0 | $\dfrac{\pi}{6}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{3}$ | $\dfrac{\pi}{2}$ |
| $arccosa$ | $\dfrac{\pi}{2}$ | $\dfrac{\pi}{3}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{6}$ | 0 |

13.6a jadval.

| $b$ | $0$ | $\dfrac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ |
|---|---|---|---|---|
| $arctgb$ | $0$ | $\dfrac{\pi}{6}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{3}$ |
| $arcctgb$ | $\dfrac{\pi}{2}$ | $\dfrac{\pi}{3}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{6}$ |

13.6b jadval.

Teskari trigonometrik funksiyalarning asosiy xossalarini keltiramiz.

1. **Aniqlanish sohasi:**
   $D(\arcsin) = D(\arccos) = [-1;\ 1]$,
   $D(arctg) = D(arcctg) = R$.

2. **Arksinus va arktangenslar toq funksiya:**
   $\arcsin(-x) = -\arcsin x,\ arctg(-x) = -arctgx$

3. **Arkkosinus va arkkotangenslar uchun**
   $\arccos(-x) = \pi - \arccos x$,
   $arcctg(-x) = \pi - arcctgx$
   **tengliklar o'rinli.**
   **To'g'ri va teskari funksiya ta'rifidan quyidagi 4-11 xossalar kelib chiqadi:**

4. $\arcsin(\sin x) = x,\quad -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

5. $\sin(\arcsin x) = x,\quad -1 \le x \le 1$.

6. $\arccos(\cos x) = x,\quad 0 \le x \le \pi$.

7. $\cos(\arccos x) = x,\quad -1 \le x \le 1$.

8. $arctg(tgx) = x,\quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

9. $tg(arctgx) = x,\quad x \in R$.

10. $arcctg(ctgx) = x,\quad 0 < x < \pi$.

11. $ctg(arcctgx) = x,\quad x \in R$.
    **Quyidagi 12-20 tengliklar o'rinli:**

12. $\sin(\arccos x) = \cos(\arcsin x) = \sqrt{1-x^2}$.

13. $\arcsin a + \arccos a = \dfrac{\pi}{2}$.

14. $arctgx + arcctgx = \dfrac{\pi}{2}$.

15. $\arcsin x + \arcsin y = \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2})$.

16. $\arccos x + \arccos y = \arccos(x\sqrt{1-y^2} - y\sqrt{1-x^2})$.

17. $\arccos x - \arccos y = \arccos(x\sqrt{1-y^2} + y\sqrt{1-x^2})$.

18. $\arcsin x - \arcsin y = \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2})$.

19. $arctgx + arctgy = arctg\dfrac{x+y}{1-xy}$.

20. $arctgx - arctgy = arctg\dfrac{x-y}{1+xy}$.

1. (98-9-20) Hisoblang.
$$\arccos\left(-\frac{1}{2}\right) - \arcsin\left(-\frac{\sqrt{2}}{2}\right)$$
A) $\dfrac{11\pi}{12}$    B) $\dfrac{7\pi}{4}$    C) $\dfrac{\pi}{12}$    D) $\dfrac{5\pi}{6}$

**Yechish:** 2 va 3-xossalardan foydalansak, berilgan ifodani $\pi - \arccos\dfrac{1}{2} + \arcsin\dfrac{\sqrt{2}}{2}$ shaklda yozish mumkin. $13.6a-$ jadvaldan $\arccos\dfrac{1}{2} = \dfrac{\pi}{3}$ va $\arcsin\dfrac{\sqrt{2}}{2} = \dfrac{\pi}{4}$ qiymatlarni topib, ularni bu ifodaga qo'yib $\dfrac{11\pi}{12}$ ni olamiz. **Javob:** $\dfrac{11\pi}{12}$ (A).

2. (98-2-22) Hisoblang.
$$\arccos\left(-\frac{\sqrt{2}}{2}\right) - arctg\frac{1}{\sqrt{3}}$$
A) $-75^0$    B) $75^0$    C) $-105^0$    D) $105^0$

3. (98-9-23) Hisoblang.
$$\arccos\left(-\frac{\sqrt{2}}{2}\right) - \arcsin\left(-\frac{\sqrt{3}}{2}\right)$$
A) $\dfrac{7\pi}{12}$    B) $\dfrac{\pi}{12}$    C) $\dfrac{13}{12}\pi$    D) $\dfrac{5\pi}{12}$

4. (99-8-68) Hisoblang.
$$2\arcsin\left(-\frac{1}{2}\right) + \frac{1}{2}\arccos\frac{\sqrt{3}}{2}$$
A) $-\dfrac{\pi}{4}$    B) $\dfrac{\pi}{6}$    C) $0$    D) $\dfrac{\pi}{3}$

5. (01-6-31) Hisoblang.
$$\sin(2\arccos\frac{\sqrt{3}}{2})$$
A) $\dfrac{\sqrt{2}}{2}$    B) $\dfrac{1}{2}$    C) $\dfrac{\sqrt{3}}{2}$    D) $\dfrac{\sqrt{6}}{2}$

6. (98-3-57) Hisoblang.
$$\arcsin\left(\sin\frac{5\pi}{8}\right) + \arccos\left(\cos\frac{8\pi}{7}\right)$$
A) $\dfrac{99\pi}{56}$    B) $\dfrac{83\pi}{56}$    C) $\dfrac{85\pi}{56}$    D) $\dfrac{69\pi}{56}$

**Yechish:** $\sin\alpha = \sin(\pi - \alpha)$ formuladan foydalanib

1) $\sin\dfrac{5\pi}{8} = \sin(\pi - \dfrac{5\pi}{8}) = \sin\dfrac{3\pi}{8}$

ni hosil qilamiz. $-\dfrac{\pi}{2} \le \alpha \le \dfrac{\pi}{2}$ oraliqda $\arcsin(\sin\alpha) = \alpha$ ekanidan foydalanamiz.

$$-\frac{\pi}{2} < \frac{3\pi}{8} < \frac{\pi}{2}$$

munosabatlarni tekshirish qiyin emas. Shuning uchun

$$\arcsin(\sin\frac{5\pi}{8}) = \arcsin(\sin\frac{3\pi}{8}) = \frac{3\pi}{8}.$$

$\cos\alpha = \cos(2\pi - \alpha)$ ekanidan

$$2)\quad \cos\frac{8\pi}{7} = \cos(2\pi - \frac{8\pi}{7}) = \cos\frac{6\pi}{7}$$

bo'ladi. $0 \le \alpha \le \pi$ oraliqda $\arccos(\cos\alpha) = \alpha$ bo'lgani uchun

$$\arccos(\cos\frac{8\pi}{7}) = \arccos(\cos\frac{6\pi}{7}) = \frac{6\pi}{7}$$

bo'ladi, chunki

$$0 < \frac{6\pi}{7} < \pi.$$

Shuning uchun berilgan ifoda $\frac{3\pi}{8} + \frac{6\pi}{7} = \frac{69\pi}{56}$ ga teng.

**Javob:** $\frac{69\pi}{56}$ (D).

7. (98-5-47) Hisoblang.

$$\sin\left(\arcsin\frac{\sqrt{2}}{2} - \arccos\frac{\sqrt{2}}{2}\right)$$

A) 0      B) 1      C) $\frac{\sqrt{2}}{2}$      D) $\frac{\sqrt{3}}{2}$

8. (99-7-46) Hisoblang.

$$tg\left(\arcsin\frac{\sqrt{3}}{2} + arctg\sqrt{3}\right)$$

A) $\sqrt{3}$      B) $-\frac{\sqrt{3}}{3}$      C) $-\frac{\sqrt{2}}{2}$      D) $-\sqrt{3}$

9. (98-4-16) Hisoblang.

$$\arccos\left(\sin\frac{\pi}{8}\right)$$

A) $1 - (\frac{\pi}{8})^2$   B) $\frac{5\pi}{8}$   C) $\frac{7\pi}{8}$   D) $\frac{3\pi}{8}$

10. (98-10-104) Hisoblang.

$$arctg(tg(-\frac{3\pi}{5})) + arcctg(ctg(-\frac{3\pi}{5}))$$

A) $-\frac{6\pi}{5}$      B) $-\frac{7\pi}{10}$      C) $\frac{4\pi}{5}$      D) $-\frac{4\pi}{5}$

11. (00-4-46) $\sin(2arctg3)$ ning qiymatini toping.
A) 0,6      B) 0,8      C) 0,75      D) 0,36

**Yechish:** $\alpha = arctg3$ bo'lsin. U holda $tg\alpha = 3$ va

$$\sin 2\alpha = \frac{2tg\alpha}{1 + tg^2\alpha} = \frac{2\cdot 3}{1+9} = \frac{6}{10} = 0,6$$

**Javob:** 0,6 (A).

12. (99-2-25) $m = \arcsin\frac{\sqrt{3}}{2}$, $n = \arccos(-\frac{1}{2})$ va $p = arctg1$ sonlarni kamayish tartibida joylashtiring.
A) $m > p > n$      B) $m > n > p$
C) $n > m > p$      D) $p > n > m$

13. (01-12-31) Qaysi ifoda ma'noga ega emas?

$$1)\ \sqrt{\lg\frac{11\pi}{8}};\quad 2)\ \sqrt{\sin\frac{19\pi}{12}};\quad 3)\ \log_{\sqrt{\frac{\pi}{3}}}\sqrt[3]{\frac{3\pi}{8}}.$$

A) 1; 3      B) 3      C) 2      D) 1; 2

**Yechish:** 1) ifodada $1 < \frac{11\pi}{8}$ bo'lgani sababli $\lg\frac{11\pi}{8}$ musbat. Shu sababli $\sqrt{\lg\frac{11\pi}{8}}$ ma'noga ega. $\sin\frac{19\pi}{12} = \sin\left(2\pi - \frac{5\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right) < 0$ bo'lgani uchun 2) dagi ildiz ma'noga ega emas. 3) ifodada $\sqrt[3]{\frac{3\pi}{8}} > 0$ bo'lgani uchun, $\log_{\sqrt{\frac{\pi}{3}}}\sqrt[3]{\frac{3\pi}{8}}$ ma'noga ega. **Javob:** 2 (C).

14. (97-4-37) Ma'noga ega ifodalarni ko'rsating.
1) $\arcsin(\log_2 5)$;     2) $\arccos\frac{\pi}{\sqrt{17}}$
3) $\arccos\frac{a^2+b^2}{a^2+b^2+c^2}$     4) $\arcsin\frac{a^2+b^2+\sqrt{2}}{a^2+b^2+1}$
A) 1);2)      B) 1); 3)      C) 2);3)      D) 3);4)

15. (97-9-97) Ma'noga ega ifodalarni ko'rsating.
1) $\lg(\arccos 1)$;     2) $\arcsin(\lg\frac{1}{2})$
3) $\arccos\left(\frac{a^4+1}{(a^2+1)^2}\right)$     4) $\arcsin(\sqrt[10]{2})$
A) 1); 2)      B) 2); 4)      C) 3); 4)      D) 2); 3)

16. (01-9-19) Hisoblang.

$$\frac{\sin(\pi + \arcsin\frac{\sqrt{3}}{2})}{\cos(0,5\pi + \arcsin\frac{1}{2})}$$

A) $\sqrt{3}$      B) $-\frac{\sqrt{3}}{2}$      C) $-\frac{1}{2}$      D) $\frac{\sqrt{3}}{2}$

17. (97-9-30) Hisoblang.

$$arcctg(ctg(-3))$$

A) $\pi + 3$    B) $2\pi - 3$    C) $\frac{2\pi}{3} - 3$    D) $\pi - 3$

**Yechish:** $ctg(-3) = -ctg3$ tenglikdan hamda 3-xossadan $arcctg(ctg(-3)) = \pi - arcctg(ctg3)$ kelib chiqadi. Endi $0 < 3 < \pi$ bo'lganligi uchun 10-xossadan foydalansak

$$arcctg(ctg(-3)) = \pi - arcctg(ctg3) = \pi - 3$$

ni olamiz. **Javob:** $\pi - 3$ (D).

18. (96-7-60) Hisoblang.

$$\sin\left(2\arcsin\frac{1}{3}\right)$$

A) $\frac{2}{3}$      B) $\frac{2\sqrt{2}}{3}$      C) $\frac{4\sqrt{2}}{9}$      D) $\frac{2\sqrt{2}}{9}$

19. (97-4-63) Hisoblang.

$$\sin\left(2\arccos\frac{1}{3}\right)$$

A) $\frac{2}{3}$   B) $\frac{2}{9}$   C) $\frac{4\sqrt{2}}{9}$   D) $\frac{4\sqrt{2}}{3}$

20. (00-6-54) Ifodaning qiymatini toping.

$$\cos\left(2\arcsin\frac{2}{5}\right)$$

A) $\frac{9}{25}$   B) $\frac{1}{5}$   C) $\frac{4}{5}$   D) $\frac{17}{25}$

21. (98-12-76) Hisoblang.

$$\sin\left(\frac{1}{2}\arccos\frac{1}{9}\right)$$

A) $\frac{2}{3}$   B) $\frac{4}{9}$   C) $\frac{8}{9}$   D) $\frac{3}{4}$

22. (01-1-47) Ifodaning qiymatini toping:

$$arctg3 - \arcsin\frac{\sqrt{5}}{5}$$

A) $0$   B) $\frac{\pi}{6}$   C) $\frac{\pi}{3}$   D) $\frac{\pi}{4}$

23. (02-1-54) Hisoblang.

$$\cos(arctg\sqrt{3} + \arccos\frac{\sqrt{3}}{2})$$

A) $1$   B) $\frac{\sqrt{3}}{2}$   C) $\frac{1}{2}$   D) $0$

24. (02-2-50) Tenglamani yeching.

$$\frac{\pi}{24}(8x+1) = \arccos(-\frac{1}{2}) + \arcsin\frac{1}{2} - \frac{1}{2}arctg1$$

A) $4$   B) $6$   C) $5$   D) $2$

25. (02-4-31) Hisoblang.

$$12\arcsin(-\frac{1}{2})/\pi$$

A) $0$   B) $-2$   C) $2$   D) $1$

26. (02-4-32) $tg(arctg3 + arctg7)$ ni hisoblang.
A) $0$   B) $0,5$   C) $-0,5$   D) $0,25$

**Yechish:** Agar $arctg3 = \alpha$, $arctg7 = \beta$ desak, 9-xossadan $tg\alpha = 3$, $tg\beta = 7$ ni olamiz. U holda qo'shish formulalarining 5-dan

$$tg(\alpha + \beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha\, tg\beta} = \frac{3+7}{1 - 3\cdot 7} = -\frac{1}{2}$$

kelib chiqadi.   **Javob:** $-0,5$ (C).

27. (02-7-19) Hisoblang.

$$arcctg3 - arctg2$$

A) $-\frac{\pi}{4}$   B) $\frac{\pi}{6}$   C) $\frac{\pi}{4}$   D) $\frac{\pi}{2}$

28. (02-7-34)   $tg(\pi - \arcsin\frac{3}{5})$ ni hisoblang.

A) $-\frac{3}{4}$   B) $\frac{3}{4}$   C) $\frac{1}{4}$   D) $\frac{1}{5}$

29. (02-12-39) Hisoblang.

$$\cos\left(2\arcsin\frac{4}{5}\right)$$

A) $-0,28$   B) $0,28$   C) $-0,26$   D) $0,26$

30. (03-5-45) Hisoblang.

$$\sin(2arctg3) - \cos(2arctg2)$$

A) $1,2$   B) $0,4$   C) $-0,8$   D) $0,8$

## 13.4   Trigonometrik tenglamalar

**1. Eng sodda trigonometrik tenglamalar**

$\sin x = a$, $\cos x = a$, $tgx = a$, $ctgx = a$ ko'rinishdagi tenglamalarga eng sodda trigonometrik tenglamalar deyiladi. $\sin x = a$, $\cos x = a$ tenglamalar $|a| > 1$ da yechimga ega emas. $tgx = a$, $ctgx = a$ tenglamalar ixtiyoriy $a \in R$ da yechimga ega. Endi sodda trigonometrik tenglamalarning yechimlari beramiz:

1. $\sin x = a$,   $|a| \le 1$, **yechimi:**
   $x = (-1)^n \arcsin a + \pi n$, $n \in Z$.

2. $\sin x = 0$, **yechimi:** $x = \pi n$, $n \in Z$.

3. $\sin x = -1$, **yechimi:** $x = -\frac{\pi}{2} + 2\pi n$.

4. $\sin x = 1$, **yechimi:** $x = \frac{\pi}{2} + 2\pi n$.

5. $\cos x = a$,   $|a| \le 1$, **yechimi:**
   $x = \pm\arccos a + 2\pi n$,   $n \in Z$.

6. $\cos x = 0$, **yechimi:** $x = \frac{\pi}{2} + \pi n$.

7. $\cos x = -1$, **yechimi:** $x = \pi + 2\pi n$.

8. $\cos x = 1$, **yechimi:** $x = 2\pi n$.

9. $tgx = a$, **yechimi:** $x = arctga + \pi n$.

10. $ctgx = a$, **yechimi:** $x = arcctga + \pi n$.

11. $\cos x = \cos y$, **yechimi:** $y = \pm x + 2\pi n$.

12. $\sin x = \sin y$, **yechimi:** $y = (-1)^n x + \pi n$.

13. $tgx = tgy$, **yechimi:** $y = x + \pi n$.

14. $ctgx = ctgy$, **yechimi:** $y = x + \pi n$.

15. $\sin^2 x = a$, $0 \le a \le 1$, **yechimi:**
    $x = \pm\arcsin\sqrt{a} + \pi n$, $n \in Z$.

16. $\cos^2 x = a$, $0 \le a \le 1$, **yechimi:**
    $x = \pm\arccos\sqrt{a} + \pi n$, $n \in Z$.

1. (96-6-43) Tenglamani yeching.

$$2\sin x = -1$$

A) $-\dfrac{\pi}{6} + 2\pi k, k \in Z$

B) $-\dfrac{\pi}{6} + \pi k, k \in Z$

C) $(-1)^k \dfrac{\pi}{6} + \pi k, k \in Z$

D) $(-1)^{k+1} \dfrac{\pi}{6} + \pi k, k \in Z$

**Yechish:** Berilgan tenglama $\sin x = -0,5$ tenglamaga teng kuchli. Bu yerda $a = -0,5 \in [-1; 1]$ bo'lganligi uchun tenglama yechimga ega. Tenglamaning yechimini 1-formula yordamida topamiz:

$$x = (-1)^k \arcsin(-0,5) + \pi k = (-1)^{k+1}\dfrac{\pi}{6} + \pi k, \ k \in Z.$$

**Javob:** $(-1)^{k+1}\dfrac{\pi}{6} + \pi k, \ k \in Z.$ (D).

2. (97-12-42) Tenglamani yeching.

$$2\sin x = -\sqrt{3}$$

A) $(-1)^k \dfrac{\pi}{3} + \pi k, \quad k \in Z$

B) $\pm\dfrac{\pi}{6} + 2\pi k, \quad k \in Z$

C) $(-1)^k \dfrac{\pi}{6} + \pi k, \quad k \in Z$

D) $(-1)^{k+1} \dfrac{\pi}{3} + \pi k, \quad k \in Z$

3. (96-11-60) Tenglamani yeching.

$$\sin\left(3x - \dfrac{\pi}{2}\right) = 0$$

A) $\dfrac{\pi}{3}n, n \in Z$ 　　 B) $\dfrac{\pi}{6} + \dfrac{\pi}{3}n, n \in Z$

C) $3\pi n, n \in Z$ 　　 D) $\dfrac{\pi}{2} + \dfrac{\pi}{3}n, n \in Z$

4. (96-12-44) Tenglamaning yechimini toping.

$$\cos\left(2x - \dfrac{\pi}{2}\right) = 0$$

A) $\dfrac{\pi}{2}n, \quad n \in Z$ 　　 B) $\dfrac{\pi}{2}$

C) $\pi n, \quad n \in Z$ 　　 D) $\dfrac{\pi}{2} + \dfrac{\pi}{4}n, \quad n \in Z$

5. Tenglamaning yechimini toping.

$$tg\left(x + \dfrac{\pi}{4}\right) = 1$$

A) $\dfrac{\pi}{2}n, \quad n \in Z$ 　　 B) $\dfrac{\pi}{2}$

C) $\pi n, \quad n \in Z$ 　　 D) $\dfrac{\pi}{2} + \dfrac{\pi}{4}n, \quad n \in Z$

6. Tenglamaning yechimini toping.

$$ctg\left(2x + \dfrac{\pi}{6}\right) = \sqrt{3}$$

A) $\dfrac{\pi}{2}n, \quad n \in Z$ 　　 B) $\dfrac{\pi}{2}$

C) $\pi n, \quad n \in Z$ 　　 D) $\dfrac{\pi}{2} + \dfrac{\pi}{4}n, \quad n \in Z$

7. (98-12-58) Tenglamani yeching.

$$2\sin 2x = -1$$

A) $(-1)^{n+1}\dfrac{\pi}{12} + \dfrac{\pi n}{2}, \quad n \in Z$

B) $(-1)^{n}\dfrac{\pi}{12} + \dfrac{\pi n}{2}, \quad n \in Z$

C) $(-1)^{n+1}\dfrac{\pi}{6} + \pi n, \quad n \in Z$

D) $(-1)^{n+1}\dfrac{\pi}{6} + \dfrac{\pi n}{2}, \quad n \in Z$

8. (96-3-58) Tenglamani yeching.

$$\sin\left(2x - \dfrac{\pi}{2}\right) = 0$$

A) $\dfrac{\pi}{4} + \pi n, \quad n \in Z$ 　　 B) $\dfrac{\pi}{2}n, \quad n \in Z$

C) $\dfrac{\pi}{4} + \dfrac{\pi}{2}n, \quad n \in Z$ 　　 D) $\pi n, \quad n \in Z$

9. (97-1-53) Quyidagi sonlardan qaysi biri

$$\sin \dfrac{\pi x}{2} = 1$$

tenglamaning ildizi emas.

A) 2005 　　 B) 2010 　　 C) 2001 　　 D) 2009

10. (97-6-52) Quyidagi sonlardan qaysi biri

$$\cos \dfrac{\pi x}{2} = 1$$

tenglamaning ildizi emas.

A) 2000 　　 B) 2010 　　 C) 2004 　　 D) 2012

11. (97-4-40) 　 $\cos x = \dfrac{\sqrt{2}}{2}$ tenglamaning $(0; 2\pi)$ oraliqqa tegishli yechimlarini toping.

A) $\dfrac{3\pi}{4}; \dfrac{5\pi}{4}$ 　 B) $\dfrac{\pi}{4}; \dfrac{7\pi}{4}$ 　 C) $\dfrac{3\pi}{4}; \dfrac{7\pi}{4}$ 　 D) $\dfrac{5\pi}{4}; \dfrac{7\pi}{6}$

**Yechish:** 5-formuladan foydalanib $\cos x = \dfrac{\sqrt{2}}{2}$ tenglamaning barcha yechimlarini topamiz:

$$x = \pm\dfrac{\pi}{4} + 2\pi k, \quad k \in Z.$$

Bu yechimlar ichidan $k = 0$ bo'lganda $\dfrac{\pi}{4}$, $k = 1$ bo'lganda $-\dfrac{\pi}{4} + 2\pi = \dfrac{7\pi}{4}$ largina $(0; 2\pi)$ oraliqqa tegishli bo'ladi. **Javob:** $\dfrac{\pi}{4}; \dfrac{7\pi}{4}$ (B).

12. (01-5-17) Ushbu

$$\sin \dfrac{\pi}{x} = 1$$

tenglamaning $[0,05; 0,1]$ oraliqda nechta ildizi bor?

A) 5 　　 B) 1 　　 C) 2 　　 D) 3

13. (98-3-59) Tenglamaning $[0; 2\pi]$ kesmada nechta ildizi bor?

$$\sin 2x = (\cos x - \sin x)^2$$

A) 4 　　 B) 8 　　 C) 2 　　 D) 1

14. (02-9-40) Tenglamaning $(1;5)$ oraliqda nechta ildizi bor?

$$ctg\left(\frac{\pi}{2}(x-1)\right)=0$$

A) 1     B) 2     C) 3     D) 4

15. (03-5-43) Tenglama $[\pi;2\pi]$ kesmada nechta ildizga ega?

$$\frac{|\cos x|}{\cos x}=\cos 2x-1$$

A) 1     B) 2     C) 3     D) 4

16. (98-11-102) Tenglamaning eng kichik musbat ildizini toping.

$$tg\pi x^2 = tg(\pi x^2+2\pi x)$$

A) $\frac{1}{2}$     B) $\frac{1}{3}$     C) 1     D) $\frac{3}{4}$

**Yechish:** 13-formulaga ko'ra, berilgan tenglamaning barcha yechimlari

$$\pi x^2+2\pi x=\pi x^2+n\pi \Longleftrightarrow x=\frac{n}{2},\quad n\in Z$$

ko'rinishda bo'ladi. Musbat yechimlar $x=\frac{n}{2}$, $n\in N$ shaklda bo'ladi. Eng kichik natural son 1 bo'lganligi uchun, eng kichik musbat ildiz $\frac{1}{2}$ bo'ladi.

**Javob:** $\frac{1}{2}$ (A).

17. Tenglamaning eng kichik musbat ildizini toping.

$$\sin^2\pi x=1$$

A) $\frac{1}{2}$     B) $\frac{1}{3}$     C) 1     D) $\frac{3}{4}$

18. $\cos x=\cos(2x+\pi)$ tenglamaning eng kichik musbat ildizini toping.

A) $\frac{\pi}{3}$     B) $\frac{\pi}{6}$     C) $\pi$     D) $2\pi$

19. (02-7-12) Tenglamani yeching.

$$\sin(\pi\cos x)=0$$

A) $\frac{n\pi}{2}$, $n\in Z$    B) $\pi+2n\pi$, $n\in Z$
C) $\frac{\pi}{2}+n\pi$, $n\in Z$    D) $2n\pi$, $n\in Z$

20. $ctgx=ctg2x$ tenglamani yeching.
A) $n\pi$, $n\in Z$    B) $\pi+2n\pi$, $n\in Z$
C) $\frac{\pi}{2}+n\pi$, $n\in Z$    D) $2n\pi$, $n\in Z$

**2. Qo'shish formulalari yordamida yechiladigan tenglamalar**

21. Tenglamani yeching.

$$\cos 3x\cdot\cos x+1=\sin 3x\cdot\sin x$$

A) $\frac{\pi}{4}+\frac{\pi k}{2}$, $k\in Z$    B) $\frac{\pi}{6}+2\pi k$, $k\in Z$
C) $\frac{\pi}{6}+\pi k$, $k\in Z$    D) $\pm\frac{\pi}{6}+\frac{\pi k}{2}$, $k\in Z$

**Yechish:** Berilgan tenglama

$$\cos 3x\cdot\cos x-\sin 3x\cdot\sin x=-1$$

tenglamaga teng kuchli. 13.2.3-dagi 3-formulaga ko'ra, bu tenglamani $\cos 4x=-1$ ko'rinishda yozish mumkin. Bu tenglamaning yechimlarini 7-formuladan foydalanib topamiz:

$$4x=\pi+2\pi n\Longleftrightarrow x=\frac{\pi}{4}+\frac{\pi n}{2},\ n\in Z.$$

**Javob:** $\frac{\pi}{4}+\frac{\pi n}{2}$, $n\in Z$ (A).

22. (96-3-60) Tenglamaning yechimini ko'rsating.

$$\sin x\cdot\cos 2x+\cos x\cdot\sin 2x=0$$

A) $\frac{\pi n}{4}$, $n\in Z$    B) $\frac{\pi n}{3}$, $n\in Z$
C) $\frac{\pi n}{2}$, $n\in Z$    D) $\frac{\pi n}{5}$, $n\in Z$

23. (96-10-28) Tenglamaning ildizlarini ko'rsating.

$$\sin 5x\cdot\cos 2x=\cos 5x\cdot\sin 2x-1$$

A) $\pm\frac{\pi}{3}+2\pi k$, $k\in Z$    B) $\frac{\pi}{3}+\frac{2\pi k}{3}$, $k\in Z$
C) $-\frac{\pi}{6}+\frac{2\pi k}{3}$, $k\in Z$    D) $\frac{\pi}{4}+\pi k$, $k\in Z$

24. (96-11-10) Tenglamani yeching.

$$\cos 2x\cdot\sin 3x+\sin 2x\cdot\cos 3x=\frac{1}{2}$$

A) $(-1)^n\frac{\pi}{5}+\frac{\pi}{5}n$, $n\in Z$    B) $\frac{\pi}{4}n$, $n\in Z$
C) $\frac{\pi}{30}n$, $n\in Z$    D) $(-1)^n\frac{\pi}{30}+\frac{\pi}{5}n$, $n\in Z$

25. (96-12-53) Tenglamani yeching.

$$\sin x\cdot\cos 3x+\cos x\cdot\sin 3x=1$$

A) $\frac{\pi}{2}n$, $n\in Z$    B) $\frac{\pi}{8}+n\pi$, $n\in Z$
C) $\frac{\pi}{5}n$, $n\in Z$    D) $\frac{\pi}{8}+\frac{\pi}{2}n$, $n\in Z$

26. (97-4-42) $k$ ning quyida ko'rsatilgan qiymatlaridan qaysi birida

$$\sin kx\cdot\cos x-\sin x\cdot\cos kx=0$$

tenglamaning ildizlari $\frac{\pi n}{5}$, $n\in Z$ bo'ladi?
A) 5    B) 4    C) 6    D) 7

**3. Ko'paytmaga keltiriladigan tenglamalar**

27. (97-1-51) Tenglamaning eng kichik musbat ildizini toping.

$$(3\cos\pi x-\pi)\cdot(2\sin\pi x-\sqrt{3})=0$$

A) $\frac{\pi}{6}$    B) $\frac{1}{4}$    C) $\frac{1}{3}$    D) $\frac{1}{2}$

**Yechish:** Berilgan tenglamaning ildizlari

$$1)\ \cos\pi x=\frac{\pi}{3};\quad 2)\ \sin\pi x=\frac{\sqrt{3}}{2}$$

tenglama ildizlari birlashmasidan iborat. 1-tenglama yechimga ega emas, chunki $\frac{\pi}{3} > 1$. 2-tenglamaning ildizlari 1-formulaga ko'ra,

$$x = (-1)^n \frac{1}{3} + n, \ n \in Z$$

ko'rinishda bo'ladi. $n = 0$ da $x_0 = 1/3$ ildiz, tenglamaning eng kichik musbat ildizi bo'ladi.
**Javob:** $\frac{1}{3}$ (C).

28. (97-6-49) Tenglamaning $(90^0; 180^0]$ oraliqdagi ildizini toping.

$$\cos 2x \cdot \sin x - \cos 2x = 0$$

A) $120^0$     B) $135^0$     C) $150^0$     D) $180^0$

29. (97-6-50) Tenglamaning $[0; 3]$ oraliqda nechta ildizi bor?

$$(3\sin \pi x - \pi)(2\cos \pi x - 1) = 0$$

A) 1     B) 2     C) 3     D) 4

30. (97-6-54) Tenglamani yeching:

$$\sqrt{\cos x} \cdot \sin x = 0$$

A) $\frac{\pi}{2} + \pi k, \quad k \in Z$     B) $\pi k, \ k \in Z$
C) $2\pi k; \frac{\pi}{2} + \pi k, \quad k \in Z$     D) $\frac{\pi}{2} + 2\pi k, \ k \in Z$

31. (97-8-42) Tenglamani yeching.

$$tgx \cdot \cos x = 0$$

A) $2\pi k, \quad k \in Z$     B) $\frac{\pi}{4} + \pi k; \frac{\pi}{2} + 2\pi k, \quad k \in Z$

C) $\pi k, \quad k \in Z$     D) $\frac{\pi}{2} + \pi k, \quad k \in Z$

32. (97-12-63) Tenglamaning $[0^0; 60^0]$ oraliqdagi ildizini toping.

$$\cos x - \sin 2x \cos x = 0$$

A) $0^0$     B) $30^0$     C) $45^0$     D) $15^0$

33. (98-2-27) Tenglama yechimga ega bo'ladigan $b$ ning barcha qiymatlarini toping.

$$\cos x + \cos(120^0 - x) = b$$

A) $0 \leq b \leq 1$     B) $-1 \leq b \leq 1$
C) $-1 < b < 1$     D) $b \leq 1$

**Yechish:** Kosinuslar yig'indisini ko'paytmaga almashtirib, berilgan tenglamaga teng kuchli bo'lgan

$$2\cos 60^0 \cdot \cos(x - 60^0) = b \Longleftrightarrow \cos(x - 60^0) = b$$

tenglamaga ega bo'lamiz. Bu tenglama faqat $b \in [-1; 1]$ larda yechimga ega. **Javob:** $-1 \leq b \leq 1$ (B).

34. (98-9-25) Tenglama $k$ ning qanday qiymatlarida yechimga ega?

$$\sin(60^0 + x) - \sin(60^0 - x) = k$$

A) $k \in (-1; 1)$     B) $k \in [-1; 1]$
C) $k \leq 1$     D) $k \leq -1$

35. (02-7-18) Tenglamani yeching.

$$\sin 5x + \sin 3x + \sin x = 0$$

A) $\frac{\pi n}{3}; \quad \pm\frac{\pi}{3} + \pi n, \quad n \in Z$
B) $\frac{n\pi}{3}; \quad \frac{\pi}{2} + \frac{\pi n}{2}, \quad n \in Z$
C) $\frac{\pi}{2} + \frac{\pi n}{2}, \quad n \in Z$
D) $\frac{\pi n}{3}, \quad n \in Z$

**Yechish:** Sinuslar yig'indisi $\sin 5x + \sin x$ ni ko'paytmaga almashtirib, berilgan tenglamaga teng kuchli bo'lgan $2\sin 3x \cdot \cos 2x + \sin 3x = 0$ yoki $\sin 3x(2 \cdot \cos 2x + 1) = 0$ tenglamaga ega bo'lamiz. Bu tenglamaning yechimlari

$$\sin 3x = 0 \quad \text{va} \quad \cos 2x = -\frac{1}{2}$$

tenglama yechimlari birlashmasidan iborat. Bu tenglamalar eng sodda trigonometrik tenglamalar bo'lib, ularning yechimlari 2 va 5-formulalar yordamida topiladi: $x = \frac{\pi n}{3}$, $x = \pm\frac{\pi}{3} + \pi n, \ n \in Z$.
**Javob:** $\frac{\pi n}{3}; \quad \pm\frac{\pi}{3} + \pi n, \quad n \in Z$. (A).

36. (00-10-57) Ushbu

$$\sin 2x + \sin 4x = 0$$

tenglama $[0; 2\pi]$ oraliqda nechta ildizga ega?
A) 0     B) 7     C) 4     D) 9

37. (02-1-61) Tenglamani yeching.

$$\sin 6x + \sin 2x = \sin 4x$$

A) $\frac{\pi n}{4}, \quad n \in Z$     B) $\frac{\pi}{3} + 2\pi n, \quad n \in Z$
C) $-\frac{\pi}{3} + \pi n, \ n \in Z$     D) $\frac{\pi n}{4}, \pm\frac{\pi}{6} + \pi n, \ n \in Z$

38. (03-6-63) Qanday eng kichik o'tkir burchak

$$\sin(2x + 45^0) = \cos(30^0 - x)$$

tenglamani qanoatlantiradi?
A) $25^0$     B) $5^0$     C) $45^0$     D) $15^0$

39. (98-1-59) Tenglama $[0; \pi]$ kesmada nechta ildizga ega?.

$$\cos x \cdot \cos 4x - \cos 5x = 0$$

A) 1     B) 2     C) 4     D) 5

**Yechish:** $\cos 5x = \cos(x + 4x)$ ga qo'shish formulasini (13.2.3-ning 3-ga qarang) qo'llab, keyin

o'xshash hadlarni ixchamlab berilgan tenglamaga teng kuchli bo'lgan

$$\sin x \cdot \sin 4x = 0$$

tenglamaga ega bo'lamiz. Bu tenglamadan $\sin x = 0$ yoki $\sin 4x = 0$ ni olamiz. $\sin x = 0$ tenglama $[0; \pi]$ kesmada ikkita 0 va $\pi$ ildizlarda ega, $\sin 4x = 0$ tenglama esa $[0; \pi]$ kesmada 5 ta 0, $\frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}$ va $\frac{4\pi}{4} = \pi$ ildizlarda ega. 1-tenglamaning ildizlari 0 va $\pi$ lar, 2-tenglamaning yechimlari ichida uchraganligi uchun ular bir marta sanaladi. Demak, berilgan tenglama $[0; \pi]$ kesmada 5 ta ildizga ega ekan. **Javob:** 5 (D).

40. (98-8-59) Tenglama $[0; 2\pi]$ oraliqda nechta ildizga ega?
$$\cos x \cdot \cos 2x = \cos 3x$$

A) 5      B) 4      C) 3      D) 2

41. Tenglamaning I va II chorakdagi ildizlari yig'indisini toping?
$$\sin(3x - 45^0) = 0$$

A) $135^0$      B) $150^0$      C) $210^0$      D) $225^0$

42. (02-1-19) Tenglamaning $[0; \frac{\pi}{2}]$ kesmadagi ildizlari yig'indisini toping?
$$\cos 4x \cdot \cos 5x = \cos 6x \cdot \cos 7x$$

A) $\frac{41\pi}{22}$    B) $\frac{31\pi}{22}$    C) $\frac{30\pi}{11}$    D) $\frac{43\pi}{22}$

43. (02-10-60) Tenglamani yeching.
$$\cos\left(\frac{3\pi + x}{3}\right) \cdot \cos\left(\frac{9\pi + 2x}{6}\right) = \frac{1}{4}$$

A) $(-1)^{n+1}\frac{\pi}{3} + 2\pi n; \quad n \in Z$

B) $(-1)^{n+1}\frac{\pi}{6} + \pi n; \quad n \in Z$

C) $(-1)^n\frac{\pi}{3} + 2\pi n; \quad n \in Z$

D) $(-1)^n\frac{\pi}{6} + \frac{3\pi n}{2}; \quad n \in Z$

**4. Bir xil ismga keltiriladigan tenglamalar**

44. (97-1-46) Tenglamani yeching.
$$2\cos^2(x - \pi) + 3\sin(\pi + x) = 0$$

A) $\frac{\pi}{2} + \pi n, \quad n \in Z$      B) $(-1)^n\frac{\pi}{6} + \pi n, \quad n \in Z$

C) $\pm\frac{\pi}{3} + 2\pi n, \quad n \in Z$      D) $\pm\frac{\pi}{6} + 2\pi n, \quad n \in Z$

**Yechish:** Keltirish formulalari

$$\cos(x - \pi) = -\cos x; \; \sin(\pi + x) = -\sin x$$

ni qo'llab, berilgan tenglamani

$$2\cos^2 x - 3\sin x = 0$$

shaklda yozib olamiz. Bu tenglamani xil ismga keltirish uchun $\cos^2 x = 1 - \sin^2 x$ ayniyatdan foydalanamiz, natijada

$$2\sin^2 x + 3\sin x - 2 = 0$$

tenglamani olamiz. Agar $\sin x = y$ belgilash olsak oxirga tenglama $2y^2 + 3y - 2 = 0$ kvadrat tenglamaga keladi. Bu kvadrat tenglamaning ildizlari $y_1 = -2$ va $y_2 = 0,5$ lardir. $\sin x = y_1 = -2$ tenglama ildizga ega emas, chunki $|-2| > 1$. $\sin x = y_2 = 0,5$ tenglamaning ildizlari

$$x = (-1)^n\frac{\pi}{6} + \pi n, \quad n \in Z$$

ko'rinishga ega. **Javob:** (B).

45. (97-11-45) Tenglamani yeching.
$$2\sin^2(\pi - x) + 5\sin(1,5\pi + x) = 2$$

A) $\pi n, \quad n \in Z$      B) $\frac{\pi}{2} + \pi n, \quad n \in Z$

C) $\frac{\pi}{2} + 2\pi n, n \in Z$    D) $(-1)^n \cdot \frac{\pi}{6} + \pi n, \quad n \in Z$

46. (97-1-50) Tenglamaning $(0^0; 90^0]$ oraliqdagi ildizini toping.
$$2\sin^2 x - \sqrt{3}\sin 2x = 0$$

A) $30^0$      B) $45^0$      C) $60^0$      D) $90^0$

47. (00-3-52) Tenglamaning $[0; 2\pi]$ kesmadagi eng katta va eng kichik ildizlari ayirmasini toping?
$$\cos^2 x - \frac{1}{2}\sin 2x = 0$$

A) $\frac{\pi}{2}$    B) $\frac{3\pi}{4}$    C) $\pi$    D) $\frac{5\pi}{4}$

48. (02-3-79) Tenglama $[-2\pi; \pi]$ kesmada nechta ildizga ega?.
$$tgx + \frac{1}{tgx} = 2$$

A) 3     B) 5     C) 4     D) 6

49. (00-5-41) Tenglamani yeching.
$$\cos 2x - 5\sin x - 3 = 0$$

A) $(-1)^n\frac{\pi}{6} + \pi n, \quad n \in Z$

B) $(-1)^{n+1}\frac{\pi}{6} + \pi n, \quad n \in Z$

C) $(-1)^n\frac{\pi}{6} + 2\pi n, \quad n \in Z$

D) $(-1)^{n+1}\frac{\pi}{6} + 2\pi n, \quad n \in Z$

**Yechish:** $\cos 2x = 1 - 2\sin^2 x$ ayniyatdan foydalanib, berilgan tenglamani

$$2\sin^2 x + 5\sin x + 2 = 0$$

shaklda yozib olamiz. Bu tenglamada $\sin x = y$ belgilash olib, uni $2y^2 + 5y + 2 = 0$ kvadrat tenglamaga keltiramiz. Bu kvadrat tenglamaning ildizlari $y_1 = -2$ va $y_2 = -0,5$ lardir. $\sin x =$

$y_1 = -2$ tenglama ildizga ega emas, chunki $|-2| > 1$. $\sin x = y_2 = -0,5$ tenglamaning ildizlari

$$x = (-1)^{n+1}\frac{\pi}{6} + \pi n, \quad n \in Z$$

ko'rinishga ega. **Javob:** (B).

50. (02-11-43) Tenglamaning $(-90^0; 180^0)$ intervalga tegishli ildizlari yig'indisini toping.

$$3\sin^2 2x + 7\cos 2x - 3 = 0$$

A) $90^0$     B) $105^0$     C) $180^0$     D) $135^0$

51. (02-11-44) Tenglamaning $[-4\pi; 4\pi]$ kesmaga tegishli ildizlari nechta?

$$\cos 2x + 5\cos x = 6$$

A) 4     B) 5     C) 6     D) 8

52. (02-12-40) Tenglamaning $[0; 2\pi]$ kesmadagi ildizlari yig'indisini hisoblang.

$$\cos 2x - 2\sin^2 x = 0$$

A) $3,5\pi$     B) $3\frac{1}{6}\pi$     C) $4\pi$     D) $4\frac{1}{6}\pi$

53. (03-4-25) Tenglama ildizlari yig'indisini toping.

$$1 - \sin x - \cos 2x = 0 \quad (x \in [0; 2\pi])$$

A) $3,5\pi$     B) $4,2\pi$     C) $4\pi$     D) $3,8\pi$

**5. Darajani pasaytirish usuli yordamida yechiladigan tenglamalar**

Darajani pasaytirish formulalari quyidagilardir:

- $\sin^2 x = \dfrac{1 - \cos 2x}{2} \Longleftrightarrow 1 - \cos 2x = 2\sin^2 x$.
- $\cos^2 x = \dfrac{1 + \cos 2x}{2} \Longleftrightarrow 1 + \cos 2x = 2\cos^2 x$.

54. (98-2-26) Tenglamani yeching.

$$2\cos^2 x - 1 = -\frac{1}{2}$$

A) $(-1)^k\dfrac{\pi}{6} + \dfrac{\pi}{2}k; \quad k \in Z$

B) $(-1)^{k+1}\dfrac{\pi}{6} + \pi k, \quad k \in Z$

C) $\pm\dfrac{\pi}{6} + \pi k, \quad k \in Z$

D) $\pm\dfrac{\pi}{3} + \pi k, \quad k \in Z$

**Yechish:** Darajani pasaytirish formulasining 2-dan foydalanib, berilgan tenglamani $\cos 2x = -0,5$ shaklda yozib olamiz. Bu tenglama eng sodda trigonometrik tenglama bo'lib, uning ildizlari

$$x = \pm\frac{\pi}{3} + \pi n, \quad n \in Z$$

ko'rinishga ega. **Javob:** (D).

55. (98-6-50) Tenglamani yeching.

$$4\cos^2 2x - 1 = \cos 4x$$

A) $\dfrac{\pi}{4} + \dfrac{\pi n}{2}, \quad n \in Z$     B) $\dfrac{\pi n}{2}, \quad n \in Z$

C) $\dfrac{\pi}{6} + \dfrac{\pi n}{2}, \quad n \in Z$     D) $\dfrac{\pi}{3} + \dfrac{\pi n}{2}, \quad n \in Z$

56. (96-9-50) Ushbu

$$4\sin\frac{x}{2} - \cos x + 1 = 0$$

tenglamaning $[0; 2\pi]$ kesmada nechta ildizi bor?
A) 0     B) 2     C) 3     D) 1

57. (96-12-97) Ushbu

$$\sin\frac{x}{2} + \cos x - 1 = 0$$

tenglama $[0; 2\pi]$ oraliqda nechta yechimga ega?
A) 3     B) 4     C) 0     D) 2

58. (96-13-43) Tenglamaning $[0; 2\pi]$ kesmada nechta ildizi bor?

$$4\cos\frac{x}{2} + \cos x + 1 = 0$$

A) 1     B) 2     C) 0     D) 3

59. (98-11-99) Tenglamani yeching.

$$2\cos^2\frac{x}{2} = 1 + \cos x + \cos 2x$$

A) $\dfrac{\pi}{4} + \dfrac{\pi k}{2}, \quad k \in Z$     B) $\dfrac{\pi}{4} + \pi k, \quad k \in Z$

C) $\dfrac{\pi k}{2}, \quad k \in Z$     D) $\pi k, \quad k \in Z$

60. (01-1-48) Tenglamani yeching.

$$4\sin^2 x(1 + \cos 2x) = 1 - \cos 2x$$

A) $\pi n, n \in Z$     B) $\pi n; \pm\dfrac{\pi}{3} + \pi n, n \in Z$

C) $\pm\dfrac{\pi}{3} + \pi n, n \in Z$     D) $\pi n; \pm\dfrac{\pi}{3} + 2\pi n, n \in Z$

**Yechish:** Darajani pasaytirish formulasining 1-dan foydalanib, berilgan tenglamani

$$2(1 - \cos 2x)(1 + \cos 2x) = 1 - \cos 2x$$

shaklda yozib olamiz. Bu ifodada $1 - \cos 2x$ umumiy ko'paytuvchini qavs oldiga chiqarib

$$(1 - \cos 2x)(1 + 2\cos 2x) = 0$$

tenglamani olamiz. Bu yerdan $\cos 2x = 1$ yoki $\cos 2x = -0,5$ tenglamaga kelamiz. Bular eng sodda trigonometrik tenglamalar bo'lib, ularning ildizlari

$$x = \pi n, \; n \in Z; \quad x = \pm\frac{\pi}{3} + \pi n, n \in Z$$

ko'rinishga ega. **Javob:** (B).

61. (99-10-34) Tenglamani yeching.

$$(1 + \cos x)tg\frac{x}{2} = 0$$

A) $\pi k, \quad k \in Z$      B) $\pi + 2\pi k, \quad k \in Z$
C) $2\pi k, \quad k \in Z$      D) $\pi + \pi k, \quad k \in Z$

62. (01-2-81) Ushbu

$$7\cos 2x - 6 = \cos 4x$$

tenglamaning $[0; 628]$ kesmaga tegishli ildizlari yig'indisini toping.
A) $200\pi$    B) $199\pi$    C) $20100\pi$    D) $19900\pi$

63. (02-6-44) Tenglama $[0; 2\pi]$ kesmada nechta ildizga ega?

$$3\sin 2x - 2\cos 2x = 2$$

A) 5      B) 1      C) 2      D) 4

64. (03-10-41) Tenglamani yeching.

$$\sin^2 x + \sin^2 4x = \sin^2 2x + \sin^2 3x$$

A) $\dfrac{\pi n}{2}, \quad n \in Z$

B) $\dfrac{\pi}{5} + \dfrac{2\pi n}{5}, \quad n \in Z$

C) $\dfrac{\pi}{10} + \dfrac{2\pi n}{5}, \quad n \in Z$

D) $\dfrac{\pi}{10} + \dfrac{\pi n}{5}; \dfrac{\pi n}{2}, \quad n \in Z$

**6. Quyidagi tenglamalarni yechishda uning aniqlanish sohasiga e'tibor bering**

65. (98-1-56) Tenglamani yeching.

$$\frac{\sin 2x}{tgx - 1} = 0$$

A) $\dfrac{\pi k}{2}, \quad k \in Z$      B) $\dfrac{\pi}{2} + \pi k, k \in Z$
C) $2\pi k, \quad k \in Z$      D) $\pi k, k \in Z$

**Yechish:** Berilgan tenglama

$$tgx - 1 \neq 0, \quad \cos x \neq 0$$

shartda aniqlangan. Kasr nolga aylanishi uchun uning surati, ya'ni $\sin 2x = 0$ bo'lishi kerak. Bu tenglamani $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ekanidan foydalanib $2\sin x\cos x = 0$ ko'rinishda yozamiz. Bu yerdan $\cos x \neq 0$ ni e'tiborga olib, $\sin x = 0$ tenglamani, bundan esa $x = \pi k, \ k \in Z$ ekanini hosil qilamiz. Bu nuqtalarda $tgx - 1 \neq 0$ shart ham bajariladi. **Javob:** $\pi k, \quad k \in Z$ (D).

66. (97-7-59) Tenglama $[0; 4\pi]$ oraliqda nechta ildizga ega?

$$\frac{\sin^2 x + \sin x}{\cos x} = 0$$

A) 5      B) 4      C) 7      D) 2

67. (97-12-65) Tenglama $[-2\pi; 2\pi]$ oraliqda nechta yechimga ega?

$$\frac{\cos^2 x - \cos x}{\sin x} = 0$$

A) 6      B) 4      C) 3      D) 2

68. (98-9-26) Tenglamani yeching.

$$\frac{1}{\cos^2 x} = 2tg^2 x$$

A) $\pm\dfrac{\pi}{4} + 2\pi k, k \in Z$      B) $\pm\dfrac{\pi}{4} + \pi k, k \in Z$
C) $\pm\dfrac{\pi}{3} + \pi k, \quad k \in Z$      D) $\pm\dfrac{\pi}{3} + 2\pi k, \quad k \in Z$

69. (99-1-44) $ctg(x+1) \cdot tg(2x-3) = 1$ tenglamaning $[\pi; 2\pi]$ oraliqdagi yechimini toping.
A) 4      B) 2      C) 3      D) 5

70. (00-4-47) Tenglamaning $[\pi; 3\pi]$ kesmadagi ildizlari yig'indisini toping.

$$\sqrt{1 - \cos x} = \sin x$$

A) $2\pi$      B) $5\pi$      C) $6\pi$      D) $4, 5\pi$

71. (98-10-105) Tenglamaning $[0; 2\pi]$ kesmada nechta ildizi bor?
$$\frac{1 + \cos x}{\sin x} = \cos\frac{x}{2}$$

A) 0      B) 1      C) 2      D) 3

72. (01-2-32) Tenglamani yeching.

$$\frac{\cos 3x}{\sin 3x - 2\sin x} = tgx$$

A) $\dfrac{\pi}{4} + \pi n, \quad n \in Z$      B) $\dfrac{\pi}{4} + 2\pi n, n \in Z$
C) $\dfrac{\pi}{4} + \dfrac{\pi}{2}n, \quad n \in Z$      D) $\dfrac{\pi}{3} + \dfrac{\pi}{2}n, n \in Z$

73. (01-6-30) Tenglamaning $[0; 4\pi]$ kesmadagi ildizlari yig'indisini toping.

$$tg^2 x - \frac{2}{\cos x} + 1 = 0$$

A) $7\pi$      B) $7\dfrac{2}{3}\pi$      C) $8\pi$      D) $7\dfrac{1}{3}\pi$

**Yechish:** Berilgan tenglama $\cos x \neq 0$ shartda aniqlangan. $1 + tg^2 x = \dfrac{1}{\cos^2 x}$ ayniyatdan foydalanib berilgan tenglamani

$$\frac{1}{\cos^2 x} - \frac{2}{\cos x} = 0 \Longleftrightarrow \frac{1 - 2\cos x}{\cos x} = 0$$

ko'rinishda yozamiz. Bu yerdan $\cos x = 0, 5$ ni, bundan esa $x = \pm\dfrac{\pi}{3} + 2\pi k, \ k \in Z$ ekanini hosil qilamiz. Bu yechimlardan 4 tasi $\dfrac{\pi}{3}; 2\pi \pm \dfrac{\pi}{3}; \ 4\pi - \dfrac{\pi}{3}$ lar $[0; 4\pi]$ kesmada yotadi. Ularning yig'indisi $8\pi$. **Javob:** $8\pi$ (C).

74. (01-10-37) Tenglamaning $[-\frac{\pi}{2}; \frac{\pi}{2}]$ kesmada nechta ildizi bor?
$$\cos 4x + \frac{10tgx}{1 + tg^2x} = 3$$
A) 0     B) 1     C) 2     D) 3

75. (01-11-21) Tenglamani yeching.
$$tgx\,tg3x = -1$$
A) $\frac{\pi}{2}k, k \in Z$     B) $\pi k, k \in Z$
C) $\frac{\pi}{4} + \frac{\pi}{2}k, k \in Z$     D) $\frac{\pi}{4} + \pi k, k \in Z$

76. (98-3-58) Tenglamaning $[0; 4\pi]$ kesmada nechta ildizi bor?
$$\frac{\cos 2x}{\frac{\sqrt{2}}{2} + \sin x} = 0$$
A) 8     B) 6     C) 4     D) 2

77. (03-7-39) Tenglamani yeching.
$$\sqrt{\cos 2x + \sqrt{3}\sin x} = -2\cos x$$
A) $\frac{2\pi}{3} + 2k\pi, k \in Z$
B) $\frac{\pi}{3} + 2k\pi, k \in Z$
C) $(-1)^k\frac{\pi}{3} + \pi k, k \in Z$
D) $(-1)^k\frac{2\pi}{3} + 2\pi k, k \in Z$

**7. Turli tenglamalar**

78. (98-5-50) Tenglamani yeching.
$$4^{\cos^2 x + 2\cos x} = 1$$
A) $\pi n; \frac{\pi}{2} + 2\pi n, n \in Z$     B) $\frac{\pi}{2} + \pi n, n \in Z$
C) $\pi n; -\frac{\pi}{2} + 2\pi n, n \in Z$     D) $2\pi n, n \in Z$

**Yechish:** Berilgan tenglamani $4^{\cos^2 x + 2\cos x} = 4^0$ shaklda yozamiz. Bu tenglama
$$\cos^2 x + 2\cos x = 0 \iff (\cos x + 2)\cos x = 0$$
tenglamaga teng kuchli. $\cos x + 2 \neq 0$ bo'lganligi uchun $\cos x = 0$ bo'lib, uning yechimlari $x = \frac{\pi}{2} + \pi n, n \in Z$. **Javob:** $\frac{\pi}{2} + \pi n, n \in Z$ (B).

79. (99-7-48) Tenglamani yeching.
$$5 \cdot 5^{\sin^2 x + \cos 2x} = \frac{1}{25}$$
A) $\emptyset$     B) $\pi n, \quad n \in Z$
C) $\frac{\pi}{2} + 2\pi n, n \in Z$     D) $2\pi n, \quad n \in Z$

80. (97-3-58) Tenglamani yeching.
$$2^{1 - \log_2 \sin x} = 4$$
A) $\frac{\pi}{6} + 2\pi n; n \in Z$     B) $(-1)^n\frac{\pi}{6} + \pi n, n \in Z$
C) $(-1)^n\frac{\pi}{3} + \pi n, n \in Z$     D) $\frac{\pi}{4} + 2\pi n, n \in Z$

81. (97-7-58) Tenglamani yeching.
$$3^{1 + \log_3 ctgx} = \sqrt{3}$$
A) $\frac{\pi}{6} + \pi n; \quad n \in Z$     B) $\frac{\pi}{3} + \pi n, n \in Z$
C) $\frac{\pi}{3} + 2\pi n, \quad n \in Z$     D) $\frac{\pi}{4} + \pi n, n \in Z$

82. (99-2-37) $a$ ning qanday qiymatlarida $\log_a \sin x = 1$ tenglama yechimga ega?
A) $a \in [-1; 1]$     B) $a \in (-1; 1)$
C) $a \in (0; 1]$     D) $a \in (0; 1)$

83. (02-9-36) Tenglamani yeching.
$$9^{\cos x} + 2 \cdot 3^{\cos x} = 15$$
A) $\pi n, \quad n \in Z$     B) $2\pi n, \quad n \in Z$
C) $\frac{\pi}{3} + 2\pi n, \quad n \in Z$     D) $\frac{\pi}{2} + \pi n, n \in Z$

84. (03-5-41) Tenglamani yeching.
$$8^{\sin^2 x} - 2^{\cos^2 x} = 0$$
A) $\pm\frac{\pi}{6} + \pi n, \quad n \in Z$     B) $\frac{\pi}{6} + \pi n, \quad n \in Z$
C) $-\frac{\pi}{6} + \pi n, \quad n \in Z$     D) $\frac{\pi}{4} + \pi n, \quad n \in Z$

85. (03-12-61) $a$ parametrning qanday qiymatlarida
$$\sin^6 x + \cos^6 x = a$$
tenglama yechimga ega?
A) $[0; 1]$     B) $[0, 5; 1]$
C) $[0, 25; 0, 5]$     D) $[0, 25; 1]$

## 13.5   Trigonometrik tengsizliklar

$\sin x \geq a$, $\cos x \geq a$, $tgx \geq a$, $ctgx \geq a$ tengsizliklar sodda trigonometrik tengsizliklar deyiladi. Bu yerda tengsizlik $>$ yoki $\leq$ yoki $<$ belgilaridan ixtiyoriy biri bo'lishi mumkin. Biz asosan ayniy almashtirishlar yordamida sodda trigonometrik tengsizliklarga yoki shu tipdagi tengsizliklar sistemasiga keladigan tengsizliklarni yechish usullarini beramiz. Sodda trigonometrik tengsizliklarning yechimlarini keltiramiz.

1. $\sin x \geq a$,     $-1 \leq a \leq 1$
$2n\pi + \arcsin a \leq x \leq -\arcsin a + (2n+1)\pi, n \in Z$.

2. $\sin x \leq a$,     $-1 \leq a \leq 1$
$(2n-1)\pi - \arcsin a \leq x \leq \arcsin a + 2n\pi, n \in Z$.

3. $\cos x \geq a$,     $-1 \leq a \leq 1$
$2n\pi - \arccos a \leq x \leq \arccos a + 2n\pi, n \in Z$.

4. $\cos x \leq a$,     $-1 \leq a \leq 1$
$2n\pi + \arccos a \leq x \leq 2(n+1)\pi - \arccos a, n \in Z$.

5. $tgx \geq b$,     $arctgb + n\pi \leq x < \frac{\pi}{2} + n\pi, \quad n \in Z$.

6. $tgx \leq b$,     $-\frac{\pi}{2} + \pi n < x \leq arctgb + n\pi, \quad n \in Z$.

7. $ctgx \geq b$,     $n\pi < x \leq arcctgb + n\pi, \quad n \in Z$.

**8.** $ctgx \leq b$, $\quad arcctgb + n\pi \leq x < \pi + n\pi$, $\quad n \in Z$.

**9.** $\sin x \geq a$, $\quad a > 1$ **bo'lsa** $\quad x \in \emptyset$.

**10.** $\sin x \leq a$, $\quad a \geq 1$ **bo'lsa** $\quad x \in (-\infty; \infty)$.

**11.** $\cos x \geq a$, $\quad a > 1$ **bo'lsa** $\quad x \in \emptyset$.

**12.** $\cos x \leq a$, $\quad a \geq 1$ **bo'lsa** $\quad x \in (-\infty; \infty)$.

Agar $\sin x \leq a$, $\cos x \leq a$, $tgx \leq a$, $ctgx \leq a$ tengsizliklarda $\leq$ belgisi $<$ belgi bilan almashsa, u holda yechimlarda ham $\leq$ belgisi o'rniga $<$ belgisi qo'yiladi.

1. (97-6-47) Ushbu $y = \sqrt{2\sin x - 1}$ funksiyaning aniqlanish sohasini toping.
   A) $\left(-\dfrac{\pi}{6} + 2\pi n; \dfrac{\pi}{6} + 2\pi n\right)$, $\quad n \in Z$
   B) $[\dfrac{\pi}{6} + 2\pi n; \dfrac{5\pi}{6} + 2\pi n]$, $\quad n \in Z$
   C) $\left(\dfrac{\pi}{6} + 2\pi n; \dfrac{5\pi}{6} + 2\pi n\right)$, $\quad n \in Z$
   D) $[-\dfrac{\pi}{6} + 2\pi n; \dfrac{\pi}{6} + 2\pi n]$, $\quad n \in Z$

   **Yechish:** $y = \sqrt{2\sin x - 1}$ funksiya $2\sin x - 1 \geq 0$ bo'lganda aniqlangan. Bu tengsizlikni

   $$\sin x \geq \frac{1}{2}$$

   ko'rinishda yozamiz. 1-formulaga ko'ra uning javobi $2\pi n + \dfrac{\pi}{6} \leq x \leq \dfrac{5\pi}{6} + 2\pi n$, $n \in Z$. **Javob:** (B).

2. (96-9-51) Ushbu $\sin^2 x - \dfrac{5}{2}\sin x + 1 < 0$ tengsizlik $x$ $(x \in [0; 2\pi])$ ning qanday qiymatlarida o'rinli?
   A) $[0; \dfrac{\pi}{6}] \cup [\dfrac{5\pi}{6}; 2\pi]$ $\qquad$ B) $(\dfrac{\pi}{6}; \dfrac{5\pi}{6})$
   C) $(0; \dfrac{\pi}{3}) \cup (\dfrac{2\pi}{3}; 2\pi]$ $\qquad$ D) $[0; \dfrac{\pi}{3}) \cup (\dfrac{2\pi}{3}; 2\pi]$

3. (99-1-43) Tengsizlikni yeching.

   $$2\sin x \geq \sqrt{2}$$

   A) $\dfrac{\pi}{4} + 2\pi n \leq x \leq \dfrac{3\pi}{4} + 2\pi n$, $\quad n \in Z$
   B) $-\dfrac{5\pi}{4} + 2\pi n \leq x \leq \dfrac{\pi}{4} + 2\pi n$, $\quad n \in Z$
   C) $\dfrac{\pi}{4} + 2\pi n \leq x \leq \dfrac{3\pi}{4} + 2\pi n$, $\quad n \in Z$
   D) $\dfrac{\pi}{4} + \pi n \leq x \leq \dfrac{3\pi}{4} + \pi n$, $\quad n \in Z$

4. (96-9-105) Tengsizlikni yeching.

   $$2\sin 2x \geq ctg\frac{\pi}{4}$$

   A) $[\dfrac{\pi}{6} + 2\pi n; \dfrac{5\pi}{6} + 2\pi n]$, $\quad n \in Z$
   B) $(\dfrac{\pi}{12} + \pi n; \dfrac{5\pi}{12} + \pi n)$, $\quad n \in Z$
   C) $[\dfrac{\pi}{12} + \pi n; \dfrac{5\pi}{12} + \pi n]$, $\quad n \in Z$
   D) $[\dfrac{\pi}{12} + 2\pi n; \dfrac{5\pi}{12} + 2\pi n]$, $\quad n \in Z$

5. (97-9-101) Tengsizlikni yeching.

   $$\sin x \cdot \cos x > \frac{\sqrt{2}}{4}$$

   A) $\dfrac{\pi}{8} + 2\pi k < x < \dfrac{3\pi}{8} + 2\pi k$, $\quad k \in Z$
   B) $\dfrac{\pi}{4} + \pi k < x < \dfrac{3\pi}{4} + \pi k$, $\quad k \in Z$
   C) $\dfrac{\pi}{8} + \pi k < x < \dfrac{3\pi}{8} + \pi k$, $\quad k \in Z$
   D) $\dfrac{\pi}{8} + \pi k \leq x \leq \dfrac{3\pi}{8} + \pi k$, $\quad k \in Z$

6. (98-5-51) Tengsizlikni yeching.

   $$\sin 5x \cdot \cos 4x + \cos 5x \cdot \sin 4x > \frac{1}{2}$$

   A) $\dfrac{\pi}{6} + 2\pi n < x < \dfrac{5\pi}{6} + 2\pi n$, $\quad n \in Z$
   B) $\dfrac{\pi}{54} + 2\pi n < x < \dfrac{5\pi}{54} + 2\pi n$, $\quad n \in Z$
   C) $\dfrac{\pi}{36} + \dfrac{2\pi n}{9} < x < \dfrac{5\pi}{36} + \dfrac{2\pi n}{9}$, $\quad n \in Z$
   D) $\dfrac{\pi}{54} + \dfrac{2\pi n}{9} < x < \dfrac{5\pi}{54} + \dfrac{2\pi n}{9}$, $\quad n \in Z$

7. (98-8-60) Tengsizlikni yeching.

   $$1 - 2\sin 4x < \cos^2 4x$$

   A) $(\pi k; \dfrac{\pi}{2} + \pi k)$, $\quad k \in Z$
   B) $(-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k)$, $\quad k \in Z$
   C) $(\dfrac{\pi k}{2}; \dfrac{\pi}{4} + \dfrac{\pi k}{2})$, $\quad k \in Z$
   D) $(-\dfrac{\pi}{4} + 2\pi k; \dfrac{\pi}{4} + 2\pi k)$, $\quad k \in Z$

   **Yechish:** Agar $\cos^2 4x = 1 - \sin^2 4x$ ayniayatdan foydalansak, berilgan tengsizlikni

   $$1 - 2\sin 4x < 1 - \sin^2 4x \Longleftrightarrow \sin 4x(\sin 4x - 2) < 0$$

   ko'rinishda yozishimiz mumkin. Barcha $x \in R$ lar uchun $\sin 4x - 2 < 0$ tengsizligi o'rinli, shuning uchun berilgan tengsizlik $\sin 4x > 0$ tengsizlikka teng kuchli. 1-formulaga ko'ra uning yechimi $2\pi n < 4x < \pi(2n+1)$, $n \in Z$. Bu tengsizlikning barcha qismlarini 4 ga bo'lib $\dfrac{\pi n}{2} < x < \dfrac{\pi}{4} + \dfrac{\pi n}{2}$, $n \in Z$.
   **Javob:** (C).

8. (98-1-60) Tengsizlikni yeching.

   $$1 - 2\cos 2x > \sin^2 2x$$

   A) $(\dfrac{\pi}{2} + \pi k; \pi + \pi k)$, $\quad k \in Z$
   B) $(\dfrac{\pi}{3} + 2\pi k; \dfrac{2\pi}{3} + 2\pi k)$, $\quad k \in Z$
   C) $(\dfrac{\pi}{4} + \pi k; \dfrac{3\pi}{4} + \pi k)$, $\quad k \in Z$
   D) $(-\dfrac{\pi}{2} + \pi k; \dfrac{\pi}{2} + \pi k)$, $\quad k \in Z$

9. (98-12-59) Tengsizlikni yeching.

   $$\sin^2 3x - \cos^2 3x \leq -\frac{\sqrt{3}}{2}$$

A) $\left[-\dfrac{\pi}{36}+\dfrac{\pi n}{3};\dfrac{\pi}{36}+\dfrac{\pi n}{3}\right], \quad n \in Z$

B) $\left(-\dfrac{\pi}{36}+\dfrac{\pi n}{3};\dfrac{\pi}{36}+\dfrac{\pi n}{3}\right), \quad n \in Z$

C) $\left[-\dfrac{\pi}{6}+2\pi n;\dfrac{\pi}{6}+2\pi n\right], \quad n \in Z$

D) $\left(-\dfrac{\pi}{6}+2\pi n;\dfrac{\pi}{6}+2\pi n\right], \quad n \in Z$

10. (99-3-38) Tengsizlikni yeching.

$$4\cos^2 x - 3 \geq 0$$

A) $[-\dfrac{\pi}{3}+2\pi k;\dfrac{\pi}{3}+2\pi k], \quad k \in Z$

B) $[-\dfrac{\pi}{3}+\pi k;\dfrac{\pi}{3}+\pi k], \quad k \in Z$

C) $[-\dfrac{\pi}{6}+\pi k;\dfrac{\pi}{6}+\pi k], \quad k \in Z$

D) $[-\dfrac{\pi}{6}+2\pi k;\dfrac{\pi}{6}+2\pi k], \quad k \in Z$

11. (99-7-49) Tengsizlikni yeching.

$$\cos 5x \cdot \cos 4x + \sin 5x \cdot \sin 4x < \dfrac{\sqrt{3}}{2}$$

A) $\dfrac{\pi}{3}+2\pi n < x < \dfrac{5\pi}{3}+2\pi n, \quad n \in Z$

B) $\dfrac{\pi}{6}+2\pi n < x < \dfrac{11\pi}{6}+2\pi n, \quad n \in Z$

C) $\dfrac{\pi}{3}+\pi n < x < \dfrac{5\pi}{3}+\pi n, \quad n \in Z$

D) $\dfrac{\pi}{6}+\pi n < x < \dfrac{11\pi}{6}+\pi n, \quad n \in Z$

12. (96-12-111) $x\,(x \in [0;2\pi])$ ning qanday qiymatlarida tengsizlik to'g'ri?

$$\cos^2 x - \dfrac{5}{2}\cos x + 1 > 0$$

A) $[0;\dfrac{\pi}{3}]\cup(\dfrac{5\pi}{3};2\pi]$     B) $(\dfrac{\pi}{3};\dfrac{\pi}{2}]\cup[\dfrac{3\pi}{2};\dfrac{5\pi}{3})$

C) $(\dfrac{\pi}{3};\dfrac{5\pi}{3})$                   D) $(\dfrac{\pi}{3};\dfrac{\pi}{2}]$

**Yechish:** Berilgan tengsizlikda $\cos x = y$ belgilash olib, uni quyidagicha yozib olamiz

$$y^2 - 2,5y + 1 > 0 \Longleftrightarrow (y - 0,5)(y - 2) > 0.$$

Yana eski belgilashga qaytib berilgan tengsizlikka teng kuchli bo'lgan

$$(\cos x - 0,5)(\cos x - 2) > 0$$

tengsizlikka ega bo'lamiz. Barcha $x \in R$ larda $\cos x - 2 < 0$ tengsizligi o'rinli bo'lgani uchun, berilgan tengsizlik

$$\cos x - 0,5 < 0 \Longleftrightarrow \cos x < \dfrac{1}{2}$$

tengsizlikka teng kuchli. 4-formulaga ko'ra uning yechimi $2\pi n + \dfrac{\pi}{3} < x < 2\pi(n+1) - \dfrac{\pi}{3}$, $n \in Z$ ko'rinishda bo'ladi. Bu yechimning $[0; 2\pi]$ kesmadagi qismini olish uchun $n = 0$ deymiz, u holda $(\dfrac{\pi}{3}; \dfrac{5\pi}{3})$ oraliq $[0; 2\pi]$ kesmaning qismi bo'ladi.

Agar $n \neq 0$ bo'lsa, u holda $(2\pi n + \dfrac{\pi}{3}; 2\pi(n+1) - \dfrac{\pi}{3})$ interval $[0; 2\pi]$ kesma bilan umumiy qismga ega emas. **Javob:** $(\dfrac{\pi}{3}; \dfrac{5\pi}{3})$ (C).

13. (96-13-26) Ushbu

$$\cos^2 x - \dfrac{5}{2}\cos x + 1 \leq 0$$

tengsizlik $x\,(x \in [0;2\pi])$ ning qanday qiymatlarida o'rinli?

A) $[0;\dfrac{\pi}{3}]\cup[\dfrac{5\pi}{3};2\pi]$       B) $[0;\dfrac{\pi}{3}]$

C) $[\dfrac{5\pi}{3};2\pi]$          D) $[\dfrac{\pi}{3};\dfrac{\pi}{2}]\cup[\dfrac{3\pi}{2};\dfrac{5\pi}{3}]$

14. (97-4-41) Tengsizlikni yeching.

$$\cos^2 x < \dfrac{\sqrt{2}}{2} + \sin^2 x$$

A) $\dfrac{\pi}{8}+2\pi n < x < \dfrac{7\pi}{8}+2\pi n, \quad n \in Z$

B) $\dfrac{\pi}{8}+\pi n < x < \dfrac{7\pi}{8}+\pi n, \quad n \in Z$

C) $-\dfrac{\pi}{8}+2\pi n < x < \dfrac{\pi}{8}+2\pi n, \quad n \in Z$

D) $\dfrac{\pi}{4}+2\pi n < x < \dfrac{7\pi}{4}+2\pi n, \quad n \in Z$

15. (98-6-55) Ushbu

$$\cos 2x \leq -\dfrac{1}{2}$$

tengsizlikning $[0;\pi]$ kesmadagi yechimini toping.

A) $[\dfrac{\pi}{3};\dfrac{2\pi}{3}]$          B) $[0;\dfrac{2\pi}{3}]$

C) $[-\dfrac{2\pi}{3};\dfrac{4\pi}{3}]$       D) $[\dfrac{4\pi}{3};2\pi]$

16. (00-3-55) Quyidagi tengsizlik

$$-1 - \dfrac{2}{\sqrt{3}}\cos x > 0$$

$[-\pi;\pi]$ kesmada nechta butun yechimga ega?
A) 4      B) 3      C) 6      D) 2

17. (00-6-56) Tengsizlikni yeching.

$$\cos x < \sin x$$

A) $(\dfrac{\pi}{4}+\pi k;\dfrac{3\pi}{4}+\pi k), \quad k \in Z$

B) $(\dfrac{\pi}{4}+\pi k;\dfrac{5\pi}{4}+\pi k), \quad k \in Z$

C) $(\dfrac{\pi}{4}+2\pi k;\dfrac{3\pi}{4}+2\pi k), \quad k \in Z$

D) $(\dfrac{\pi}{4}+2\pi k;\dfrac{5\pi}{4}+2\pi k), \quad k \in Z$

18. (96-1-59) Tengsizlikni yeching.

$$tg\left(x+\dfrac{\pi}{4}\right) \geq 1$$

A) $\left[-\dfrac{\pi}{4}+\pi k;\dfrac{\pi}{2}+\pi k\right], \quad k \in Z$

B) $[\pi k;\dfrac{\pi}{2}+\pi k) \quad k \in Z$

C) $\left[\dfrac{\pi}{4} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right]$, $\quad k \in Z$

D) $\left[\pi k; \dfrac{\pi}{4} + \pi k\right)$, $\quad k \in Z$

19. (96-12-91) $x\,(x \in [0; 2\pi])$ ning qaysi qiymatlarida funksiya aniqlangan?

$$y = \sqrt{1 - \log_{\frac{1}{2}} \cos x}$$

A) $[\dfrac{\pi}{3}; \dfrac{\pi}{2})$     B) $(\dfrac{3\pi}{2}; \dfrac{5\pi}{3}]$

C) $[0; \dfrac{\pi}{3}]$     D) $[\dfrac{5\pi}{3}; 2\pi] \cup [0; \dfrac{\pi}{3}]$

20. (96-13-34) Ushbu $y = \sqrt{1 + \log_{\frac{1}{2}} \sin x}$ funksiyaning aniqlanish sohasiga tegishli bo'lgan $x$ ning $[0; 2\pi]$ kesmadagi barcha qiymatlarini aniqlang.

A) $\left[\dfrac{\pi}{6}; \dfrac{5\pi}{6}\right]$    B) $\left(0; \dfrac{\pi}{6}\right] \cup \left[\dfrac{5\pi}{6}; \pi\right)$

C) $\left(0; \dfrac{\pi}{6}\right]$     D) $(0; \pi)$

21. (01-4-3) Ushbu $y = \arccos(2 \sin x)$ funksiyaning aniqlanish sohasiga tegishli bo'lgan $x$ ning $[-\pi; \pi]$ kesmadagi barcha qiymatlarini aniqlang.

A) $[-\dfrac{\pi}{6}; \dfrac{\pi}{6}]$    B) $[-\dfrac{\pi}{4}; \dfrac{\pi}{4}]$    C) $[-\dfrac{\pi}{3}; \dfrac{\pi}{3}]$

D) $[-\pi; -\dfrac{5\pi}{6}] \cup [-\dfrac{\pi}{6} \dfrac{\pi}{6}] \cup [\dfrac{5\pi}{6}; \pi]$

22. (01-10-39) Tengsizlikni yeching.

$$\sin 2x < \cos 2x$$

A) $(-\dfrac{3\pi}{8} + 2\pi n; \dfrac{\pi}{8} + 2\pi n)$, $\quad n \in Z$

B) $(-\dfrac{3\pi}{4} + 2\pi n; \dfrac{\pi}{4} + 2\pi n)$, $\quad n \in Z$

C) $(-\dfrac{\pi}{8} + \pi n; \dfrac{\pi}{8} + \pi n)$, $\quad n \in Z$

D) $(-\dfrac{3\pi}{8} + \pi n; \dfrac{\pi}{8} + \pi n)$, $\quad n \in Z$

23. (01-11-22) Tengsizlikning $[0; 2\pi]$ kesmadagi eng katta va eng kichik yechimlari yig'indisini hisoblang.

$$2^{\frac{1}{2}} \le 2^{\sin x} \le 2^{\frac{\sqrt{3}}{2}}$$

A) $\dfrac{2\pi}{3}$    B) $\pi$    C) $\dfrac{4\pi}{5}$    D) $\dfrac{\pi}{2}$

24. (02-1-62) Tengsizlikni yeching.

$$\cos(\sin x) < 0$$

A) $(\dfrac{\pi}{2} + 2\pi n; \dfrac{3\pi}{2} + 2\pi n)$, $\quad n \in Z$

B) $(\dfrac{\pi}{2} + \pi n; \dfrac{3\pi}{2} + \pi n)$, $\quad n \in Z$

C) $(0; \dfrac{3\pi}{2} + 2\pi n)$, $\quad n \in Z$

D) yechimga ega emas

**Yechish:** Ma'lumki, ixtiyoriy $x \in R$ da $\sin x \in [-1; 1]$ bo'ladi. Kosinus funksiya esa $(-\dfrac{\pi}{2}; \dfrac{\pi}{2})$ da musbat qiymatlar qabul qiladi. Demak, ixtiyoriy $t = \sin x \in (-\dfrac{\pi}{2}; \dfrac{\pi}{2})$ da ham $\cos(\sin x) = \cos t > 0$ bo'ladi. Bu yerdan berilgan tengsizlik yechimga ega emasligi kelib chiqadi. **Javob:** yechimga ega emas. (D).

25. (02-6-45) Tengsizlikni yeching.

$$\sin x > \sqrt{3} \cdot \cos x$$

A) $(\dfrac{\pi}{3} + 2\pi n; \dfrac{4\pi}{3} + 2\pi n)$, $\quad n \in Z$

B) $(\dfrac{\pi}{6} + \pi n; \dfrac{2\pi}{3} + \pi n)$, $\quad n \in Z$

C) $(\dfrac{\pi}{6} + 2\pi n; \dfrac{7\pi}{6} + 2\pi n)$, $\quad n \in Z$

D) $(\dfrac{\pi}{4} + \pi n; \dfrac{3\pi}{4} + \pi n)$, $\quad n \in Z$

26. (02-8-19) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\log_3 \sin x}$$

A) $\dfrac{\pi}{2} + 2\pi n, n \in Z$    B) $\dfrac{\pi}{2} + \pi n, n \in Z$

C) $(0; 1)$      D) $(0; \pi)$

27. (02-10-62) Tengsizlikni yeching.

$$\sqrt{\cos^2 x - \cos x + \dfrac{1}{4}} \ge \dfrac{1}{2}$$

A) $[\dfrac{\pi}{2} + 2\pi n; \dfrac{3\pi}{2} + 2\pi n] \cup \{2\pi n\}$, $\quad n \in Z$

B) $[-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n] \cup \{2\pi n\}$, $\quad n \in Z$

C) $(-\dfrac{\pi}{2} + 2\pi n; \pi + 2\pi n] \cup \{2\pi n\}$, $\quad n \in Z$

D) $[\dfrac{2\pi}{3} + \pi n; \dfrac{7\pi}{6} + \pi n]$, $\quad n \in Z$

28. (03-2-31) Tengsizlikni yeching.

$$\cos(\pi \sin x) > 0$$

A) $\left(\pi k; \dfrac{\pi}{3} + \pi k\right)$, $\quad k \in Z$

B) $\left(-\dfrac{\pi}{6} + \pi k; \dfrac{\pi}{6} + \pi k\right)$, $\quad k \in Z$

C) $\left(-\dfrac{\pi}{3} + 2\pi k; \dfrac{\pi}{3} + 2\pi k\right)$, $\quad k \in Z$

D) $\left(\pi k; \dfrac{\pi}{6} + \pi k\right)$, $\quad k \in Z$

## 13.6   Aralash tipdagi masalalar

1. $y = \arcsin x$ **funksiyaning aniqlanish sohasi** $[-1; 1]$, **qiymatlar sohasi esa -** $[-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$.
   $y = \arcsin x$ **funksiya** $[-1; 1]$ **da o'suvchi.**

2. $y = \arccos x$ **funksiyaning aniqlanish sohasi** $[-1; 1]$, **qiymatlar sohasi esa -** $[0; \pi]$.
   $y = \arccos x$ **funksiya** $[-1; 1]$ **da kamayuvchi.**

3. $y = arctgx$ **funksiyaning aniqlanish sohasi** $(-\infty; \infty)$, **qiymatlar sohasi esa -** $(-\dfrac{\pi}{2}; \dfrac{\pi}{2})$.
   $y = arctgx$ **funksiya** $(-\infty; \infty)$ **da o'suvchi.**

4. $y = arcctgx$ **funksiyaning aniqlanish sohasi** $(-\infty; \infty)$, **qiymatlar sohasi esa -** $(0; \pi)$.
   $y = arcctgx$ **funksiya** $(-\infty; \infty)$ **da kamayuvchi.**

5. $y = \arcsin x$ **va** $y = arctgx$ **- toq funksiyalar,** $y = \arccos x$ **va** $y = arcctgx$ **funksiyalar esa juft ham emas, toq ham emas va ular uchun** $\arccos(-x) = \pi - \arccos x,\quad arcctg(-x) = \pi - arcctgx$ **tengliklar o'rinli.**

6. $\arcsin x + \arccos x = \dfrac{\pi}{2}, \quad x \in [-1; 1]$.

7. $\arcsin a > \arcsin b \Leftrightarrow \begin{cases} a > b \\ b \geq -1 \\ a \leq 1. \end{cases}$

8. $\arccos a > \arccos b \Leftrightarrow \begin{cases} a < b \\ a \geq -1 \\ b \leq 1. \end{cases}$

9. $arctga > arctgb \Leftrightarrow a > b$.

10. $arcctga > arcctgb \Leftrightarrow a < b$.

1. (98-6-51) Tengsizlikni yeching.

$$\arcsin x < \arcsin(1 - x)$$

A) $[0; \dfrac{1}{2})$    B) $[-1; 1]$    C) $(-\infty; \dfrac{1}{2}]$    D) $[0; 2]$

**Yechish:** $y = \arcsin x, \quad -1 \leq x \leq 1$ funksiya o'suvchi ekani ma'lum. U holda berilgan tengsizlik quyidagi

$$\begin{cases} x < 1 - x \\ -1 \leq x \leq 1 \\ -1 \leq 1 - x \leq 1 \end{cases}$$

sistemaga ekvivalent bo'ladi. Uni yechamiz.

$$\begin{cases} 2x < 1 \\ -1 \leq x \leq 1 \\ 0 \leq x \leq 2 \end{cases}$$

Demak,

$$0 \leq x < \dfrac{1}{2}$$

**Javob:** $[0; \dfrac{1}{2})$ (A).

2. (98-6-53) Tenglamaning eng kichik musbat ildizini toping.

$$\arcsin(2\sin x) = \dfrac{\pi}{2}$$

A) $\dfrac{1}{3}$    B) $\dfrac{5\pi}{6}$    C) $\dfrac{1}{2}$    D) $\dfrac{\pi}{6}$

3. (98-11-30) Tenglamaning yechimi nechta?

$$arctg|x| = -\dfrac{\pi}{6}$$

A) 1    B) 0    C) 2    D) cheksiz ko'p

4. (98-11-74) Tengsizlikni yeching.

$$\arccos x > \arccos x^2$$

A) $(0; 1)$    B) $[-1; 0)$
C) $[-1; 1]$    D) $(-\infty; 0) \cup (1; \infty)$

5. (00-1-33) Tenglamaning ildizlari yig'indisini toping.

$$2(\arccos x)^2 + \pi^2 = 3\pi \arccos x$$

A) $\dfrac{\sqrt{2}}{2}$    B) $-1$    C) $1$    D) $-\dfrac{\sqrt{2}}{2}$

6. (01-4-4) Ushbu

$$\arccos^2 x - \dfrac{5\pi}{6} \cdot \arccos x + \dfrac{\pi^2}{6} \leq 0$$

tengsizlik o'rinli bo'ladigan kesmaning o'rtasini toping.
A) 0,5    B) 0,4    C) 0,25    D) $\dfrac{\pi}{4}$

**Yechish:** Berilgan tengsizlikning chap qismini ko'paytuvchilarga ajratamiz

$$(\arccos x - \dfrac{\pi}{3})(\arccos x - \dfrac{\pi}{2}) \leq 0.$$

Bu tengsizlikka oraliqlar usulini qo'llab

$$\dfrac{\pi}{3} \leq \arccos x \leq \dfrac{\pi}{2}$$

tengsizlikni olamiz. Kosinus funksiyaning $[0; \dfrac{\pi}{2}]$ da kamayuvchi ekanligidan

$$\cos\dfrac{\pi}{3} \geq x \geq \cos\dfrac{\pi}{2} \iff \dfrac{1}{2} \geq x \geq 0$$

ni olamiz. Demak, berilgan tengsizlikning yechimi $[0; \dfrac{1}{2}]$ kesmadan iborat. Bu kesmaning o'rtasi $0,25$ dir. **Javob:** $0,25$ (C).

7. (01-5-18) Tenglama nechta ildizga ega?

$$x \cdot arctgx = 1$$

A) 2    B) 1    C) 0    D) 3

8. (01-5-19) Tenglama nechta ildizga ega?

$$\cos(10arctgx) = 1$$

A) 5    B) cheksiz ko'p    C) 1    D) 3

9. (01-9-14) Tenglama ildizlari ko'paytmasini toping.

$$4arctg(x^2 - 3x + 3) - \pi = 0$$

A) 2    B) 3    C) $-3$    D) 1

10. (01-12-27) Tengsizlikni yeching.

$$\lg(\arcsin x) > -1$$

A) $(0; \dfrac{\pi}{2}]$    B) $[\sin 0,1; 1]$
C) $(\sin 0,1; 1)$    D) $(\sin 0,1; 1]$

11. (00-10-25) Tenglamaning nechta ildizi bor?

$$arctg|x| = \dfrac{\pi}{2}$$

A) 2    B) 1    C) 0    D) cheksiz ko'p

12. (02-4-37) Tengsizlikni qanoatlantiruvchi $x$ ning eng katta butun qiymatini toping.

$$arctgx < 0$$

A) $-2$      B) $-1$      C) $0$      D) $1$

13. (99-8-35) Ushbu $y = \arcsin x + \dfrac{\pi}{2}$ funksiyaning qiymatlar to'plamini toping.

A) $[0; \pi]$        B) $[-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$

C) $[\dfrac{\pi}{2} - 1; \dfrac{\pi}{2} + 1]$    D) $[0; \dfrac{\pi}{2}]$

**Yechish:** 1-xossaga ko'ra $y = \arcsin x$ funksiyaning qiymatlar to'plami $[-\dfrac{\pi}{2}; \dfrac{\pi}{2}]$ kesmadan iborat. Shuning uchun $y = \arcsin x + \dfrac{\pi}{2}$ funksiyaning qiymatlar to'plami $[0; \pi]$ kesmadan iborat bo'ladi.
**Javob:** $[0; \pi]$ (A).

14. (98-6-49) Ushbu $x = \arccos 0,9$; $y = \arccos(-0,7)$; $z = \arccos(-0,2)$ sonlarni o'sib borish tartibida yozing.

A) $y < z < x$      B) $x < y < z$
C) $y < x < z$      D) $x < z < y$

15. (07-156-36) $\cos(2\arccos\dfrac{4}{5})$ ning qiymatini toping.

A) $\dfrac{7}{25}$    B) $\dfrac{24}{25}$    C) $-\dfrac{24}{25}$    D) $-\dfrac{7}{25}$

16. (07-158-36) $\cos(2\arccos\dfrac{4}{9})$ ning qiymatini toping.

A) $\dfrac{49}{81}$    B) $\dfrac{8}{9}$    C) $-\dfrac{49}{81}$    D) $-\dfrac{8}{9}$

17. (99-3-30) Funksiyaning aniqlanish sohasini toping.

$$y = \arcsin\frac{x-3}{2} - \lg(4-x)$$

A) $[1; 4]$   B) $[1; 5]$   C) $(1; 4)$   D) $[1; 4)$

**Yechish:** $y = \arcsin x$ funksiyaning aniqlanish sohasi $[-1; 1]$ kesmadan, $y = \lg x$ funksiyaning aniqlanish sohasi $(0; \infty)$ dan iborat. Bularga ko'ra berilgan funksiyaning aniqlanish sohasi

$$\begin{cases} -1 \le \dfrac{x-3}{2} \le 1 \\ 4 - x > 0 \end{cases}$$

sistema yechimidan iborat bo'ladi. Bu sistemaning yechimlari $[1; 4)$ kesmadan iborat. **Javob:** $[1; 4)$ (D).

18. (99-8-73) $y = \arcsin\dfrac{x^3}{8}$ funksiyaning aniqlanish sohasini toping.

A) $[-2; 2]$   B) $[-1; 1]$   C) $(-2; 2)$   D) $[1; 2]$

19. (99-10-41) Funksiyaning aniqlanish sohasini toping.

$$y = \frac{\sqrt{x + 0,2}}{\arccos x}$$

A) $(-0,2; 1)$     B) $(-0,2; 1]$
C) $[-0,2; 1]$     D) $[-0,2; 1)$

20. (03-6-62) Funksiyaning aniqlanish sohasini toping.

$$y = \arcsin\frac{2}{2 + \sin x}$$

A) $-\pi + 2\pi k \le x \le \pi + 2\pi k, \quad k \in Z$
B) $x \le \pi + 2\pi k, \quad k \in Z$
C) $x > 2\pi k, \quad k \in Z$
D) $2\pi k \le x \le \pi + 2\pi k, \quad k \in Z$

21. (03-6-67) $y = \arccos|x - 2|$ funksiyaning aniqlanish sohasini toping.
A) $1 \le x \le 3$      B) $x > 1$
C) $x < 3$          D) $2 \le x \le 3$

22. (03-7-58) Funksiyaning aniqlanish sohasini toping.

$$y = \frac{\sqrt{x^2 - 5x + 6}}{lg(x+5)^2} + \frac{1}{\arccos(x+3)}$$

A) $(-4; -2]$      B) $(-\infty; 2) \cup [3; \infty)$
C) $(-\infty; -3) \cup (-3; 2]$    D) $(-4; -2)$

23. (02-4-35) $y = (x - 10)arctgx$ funksiya grafigining $Ox$ o'qi bilan kesishish nuqtasi abssissasining eng kichik qiymatini toping.
A) $-2$     B) $-1$     C) $0$     D) $1$

**Yechish:** $Ox$ o'qidagi nuqtalarning ordinatasi, ya'ni $y = 0$ bo'ladi. Shunday qilib, $(x-10)arctgx = 0$ tenglama yechimlarini topamiz. Bu tenglamaning yechimlari $x - 10 = 0$ va $arctgx = 0$ tenglama yechimlaridan iborat. Demak, berilgan tenglama $x_1 = 10$ va $x_2 = 0$ yechimlarga ega. Ularning kichigi $x = 0$. **Javob:** 0 (C).

24. (02-7-5) $y = \arcsin(3x - 7)$ funksiyaning aniqlanish sohasiga tegishli $x$ ning butun qiymatlari nechta?
A) $2$     B) $3$     C) $1$     D) $4$

25. (02-11-48) $y = \arccos(\log_3 x - 1)$ funksiyaning aniqlanish sohasiga tegishli butun sonlar nechta?
A) $12$     B) $9$     C) $8$     D) $7$

# 14 -bob. Hosila va integral

Funksiyaning uzluksizligi, hosilasi va integrali funksiya limiti tushunchasi bilan uzviy bog'liq. Shuning uchun funksiya limiti tushunchasi beramiz. Bizga $f : [a; b] \to R$ funksiya va $x_0 \in [a; b]$ berilgan bo'lsin. Agar ixtiyoriy $\varepsilon > 0$ uchun shunday $\delta > 0$ mavjud bo'lib, $0 < |x - x_0| < \delta$ shartni qanoatlantiruvchi batcha $x \in [a; b]$ larda $|f(x) - A| < \varepsilon$ tengsizligi bajarilsa, $f$ funksiya $x \to x_0$ da $A$ limitga ega deyiladi va quyidagicha yoziladi:

$$\lim_{x \to x_0} f(x) = A.$$

Xususan, agar $\lim_{x \to x_0} f(x) = f(x_0)$ bo'lsa, $f$ funksiya $x_0$ nuqtada *uzluksiz* deyiladi. Agar funksiya aniqlanish sohasining barcha nuqtalarida uzluksiz bo'lsa, u *uzluksiz* funksiya deyiladi. Uzluksiz funksiyaga misollar: $y = ax + b-$ chiziqli funksiya, $y = ax^2 + bx + $

$c-$ kvadratik funksiya, ixtiyoriy ko'phad $y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$, ko'rsatkichli funksiya $y = a^x$, logarifmik funksiya $y = \log_a x$ va trigonometrik funksiyalar $y = \sin x$, $y = \cos x$, $y = tgx$, $y = ctgx$ funksiyalar o'zlarining aniqlanish sohasida uzluksizdir. Ishora funksiyasi

$$y = signx = \begin{cases} -1 & agar\, x < 0 \\ 0 & agar\, x = 0 \\ 1 & agar\, x > 0 \end{cases}$$

yagona $x = 0$ nuqtada uzilishga ega, qolgan barcha nuqtalarda uzluksiz. $x$ ning butun qismi $y = [x]$ va $x$ ning kasr qismi $y = \{x\}$ lar barcha butun nuqtalarda uzilishga ega. Endi hosila ta'rifini beramiz. $x$ va $x + \Delta x$ lar $[a; b]$ kesmaga qarashli bo'lsin. Agar

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

nisbat $\Delta x \to 0$ da chekli limitga ega bo'lsa, u holda $f$ funksiya $x$ nuqtada hosilaga ega deyiladi va bu quyidagicha yoziladi:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Matematika, mexanika va fizikada hosilaning ko'pgina tadbiqlari uchraydi. Endi elementar funksiyalar uchun hosilalar jadvalini va hosilani hisoblash qoidalarini keltiramiz.

## 14.1 Elementar funksiyalarning hosilasi

**Hosilalar jadvali**

1. $c' = 0, \quad c = const.$

2. $(x^\alpha)' = \alpha\, x^{\alpha-1}, \quad x' = 1.$

3. $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}.$

4. $(\dfrac{1}{x})' = -\dfrac{1}{x^2}.$

5. $(a^x)' = a^x \ln a, \quad (e^x)' = e^x.$

6. $(\log_a x)' = \dfrac{1}{x \ln a}, \quad (\ln x)' = \dfrac{1}{x}.$

7. $(\sin x)' = \cos x, \quad (\cos x)' = -\sin x.$

8. $(tgx)' = \dfrac{1}{\cos^2 x}, \quad (ctgx)' = -\dfrac{1}{\sin^2 x}.$

   **Yig'indi va ayirmaning hosilasi**

9. $(u(x) \pm v(x))' = u'(x) \pm v'(x)$

10. $(C \cdot u(x))' = C \cdot u'(x), \quad C$- **o'zgarmas son.**

1. Agar $f(x) = 2x + 3$ bo'lsa, $f'(x)$ ni hisoblang.

   A) 1     B) 2     C) 0     D) 5

   **Yechish:** 9 va 10-qoidalarga ko'ra $f'(x) = (2x + 3)' = 2x' + 3'$. Endi 1 va 2-formulalardan foydalanamiz: $f'(x) = 2 \cdot 1 + 0 = 2$. **Javob:** 2 (B).

2. Agar $f(x) = 2x^2 + 3x + 7$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $2x + 3$     B) $4x + 3$     C) $4x$     D) 3

3. Agar $f(x) = x^2 + 2\sqrt{x}$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $2x + \dfrac{2}{\sqrt{x}}$   B) $2x + \sqrt{x}$   C) $x$   D) $2x + x^{-1/2}$

4. Agar $f(x) = -\dfrac{2}{x}$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $2x^{-2}$   B) $\dfrac{-2}{x^2}$   C) $\dfrac{4}{x^2}$   D) $x^{-1/2}$

5. Agar $f(x) = 2^x + 7$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $2^x \cdot \ln 2$   B) $\dfrac{2^x}{\ln 2}$   C) $2^{x-1}$   D) $-2^x \cdot \ln 2$

6. Agar $f(x) = \ln x + 5$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $\dfrac{1}{x}$    B) $\dfrac{1}{x \cdot \ln 2}$    C) $-\dfrac{1}{x}$    D) $\dfrac{\ln 2}{x}$

7. Agar $f(x) = x - tgx$ bo'lsa, $f'(x)$ ni hisoblang.

   A) $-tg^2 x$           B) $1 + \dfrac{1}{\cos^2 x}$

   C) $1 - \dfrac{1}{\sin^2 x}$      D) $1 - ctgx$

8. (96-7-28) Agar $f(x) = 5\sin x + 3\cos x$ bo'lsa, $f'(\dfrac{\pi}{4})$ ni hisoblang.

   A) $-\sqrt{2}$     B) $\sqrt{2}$     C) $-2\sqrt{3}$     D) $4\sqrt{2}$

   **Yechish:** 7 va 10-qoidalarga ko'ra

   $$f'(x) = 5(\sin x)' + 3(\cos x)' = 5\cos x - 3\sin x.$$

   Endi $x = \dfrac{\pi}{4}$ deb $f'(\dfrac{\pi}{4})$ ni hisoblaymiz:

   $$f'(\dfrac{\pi}{4}) = 5\cos\dfrac{\pi}{4} - 3\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}(5 - 3) = \sqrt{2}.$$

   **Javob:** $\sqrt{2}$ (B).

9. (97-6-19) Agar

   $$g(x) = ctgx + \dfrac{12x^3}{\pi^2} + \pi$$

   bo'lsa, $g'(\dfrac{\pi}{6})$ ni hisoblang.

   A) $-1$     B) $-3$     C) $5$     D) $3$

10. (97-7-28) Agar $f(x) = 2\sin x - 4\sqrt{3}\cos x$ bo'lsa, $f'(\dfrac{\pi}{3})$ ni hisoblang.

    A) 7     B) $-5$     C) $2 + 4\sqrt{3}$     D) $2\sqrt{3} - 2$

11. (97-9-34) Ushbu $y = \dfrac{1}{3}6^x - 6$ funksiyaning $x = 1$ nuqtadagi hosilasini toping.

    A) $\ln 12$     B) $\ln 36$     C) $\ln 6$     D) $\ln\dfrac{6}{e}$

12. (97-12-55) Agar

    $$f(x) = \dfrac{1}{3}x^3 - 16x$$

    bo'lsa, $f'(4)$ ni toping.

    A) 1     B) 2     C) 3     D) 0

13. (98-1-28) Agar

$$f(x) = e^x + 5x$$

bo'lsa, $f'(\ln 3)$ ni hisoblang.
A) 8     B) 5     C) $e^3 + 5$     D) $e^3$

14. (98-5-26) Ushbu $y = \sin^2 x + \cos^2 x$ funksiyaning hosilasini toping.
A) $2\sin 2x$     B) 0     C) $4\sin x$     D) $\sin 4x$

15. (99-7-27) Ushbu $y = tgx \cdot ctgx$ funksiyaning hosilasini toping.
A) 1     B) 2     C) $-\dfrac{1}{\sin^2 x \cdot \cos^2 x}$     D) 0

16. (96-1-30) Agar $f(x) = x^3 - 3x - 4$ bo'lsa,

$$\frac{f'(x)}{x-5} \geq 0$$

tengsizlikning eng kichik butun yechimini toping.
A) 1     B) $-1$     C) $-5$     D) 0

**Yechish:** 2-qoidaga ko'ra $f'(x) = 3x^2 - 3$. Endi

$$\frac{3x^2 - 3}{x - 5} \geq 0 \Longleftrightarrow \frac{3(x-1)(x+1)}{x-5} \geq 0$$

tengsizlikni yechamiz. Bu tengsizlik oraliqlar usuli bilan oson yechiladi (14.1-chizma) va uning yechimi $[-1;\,1] \cup (5;\,\infty)$ to'plamdan iborat. Bu to'plamdagi eng kichik butun son $-1$ dir. **Javob:** $-1$ (B).

17. (96-10-32) Agar $f(x) = x^3 - 12x + 7$ bo'lsa,

$$\frac{f'(x)}{x-4} \leq 0$$

tengsizlikning eng katta butun yechimini toping.
A) 2     B) $-4$     C) 3     D) $-2$

18. (98-5-25) $x$ ning qanday qiymatlarida $f(x) = \sin x$ va $g(x) = 5x + 3$ funksiyalar uchun $f'(x) < g'(x)$ tengsizlik bajariladi?
A) $(-\infty; 5)$     B) $(2\pi n; \dfrac{\pi}{2} + 2\pi n), \quad n \in Z$
C) $(-\infty; \infty)$     D) $(0; \infty)$

19. (00-6-27) Agar $f(x) = -4x^3 - 11x^2 - 8x + 7$ bo'lsa, $f'(x) \geq 0$ tengsizlikning nechta butun yechimi bor?
A) 4     B) 3     C) 2     D) 1

20. (97-5-34) Ushbu $y = 2^x - 1$ funksiyaning $x = 1$ nuqtadagi hosilasini toping.
A) 1     B) $\ln 2$     C) $\ln \dfrac{4}{e}$     D) $\ln 4$

21. (01-6-42) Agar $f(x) = x^{2/3} + 85\dfrac{1}{3} \ln x$ bo'lsa, $f'(8)$ ning qiymatini toping.
A) 10     B) 12     C) 9     D) 11

22. (02-2-29) Agar $f(x) = 3x + \dfrac{3}{x}$ bo'lsa, $f'(x) < 0$ tengsizlikni yeching.
A) $(-1; 0) \cup (0; 1)$     B) $(-\infty; -1)$
C) $(1; \infty)$     D) $(0; 1)$

23. (02-2-32) Nechta nuqtada $f(x) = x^3$ funksiya va uning hosilasi qiymatlari teng bo'ladi?
A) 2     B) 1     C) $\emptyset$     D) 3

**Yechish:** 2-qoidaga ko'ra $f'(x) = 3x^2$ bo'ladi. Masala shartiga ko'ra

$$f(x) = f'(x) \Longleftrightarrow x^3 = 3x^2 \Longleftrightarrow x^2(x-3) = 0$$

tenglamani yechamiz. Bu tenglamaning ildizlari $x_1 = 0$ va $x_2 = 3$ lardir. **Javob:** 2 (A).

24. (03-2-9) Agar

$$f(x) = x^3 + x - \sqrt{2}, \quad g(x) = 3x^2 + x + \sqrt{2}$$

bo'lsa, $f'(x) > g'(x)$ tengsizlikning eng kichik natural yechimini toping.
A) 3     B) 2     C) 6     D) 5

25. (03-8-48) $f(x) = x^4 + x^3 - 13,5x^2 + 2003$ bo'lsa, $f'(x) \leq 0$ tengsizlikning eng kichik natural yechimini toping.
A) 1     B) 2     C) 3     D) 4

26. (03-12-20) Agar $f(x) = \ln x$ bo'lsa, $f'(x) \leq x$ tengsizlikni yeching.
A) $[-1; 0) \cup [1; \infty)$     B) $(-1; 0) \cup [1; \infty)$
C) $(-\infty; -1] \cup [1; \infty)$     D) $[1; \infty)$

27. (03-12-73) Agar $f(x) = x^3 + 5x^2 + 4x + 2$ bo'lsa, $f'(x) = f(1)$ tenglamaning eng kichik ildizini toping.
A) $-6$     B) $-\dfrac{1}{3}$     C) $-2$     D) $-4$

### 14.1.1     Murakkab funksiyaning hosilasi

**Ko'paytma va bo'linmaning hosilasi**

**1.** $(u(x) \cdot v(x))' = u'(x)v(x) + u(x)v'(x)$.

**2.** $\left(\dfrac{u(x)}{v(x)}\right)' = \dfrac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}, \quad v(x) \neq 0$.

**Murakkab funksiyalarning hosilalari**

**3.** $(f(g(x)))' = f'(g(x))g'(x)$ **umumiy qoida.**

**4.** $(u^\alpha(x))' = \alpha\, u^{\alpha-1}(x)\, u'(x)$.

**5.** $(e^{u(x)})' = e^{u(x)}\, u'(x), \quad (a^{u(x)})' = a^{u(x)} \ln a\, u'(x)$.

**6.** $(\log_a u(x))' = \dfrac{u'(x)}{u(x) \ln a}, \quad (\ln u(x))' = \dfrac{u'(x)}{u(x)}$.

**7.** $(\sin u(x))' = \cos u(x)\, u'(x)$.

**8.** $(\cos u(x))' = -\sin u(x)\, u'(x).$

**9.** $(tgu(x))' = \dfrac{u'(x)}{\cos^2 u(x)}.$

**10.** $(ctgu(x))' = -\dfrac{u'(x)}{\sin^2 u(x)}.$

1. (96-9-79) Agar $f(x) = 3x \cdot 2^x$ bo'lsa, $f'(0)$ ni toping.
   A) $-3$      B) $3$      C) $1$      D) $-1$
   **Yechish:** 1-qoidaga ko'ra

   $$f'(x) = (3x)' \cdot 2^x + 3x \cdot (2^x)' = 3 \cdot 2^x + 3x \cdot 2^x \cdot \ln 2$$

   bo'ladi. Endi $f'(0)$ ni hisoblaymiz:

   $$f'(0) = 3 \cdot 2^0 + 3 \cdot 0 \cdot 2^0 \cdot \ln 2 = 3 + 0 = 3.$$

   **Javob:** 3 (B).

2. (96-10-30) Agar $f(x) = 2x \cdot 3^x$ bo'lsa, $f'(0)$ ni toping.
   A) $-1$      B) $2$      C) $-2$      D) $3$

3. (98-12-39) Ushbu $y = e^x \cdot x^2$ funksiyaning hosilasini toping.
   A) $e^x(x^2 + 2x)$      B) $e^x(x^2 + 2)$
   C) $e^x(2x + 1)$      D) $e^x(x^2 + x)$

4. (00-5-46) $y = (x-3)(x^2 + 3x + 9)$ funksiyaning $x = 3$ nuqtadagi hosilasini toping.
   A) $0$      B) $3$      C) $27$      D) $-27$

5. (96-9-22) Ushbu

   $$f(x) = \frac{x^2}{x^2 - 1}$$

   funksiya uchun $f'(-2)$ ni hisoblang.
   A) $\dfrac{4}{9}$      B) $-\dfrac{4}{9}$      C) $\dfrac{3}{4}$      D) $-\dfrac{3}{4}$
   **Yechish:** 2-qoidaga ko'ra

   $$f'(x) = \frac{(x^2)' \cdot (x^2 - 1) - x^2(x^2 - 1)'}{(x^2 - 1)^2}.$$

   Endi hosilalarni hisoblaymiz va soddalashtiramiz:

   $$f'(x) = \frac{-2x}{(x^2 - 1)^2} \implies f'(-2) = \frac{-2(-2)}{((-2)^2 - 1)^2} = \frac{4}{9}.$$

   **Javob:** $\dfrac{4}{9}$ (A).

6. (96-3-81) Agar

   $$f(x) = \frac{x}{1 - x}$$

   bo'lsa, $f'(2)$ ni toping.
   A) $-1$      B) $-2$      C) $2$      D) $1$

7. (96-13-22) $F'(1)$ ni toping.

   $$F(x) = \frac{x^2}{x^2 + 1}$$

   A) $-\dfrac{1}{2}$      B) $\dfrac{1}{2}$      C) $\dfrac{2}{3}$      D) $-\dfrac{2}{3}$

8. (98-10-69) $f'(1)$ ni toping.

   $$f(x) = \frac{\sqrt{x} + 1}{\sqrt{x}}$$

   A) $\dfrac{1}{2}$      B) $2$      C) $-\dfrac{1}{2}$      D) $1$

9. (02-1-27) $f'(1)$ ni toping.

   $$f(x) = \frac{8x\sqrt{x} + 2}{x}$$

   A) $2$      B) $1$      C) $0$      D) $3$

10. (97-5-33) $y = e^{\sin^2 x}$ funksiya hosilasini toping.
    A) $e^{\sin^2 x}$      B) $e^{\sin^2 x} \cdot \sin 2x$
    C) $2e^{\sin^2 x} \cdot \sin x$      D) $\sin^2 x \cdot e^{\sin^2 x - 1}$

    **Yechish:** Murakkab funksiyaning hosilasini hisoblash qoidasi 5-ga ko'ra

    $$y' = e^{\sin^2 x} \cdot 2 \sin x \cdot \cos x$$

    ekanini topamiz. $2 \sin x \cdot \cos x = \sin 2x$ formulaga ko'ra $y' = e^{\sin^2 x} \cdot \sin 2x$ bo'ladi.
    **Javob:** $e^{\sin^2 x} \cdot \sin 2x$ (B).

11. (96-3-34) $f(x) = e^{\sin 2x}$ funksiya hosilasini toping.
    A) $\sin 2x \cdot e^{\sin 2x - 1}$      B) $2 \cos 2x \cdot e^{\sin 2x}$
    C) $2 \cos 2x \cdot e^{\cos 2x}$      D) $\cos 2x \cdot e^{\sin 2x}$

12. (96-12-36) $f(x) = e^{\cos 2x}$ funksiyaning hosilasini toping.
    A) $2 \sin 2x \cdot e^{\cos 2x}$      B) $\cos 2x \cdot e^{\cos 2x - 1}$
    C) $-2 \sin 2x \cdot e^{-2 \sin 2x}$      D) $-2 \sin 2x \cdot e^{\cos 2x}$

13. (97-6-48) $g(x) = \dfrac{1}{3} ctg 3x$ funksiyaning $x = \dfrac{\pi}{18}$ dagi hosilasini hisoblang.
    A) $-2$      B) $\dfrac{4}{3}$      C) $4$      D) $-4$

14. (00-6-26) Agar $f(x) = 3x^2 \cdot e^{\sin x} - 8$ bo'lsa, $f'(\pi)$ ning qiymatini toping.
    A) $3\pi(2 + \pi)$      B) $3\pi^2(3 - \pi)$
    C) $2\pi(3 + \pi)$      D) $3\pi(2 - \pi)$

15. (99-4-33) Agar $f(x) = (x-2)^2 \cdot (x+4)$ bo'lsa, $f'(x) \leq 0$ tengsizlikni eching.
    A) $[-4; 2]$      B) $[2; 4]$
    C) $[-2; 2]$      D) $[-3; 2]$

16. (96-1-28) Agar $f(x) = x \cdot 2^{x+1}$ bo'lsa, $f'(0)$ ni toping.
    A) $-2$      B) $-1$      C) $1$      D) $2$

17. (96-3-29) $y = \cos(x^3 - 5)$ funksiyaning hosilasini toping.
    A) $-3x^2 \sin(x^3 - 5)$      B) $3x^2 \sin(x^3 - 5)$
    C) $-\sin(3x^2 - 5)$      D) $\sin(3x^2 - 5)$

18. (96-3-33) $f(x) = \ln(x^2 - 3\sin x)$ funksiyaning hosilasini toping.

A) $\dfrac{3}{x^2 - 3\sin x}$     B) $\dfrac{2x + 3\cos x}{x^2 - 3\sin x}$

C) $\dfrac{2x - 3\cos x}{x^2 - 3\sin x}$     D) $\dfrac{2x}{x^2 - 3\sin x}$

**Yechish:** Murakkab funksiyaning hosilasini hisoblash qoidasi 6-ga ko'ra

$$f'(x) = \frac{(x^2 - 3\sin x)'}{x^2 - 3\sin x} = \frac{2x - 3\cos x}{x^2 - 3\sin x}$$

ekanini topamiz. **Javob:** (C).

19. (96-12-34) Funksiyaning hosilasini toping.

$$f(x) = \ln(x^2 + 3\sin x)$$

A) $\dfrac{3}{x^2 + 3\sin x}$     B) $\dfrac{2x + 3\sin x}{x^2 + 3\sin x}$

C) $\dfrac{2x + 3\cos x}{x^2 + 3\sin x}$     D) $\dfrac{2x - 3\cos x}{x^2 + 3\sin x}$

20. (96-6-56) Agar $f(x) = \ln\sin x$ bo'lsa, $f'(\frac{\pi}{6})$ ni toping.

A) $-\sqrt{3}$     B) $\dfrac{\sqrt{3}}{3}$     C) $\sqrt{3}$     D) $-\dfrac{\sqrt{3}}{3}$

21. (97-3-28) Agar $f(x) = 2\sqrt{3}\cos 4x - 2\cos x$ bo'lsa, $f'(\frac{\pi}{6})$ ni hisoblang.

A) $-11$     B) $13$     C) $\sqrt{3}+1$     D) $\sqrt{3}-2$

22. (97-6-62) Agar $f(x) = \ln\sin x$ bo'lsa, $f'(\frac{\pi}{4})$ ni hisoblang.

A) $-1$     B) $3$     C) $-\sqrt{3}$     D) $1$

23. (97-8-57) Agar $f(x) = (x^2 + 1)^2$ bo'lsa, $f'(\frac{1}{2})$ ni toping.

A) $2,5$     B) $-1\dfrac{2}{5}$     C) $-1\dfrac{4}{5}$     D) $\dfrac{2}{5}$

24. (97-10-28) Agar $f(x) = 3\cos 2x - \sin 2x$ bo'lsa, $f'(\frac{\pi}{8})$ ni hisoblang.

A) $-4\sqrt{2}$     B) $\sqrt{2}$     C) $2\sqrt{2}$     D) $4\sqrt{3}$

25. (97-12-62) Agar $f(x) = 0,5tg2x$ bo'lsa, $f'(\frac{\pi}{6})$ ni toping.

A) $\dfrac{4}{3}$     B) $-\dfrac{1}{4}$     C) $4$     D) $2$

26. (98-7-39) Ushbu $y = -\dfrac{1}{7}\sin(7x - 5)$ funksiyaning hosilasini toping.

A) $-\dfrac{1}{7}\cdot\cos(7x - 5)$     B) $-7\cos(7x - 5)$

C) $\cos(7x - 5)$     D) $-\cos(7x - 5)$

27. (98-7-40) Ushbu $y = \log_5 2x$ funksiyaning hosilasini toping.

A) $\dfrac{1}{x\ln 2}$     B) $\dfrac{1}{x\ln 5}$     C) $\dfrac{2}{x\ln 5}$     D) $\dfrac{2}{x\ln 2}$

28. (98-8-28) Agar $f(x) = 3x - 2e^{-x}$ bo'lsa, $f'(\ln 2)$ ni hisoblang.

A) $1$     B) $2$     C) $5$     D) $4$

29. (02-1-64) Agar $f(x) = x\cdot\sin 2x$ bo'lsa, $f'(\pi) + f(\pi) + 2$ ni hisoblang.

A) $2\pi$     B) $2$     C) $2 + 2\pi$     D) $2 - 2\pi$

30. (99-1-24) Funksiyaning hosilasini hisoblang.

$$y = 2 - \cos 2x$$

A) $2\sin 2x$     B) $\sin 2x$

C) $4\cos 2x$     D) $-\sin 2x$

31. (99-1-25) $y_1 = \cos^2 3x$,   $y_2 = -\sin^2 3x$   va $y_3 = 2\sin 6x$ funksiyalardan qaysilarining hosilalari teng?

A) $y_1; y_2$   B) $y_1; y_3$   C) $y_2; y_3$   D) $y_1; y_2; y_3$

32. (99-10-43) Agar $f(x) = \sin^2 3x$ bo'lsa, $f'(\frac{\pi}{12})$ ni hisoblang.

A) $-3$     B) $3$     C) $2$     D) $-2$

33. (00-2-27) Agar $f(x) = 5\sin(2x + \frac{2}{x})$ bo'lsa, $f'(1)$ ni toping.

A) $5$     B) $0$     C) $2,5$     D) $-\dfrac{1}{5}$

34. (00-3-62) Ushbu $f(x) = \sin 2x + \ln\cos 2x$ funksiya uchun $f'(\frac{\pi}{6})$ ni toping.

A) $\dfrac{1}{2}(1 - \sqrt{3})$     B) $1 - 2\sqrt{3}$     C) $-\dfrac{3}{2}$     D) $\dfrac{3}{2}$

35. (00-8-67) Ushbu $f(x) = \sin\left(\frac{1}{x} - 1\right)$ funksiyaning hosilasini toping.

A) $\dfrac{1}{x}\cos(\frac{1}{x} - 1)$     B) $-\dfrac{1}{x}\cos(\frac{1}{x} - 1)$

C) $\dfrac{1}{x}\cos(\frac{1}{x} + 1)$     D) $-\dfrac{1}{x^2}\cos(\frac{1}{x} - 1)$

36. (01-8-26) Agar $f(x) = e^{1-2x}\cdot\cos 2x$ bo'lsa, $f'(0)$ ning qiymatini toping.

A) $-2e$     B) $0$     C) $e$     D) $2e$

37. (01-10-49) Agar

$$f(x) = \sin^4 3x, \quad \varphi(x) = 6\sin 6x$$

bo'lsa, $f'(x) = \varphi(x)$ tenglik o'rinli bo'ladigan $x$ ning barcha qiymatlarini toping.

A) $\dfrac{\pi n}{3}$,   $n \in Z$     B) $\dfrac{\pi n}{6}$,   $n \in Z$

C) $\dfrac{\pi n}{4}$,   $n \in Z$     D) $\dfrac{\pi}{3} + \dfrac{\pi n}{4}$,   $n \in Z$

38. (02-1-66) $f(x) = \sin^2 2x$ funksiya berilgan. $\dfrac{f'(x)}{2\cos 2x}$ ni toping.

A) $\sin 2x$   B) $\cos 2x$   C) $-\sin 2x$   D) $2\sin 2x$

39. (02-2-28) Agar $f(x) = \sqrt{\sin 2x}$ bolsa, $f'(\frac{\pi}{4})$ ni toping.

A) $0$     B) $1$     C) $\dfrac{1}{2}$     D) $\dfrac{\sqrt{2}}{2}$

**Yechish:** Berilgan funksiyani $f(x) = (\sin 2x)^{1/2}$ shaklda yozib, unga murakkab funksiyalarning hosilalarini hisoblashdagi 6-qoidani qo'llab

$$f'(x) = \frac{1}{2}\frac{2\cos 2x}{\sqrt{\sin 2x}} = \frac{\cos 2x}{\sqrt{\sin 2x}}$$

ekanini topamiz. Endi hosilaning $\frac{\pi}{4}$ nuqtadagi qiymatini hisoblaymiz:

$$f'(\frac{\pi}{4}) = \frac{\cos(2\cdot\frac{\pi}{4})}{\sqrt{\sin(2\cdot\frac{\pi}{4})}} = \frac{\cos\frac{\pi}{2}}{\sqrt{\sin\frac{\pi}{2}}} = \frac{0}{1} = 0.$$

**Javob:** 0 (A).

40. (02-3-46) Agar $f(x) = \sqrt{tgx}$ bolsa, $f'(\frac{\pi}{4})$ ni toping.

A) 1        B) $\frac{1}{2}$        C) $\frac{1}{4}$        D) $\frac{3}{4}$

41. (02-9-32) Agar

$$f(x) = \cos(x + \frac{\pi}{2}), \quad tg(\frac{\alpha}{2}) = \frac{1}{2}$$

bolsa, $f'(\alpha)$ ni hisoblang.

A) $-0,6$        B) $\frac{3}{5}$        C) $0,8$        D) $-\frac{1}{3}$

42. (03-1-50)
$$y = \sin^4 2x, \quad y' =?$$

A) $2\sin^2 2x \sin 4x$        B) $4\sin^2 4x \sin 2x$
C) $4\sin 2x \sin^2 4x$        D) $4\sin^2 2x \sin 4x$

43. (03-2-10) Agar

$$f(x) = e^{1-x}\cdot\sin\frac{\pi x}{2}$$

bolsa, $f'(1)$ ning qiymatini toping.
A) 1        B) 2        C) $-\sqrt{2}$        D) $-1$

44. (03-6-21)

$$f(x) = |x^2 - 14x + 45|. \quad f'(6)-?$$

A) 0        B) 5        C) 2        D) 7

## 14.2   Funksiyani hosila yordamida tekshirish. Maksimum va minimum

Funksiyalarni tekshirishda ularning o'sish yoki kamayish oraliqlarini topish muhim ahamiyatga ega. Bizga $[a; b]$ kesmada aniqlangan va uning har nuqtasida hosilasi mavjud bo'lgan $f : [a; b] \to R$ funksiya berilgan bo'lsin. Aniqlanish sohasining har nuqtasida hosilasi mavjud bo'lgan funksiyalar, *differensiallanuvchi funksiyalar* deyiladi.

Agar biror $\delta > 0$ va barcha $x \in (x_0-\delta; x_0+\delta)\cap[a; b]$ lar uchun $f(x) \le f(x_0)$ $(f(x) \ge f(x_0))$ tengsizligi bajarilsa, $x = x_0$ nuqta $f : [a; b] \to R$ funksiyaning *maksimum nuqtasi* (*minimum nuqtasi*) deyiladi. Funksiyaning minimum va maksimum nuqtalari shu funksiyaning *ekstrimum nuqtalari*, funksiyaning bu nuqtalardagi qiymatlari esa funksiyaning *ekstrimumlari* deyiladi.

Funksiyaning hosilasi nolga aylanadigan yoki hosilasi mavjud bo'lmagan nuqtalari funksiyaning *kritik nuqtalari* deyiladi.

Ekstrimumning zaruriy sharti haqida Ferma teoremasini keltiramiz. Agar differensiallanuvchi $y = f(x)$ funksiya $x = x_0$ nuqtada minimumga yoki maksimumga erishsa, u holda $f'(x_0) = 0$ bo'ladi. Demak, funksiyaning ekstrimum nuqtalarini, uning hosilasi nolga aylanadigan nuqtalari ichidan izlashimiz kerak ekan.

1. **Agar $[a; b]$ kesmada differensiallanuvchi $y = f(x)$ funksiya uchun $f'(x) > 0$, $x \in (a_1; b_1) \subset [a; b]$ bo'lsa, u holda $f$ funksiya $(a_1; b_1)$ oraliqda o'suvchi bo'ladi.**

2. **Agar $[a; b]$ kesmada differensiallanuvchi $y = f(x)$ funksiya uchun $f'(x) < 0$, $x \in (a_1; b_1) \subset [a; b]$ bo'lsa, u holda $f$ funksiya $(a_1; b_1)$ oraliqda kamayuvchi bo'ladi.**

3. **Agar $x_0 \in (a; b)$ nuqtaning shunday $\delta > 0$ atrofi mavjud bo'lib, barcha $x \in (x_0 - \delta; x_0)$ larda $f'(x) < 0$ va barcha $x \in (x_0; x_0 + \delta)$ larda $f'(x) > 0$ bo'lsa, u holda $x = x_0$ nuqta $f$ funksiya uchun minimum nuqta bo'ladi.**

4. **Agar $x_0 \in (a; b)$ nuqtaning shunday $\delta > 0$ atrofi mavjud bo'lib, barcha $x \in (x_0 - \delta; x_0)$ larda $f'(x) > 0$ va barcha $x \in (x_0; x_0 + \delta)$ larda $f'(x) < 0$ bo'lsa, u holda $x = x_0$ nuqta $f$ funksiya uchun maxsimum nuqta bo'ladi.**

5. **Agar $f'(x_0) = 0$ bo'lib, biror $\delta > 0$ va barcha $x \in (x_0 - \delta; x_0) \cup (x_0; x_0 + \delta)$ larda $f'(x) > 0$ ( yoki $f'(x) < 0$ ) bo'lsa, u holda $x = x_0$ nuqta $f$ funksiya uchun bo'rilish nuqtasi bo'ladi. Bo'rilish nuqta funksiya uchun ekstrimum nuqta bo'la olmaydi.**

1. (98-6-18) Ushbu $y = \frac{x^2}{2} - \ln x$ funksiyaning o'sish oraliqlarini toping.
A) $[-1;0) \cup [1;\infty)$        B) $[1;\infty)$
C) $[-1;\infty)$        D) $(-\infty;-1) \cup [1;\infty)$

**Yechish:** 1-xossaga ko'ra, agar $f(x)$ funksiya uchun $f'(x) > 0$, $x \in (a; b)$ bo'lsa, u holda $f(x)$ funksiya $(a; b)$ oraliqda o'suvchi bo'ladi. Berilgan funksiyaning hosilasini topamiz.

$$f'(x) = x - \frac{1}{x} = \frac{x^2 - 1}{x} = \frac{(x-1)(x+1)}{x}$$

$f'(x) > 0$ tengsizlik oraliqlar usuli bilan oson yechiladi. Uning yechimi $(-1;0) \cup (1;\infty)$. Berilgan funksiyaning aniqlanish sohasi $x > 0$ bo'lgani uchun $(-1;0)$ oraliqni chiqarib tashlaymiz. Bundan tashqari $x = 1$ nuqta funksiyaning aniqlanish sohasiga tegishli bo'lgani uchun uni ham funksiyaning o'sish oralig'iga qo'shib qo'yamiz. **Javob:** $[1; \infty)$ (B).

2. (97-9-25) Ushbu $y = x^2 - 2$ funksiyaning kamayish oralig'ini ko'rsating.
   A) $(-\infty; -2)$      B) $(-\infty; 2)$
   C) $(2; \infty)$        D) $(-\infty; 0]$

3. (97-11-20) Ushbu $y = 2x^3 + 3x^2 - 12x + 7$ funksiyaning kamayish oralig'ini aniqlang.
   A) $(-\infty; -2] \cup [1; \infty)$    B) $[-2; 1]$
   C) $[-1; 2]$              D) $[-2; \infty)$

4. (96-11-21) Ushbu $f(x) = x^2 + 2x + 4$ funksiyaning o'sish oralig'ini toping.
   A) $(-\infty; -1)$      B) $[-1; \infty)$
   C) $(1; \infty;)$       D) $(0; \infty)$

5. (96-12-21) $f(x) = x^2 - 2x + 3$ funksiyaning o'sish oralig'ini toping.
   A) $(0; \infty)$        B) $(-\infty; 1)$
   C) $[1; \infty)$        D) $(-\infty; -1)$

6. (01-2-35) Ushbu $y = x + \dfrac{1}{x-1}$ funksiyaning kamayish oraliqlarini toping.
   A) $[0; 1) \cup (1; 2]$    B) $(0; 2)$    C) $(0; 1)$    D) $(1; 2)$

7. (96-3-20) Ushbu $f(x) = -x^2 + 2x - 1$ funksiyaning o'sish oralig'ini toping.
   A) $(1; \infty)$        B) $(0; \infty)$
   C) $(-\infty; -1)$      D) $(-\infty; 1]$

8. (96-6-44) $a$ ning qanday qiymatlarida $f(x) = ax + \sin x$ funksiya o'zining aniqlanish sohasida o'sadi. Shunday $a$ larning barchasini toping.
   A) $|a| > 1$    B) $0 < a < 1$    C) $a \geq 1$    D) $a = 0$

   **Yechish:** Berilgan funksiyaning aniqlanish sohasi $D(f) = R$. 1-xossaga ko'ra, $f'(x) = a + \cos x > 0$, $x \in R$ bo'lishi kerak. Bu tengsizlik $a > 1$ shartda bajariladi. $a = 1$ da bu funksiya o'suvchidir. **Javob:** $a \geq 1$ (C).

9. (96-10-14) Quyidagi funksiyalardan qaysi biri $(0; \infty)$ oraliqda kamayuvchi bo'ladi?
   A) $y = x + 8$        B) $y = 3 - x$
   C) $y = -\dfrac{4}{x}$          D) $y = 2x^2$

10. (96-1-14) Quyidagi funksiyalardan qaysi biri $(-\infty; 0)$ oraliqda o'suvchi bo'ladi?
    A) $y = 3x + 2$        B) $y = \dfrac{3}{x}$
    C) $y = 6 - 3x$        D) $y = x^2$

11. (96-9-64) Quyidagi funksiyalardan qaysi biri $(-\infty; 0)$ oraliqda o'suvchi bo'ladi?
    A) $y = 0,5 - 2x$      B) $y = \dfrac{5}{x}$
    C) $y = 2 + 3x$        D) $y = 2\sqrt{-x}$

12. (97-8-44) $k$ ning qanday qiymatlarida $f(x) = \sin x - kx$ funksiya o'zining aniqlanish sohasida o'sadi?
    A) $(-\infty; 1)$      B) $(1; \infty)$
    C) $(-1; 0)$          D) $(-\infty; -1]$

13. (02-11-54) Quyidagi funksiyalardan qaysi biri o'zining aniqlanish sohasida o'suvchi bo'ladi?
    A) $y = \sin x$        B) $y = \dfrac{\ln x}{x}$
    C) $y = \dfrac{1}{x^2 + 1}$        D) $y = 2x^7 - 8$

14. (01-1-35) Ushbu $y = x^2 e^{-2x}$ funksiyaning o'sish oraliqlarini toping.
    A) $(-\infty; -1)$              B) $[-1; 1]$
    C) $(-\infty; -1) \cup [0; 1]$    D) $[0; 1]$

    **Yechish:** Ko'paytmaning hosilasini hisoblash qoidasidan foydalanib, berilgan funksiyaning hosilasini topamiz:

    $$f'(x) = 2xe^{-2x} - 2x^2 e^{-2x} = 2xe^{-2x}(1 - x).$$

    1-xossaga ko'ra, $f'(x) = 2xe^{-2x}(1 - x) > 0$ tengsizlikni yechamiz. Barcha $x \in R$ lar uchun $2e^{-2x} > 0$ ekanligini hisobga olsak, $f'(x) > 0$ tengsizlik $x(1 - x) > 0$ tengsizlikka teng kuchli bo'ladi. Bu tengsizlikni oraliqlar usuli bilan yechib $(0; 1)$ ni olamiz. $[0; 1]$ kesmada ham funksiyaning o'sish xossasi saqlanadi. **Javob:** $[0; 1]$ (D).

15. (00-1-44) Qaysi oraliqda $f(x) = \ln(4x - x^2)$ funksiya kamayadi?
    A) $(0; 2)$    B) $(-\infty; 0)$    C) $(0; 4)$    D) $(2; 4)$

16. (00-7-37) Ushbu $f(x) = \dfrac{2}{3}x^3 - 4x^2 + 3$ funksiya kamayadigan oraliqdagi barcha butun qiymatlar yig'indisini toping.
    A) $9$        B) $8$        C) $10$        D) $7$

17. (01-3-13) Funksiyaning kamayish oralig'ini toping.
    $$y = \dfrac{x^2}{2} - 12\ln(x - 4)$$
    A) $[6; \infty)$    B) $(4; \infty)$    C) $(2; 4)$    D) $(4; 6]$

18. (01-11-37) Qaysi oraliqda $f(x) = \dfrac{1}{5}x^5 - 4x^2$ funksiya kamayadi?
    A) $[0; 2]$    B) $(0; 2]$    C) $[0; 2)$    D) $(0; 2)$

19. (02-1-65) $y = 2x^3 + 3x^2 - 2$ funksiyaning kamayish oraliqlarini aniqlang.
    A) $(0; 8)$    B) $(-\infty; -1]$    C) $[-1; \infty)$    D) $[-1; 0]$

20. (02-5-43) $y = \dfrac{1}{4}x^4 - \dfrac{5}{3}x^3 + 3x^2 + 10$ funksiyaning kamayish oraliqlarini aniqlang.
    A) $(2; 3)$              B) $(-\infty; 0] \cup [2; 3]$
    C) $(-\infty; 3)$        D) $(-\infty; 0) \cup (3; +\infty)$

21. (02-9-31) $f(x) = -2x^3 + 15x^2 + 12$ funksiya o'sadigan kesmaning uzunligini aniqlang.
    A) $5$        B) $4$        C) $6$        D) $4,5$

22. (02-12-55) Funksiya qaysi oraliqda kamayadi?
    $$f(x) = \dfrac{1}{5}x^5 - 4x^2$$
    A) $(-2; 0]$    B) $[0; 2]$    C) $[-2; 0)$    D) $(0; 3)$

**Funksiyaning ekstremumlari**

23. (96-7-29) $f(x) = 3x - x^3$ funksiyaning maksimumini toping.
    A) $-1$     B) 2     C) $-2$     D) 4

    **Yechish:** Ferma teoremasiga ko'ra, funksiya maksimumga erishadigan nuqtalarda uning hosilasi nolga aylanadi. Shu maqsadda

    $$f'(x) = 3 - 3x^2 = 3(1-x^2) = 3(1-x)(1+x) = 0$$

    tenglamani yechamiz. Bu tenglamaning yechimi $x_1 = -1$ va $x_2 = 1$ lardir. Oson tekshirish mumkinki $(-\infty; -1)$ oraliqda $f'(x) > 0$ va $(-1; 1)$ oraliqda $f'(x) < 0$. 4-xossaga ko'ra, berilgan funksiya $x = -1$ nuqtada maksimumga erishadi. Demak, $f(-1) = -2$ funksiyaning maksimumi bo'ladi. **Javob:** $-2$ (C).

24. (97-3-29) Ushbu $g(x) = 12x - x^3$ funksiyaning minimumini toping.
    A) $-32$     B) $-16$     C) 0     D) 16

25. (97-10-29) Ushbu $y = -4x^3 + 12x$ funksiyaning minimumini toping.
    A) 0     B) $-8$     C) $-16$     D) 8

26. (97-11-15) Ushbu $y = x^2 - 8x + 7$ funksiyaning qiymatlari sohasini toping.
    A) $(2; \infty)$   B) $[-9; \infty)$   C) $[9; \infty)$   D) $[-4; \infty)$

27. (98-1-29) Ushbu $f(x) = x^3 + 2,5x^2 - 2x$ funksiyaning maksimum nuqtasidagi qiymatini hisoblang.
    A) $-8$     B) 6     C) 10,5     D) $-12$

28. (98-9-8) $t$ ning qanday qiymatida $-t^2 + 14t - 31$ uchhad eng katta qiymatga erishadi?
    A) 6     B) 5     C) 8     D) 7

29. (99-3-16) Ushbu

    $$x^2 - ax + a - 1 = 0$$

    tenglamaning ildizlari $x_1$ va $x_2$ bo'lsin. $a$ ning qanday qiymatida $x_1^2 + x_2^2$ yig'indi eng kichik qiymatga ega bo'ladi?
    A) 1     B) 2     C) 1,5     D) 2,5

30. (99-4-21) Agar $2x + y = 6$ bo'lsa, $xy$ ning eng katta qiymati nechaga teng bo'ladi?
    A) 2,5     B) 4,5     C) 3     D) $-2,5$

31. (99-9-48) Ushbu $y = -x^2 + 6x - 12$ funksiyaning qiymatlari sohasini toping.
    A) $(-3; \infty)$         B) $[-3; \infty)$
    C) $(-\infty; -3)$     D) $(-\infty; -3]$

32. (99-4-25) Ushbu $f(x) = \sqrt{2 - x - x^2}$ funksiyaning eng katta qiymatini toping.
    A) $\sqrt{2}$     B) 1,5     C) 3     D) $2\sqrt{2}$

    **Yechish:** $y = \sqrt{t}$ funksiya $[0; \infty)$ o'suvchi bo'lganligi uchun, berilgan funksiyaning eng katta qiymati ildiz ostidagi $g(x) = 2 - x - x^2$ funksiyaning eng katta qiymatida erishadi. Ferma teoremasiga ko'ra, $g$ funksiya maksimumga erishadigan nuqtalar $g'(x) = -1 - 2x = 0$ tenglama ildizlari

ichida bo'ladi. Bu tenglama esa yagona $x_0 = -0,5$ ildizga ega. Shuning uchun $f(x_0) =$

$$= \sqrt{2 - (-0,5) - (0,5)^2} = \sqrt{2,5 - 0,25} = 1,5$$

$f$ ning eng katta qiymati bo'ladi. **Javob:** $1,5$ (B).

33. (99-3-28) Funksiyaning qiymatlar sohasini toping.

    $$y = \sqrt{3x^2 - 4x + 5}$$

    A) $[0; \infty)$         B) $[\sqrt{3}; \infty)$
    C) $[\sqrt{\dfrac{3}{2}}; \infty)$     D) $[\sqrt{\dfrac{11}{3}}; \infty)$

34. (03-2-7) $y = \sqrt{x^2 + 2x + 4}$ funksiyaning qiymatlar sohasini ko'rsating.
    A) $[0; \infty)$     B) $[2; \infty)$     C) $(0; \infty)$     D) $[\sqrt{3}; \infty)$

35. (99-8-37) Ushbu $y = \sqrt{3 - x^2 - 2x}$ funksiyaning eng katta qiymatini toping.
    A) $-2$     B) 4     C) 2     D) 3

36. (00-7-35) Ushbu $f(x) = 3^{1+x} + 3^{1-x}$ funksiyaning eng kichik qiymatini toping.
    A) 9     B) 4     C) 8     D) 6

37. (99-10-42) Ushbu $y = \sqrt{x^2 - 2x + 10}$ funksiyaning qiymatlar sohasini toping.
    A) $[3; \infty)$   B) $(3; \infty)$   C) $[5; \infty)$   D) $[2; \infty)$

38. (01-1-34) Ushbu $f(x) = 3x^5 - 5x^3 - 3$ funksiyaning ekstremum nuqtalaridagi qiymatlari yig'indisini hisoblang.
    A) $-9$     B) $-6$     C) $-8$     D) $-4$

    **Yechish:** Dastlab berilgan funksiyaning ekstremum nuqtalari topamiz. Ferma teoremasiga ko'ra, funksiyaning ekstremum nuqtalari

    $$f'(x) = 15x^4 - 15x^2 = 15x^2(x-1)(x+1) = 0$$

    tenglama ildizlari ichida bo'ladi. Bu tenglama $x_0 = -1$, $x_1 = 0$, $x_2 = 1$ ildizlarga ega. Argument $x$  $x_0 = -1$ yoki $x_2 = 1$ nuqtadan o'tganda funksiya hosilasi ishorasini o'zgartiradi. 3 va 4-xossalarga ko'ra, funksiya bu nuqtalarda ekstremumga erishadi. Argument $x$  $x_1 = 0$ nuqtadan o'tganda funksiya hosilasi ishorasini o'zgartirmaydi. 5-xossaga ko'ra, bu nuqta funksiya uchun ekstremum emas. Masalani oxiriga yetkazish uchun $f(x_0) + f(x_2)$ yig'indini hisoblashimiz kerak.

    $$f(-1) + f(1) = 3(-1)^5 - 5(-1)^3 - 3 + 3 - 5 - 3 = -6.$$

    **Javob:** $-6$ (B).

39. (01-2-60) $x(x+1)(x+2)(x+3)$ ko'paytmaning eng kichik qiymatini toping.
    A) 3     B) 2     C) 1     D) $-1$

40. (01-12-38) $y = -x^2 + bx + c$ funksiya $x = -1$ nuqtada 5 ga teng eng katta qiymatni qabul qilsa, $y(1)$ ni toping.
    A) $-1$     B) 0     C) 1     D) 1,5

41. (02-4-6) $y = x^2 + 4x + 11$ funksiyaning eng kichik qiymatini toping.

A) 4      B) 11      C) $\dfrac{11}{4}$      D) 7

42. (02-2-4) $a$ ning qanday qiymatida

$$(a-7)^2 + (a-8)^2 + (a-12)^2$$

ifoda eng kichik qiymatga ega bo'ladi?
A) 9      B) 10      C) 8      D) 11

43. (02-11-52) $f(x) = 0,9x^5 - 4,5x^3 + 4$ funksiyaning minimum nuqtasini toping.
A) $-1$      B) 1      C) $\sqrt{2}$      D) $\sqrt{3}$

44. (03-5-30) $f(x) = 9^x + 5 \cdot 3^{-2x}$ funksiyaning qiymatlar to'plamini ko'rsating.
A) $[2\sqrt{5}; \infty)$   B) $(0; \infty)$   C) $[5; \infty)$   D) $[6; \infty)$

**Yechish:** Ikkala $g(x) = 9^x$ va $\varphi(x) = 5 \cdot 3^{-2x}$ funksiyalarning qiymatlari to'plami $(0; \infty)$ dan iborat. Shuning uchun yig'indi $f(x) = 9^x + 5 \cdot 3^{-2x}$ funksiyaning qiymatlar to'plami $[m; \infty)$, bu yerda $m$ berilgan funksiyaning eng kichik qiymati. Berilgan funksiyani $f(x) = 9^x + 5 \cdot 9^{-x}$ shaklda yozib, uning kritik nuqtalari topamiz. Shu maqsadda

$$f'(x) = 9^x \cdot \ln 9 - 5 \cdot 9^{-x} \cdot \ln 9 = \ln 9(9^x - 5 \cdot 9^{-x}) = 0$$

tenglamani yechamiz. Bu ko'rsatkichli tenglama bo'lib, uning ildizi $x_0 = \log_9 \sqrt{5}$ dir. Funsiya hosilasi yagona $x_0$ nuqtada nolga aylanyapti, shuning uchun u minimum nuqta bo'ladi. Uni hisoblaymiz:

$$f(x_0) = 9^{\log_9 \sqrt{5}} + 5 \cdot 9^{-\log_9 \sqrt{5}} = \sqrt{5} + 5 \cdot \frac{1}{\sqrt{5}} = 2\sqrt{5}.$$

Berilgan funksiyaning qiymatlar to'plami $[2\sqrt{5}; \infty)$ ekan. **Javob:** $[2\sqrt{5}; \infty)$ (A).

45. (03-1-57) Qaysi sonni o'zining kvadrati bilan yig'indisi eng kichik bo'ladi?
A) $-1$   B) $-0,4$   C) $-0,8$   D) $-0,5$

46. (03-3-52) Agar $m$ va $M$ sonlar $y = x + \frac{1}{x}$ funksiyaning mos ravishda minimum va maksimum nuqtalaridagi qiymatlari bo'lsa, $m - 2M$ ning qiymatini toping.
A) $-6$      B) 6      C) $-4$      D) 4

47. (03-7-81) $y = -x^4 + 2x^2 + 5$ funksiyaning qiymatlar to'plamini toping.
A) $(-\infty; 6]$   B) $(-\infty; 6)$   C) $[5; 6]$   D) $(-\infty; 5]$

48. (03-9-46) $f(x) = 0,6x^5 - 2x^3 - 1$ funksiyaning maksimum va minimum nuqtalaridagi qiymatlari yig'indisini toping.
A) $-3$      B) $-2$      C) $-1$      D) 1

49. (03-11-7) $m$ va $n$ natural sonlar. $\dfrac{6}{x} = \dfrac{1}{m} + \dfrac{1}{n}$ va $m + n = 18$ bo'lsa, $x$ ning eng katta qiymatini toping.

A) 27      B) 24      C) 18      D) 30

### Eng katta va eng kichik qiymatlar

Berilgan $f(x)$ funksiyaning $[a; b]$ kesmadagi eng katta va eng kichik qiymatlarini topish uchun, dastlab uning shu oraliqqa tegishli kritik nuqtalari topiladi, ya'ni $f'(x) = 0$ tenglamaning $[a; b]$ oraliqqa tegishli ildizlari topilib, so'ngra berilgan funksiyaning bu ildizlardagi qiymatlari va kesmaning chetki $x = a$, $x = b$ nuqtalardagi qiymatlari hisoblanib, ular o'zaro taqqoslanadi. Bu qiymatlardan eng kattasi funksiyaning $[a; b]$ kesmadagi eng katta qiymati, eng kichigi esa funksiyaning eng kichik qiymati bo'ladi.

50. (97-9-90) Ushbu $f(x) = 3x^2 - 4x - 4$ funksiyaning $[0; 3]$ kesmadagi eng katta qiymatini toping.
A) 10      B) 20      C) 11      D) 16

**Yechish:** Yuqorida keltirilgan qoidaga ko'ra

$$f'(x) = 6x - 4 = 0$$

tenglamani yechamiz. Uning yechimi $x = \dfrac{2}{3} \in [0; 3]$. Endi berilgan funksiyanung $x = 0$, $x = 3$, $x = \dfrac{2}{3}$ nuqtalardagi qiymatlarini hisoblaymiz:

$$f(0) = -4, \quad f(3) = 11, \quad f\left(\frac{2}{3}\right) = -\frac{16}{3}.$$

Sonlardan eng kattasi 11 bo'lganligi uchun funksiyaning $[0; 3]$ kesmadagi eng katta qiymati 11 ga teng bo'ladi. **Javob:** 11 (C).

51. (97-4-30) Ushbu $f(x) = x^2 - 3x + 1,25$ funksiyaning $[-1; 1]$ oraliqdagi eng katta qiymatini toping.
A) 0      B) $-0,75$      C) 5,25      D) 6,25

52. (98-5-27) Ushbu $y = x^2 - 2x + 5$ funksiyaning $[0; 1]$ kesmadagi eng katta qiymatini toping.
A) 5      B) 4      C) $-2$      D) 0

53. (98-9-38) Ushbu $f(x) = x^3 + 2x - 5$ funksiyaning $[-1; 1]$ kesmadagi eng katta va eng kichik qiymatlari ayirmani toping.
A) $-6$      B) 6      C) $-5$      D) 5

54. (98-10-72) $y = 2x^3 + 3x^2 - 12x$ funksiyaning $[0; 2]$ kesmadagi eng kichik qiymatini toping.
A) 0      B) $-2$      C) $-5$      D) $-7$

55. (98-11-33) $y = 0,25x^4 - \dfrac{x^3}{3} - x^2$ funksiyaning $[-2,5; \infty)$ oraliqdagi eng kichik qiymatini aniqlang.
A) $-\dfrac{3}{8}$      B) $\dfrac{3}{8}$      C) $\dfrac{8}{3}$      D) $-\dfrac{8}{3}$

56. (99-2-42) $y = 3x^4 - 4x^3$ funksiyaning $[0; 2]$ kesmadagi eng kichik qiymatini toping.
A) 0      B) $-16$      C) $-1$      D) 1

57. (99-3-53) Ushbu $y = x^3 - 3x^2 + 1$ funksiyaning $[-1; 4]$ kesmadagi eng katta va eng kichik qiymatlari ayirmasini toping.
   A) 20      B) 14      C) 15      D) 18

58. (99-7-28) Ushbu $y = x^2 - 2x - 1$ funksiyaning $[-1; 1]$ kesmadagi eng katta qiymatini toping.
   A) 4      B) 2      C) 0      D) 6

59. (00-1-15) Agar $m > 0$, $n > 0$ va $m + n = 16$ bo'lsa, $mn$ ning eng katta qiymatini toping.
   A) 62      B) 72      C) 64      D) 60

   **Yechish:** Masala shartlaridan $m = 16 - n$ va $mn = (16 - n)n$, $n \in (0; 16)$ kelib chiqadi. Agar $f(n) = (16 - n)n = 16n - n^2$ deb belgilab olsak, bu funksiyaning $(0; 16)$ intervaldagi eng katta qiymatini topish talab qilinadi. $f'(n) = 16 - 2n = 0$ tenglama yagona $n = 8$ yechimga ega. $f(0) = f(16) = 0$, $f(8) = 8 \cdot 8 = 64$. Demak, $mn$ ning eng katta qiymati 64 ekan. **Javob:** 64 (C).

60. (00-3-66) Ushbu $f(x) = 3x - x^3$ funksiyaning $[-2; 3]$ kesmadagi eng katta va eng kichik qiymatlari ayirmasini toping.
   A) 20      B) 18      C) 16      D) 12

61. (00-3-67) Bir tomondan imorat bilan chegaralangan, qolgan tomonlari uzunligi 120 m panjaradan iborat to'g'ri to'rtburchak shaklidagi yer maydonining eng katta yuzini toping.
   A) 1600      B) 1500      C) 1800      D) 2000

62. (00-4-52) Ushbu $y = 4x^2 + \dfrac{1}{x}$ funksiyaning $[0, 25; 1]$ kesmadagi eng katta qiymatini toping.
   A) 3      B) 4,25      C) 4,5      D) 5

63. (00-10-28) $y = 12x - x^3$ funksiyaning $[-1; 3]$ kesmadagi eng katta va eng kichik qiymatlari ayirmasini toping.
   A) 27      B) 15      C) 5      D) 32

64. (01-3-18) Ushbu $f(x) = x^2(x - 6)$ funksiyaning $[-1; 3]$ dagi eng katta va eng kichik qiymatlarini aniqlang.
   A) $2; -4$    B) $0; -32$    C) $6; -21$    D) $0; -27$

65. (01-7-49) Ushbu $y = 4x^2 + \dfrac{1}{x}$ funksiyaning $[\dfrac{1}{4}; 1]$ kesmadagi eng katta va eng kichik qiymatlari yig'indisini toping.
   A) $7\dfrac{1}{4}$      B) $9\dfrac{1}{4}$      C) $10\dfrac{1}{4}$      D) 8

66. (01-9-49) Ushbu $y = \dfrac{1}{3}x^3 + x^2 - 8x$ funksiyaning $[1; 3]$ kesmadagi eng katta va eng kichik qiymatlarining ko'paytmasini toping.
   A) 48      B) $-37$      C) 50      D) 56

67. (01-11-39) Ushbu $y = \dfrac{1}{3}x^3 - 4x$ funksiyaning $[0; 2]$ kesmadagi eng katta va eng kichik qiymatlarining ayirmasini toping.
   A) $5\dfrac{1}{3}$      B) $15\dfrac{2}{3}$      C) $10\dfrac{2}{3}$      D) $15\dfrac{1}{5}$

## 14.3 Hosilaning geometrik va mexanik ma'nosi. Urinma va tezlik

$y = f(x)$ funksiya grafigiga $(x_0; y_0)$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti $k$, bu urinmaning $Ox$ o'qining musbat yo'nalishi bilan tashkil qilgan burchagi $\alpha$ bo'lsin (14.2-chizma). U holda quyidagilar o'rinli.

1. $k = f'(x_0) = tg\alpha$.

2. $y = f(x)$ **funksiya grafigining** $(x_0; y_0)$ **nuqtasidan o'tuvchi urinma tenglamasi**

$$y = y_0 + f'(x_0)(x - x_0). \qquad (1)$$

3. $y = f(x)$ **va** $y = g(x)$ **funksiyalar grafiklariga abssissasi** $x_0$ **bo'lgan nuqtada o'tkazilgan urinmalarning parallellik sharti:**

$$f'(x_0) = g'(x_0).$$

   $S(t)$ qonuniyat bo'yicha harakatlanayotgan moddiy nuqtaning tezligi $\vartheta(t)$, tezlanishi esa $a(t)$ bo'lsin. U holda quyidagilar o'rinli.

4. $\vartheta(t) = S'(t)$.

5. $a(t) = \vartheta'(t)$.

1. (96-13-23)   $y = \dfrac{x}{1 - x}$ funksiya grafigiga abssissasi $x_0 = 3$ bo'lgan nuqtada o'tkazilgan urinmaning $Ox$ o'qi bilan tashkil qilgan burchagi $\alpha$ bo'lsa, $tg2\alpha$ ni toping.
   A) $\dfrac{7}{15}$      B) $\dfrac{2}{5}$      C) $\dfrac{8}{15}$      D) $\dfrac{3}{5}$

   **Yechish:** 1-xossaga ko'ra $tg\alpha = f'(x_0)$ ekani ma'lum. Endi funksiyaning hosilasini topamiz:

$$y' = \frac{1 - x + x}{(1 - x)^2} = \frac{1}{(1 - x)^2}$$

   So'ngra $x$ ning o'rniga $x = 3$ ni qo'yib

$$tg\alpha = \frac{1}{(1 - 3)^2} = \frac{1}{4}$$

   ni topamiz. Ikkilangan burchak formulasidan

$$tg2\alpha = \frac{2tg\alpha}{1 - tg^2\alpha} = \frac{2 \cdot \dfrac{1}{4}}{1 - \dfrac{1}{16}} = \frac{8}{15}$$

   ni hosil qilamiz. **Javob:** $\dfrac{8}{15}$ (C).

2. (96-6-46) Ushbu $y = x^2 - 3x + 2$ parabolaga abssissasi $x_0 = 2$ bo'lgan nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti nimaga teng.
   A) 1        B) 2        C) $-3$        D) 3

3. (97-2-46) $y = \ln x + x^2$ funksiyaning grafigiga $x_0 = \dfrac{1}{2}$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyentini toping.
   A) 3        B) 6        C) 4        D)6,5

4. (97-8-46) Ushbu $y = \dfrac{1}{3}x^3 - \ln x$ funksiyaning grafigiga $x_0 = 2$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyentini toping.
   A) 4        B) 3        C) 2        D) 3,5

5. (97-4-29) $y = 3x^2 + 2x$ funksiya grafigiga abssissasi $x_0 = -3$ nuqtada o'tkazilgan urinma $Ox$ o'qining musbat yo'nalishi bilan qanday burchak hosil qiladi?
   A) $arctg3$              B) $\pi - arctg16$
   C) $\pi - arctg3$        D) $-arctg16$

6. (98-9-39) Ushbu $f(x) = \dfrac{\sqrt{3}}{3} \cdot x^3 - 1$ funksiyaning grafigiga $x_0 = 1$ nuqtada o'tkazilgan urinmaning $Ox$ o'qi bilan tashkil qilgan burchagini toping.
   A) $60^0$        B) $30^0$        C) $45^0$        D) $120^0$

7. (98-11-76) Qaysi nuqtada $y = 1 + e^{x-1}$ funksiyaning grafigiga o'tkazilgan urinma $Ox$ o'qi bilan $45^0$ li burchak hosil qiladi?
   A) $x = 1$    B) $x = 0$    C) $x = -1$    D) $x = 2$

8. (99-2-41) Abssissasi $x_0 = 3$ bo'lgan nuqtadan $f(x) = \sqrt{3}\ln x$ funksiya grafigiga o'tkazilgan urinma $Oy$ o'qi bilan qanday burchak tashkil qiladi?
   A) $arctg3$        B) $60^0$        C) $30^0$        D) $arctg2$

9. (99-10-44) Ushbu $y = \sqrt{3} \cdot x^2 - 3\sqrt{3} \cdot x + 4$ funksiyaning grafigiga $x_0 = 2$ nuqtada o'tkazilgan urinma $Oy$ o'qi bilan qanday burchak tashkil qiladi?
   A) $120^0$        B) $60^0$        C) $30^0$        D) $150^0$

10. (98-10-73) $y = 2x^3 + 3x^2 - 6x$ funksiyaning grafigiga o'tkazilgan urinma $x$ ning qanday qiymatlarida $y = 6x + 1$ to'g'ri chiziqqa parallel bo'ladi?
    A) $-2$ va 3        B) 1 va 3
    C) $-2$ va 1        D) 2 va $-1$

    **Yechish:** Parallellik sharti 3-xossaga ko'ra $6 = f'(x_0) = 6x^2 + 6x - 6$ tenglik bajarilishi kerak. Bu tenglik bajarilishi uchun $x = -2$ yoki $x = 1$ bo'lishi kerak. **Javob:** $-2$ va 1 (C).

11. (98-11-37) $y = x^2 - 2x + 1$ funksiya grafigining qanday nuqtasidan o'tkazilgan urinma $y = -4(x+1)$ to'g'ri chiziqqa parallel bo'ladi?
    A) $(-1; \dfrac{1}{4})$    B) $(-1; 4)$    C) $(1; \dfrac{1}{4})$    D) $(1; 4)$

12. (98-11-77) Agar $f(x)$ funksiyaning grafigiga $x_0 = 2$ nuqtada o'tkazilgan urinmaning tenglamasi $2x -$

$3y = 6$ bo'lsa, $f'(2)$ qanchaga teng bo'ladi?
   A) $\dfrac{2}{3}$        B) $\dfrac{3}{2}$        C) 2        D) 3

13. (99-3-54) $y = x^2 + 1$ egri chiziqqa o'tkazilgan urinma $y = 2x + 3$ to'g'ri chiziqqa parallel. Urinish nuqtasining ordinatasini toping.
    A) 0        B) 2        C) 4        D) $\dfrac{1}{2}$

14. (00-9-41) $y = (2x+1)^2$ egri chiziqqa $(x_0; y_0)$ nuqtadan o'tkazilgan urinma $y = 2x + \dfrac{1}{2}$ to'g'ri chiziqqa parallel. Shu nuqtadan koordinata boshigacha bi'lgan masofani toping.
    A) $\dfrac{\sqrt{2}}{2}$        B) $\dfrac{\sqrt{2}}{4}$        C) $\dfrac{\sqrt{2}}{8}$        D) 1

15. (00-10-32) Qaysi nuqtada $y = x^2 + 2x + 8$ funksiyaning grafigiga o'tkazilgan urinma $y + 2x - 8 = 0$ to'g'ri ciziqqa parallel bo'ladi?
    A) $(-2; 8)$    B) $(2; 8)$    C) $(-2; -8)$    D) $(2; -8)$

16. (01-3-25) Ushbu $y = x^2 + \ln(x - 1)$ funksiyaning grafigiga $x = 2$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti toping.
    A) 12        B) 5        C) 3        D)1

    **Yechish:** Agar abssissasi $x = 2$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyentini $k$ desak, u holda 1-xossaga ko'ra

    $$k = f'(2) = (2x + \dfrac{1}{x - 1})|_{x=2} = 2 \cdot 2 + \dfrac{1}{2 - 1} = 5$$

    tenglik o'rinli bo'ladi. **Javob:** 5 (B).

17. (02-6-53) $f(x) = 0,5x^2 + x - 1,5$ funksiya grafigining abssissasi 2 ga teng bo'lgan nuqtasiga o'tkazilgan urinmaning burchak koeffitsiyenti toping.
    A) 1        B) 2        C) 3        D)4

18. (02-12-54) $y = -5x + 3$ to'g'ri chiziq, $f(x) = x^2 - x$ funksiyaning grafigiga o'tkazilgan urinmaga parallel. Urinish nuqtasining koordinatalarini toping.
    A) $(-2; 6)$    B) $(1; 0)$    C) $(2; 4)$    D) $(0; 0)$

19. (03-2-8) Qaysi to'g'ri chiziq $y = 4 - x^2$ funksiya grafigiga $x_0 = 2$ nuqtada o'tkazilgan urinmaga parallel bo'ladi?
    A) $y = 4 - 4x$        B) $y = 2x + 8$
    C) $y = x + 8$        D) $y = 4x + 8$

20. (03-3-51) $y = \sin \dfrac{x}{2}$ ($x \in (0; \pi)$) funksiyaning grafigiga $(x_0, y_0)$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti $\dfrac{\sqrt{3}}{4}$ ga teng. $x_0 \cdot y_0$ ni hisoblang.
    A) $\dfrac{2}{3}$        B) $\dfrac{1}{6}$        C) $\dfrac{2\pi}{3}$        D) $\dfrac{\pi}{6}$

21. (03-11-19) $(x + 3)^2 + (y - 5)^2 = 45$ aylananing $A(0; 11)$ nuqtasiga o'tkazilgan urinmaning burchak koeffitsiyentini toping.
    A) $-\dfrac{1}{2}$        B) $-2$        C) $\dfrac{1}{2}$        D) 2

### Urinma tenglamasi

22. (99-4-31) $y = e^{2-x} \cdot \cos \dfrac{\pi x}{2}$ funksiyaga abssissasi $x_0 = 2$ bo'lgan nuqtada o'tkazilgan urinmaning tenglamasini ko'rsating.
    A) $y = x - 1$  B) $y = 1 - x$
    C) $y = 2x - 1$  D) $y = x - 3$

    **Yechish:** Masalani yechishda 2-xossadan foydalanamiz. Shu maqsadda $y_0$ va $f'(x_0)$ larni hisoblaymiz:

    $$y_0 = e^{2-2} \cdot \cos \frac{\pi 2}{2} = e^0 \cdot \cos \pi = -1,$$

    $$f'(2) = (-e^{2-x} \cdot \cos \frac{\pi x}{2} - \frac{\pi}{2} e^{2-x} \cdot \sin \frac{\pi x}{2})|_{x=2} = 1.$$

    Endi topilganlarni (1) formulaga qo'yamiz:

    $$y = -1 + 1 \cdot (x - 2) = x - 3.$$

    **Javob:** $y = x - 3$ (D).

23. (96-1-29) Ushbu $f(x) = 2x^2 - 1$ funksiya grafigiga abssissasi $x_0 = 0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini ko'rsating.
    A) $y = -1$  B) $y = 2$
    C) $y = 2x + 1$  D) $y = 1$

24. (96-9-80) $y = -2x^2 - 1$ funksiya grafigiga abssissasi $x_0 = 0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini ko'rsating.
    A) $y = 1$  B) $y = -2x$
    C) $y = x - 1$  D) $y = -1$

25. (96-10-31) $y = 1 - 2x^2$ funksiya grafigiga abssissasi $x_0 = 0$ nuqtada o'tkazilgan urinma tenglamasini ko'rsating.
    A) $y = 1$  B) $y = -1$
    C) $y = -x$  D) $y = 1 - 4x$

26. (99-9-52) Abssissasi $x_0 = 0$ nuqtadan $y = x^3$ funksiya grafigiga o'tkazilgan urinmaning tenglamasini ko'rsating.
    A) $y = x$  B) $y = -0,5x$
    C) $y = 0$  D) $y = 0,5x$

27. (00-5-48) $y = 4 - x^2$ parabolaga abssissasi $x_0 = 1$ nuqtada urinma o'tkazilgan. Bu urinmaning $Oy$ o'qi bilan kesishadigan nuqtasining koordinatasini toping.
    A) $(0; 5)$  B) $(0; 1)$  C) $(0; -5)$  D) $(0; -1)$

    **Yechish:** Dastlab urinma tenglamasini topamiz. Shu maqsadda $y_0$ va $f'(x_0)$ larni hisoblaymiz:

    $$y_0 = 4 - 1^2 = 3, \quad f'(1) = -2x|_{x=1} = -2.$$

    Bu qiymatlarni (1) formulaga qo'yib, $y = 3 - 2(x-1)$ urinma tenglamasini olamiz. $Oy$ o'qining tenglamasi $x = 0$. Urinma tenglamasida $x = 0$ deb $y = 5$ ni olamiz. **Javob:** $(0; 5)$ (A).

28. (00-6-28) $y = 3\ln x - 0,5x$ funksiya grafigiga abssissasi $x_0 = 3$ nuqtada o'tkazilgan urinmaning tenglamasini tuzing.
    A) $y = 0,5x - 1,5$  B) $y = 3x - \ln 3$
    C) $y = x - 3\ln 3$  D) $y = 0,5x + 3\ln 3 - 3$

29. (00-10-58) Ushbu $f(x) = \cos 2x$ funksiyaga $(\frac{\pi}{4}; f(\frac{\pi}{4}))$ nuqtadan o'tkazilgan urinma tenglamasini ko'rsating.
    A) $y = \dfrac{\pi}{2} - 2x$  B) $y = \pi - 3x$
    C) $y = \dfrac{\pi}{2} + 3x$  D) $y = \pi - 2x$

30. (01-4-35) $f(x) = x^3$ funksiya grafigining $A(-1; -1)$ nuqtasiga o'tkazilgan urinma tenglamasini ko'rsating.
    A) $y = 3x - 2$  B) $y = 3x + 2$
    C) $y = x + 2$  D) $y = x - 2$

31. (02-1-67) $y = x - 3x^2$ funksiyaning grafigiga $x_0 = 2$ nuqtada o'tkazilgan urinmaning tenglamasini yozing.
    A) $y = 1 - 6x$  B) $y = -11x + 12$
    C) $y = 3x + 1$  D) $y = x - 3$

32. (03-6-69) $y = x^2 - 2x$ parabolaga uning biror nuqtasidan o'tkazilgan urinmaning burchak koeffisenti 4 ga teng. Shu urinmaning tenglamasini toping.
    A) $y = 4x - 4$  B) $y = 4x + 9$
    C) $y = 4x + 4$  D) $y = 4x - 9$

### Hosilaning mexanik ma'nosi

33. (99-2-39) Moddiy nuqta $S(t) = 3t^3 - 3t^2 + 12t(m)$ qonuniyat bo'yicha harakatlanyapti. Uning tezlanishi 0 ga teng bo'lgan paytda tezligi necha $m/min$ bo'ladi?
    A) 8  B) 7  C) 9  D) 11

    **Yechish:** 4 va 5-xossalarga ko'ra, nuqtaning tezligi uchun $\vartheta(t) = S'(t)$, tezlanishi uchun esa, $a(t) = \vartheta'(t)$ formulalar o'rinlidir. Demak, $\vartheta(t) = 9t^2 - 6t + 12$, $a(t) = \vartheta'(t) = 18t - 6$. Tezlanish 0 ga tengligidan $18t - 6 = 0 \Longleftrightarrow t = \dfrac{6}{18} = \dfrac{1}{3}$ ekanini topamiz. Uni tezlikning ifodasiga qo'yib

    $$\vartheta\left(\frac{1}{3}\right) = 9 \cdot \left(\frac{1}{3}\right)^2 - 6 \cdot \frac{1}{3} + 12 = 1 - 2 + 12 = 11$$

    ni topamiz. **Javob:** 11 (D).

34. (96-3-83) To'g'ri chiziq bo'ylab $x(t) = -t^3 + 6t^2 + 15t$ qonuniyat bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekund o'tgach to'xtaydi.
    A) 1  B) 2  C) 3  D) 5

35. (96-9-14) To'g'ri chiziq bo'ylab

    $$x(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 4t$$

    qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekund o'tgach to'xtaydi.
    A) 5  B) 3  C) 2  D) 4

36. (98-9-40) Moddiy nuqta $S(t) = e^t + \cos t + 5t$ qonuniyat bo'yicha harakatlanyapti. Shu nuqtaning $t = 0$ dagi tezligini toping.
    A) 5  B) 8  C) 4  D) 6

**Yechish:** 4-xossaga ko'ra, nuqtaning tezligi uchun $\vartheta(t) = S'(t) = e^t - \sin t + 5$ formula o'rinlidir. Demak, $\vartheta(0) = e^0 - \sin 0 + 5 = 1 - 0 + 5 = 6$. **Javob:** 6 (D).

37. (98-12-107) Moddiy nuqta $S(t) = -\dfrac{1}{6}t^3 + 3t^2 - 5$ qonuniyat bo'yicha harakatlanyapti. Uning tezlanishi nolga teng bo'lganda, tezligi qanchaga teng bo'ladi?
    A) 24      B) 18      C) 12      D) 6

38. (99-3-57) Ikki moddiy nuqta $S_1(t) = 2,5t^2 - 6t + 1$ va $S_2(t) = 0,5t^2 + 2t - 3$ qonuniyat bo'yicha harakatlanyapti. Qaysi vaqtda birinchi nuqtaning tezligi ikkinchisinikidan uch marta ko'p bo'lishi mumkin?
    A) 2      B) 3      C) 4      D) 6

39. (99-9-51) Moddiy nuqta to'g'ri chiziq bo'ylab $S(t) = 6t^2 - 2t^3 + 5$ qonuniyat bo'yicha harakatlanyapti. Uning tezlanishi 0 ga teng bo'lgandagi oniy tezligi nimaga teng.
    A) 8      B) 6      C) 7      D) 9

40. (02-3-50) $S(t) = t\sqrt{t}$ qonuniyat bilan harakatlanayotgan moddiy nuqtaning $t = 2$ sekunddagi tezlanishini hisoblang.
    A) $\dfrac{3}{8}\sqrt{2}$      B) $\dfrac{3}{4}\sqrt{2}$      C) $\dfrac{3}{16}\sqrt{2}$      D) $3\sqrt{2}$

41. (02-11-51) To'g'ri chiziq bo'ylab $S(t) = \dfrac{3t + 2}{t + 3}$ qonuniyat bo'yicha harakatlanayotgan moddiy nuqtaning $t = 2$ sekunddagi tezligini (m/sek) aniqlang.
    A) 0,2      B) 0,25      C) 0,28      D) 0,32

42. (03-4-44) Ikki moddiy nuqta $S_1(t) = 2t^3 - 5t^2 - 3t (m)$ va $S_2(t) = 2t^3 - 3t^2 - 11t + 7 (m)$ qonuniyatlar bo'yicha harakatlanyapti. Bu ikki nuqtaning tezliklari teng bo'lgan paytda birinchi nuqtaning tezlanishini $(m/s^2)$ toping.
    A) 10      B) 8      C) 14      D) 9

## 14.4   Boshlang'ich funksiya va integral

Biror intervalda ikki $f$ va $F$ funksiyalar berilgan bo'lib, ular

$$F'(x) = f(x) \qquad (1)$$

munosabat bilan bog'langan bo'lsin. Bobning boshida ta'kidlanganidek, $f$ funksiya $F$ funksiyaning hosilasi deyiladi. Funksiya hosilasi mavzusida $F$ funksiyani bilgan holda $f$ funksiyani topish usullarini ko'rib chiqdik. Endi teskari masalani o'rganamiz, ya'ni $f$ funksiya ma'lum bo'lsa, hosilasi $f$ ga teng bo'lgan $F$ funksiyani topish usullari bilan tanishamiz. Agar $F$ funksiya biror intervalda differensiallanuvchi bo'lib, (1) tenglik bajarilsa, $F$ funksiya shu intervalda $f$ funksiya uchun *boshlang'ich funksiya* deyiladi. Ko'p hollarda biror amalga teskari amal kiritilganda, u yagona ravishda aniqlanmaydi. Shunga o'xshash hol berilgan funksiyaga boshlang'ich funksiyani topishda ham sodir bo'ladi. Agar $F$ funksiya $f$ funksiya uchun boshlang'ich funksiya bo'lsa,

u holda ixtiyoriy $C$ o'zgarmas uchun $F(x) + C$ ham $f$ funksiya uchun boshlang'ich funksiya bo'ladi. Berilgan $f$ funksiya uchun boshlang'ich funksiyani topish jarayoni *$f$ funksiyani integrallash* deyiladi.

Agar $F$ funksiya $f$ funksiya uchun boshlang'ich funksiya bo'lsa, u holda ixtiyoriy boshqa boshlang'ich funksiya $F(x) + C$ ko'rinishga ega bo'ladi va u *aniqmas integral* degan maxsus nomga ega bo'lib, quyidagicha yoziladi:

$$\int f(x)dx = F(x) + C.$$

Endi elementar funksiyalar uchun aniqmas integrallar jadvalini keltiramiz.

1. $\displaystyle\int x^\alpha dx = \dfrac{x^{\alpha+1}}{\alpha + 1} + C, \quad (\alpha \neq -1).$

2. $\displaystyle\int x^{-1}dx = \ln|x| + C.$

3. $\int \sin x dx = -\cos x + C;$

4. $\int \cos x dx = \sin x + C.$

5. $\displaystyle\int \dfrac{1}{\sin^2 x}dx = -ctg x + C.$

6. $\displaystyle\int \dfrac{1}{\cos^2 x}dx = tg x + C.$

7. $\displaystyle\int a^x dx = \dfrac{a^x}{\ln a} + C.$

8. $\int e^x dx = e^x + C;$

9. $\int tg x dx = -\ln|\cos x| + C.$

10. $\int ctg x dx = \ln|\sin x| + C.$

    **Boshlang'ich funksiyani hisoblash qoidalari**

11. $\int f'(x)dx = f(x) + C.$

12. $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx.$

13. $\int Cf(x)dx = C \int f(x)dx.$

14. **Agar** $\int f(x)dx = F(x) + C$ **bo'lsa,**

$$\int f(ax + b)dx = \frac{1}{a}F(ax + b) + C.$$

1. Ushbu $f(x) = 2x - 1$ funksiyaning boshlang'ich funksiyasining umumiy ko'rinishini toping.
   A) $2x^2 - x + C$          B) $x^2 - x + C$
   C) $x^2 - 1 + C$          D) $x^2 + x + C$

   **Yechish:** Berilgan funksiyani $f(x) = 2x^1 - x^0$ shaklda yozib, 1 va 12-13 formulalardan foydalanib

   $$\int f(x)dx = 2 \cdot \frac{x^{1+1}}{2} - \frac{x}{1} + C = x^2 - x + C$$

   ni olamiz. **Javob:** $x^2 - x + C$ (B).

2. Ushbu $f(x) = 3x^2 - \sin x$ funksiyaning aniqmas integralini toping.
   A) $x^3 + \cos x + C$          B) $x^3 - \cos x + C$
   C) $3x^3 - \cos x + C$          D) $x^3 + tg x + C$

3. Ushbu $f(x) = e^x + \cos x$ funksiyaning boshlang'ich funksiyasining umumiy ko'rinishini toping.
   A) $e^x - \cos x + C$     B) $e^x + \sin x + C$
   C) $e^x - \sin x + C$     D) $e^x + \cos x + C$

4. Ushbu $f(x) = 1 + \cos^{-2} x$ funksiyaning boshlang'ich funksiyasini toping.
   A) $x + 2\cos^{-3} x + C$     B) $x - tgx + C$
   C) $x - ctgx + C$     D) $x + tgx + C$

5. (99-8-40) Ushbu $f(x) = \dfrac{3}{4\sqrt{x}}$ funksiyaning boshlang'ich funksiyasini toping.
   A) $\dfrac{3}{2}\sqrt{x} + C$     B) $3\sqrt{x} + C$
   C) $\dfrac{4}{3}\sqrt{x} + C$     D) $-\dfrac{3}{2}\sqrt{x} + C$

6. Ushbu $f(x) = x^{-1} - tgx$ funksiyaning boshlang'ich funksiyasining umumiy ko'rinishini toping.
   A) $\ln|x \cos x| + C$     B) $\ln|x| + ctgx + C$
   C) $\ln\left|\dfrac{\cos x}{x}\right| + C$     D) $\ln|x| - ctgx + C$

7. Ushbu $f(x) = 3(x-1)(x+3)$ funksiyaning boshlang'ich funksiyasini toping.
   A) $x^3 + 3x^2 - 9x + C$     B) $x^3 - 3x^2 - 9x + C$
   C) $x^3 + 3x^2 + 9x + C$     D) $x^3 + 3x^2 - 3x + C$

   **Yechish:** Qavsni ochib berilgan funksiyani $f(x) = 3x^2 + 6x - 9$ shaklda yozib, 1 va 12-13 formulalardan foydalanib

   $$\int f(x)dx = x^3 + 3x^2 - 9x + C$$

   ni olamiz. **Javob:** $x^3 + 3x^2 - 9x + C$ (A).

8. Ushbu $f(x) = (3x+1)^2$ funksiyaning aniqmas integralini toping.
   A) $x^3 + 3x^2 + x + C$     B) $x^3 - 3x^2 - x + C$
   C) $x^3 - 3x^2 + x + C$     D) $x^3 - 3x^2 + x + C$

9. Ushbu $f(x) = (e^{0,5x} + e^{-0,5x})^2$ funksiyaning aniqmas integralini toping.
   A) $e^x + e^{-x} + x + C$     B) $e^x - e^{-x} + x + C$
   C) $e^x + e^{-x} + 2x + C$     D) $e^x - e^{-x} + 2x + C$

10. Ushbu $f(x) = \sqrt{x}(x+1)$ funksiyaning boshlang'ich funksiyasini toping.
    A) $\dfrac{2}{5} x^2\sqrt{x} + \dfrac{2}{3} x\sqrt{x} + C$     B) $\dfrac{2}{5} x^{5/2} - \dfrac{2}{3} x^{3/2} + C$
    C) $\dfrac{5}{2} x^2\sqrt{x} + \dfrac{3}{2} x\sqrt{x} + C$     D) $\dfrac{2}{5} x^{5/2} - \dfrac{3}{2} x^{3/2} + C$

11. Ushbu $f(x) = \dfrac{x-1}{\sqrt{x}}$ funksiyaning boshlang'ich funksiyasini umumiy ko'rinishini toping.
    A) $\dfrac{2}{3} x\sqrt{x} - 2\sqrt{x} + C$     B) $\dfrac{3}{2} x\sqrt{x} - 2\sqrt{x} + C$
    C) $\dfrac{2}{3} x\sqrt{x} + 2\sqrt{x} + C$     D) $\dfrac{2}{3} x\sqrt{x} - \dfrac{1}{2}\sqrt{x} + C$

12. (00-3-70) Ushbu

    $$f(x) = 2x - \frac{1}{x^2} - \cos 2x$$

funksiyaning boshlang'ich funksiyasini toping.
A) $x^2 + \dfrac{1}{x} + \dfrac{1}{2} \sin 2x + C$

B) $x^2 - \dfrac{1}{x} + \dfrac{1}{2} \sin 2x + C$

C) $x^2 + \dfrac{1}{x} - \dfrac{1}{2} \sin 2x + C$

D) $x^2 + \dfrac{1}{x} - \sin 2x + C$

**Yechish:** Boshlang'ich funksiyalarni hisoblashning 1 va 12-qoidasidan foydalanib

$$\int f(x)dx = x^2 + x^{-1} - \int \cos 2x dx \quad (2)$$

ni olamiz. 4 va 14-qoidalardan foydalanib

$$\int \cos 2x dx = \frac{1}{2} \sin 2x + C$$

ekanligini olamiz. Bu ifodani (2) ga qo'yib

$$\int f(x)dx = x^2 + \frac{1}{x} - \frac{1}{2} \sin 2x + C$$

ni olamiz. **Javob:** (C).

13. (98-8-31) Ushbu $y = \dfrac{2}{e^x}$ funksiyaning boshlang'ich funksiyasini toping.
    A) $\dfrac{2}{e^x} + C$     B) $2\ln x + C$
    C) $e^{-x} + C$     D) $-2e^{-x} + C$

14. (96-1-32) Ushbu

    $$f(x) = 1 - \frac{1}{\cos^2 3x}$$

    funksiya uchun boshlang'ich funksiyasining umumiy ko'rinishini toping.
    A) $x + \dfrac{1}{3}ctgx + C$     B) $x - \dfrac{1}{3}tgx + C$
    C) $x - \dfrac{1}{3}tg3x + C$     D) $tg3x + C$

15. (96-3-31) Ushbu $f(x) = 2\sin 3x$ funksiya uchun boshlang'ich funksiyaning umumiy ko'rinishini toping.
    A) $-\dfrac{2}{3} \cos 3x + C$     B) $\dfrac{2}{3} \cos 3x + C$
    C) $-\dfrac{3}{2} \sin 2x + C$     D) $\dfrac{3}{2} \sin 2x + C$

16. (96-7-32) $f(x) = 2\cos^2 x$ funksiya boshlang'ich funksiyasining umumiy ko'rinishini ko'rsating.
    A) $2\sin^2 x + C$     B) $x + \dfrac{1}{2} \sin 2x + C$
    C) $\dfrac{2}{3} \cos^3 x + C$     D) $2x - \dfrac{1}{2} \sin 2x + C$

    **Yechish:** Darajani pasaytirish formulasi $2\cos^2 x = 1 + \cos 2x$ dan hamda 4 va 14-qoidalardan foydalanib

    $$\int f(x)dx = \int (1 + \cos 2x)dx = x + \frac{1}{2} \sin 2x + C$$

    ni olamiz. **Javob:** (B).

17. (97-5-35) Ushbu $f(x) = \sin^2 x$ funksiyaning boshlang'ich funksiyasini toping.

A) $-\dfrac{1}{2}x + \dfrac{1}{4}\sin 2x + C$    B) $\dfrac{1}{2}x - \dfrac{1}{4}\sin 2x + C$

C) $\dfrac{1}{4}\sin 2x + C$    D) $-\dfrac{1}{4}\sin 2x + C$

18. (96-10-34) Ushbu $f(x) = 1 + \dfrac{1}{\sin^2 4x}$ funksiya boshlang'ich funksiyasining umumiy ko'rinishini toping.

A) $x - \dfrac{1}{4}ctg4x + C$    B) $x + \dfrac{1}{4}tg4x + C$

C) $x - ctg4x + C$    D) $x + \dfrac{1}{4}ctgx + C$

19. (96-11-32) Ushbu $f(x) = 3\sin 2x$ funksiya uchun boshlang'ich funksiyasining umumiy ko'rinishini ko'rsating.

A) $-\dfrac{3}{2}\cdot\cos 2x + C$    B) $-\dfrac{2}{3}\cdot\cos 2x + C$

C) $\dfrac{3}{2}\cdot\sin 2x + C$    D) $-\dfrac{3}{2}\cdot\sin 2x + C$

20. (96-12-82) $f(x) = x^2$ funksiyaning $(3; 2)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping.

A) $\dfrac{x^3}{3} + 7$    B) $\dfrac{x^3}{3} - 7$    C) $2x - 4$    D) $2x + 4$

**Yechish:** 1-qoidadan foydalanib

$$\int f(x)dx = \int x^2 dx = \frac{1}{3}x^3 + C = F(x)$$

ni olamiz. $F$ funksiyaning grafigi $(3; 2)$ nuqtadan o'tishidan foydalansak $F(3) = \dfrac{1}{3}3^3 + C = 2$ tenglikni olamiz. Bu yerdan $C = -7$ ekanligi kelib chiqadi. **Javob:** $\dfrac{x^3}{3} - 7$ (B).

21. (96-13-25) $f(x) = x - \dfrac{x^2}{2}$ funksiyaning $(6; 0)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping.

A) $1 - x + 5$    B) $1 - x - 5$

C) $\dfrac{x^2}{2} - \dfrac{x^3}{6} - 18$    D) $\dfrac{x^2}{2} - \dfrac{x^3}{6} + 18$

22. (97-6-23) Agar $F'(x) = 2x - 1$ va $F(1) = 2$ bo'lsa, $F(x)$ ni toping.

A) $F(x) = 3x^2 - 3x + 2$    B) $F(x) = x^2 - x + 2$

C) $F(x) = x^2 + x$    D) $F(x) = \dfrac{x^2}{2} - x + 2\dfrac{1}{2}$

23. (97-7-32) Ushbu $f(x) = \sin x \cdot \cos 2x$ funksiya boshlang'ich funksiyasining umumiy ko'rinishini ko'rsating.

A) $\dfrac{1}{3}\sin 3x + \dfrac{1}{2}\sin x + C$

B) $\dfrac{1}{2}\cos x - \dfrac{1}{3}\cos 3x + C$

C) $\dfrac{1}{2}\cos x - \dfrac{1}{6}\cos 3x + C$

D) $-\cos x \cdot \sin 2x + C$

24. (97-10-32) Quyidagilardan qaysi biri $f(x) = \sin 2x \cdot \cos x$ funksiya boshlang'ich funksiyasining umumiy ko'rinishi?

A) $-\dfrac{1}{2}\cos 2x \cdot \sin x + C$

B) $\dfrac{1}{6}\cos 3x + \dfrac{1}{2}\cos x + C$

C) $-\dfrac{1}{6}\cos 6x - \dfrac{1}{6}\cos x + C$

D) $-\dfrac{1}{2}\cos x - \dfrac{1}{6}\cos 3x + C$

25. (98-1-31) Ushbu $y = e^{1-3x}$ funksiyaning boshlang'ich funksiyasini ko'rsating.

A) $-3e^x + C$    B) $e^{1-3x} + C$

C) $-3e^{1-3x} + C$    D) $-\dfrac{1}{3}e^{1-3x} + C$

26. (96-6-47) Quyidagi funksiyalarning qaysi biri uchun $F(x) = 2\cos x + \sin x + C$ funksiya boshlang'ich funksiya bo'ladi?

A) $f(x) = -2\sin x - \cos x$

B) $f(x) = 2\sin x + \cos x$

C) $f(x) = -2\sin x + \cos x$

D) $f(x) = 2\sin x - \cos x$

**Yechish:** $F$ funksiyaning hosilasini hisoblaymiz:

$$F'(x) = -2\sin x + \cos x = f(x).$$

**Javob:** $f(x) = -2\sin x + \cos x$   (C).

27. (98-2-43) Ushbu $F(x) = e^x - \dfrac{1}{3}\sin 3x + ctgx + C$ funksiya quyidagi funksiyalardan qaysi birining boshlang'ich funksiyasi?

A) $f(x) = e^x - \cos 3x - \dfrac{1}{\sin^2 x}$

B) $f(x) = e^x + \cos 3x - \dfrac{1}{\sin^2 x}$

C) $f(x) = e^x - \cos 3x + \dfrac{1}{\sin^2 x}$

D) $f(x) = e^x + \cos 3x + \dfrac{1}{\sin^2 x}$

28. (96-6-48) Agar $y = f(x)$ funksiyaning boshlang'ich funksiyasi $F(x)$ bo'lsa, $2f(2x)$ funksiyaning boshlang'ich funksiyasini toping.

A) $2F(2x)$    B) $\dfrac{1}{2}F(2x)$    C) $F(2x)$    D) $2F(x)$

29. (98-9-41) $F(x) = 2\cos 2x + \sin x + C$ funksiya quyidagi funksiyalardan qaysi birining boshlang'ich funksiyasi hisoblanadi?

A) $-4\sin 2x - \cos x$    B) $4\sin x + \cos x$

C) $-2\sin 2x + \cos x$    D) $-4\sin 2x + \cos x$

30. (99-2-43) $F(x) = \dfrac{1}{2}x^2 + \cos x + C$ funksiya $y = f(x)$ funksiyaning boshlang'ich funksiyasi. $y = f(x)$ funksiyaning hosilasini toping.

A) $2\cos^2\dfrac{x}{2}$    B) $2\sin^2\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

C) $1 + 2\cdot\cos x$    D) $2\cdot\sin^2\dfrac{x}{2}$

31. (99-3-59) Ushbu $f(x) = x + ctg^2 x$ funksiyaning boshlang'ich funksiyasini toping.

A) $\dfrac{x^2}{2} + \dfrac{1}{3}ctg^3 x + C$    B) $\dfrac{x^2}{2} - \dfrac{1}{3}ctg^3 x + C$

C) $\dfrac{x^2}{2} - x - ctgx + C$    D) $\dfrac{x^2}{2} - x + ctgx + C$

32. (01-12-50) Ushbu $f(x) = (\ln\sin x + 1)\cdot\cos x$ funksiya uchun boshlang'ich funksiyani toping.
    A) $\cos x \cdot \ln\sin x + C$       B) $\sin x \cdot \ln\sin x + C$
    C) $\sin x \cdot \ln\cos x + C$       D) $x + \ln\sin x + C$

33. (02-3-51) $f(x) = (tgx + ctgx)^2$ funksiyaning boshlang'ich funksiyasini toping.
    A) $tgx - ctgx + C$       B) $tgx - ctgx + 2x + C$
    C) $tgx - ctgx + 4x + C$       D) $tgx - ctgx - 4x + C$

34. (99-8-41) Ushbu $f(x) = 3x^2 - 2$ funksiyaning boshlang'ich funksiyalaridan qaysi birining grafigi $M(2;4)$ nuqtadan o'tadi?
    A) $F(x) = x^3 - 2x$       B) $F(x) = x^3 - 2x + 1$
    C) $F(x) = x^3 - 2x + 5$    D) $F(x) = x^3 - 2x + 8$

35. (99-10-45) Ushbu $f(x) = 2\cos^2(\frac{x}{2})$ funksiyaning $M(0;3)$ nuqtadan o'tadigan boshlang'ich funksiyasini toping.
    A) $F(x) = x - \sin x + 3$
    B) $F(x) = -x + \sin x + 3$
    C) $F(x) = x + \sin x + 3$
    D) $F(x) = x + \cos x + 3$

36. (02-10-32) $f(x) = 6x^2 - 6x + 7$ funksiyaning $M(1;0)$ nuqtadan o'tuvchi boshlang'ich funksiyasini ko'rsating.
    A) $2x^3 - 3x^2 + 7x - 6$       B) $6x^2 - 6x$
    C) $6x^3 - 6x^2 + 7x - 7$       D) $3x^3 - 3x^2 + 7x - 7$

37. (01-1-36) $f(x) = 3x^2 - 2\cos(2x + \frac{\pi}{3})$ funksiyaning, grafigi koordinata boshidan o'tuvchi boshlang'ich funksiyasini toping.
    A) $x^3 - \frac{1}{2}\sin(2x + \frac{\pi}{3}) - \frac{\sqrt{3}}{2}$
    B) $3x^3 - \sin 2x - \frac{\sqrt{3}}{2}$
    C) $x^3 - \sin x + \frac{1}{2}$
    D) $x^3 - \sin(2x + \frac{\pi}{3}) + \frac{\sqrt{3}}{2}$

38. (01-4-24) $f(x) = \frac{1}{x}$ funksiyaning, grafigi $(e;2)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping.
    A) $2\ln|x|$       B) $3 - \ln|x|$
    C) $e\ln|x|$       D) $\ln|x| + 1$

39. (01-7-51) $f(x) = \frac{1}{\sqrt{x-2}}$ funksiyaning, grafigi $A(3;5)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping.
    A) $\sqrt{x-2} + 4$       B) $2\sqrt{x-2} + 3$
    C) $\sqrt{x-2} + 3$       D) $2\sqrt{x-2} + 4$

40. (01-8-30) Agar $F'(x) = e^{-3x}$ va $F(1) = 0$ bo'lsa, $F(x)$ ni toping.
    A) $-3e^{-3x} + 1$       B) $-\frac{1}{3}e^{-3x} + \frac{1}{3}$
    C) $\frac{1}{3}e^{-3x} + e$       D) $-\frac{1}{3}e^{-3x} + \frac{1}{3}e^{-3}$
    **Yechish:** 11-qoidaga ko'ra,
    $$F(x) = \int F'(x)dx = -\frac{1}{3}e^{-3x} + C. \quad (3)$$

Endi $F(1) = 0$ shartdan foydalanamiz: $0 = -\frac{1}{3}e^{-3} + C$. Bu yerdan $C = \frac{1}{3}e^{-3}$ topib, uni (3) ga qoyamiz, natijada D) javobni olamiz. **Javob:** (D).

41. (01-1-37) Agar $F'(x) = e^x + \sin 2x$ va $F(0) = 3,5$ bo'lsa, $F(x)$ ini toping.
    A) $e^x - \frac{1}{2}\cos 2x + 3$       B) $e^x - \frac{1}{2}\cos 2x + 4$
    C) $e^x - \cos 2x + 4,5$       D) $e^x - \cos x + 3$

42. (01-11-41) Agar $F'(x) = 3x^2 - 2x$ va $F(0) = 4$ bo'lsa, $F(x)$ ni toping.
    A) $F(x) = x^4 + 2x^2 - 4$     B) $F(x) = x^4 - 2x^2 + 4$
    C) $F(x) = x^4 - x^2 - 4$     D) $F(x) = x^3 - x^2 + 4$

43. (97-11-23) Agar $F'(x) = x - 4$, $F(2) = 0$ bo'lsa, $F(x)$ ni toping.
    A) $F(x) = x^2 - 2x$       B) $F(x) = x^2 - 4x + 4$
    C) $F(x) = 2x^2 - 4x$       D) $F(x) = \frac{1}{2}x^2 - 4x + 6$

44. (02-2-31) Agar $f'(x) = 6x^2 - 3x + 5$ va $f(4) = 130$ bo'lsa, $f(0) = ?$
    A) 6       B) 4       C) $-4$       D) $-6$

### 14.4.1 Aniq integral

Aniq integralning ta'rifiga to'xtalmaymiz, ammo uning xossalari va tadbiqlarini qarab chiqamiz. $f$ funksiyadan $[a; b]$ kesma bo'yicha olingan aniq integral quyidagicha belgilanadi:

$$\int_a^b f(x)dx.$$

Aniq integral tushunchasi $[a; b]$ kesma $f$ funksiya grafigi va abssissalar o'qi bilan chegaralangan geometrik figura yuzasini hisoblash masalasi bilan uzviy bog'liqdir. Faraz qilaylik, $F$ funksiya $f$ funksiyaning $[a; b]$ kesmadagi boshlang'ich funksiyasi bo'lsin, ya'ni $F'(x) = f(x)$, $x \in [a; b]$. Endi integral hisobning asosiy formulasi − Nyuton-Lebnist formulasini keltiramiz:

1. **Nyuton-Lebnist formulasi:**

$$\int_b^a f(x)dx = F(x)|_a^b = F(b) - F(a). \quad (14.4)$$

Bizga $[a; b]$ kesmada aniqlangan manfiymas $f$ funksiya berilgan bo'lsin. Yuqoridan $f$ funksiyaning grafigi, quyidan abssissalar o'qi va yon tomonlardan $x = a$ hamda $x = b$ vertikal to'g'ri chiziqlar bilan chegaralangan figura egri chiziqli trapetsiya (14.3-chizma) deyiladi. Egri chiziqli trapetsiya yuzasi $S$ uchun quyidagi tenglik o'rinli:

$$S = \int_b^a f(x)dx = F(b) - F(a). \quad (14.5)$$

1. (98-8-32) Hisoblang.

$$\int_{-\frac{\pi}{2}}^{\pi} |\cos x| dx$$

A) 1     B) 3     C) −1     D) 4

**Yechish:** $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ da $\cos x \ge 0$ ekanidan bu oraliqda $|\cos x| = \cos x$ tenglik, $\frac{\pi}{2} \le x \le \pi$ da esa $\cos x \le 0$ ekanidan bu oraliqda $|\cos x| = -\cos x$ tenglik o'rinli ekani kelib chiqadi. Shu sababli berilgan integralni ikkita integralga ajratib hisoblaymiz.

$$\int_{-\frac{\pi}{2}}^{\pi} |\cos x| dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\pi} \cos x dx =$$

$$= \sin x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \sin x\Big|_{\frac{\pi}{2}}^{\pi} = \sin\frac{\pi}{2} - \sin(-\frac{\pi}{2}) -$$

$$- \sin \pi + \sin\frac{\pi}{2} = 1 + 1 - 0 + 1 = 3$$

**Javob:** 3 (B).

2. (97-6-63) Hisoblang.

$$\int_{-2}^{3} |3 - x| dx$$

A) 9     B) 8     C) 4     D) 12,5

3. (96-1-31) Integralni hisoblang.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x dx$$

A) $\frac{\sqrt{3}}{2}$     B) $\frac{\sqrt{2}}{2}$     C) $\frac{1}{2}$     D) $-\sqrt{2}$

4. (96-6-49) Integralni hisoblang.

$$\int_{0}^{e^2-1} \frac{dx}{x+1}$$

A) 3     B) 2     C) −2     D) −3

5. (96-7-31) Hisoblang.

$$\int_{0}^{2} (1 - 2x)^2 dx$$

A) $4\frac{1}{2}$     B) $-3\frac{1}{3}$     C) 9     D) $4\frac{2}{3}$

6. (97-3-31) Hisoblang.

$$\int_{0}^{1} (3x - 1)^2 dx$$

A) 3     B) 1     C) $-\frac{1}{3}$     D) $\frac{7}{9}$

7. (96-9-82) Hisoblang.

$$\int_{0}^{\frac{\pi}{4}} \sin 2x dx$$

A) $\frac{1}{2}$     B) −1     C) $-\frac{1}{2}$     D) 1

**Yechish:** Nyuton-Lebnist formulasiga ko'ra,

$$\int_{0}^{\frac{\pi}{4}} \sin 2x dx = -\frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{4}} = -\frac{1}{2}\cdot 0 + \frac{1}{2} = \frac{1}{2}.$$

**Javob:** $\frac{1}{2}$ (A).

8. (96-10-33) Integralni hisoblang.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x dx$$

A) $\frac{1}{2}$     B) $-\frac{\sqrt{3}}{4}$     C) 0     D) $\frac{\sqrt{3}}{4}$

9. (97-1-22) Integralni hisoblang.

$$\int_{-\frac{\pi}{2}}^{0} \cos 3x dx$$

A) $\frac{1}{3}$     B) 0     C) $-\frac{1}{3}$     D) $\frac{2}{3}$

10. (97-6-22) Hisoblang.

$$\int_{\frac{\pi}{4}}^{-\frac{\pi}{4}} \cos 2x dx$$

A) 0     B) −2     C) −1     D) $\sqrt{2}$

11. (97-8-49) Hisoblang.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + ctg^2 x) dx$$

A) $\frac{\sqrt{3}}{3}$     B) 1     C) $\sqrt{3} - 1$     D) −1

12. (97-7-31) Hisoblang.

$$\int_{-1}^{0} (2x + 1)^2 dx$$

A) $\frac{1}{6}$     B) $\frac{2}{3}$     C) 1     D) $\frac{1}{3}$

13. (97-10-31) Hisoblang.

$$\int_{-1}^{0} (1 + 3x)^2 dx$$

A) 1     B) −1     C) $\frac{7}{9}$     D) $-\frac{1}{3}$

14. (97-11-22) Integralni hisoblang.

$$\int_0^{\frac{\pi}{2}} \sin 5x\, dx$$

A) $\dfrac{1}{5}$    B) $-\dfrac{2}{5}$    C) 1    D) $-1$

15. (98-4-43) $a$ ning qanday qiymatlarida

$$\int_0^2 (t - \log_2 a)\, dt = 2\log_2 \frac{2}{a}$$

tenglik o'rinli bo'ladi?
A) $a \in (2; \infty)$    B) $a \in (1; 2)$
C) $a \in (0; \infty)$    D) $a \in (-1; 1)$

16. (98-7-41) Hisoblang.

$$\int_{-2}^0 (|x| + 1)\, dx$$

A) 3    B) 2    C) 4    D) $-4$

17. (98-11-41) Hisoblang.

$$\int_0^{\ln 3} (e^{2t} - e^{-t/2})\, dt$$

A) $2 + \dfrac{2}{\sqrt{3}}$    B) $2 - \dfrac{2}{\sqrt{3}}$

C) $\dfrac{2}{\sqrt{3}} - 2$    D) $2 + \dfrac{1}{\sqrt{3}}$

**Yechish:** Integral ostidagi $f(t) = e^{2t} - e^{-t/2}$ funksiya uchun boshlang'ich funksiya

$$F(t) = \frac{1}{2} \cdot e^{2t} + 2e^{-t/2}$$

dir. Nyuton-Lebnist formulasiga ko'ra,

$$F(\ln 3) - F(0) = \frac{1}{2} \cdot e^{2\ln 3} + 2e^{-\ln 3/2} =$$

$$= \frac{9}{2} + \frac{2}{\sqrt{3}} - 2,5 = 2 + \frac{2}{\sqrt{3}}$$

son integralning qiymati bo'ladi. **Javob:** (A).

18. (98-12-40) Hisoblang.

$$\int_0^2 (|x| + 1)\, dx$$

A) 4    B) 2    C) 3    D) 8

19. (99-1-27) Hisoblang.

$$\int_0^2 x^3\, dx$$

A) 4    B) $-4$    C) $\dfrac{16}{3}$    D) 2

20. (99-2-44) Hisoblang.

$$\int_{\frac{5\pi}{3}}^{\frac{4\pi}{3}} |\sin x|\, dx$$

A) 1,5    B) $-2$    C) 1    D) $-1$

21. (99-6-24) Hisoblang.

$$\int_{\frac{2\pi}{3}}^{2\pi} \cos(0,25x)\, dx$$

A) $-2$    B) 1    C) $-1$    D) 2

22. (00-2-29) Hisoblang.

$$\int_0^{2\pi} \cos 7x \cdot \cos 2x\, dx$$

A) 0,5    B) 1    C) 2    D) 0

**Yechish:** Kosinuslar ko'paytmasini yig'indiga almashtirish formulasidan foydalanib $\cos 7x \cdot \cos 2x = \dfrac{1}{2}(\cos 5x + \cos 7x)$ ni olamiz. Demak, integral ostidagi funksiyaning boshlang'ich funksiyasi

$$F(x) = \frac{1}{10} \cdot \sin 5x + \frac{1}{14} \sin 7x$$

dir. $F$ funksiya uchun $F(2\pi) = F(0) = 0$ bo'lganligi uchun, Nyuton-Lebnist formulasiga ko'ra integralning qiymati ham nol bo'ladi. **Javob:** 0 (D).

23. (00-2-44) $b$ ning qanday qiymatida

$$\int_{-1}^1 (4x + b)\, dx$$

integralning qiymati 1 ga teng bo'ladi?
A) $\dfrac{1}{2}$    B) $\dfrac{1}{4}$    C) $\dfrac{1}{3}$    D) 2

24. (00-3-68) Integralni hisoblang.

$$\int_{-3}^6 x|x|\, dx$$

A) 81    B) 63    C) 60    D) 84

25. (00-3-71) Integralni hisoblang.

$$\int_{-\pi/2}^{-\pi/4} \frac{dx}{\cos^2\left(\frac{\pi}{2} + x\right)}$$

A) $\sqrt{3}$    B) $\sqrt{3} - 1$    C) 0    D) 1

26. (00-4-54) Hisoblang.

$$\frac{1}{16} \int_0^{\pi} \frac{dx}{\cos^2 \frac{x}{4}}$$

A) 1    B) 0,5    C) 0,25    D) 2

27. (00-10-36) Integralni hisoblang.

$$\int_0^1 \frac{e^x + e^{-1}}{e^{x-1}}\, dx$$

A) $\dfrac{e^2 - e + 1}{e}$    B) $\dfrac{e^2 - e - 1}{e}$

C) $\dfrac{-e^2 + e - 1}{e}$    D) $\dfrac{e^2 + e - 1}{e}$

28. (01-1-38) Integralni hisoblang.

$$\int_{-4}^{4} x|x|dx$$

A) 0    B) $\dfrac{1}{2}$    C) $-\dfrac{1}{2}$    D) $\dfrac{1}{4}$

29. (01-7-52) Hisoblang.

$$\int_{0}^{\pi/2} \sin x \cos x dx$$

A) $\dfrac{1}{2}$    B) $\dfrac{1}{4}$    C) 1    D) $\dfrac{1}{8}$

**Egri chiziqli trapetsiyaning yuzi**

30. (97-9-92) $y = x^2$ va $y = 2x$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) $1\dfrac{1}{3}$    B) 1    C) $1\dfrac{1}{4}$    D) $1\dfrac{1}{2}$

**Yechish:** $y = x^2$, $y = 2x$ funksiyalar grafiklarining kesishgan nuqtalarining abssissalarini topamiz. Buning uchun $x^2 = 2x$ tenglamani yechamiz. Uning ildizlari $x_1 = 0$, $x_2 = 2$. Suning uchun berilgan chiziqlar bilan chegaralangan yuza

$$S = \int_{0}^{2} (2x - x^2)dx = \left(x^2 - \frac{x^3}{3}\right)\Big|_{0}^{2} =$$

$$= 4 - \frac{8}{3} = \frac{12 - 8}{3} = \frac{4}{3} = 1\frac{1}{3}$$

ga teng. **Javob:** $1\dfrac{1}{3}$ (A).

31. (96-3-32) $y = x^2$, $y = 0$, $x = 0$ va $x = 2$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) $\dfrac{1}{2}$    B) 2    C) 4    D) $2\dfrac{2}{3}$

32. (96-11-33) $y = x^2$, $y = 0$ va $x = -2$ chiziqlar bilan chegaralangan figuraning yuzini toping.
A) $2\dfrac{2}{3}$    B) $2\dfrac{1}{3}$    C) $2\dfrac{5}{6}$    D) 2

33. (96-12-33) $y = x^3$, $y = 0$ va $x = 2$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) 8    B) 4    C) $\dfrac{1}{2}$    D) $2\dfrac{2}{3}$

34. (97-4-32) $y = \sqrt{x}$, $y = 0$ va $x = 4$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) $5\dfrac{1}{3}$    B) $5\dfrac{2}{3}$    C) 5    D) $6\dfrac{1}{4}$

35. (97-5-36) $y = \dfrac{1}{\sqrt{x}}$, $y = 0$, $x = 1$, $x = 4$ chiziqlar bilan chegaralangan figuraning yuzini toping.
A) 5    B) 2    C) 3    D) 1

36. (97-9-36) $y = \dfrac{3}{\sqrt{x}}$, $y = 0$, $x = 1$ va $x = 4$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) 6    B) 7    C) 5    D) 4

37. (99-8-75) Chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

$$y = \sin 2x, \quad y = 0, \quad x = 0 \quad \text{va} \quad x = \frac{\pi}{2}$$

A) 1    B) $\dfrac{1}{2}$    C) 2    D) $\dfrac{3}{2}$

38. (99-10-46) $t$ ning qanday qiymatlarida $y = x^2$, $x = 0$ va $x = t$ chiziqlar bilan chegaralangan figuraning yuzi 9 ga teng bo'ladi?
A) 6    B) 4    C) 5    D) 3

39. (01-4-22) $y = -x^2$, $y = 0$, $x = 1$ va $x = 2$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) $\dfrac{7}{3}$    B) $\dfrac{3}{7}$    C) $\dfrac{3}{2}$    D) $\dfrac{5}{2}$

40. (01-4-29) Ushbu $y = \dfrac{x^2}{2}$ va $y = \dfrac{x^3}{2}$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) $\dfrac{1}{12}$    B) $\dfrac{1}{24}$    C) $\dfrac{1}{6}$    D) $\dfrac{1}{13}$

41. (01-9-53) $y = 2x^2$, $y = \dfrac{2}{x}$, $y = 0$ va $x = e$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) 2    B) $2\dfrac{1}{3}$    C) 1,5    D) $2\dfrac{2}{3}$

42. (02-2-34) Ushbu $y = 2x^2$, $y = 0$ va $x = 3$ chiziqlar bilan chegaralangan figuraning yuzi necha kvadrat birlik bo'ladi?
A) 18    B) 27    C) 54    D) 36

43. (02-3-52) Ushbu $2x - 3y + 2 = 0$, $y = 0$, $x = 2$ va $x = 5$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) 9    B) 7    C) 11    D) 10

44. (02-6-55) $y = x^3$ va $y = \sqrt{x}$ chiziqlar bilan chegaralangan shaklning yuzini hisoblang.
A) $\dfrac{2}{5}$    B) $\dfrac{3}{7}$    C) $\dfrac{7}{12}$    D) $\dfrac{5}{12}$

45. (03-6-23) $x = 0$, $y = 9 - x^2$ va $y = x^2 + 1$ chiziqlar bilan chegaralangan sohaning yuzini toping.
A) $10\dfrac{1}{3}$    B) $10\dfrac{2}{3}$    C) $13\dfrac{2}{3}$    D) $21\dfrac{1}{3}$

46. (03-6-24)

$$x = 1, \quad y = 1 - |x - 1|, \quad \text{va} \quad y = -1 + |x - 1|$$

chiziqlar bilan chegaralangan sohaning yuzini toping.
A) $\dfrac{1}{2}$    B) $\dfrac{2}{3}$    C) 1    D) 2

47. (Q) Quyidagi chiziqlar bilan chegaralangan shaklning yuzini toping:

$$x = 0, \quad x = \ln 3, \quad y = 0 \quad \text{va} \quad y = e^x.$$

<u>A) 2</u>    B) $\ln 3 - 1$    C) 1    D) 3

## 14.5 Maxsus yo'l bilan yechiladigan masalalar

1. **Agar** $A + B + C = 0$ **bo'lib,** $A \geq 0,$ $B \geq 0,$ $C \geq 0$ **bo'lsa, u holda** $A = 0,$ $B = 0,$ $C = 0$ **tengliklar bir vaqtda bajariladi.**

2. **Agar** $A$ **soni uchun** $f(x) \leq A,$ $g(x) \geq A$ **bo'lsa,**
   $f(x) = g(x)$ **tenglama** $\begin{cases} f(x) = A \\ g(x) = A \end{cases}$ **sistemaga teng kuchli.**

3. $P(x)$ **ko'phad koeffitsiyentlari yig'indisi** $P(1)$ **ga,** $x$ **ning juft darajalari oldidagi koeffitsiyentlari yig'indisi**
   $\frac{1}{2}(P(1) + P(-1))$ **ga;** $x$ **ning toq darajalari oldidagi koeffitsiyentlari yig'indisi**
   $\frac{1}{2}(P(1) - P(-1))$ **ga teng.**

4. $x^3 + ax^2 + bx + c = 0$ **tenglamaning ildizlari** $x_1, x_2, x_3$ **bo'lsin. U holda** $x_1 + x_2 + x_3 = -a,$ $x_1 x_2 + x_2 x_3 + x_3 x_1 = b,$ $x_1 x_2 x_3 = -c.$

5. **Ifodalarning eng katta yoki eng kichik qiymatlarini topishda quyidagi tengsizliklardan foydalaniladi:**
   **1)** $a + b \geq 2\sqrt{ab},$ $a, b \geq 0;$
   **2)** $p^2 + q^2 + r^2 \geq pq + qr + pr,$ $p, q, r-$ **ixtiyoriy sonlar.**

6. $P(x)$ **ko'phadni** $x - a$ **ga bo'lgandagi qoldiq** $P(a)$ **ga teng.**

1. (99-5-16) Tenglamaning ildizlari nechta?
   $$\cos(lg(2 - 3^{x^2})) = 3^{x^2}$$
   A) $\emptyset$  B) cheksiz ko'p  C) 1  D) 2

   **Yechish:** $-1 \leq \cos x \leq 1$ bo'lgani uchun tenglama chap qismining eng katta qiymati 1 ga teng. $3^{x^2} \geq 3^0 = 1$ bo'lgani uchun tenglama o'ng qismining eng kichik qiymati 1 ga teng. Tenglik bajarilishi uchun
   $$\begin{cases} \cos(lg(2 - 3^{x^2})) = 1 \\ 3^{x^2} = 1 \end{cases}$$
   bo'lishi kerak ekan. Ikkinchi tenglamadan $x^2 = 0$, ya'ni $x = 0$ ni topamiz. $x = 0$ son 1-tenglamani ham qanoatlantiradi. Shuning uchun berilgan tenglama yagona $x = 0$ yechimga ega ekan. **Javob:** 1 (C).

2. (97-12-10) Agar $(a - |b|)^2 + (a - 2)^2 = 0$ bo'lsa, $2a - 3b$ ning qiymatini toping.
   A) $-2$  B) 10  C) 2 va 10  D) $-2$ va 10

3. (98-11-61) Agar $x$ va $y$ sonlari
   $$x^2 + y^2 + (y - 1)^2 = 2xy$$
   tenglikni qanoatlantirsa, $x + y$ qanchaga teng bo'ladi?
   A) 4  B) 1  C) 3  D) 2

4. (98-12-80) Agar
   $$x^2 + y^2 + 2(2x - 3y) + |z - xy| + 13 = 0$$
   bo'lsa, $x + y + z$ ni toping.
   A) 8  B) 11  C) $-5$  D) $-7$

5. (99-9-8) Agar $n - m = (a - 2)^2,$ $p - n = (b - 3)^2$ va $m - p = (c - 4)^2$ bo'lsa, $a + b + c$ yig'indi nechaga teng?
   A) 8  B) 10  C) 11  D) 9

6. (99-10-8) Agar $m - n = (2x + y)^2,$ $n - m = (4x - y - 12)^2$ bo'lsa, $x \cdot y$ ni toping.
   A) $-6$  B) 6  C) $-8$  D) 8

7. (00-6-14) Tenglamalar sistemasi nechta yechimga ega?
   $$\begin{cases} y = x^2 + 7x + 11 \\ y = x^2 + 3x + 15 \end{cases}$$
   A) 4  B) 3  C) 2  D) 1

8. (00-9-39) $x^2 + y^2$ ni hisoblang.
   $$9(x^4 + y^4) - 6(x^2 + y^2) + 2 = 0$$
   A) $\frac{1}{3}$  B) 1  C) $\frac{2}{3}$  D) 3

9. (02-9-8) Agar
   $$16a^2 + 9b^2 + 4c^2 + 3 = 8a + 6b + 4c$$
   bo'lsa, $a + b + c$ ga teskari sonni toping.
   A) $-1\frac{1}{12}$  B) $\frac{12}{13}$  C) $\frac{12}{11}$  D) $-\frac{11}{12}$

10. (01-9-44) Tenglamani yeching.
    $$\log_7^2(x^2 + 5x - 13) + \log_{1/7}^2(x^2 - 8x + 13) = 0$$
    A) 3  B) 2  C) 5  D) 1

11. (03-5-42) Tenglamani yeching.
    $$\cos^2\left(\frac{\pi x}{3}\right) + \sqrt{2x^2 - 5x - 3} = 0$$
    A) 3  B) $\frac{3}{2}$  C) $-\frac{1}{2}$  D) $-3$

12. (99-5-31) Tenglama $[-3\pi; 3\pi]$ oraliqda nechta yechimga ega?
    $$\sin\left(\frac{\pi\sqrt{5}}{20} \cdot x\right) = 21 - 4\sqrt{5}x + x^2$$
    A) $\emptyset$  B) 1  C) 2  D) 3

13. (00-5-42) Tenglamani yeching.
    $$\sin 5x - 3 \cdot \cos 2x = 4$$
    A) $-\frac{\pi}{2} + 2\pi n,$ $n \in Z$  B) $\frac{\pi}{2} + \pi n,$ $n \in Z$
    C) $\pi + \pi n,$ $n \in Z$  D) $\frac{\pi}{2} + 2\pi n,$ $n \in Z$

14. (00-6-55) Tenglama $[-2\pi; 2\pi]$ kesmada nechta ildizga ega?
$$\cos x \cos 2x \cos 4x = 1$$
A) 1    B) 2    C) 3    D) 4

15. (00-9-24) Tenglamaning ildizi nechta?
$$\log_3 x + \log_x 3 = 2\cos(6\pi x^2)$$
A) $\emptyset$    B) 1    C) 2    D) 3

16. (01-2-31) Tengsizlikni yeching.
$$\cos^2(x+1) \cdot lg(9 - 2x - x^2) \geq 1$$
A) $(-\infty; -1]$    B) $\{-1\}$    C) $[-1; 0)$    D) $(0; 1)$

17. (01-2-67) Tenglamaning nechta ildizi bor?
$$\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} = 4 - 2x - x^2$$
A) 0    B) 1    C) 2    D) 3

18. (01-8-34) Tenglama ildizlari yig'indisini toping.
$$3 - 4x - 4x^2 = 2^{4x^2 + 4x + 3}$$
A) 2    B) $-0,5$    C) 6    D) 4,5

19. (01-12-22) Tenglama $[-\pi; \pi]$ kesmada nechta ildizga ega?
$$\cos^2 \frac{x}{2} - \sin^2\left(\frac{\sqrt{3}x}{2}\right) = 1$$
A) 1    B) 2    C) 3    D) yechimi yo'q

20. (03-2-19) Tenglama ildizlari yig'indisini toping.
$$6x - x^2 - 5 = 2^{x^2 - 6x + 11}$$
A) $-5$    B) $-3$    C) 6    D) 4

21. (03-9-15) Tenglamaning ildizlari quyida keltirilgan oraliqlarning qaysi biriga tegishli?
$$\sqrt{25 - x^2} + \sqrt{9 - x^2} = 9x^4 + 8$$
A) $[-3; -1]$    B) $(-2; 0)$    C) $[0; 2]$    D) $(0; 2)$

22. (99-10-6) Ushbu
$$x^3 - px^2 - qx + 4 = 0$$
tenglamaning ildizlaridan biri 1 ga teng. Shu tenglama barcha koeffitsiyentlari yig'indisini toping.
A) $-1$    B) 0    C) 1    D) 1,5

23. (03-3-26) $f(x) = (x^3 + 2x^2 - 1)^2 - 3x^2$ ko'phadning juft darajali hadlari koeffitsiyentlarining yig'indisini toping.
A) $-6$    B) $-2$    C) 3    D) $-1$

24. (97-1-12) Tenglama ildizlari yig'indisini toping.
$$x^3 + 2x^2 - 9x - 18 = 0$$
A) 9    B) $-2$    C) 6    D) 2

25. (97-6-12) Tenglama ildizlari ko'paytmasini toping.
$$x^3 - 3x^2 - 4x + 12 = 0$$
A) 6    B) $-4$    C) 12    D) $-12$

26. (97-11-12) Tenglama ildizlari ko'paytmasini toping.
$$x^3 + 5x^2 - 4x - 20 = 0$$
A) $-10$    B) 20    C) $-4$    D) $-20$

27. (00-8-12) Tenglama ildizlari yig'indisini toping.
$$x^3 + 3x^2 - 4x - 12 = 0$$
A) $-3$    B) $-7$    C) 4    D) 12

28. (02-11-22) Tenglama ildizlari ko'paytmasini toping.
$$x^3 - 3x^2 - 2x + 6 = 0$$
A) 3    B) $-6$    C) 6    D) $-3$

29. (99-8-22) Ko'phadning eng kichik qiymatini toping.
$$x^2 - 2x + 2y^2 + 8y + 9$$
A) 0    B) 8    C) 1    D) 9

30. (00-1-17) Ushbu
$$2x^2 + 2xy + 2y^2 + 2x - 2y + 3$$
ko'phad eng kichik qiymatga erishganda, $xy$ ning qiymati qanday bo'ladi?
A) 1    B) $-2$    C) 2    D) $-1$

31. (97-9-56) 18 ta gugurt chupidan ularni sindirmay eng katta yuzali to'g'ri to'rtburchak yasalgan. Shu to'rtburchakning yuzini toping.
A) 16    B) 20    C) 24    D) 28

32. (00-3-20) Ifodalarni taqqoslang.
$$p = a^2 + b^2 + c^2, \quad q = ab + ac + bc$$
A) $p < q$    B) $p = q$    C) $p > q$    D) $p \geq q$

33. (98-11-64) Agar $|a| \leq 1$, $|b| \leq 1$ bo'lsa, $\arccos a - 4\arcsin b$ ifodaning eng katta qiymati qanchaga teng bo'ladi?
A) $2\pi$    B) 1    C) $3\pi$    D) $5\pi$

34. (98-12-77) Ushbu
$$\frac{10}{x^2 + 8x + 41} + \cos 5y$$
ifodaning eng katta qiymati nechaga teng bo'lishi mumkin?
A) 1,8    B) 1,5    C) 1,4    D) 2

35. (02-6-39) Ifodaning eng kichik qiymatini toping.
$$\frac{2\sin\alpha - 1}{5 - 2\sin\beta} + \frac{tg^2\gamma + ctg^2\gamma}{2}$$
A) 0    B) 1    C) $-1$    D) $\frac{4}{7}$

36. (02-9-17) Ifodaning eng kichik qiymatini toping.

$$2a^2 - 2ab + b^2 - 2a + 2$$

A) $-2$    B) 1    C) 2    D) 4

37. (00-2-23) Yig'indini hisoblang.

$$\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + ...+$$

$$+\frac{1}{\sqrt{79}+\sqrt{81}}$$

A) 6    B) 5    C) 3    D)4

38. (00-10-54) Ifodaning qiymatini hisoblang.

$$\sqrt{2^3\sqrt{5^3\sqrt{2^3\sqrt{5^3...}}}}$$

A) 17    B) 12    C) 14    D) 20

39. (97-5-15) Tenglamaning natural sonlardagi yechimida $z$ nimaga teng.

$$x + \frac{1}{y+\frac{1}{z}} = \frac{10}{7}$$

A) 3    B) 4    C) 1    D) 2

40. (97-5-18) Tenglamani eching.

$$[x^2] = 9$$

A) 3    B) $(-\sqrt{10};-3) \cup (3;\sqrt{10})$
C) $-3$    D) $(-\sqrt{10};-3] \cup [3;\sqrt{10})$

41. (99-3-12) n ning qanday qiymatlarida

$$4x^2 - 3nx + 36 = 0$$

tenglama ikkita manfiy ildizga ega bo'ladi?
A) $|n| \geq 8$   B) $n \leq -8$   C) $n < 8$   D) $n < -8$

42. (97-5-30) Hisoblang.

$$\arcsin(\sin 10)$$

A) $\pi - 10$    B) $2\pi - 10$
C) $3\pi - 10$    D) $\frac{3\pi}{2} - 10$

43. (98-12-18) $a$ ning qanday qiymatida $\frac{a^3}{a^2-1}$
kasrning qiymati $\frac{27}{8}$ ga teng bo'ladi.
A) 3    B) 2    C) 27    D) 8

44. (99-6-42) Agar

$$\begin{cases} x^3 + y^3 = 10 \\ 3xy^2 + 3x^2y = 17 \end{cases}$$

bo'lsa, $x + y$ ni toping.
A) 3    B) 2    C) $\sqrt{3}$    D) $3\sqrt{3}$

45. (99-8-13) Nechta $(x;y)$ butun sonlar jufti

$$(x+1)(y-2) = 2$$

tenglikni qanoatlantiradi.
A) 4    B) 2    C) 1    D) 3

46. (00-10-49) m ning qanday qiymatida

$$x(x+a)(x+b)(x+a+b) + 4m^2$$

ifoda to'la kvadrat bo'ladi?
A) $\frac{a^2b^2}{4}$    B) $\pm\frac{ab}{4}$    C) $\pm\frac{a+b}{4}$    D) $\frac{ab^2}{4}$

47. (98-5-30) Ushbu

$$\left(\frac{4}{5}\right)^x = 4$$

tenglamaning yechimi qaysi oraliqqa tegishli?
A) $(-\infty;-1)$    B) $(0;1)$    C) $[2;\infty)$    D) $(-1;0)$

48. (98-6-17) Ushbu $y = 2^{x+\frac{1}{x}}$ funksiyaning qiymatlar sohasini toping.
A) $(-\infty;\infty)$    B) $(0;\infty)$
C) $[2;\infty)$    D) $(0;\frac{1}{4}] \cup [4;\infty)$

49. (98-12-81) $k$ ning qanday qiymatida $f(x) = |\log_5(k-x)|$ va $g(x) = -|x-7|$ funksiyalarning grafiklari OX o'qida yotgan nuqtada kesishadi?
A) 1    B) 4    C) 5    D) 8

50. (99-7-31) Tenglamaning yechimi qaysi oraliqqa tegishli?

$$\left(\frac{2}{3}\right)^x = 2$$

A) $(-\infty;-2)$   B) $(-1;0)$   C) $(1;\infty)$   D) $(-2;-1)$

51. (98-11-65) Agar

$$(x-2)f(x-2) + f(2x) + f(x+2) = x+6$$

bo'lsa, $f(4)$ qanchaga teng bo'ladi?
A) 13    B) 2    C) 3    D) 4

52. (03-1-18) $\sin x < 1 + \frac{x^2}{4}$ tengsizlikni yeching.
A) $\emptyset$    B) $\left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right), n \in Z$
C) $[-\pi;\pi]$    D) $(-\infty;\infty)$

### 1.1.1 Hisoblashga oid misollar

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | C | A | D | C | B | C | B | D |
| 1 | B | D | A | B | C | B | D | B | A | D |

### 1.1.2

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | C | B | C | D |   |   |   |

### 1.1.3

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | C | B | A | B | A | D | B | C |
| 1 | B | D | B | D | D | D | D | A | B | C |
| 2 | A | C | C | D | C | B | D | A | D | D |
| 3 | C | B | A | A | D | A | A |   |   |   |

### 1.1.4

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | D | C | D | C | C | C | B | D |
| 1 | B | D | C | D | C | D | D | D | A | D |
| 2 | B | B | C | A | B | D | C | C | D | D |
| 3 | A | D | C | D | C | B | D | B | B | D |
| 4 | B | D | D | D | D | D | A | C | B | B |

### 1.1.5

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | A | D | A | B | B | C | D | A |
| 1 | D | C | B | D | C | A | A | A | A | B |
| 2 | B | D | B | D | D | A | C | D | D | B |
| 3 | A | D | C | A | D | B | D | B | B |   |

### 1.1.6

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | B | D | B | D | B | C | D | C |
| 1 | B | C | D | B | A | D | B |   |   |   |

### 1.1.7

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | B | D | A | D | B | A | A | C |
| 1 | C | C |   |   |   |   |   |   |   |   |

### 1.2.1

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | A | A | B | B | D | D | C |
| 1 | B | C | B | A | A | A | D | D | D | D |
| 2 | A | B | A | D | A | C | B | D | C | D |
| 3 | B | C | A | A | A | D | A | B | C | C |

### 2.2.2

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | A | C | B | B | D | C | A | D |
| 1 | C | B | C | C | B | C | A | B | A | B |
| 2 | C | A | A | B | C | C | A | A | C | B |
| 3 | C | C | B | D | B | B | B | D | D | D |
| 4 | A |   |   |   |   |   |   |   |   |   |

### 2.2.3

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | A | B | D | D | A | B | C | B |
| 1 | A | D | C | A | B | C | B | D | C | C |
| 2 | A | C | B | B | A | A | C | D | B | A |
| 3 | C | D | D | A | D | A | A | A | A | A |
| 4 | D | B | D | A | A | C | C | A |   |   |

### 2-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | C | D | C | B | A | D | D |
| 1 | C | B | C | D | C | B | D | D | D | A |
| 2 | C | D | D | C | A | C | B | D | D | A |
| 3 | A | C | D | D | B | D | C | D | C | A |
| 4 | D | D | D | C | A | C | A | D | C | B |
| 5 | D | A | B | D | D | C | B | C | D |   |

### 3-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | D | A | D | C | A | C | D | B |
| 1 | C | D | A | D | C | B | C | A | C | A |
| 2 | B | C | B | C | C | D | D | B | A | B |
| 3 | D | B | C | A | B | A | B | D | C | C |
| 4 | D | D | D | B | C | B | C | B | D | D |
| 5 | A | D | B | C | C | D | A | A | B | C |

### 4-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | C | B | D | C | D | D | C | A |
| 1 | B | D | D | D | C | B | A | B | D | D |
| 2 | C | D | D | B | C | B | A | D | D | B |
| 3 | C | D | A | D | D | B | D | C | C | D |
| 4 | B | B | C | D | D | D | B | A | B | D |
| 5 | D | C | D | D | D | B | C | D | D | B |
| 6 | A | B | C | C |   |   |   |   |   |   |

### 5-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | D | D | A | D | A | C | D |
| 1 | C | B | B | B | C | A | B | A | D | C |
| 2 | C | D | B | D | A | C | D | C | C | A |
| 3 | C | D | A | B | D | B | B | A | D | D |
| 4 | B | B | B | B | D | C | D | B | A | B |
| 5 | B | C | C | B | D |   |   |   |   |   |

### 6-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | D | B | C | C | C | A | D | D |
| 1 | C | B | D | D | A | C | A | D | D | B |
| 2 | D | A | C | B | A | D | C | D | B | A |
| 3 | D | A | C | C | A | B | A | D | D | B |
| 4 | D | A | B | B | D | A | A | B | D | B |
| 5 | C | D | B | B | C | D | B | D | B | C |
| 6 | C | C | D | D | D | A | B | B | D |   |
| 7 | A | B | C | D | D | D |   |   |   |   |

### 7-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | D | D | D | C | A | C | B | C | B |
| 1  | C | D | A | C | C | D | A | C | B | D |
| 2  | D | D | C | B | C | C | C | B | A | C |
| 3  | C | D | C | B | D | B | B | B | D | C |
| 4  | D | D | C | C | B | C | B | B | D | B |
| 5  | D | C | C | B | D | D | C | D | B | A |
| 6  | C | A | A | C | B | A | D |   |   |   |

### 8-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | B | D | D | C | C | D | A | D |
| 1  | C | D | C | D | A | D | A | B | D | D |
| 2  | D | D | B | B | C | D | D | D | B | C |
| 3  | D | D | C | B | D | A | C | B | C | D |
| 4  | A | A | B | D | D | A | D | D | B | B |
| 5  | D | D | B | C | D | D | B | A | C | D |
| 6  | D |   |   |   |   |   |   |   |   |   |

### 9-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | B | D | C | C | B | D | D | A | A |
| 1  | B | D | A | D | D | D | C | B | C | D |
| 2  | D | A | C | A | C | D | A | A | B | D |
| 3  | D | C | B | C | B | B | A | D | B | A |
| 4  | C | B | B | D | A | A | D | D | B | C |
| 5  | D | D | D | D | B | B | B | A | B | A |
| 6  | B | C | B | C | D | A | D | D | B | C |
| 7  | B | D | C | D | A | D | B | D | B | B |
| 8  | A | D | D | D | A | C | D | D | B | A |
| 9  | C | D | A | C | A | B | C | D | A | D |
| 10 | B | C | D | D | A | C | D | D | B | C |
| 11 | D | D | A | C | D | C | C | D |   |   |

### 10-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | B | D | C | C | B | C | C | D | D |
| 1  | B | B | C | D | B | D | D | D | B | D |
| 2  | C | C | B | C | A | D | D | D | A | B |
| 3  | B | A | D | C | C | B | D | C | A | B |
| 4  | B | D | C | C | B | D | B | C | C | C |
| 5  | A | B | A | A | A | D | B | B |   |   |

### 11-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | B | B | D | B | D | D | A | C | D |
| 1  | B | A | B | D | C | B | B | D | D | C |
| 2  | B | D | A | D | D | D | D | A | D | D |
| 3  | A | B | D | C | D | A | B | A | C | B |
| 4  | D | A | B | D | D | B | A | C |   |   |

### 12-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | D | B | D | C | B | D | C | D |
| 1  | D | C | D | C | D | A | C | A | D | B |
| 2  | D | B | C | B | D | C | B | D | C | D |
| 3  | D | C | B | C | D | D | C | D | B | D |
| 4  | C | C | D | A | C | D | C | D | D | B |
| 5  | C | D | D | C | B | D | D | C | C | B |
| 6  | A | C | C | C | B | B | D |   |   |   |

## 1998 yil - Matematika

### 1-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | D | B | B | C | D | B | D | C |
| 1  | B | D | A | C | C | D | C | A | B | B |
| 2  | C | D | C | D | D | D | C | B | A | B |
| 3  | D | D | B | D | D | B | C | A | D | A |
| 4  | C | D | B | C | A | A | D | C | B | B |
| 5  | C | D | A | D | B | D | D | A | D | D |
| 6  | C |   |   |   |   |   |   |   |   |   |

### 2-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | D | C | D | B | C | A | D | D | B |
| 1  | C | B | D | D | B | D | C | D | B | C |
| 2  | B | A | D | D | C | D | D | B | B | C |
| 3  | C | C | D | D | D | C | D | B | D | D |
| 4  | B | C | D | A | B | D | B | D | C | D |
| 5  | A | C | B | B | D | C | D | B | C | D |

### 3-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | D | D | D | B | D | D | B | D |
| 1  | B | C | A | C | C | D | D | D | C | C |
| 2  | D | C | B | D | C | B | B | B | C | C |
| 3  | C | D | D | C | D | D | C | D | B | A |
| 4  | D | C | A | D | D | C | C | A | A | C |
| 5  | B | B | B | A | B | D | D | D | C | A |
| 6  | A |   |   |   |   |   |   |   |   |   |

### 4-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | B | C | D | D | D | D | C | B |
| 1  | A | B | D | A | C | D | D | C | C | B |
| 2  | A | A | B | C | D | D | D | C | C | B |
| 3  | A | A | B | C | D | D | A | D | C | B |
| 4  | A | A | B | C | D | D | A | D |   |   |

### 5-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | D | A | C | B | D | A | B | B | D |
| 1  | A | D | C | C | B | D | A | D | D | C |
| 2  | A | B | A | D | B | C | B | A | D | B |
| 3  | A | D | C | C | D | D | B | C | B | A |
| 4  | D | D | B | B | B | A | D | A | B | C |
| 5  | B | D | D | C | B | A |   |   |   |   |

### 6-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | B | C | D | D | D | D | C | B |
| 1  | A | D | D | A | C | B | A | D | B | C |
| 2  | D | A | C | A | C | B | D | A | A | D |
| 3  | A | C | B | A | D | D | B | D | A | A |
| 4  | D | C | B | A | C | D | A | D | B | D |
| 5  | A | A | D | D | C | A | B | C | D |   |

### 7-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | D | C | D | D | C | A | D | B |
| 1  | C | D | B | B | C | A | A | D | D | B |
| 2  | A | D | C | D | C | D | A | D | C | D |
| 3  | A | C | D | D | D | D | D | B | A | D |
| 4  | B | C | D | D | A | D | B | A | C | D |
| 5  | A | D | D | C | D |   |   |   |   |   |

## 8-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | B | D | A | C | C | D | D | D | B |
| 1  | C | B | C | D | D | D | D | B | A | B |
| 2  | C | B | D | C | D | D | C | D | D | A |
| 3  | C | D | B | C | D | D | B | A | C | D |
| 4  | A | B | A | D | B | C | C | A | C | D |
| 5  | D | C | D | B | B | D | A | C | B | A |
| 6  | C | C | B | A |   |   |   |   |   |   |

## 9-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | D | C | A | B | C | A | C | D | B |
| 1  | D | D | C | A | B | D | D | C | C | B |
| 2  | A | D | C | C | A | B | B | D | D | C |
| 3  | D | D | C | B | D | D | D | D | B | A |
| 4  | D | D | C | D | C | D | D | A | D | B |
| 5  | D | C | D | C | B | D | C | C |   |   |

## 10-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | C | A | D | D | B | C | C | D |
| 1  | B | D | B | D | D | C | D | D | C | A |
| 2  | C | D | C | D | B | D | D | B | C | D |
| 3  | C | D | B | D | C | D | B | D | D | C |
| 4  | D | B | D | B | C | D | D | D | C | A |
| 5  | A | B | D | A | D | A | C | A | B | C |
| 6  | C | D | A | C | C | C | A | C | B | C |
| 7  | C | B | D | C | A | B | D | D | D | A |
| 8  | B | B | C | D | D | C | D | D | D | C |
| 9  | A | D | B | C | C | D | C | D | D | C |
| 10 | D | B | A | D | C | A |   |   |   |   |

## 11-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | D | C | A | C | B | B | A | A | C |
| 1  | D | D | C | C | C | A | D | D | B | D |
| 2  | B | C | D | D | D | C | A | B | B | C |
| 3  | B | A | A | D | D | C | D | B | B | C |
| 4  | D | A | D | B | B | A | A | C | C | D |
| 5  | B | A | B | D | A | D | A | C | D | A |
| 6  | A | D | A | B | C | D | D | A | A | B |
| 7  | A | D | A | D | B | D | A | A | A | D |
| 8  | A | D | A | D | A | D | A | B | D | D |
| 9  | B | A | D | D | A | A | B | A | D | A |
| 10 | A | D | A | A | A | D |   |   |   |   |

## 12-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | A | A | A | C | A | A | A | A |
| 1  | A | C | A | A | A | A | A | A | A | A |
| 2  | A | A | A | A | A | A | A | A | A | A |
| 3  | A | A | A | A | B | A | A | A | A | A |
| 4  | A | A | A | A | A | A | A | A | A | A |
| 5  | A | A | A | A | A | A | A | A | A | A |
| 6  | A | C | D | A | A | A | D | C | D | A |
| 7  | C | D | D | A | B | A | A | C | A | A |
| 8  | C | D | C | B | D | A | B | D | A | B |
| 9  | B | A | C | D | A | C | A | D | A | C |
| 10 | A | C | A | C | A | D | A | B |   |   |

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## 1-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | A | A | A | A | A | B | A | A |
| 1  | A | A | A | A | A | A | A | A | A | A |
| 2  | A | A | A | A | A | A | A | A | A | A |
| 3  | A | A | A | A | A | A | A | A | A | A |
| 4  | A | A | A | A | A | A | A | A | A | C |
| 5  | A |   |   |   |   |   |   |   |   |   |

## 2-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | B | C | D | D | B | C | D | B |
| 1  | D | C | A | C | D | D | C | D | D | C |
| 2  | D | D | A | B | C | C | D | D | B | C |
| 3  | D | C | D | D | C | B | D | D | B | D |
| 4  | D | B | C | D | D | C | D | C | B | C |
| 5  | D | A | D | B | C | D | C |   |   |   |

## 3-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | B | C | C | B | D | A | D | D | D |
| 1  | D | B | D | C | A | B | A | D | C | C |
| 2  | B | A | D | B | D | C | D | D | D | D |
| 3  | D | D | A | D | D | C | D | B | C | D |
| 4  | A | D | D | B | C | C | A | B | A | A |
| 5  | B | D | D | A | B | B | A | D | A | C |
| 6  | D |   |   |   |   |   |   |   |   |   |

## 4-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | C | D | A | C | B | D | C | A | C |
| 1  | D | B | B | C | D | D | D | A | D | D |
| 2  | A | B | C | D | C | B | B | C | A | D |
| 3  | B | D | D | C | B | B | A | B | D | C |
| 4  | C | D | D | D | D | B | A | B | C | C |
| 5  | C | B | D | D | B | C | C |   |   |   |

## 5-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | D | D | A | C | C | D | D | B |
| 1  | D | D | B | D | D | D | C | D | D | D |
| 2  | D | C | B | D | D | B | D | A | D | D |
| 3  | D | B | B | A | C | D | C | C | B | B |
| 4  | C | C | D | B | B | D | D | B | C | A |
| 5  | B | B | D | D | B | C | D | D |   |   |

## 6-axborotnoma

|    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|---|---|---|---|
| 0  |   | A | B | A | C | D | D | B | C | D |
| 1  | D | A | A | A | A | A | A | D | A | A |
| 2  | B | A | D | A | D | A | D | C | D | A |
| 3  | A | A | A | D | A | D | A | A | D | A |
| 4  | D | A | A | A | C | A | A | D | A | D |
| 5  | D | C | A | D | A | A | A | B | A | A |

### 7-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | D | B | C | D | A | A | D | D |
| 1 | C | A | B | D | D | D | A | C | D | D |
| 2 | C | B | A | D | D | A | B | D | B | D |
| 3 | A | D | A | C | D | D | A | D | D | D |
| 4 | C | D | D | B | B | C | D | D | A | B |

### 8-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | A | C | D | A | C | A | A |
| 1 | D | C | A | A | A | A | A | A | B | A |
| 2 | D | A | A | A | A | D | D | A | A | A |
| 3 | C | D | B | A | A | A | D | C | A | A |
| 4 | A | A | D | A | A | D | A | A | C | A |
| 5 | C | B | B | A | D | A | B | A | A | D |
| 6 | C | A | A | A | A | A | A | A | A | A |
| 7 | A | A | A | A | A | A | D | B | D | D |
| 8 | D |   |   |   |   |   |   |   |   |   |

### 9-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | B | D | C | D | C | D | C |
| 1 | A | C | D | B | D | C | C | D | B | D |
| 2 | B | D | A | C | D | D | B | C | D | C |
| 3 | C | A | D | B | D | D | D | C | A | D |
| 4 | D | D | D | D | D | D | B | B | D | C |
| 5 | D | B | C |   |   |   |   |   |   |   |

### 10-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | A | D | D | D | B | B | C | B |
| 1 | D | B | C | D | B | C | D | B | C | C |
| 2 | D | D | C | D | D | C | D | D | C | A |
| 3 | D | D | D | D | C | D | B | C | D | D |
| 4 | B | D | A | B | C | C | D | D | B | C |

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### 1-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | D | C | D | B | A | C | A |
| 1 | B | B | D | C | D | C | A | D | C | B |
| 2 | B | D | D | D | A | D | A | A | C | D |
| 3 | C | C | D | B | D | D | D | C | C | B |
| 4 | B | C | D | C | D | A | C | D | C | D |
| 5 | B | D | A | D | D | C | B | C | D | D |

### 2-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | C | B | D | D | A | D | A | D |
| 1 | A | B | D | A | D | D | A | C | A | A |
| 2 | C | D | C | D | A | C | A | B | A | D |
| 3 | A | A | C | A | A | D | A | C | D | A |
| 4 | C | C | A | B | A | A | A | A |   |   |

### 3-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | D | D | D | D | D | D | C | C |
| 1 | B | B | A | D | D | D | D | A | D | D |
| 2 | D | C | D | D | D | A | D | D | C | A |
| 3 | C | D | D | C | A | A | B | A | B | A |
| 4 | D | D | C | D | D | D | B | D | A | D |
| 5 | C | C | D | D | D | D | D | A | B | C |
| 6 | D | B | B | C | D | D | A | C | B | A |
| 7 | C | D | D | D | D | A | B | B | C | C |
| 8 | D | C | D | D | D |   |   |   |   |   |

### 4-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | D | B | B | C | B | A | D | C |
| 1 | D | D | D | C | D | D | C | D | C | A |
| 2 | D | D | C | A | D | C | D | D | D | C |
| 3 | C | C | B | D | D | B | D | A | C | C |
| 4 | D | C | C | B | C | C | A | D | A | B |
| 5 | A | D | D | B | C |   |   |   |   |   |

### 5-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | B | B | A | D | D | C | B | D |
| 1 | A | D | A | C | C | B | B | C | A | D |
| 2 | C | A | B | D | D | C | B | A | C | D |
| 3 | A | A | C | B | A | D | C | D | C | C |
| 4 | A | B | D | C | D | D | C | B | A | D |
| 5 | D | C | D | B | A | D | C | B | D | B |
| 6 | A | D | C | B | D | D | B | C | D | D |
| 7 | D | B |   |   |   |   |   |   |   |   |

### 6-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | D | D | D | C | A | B | D |
| 1 | D | A | B | A | D | C | C | C | C | B |
| 2 | D | B | A | A | D | C | D | D | D | C |
| 3 | C | D | A | D | B | C | D | A | D | C |
| 4 | D | D | A | C | B | B | D | C | B | A |
| 5 | B | D | D | B | D | C | D |   |   |   |

### 7-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | D | D | B | D | A | B | C |
| 1 | C | C | C | B | D | D | B | C | D | D |
| 2 | D | D | C | D | C | A | B | D | D | A |
| 3 | B | D | D | B | C | D | B | C | D | A |
| 4 | C | D | B | A | D | D | D | B | C | C |
| 5 | D | B | B |   |   |   |   |   |   |   |

### 8-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | B | D | B | B | A | C | D | D | A |
| 6 | A | C | B | B | A | D | D | D | A | A |

### 9-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | D | D | D | D | D | D | D |
| 1 | A | A | A | C | B | D | D | C | B | D |
| 2 | D | A | B | D | B | D | C | A | D | C |
| 3 | C | D | C | A | D | D | C | A | C | C |
| 4 | C | B | C | B | D | D | D | B | C | C |
| 5 | A | D | C | D | C | B | D | D | A | C |
| 6 | C | B | B | B | C | A | D | C |   |   |

### 10-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | D | D | D | A | D | D | D |
| 1 | C | B | B | C | C | D | C | A | D | D |
| 2 | D | D | C | D | C | C | C | A | A | D |
| 3 | A | B | A | A | C | A | D | B | C | D |
| 4 | C | A | B | B | C | B | B | B | A | B |
| 5 | A | D | A | C | D | C | A | D | A | A |
| 6 | A | D | A | A | D | D | A | A | A | A |
| 7 | A | A | A | A | C | A | A | D | A | A |
| 8 | A | A | A |   |   |   |   |   |   |   |

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### 1-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | C | D | D | A | C | D | D | D |
| 1 | D | D | D | D | D | B | C | C | B | D |
| 2 | D | B | D | B | D | D | D | B | C | B |
| 3 | C | B | A | C | B | D | D | A | A | D |
| 4 | C | D | C | B | A | D | A | D | B | A |
| 5 | C | B | C | C | D | B | D | B | A | C |
| 6 | D | D | D | B | D | D | B | B | D | C |
| 7 | D | D | A |   |   |   |   |   |   |   |

### 2-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | B | D | D | C | B | D | B |
| 1 | C | D | B | D | D | D | D | C | D | C |
| 2 | B | C | A | D | B | C | B | D | D | A |
| 3 | D | B | C | B | D | A | C | D | B | C |
| 4 | D | A | D | C | D | B | D | B | A | C |
| 5 | D | B | D | A | C | D | C | D | B | D |
| 6 | D | C | C | D | D | C | A | B | D | D |
| 7 | A | D | D | D | B | D | B | A | B | B |
| 8 | D | D | D | D | D | C | C | C |   |   |

### 3-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | C | A | D | B | A | D | B |
| 1 | C | D | A | D | C | D | C | A | D | A |
| 2 | D | C | B | B | D | B | C | C | A | B |
| 3 | D | B | A | C | C | B | D | A | A | B |
| 4 | C | A | B | C |   |   |   |   |   |   |

### 4-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | D | C | A | C | D | B | C |
| 1 | B | D | C | B | D | D | A | D | D | D |
| 2 | D | A | A | C | D | D | B | A | B | B |
| 3 | A | A | C | A | A | B |   |   |   |   |

### 5-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |

### 6-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | B | B | D | D | D | C | B | C |
| 1 | D | D | B | B | D | A | D | B | C | C |
| 2 | D | C | A | D | B | C | D | B | D | D |
| 3 | C | C | C | B | C | D | D | D | D | D |
| 4 | C | D | D | B | B | A | D | D | C |   |
| 5 | B | C | B | A | C | D |   |   |   |   |

## 7-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | C | D | C | D | D | D | B |
| 1 | D | D | C | B | D | D | D | D | B | D |
| 2 | D | D | C | C | B | D | D | C | A | D |
| 3 | D | C | B | D | D | B | D | C | B | D |
| 4 | C | A | B | D | D | C | A | D | D | D |
| 5 | D | B | A | B | D | B | D | A | D |   |

## 8-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | D | B | B | A | D | C | B | C |
| 1 | D | C | D | D | B | D | B | A | A | D |
| 2 | D | D | D | C | D | A | A | B | C | D |
| 3 | D | D | D | C | B | B | A | C | B | C |
| 4 | D | D | B | B | B | D | D | A | C | D |
| 5 | A | A | D | D | C | B | A | D |   |   |

## 9-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | C | C | C |
| 3 | A | B | D | A | D | C | D | B | D | D |
| 4 | B | B | D | A | B | D | A | D | C | D |
| 5 | C | D | C | D | D | D | C |   |   |   |

## 10-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | C | C | C | D | D | D | D |
| 1 | C | C | C | D | B | C | C | D | D | C |
| 2 | B | D | B | D | B | D | A | A | D | B |
| 3 | D | D | B | C | C | C | A | C | B | D |
| 4 | C | D | B | D | C | C | A | D | B | B |
| 5 | A | C | B |   |   |   |   |   |   |   |

## 11-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | B | D | D | B | C | D | D |
| 1 | C | D | D | C | D | B | A | D | D | D |
| 2 | C | C | B | D | D | C | D | D | B | D |
| 3 | D | D | C | D | D | B | C | A | D | A |
| 4 | C | D | B | B | D | D | C | A | D | D |
| 5 | D | B | D | D | C | D |   |   |   |   |

## 12-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | A | B | D | C | D | A | D |
| 1 | D | B | C | C | A | C | A | C | D | D |
| 2 | B | B | A | D | D | B | C | D | A | D |
| 3 | D | C | B | D | C | B | A | C | C | C |
| 4 | D | D | B | C | D | C | D | B | B | A |
| 5 | B | D | B | A | B | B |   |   |   |   |

## 2002 yil - Matematika

### 1-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | B | A | A | A | C | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | C | C |
| 3 | A | B | B | C | D | A | D | C | D | C |
| 4 | D | B | B | D | A | A | D | B | D | A |
| 5 | D | D | D | B | D | D | D | B | A | D |
| 6 | D | D | D | A | C | D | D | B | D | C |
| 7 | D | C | C | C | A | C | B |   |   |   |

### 2-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | D | A | A | A | A | A | A | A | A | A |
| 6 | A | A | A | A |   |   |   |   |   |   |

### 3-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | A | A | A | A | A | A | A | A | A |
| 6 | A | A | A | A | A | A | A | A | A | A |
| 7 | A | A | A | C | A | A | A | A | A | A |
| 8 | A | A |   |   |   |   |   |   |   |   |

### 4-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | D | B | B | D | C | B | A |
| 1 | B | C | D | C | C | B | C | D | C | D |
| 2 | B | C | B | B | C | C | C | B | D | C |
| 3 | D | B | C | B | C | C | C | B | B | D |
| 4 | D | B | D | D | B | D | A | A | B | B |
| 5 | C | B | D |   |   |   |   |   |   |   |

### 5-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | C | B | B | D | D | B | C | A |
| 1 | D | D | C | D | A | A | D | D | D | D |
| 2 | C | B | C | A | A | D | D | C | D | A |
| 3 | D | B | D | D | B | D | A | D | C | C |
| 4 | D | D | C | B | C | D | A | C | B | C |
| 5 | B | B | C | C |   |   |   |   |   |   |

### 6-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | D | B |
| 1 | B | D | D | D | B | C | C | B | C | A |
| 2 | C | D | D | C | D | B | A | C | B | D |
| 3 | C | A | D | D | B | B | D | C | B | A |
| 4 | D | A | C | D | D | A | D | D | D | B |
| 5 | B | D | D | C | C | D |   |   |   |   |

## 7-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | C | A | A | A | A |
| 1 | A | A | B | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | A | A | A | A | A | A |   |   |   |

## 8-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | B | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A |   |   |   |   |   |   |   |

## 9-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | B | B | A | D | B | D |
| 1 | A | D | C | C | B | B | D | B | C | D |
| 2 | B | D | C | C | D | C | A | D | B | B |
| 3 | B | A | A | D | B | B | B | A | D | A |
| 4 | B | D | C | C | A | A | B | B | D | B |
| 5 | D | D | D | D | D | D | B | D | D | D |
| 6 | C |   |   |   |   |   |   |   |   |   |

## 10-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | D | A |
| 1 | A | A | A | A | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | D | A | A |
| 5 | A | A | A | A | A | A | A | A | A | A |
| 6 | A | A | A | A | A | A | A | A | A | A |
| 7 | A | A | A | A | A | A | A | A | A | A |
| 8 | A |   |   |   |   |   |   |   |   |   |

## 11-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | D | C | B | D | D | D | D | D |
| 1 | A | A | D | B | D | C | D | D | D | A |
| 2 | D | B | B | A | B | B | D | A | C | D |
| 3 | D | C | D | D | D | D | D | B | C | D |
| 4 | C | C | D | D | B | D | A | D | B | D |
| 5 | A | C | D | C | D | D | A | B | D | B |
| 6 | C | B | D | D |   |   |   |   |   |   |

## 12-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | A | B | D | C | C | A | D | D |
| 1 | C | A | B | C | A | B | D | D | C | B |
| 2 | C | D | B | B | C | D | D | B | D | D |
| 3 | B | D | A | C | C | B | D | D | B | A |
| 4 | C | B | D | C | A | C | D | D | D | D |
| 5 | B | D | D | A | A | B | C | D | D | C |
| 6 | B | C | A | A | D |   |   |   |   |   |

## 2003 yil - Matematika

### 1-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | D | B | C | A | B | D | D | B |
| 1 | C | B | A | B | D | D | C | D | D | B |
| 2 | D | D | D | C | D | A | D | D | D | C |
| 3 | D | D | B | D | D | D | A | B | B | D |
| 4 | A | D | A | B | A | A | C | C | C | B |
| 5 | D | D | B | D | D | D | C | D | C | A |
| 6 | D | B | D | C | D | A | C | C | B | C |
| 7 | D |   |   |   |   |   |   |   |   |   |

### 2-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | C | D | D | B | D | A | A |
| 1 | D | C | C | A | C | D | A | C | B | D |
| 2 | C | D | B | D | D | D | C | D | D | B |
| 3 | A | B | B | D | C | B | D | A | C | D |
| 4 | C | C | C | B | C | D | D | B | C | D |
| 5 | A | D | B | D | D | B | D | D | D | B |
| 6 | D | A | B | D | C | B | C | C | A | D |

### 3-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | D | A | D | B | B | B | C | D | D |
| 1 | C | A | D | C | C | B | C | C | D | D |
| 2 | C | B | C | C | D | B | D | D | D | D |
| 3 | C | C | D | C | B | D | C | D | D | B |
| 4 | D | A | C | D | A | D | D | A | D | B |
| 5 | D | D | B | A | C | D | D | D | C | B |
| 6 | C |   |   |   |   |   |   |   |   |   |

### 4-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | D | D | D | D | B | C | D | C |
| 1 | D | D | B | D | C | D | D | B | C | B |
| 2 | D | A | B | D | B | C | D | B | B | D |
| 3 | C | D | C | B | C | B | C | C | C | C |
| 4 | D | D | B | C | C | C | D | C | B | C |
| 5 | B | D | D | D | C | D |   |   |   |   |

### 5-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | B | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | A | A | D | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | A | A | A | A | A | A | A | A | A |
| 6 | A | A | A | A |   |   |   |   |   |   |

### 6-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | C | D | A | B | C | B | D |
| 1 | C | B | D | D | D | C | D | B | A | C |
| 2 | B | C | D | D | D | B | D | D | C | B |
| 3 | D | B | D | A | D | B | C | C | D | B |
| 4 | D | D | B | D | D | B | C | C | C | B |
| 5 | D | D | A | D | B | D | D | B | C | D |
| 6 | A | C | D | D | B | D | C | A | B | D |
| 7 | C | B | A | C | D | A | D | D | A | B |
| 8 | D | C | D | D |   |   |   |   |   |   |

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Foydalanilgan adabiyotlar.

1. R.Azimov, N.Sherboyev, Sh.Mirhamidov, A.Karimova. Matematika (Algebra va analiz asoslari). T.: O'qituvchi. 1992.

2. F.Usmonov, R.Isomov, B.Xo'jayev. Matematikadan qo'llanma. T.: Yangi asr avlodi. I qism. 2006.

3. 1996-2003 yilgi "Axborotnomalar".

4. 2006-2008 yillarda abituriyentlarni oliy ta'lim muassasalariga qabul qilish uchun foydalanilgan test kitoblari.

#### 7-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | C | C | B | D | B | D | B | C |
| 1 | D | B | B | A | D | B | A | C | A | B |
| 2 | D | C | C | D | D | B | B | C | D | D |
| 3 | A | D | D | D | C | D | D | A | A | A |
| 4 | D | B | A | D | D | C | A | D | D | C |
| 5 | B | D | A | C | D | B | C | D | D | D |
| 6 | B | C | A | A | D | D | D | D | D | D |
| 7 | B | D | A | D | C | C | D | B | C | C |
| 8 | A | A | D | D | B | A |   |   |   |   |

#### 8-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | C | B | C | D | B | C | B | A |
| 1 | D | B | C | B | D | D | D | D | C | D |
| 2 | A | A | A | A | A | A | A | A | D | A |
| 3 | A | A | A | A | A | A | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | A | A | A | A | A | A | A | A | A |
| 6 | A |   |   |   |   |   |   |   |   |   |

#### 9-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | C | B | D | A | D | D | B | D | C |
| 1 | B | C | A | B | C | C | D | B | B | D |
| 2 | B | A | C | D | C | C | D | D | D | D |
| 3 | D | C | D | C | D | C | A | B | A | C |
| 4 | D | C | A | D | D | C | B | B | B | D |
| 5 | C | D | D | D | D | B | D | C | C | C |
| 6 | A | A | A | A | A | A | A |   |   |   |

#### 10-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | A | A | A | B | D | C | B |
| 2 | D | B | D | B | A | D | B | D | C | B |
| 3 | A | A | D | D | D | C | B | D | B | D |
| 4 | D | D | A | C | D | C | C | C | A | C |
| 5 | D | D | C | D | A | D | C | B | D | A |
| 6 | C | B | A | D | B |   |   |   |   |   |

#### 11-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | A | A | A | A | A | A | A | A | A |
| 1 | A | A | A | D | A | A | A | A | A | A |
| 2 | A | A | A | A | A | A | A | A | A | A |
| 3 | A | A | A | A | A | D | A | A | A | A |
| 4 | A | A | A | A | A | A | A | A | A | A |
| 5 | A | A | A | B | C | A | D | B | D | A |
| 6 | D | B | C | B | C | D | A | B | C | D |
| 7 | A | C | C | B | D | C | B | A | D | D |
| 8 | C | A | D | D |   |   |   |   |   |   |

#### 12-axborotnoma

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 |   | B | D | C | D | A | C | A | A | D |
| 1 | C | C | B | A | D | B | D | C | D | B |
| 2 | D | B | B | D | D | B | C | C | C | D |
| 3 | B | C | A | D | A | D | B | C | C | D |
| 4 | B | D | A | D | C | B | D | B | B | C |
| 5 | C | C | B | A | A | C | D | D | D | D |
| 6 | C | D | D | C | D | A | D | D | D | C |
| 7 | A | D | A | D | A | D | C | D | D | D |
| 8 | D | B | D | C | B | D | A |   |   |   |