

Proposition: A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false but not both.

Example: The sky is blue.

This proposition is either true or false.

Propositional logic: The area of logic that deals with propositions is called the propositional calculus or propositional logic.

List of propositional logic: ① Translating English sentences into logical statements

② System specifications.

③ Boolean searches.

④ Logic puzzles

⑤ Logic circuits

⑥ Inference and decision making

⑦ Artificial intelligence.

NP. we (1) statement that is
Compound proposition: A ~~expression~~ ^{statement} formed by
combining ^{two or more} ~~propositions~~ ^{propositions} variables using logical connectives
operators, such as \wedge or \vee .

Tautology: A compound proposition that
is always true, no matter is called
a tautology.

Contradiction: A compound proposition
that is always false is called a
contradiction.

Contingency: A compound proposition that
is neither a tautology nor a
contradiction is called a contingency.

Logical Equivalence: Compound pro-
positions that have the same truth
values in all possible cases are
called logically equivalent.

De Morgan's Laws: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
 $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Example 6: Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$
are logically equivalent.

Ans: To solve this problem we ~~can~~ ^{will} use
logical ~~identities~~ ^{equivalences} starting with

Here are some types of problems that are commonly solved using discrete math

- ② Logic and Propositional Calculus.
- ③ Set theory
- ④ Algorithms and Data Structures.
- ⑤ Number Theory
- ⑥ Graph Theory
- ⑦ Probability and Statistics
- ⑧ Coding Theory
- ⑨ Game Theory
- ⑩ Combinatorics
- ⑪ Operations Research.
- ⑫ Finite Mathematics.

Q1 Rules of Inference.

The context is the well-known [n-Queens problem](#) and on the textbook, the following compound preposition is given:

Let $p(i, j)$ be a proposition that is *True* iff there's a queen in the i th row and j th column, where $i = 1 \dots n$ and $j = 1 \dots n$.

- to check all row contains at least one queen: $Q_1 = \bigwedge_{i=1}^n \bigvee_{j=1}^n p(i, j)$
- to check at most one queen per row: $Q_2 = \bigwedge_{i=1}^n \bigwedge_{j=1}^{n-1} \bigwedge_{k=j+1}^n (\neg p(i, j) \vee \neg p(k, j))$
 - Here comes my first question. I believe it's wrong and should be $Q_2 = \bigwedge_{i=1}^n \bigwedge_{j=1}^{n-1} \bigwedge_{k=j+1}^n (\neg p(i, j) \vee \neg p(i, k))$ but I couldn't find any public errata. Does it make sense?
- to check at most one queen per column: $Q_3 = \bigwedge_{j=1}^n \bigwedge_{i=1}^{n-1} \bigwedge_{k=i+1}^n (\neg p(i, j) \vee \neg p(k, j))$
- to assert at most one queen on the diagonals:
 - $Q_4 = \bigwedge_{i=2}^n \bigwedge_{j=1}^{n-1} \bigwedge_{k=1}^{\min(i-1, n-j)} (\neg p(i, j) \vee \neg p(i-k, k+j))$
 - $Q_5 = \bigwedge_{i=1}^{n-1} \bigwedge_{j=1}^{n-1} \bigwedge_{k=1}^{\min(n-i, n-j)} (\neg p(i, j) \vee \neg p(i+k, j+k))$

So, to find valid results we need: $Q = Q_1 \wedge Q_2 \wedge Q_3 \wedge Q_4 \wedge Q_5$

I understand all of the proposed compound propositions and how they work. I could even easily convert them into an algorithm.

- ▶ For each cell with a given value, we assert $p(i, j, n)$ when the cell in row i and column j has the given value n .
- ▶ We assert that every row contains every number:

$$\bigwedge_{i=1}^9 \bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$$

- ▶ We assert that every column contains every number:

$$\bigwedge_{j=1}^9 \bigwedge_{n=1}^9 \bigvee_{i=1}^9 p(i, j, n)$$

- ▶ We assert that each of the nine 3×3 blocks contains every number:

$$\bigwedge_{r=0}^2 \bigwedge_{s=0}^2 \bigwedge_{n=1}^9 \bigvee_{i=1}^3 \bigvee_{j=1}^3 p(3r + i, 3s + j, n)$$

- ▶ To assert that no cell contains more than one number, we take the conjunction over all values of n, n', i , and j , where each variable ranges from 1 to 9 and $n \neq n'$ of $p(i, j, n) \rightarrow \neg p(i, j, n')$.

We now explain how to construct the assertion that every row contains every number. First, to assert that row i contains the number n , we form $\bigvee_{j=1}^9 p(i, j, n)$. To assert that row i contains all n numbers, we form the conjunction of these disjunctions over all nine possible values of n , giving us $\bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$. Finally, to assert that every row contains

every number, we take the conjunction of $\bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$ over all nine rows. This gives us $\bigwedge_{i=1}^9 \bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$. (Exercises 71 and 72 ask for explanations of the assertions that every column contains every number and that each of the nine 3×3 blocks contains every number.)

Given a particular Sudoku puzzle, to solve this puzzle we can find a solution to the satisfiability problems that asks for a set of truth values for the 729 variables $p(i, j, n)$ that makes the conjunction of all the listed assertions true. ◀

Definition of rules of inference?

In logic, rules of inference are like blueprints for constructing valid arguments. They provide guidelines for deriving new statements (conclusions) from already established ones (premises), ensuring those conclusions logically follow from the truth of the premises.

Modular arithmetic is a system of arithmetic that **focuses on remainders after division by a fixed number**, called the modulus (denoted by n). It's like a clock that "wraps around" when it reaches a certain point.

Key Concepts:

Modulus (n): The fixed number that determines the "cycle" or "wrap-around" point.

Congruence: Two integers a and b are congruent modulo n if they have the same remainder when divided by n . We write this as $a \equiv b \pmod{n}$.

Modulo Operator ($\%$): The modulo operator gives the remainder of a division. For example, $17 \% 5 = 2$ because 17 divided by 5 leaves a remainder of 2.

Part II: Solving Congruences | Chinese Remainder Theorem | Fermat's Little Theorem

$$x \equiv 2 \pmod{3} \quad , a_1 = 2, a_2 = 3, a_3 = 2$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

Solution

$$M = m_1 \cdot m_2 \cdot m_3 = 3 \cdot 5 \cdot 7 = 105$$

$$M_1 = M/m_1 = 105/3 = 35 \quad M_2 = 105/5 = 21 \quad M_3 = 105/7 = 15$$

$$M_1^{-1}$$

$$M_2^{-1}$$

$$M_3^{-1}$$

$$M_1 \times \underline{\quad} = 1 \pmod{m_1} \quad M_2 \times \underline{\quad} = 1 \pmod{5} \quad 15 \times \underline{\quad} = 1 \pmod{7}$$

$$35 \times \underline{2} = 1 \pmod{3} \quad 21 \times \underline{1} = 1 \pmod{5} \quad \boxed{M_3^{-1} = 1}$$

$$\boxed{M_1^{-1} = 2}$$

$$\boxed{M_2^{-1} = 1}$$

$$x = (a_1 \dot{M}_1 y_1 + a_2 \dot{M}_2 y_2 + \cdots + a_n \dot{M}_n y_n) \pmod{M}$$

$$35 \times 2 = 1 \pmod{3} \quad 21 \times 1 = 1 \pmod{5} \quad M_1^{-1} = 1$$

$$M_1^{-1} = 2$$

$$M_2^{-1} = 1$$

$$x = (a_1 M_1 y_1 + a_2 M_2 y_2 + \dots + a_n M_n y_n) \pmod{M}$$

$$= 2 \cdot 35 \cdot 2 + 3 \cdot 21 \cdot 1 + 2 \cdot 15 \cdot 1 \pmod{105}$$

$$\Rightarrow 233 \pmod{105}, \quad x = 23$$

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