2.2 - Rosenbrock

$$f(x_1,\dots,x_n) = \sum_{i=1}^{n-1}\left[(1-x_i)^2 + 100(x_{i+1}-x_i^2)^2\right] =$$

$$f(x_1,\dots x_n) = (1-x_1)^2 + 100(x_2-x_1^2)^2 + (1-x_2)^2 + 100(x_3-x_2^2)^2 + \dots + 100(x_n - x_{n-1}^2)^2$$

$$\nabla f(x_1,\dots,x_n) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_i} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} -2(1-x_1) - 400x_1(x_2 - x_1^2) \\ \vdots \\ -2(1-x_i) - 400x_i(x_{i+1}-x_i^2) + 200(x_i - x_{i-1}^2) \\ \vdots \\ 200(x_n - x_{n-1}^2) \end{bmatrix}$$

Hessian:

$$\frac{\partial^2 f}{\partial x_1} = \dots 2 \cdot + 1200x_1^2 - 400x_2$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -400x_1$$

$$(1 < i < n) \quad \frac{\partial^2 f}{\partial^2 x_i} = 202 + 1200x_i^2 - 400x_{i+1}$$

$$(1 \le i \le n) \quad \frac{\partial^2 f}{\partial x_i \partial x_{i-1}} = -400x_{i-1}$$

$$(1 \le i < n) \quad \partial^2 f / \partial x_i \partial x_{i+1} = -400x_i$$

$$\frac{\partial^2 f}{\partial x_n x_n} = 200$$

$$\nabla^2 f = H =$$

$$\begin{bmatrix} 2 + 1200x_1^2 - 400x_2 & -400x_1 & & & \\ -400x_1 & \ddots & \ddots & & \\ & \ddots & \ddots & -400x_{i-1} & \\ & & -400x_{i-1} & 202 + 1200x_i^2 - 400x_{i+1} & -400x_i \\ & & & \ddots & \ddots \\ & & & -400x_{n-1} & 200 \end{bmatrix}$$