Discrete Mathematics

Introduction to the Course — Class-1

Discrete = বিচ্ছিন্ন, পৃথক, স্বতন্ত্র

Discrete VS Continuous

1,2,3

1. Discrete -> Can't be broken down into fractions or decimals.

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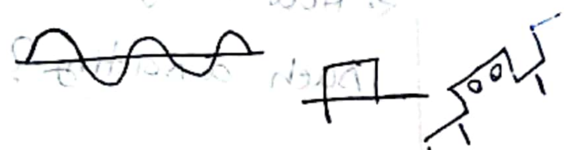
2. Number of students, purchasing tickets, text message etc.
3. Discrete elements are countable.

Discrete vs continuous

1. Continuous can be broken down into fractions or decimals.
2. Time, Temperature, weight, distance etc.
3. Continuous elements are measurable



Discrete Mathematics:

Discrete mathematics is the study of discrete objects.

Why do you need to study discrete mathematics?

1. It is a very good tool for improving problem-solving capabilities.

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program

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probability

Graph & Tree

1. logical reasoning
2. Mathematical proofs - Corre
3. Complexity of algorithms - Execution time: Best, Worst, average

Shortest path
computer Networks
Advance Data Structure

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10, 2, 25 → $\boxed{\text{Sorting program}}$ → 2, 10, 25

1. How can a list of integers be sorted so that the integers are in increasing/decreasing order? → Algorithm

2. How many steps are required to do such a sorting? → Counting techniques

3. How can it proved that a sorting algorithm correctly sorts a list? → Mathematical proofs

Most Important facts:

It has Applications to

1. Compilers,
2. Software engineering,
3. Architecture,
4. Data Bases,
5. Algorithms,
6. Data Structures, and
7. operating System.

Course plan:

1. Logic and proofs.
2. Sets, Functions, Sequences, and Sums.
3. Algorithms, the Integers and Matrices.
4. Induction and Recursion.
5. Counting
6. Discrete probability.
7. Advance Counting Techniques.
8. Relations
9. Graph
10. Trees
11. Boolean Algebra.
12. Modelling Compatation.

Tutorial-2

Introduction to propositional Logic
Logic and Proof

Application of Logic and proofs

1. In the design of computer circuits.
2. The construction of computer programs.
3. The verification of the correctness of programs.
4. Artificial Intelligence.

Either = দুইটির মধ্যে এক বা অন্যতর।
Declarative = ঘোষণাত্মক।

1. Propositional Logic (Valid logical arguments)

What is proposition?

A proposition is a declarative statement that is either true (T) or false (F), but not both.

True = T = 1

False = F = 0

সত্য বা মিথ্যা দ্বারা প্রকাশ করতে পারবো তাই ইটি একটি proposition.

Example:

1+1=2 → Truth value of this proposition: T

২+২=5 → Truth value of this proposition: F

প্রতিটি proposition এর একটি Truth value থাকতে পারে হয় তা True অথবা False — But not Both.

Sylhet is the capital of Bangladesh.

Truth value of this proposition: F

Pigs can Fly.

Truth value of this proposition: F

Pigs = শূকর

Not proposition (not valid logical arguments)

1. What time is it? → questions
2. Read this carefully. → commands / Imperative
3. Do your homework.
4. X+1=2. → Non constant values / value is not defined.
5. Bangladesh and India → Not statements
6. Shahin is the best lecturer → opinion.
7. He is a college student. → person is not defined.

propositional variables / ~~Statement variables~~

- Today is Friday. → p = Today is Friday.
- It is raining → q = It is raining.

Compound proposition:

- Compound proposition is a proposition formed by combining two or more simple proposition.
- The logical operators that are used to form compound proposition is called connectives.

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Symbol	Math Name	English Name
$\neg$	Negation	Not
$\vee$	Disjunction	OR
$\wedge$	conjunction	AND
$\oplus$	ExOR	" OR.. but not both"
$\rightarrow$	Implication	"If then"
$\Leftrightarrow$	Equivalence / biconditional	"if & only if"

Tutoria-3

Logical operator/connectives

$\neg$ (Not) Negation

$\wedge$ (AND)

$\vee$ (OR)

$\oplus$ (ExOR)

$\rightarrow$ (Implipication)

$\leftrightarrow$ (Bicondiitional)

Not

proposition: To day is Friday. $\xrightarrow{\text{Negation}}$ Today is not Friday.

P

It is not the case that today is Friday.

Negation: $\neg P$

Logical operator বুঝানোর সবচেয়ে better উপায় Truth Table ব্যবহার।

Truth Table ~~বানা~~ তৈরির সময় দেখতে হবে কতগুলো Variable আছে। then সারি সংখ্যা নিতে হবে 2^n

n= Variable সংখ্যা।

Ⓡ

$2^n = 2^1 = 2$

p	$\neg p$
F	T
T	F

Negation

① 6 is negative

— 6 is non negative / It is not the case that 6 is negative.

② $2+1=3$

$2+1 \neq 3$

③ There is no pollution in New Jersey.

There is pollution in New Jersey.

④ The summer in marine is hot and sunny.

It is not the case that the summer in marine is hot and sunny.

⑤ Today is thursday.

Today is not Thursday.

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AND / CONJUNCTION

Today is Friday and It is raining. $p \wedge q$

$\underbrace{\text{Today is Friday}}_{p}\ \underbrace{\text{and}}_{\wedge}\ \underbrace{\text{It is raining}}_{q}$

$2^n = 2^2 = 4$

p (2)	q (1)	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

দুইটি True হলে Answer True

AND এর ক্ষেত্রে output তখনই সত্য হবে যখন কোনো একটি input লাইনের সবগুলো সত্য হবে।

OR / Disjunction

Today is Friday or It is raining. $p \vee q$

$\underbrace{\text{Today is Friday}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{It is raining}}_{q}$

দুইটি বা যে কোনো একটি True হলে Answer True হবে

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

OR এর বেলায় যে কোনো একটি input সত্য হলেই output সত্য হবে।

Ⓡ Tutorial-4

ExOR ⊕ (Exclusive OR) (XOR)

" Students who have taken calculus or compute science can take this class."

$2^n = 2^2 = 4$

বিজোড় সংখ্যক input True হলে output True হবে।

T T F = F

T T T = T

p	q	p ∨ q
F	F	F
F	T	T
T	F	T
T	T	T

" Students who have taken calculus or computer science, but not both, can take the class".

Or + not both = ExOR

মাত্র যে কোনো একটি True হলেই Power True হবে।

p	q	p ⊕ q
F	F	F
F	T	T
T	F	T
T	T	F

intended = অভিপ্রেত, লক্ষ্য।

Ⓑ For each of these sentences, determine whether an exclusive OR or OR is intended:

ⓐ coffee or tea comes with dinner.
~~or~~ p ⊕ q ExOR

⑩ Ⓡ

(b) Experience with c++ or Java is required.

p v q or

(c) Lunch include soup or salad.

p ⊕ q : যেকোনো একটি দুইটি আসবে না।

(d) You can pay U.S. dollars or euros.

OR / Exor

Tutorial-5

Conditional Statement (Implication)

→ নিহিত, প্রভাব, সংশ্লেষ

→ implies

p: Today is holiday.

q: The store is closed

" if p, then q"

p → q

implies = অন্তর্নিহিত

if the day is holiday then the store is closed.

p	q	p→q
F	F	T
F	T	T
T	F	F
T	T	T

কেননা, আমি বলি নাই যে, holiday না থাকলে বন্ধ থাকবে না।

Implication এ শুধুমাত্র T → F থাকলেই False হবে। বাকি সকল ক্ষেত্রেই True হবে।

শুধুমাত্র T→F এর ক্ষেত্রে False return করবে।

Ⓡ

Determine whether each of these conditional statements is true or false:

$T \to F = F$

$TF \to F$

ⓐ If $\underbrace{1+1=2}_{p}$, then $\underbrace{2+2=5}_{q}$ $\quad T \to F = F$

$p \longrightarrow q$

ⓑ If $\underbrace{1+1=3}_{F}$, then $\underbrace{2+2=4}_{T}$ $\longrightarrow$ True

ⓒ If $\underbrace{1+1=3}_{p}$, then $\underbrace{\text{dogs can fly}}_{q}$ $\longrightarrow$ True

$F \longrightarrow F$

ⓓ If $\underbrace{1+1=2}_{T}$, then $\underbrace{\text{dogs can fly}}$ $\longrightarrow$ False

$\longrightarrow F = F$

Tutorial-6

Conditional Statement part-2

There ⟶ গুলো।

① " if p, then q"

" If I am elected, then I will lower Taxes"

$p \to q$

② " if p, q"

" If it is below freezing, it is also snowing"

$p \to q$

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③ "p is sufficient for q"

" Driving over 65 miles per hour is sufficient for getting a speeding ticket."

Driving over 65 miles per hour = p; getting a speeding ticket = q

$p \to q$

④ "q if p"

" Maria will get a good job if she learns discrete mathematics"

Maria will get a good job = q; she learns discrete mathematics = p

$p \to q$

এই ৪টি কে আমরা মুখস্থ করে রাখবো কি না?

⑤ " q whenever p"

⑥ " q is necessary for p"

⑦ " q follows from p"

⑧ " a sufficient condition for q is p"

$\left.\begin{matrix} ⑤ \\ ⑥ \\ ⑦ \\ ⑧ \end{matrix}\right\} p \to q$

⑨ "q when p"

" Maria will find a good job when she learns discrete Mathematics".

Maria will find a good job = q; she learns discrete mathematics = p

$p \to q$

⑩ " q unless $\neg p$"

" Maria will find a good job unless She does not learn discrete mathematics.

$\neg p$

$p \rightarrow q$

(11) "p only if q"

(12) if ... then

$p \rightarrow q$

Tutorial-7

Converse, Inverse, Contrapositive

Conditional → new conditional statement

Converse   Inverse   Contrapositive

$p \rightarrow q$

$q \rightarrow p$ (converse)

$\neg p \rightarrow \neg q$ (Inverse)

$\neg q \rightarrow \neg p$ (contrapositive)

converse + Inverse

p: I am in sylhet.

q: I am in Bangladesh.

If I am in sylhet then I am in Bangladesh.

$p \rightarrow q$

Converse → $q \to p$

If I am in Bangladesh then I am in Sylhet.

Inverse: $\neg p \to \neg q$

If I am not in Sylhet then I am not in Bangladesh.

Contrapositive: $\neg q \to \neg p$

If I am not in Bangladesh then I am not in Sylhet.

T→F F

Original value এর সাথে তার contrapositive ~~অবশ্যই~~ ~~মেচ~~ হবে।

$n=2$
$2^n = 2^2 = 4$

p	q	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
F	F	T	T	T	T	T
F	T	F	T	F	F	T
T	F	T	F	T	T	F
T	T	T	F	F	T	T

Converse, Inverse, contrapositive

- **$p \to q$ Truth Value = $\neg q \to \neg p$ Truth Value.**

Snow = তুষার, বরফ।

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Tutorial-8

State the converse inverse and contrapositive of each of these following statements :-

(a) If it snows today, I will ski tomorrow $p \to q$

Converse : $q \to p$ =>

I will ski tomorrow only if it snows today.

Inverse : $\neg p \to \neg q$ =>

If it does not snow today, then I will not ski tomorrow.

Contrapositive : $\neg q \to \neg p$

If I donot ski tomorrow then I will not have the snowed today.

(b) I come to class whenever there is going to be a quiz.

$\therefore p \to q$

Converse : $q \to p$

There is going to be a quiz whenever i come to class.

Inverse : $\neg p \to \neg q$

I don't come to class whenever there is not going to be a quiz.

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Implication = নিহিত প্রস্তাব

iff = if and only if

HW

Contrapositive: $\neg q \rightarrow \neg p$

There is not going to be a quiz whenever I don't come to class.

Tutorial-9

Biconditional Statement বা Bi Implication

Bi-Conditional = দ্বিশর্ত

Bi-Implicer = দ্বি-অন্তর্ভুক্তি

"You can take the flight if and only if you buy a ticket."

$p \longleftrightarrow q$

"p is necessary and sufficient for q"

"p iff q"

"If p then q, and conversely"

$p \rightarrow q \wedge q \rightarrow p$

$p \longleftrightarrow q$

$2^n = 2^2 = 4$

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

• $\longleftrightarrow$ Biconditional এর ক্ষেত্রে যে কোনো দুইটি False অথবা যে যেকোনো দুইটি True হলে তবেই শুধুমাত্র True হবে।

Ⓡ

" That it is below freezing is necessary and sufficient for it to be snowing".

$p \leftrightarrow q$

Bi-Conditional

Tutorial-10

Determine whether these biconditionals are true or false:

a) $2+2=4$ if and only if $1+1=2$

$T \leftrightarrow T = T$

b) $1+1=2$ if and only if $2+3=4$

$T \leftrightarrow F = F$

c) $1+1=3$ if and only if monkeys can fly.

$F \leftrightarrow F = T$

d) $0>1$ if and only if $2>1$.

$F \leftrightarrow T = F$

P

$\rightarrow$ Conditional.

$\leftrightarrow$ Bi-Conditional.

$2^n = 2^2 = 4$

Tutorial-11

Truth Table of compound proposition

⊛ precedence of logical

$\neg \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow$

$(p \vee \neg q) \longrightarrow (p \wedge q)$

১ ঘরের জন্য ২
২ ঘরের জন্য ৪

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

② $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

2	1				$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$\oplus$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Exor এর বেলায় যে কোনো একটি input সত্য হলে output সত্য হবে,

R

Conduct truth table for the compounds

3. $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

চলক তিনটি সুতরাং সারি হবে ৮টি

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$W = (\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	F	F
F	T	F	T	F	F	F	T
F	T	T	T	F	F	T	F
T	F	F	F	T	F	T	F
T	F	T	F	T	F	F	T
T	T	F	F	F	T	F	F
T	T	T	F	F	T	T	T

F হবে Exor ? B.H.

or → $\vee$

or.. but not both → $\oplus$ Exor.

Tutorial-12

English Sentence → proposition

Let p and q be the propositions:

p: "It is below freezing"

q: "It is snowing"

write these propositions using p and q and logical connectives:

ⓐ It is below freezing and snowing.

$p \wedge q$

ⓑ It is below freezing but not Snowing.

p $\wedge$ $\neg q$

$p \wedge (\neg q)$

* ⓒ It is either snowing or freezing but not both.

p q

$p \oplus q$

ⓓ If it is below freezing, it is also snowing.

$p \rightarrow q$

If p, q থাকলে $p \rightarrow q$ p implies q হবে।

p only if q

$p \rightarrow q$

(e) It is either below freezing or it is snowing, but it is not snowing if it is not below freezing.

$$(p \vee q) \wedge \cancel{\neg q} \; (p \rightarrow \neg q)$$

(f) That is below freezing is necessary and sufficient for it to be snowing.

$p \leftrightarrow q$

Quiz:

English Sentence $\longrightarrow$ Logical expression

"You can access the internet from campus $\boxed{\text{only}}$ $\boxed{\text{if}}$ you are a computer science major $\boxed{\text{or}}$ you are $\boxed{\text{not}}$ a freshman."

$a \rightarrow (s \vee \neg f)$

Tutorial-13

1. Let p and q be the propositions "Swimming at the New jersey is allowed" and "Sharks have been spotted near the shore", respectively Express each of these compound propositions as:

(a) $\neg p \rightarrow$ Swimming at the new Jersey is not allowed.

(b) $p \vee q \rightarrow$ Swimming at the new jersey is allowed or Sharks have been spotted near the shore.

(c) $p \rightarrow q \rightarrow$ If Swimming at the new jersey ~~at~~ is allowed then sharks have been spotted near the shore.

(d) $p \leftrightarrow q$: Swimming at the New Jersey is allowed if and only Sharks have been spotted near the shore.

(e) $\neg p \leftrightarrow \neg q$

Swimming at the new jersey is not allowed if and ~~and on~~ only if sharks have been spotted near the shore.

(f) $\neg p \vee (p \wedge q)$

Swimming at the new jersey is not allowed or Swimming at the new jersey is allowed and sharks have been spotted near the shore.

(g) $\neg p \wedge (p \vee \neg q)$

Tutorial-14

End of the propositional Logic.

Tautology = অনুলাপ, দ্বিরুক্তি, পুনরুক্তি।
Contradiction = দ্বন্দ্ব, অস্বীকার, প্রতিবাদ
Contingency = আকস্মিক ঘটনা, অনিশ্চিত ঘটনা।

Tutorial-15
Logical Equivalence

$\neg(p \wedge q)$
F
F
F
T

$\equiv$

$\neg p \vee \neg q$
F
F
F
T

যদি দুটি compound proposition এর truth value exactly same থাকে তবে তাকে Logical Equivalence বলে

compound proposition তিন ভাগে ভাগ করা যায়ঃ

i) Tautology.
ii) Contradiction.
iii) Contingency.

(i) $p \vee \neg p$ (ii) $p \wedge \neg q$ (iii) $p \rightarrow q$

p	q	$\neg p$	$\neg q$	$p \vee \neg p$	$p \wedge \neg q$	$p \rightarrow q$
F	F	T	T	T	F	T
F	T	T	F	T	F	T
T	F	F	T	T	F	F
T	T	F	F	T	F	T

Tautology = যদি কোনো compound proposition এর সবগুলো value true হয় তবে তাকে Tautology বলে। $p \vee \neg q$

Tautology এর value always true হয়।

Contradiction = যদি কোনো compound proposition এর সবগুলো value false হয় তবে তাকে contradiction বলে।

$q \wedge \neg q$
contradiction is always False.

T এবং F এর mixed up = contingency

T, F এর mixed up হবে।

- $p \to q$ এর ক্ষেত্রে শুধুমাত্র $T \to F = F$ বাকি সব ক্ষেত্রে True হবে।

$X = (p \to q) \wedge (q \to r)$

Tutorial-16

4 2 1

p	q	r	$p \to q$	$q \to r$	$q \to r$	$p \to r$	X	
F	F	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F	T
T	F	T	F	T	T	T	F	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

$Y = (p \to q) \wedge (q \to r) \to (p \to r)$

* AND এর precedence implies এর আগে।

$(p \to q) \wedge (q \to r) \to (p \to r)$

is a Tautology.

① Determine whether $(\neg p \wedge (p \to q)) \to p$ is a tautology.

p	q	$\neg p$	$p \to q$	$(\neg p \wedge (p \to q))$	$(\neg p \wedge (p \to q)) \to p$
F	F	T	T	T	F
F	T	T	T	T	F
T	F	F	F	F	T
T	T	F	T	F	T

H.W. $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology or not.

Tutorial-17

Logical Equivalence Using Truth Table

T→F হলেই p→q এর output = F

Example: 1

Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

p	q	¬p	p→q	¬p∨q
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

$p \to q$ and $\neg p \vee q$ are logically equivalent

$x = (p \to r) \vee (q \to r)$

Example 2:

$y = (p \wedge q) \to r$

Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

$row = 2^n = 2^3 = 8$

p	q	r	p→r	q→r	p∧q	x	y
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

idempotent = वरावर।

Exercise

① $(p\wedge q)\rightarrow r$ and $(p\rightarrow r)\wedge(q\rightarrow r)$,

② $(p\rightarrow q)\rightarrow r$ and $(p\rightarrow(q\rightarrow r))$

Tutorial-18
Logic Laws (part-1)

Logic Laws:

① Double Negation law:

$\neg(\neg p)\equiv p$

p	¬p	¬(¬p)
F	T	F
T	F	T

$\neg(\neg(a+b))$
$=a+b$

② Idempotent Law:

$p\vee p\equiv p$

$p\wedge p\equiv p$

p	p∨p
F	F
T	T

$Bc\wedge Bc\wedge Bc\wedge Bc\ldots\wedge nBc$
$\equiv Bc$

③ Identity Law:

$p\wedge T\equiv p$ True

$p\vee F\equiv p$ False

$(p\wedge T)\wedge(p\vee F)$
$=p\wedge p$
$=p$

④

$\bar{x}$ = complement এর কথা not বা negation একই কথা।

AND = গুণ
OR = যোগ

complement = Not

④ Domination Law:

$p \vee T \equiv T$ (T → True)

$p \wedge F \equiv F$

$(p \vee T) \wedge (p \wedge F)$

$= T \wedge F$

$= F$

⑤ Commutative Law:

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

⑥ Associative Law:

$(p \vee q) \vee r = p \vee (q \vee r)$

$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

⑦ Inverse Law:

$p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

$\neg B B = 0$

$B \cap \neg B = 0$

$A\bar{B}B\bar{C} + B\bar{C}C\bar{D} + \bar{A}DA\bar{B} + \bar{A}CD\bar{D}$

$\equiv A\neg BB\neg C + B\neg CC\neg D + \neg ADA\neg B + \neg ACD\neg D$

$= A\neg C(B \cap \neg B) +$

$= A.0.\bar{C} + B.0.\bar{D} + 0.\bar{B}.D + \bar{A}.C.0$

$= 0 + 0 + 0 + 0$

$= 0$

8. **Absorption Law:**

$$p \lor (p \land q) \equiv p$$
$$p \land (p \lor q) \equiv p$$

9. **Distributive Law:**

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$
$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

10. **Conditional Law:**

$$p \to q \equiv \neg p \lor q$$

11. **Biconditional Law:**

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

Tutorial-19
Logic Laws part-2
De Morgan's Law

De Morgan's Law

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

$x = p \land q$, $y = \neg(p \land q)$

$z = \neg p \lor \neg q$, $A = p \lor q$

$B = \neg(p \lor q)$

p	q	$\neg p$	$\neg q$	x	y	z	A	
F	F	T	T	F	T	T	F	T
F	T	T	F	F	T	T	T	F
T	F	F	T	F	T	T	T	F
T	T	F	F	T	F	F	T	F

$y = \neg(p \land q)$

De Morgan's Laws:

$\neg(R \wedge S \wedge T) = \neg R \vee \neg S \vee \neg T$

$\neg(R \vee S \vee T) = \neg R \wedge \neg S \wedge \neg T$

$\therefore \neg(R \wedge S \wedge T) \wedge \neg(R \vee S \vee T)$

$\equiv (\neg R \vee \neg S \vee \neg T) \wedge (\neg R \wedge \neg S \wedge \neg T)$

$= (\bar{R} + \bar{S} + \bar{T}) \cdot (\bar{R} \cdot \bar{S} \cdot \bar{T})$

$= \bar{R}\bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T}\bar{S} + \bar{R}\bar{S}\bar{T}\bar{T}$

$= \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T}$

$= \bar{R}\bar{S}\bar{T}$

$= \neg R \wedge \neg S \wedge \neg T$

Tutoria-20

Use De Morgan's Law to find the negation of the following statements:

ⓐ Jon is rich and happy.

$\underbrace{\text{Jon is rich}}_{p}\ \underbrace{\text{and}}_{\wedge}\ \underbrace{\text{happy.}}_{q}$

$\neg(p \wedge q) = \neg p \vee \neg q$

John is not rich or John is not happy.

ⓑ Carlos will bicycle or run tomorrow.

$\underbrace{\text{Carlos will bicycle}}_{p}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{run tomorrow}}_{q}$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Carlos will not bicycle and carlos will not run tomorrow.

Tutorial-21

⊞ Show that each of these conditional statement is a tautology without using truth table.

ⓐ $(p \wedge q) \rightarrow p$

ⓑ $(p \wedge q) \rightarrow (p \vee q)$

Rules

$$\boxed{p \rightarrow q = \neg p \vee q}$$

ⓐ $\underbrace{(p \wedge q)}_{p} \rightarrow \underbrace{p}_{q}$

$\equiv \neg(p \wedge q) \vee p$ De Morgan's Law

$= \neg p \vee \neg q \vee p$

$= (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T$

$p \to q = \neg p \vee q$

ⓑ $\underbrace{(p \wedge q)}_{p} \to \underbrace{(p \vee q)}_{q}$

$\equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv \neg p \vee \neg q \vee p \vee q$

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$

$\equiv T \vee T$

$\equiv T$

<u>Tutorial-22</u>

Logical equivalent without using truth table.

Now The each of statement are logically equivalent without using truth table.

① $\neg(p \to q) \equiv p \wedge \neg q$

② $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

① $\neg(p \to q) \equiv p \wedge \neg q$

L.H.S. $\equiv \neg(p \to q)$

$\equiv \neg(\neg p \vee q)$

$\equiv \neg(\neg p) \wedge \neg q$

$\equiv p \wedge \neg q$

$= R.H.S.$

(ii) L.H.S. $\equiv \neg(\underset{p}{\underline{p}} \vee (\underset{q}{\underline{\neg p \wedge q}}))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$

$\equiv \neg p \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv F \vee (\underset{p}{\underline{\neg p \wedge \neg q}})$

$\equiv \neg p \wedge \neg q$

Tutorial-23

End of propositional Equivalence.

Tutorial-24

Predicates & Quantifiers
[illegible] বিধেয়

1.1 Propositional Logic.

1.2 Propositional Equivalence.

Predicates

(i) "x is greater than 3" $P(x)$

(ii) "x is greater than y" $Q(x, y)$

(iii) "x is greater than y and z" → $R(x, y, z)$

(iv) "x can speak English" $P(x)$

Predicates

x is greater than 3

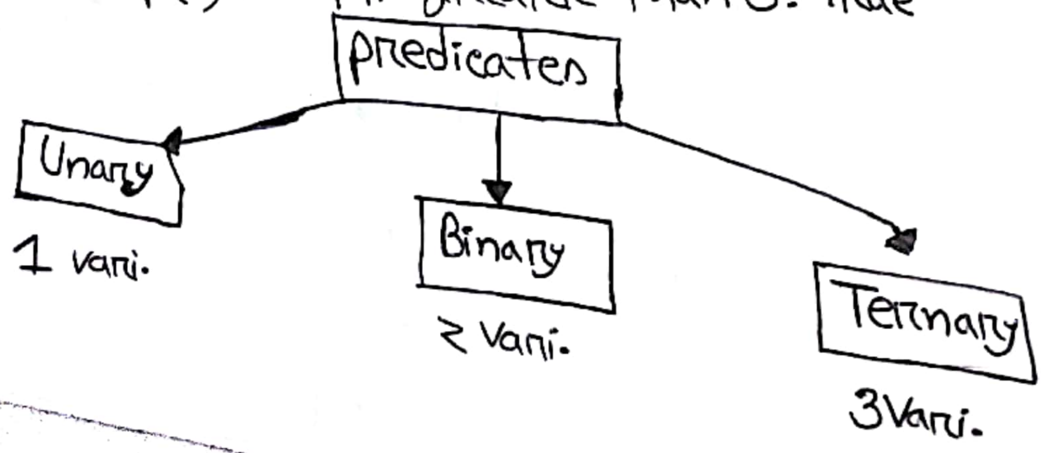
x ↓ Variable/Subject

is greater than — predicate

$P(x) =$ x is greater than 3.

$x = 2$ $P(2) =$ 2 is greater than 3. False

$x = 4$ $P(4) =$ 4 is greater than 3. True



$P(x, y) =$ x in greater than y

$(2,1) = P(2,1) =$ 2 in greater than 1.

$(2,3) = P(2,3) =$ 2 in greater than 3.

Example-1

Let P(x) denote the statement "$x>3$" what are the truth value of P(4) and P(2)?

$P(x) = x>3$

$\therefore P(4) = 4>3 = T$

$P(2) = 2>3 = F$

Example-2

Let $Q(x,y)$ denote the statement "$x = y+3$". what are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

$Q(x,y) \Rightarrow x = y+3$

$\therefore Q(1,2) \Rightarrow 1 = 2+3 \Rightarrow F$

$Q(3,0) \Rightarrow 3 = 0+3 \Rightarrow T$

Example-3 Let p(x) be the statement "the word x contains the letter a." what are these truth values?

a) P(orange).

b) P(true).

c) P(Lemon).

d) P(False).

Tutorial-25

"X can speak English." domain = Student.

$P(x) =$ x can speak English.

Quantifier

Universal $\forall$

Existential $\exists$

$P(x) =$ x can speak english.

$\forall(x)\ P(x)$

$\exists x\ P(x)$

Universal quantifier

for all, for every, for each, for any, all of, given any. ....

$\forall x\ P(x) \rightarrow$ For all x, $P(x)$.

Every student can speak Russian.

$\forall x\ P(x)$

Existential Quantifiers

there exists, for some, for at least one, there is

$\exists x\, P(x) \rightarrow$ There is an x such that $P(x)$.

☐ There is a student who can speak Russian.

$$\exists x\ P(x)$$

Example-1

Let $Q(x)$ be the statement "$x<2$" what is the truth value of the quantification $\forall x Q(x)$, where the domain consists of ~~Rea~~ all real numbers?

$\forall x\, Q(x)$

$$Q(x) = x<2$$
$$Q(1) = 1<2$$
$$Q(2) = 2<2$$
$$Q(3) = 3<2 \ (F)$$

Example-2:

What is the truth value of $\forall x\, P(x)$, where $P(x)$ is the statement "$x^2<10$" and the domain consists of the positive integers not exceeding 4?

Tutorial-25

"x can speak English." domain = Student.

$P(x)$ = x can speak English.

Quantifier

Universal $\forall$ | Existential $\exists$

$P(x)$ = x can speak english.

$\uparrow$

$\underline{\forall(x)}\ \underline{P(x)}$

$\exists x\ P(x)$

Universal quantifiers

for all, for every, for each, for any, all of, given any. ....

$\forall x\ P(x) \rightarrow$ For all x, $P(x)$.

Ex: Every student can speak Russian.

$\forall x\ P(x)$

Existential Quantifiers

there exists, for some, for at least one, there is

$\exists x\, P(x) \rightarrow$ There is an x such that $P(x)$.

Ex There is a student who can speak Russian.

$\exists x\; P(x)$

Example-1

Let $Q(x)$ be the statement "$x<2$" what is the truth value of the quantification $\forall x Q(x)$, where the domain consists of ~~rea~~ all real numbers?

$\forall x Q(x)$

$Q(x) = x<2$

$Q(1) = 1<2$

$Q(2) = 2<2$

$Q(3) = 3<2$ (F)

Example-2:

What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement "$x^2<10$" and the domain consists of the positive integers not exceeding 4?

$P(x) = x^2 < 10$

domain; $x = \{0, 1, 2, 3, 4\}$

$\forall x\, P(x) = x^2 < 10$

Truth value:

$4^2 < 10$ (F) False

Example-3:

what is the truth value of $\exists x\, P(x)$ where $P(x)$ is the statement "$x^2 > 10$" and the domain consists of the positive integers not exceeding 4?

$\exists x\, P(x) = x^2 > 10 \quad x = \{0, 1, 2, 3, 4\}$

(T)

$0^2 > 10$

$1^2 > 10$

$2^2 > 10$

$3^2 > 10$

$4^2 > 10$

Existential এর ক্ষেত্রে যে কোনো একটির value true হলেই সেটি true value return করবে।

Home Work:

Let $P(x)$ be the statement "$x = x^2$". If the domain consists of the integers, what are the truth values?

ⓐ $P(0)$

ⓑ $P(1)$

ⓒ $P(2)$

ⓓ $P(-1)$

ⓔ $\exists x P(x)$

ⓕ $\forall x P(x)$

• Tutorial-26

Translating ~~in~~ from english into Logical expression

Using prediction & quantifiers:

① Every student in the class has studied calculus.

$\forall x$ $C(x)$

$\forall x C(x)$

for every student x in the class, x has studied calculus.

$\forall x C(x)$

For every person x, if person x is a student in the class then x has studied calculus.

domain → person

$S(x) = x$ is a student.

$C(x) =$ x has studied calculus.

$\forall x \, (S(x) \rightarrow C(x))$

② Some student in this class has visited mexico.

$\exists$ $M(x) =$ x has visited Mexico.

$\exists x \; M(x)$

There is a student x in the class having the property that x has visited mexico.

$\exists x$ $M(x)$

$\exists_x M(x)$

There is a person x, having properties that x is a student in the class and x has visited Mexico.

$\exists_x (S(x) \wedge M(x))$

③ For every person x, if x is a student in the cla[ss] then x has visited mexico or x has visited cana[da]

$\forall x \; (S(x) \longrightarrow M(x) \vee C(x))$

Using quantifier in system Specifications:

Every mail message larger than one megabyte will be compressed.

$S(m, y)$ = m message larger than y megabyte.

$C(m)$ = m is a message that will be compressed.

$\forall m\, (S(m,1) \rightarrow C(m))$

Tutoria-27

English to logical Expression (part-2)

Logic $P(x)$ be the statement "x can speak Russian" and Let $Q(x)$ be the statement "x knows the computer language C++".

Express each of these in sentences in terms of $P(x)$, $Q(x)$ quantifiers and logical connectives.

Domain → All students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

$\exists x (P(x) \wedge Q(x))$

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(b) There is a student at your school can speak Russian but who does not know C++

$$\underbrace{\text{can speak Russian}}_{P(x)} \quad \wedge \quad \underbrace{\text{who does not know C++}}_{\phi(x)}$$

$$\exists_x (P(x) \wedge \neg \phi(x))$$

(c) No student at your school can $\underbrace{\text{speak Ru}}_{P(x)}$

or $\underbrace{\text{knows C++.}}_{\phi(x)}$

$$\exists_x \neg (P(x) \vee \phi(x))$$

(d) Every student at your either can $\underbrace{\text{speak R}}_{P(x)}$

or $\underbrace{\text{knows C++.}}_{\phi(x)}$

$$\forall_x (P(x) \vee \phi(x))$$

H.W.:

$C(x) \rightarrow$ x has a cat.

$D(x) \rightarrow$ x has a dog.

$F(x) \rightarrow$ x has a Ferret.

Domain $\rightarrow$ All student in your class.

(a) All students in your class has a cat, a dog a

ferret.

ⓑ Some students in your class has a cat and a ferret but not a dog.

ⓒ No students in your class has a cat, a dog and a ferret.

Tutorial-28

Logical Expression to English

C(x) is "x is a comedian".
F(x) is "x is funny".
domain → all people.

ⓐ $\forall x\,(C(x) \rightarrow F(x))$

For all x if x is a comedian then x is a funny person.

All / Every comedian is funny.

ⓑ $\forall x\,(C(x) \wedge F(x))$

For all x, x is a comedian and x is funny.

Every person is a funny comedian.

H.W.

ⓐ $\exists x\,(C(x) \longrightarrow F(x))$

*

ⓑ $\exists x\,((C(x) \land F(x))$

Tutorial-29

Negating Quantified Expression

$\forall x\, P(x)$

$= \neg(\forall x\, P(x))$

$= \exists x\, \neg P(x)$

$\exists x\, P(x)$

$= \neg(\exists x\, P(x))$

$= \forall x\, \neg P(x)$

ⓓ "Every student in your class has taken a course in calculus."

$\forall x\, C(x)$

$\neg(\forall x\, C(x))$

$= \exists x\, \neg C(x)$

There is a student in your class who has not taken a course in calculus.

"There is an honest politician".

$H(x) \rightarrow x$ is honest.

$\exists x\ H(x)$

$= \neg(\exists x\ H(x))$

$= \forall x\ \neg H(x)$

Every politeion is dishonent.

Not all politician are honest.

① Some old dogs can learn new tricks.

domain $\rightarrow$ all dogs.

$T(x) = x$ can learn new tricks.

$\exists x\ T(x)$

$\equiv \neg(\exists x\ T(x))$

$\equiv \forall x\ \neg T(x)$

No old dogs can learn new ~~th~~ tricks.

② No Rabbit knows calculus.

$c(x) =$ x knows Calculus.

$\forall x\ \neg c(x)$

$\equiv \neg(\forall x\ \neg c(x))$

$\forall x =$ Universal.

$\exists x =$ Existential.

$= \exists x \neg\neg e(x)$

$= \exists x \neg(\neg e(x))$

$= \exists x\, e(x)$

There exist a rabbit that knows calculus.

③ Every $\underline{\text{bird can fly}}$.

$F(x) = x$ can fly.

$\forall x\, F(x)$

$\equiv \neg(\forall x\, F(x))$

$\equiv \exists x \neg F(x)$

There exist a bird that can not fly.

H.W. ④ There is no dog that can talk.

Tutorial-30

$\forall x \exists y\, P(x,y)$

একটি quantifier এর ভিতরে অন্য একটি quantifier থাকলে তাকে nested quantifier বলে।

☐ Translate the following statement into English

$$\forall x \forall y \; ((x>0) \wedge (y<0) \rightarrow (xy<0)).$$

Domain: real numbers.

→ For every real numbers x and y, if x is positive and y is Negative. then xy is Negative.

The product of a positive real number and a negative real number is always a negative real number.

☐ Translate the following statement into a logical expression: "The sum of two positive integers is always positive".

Ans:

$$\forall x \forall y \left((x>0) \wedge (y>0) \rightarrow x+y>0 \right)$$

Tutorial-31
Nested Quantifier (Part-2)

Translating Nested quantifiers Into English:

$\forall x (C(x) \lor \exists y (C(y) \land F(x,y)))$

$C(x)$ = "x has a computer".

$F(x,y)$ = "x and y are students"

domain → all students.

For every student x in your school, x has a compu
or there is a student y that is y has a computer a
x and y are friends.

every students in your school has a computer or has
friend who has a computer.

⇒ Use ~~of~~ quantifier to express each of these stater

$\boxed{\text{}}$ Let. $L(x,y)$ = "x loves y"

domain → all people

a) Everybody loves jerry.

$\forall x\, L(x, \text{Jerry})$

(b) Everybody Loves somebody.

$\forall x \exists y \, L(x,y)$

(c) $\exists y \forall x \, L(x,y)$

There is somebody whom everybody love.

(d) No body loves everybody.

(e) There is somebody whom Lydia does not

Tutorial-32

Rules of Inference (P-1)

অনেক গুলো propositional Logic কে একসাথে Arguments বলা হয়, এবং সেই Arguments এর মধ্যে একটি Decision থাকবে এবং একটি conclusion থাকবে।

"If you have a current password, then you can log on to the network" $p \rightarrow q$

"You have a current password"

p

Therefore,

"you can log onto the network"

$$\left.\begin{array}{l} p \to q \\ p \end{array}\right\} \text{premise}$$

$q \longrightarrow$ conclusion

$(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \longrightarrow q$

দ্বারা Argument এর value true ~~আসলে~~ ~~অ~~ tautology হলে তাকে valid argument বলে।

premise এর conclusion সহ ~~tautology~~ হলে ~~তাকে~~ vali... argument বলে।

$$\begin{array}{l} T \quad T \to T \\ p \to q \\ \therefore p \to T \\ \hline q \to T \end{array}$$

valid

$$\begin{array}{l} F \quad T \\ p \to q \to T \\ q \longrightarrow T \\ \hline p \longrightarrow F \end{array}$$

Not Va...

Rules of Inference (part-2)

Tutorial-33

① Modus Ponens:

Example:

" If it is raining, then I will study discrete math."

$\underbrace{\text{If it is raining}}_{p}$, then I $\underbrace{\text{will study discrete math}}_{q}$

" If it is raining"
$\underbrace{\text{If it is raining}}_{p}$

Therefore, I will study dis. Math.

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

$((p \to q) \land p) \to q$

p	q	$p \to q$	$(p \to q) \land p$	$((p \to q) \land p) \to p$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

Arguments true বা tautology return করলে argu-ments টা valid হবে।