

## 2> Implementation - 1

```
def fibonacci_1(n):  
    if n <= 0:  
        print("Invalid input")  
    elif n <= 2:  
        return n-1  
    else:  
        return fibonacci_1(n-1) + fibonacci_1(n-2)
```

$$T(n) = O(1) + T(n-1) + T(n-2)$$

$$\text{Best case : } T(1) = T(0) = 1$$

$$\text{Assumption : } T(n-2) \approx T(n-1)$$

$$\therefore T(n) = 2T(n-1) + c$$

$$T(n-1) = 2T(n-2) + c$$

$$= 2^2 T(n-2) + (2^{2-1})c$$

$$T(n-2) = 2T(n-3) + c$$

$$= 2^3 T(n-3) + (2^{3-1})c$$

$$\therefore T(n) = 2^K T(n-K) + (2^{K-1})c \quad T(0) = 1$$

$$n-K = 0$$

$$T(n) = 2^n T(0) + (2^{n-1})c$$

$$n = K$$

$$T(n) = 2^n + (2^{n-1})c$$

$$T(n) \propto 2^n$$

$$\therefore \text{time complexity} = O(2^n)$$

## Implementation-2

```
def fibonacci_2(n):  
    fibonacci_array = [0, 1]  
    if n < 0:  
        print("Invalid Input!")  
    elif n <= 2:  
        return fibonacci_array[n-1]  
    else:  
        for i in range(2, n):  
            fibonacci_array[i-1] = 0
```

$$\begin{aligned}\text{Time complexity} &= O(1) + O(n-2) \\ &= O(n)\end{aligned}$$

We can observe that in Implementation-1, the time complexity is  $O(2^n)$  which is in exponential time whereas in Implementation-2 the time complexity is  $O(n)$  which is in linear time. Hence, implementation-2 is more efficient.

#### 4> Pseudocode

Procedure Multiply-matrix (A, B)

Input A, B  $n \times n$  matrix

Output C  $n \times n$  matrix

begin

Initialise C as a  $n \times n$  zero matrix

for  $i = 0$  to  $(n-1)$   $O(n)$

for  $j = 0$  to  $n-1$   $O(n)$

for  $k = 0$  to  $n-1$   $O(n)$

$C[i, j] += A[i, k] \times B[k, j]$

end for

end for

end for

end Multiply-matrix

time complexity =  $O(n^k)$   $k = 3$   
=  $O(n^3)$