PATUAKHALI SCIENCE AND TECHNOLOGY UNIVERSITY

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Assignment Topic : The Foundations: Logic and Proofs

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Propositions: A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Example:

All the following declarative sentences are propositions.

1. Washington, D.C., is the capital of the United States of America.
2. Toronto is the capital of Canada.
3. 1 + 1 = 2.
4. 2 + 2 = 3.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Applications of Propositional Logic:

1. Translating English Sentences;
2. System Specifications;
3. Boolean Searches;
4. Logic Puzzles;
5. Logic Circuits;

Propositional Equivalences:

1. **Tautology:** A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology.
2. **Contradiction:** A compound proposition that is always false is called a contradiction.
3. **Contingency:** A compound proposition that is neither a tautology nor a contradiction is called a contingency.

Logical Equivalences: Compound propositions that have the same truth values in all possible cases are called logically equivalent.

Example: Show that ¬(p ∨ q) and ¬p ∧ ¬q are logically equivalent.

Solution: The truth tables for these compound propositions are displayed in Table 3. Because the truth values of the compound propositions ¬(p ∨ q) and ¬p ∧ ¬q agree for all possible combinations of the truth values of p and q, it follows that ¬(p ∨ q) ↔ (¬p ∧ ¬q) is a tautology and that these compound propositions are logically equivalent.

TABLE 3 **Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.**

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Satisfiability:

A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true (that is, when it is a tautology or a contingency). When no such assignments exists, that is, when the compound proposition is false for all assignments of truth values to its variables, the compound proposition is unsatisfiable.

Predicates and Quantifiers:

Predicates: A predicate is an expression that assigns a truth value ("true" or "false") to an element within a given domain.

Example:

"$x > 3$," "$x = y + 3$," "$x + y = z$,";

The first part, the variable x, is the subject of the statement. The second part—the predicate, "is greater than 3"—refers to a property that the subject of the statement can have.

Quantifiers:

In English, the words all, some, many, none, and few are used in quantifications. We will focus on two types of quantification here: universal quantification, which tells us that a predicate is true for every element under consideration, and existential quantification, which tells us that there is one or more element under consideration for which the predicate is true.

The universal quantification of $P(x)$ is the statement "$P(x)$ for all values of x in the domain." The notation $\forall xP(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the universal quantifier. We read $\forall xP(x)$ as "for all $xP(x)$" or "for every $xP(x)$." An element for which $P(x)$ is false is called a counterexample to $\forall xP(x)$.

The existential quantification of $P(x)$ is the proposition "There exists an element x in the domain such that $P(x)$." We use the notation $\exists xP(x)$ for the existential quantification of $P(x)$. Here $\exists$ is called the existential quantifier.

Precedence of Quantifiers:

The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus. For example, $\forall xP(x) \vee Q(x)$ is the disjunction of $\forall xP(x)$ and $Q(x)$. In other words, it means $(\forall xP(x)) \vee Q(x)$ rather than $\forall x(P(x) \vee Q(x))$.

Negating Quantified Expressions:

What are the negations of the statements $\forall x(x2 > x)$ and $\exists x(x2 = 2)$? Solution: The negation of $\forall x(x2 > x)$ is the statement $\neg\forall x(x2 > x)$, which is equivalent to $\exists x\neg(x2 > x)$. This can be rewritten as $\exists x(x2 \leq x)$. The negation of $\exists x(x2 = 2)$ is the statement $\neg\exists x(x2 = 2)$, which is equivalent to $\forall x\neg(x2 = 2)$. This can be rewritten as $\forall x(x2 \neq 2)$. The truth values of these statements depend on the domain.

Translating from English into Logical Expressions:

Express the statement “Every student in this class has studied calculus” using predicates and quantifiers.

Solution: First, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use. Doing so, we obtain:

“For every student in this class, that student has studied calculus.”

Next, we introduce a variable x so that our statement becomes

“For every student x in this class, x has studied calculus.”

Continuing, we introduce $C(x)$, which is the statement “x has studied calculus.” Consequently, if the domain for x consists of the students in the class, we can translate our statement as $\forall xC(x)$.

However, there are other correct approaches; different domains of discourse and other predicates can be used. The approach we select depends on the subsequent reasoning we want to carry out. For example, we may be interested in a wider group of people than only those in this class. If we change the domain to consist of all people, we will need to express our statement as

"For every person x, if person x is a student in this class, then x has studied calculus."

Using Quantifiers in System Specifications:

Use predicates and quantifiers to express the system specifications "Every mail message larger than one megabyte will be compressed" and "If a user is active, at least one network link will be available."

Solution: Let $S(m, y)$ be "Mail message m is larger than y megabytes," where the variable x has the domain of all mail messages and the variable y is a positive real number, and let $C(m)$ denote "Mail message m will be compressed." Then the specification "Every mail message larger than one megabyte will be compressed" can be represented as $\forall m(S(m, 1) \rightarrow C(m))$.

Let $A(u)$ represent "User u is active," where the variable u has the domain of all users, let $S(n, x)$ denote "Network link n is in state x," where n has the domain of all network links and x has the domain of all possible states for a network link. Then the specification "If a user is active, at least one network link will be available" can be represented by $\exists uA(u) \rightarrow \exists nS(n, \text{available})$.

Rules of Inference:

By an argument, we mean a sequence of statements that end with a conclusion. By valid, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises, of the argument. That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false.

An argument in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called the conclusion.

TABLE 1 Rules of Inference.		
Rule of Inference	***Tautology***	***Name***
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution