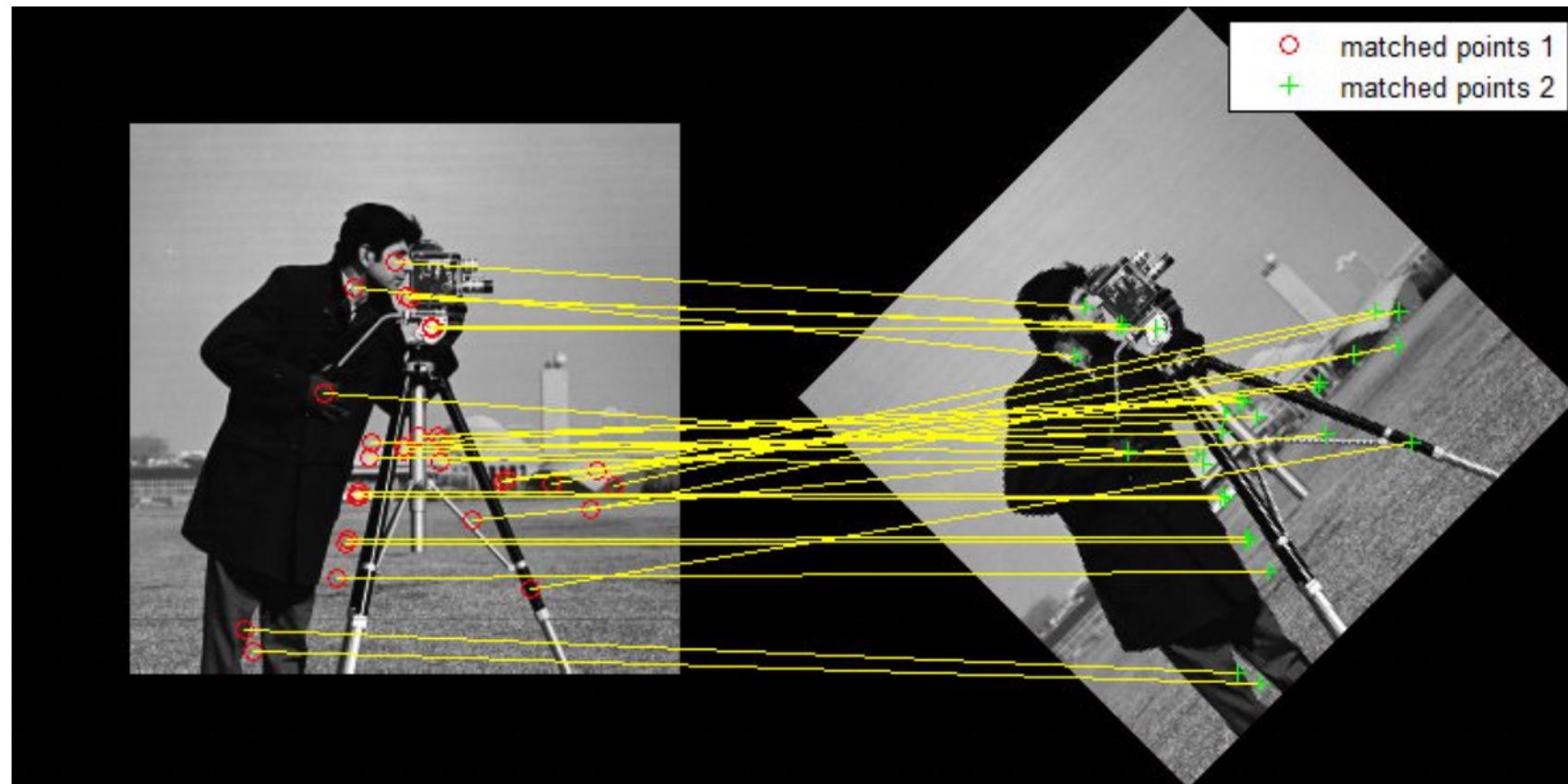


# 3. Feature Extraction and Matching

Corner Detection, SIFT

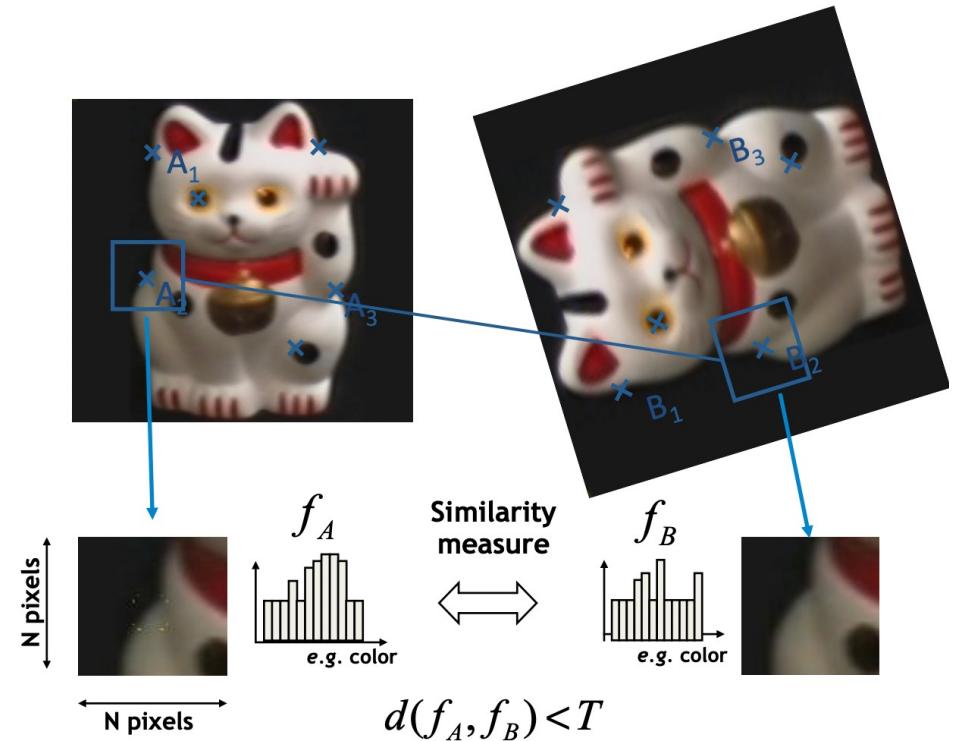
# Why do we need features?

Finding same things across images: for reconstruction/tracking/generating panoramas ...

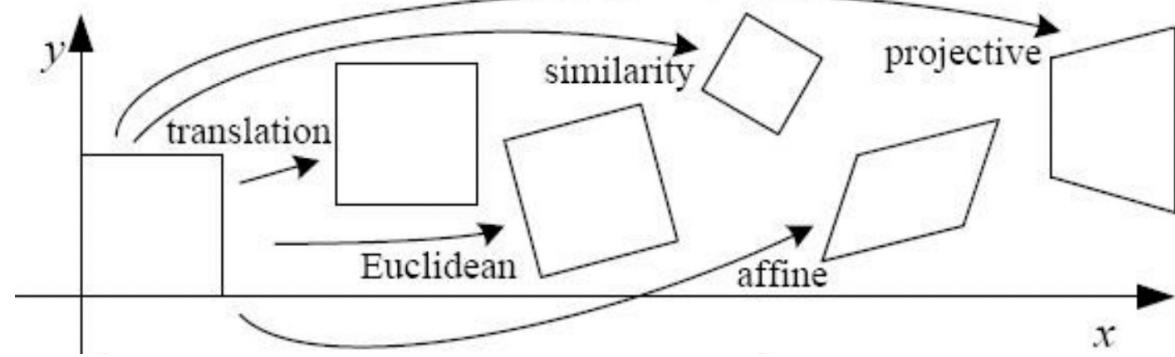


# What to do with features?

1. Use a detector to detect same scene points independently in both images and find a set of distinctive keypoints
2. Extract and normalize the region content
3. Define a region around each keypoint
4. Compute local descriptor from normalized region
5. Find correspondences by matching local descriptors



# What makes a good feature?



# What makes a good feature?

Region extraction needs to be repeatable and accurate

Invariant to translation, rotation, scale changes

Robust or covariant to out-of-plane (affine) transformations

Robust to lighting variations, noise, blur, quantization

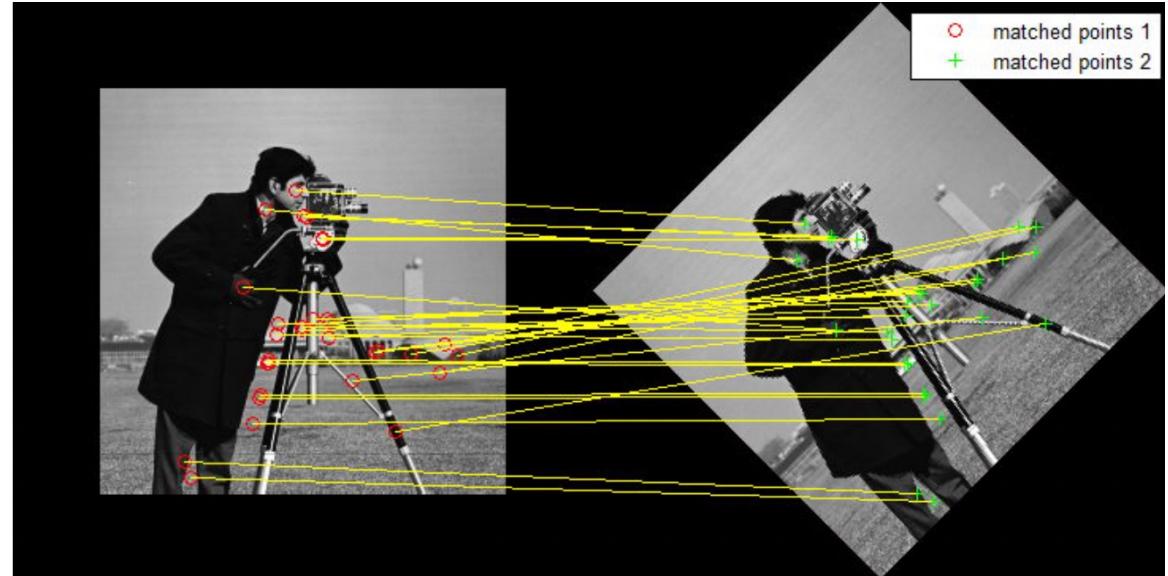
- Locality: Features are local, therefore robust to occlusion and clutter
- Quantity: We need sufficient regions to cover the object
- Distinctiveness: The regions should contain “interesting” structure
- Efficiency: Close to real-time performance

# What makes a good feature?

## Pixel Intensity

Pros: can get many, slight invariance to everting

Cons: Non-immune to photometry changes, noise, any strong geometric variation (even scale or rotation)

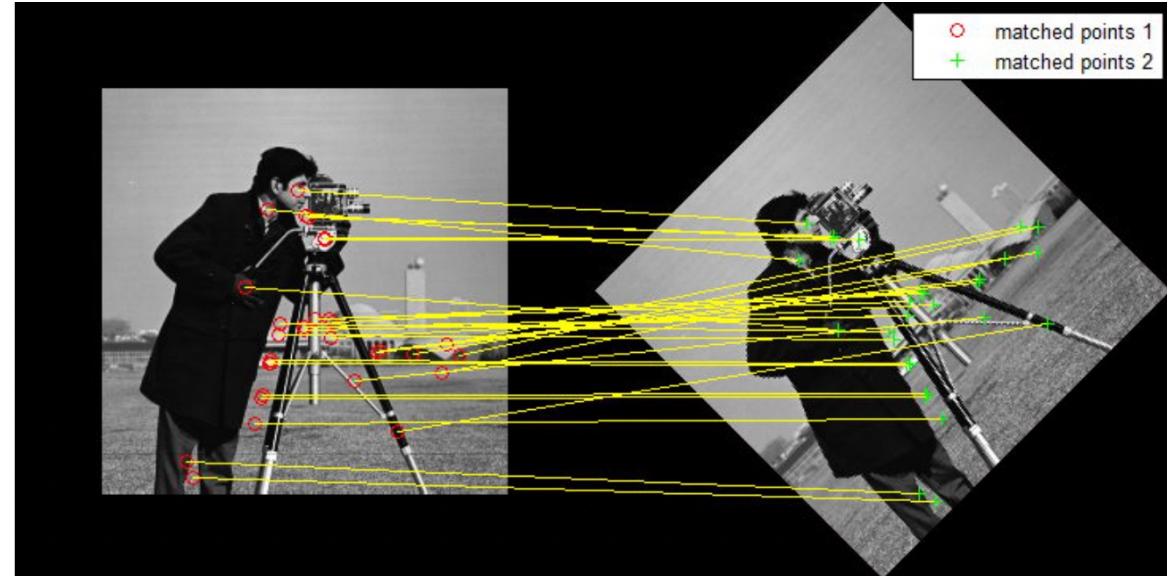


# What makes a good feature?

## Geometry

Pros: Can handle photometric changes, Invariances to some transformations, resistance to noise

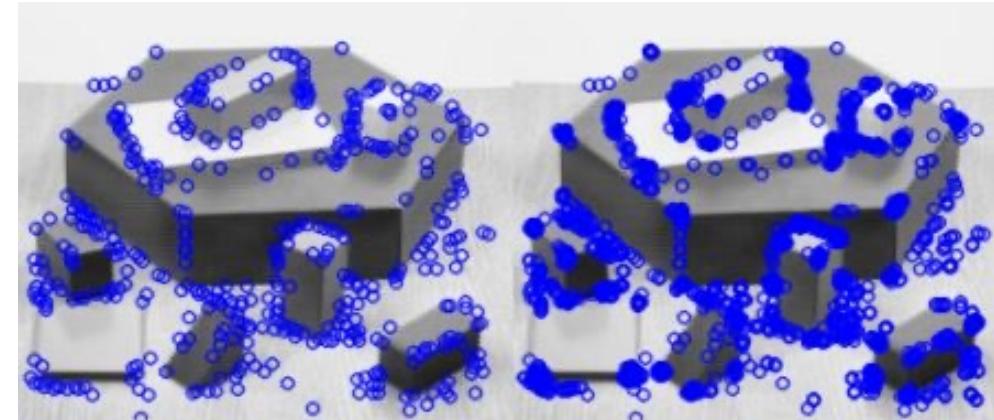
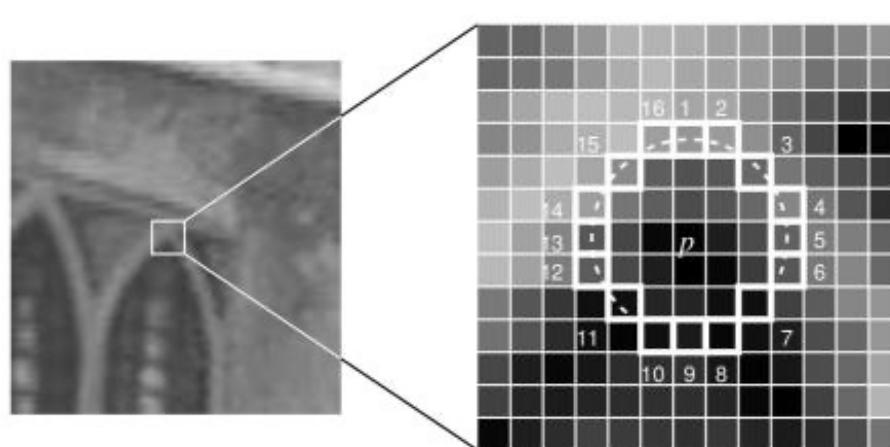
Cons: Computationally expensive



# Features from Accelerated Segment Test (FAST)

Look for a 3 neighborhood circle (Bresenhem/mid point circle) around a point = 16 points  
If a minimum of 12 points have intensity higher or smaller than the point, it's a good feature

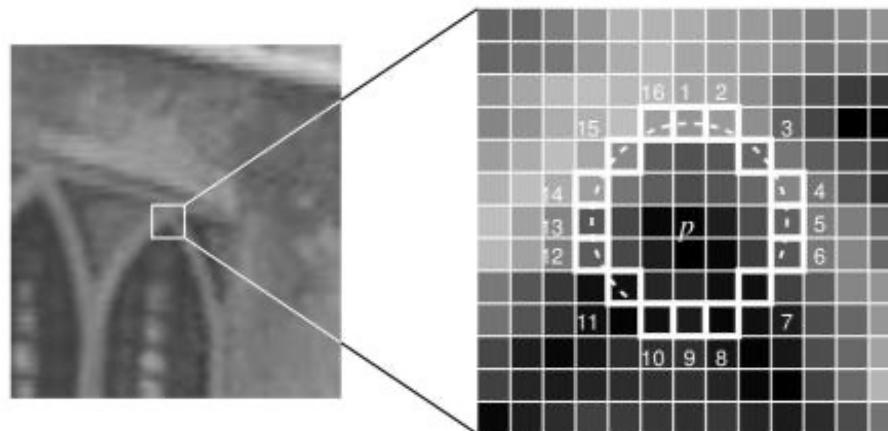
Good for detecting corners



# Features from Accelerated Segment Test (FAST)

Problems:

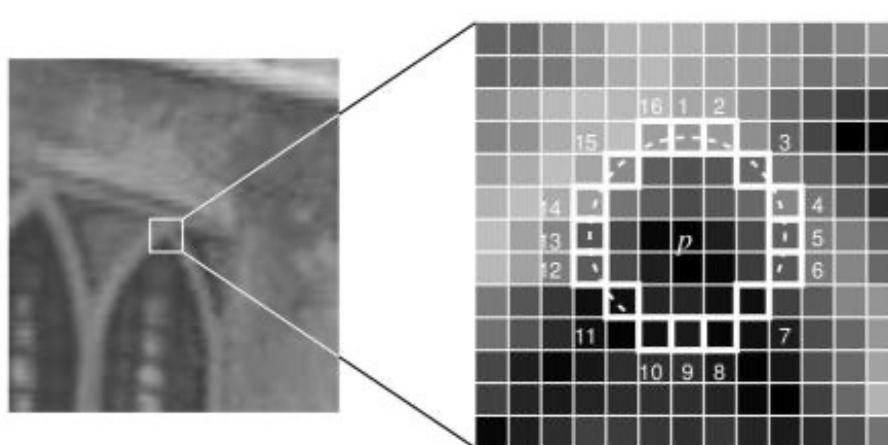
1. Slow (make fast by looking at pixels 1,5,9,13 or use machine learning)
2. Important to decide thresholds
3. Shadows/ small textual change can impact negatively



# Features from Accelerated Segment Test (FAST)

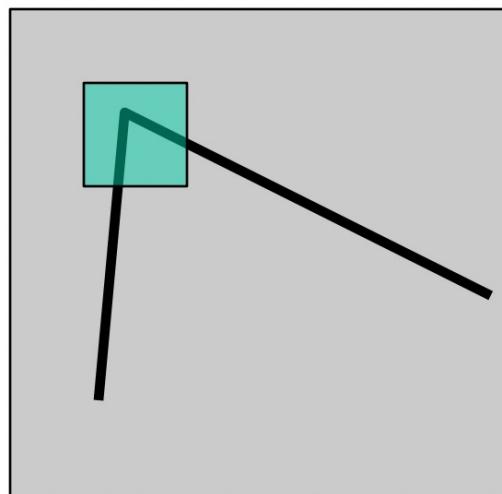
Problems:

1. Slow (make fast by looking at pixels 1,5,9,13 or use machine learning)
2. Important to decide thresholds
3. Shadows/ small textual change can impact negatively
4. Photometric variations will impact it negatively



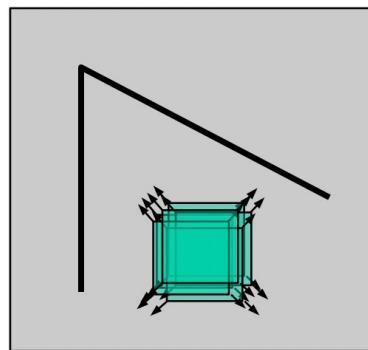
# Harris Corners

- Localise the point through a small window
- Define precise location by shifting a window in any direction such that large change in pixels intensities are observed

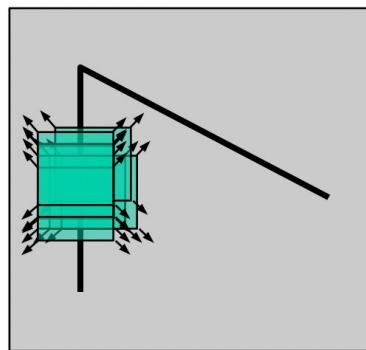


# Harris Corners

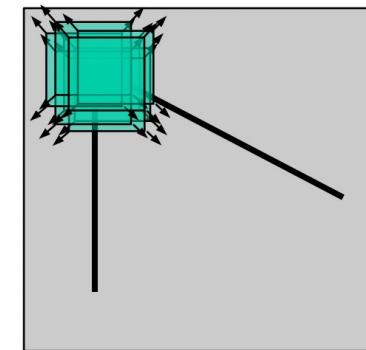
- Localise the point through a small window
- Define precise location by shifting a window in any direction such that large change in pixels intensities are observed



“flat” region:  
no change in all  
directions



“edge”:  
no change along  
the edge direction



“corner”:  
significant change  
in all directions

# Harris Corners

Window-averaged squared change of intensity induced by shifting the image data by  $[u,v]$ :

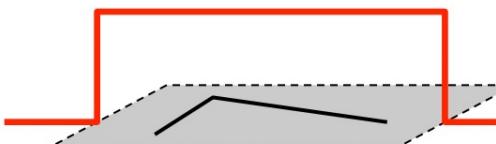
$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Window function

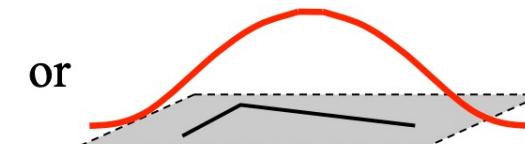
Shifted intensity

Intensity

Window function  $W(x,y) =$



1 in window, 0 outside



Gaussian

# Harris Corners

$$\begin{aligned} E(u, v) &\approx \sum_{x,y} w(x, y)[I(x, y) + uI_x + vI_y - I(x, y)]^2 \\ &= \sum_{x,y} w(x, y)[uI_x + vI_y]^2 \\ &= (u - v) \sum_{x,y} w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \\ &= \mathbf{M} \text{ (structure tensor)} \\ &\text{Its eigenanalysis reveals interesting things} \end{aligned}$$

# Harris Corners

Intensity change in shifting window: eigenvalue analysis

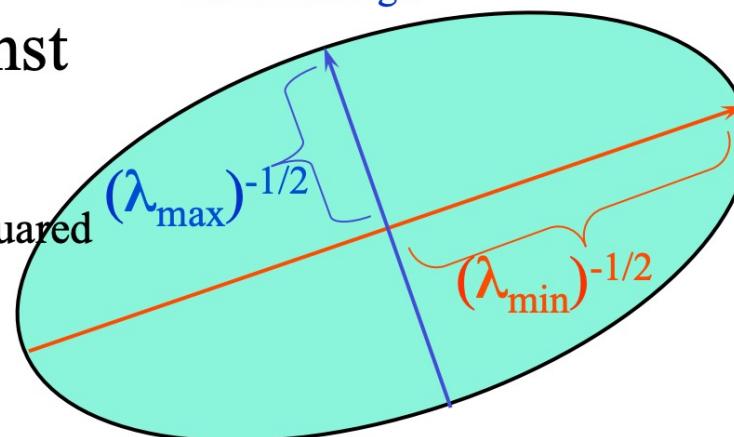
$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } M$$

Ellipse  $E(u, v) = \text{const}$

Iso-contour of the squared error,  $E(u, v)$

direction of the fastest change

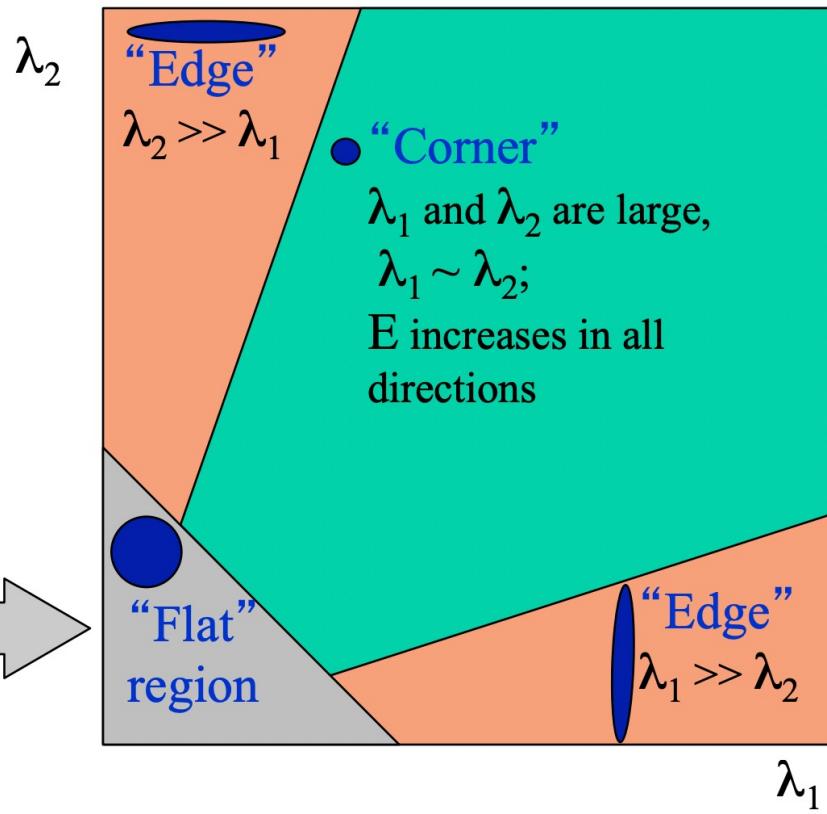
direction of the slowest change



# Harris Corners

Classification of image points using eigenvalues of M:

$\lambda_1$  and  $\lambda_2$  are small;  
E is almost constant  
in all directions



Instead of computing eigenvalues:  
 $R = \det(M) - a \text{ trace}(M)^2$

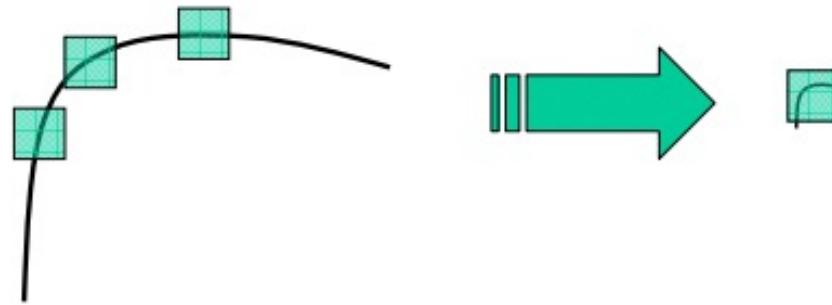
$$a = [0.04, 0.06]$$

Corner:  $R > 0$

Edge:  $R < 0$

Flat:  $\text{abs}(R)$  is small

# Harris Corners



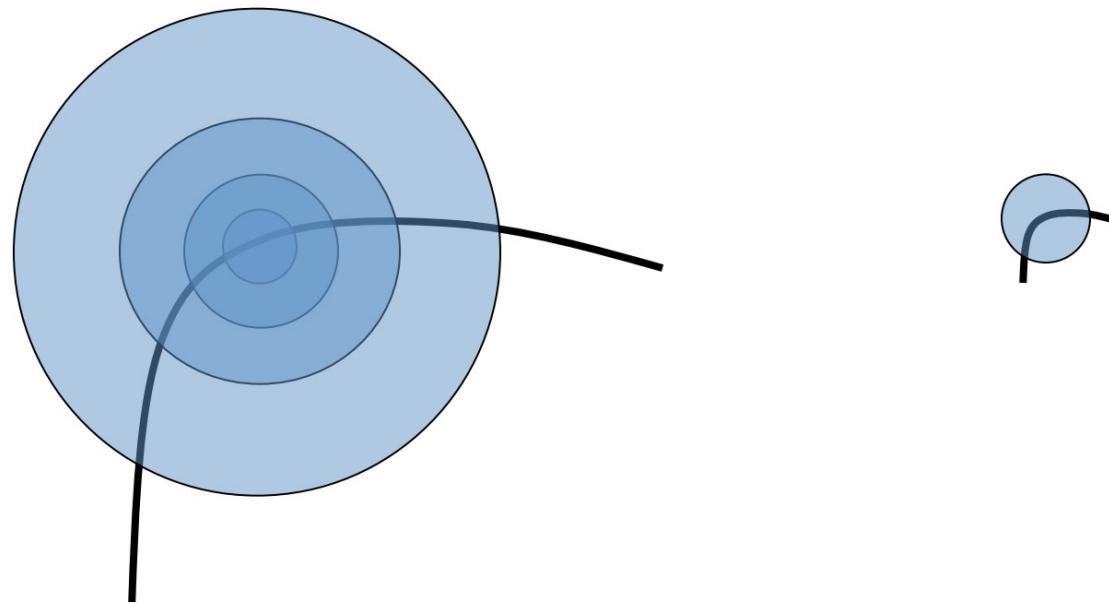
All points will be  
classified as **edges**

**Corner !**

Invariant to rotation, translation, photometric intensity changes.  
Scale? No.

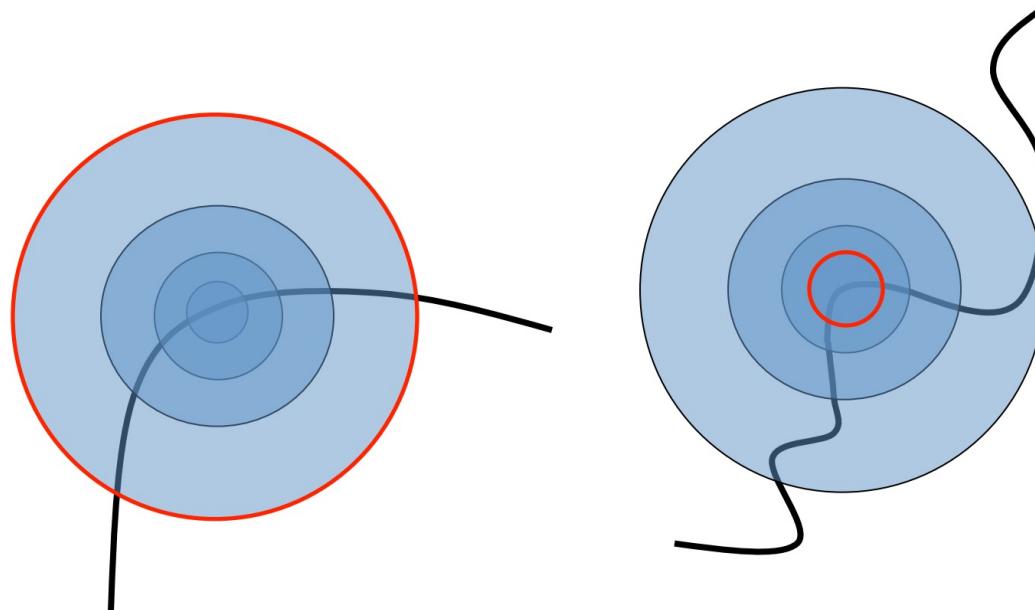
# Scale Invariant Detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



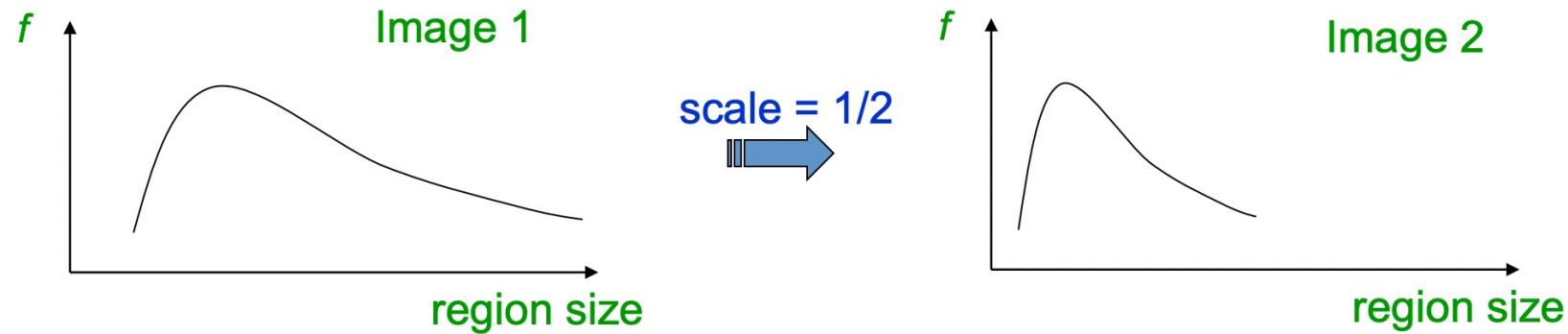
# Scale Invariant Detection

- Consider regions (e.g. circles) of different sizes around a point
  - Regions of corresponding sizes will look the same in both images
- How to choose these regions automatically?



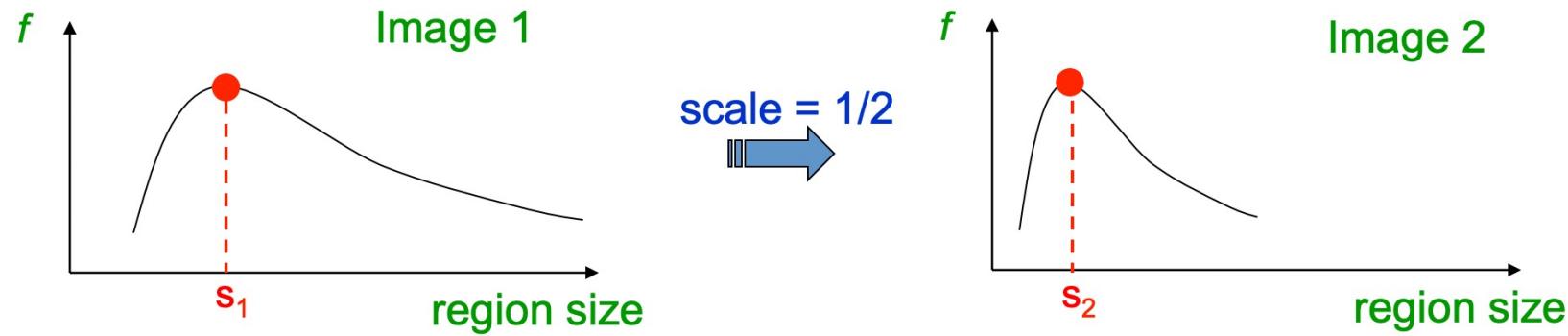
# Scale Invariant Detection

- Design a function on the region (circle), which is “scale invariant” (the same for corresponding regions, even if they are at different scales)
- Image intensity (invariant to scale, of course): For a point in one image, we can consider it as a function of region size (circle radius)



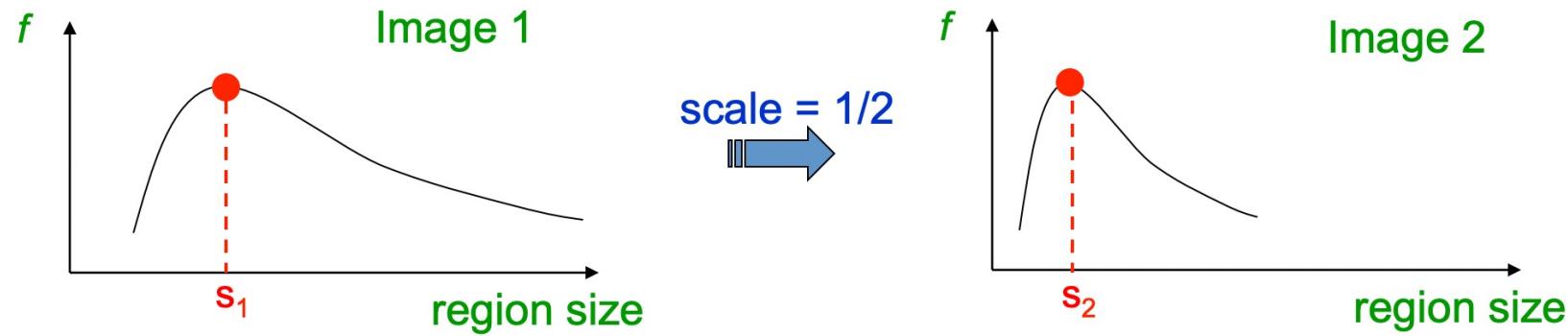
# Scale Invariant Detection

- Take a local maximum of image intensity
- Region size, for which the maximum is achieved, is normally covariant with image scale
- Find these regions independently in each image



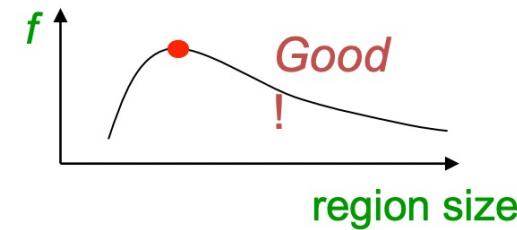
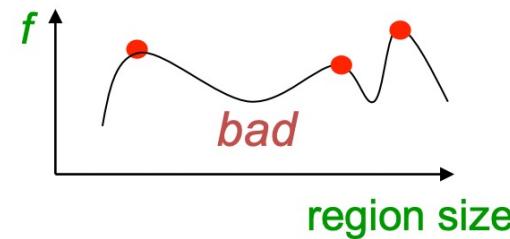
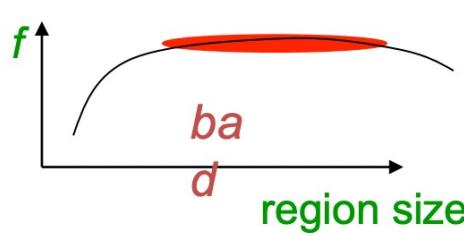
# Scale Invariant Detection

- Take a local maximum of image intensity
- Region size, for which the maximum is achieved, is normally covariant with image scale
- Find these regions independently in each image



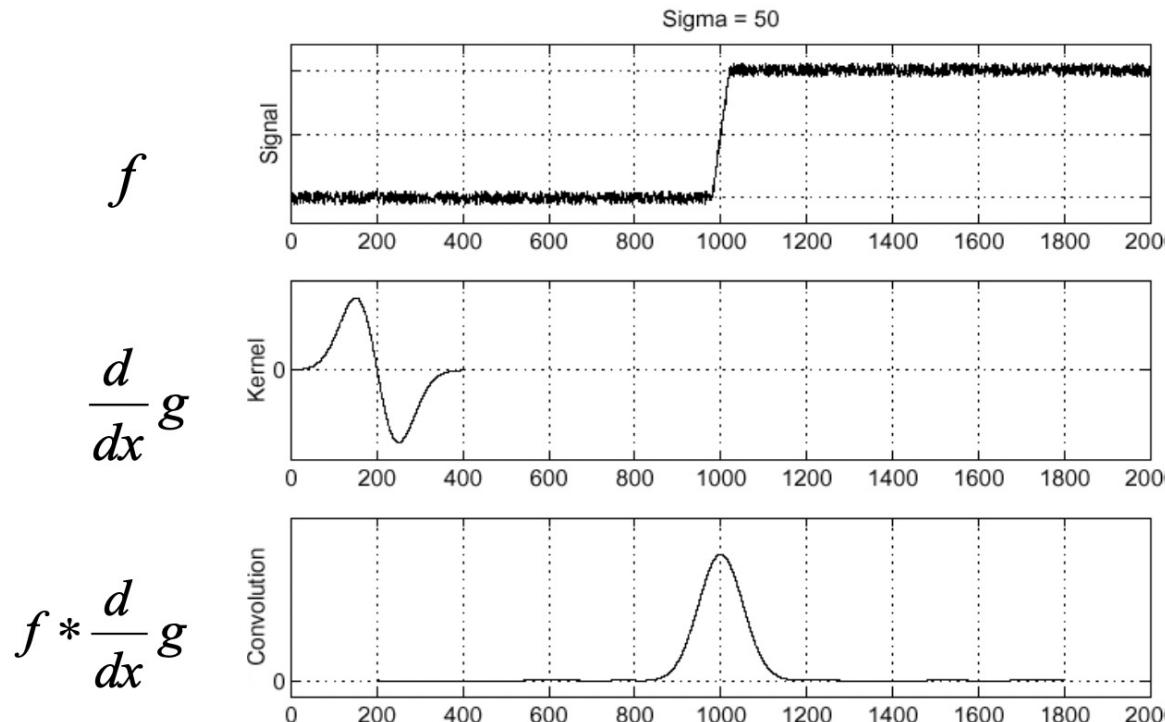
# Scale Invariant Detection

- A good function should have 1 stable peak
- Should react to sharp intensity changes



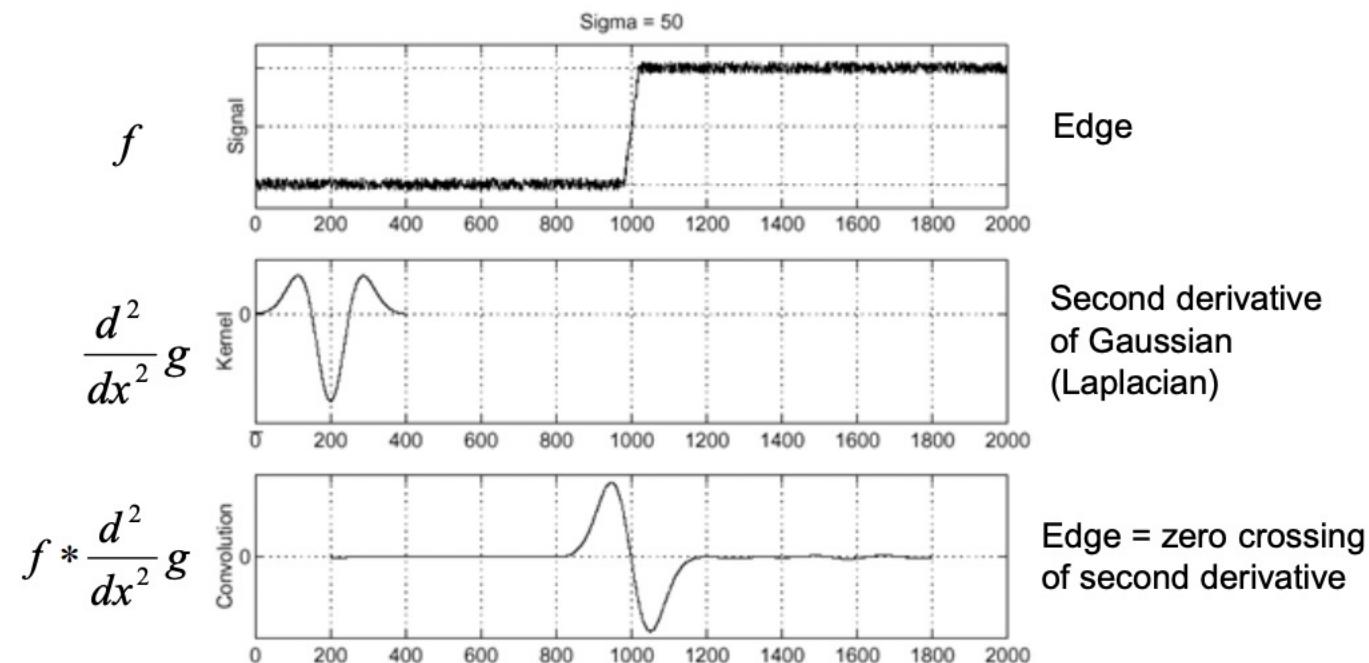
# Scale Invariant Detection

- Remember edge detection



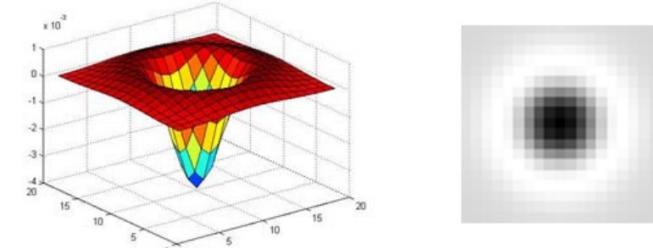
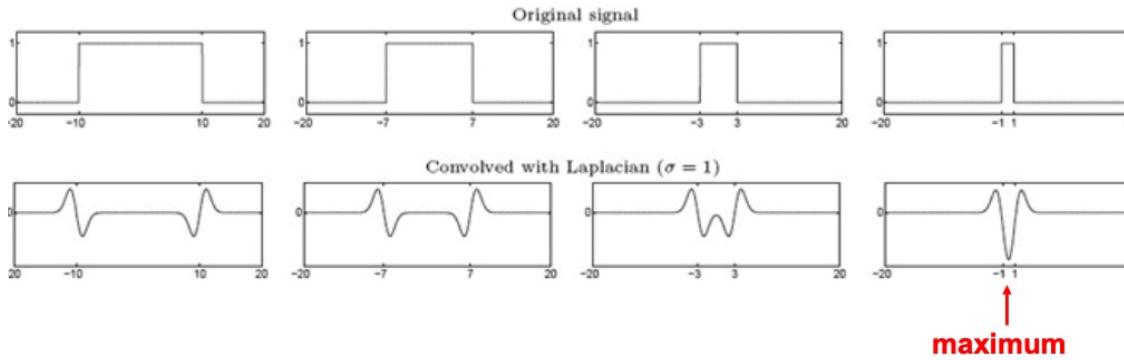
# Scale Invariant Detection

- Remember edge detection



# Scale Invariant Detection

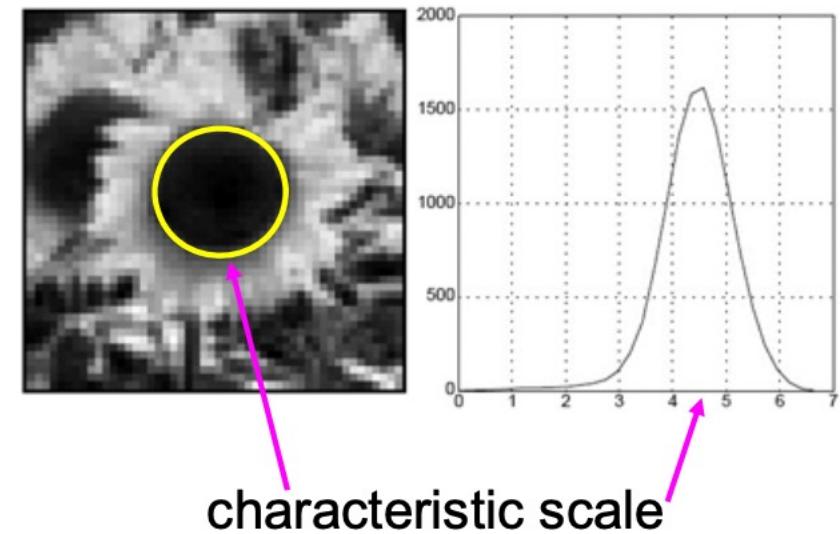
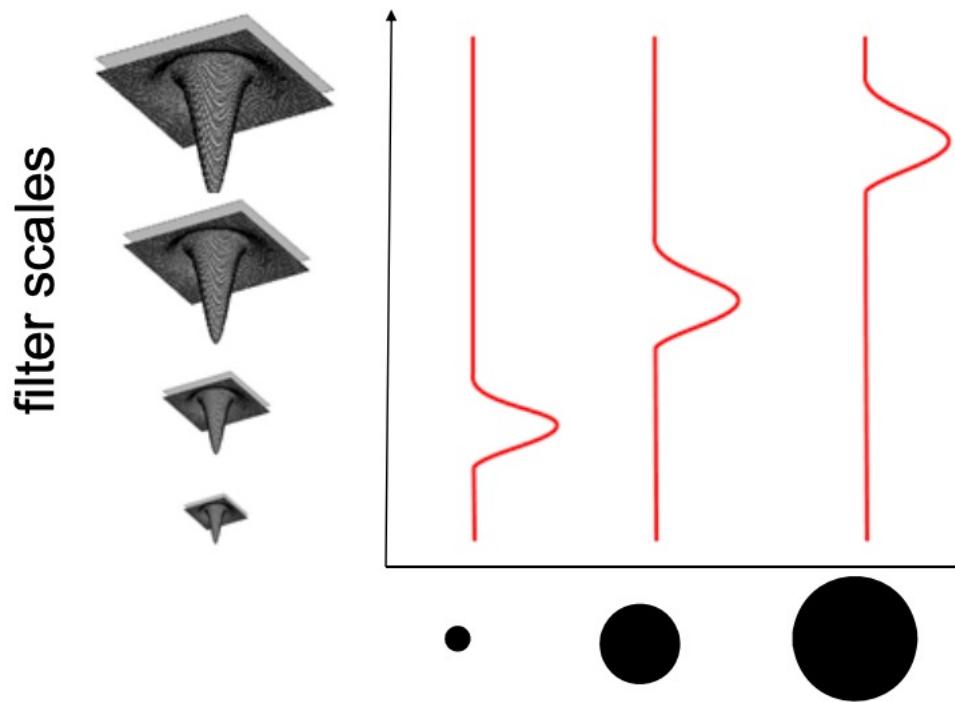
- Laplacians are good candidate, remember edge detection



$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

# Scale Invariant Detection

- Characteristic scale: scale that produces a peak



# Scale Invariant Detection

- Interest points are local maxima in both position and scale

- Functions for determining scale  $f = \text{Kernel} * \text{Image}$

Kernels:

$$L = \sigma^2 (G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma))$$

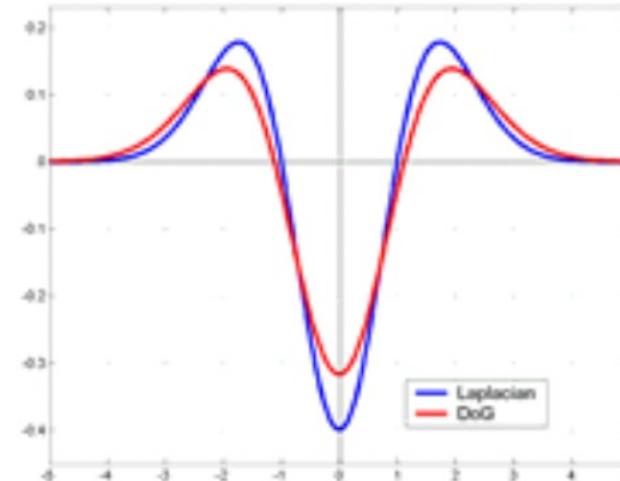
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

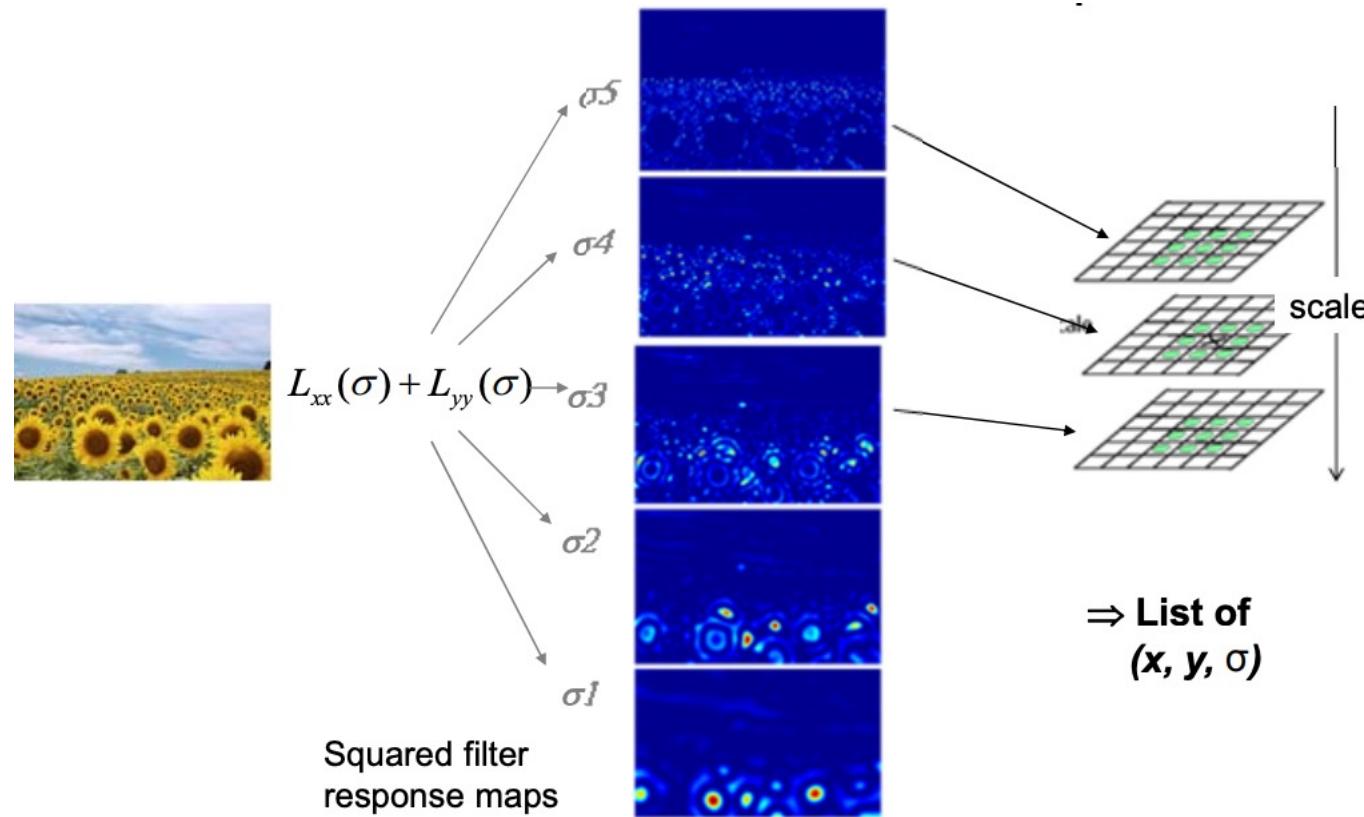
$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Note: both kernels are invariant to  
scale and rotation

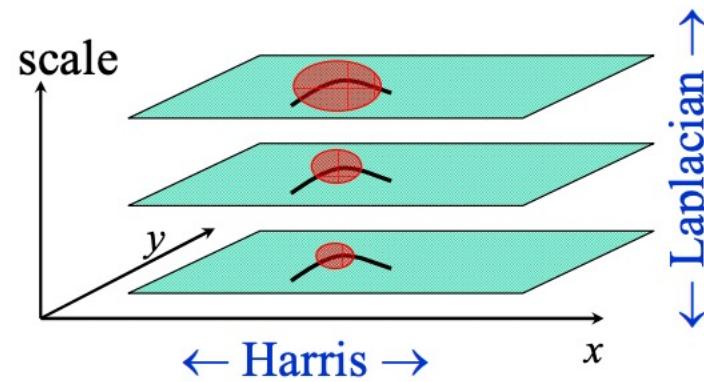
# Scale Invariant Detection

- Interest points are local maximas in both position and scale

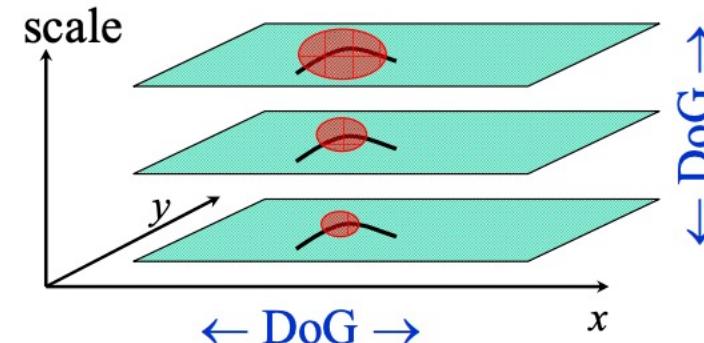


# Scale Invariant Detection

- **Harris-Laplacian<sup>1</sup>**  
*Find local maximum of:*
  - Harris corner detector in space (image coordinates)
  - Laplacian in scale



- 
- **SIFT (Lowe)<sup>2</sup>**  
*Find local maximum of:*
    - Difference of Gaussians in space and scale



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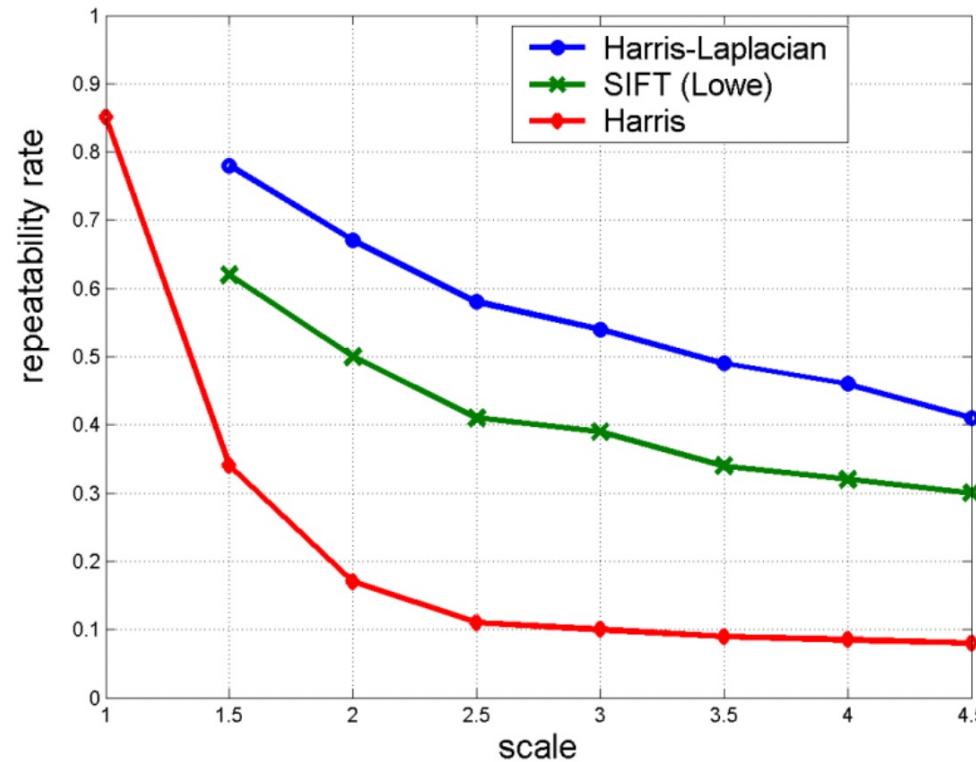
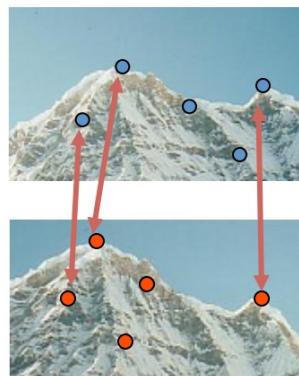
<sup>1</sup> K.Mikolajczyk, C.Schmid. “Indexing Based on Scale Invariant Interest Points”. ICCV 2001

<sup>2</sup> D.Lowe. “Distinctive Image Features from Scale-Invariant Keypoints”. Accepted to IJCV 2004

# Scale Invariant Detection

- Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:  
$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$

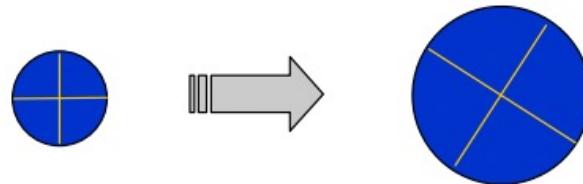


# Scale Invariant Detection

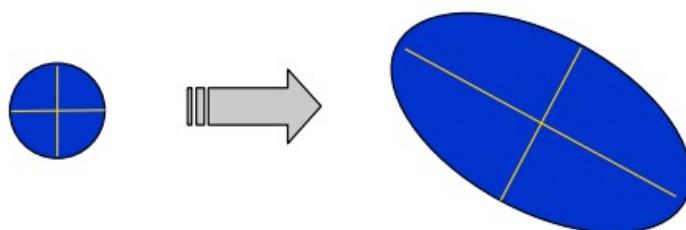
- Given: two images of the same scene with a large scale difference between them
- Goal: find the same interest points independently in each image
- Solution: search for maxima of suitable functions in scale and in space (over the image)

# Affine Invariant Detection

- **Above we considered:**  
**Similarity transform (rotation + uniform scale)**

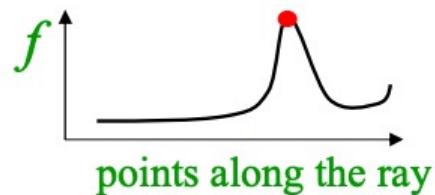


- Now we go on to:  
**Affine transform (rotation + non-uniform scale)**



# Affine Invariant Detection

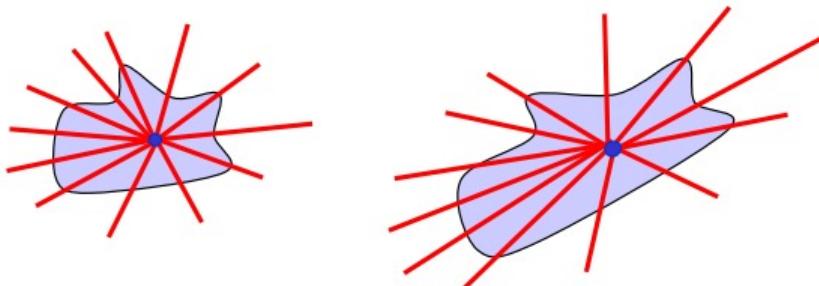
- Take a local intensity extremum as initial point
- Go along every ray starting from this point and stop when extremum of function  $f$  is reached



$$f(t) = \frac{|I(t) - I_0|}{\frac{1}{t} \int_0^t |I(t) - I_0| dt}$$

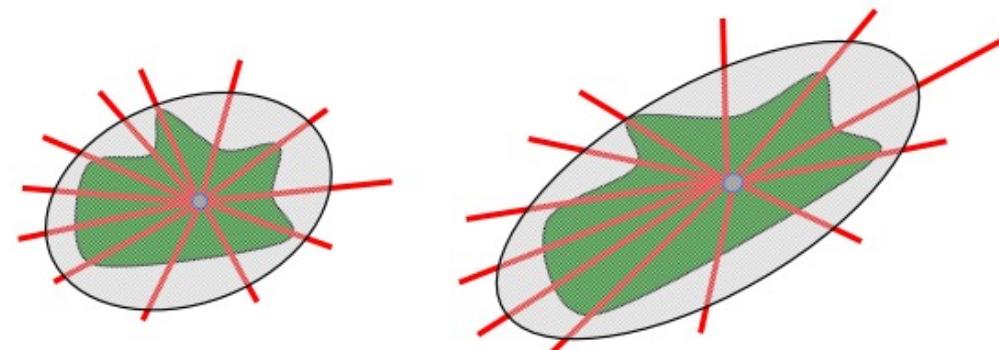
- We will obtain approximately corresponding regions

Remark: we search for scale in every direction



# Affine Invariant Detection

- **Algorithm summary (detection of affine invariant region):**
  - Start from a *local intensity extremum* point
  - Go in *every direction* until the point of extremum of some function  $f$
  - Curve connecting the points is the region boundary
  - Compute *geometric moments* of orders up to 2 for this region
  - Replace the region with *ellipse*



# Affine Invariant Detection

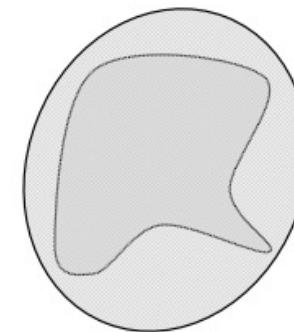
- The regions found may not exactly correspond, so we approximate them with ellipses
- Geometric Moments:

$$m_{pq} = \int_{\mathbb{R}^2} x^p y^q f(x, y) dx dy$$

Fact: moments  $m_{pq}$  uniquely determine the function  $f$

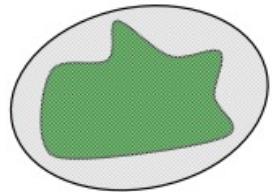
Taking  $f$  to be the characteristic function of a region (1 inside, 0 outside), moments of orders up to 2 allow to approximate the region by an ellipse

This ellipse will have the same moments of orders up to 2 as the original region

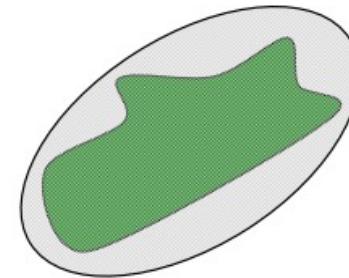


# Affine Invariant Detection

- Covariance matrix of region points defines an ellipse:



$$q = Ap$$

$$p^T \Sigma_1^{-1} p = 1$$

$$q^T \Sigma_2^{-1} q = 1$$

$$\Sigma_1 = \langle pp^T \rangle_{\text{region 1}}$$

$$\Sigma_2 = \langle qq^T \rangle_{\text{region 2}}$$

( $p = [x, y]^T$  is relative  
to the center of mass)

$$\Sigma_2 = A \Sigma_1 A^T$$

Ellipses, computed for corresponding  
regions, also correspond!

# Affine Invariant Detection

- Under affine transformation, we do not know in advance shapes of the corresponding regions
- Ellipse given by geometric covariance matrix of a region robustly approximates this region
- For corresponding regions ellipses also correspond.

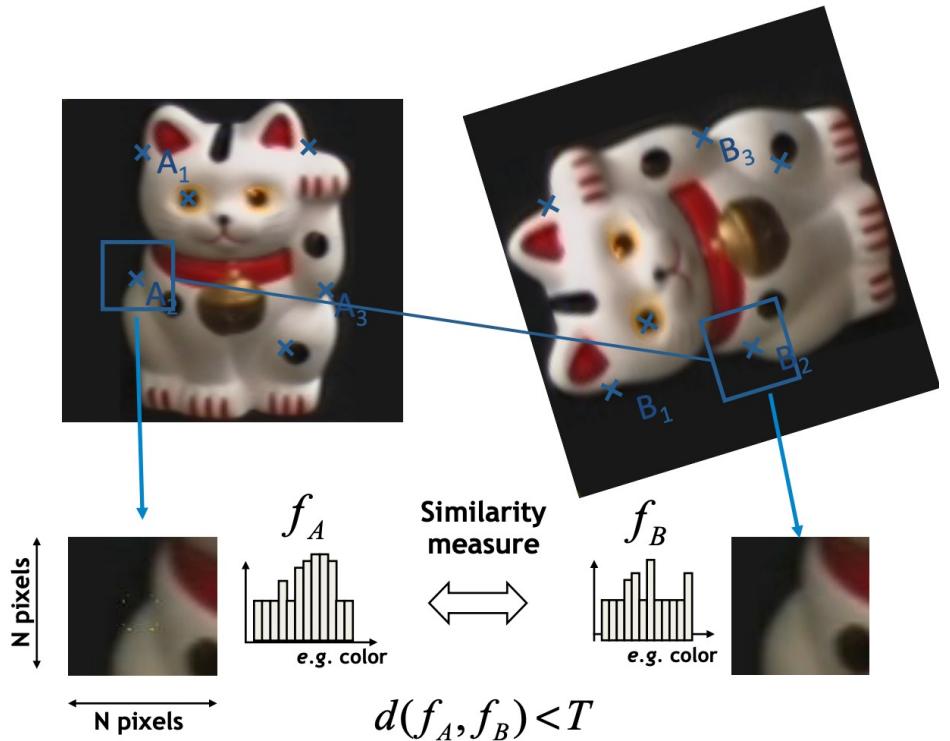
## Methods:

1. Search for extremum along rays [Tuytelaars, Van Gool]:
2. Maximally Stable Extremal Regions [Matas et.al.]

# What to do with features?

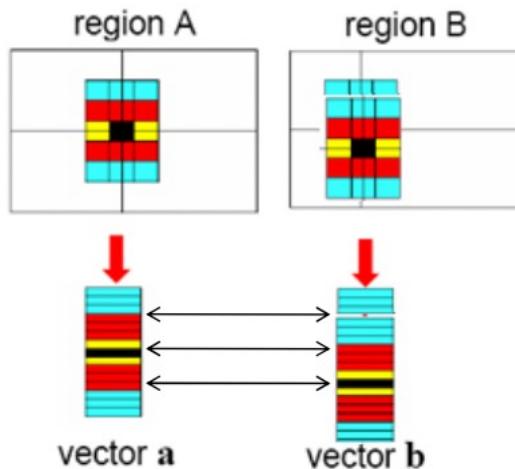
1. Use a detector to detect same scene points independently in both images and find a set of distinctive keypoints
2. Extract and normalize the region content
3. Define a region around each keypoint
4. Compute local descriptor from normalized region
5. Find correspondences by matching local descriptors

Once the detector has been decided, we need to look for a descriptor, so that the detected points can be represented distinctively and matched properly



# Patches as Descriptor

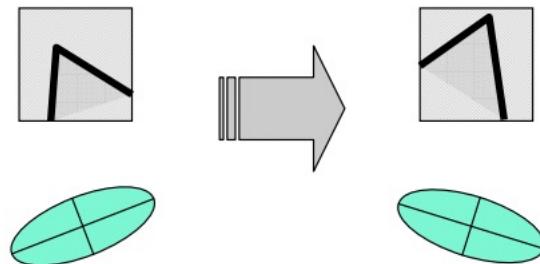
- The simplest way to describe the neighborhood around an interest point is to write down the list of intensities to form a feature vector. But this is very sensitive to even small shifts, rotations.



# Descriptors Invariant to Rotation

- **Harris corner response measure:**  
depends only on the eigenvalues of the matrix  $M$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



- **Image moments in polar coordinates**

$$m_{kl} = \iint r^k e^{-i\theta l} I(r, \theta) dr d\theta$$

Rotation in polar coordinates is translation of the angle:

$$\theta \rightarrow \theta + \theta_0$$

This transformation changes only the phase of the moments, but not its magnitude

Rotation invariant descriptor consists  
of magnitudes of moments:

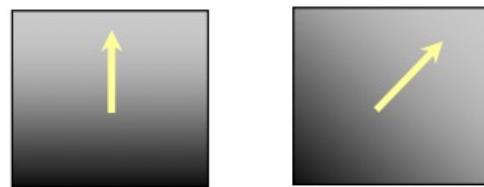
$$|m_{kl}|$$

Matching is done by comparing vectors  $[|m_{kl}|]_{k,l}$

# Descriptors Invariant to Rotation

- **Find local orientation**

Dominant direction of gradient



- Compute image derivatives relative to this orientation

# Descriptors Invariant to Scale

- **Use the scale determined by detector to compute descriptor in a normalized frame**

For example:

- moments integrated over an adapted window
- derivatives adapted to scale:  $sI_x$

# Descriptors Invariant to Affine transformation

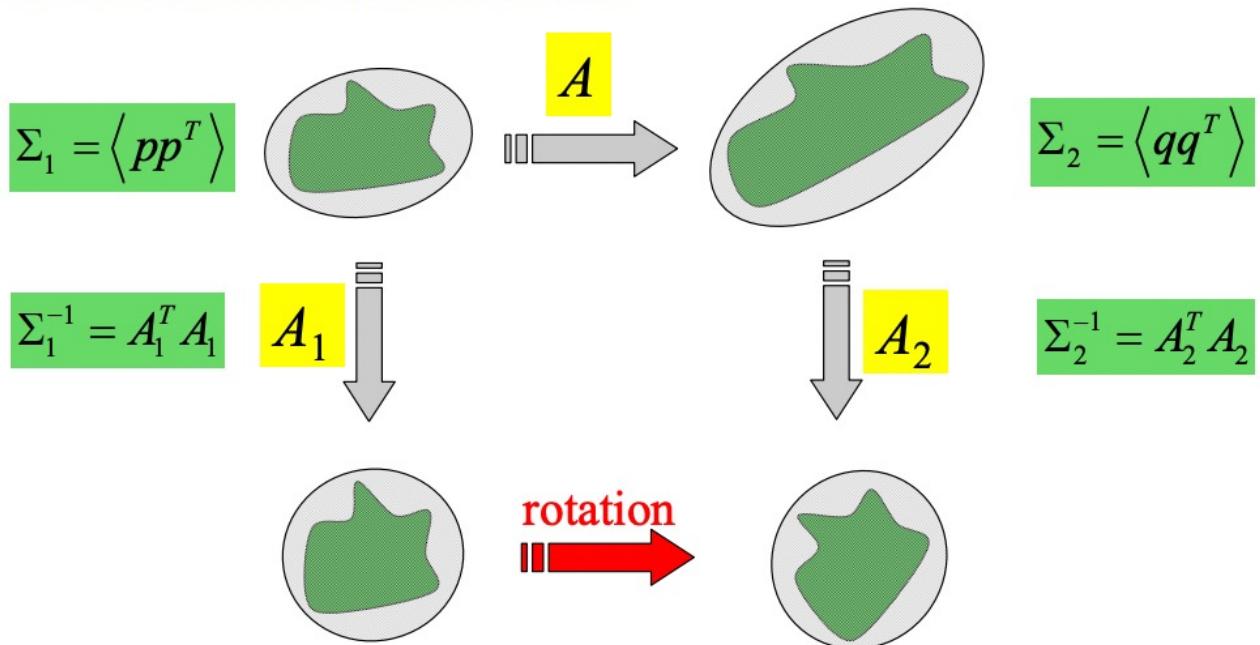
- **Affine invariant color moments**

$$m_{pq}^{abc} = \int_{region} x^p y^q R^a(x, y) G^b(x, y) B^c(x, y) dx dy$$

Different combinations of these moments are fully affine invariant

Also invariant to affine transformation of intensity  $I \rightarrow a I + b$

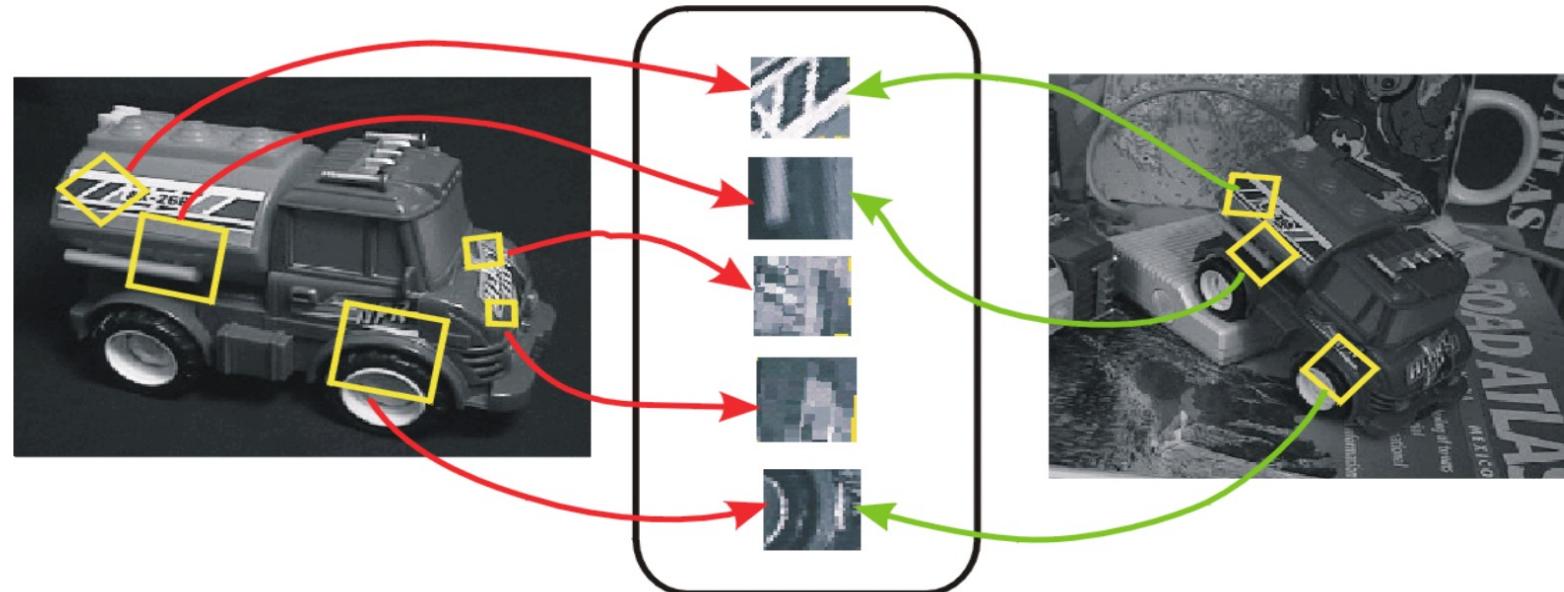
- **Find affine normalized frame**



- Compute rotational invariant descriptor in this normalized frame

# Scale Invariant Feature Transform- SIFT Descriptor

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



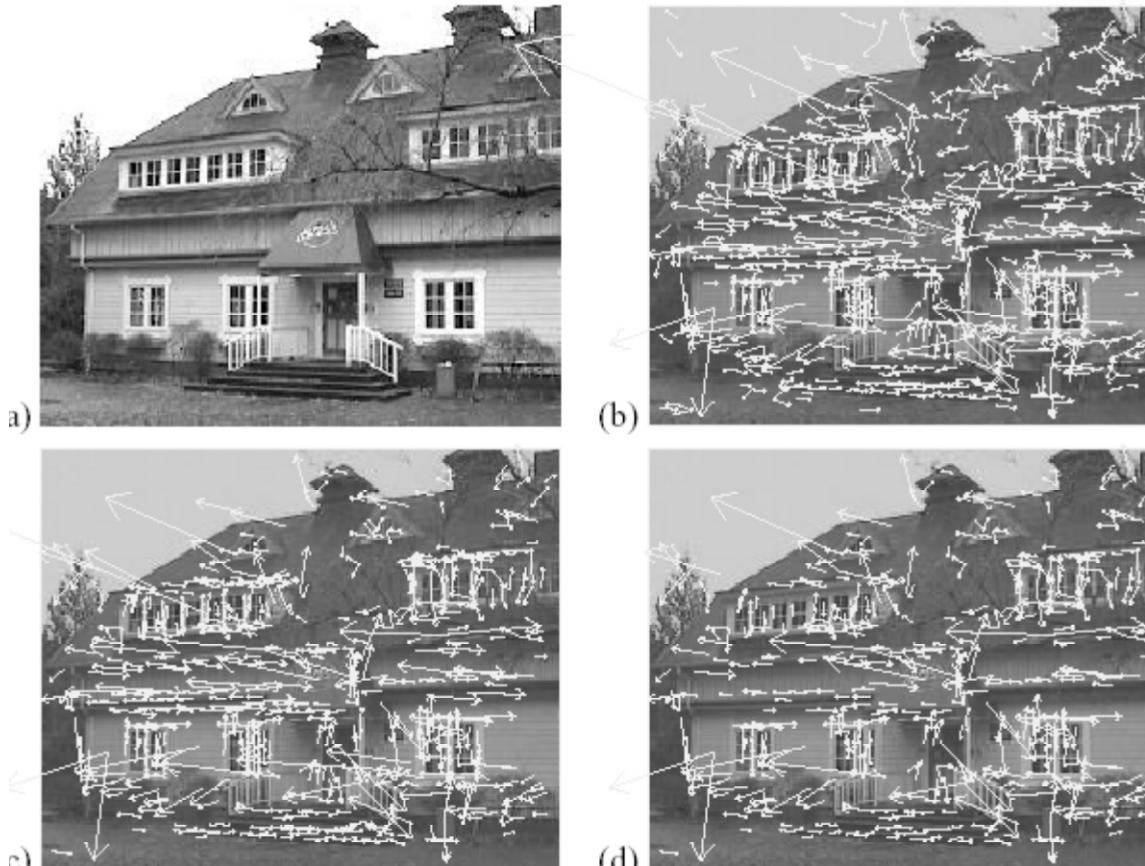
# Advantages of Invariant Local Features

- Locality: features are local, so robust to occlusion and clutter (no prior segmentation)
- Distinctiveness: individual features can be matched to a large database of objects
- Quantity: many features can be generated for even small objects
- Efficiency: close to real-time performance
- Extensibility: can easily be extended to wide range of differing feature types, with each adding robustness

# Scale Invariance

- Requires a method to repeatably select points in location and scale
- The only reasonable scale-space kernel is a Gaussian (Koenderink, 1984; Lindeberg, 1994)
- An efficient choice is to detect peaks in the difference of Gaussian pyramid (Burt & Adelson, 1983; Crowley & Parker, 1984 – but examining more scales)
- Difference-of-Gaussian with constant ratio of scales is a close approximation to Lindeberg's scale-normalized Laplacian (can be shown from the heat diffusion equation)

# Keypoint Detection



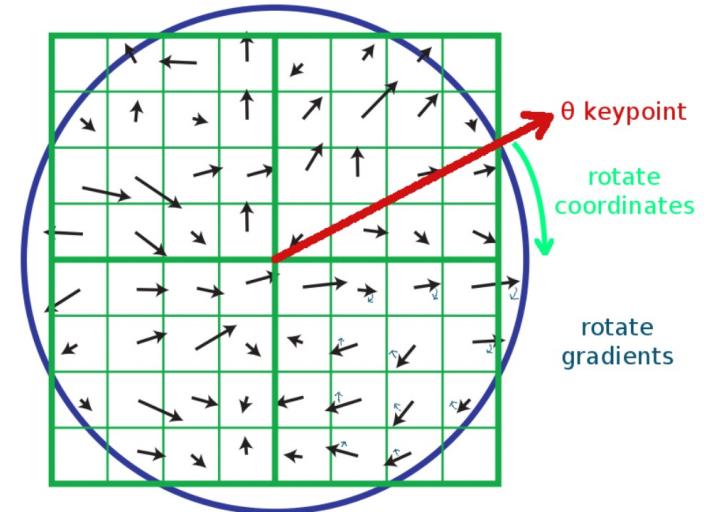
- (a)** 233x189 image
- (b)** 832 DOG extrema
- (c)** 729 left after peak value threshold
- (d)** 536 left after testing ratio of principle curvatures

Vectors indicate scale, orientation and location.

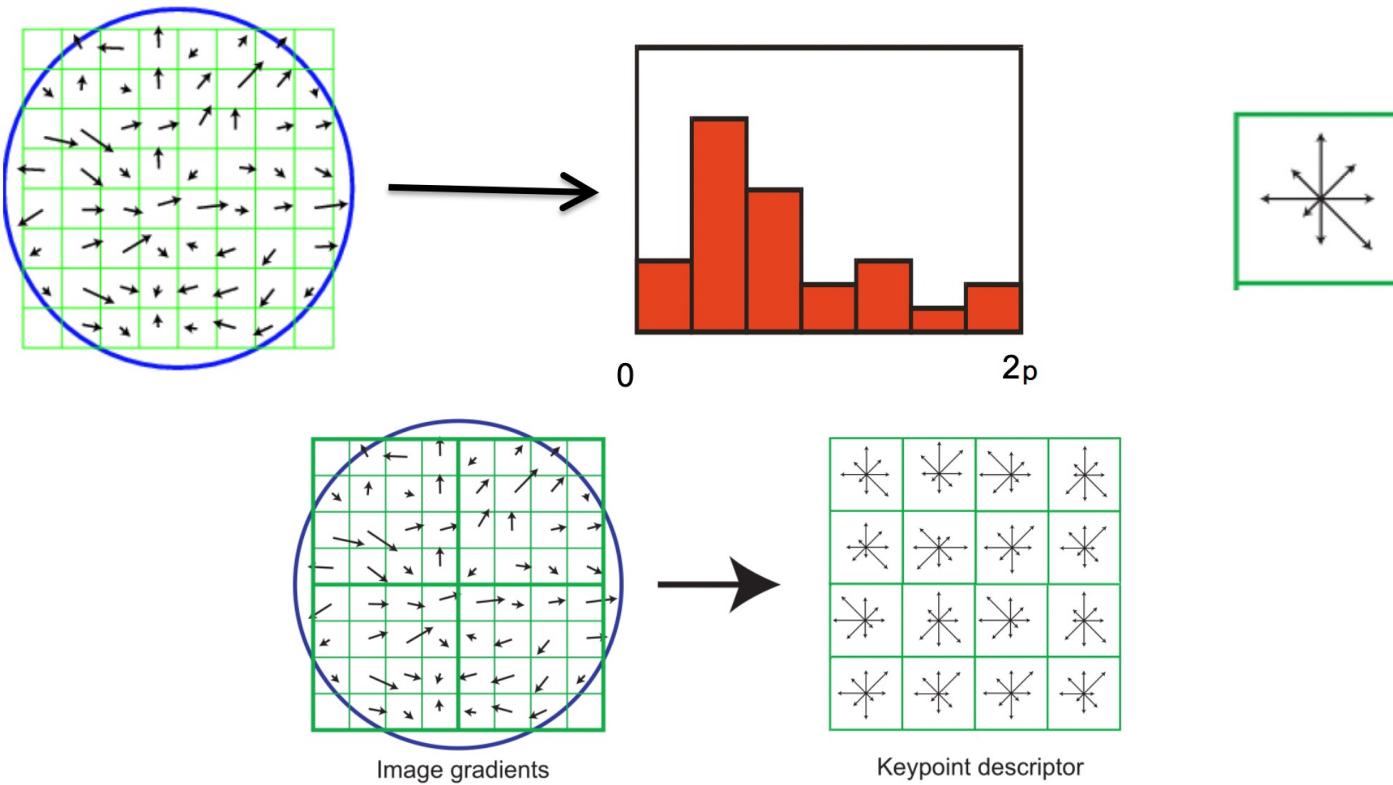
# SIFT Descriptor

- Use the blurred image associated with the keypoint's scale
- Take image gradients over the keypoint neighborhood.
- To become rotation invariant, rotate the gradient directions AND locations by (-keypoint orientation)

Now we've cancelled out rotation and have gradients expressed at locations relative to keypoint orientation  $\theta$

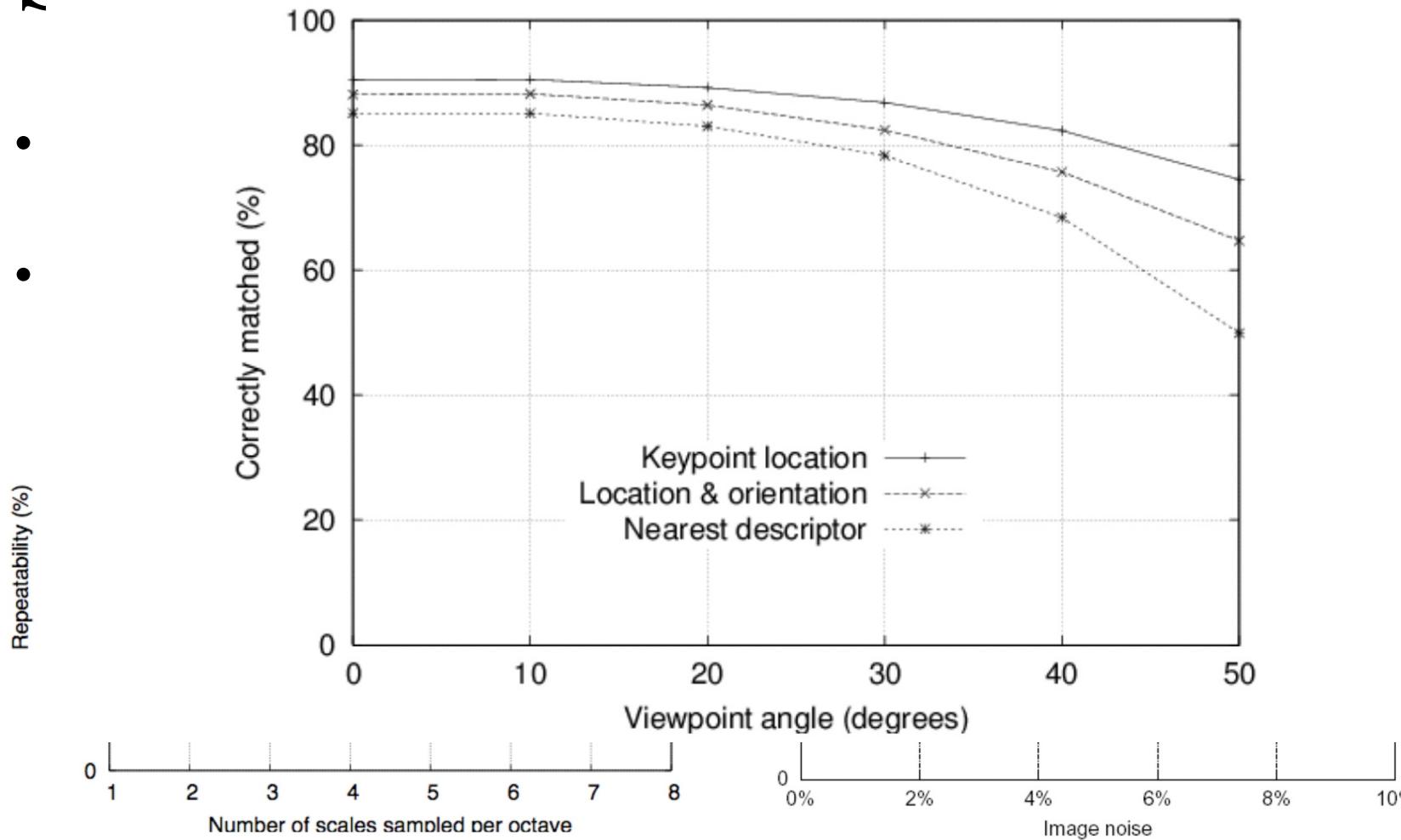


# SIFT Descriptor



- Don't use precise locations. Use blobs
- Divide into  $4 \times 4$  regions and create histograms
- Put the rotated gradients into their local orientation histograms (8 bins)

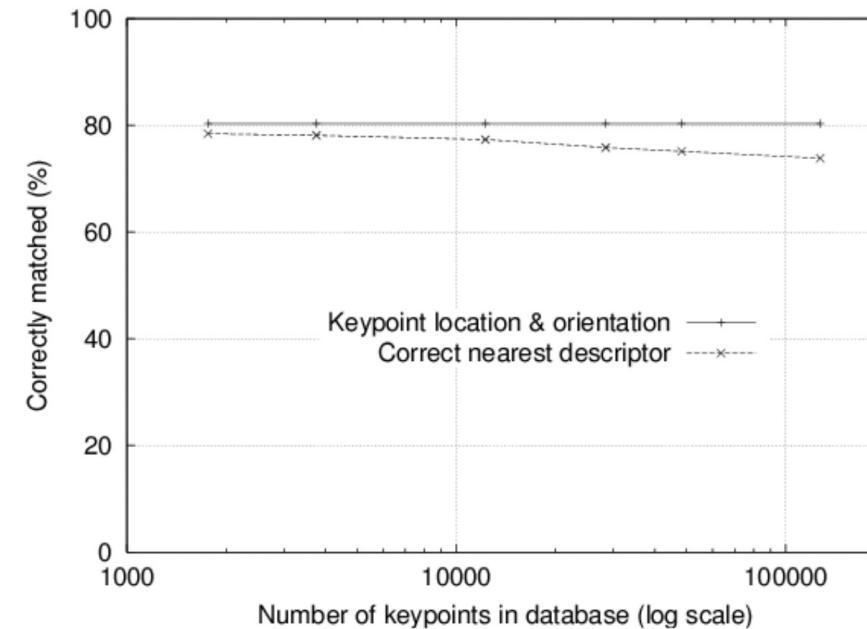
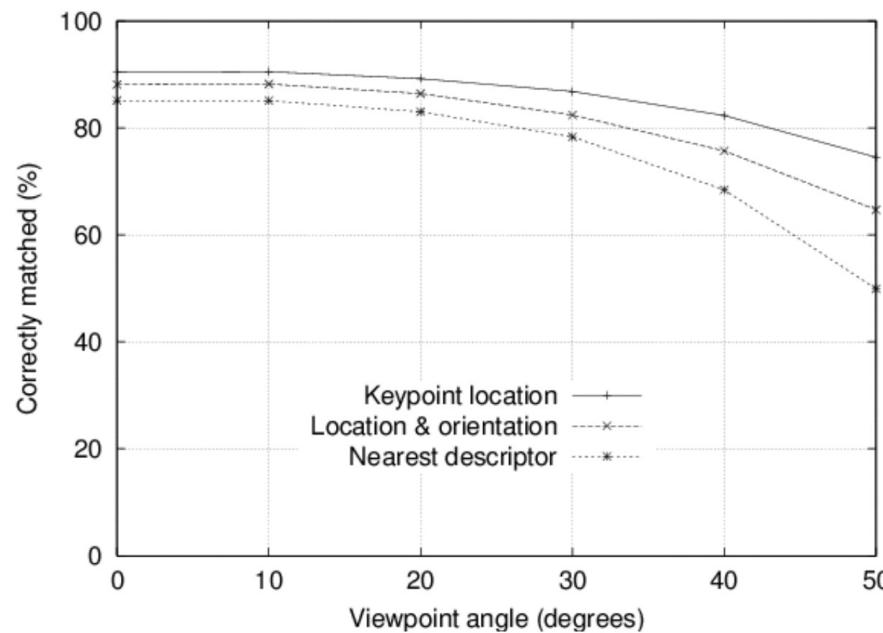
- Match features after random change in image scale & orientation, with 2% image noise, and affine distortion
- Find nearest neighbor in database of 30,000 features



elds  $8 \times 4 \times 4 =$

# SIFT Descriptor

- 8 orientation bins per histogram, and a 4x4 histogram array, yields  $8 \times 4 \times 4 = 128$  numbers.
- Find correspondences by minimizing Euclidean Distances



Affine change upto 20 degrees is ok

# Resources

- Fei-Fei Li's lectures (University of Standford)
- Alex Berg's lecture (MIT-CSAIL)

[http://alumni.media.mit.edu/~maov/classes/comp\\_photo\\_vision08f/lect/18\\_feature\\_detectors.pdf](http://alumni.media.mit.edu/~maov/classes/comp_photo_vision08f/lect/18_feature_detectors.pdf)