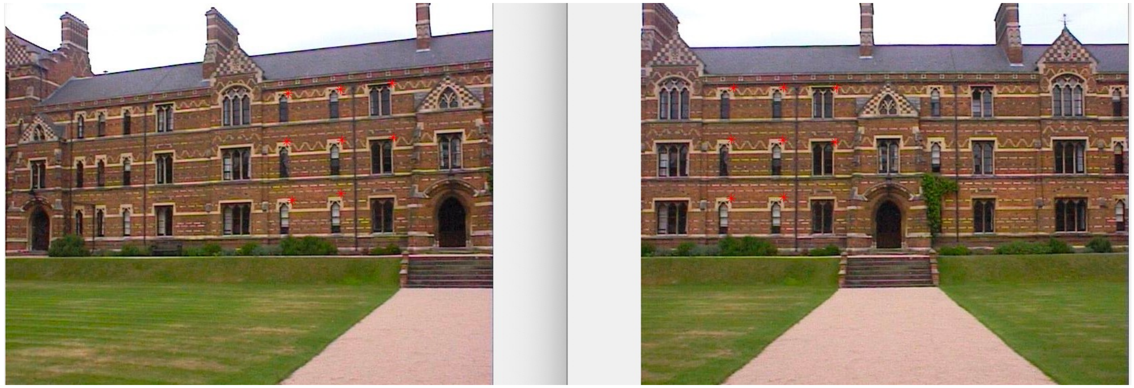
10. Fundamental Matrix Computation and Stereo Reconstruction from non-parallel images

Fundamental Matrix Estimation

- Given 8 point correspondences on a pair of images



$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_v' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

- Solution: [U,S,V] = svd(A); F = V(:,end);

Fundamental Matrix Estimation

- If we draw epipolar lines, they don't intersect on epipole.

Problem: rank(F)=3.

Solution: Force rank(F)=2. Steps:

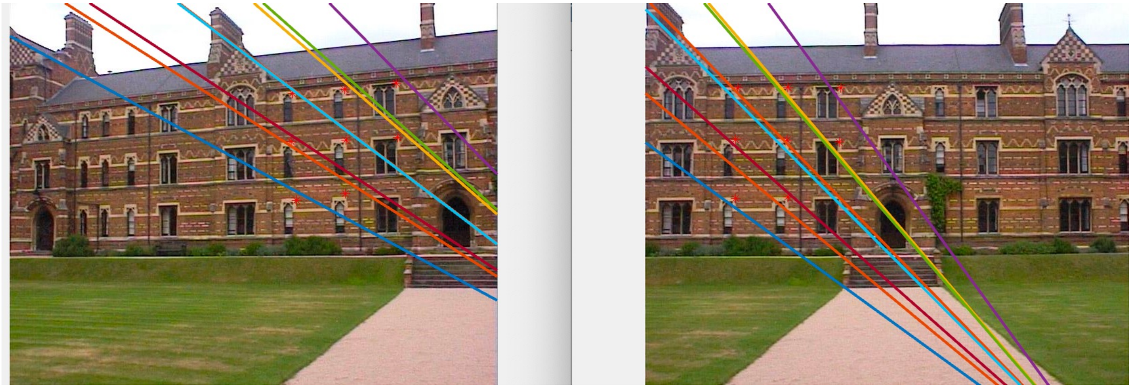
[U,S,V]= svd(F);

S(3,3)=0;

Fnew = USV';

Fundamental Matrix Estimation

- Now if we look at epipolar lines



Better convergence but epipolar lines are slightly noisy.

This is because we did not normalize data before its estimation.

Normalisation = Undo the effect of camera (image formation).

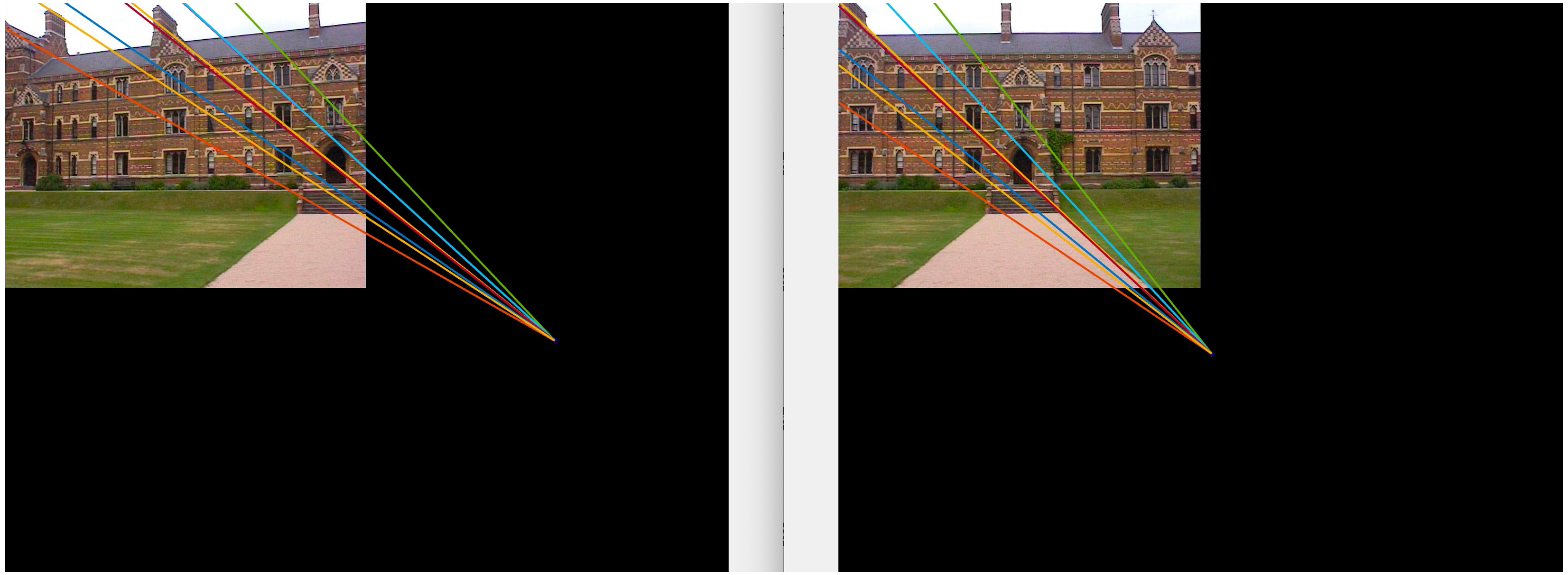
Takes image from camera plane to image plane.

You can use K matrix to do that but since we assume uncalibrated case, we don’t have it.

Normalise data: Zero-mean, standard deviation ~= (1,1,1)

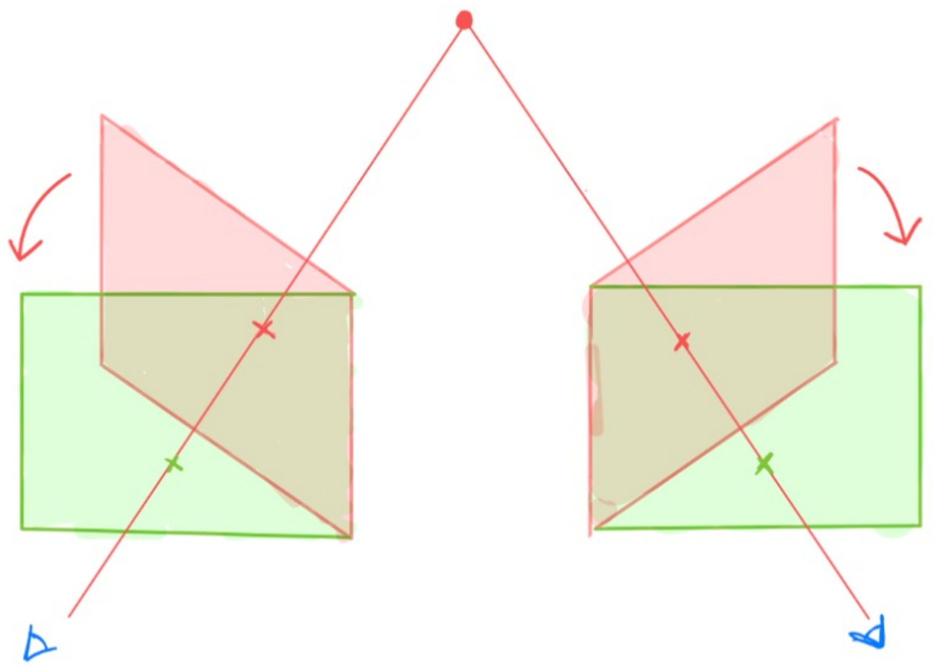
Fundamental Matrix Estimation

- Now if we look at epipolar lines



Stereo-reconstruction: Non-parallel images

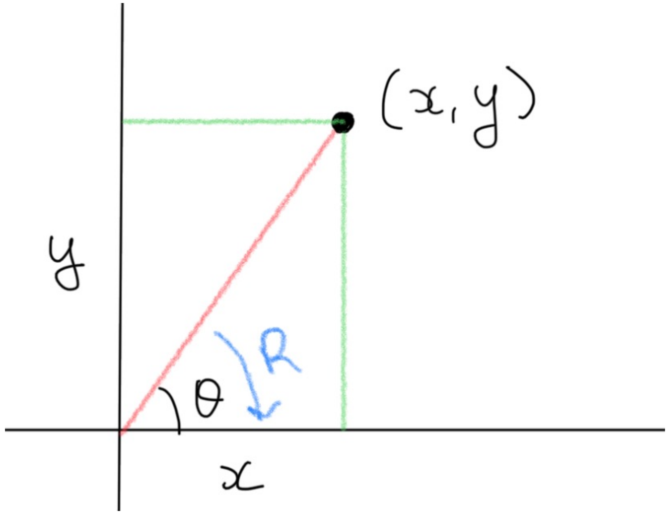
- We have non-parallel images (in red). We want to convert them to parallel ones (in green) because we know how to solve this case.



Stereo-reconstruction: Non-parallel images

- We have non-parallel images (in red). We want to convert them to parallel ones (in green) because we know how to solve this case.

- Step 1: Force epipole to lie on x-axis



Stereo-reconstruction: Non-parallel images

- We have non-parallel images (in red). We want to convert them to parallel ones (in green) because we know how to solve this case.

- Step 2: Shift epipole to infinity. Change [x;0;1] to [x;0;0]

$$G \times p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{x} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

Stereo-reconstruction: Non-parallel images

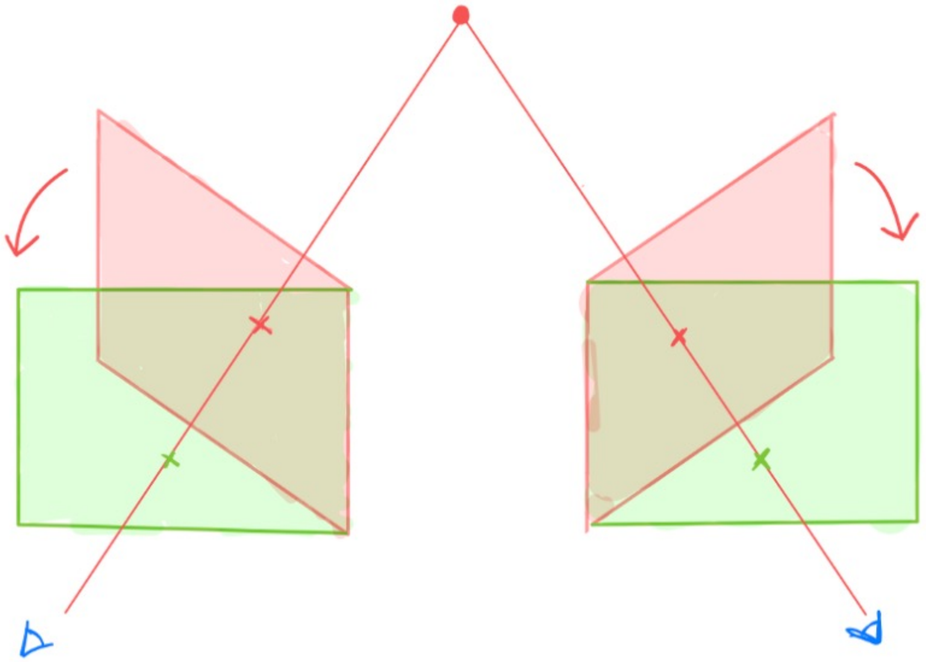
- We have non-parallel images (in red). We want to convert them to parallel ones (in green) because we know how to solve this case.

- Step 3: Move origin to image center

$$T = \begin{bmatrix} 1 & 0 & \frac{-width}{2} \\ 0 & 1 & \frac{-height}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

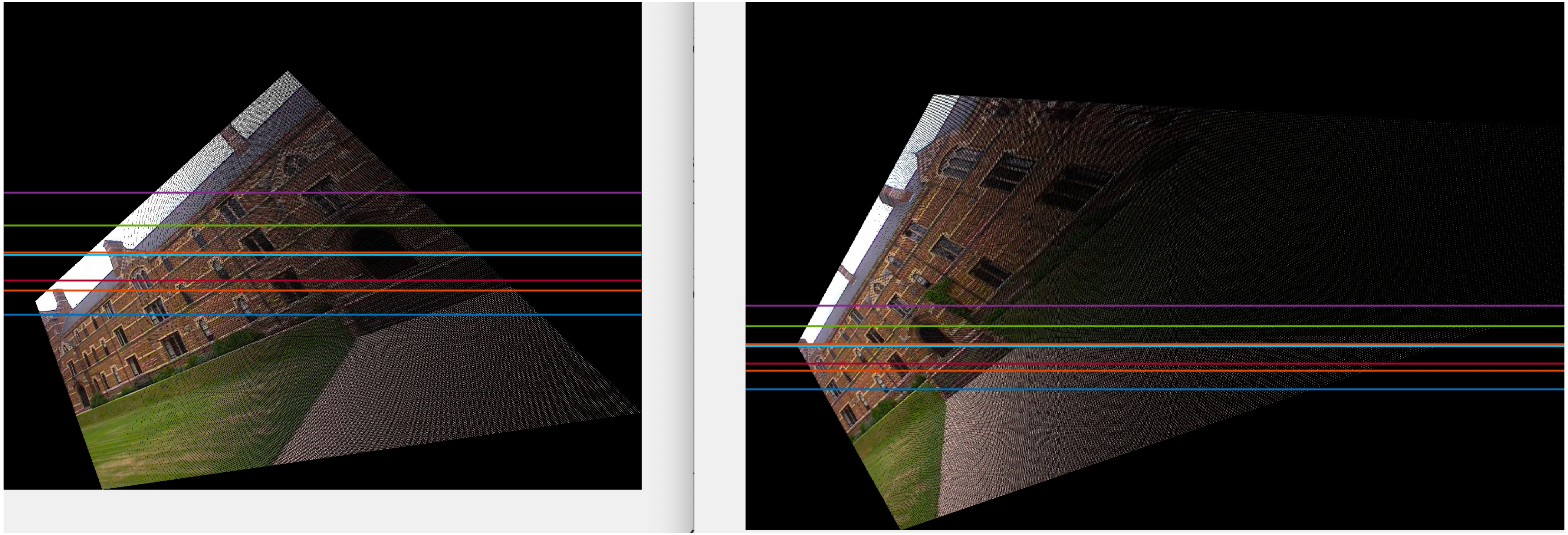
Stereo-reconstruction: Non-parallel images

- $H_{left} = inv(T)GR_{left}T,$
- $H_{right} = inv(T)GR_{right}T,$



Stereo-reconstruction: Non-parallel images

Rectification on Keble images:



There are better ways but this does the job.