Mid-Term Examination of 2nd Semesters, July-December 2021, Session: 2020-21

Course Code: CIT 121 Course Title: Discrete Mathematics

Time: 1.00 Hour [Answer all the following questions] Marks – 15

1. What is propositional logic? Show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent. Formulate satisfiability problem. How to solve a 9×9 Sudoku puzzle problem? 5

2. What is the truth value of $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4? Show that $\neg\forall x(P(x) \to Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent. 3

3. What are the applications of set theory? Shade the set $(A\cup B)\cap(A\cup C)$. 2

4. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning (A), radio (B), and power windows (W), were already installed the survey found: 5
15 had air-conditioning, 12 had radio, 5 had air-conditioning and power windows, 9 had air-conditioning and radio, 4 had radio and power windows, 3 had all three options, and 2 had no options. Find the number of cars that had: (a) only power window, (b) only air-conditioning, (c) only radio, (d) radio and power windows but not air-conditioning, (e) air-conditioning and radio but not power windows, (f) only one of the options.

V – OR
∧ → And
¬ → Not

XOR
XNOR

Patuakhali Science and Technology University

Faculty of Computer Science and Engineering

Dept. of computer Science and Information Technology

2nd Semester (Level-1, Semester-II), Final Examination of B.Sc. Engg. in (CSE) July-December-2021

Session: 2020-21

Course Code: CIT-121 Course Title: Discrete Mathematics

Full Marks: 70 Time: 3 Hours

[Figures in the right margin indicate full marks]Answer any 5 of the following questions.

1. (a) What kinds of problems are solved using discrete mathematics? 2

   (b) Define propositional logic and list the application of propositional logic. Show that $\neg(p \rightarrow q)$ and $(p \wedge \neg q)$ are logically equivalent. To model the 8-queens' problems as a satisfiability problem. Solve the problem where one queen in each row, no column contains more than one queen, and no diagonal contains two queens. 6

   (c) Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$ into English, where $C(x)$ is "x has a computer," $F(x,y)$ is "x and y are friends," and the domain for both x and y consists of all students in your school. 2

   (d) Define rules of inference, modus ponens, and resolution. Assume that "For all positive integers n, and is greater than 4, then n^2 is less than 2^n" is true. Use universal modus ponens to show that $100^2 < 2^{100}$. 4

2. (a) What is optimization problem? State Cashier's algorithm for n cents, using any set of denominations of coins, as greedy algorithm. List the applications of string matching. 4

   (b) Define Big-O notation. Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$. Demonstrate the growth of the following functions: $1, \log n, n, n\log n, n^2, 2^n, n!$ commonly used in big-O estimates. Distinguish P and NP class problem in the complexity of algorithms. 6

   (c) Write the formal definition of modular arithmetic. State the Euclidean algorithm for estimating the greatest common divisor, GCD. Find the GCD of 414 and 662 using Euclidean algorithm. 4

3. (a) Specify the following sets by listing their elements: 3

   i. $A = \{x : x \in R, -5 < x < 5\}$. ii. $B = \{x : x \in N, x \text{ is a multiple of } 3\}$.
   iii. $C = \{x : x \text{ is a U.S. citizen}, x \text{ is a teenager}\}$.

   (b) $U = \{1,2,3,\ldots,8,9\}$, $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$. 3
   Find: i. $A \cap (B \cup C)$ and ii. $(A \cap B) \cup (A \cap C)$.

   (c) Shade the set $A \cup (B \cap C)$. 2

   (d) Determine which of the following sets are finite. 6

   i. A = {seasons in the year} ii. B = {state in the Union} iii. C = {positive integers less than 1}
   iv. D = {odd integers} v. E = {positive integral divisors of 12}
   vi. F = {cats living in the United states}

4. (a) Suppose that 100 of the 120 mathematics students at a college take at least one of the languages French, German, and Russian. Also suppose 6

   65 study French 45 study German 42 study Russian
   20 study French and German 25 study French and Russian 15 study German and Russian

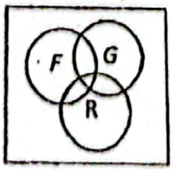


Figure:01

   i. Find the number of students who study all three languages.
   ii. Fill in the correct number of students in each of the eight regions of the Venn diagram of Fig. 01. Here F, G, and R denote the sets of students studying French, German and Russian, respectively.
   iii. Determine the number k of students who studying (x) exactly one language, and (y) exactly two languages.

   (b) Let $A = \{1, 2, 3\}$ and let $R = \{(1,1), (2,1), (3,2), (1,3)\}$ be a relation on A (i.e., a relation from A to A). 2
   Determine whether each of the following is true or false:
   i. $1R1$ ii. $1 \not R 2$ iii. $2R3$ iv. $2 \not R 1$ v. $3R2$ vi. $3 \not R 1$

Please turn over

x 3

1 2 4 2

(c) Show that the following argument is not valid

S_1: All students are lazy.
S_2: Nobody who is wealthy is a student.
S: Lazy people are not wealthy.

(d) Define a graph and Multigraph. Draw the graph $k_{2,5}$.

5. (a) Let R and S be the relation on $X = \{a, b, c\}$ defined by $R = \{(a, b), (a, c), (b, a)\}$ and $S = \{(a, c), (b, a), (b, b), (c, a)\}$ Find the composition $R \circ S$ for the relation R and S.

(b) Let S be the relation on $X = \{a, b, c, d, e, f\}$ defined by $S = \{(a, b), (b, b), (b, c), (c, f), (d, b), (e, a), (e, b), (e, f)\}$ Draw the directed graph of S.

(c) Given $A = \{1,2\}$, $B = \{x, y, z\}$, and $C = \{3, 4\}$. Find $A \times B \times C$ and $n(A \times B \times C)$.

(d) Let $B_1 = \{1,2\}$, $B_2 = \{3,4\}$, and $B_3 = \{5,6\}$. Find $\prod B_i$.

(a) Sketch the graph of $h(x) = \begin{cases} 0, & if\ x = 0 \\ \frac{1}{x}, & if\ x \neq 0 \end{cases}$.   picture

(b) Let $A = \{a, b, c, d, e\}$, and let B be the set of letters in the alphabet. Let the function f, g, and h form A into B be defined as follows:

i.	f	ii.	g	iii.	h
	$a \to r$		$a \to z$		$a \to a$
	$b \to a$		$b \to y$		$b \to c$
	$c \to s$		$c \to x$		$c \to e$
	$d \to r$		$d \to y$		$d \to r$
	$e \to e$		$e \to z$		$e \to s$

Are any of these functions one-to-one?

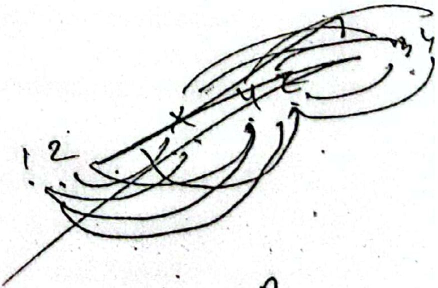
(c) Let a and b denote positive integers. Suppose a function Q is defined recursively as follows:

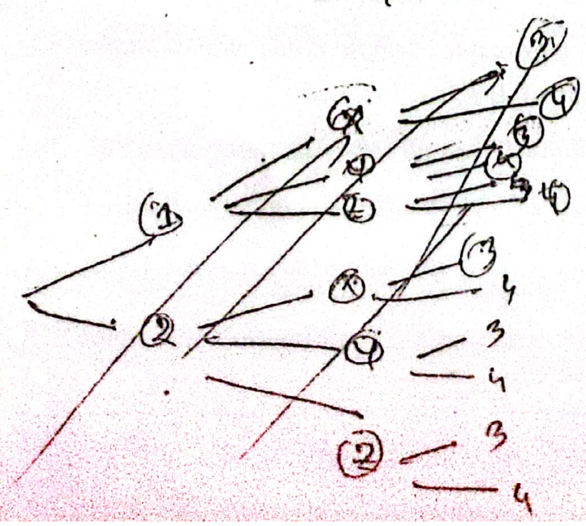
$$Q(a, b) = \begin{cases} 0, & if\ a < b \\ Q(a - b, b) + 1 & if\ b \leq a \end{cases}$$

i. Find the value of $Q(2,3)$ and $Q(14,3)$. ii. What does this function do? Find $Q(5861, 7)$.

$(\quad 2^2 \times 3^2 \times 2^2$

$= 4 \times 9 \times 4 = 144$





$2 \times 2 \times 3$

$4 \times 3 = 12$

$2 \times 2 \times$

Patuakhali Science and Technology University
2nd Semester (Level-1, Semester-II), Final Examination of B.Sc. Engg.in (CSE)
July-December: 2019, Session: 2018-19
Course Code: CIT-121 Course Title: Discrete Mathematics
Credit Hour: 3.00 Full Marks: 70 Duration: 03Hours

[Figures in the right margin indicate full marks. Split answering of any question is not recommended]

Answer any 7 of the following questions.

1. a) Rewrite the following statements using set notation:
(i) The element 1 is not a member of A.
(ii) The element 5 is a member of B. 2
(iii) A is a subset of C.
(iv) A is not a subset of D.
(v) F contains all the elements of G.
(vi) E and F contain the same elements.
b) List the elements of the following sets; here z = (*integers*).
(i) $A = \{x: x \in Z, 3 < x < 9\}$
(ii) $B = \{x: x \in Z, x^2 + 1 = 10\}$ 3
(iii) $C = \{x: x \in Z, x \text{ is odd}, -5 < x < 5\}$
c) Given that $U = N$ = (positive integers), identify which of the following sets are identical to {2,4}:
A = {*even numbers less than* 6}, $B = \{x: x < 5\}$, $C = \{x: (x-2)(x-4)(x+2) = 0\}$ 2
d) Define the set operations of:
(i) absolute complement or, simply, complement of a set, (ii) the relative complement or difference of two sets. 2
e) Describe a situation where the universal set U may be empty. 1

2. a) Find the number of elements in each finite set: 3
(i) $A = \{2, 4, 6, 8, 10\}$ (iv) $D = \{x: x \text{ is a positive integer}, x \text{ is a divisor of } 15\}$
(ii) $B = \{x: x^2 = 4\}$ (v) $E = \{\text{letters in the alphabet preceding the letter } m\}$ 13
(iii) $C = \{x: x > x + 2\}$ (vi) $F = \{x: x \text{ is a solution to } x^3 = 27\}$
(b) In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Time, and 26 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, and 8 read no magazine at all. 7
(i) Find the number of people who read all three magazines.
(ii) Fill in the correct number of people in each of the eight regions of the Venn diagram of Fig. 1-1(x). Here N, T, and F denote the set of people who read Newsweek, Time and Fortune respectively.
(iii) Determine the number of people who read exactly one magazine.

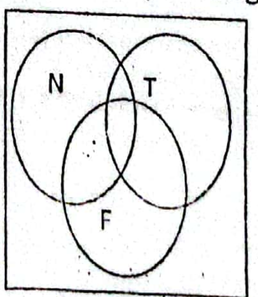


Fig. 1-1(x).

1, 3, 5
3|15
5|15
1

$x > x+2$
3, 4

3. a) Prove Theorem $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$. 2
(b) One hundred students were asked whether they had taken courses in any of the three areas, sociology, anthropology, and history. The results were: 8
45 had taken sociology 18 had taken sociology and anthropology
38 had taken anthropology 9 had taken sociology and history
21 had taken history 4 had taken history and anthropology
And 23 had taken no courses in any of the areas.
(i) Draw a Venn diagram that will show the results of survey.
(ii) Determine the number k of students who had taken classes in exactly (1) one of the areas, and (2) two of the areas

P.T.O.

4. a) Find the power set $P(A) of\ A = \{1, 2, 3, 4, 5\}$.

b) Prove the following propositions: $P(n): \frac{1}{1(3)} + \frac{1}{3(5)} + \frac{1}{5(7)} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

c) Determine the validity of the argument:

S_1: *All red meat contains cholesterol.*
S_2: *No expensive food contains cholesterol.*

S: *Read meat is not expensive.*

d) Given $A = \{1, 2\}, B = \{x, y, z\}, and\ C = \{3, 4\}. Find\ A \times B \times C\ and\ n(A \times B \times C)$. (tree diagram)

5. a) Let $A = \{1, 2, 3\}$ and $R = \{(1,1), (2,1), (3,2), (1,3)\}$ *be a relation on A (i.e., a relation from A to A).* Determine whether each of the following is true or false:
(i) $1R1$, (ii) $1\not R2$, (iii) $2R3$, (iv) $2\not R1$, (v) $3R2$, (vi) $3\not R1$.

b) Let S be the relation on $X = \{a, b, c, d, e, f\}$ *defined by* $S = \{(a,b), (b,b), (b,c), (c,f), (d,b), (e,a), (e,b), (e,f)\}$ Draw the directed graph of S

c) Let R and S be the relations on $X = \{a, b, c\}$ *defined by* $R = \{(a,b), (a,c), (b,a)\}$ *and* $S = \{(a,c), (b,a), (b,b), (c,a)\}$ Find the matrices M_R *and* M_S *representing R and S respectively.*

d) Consider functions $f: A \to B$ *and* $g: B \to C$; *that is, where the codomain of f is the domain of g. Define the composition function of f and g.*

6. a) Sketch the graph of the function $g(x) = x^2 + x - 6$.

7. b) Define an one –to-one (or injective) function and an onto (or surjective) function.

c) Let $A = \{a, b, c, d, e\}$. *and let B be the set of letters in the alphabet. Let the function* f, g and h from A into B be defined as follows:

(i)	(ii)	(iii)
$a \xrightarrow{f} r$	$a \xrightarrow{g} z$	$a \xrightarrow{h} a$
$b \to a$	$b \to y$	$b \to c$
$c \to s$	$c \to x$	$c \to e$
$d \to r$	$d \to y$	$d \to r$
$e \to e$	$e \to z$	$e \to s$

7. a) Define a graph and a multigraph. 2

b) Draw the complete graphs K_5, K_6 and also draw the complete bipartite graphs $K_{2,3}, K_{3,3}$ and $K_{2,4}$. 5

c) Define a Hamiltonian graph and draw a graph with six vertices which is Hamiltonian but not eulerian. 3

8. a) What does logically equivalent of two compound propositions mean? 2

b) Verify that the proposition $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction. 2

c) Prove that disjunction distributes over conjunction: that is, prove the distributive law $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. 4

d) Prove the theorem "If n is a composite integer, then n has a prime divisor less than or equal to $\sqrt{n}$"

9. a) Define the terminologies with example: (i) Rooted tree (ii) Ancestors of vertices (iii) Full m-ary tree.

b) Prove that a full m-ary tree with i internal vertices contain $n=mi + 1$ vertices.

c) What is the postfix form of the expression $((x + y) \uparrow 2) + ((x - 4)/ 3)$?

$(p \vee q) \wedge p$

$9 = 3 \times 3 \times 1$

$1 : 0 = 0$

$9 - 3 - 6$

$9 \div 3$

Patuakhali Science and Technology University

Mid Term Examination-2018

Course Code: CIT-121; Course Title: Discrete Mathematics

Full Time: 20 minutes Full Marks: 15

1. Describe various types of set. 5
2. Differentiate between function and relation. Explain One-to-one function, Onto function and Inverse of a Function with example. 5
3. Illustrate DeMorgan's law $(A \cup B)^C = A^C \cap B^C$ using Venn diagram. 5

Patuakhali Science and Technology University

2nd Semester (Level-1, Semester-II) Final Examination of B.Sc. Engg. (CSE) July-Dec.: 2017, Session 2016-17

Course Code: CIT-121 Course Title: Discrete Mathematics

Credit Hour: 3.0 Full Marks: 70 Duration: 3 Hours

[Figures in the right margin indicate full marks. **Split answering of any question is prohibited**]

Answer any 5 of the following questions.

1. (a) List the elements of each set where $N = \{1, 2, 3, \ldots\}$. 2
   (i) $A = \{x \in N \mid 2 < x < 7\}$ (ii) $B = \{x \in N \mid x \text{ is odd}, x < 11\}$
   (iii) $C = \{x \in N \mid 5 + x = 4\}$ (iv) $D = \{x \in N \mid x \text{ is even}, 2 + x = 4\}$.

   (b) Explain the partitioning of a set. P(NOT) 2

   (c) In a survey of 120 people, it was found that: 65 read Newsweek magazine, 20 read both Newsweek and Time, 45 read Time, 25 read both Newsweek and Fortune, 42 read Fortune, 15 read both Time and Fortune, 8 read all three magazines. 4
   (i) Draw a Venn diagram and fill in the correct number of people in each region.
   (ii) Find the number of people who read at least one of the three magazines.
   (iii) Find the number of people who read exactly one magazine.

   (d) Briefly describe various types of set.

2. (a) Let $A = \{a, b, c\}$, $B = \{x, y, z\}$, $C = \{r, s, t\}$. Let $f: A \to B$ and $g: B \to C$ be defined by: $f = \{(a, z)(b, x), (c, y)\}$ and $g = \{(x, s), (y, t), (z, r)\}$. Find composition function $g \circ f: A \to C$.

   (b) Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B: $R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$.
   (i) Find the inverse relation R^{-1} of R. (ii) Determine the domain and range of R.

   (c) Given: $A = \{1, 2\}$, $B = \{x, y, z\}$, and $C = \{3, 4\}$. Find: $A \times B \times C$.

   (d) Distinguish between function and relation. Explain One-to-one function, Onto function and Inverse of a Function with example.

3. (a) Let p be "It is cold" and let q be "It is raining". For each of the following statements make simple verbal sentence: (a) $\neg p$; (b) $p \wedge q$; (c) $p \vee q$; (d) $q \vee \neg p$.

   (b) Verify that the proposition $(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$ is a contradiction.

   (c) Briefly describe Normal Forms.

   (d) State and explain the following rules of inference with example:
   (i) Modus ponens, (ii) Hypothetical Syllogism, (iii) Destructive Dilemma and (iv) Conjunction.

4. (a) In how ways can the letters of the word 'ORANGE' be arranged so that the Consonants occupy only the even positions?

   (b) A box contains 8 blue socks and 6 red socks. Find the number of ways two socks can be drawn from the box if: (a) They can be any color. (b) They must be the same color. 2

   (c) In a certain college town, 25% of the students failed mathematics (M), 15% failed chemistry (C), and 10% failed both mathematics and chemistry. A student is selected at random. 4
   (i) If he failed chemistry, find the probability that he also failed mathematics.
   (ii) If he failed mathematics, find the probability that he also failed chemistry.
   (iii) Find the probability that he failed mathematics or chemistry. MUC
   (iv) Find the probability that he failed neither mathematics nor chemistry. $(M \cup C)^c$

   (d) State and prove Pascal's Identity. 3

   (e) A history class contains 8 male students and 6 female students. Find the number n of ways that the class can elect: (a) 1 class representative; (b) 2 class representatives, 1 male and 1 female; (c) 1 president and 1 vice president. 3

5. (a) Explain BFS algorithm for graph traversal with example. 4

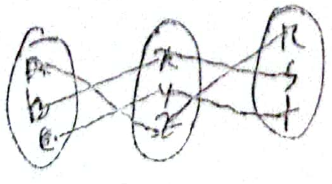
   (b) Consider three pen-stands. The first pen-stand contains 2 red pens and 3 blue pens; the second one has 3 red pens and 2 blue pens; and the third one has 4 red pens and 1 blue pen. There is equal probability of each pen-stand to be selected. If one pen is drawn at random, what is the probability that it is a red pen? 4

   (c) Minimize the following Boolean expression using Boolean identities: 6
   $$F(A, B, C) = A'B + BC' + BC + AB'C'$$

6. (a) Explain Euler graph. 2

   (b) Discuss representation of graphs. 4

   (c) What is minimum spanning tree? State and explain Kruskal's algorithm with example. 8



$g \circ f(a) = g(f(a))$
$= g(f(z))$
$= g(r)$