

2.1 $f(x) = \frac{1}{2} x^T Q x$, $df = \frac{1}{2} (dx^T Q x + x^T Q dx) = \frac{1}{2} x^T Q dx + \frac{1}{2} dx^T Q x$

$$\nabla f^T = \frac{1}{2} (x^T Q + x^T Q) \Rightarrow \nabla f = \frac{1}{2} (Qx + Q^T x)$$

$$\nabla f = \frac{1}{2} (Qx + Q^T x) = \nabla^2 f dx \Rightarrow \nabla^2 f = \frac{1}{2} Q + \frac{1}{2} Q^T$$

$$g(\alpha) = f(x + \alpha \cdot d) = \frac{1}{2} (x^T + \alpha d^T) Q (x + \alpha d) = \frac{1}{2} (x^T Q x + \alpha x^T Q d + \alpha d^T Q x + \alpha^2 d^T Q d)$$

$$g'(\alpha) = \frac{1}{2} (x^T Q d + d^T Q x + 2\alpha d^T Q d) = 0 \Leftrightarrow \alpha = - \frac{x^T Q d + d^T Q x}{2 d^T Q d}$$