**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Now, we find the z-score for X=50 minutes using the given mean (μ=45 minutes) and standard deviation (σ=8 minutes):

0.625

Z= 50−45/8

=0.625

Now, we look up this z-score in the standard normal distribution table to find the probability that the service time exceeds 50 minutes. The table gives us the probability that a standard normal random variable is less than Z, so we subtract this from 1 to get the probability that it's greater than Z.

Using a standard normal distribution table or calculator, we find that the probability corresponding to Z=0.625 is approximately 0.7340.

Therefore, the probability that the service manager cannot meet his commitment is

1−0.7340 = 0.2660,which is closest to option B: 0.2676. So, the correct answer is B.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

False.

We can use the empirical rule (or the 68-95-99.7 rule) to answer this question. The empirical rule tells us that 68% of the employees will have ages within 1 standard deviation of the mean, which is 6 years. In other words, 68% of the employees will be between 32 and 44 years old. The remaining 32% of the employees will be outside of this range, with half of them being older than 44 and the other half being younger than 32. Therefore, we can expect that there will be more employees between 38 and 44 years old than there are employees older than 44 years old.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

True.

We can use the standard normal distribution to calculate the probability that an employee is younger than 30 years old. The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1. We can convert the age of 30 years old to a standard normal variable (z) using the following formula

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Mean of 2X1: 2μ

Variance of 2X1: 4σ^2

Mean of X1 + X2: μ + μ = 2μ

Variance of X1 + X2: σ^2 + σ^2 = 2σ^2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

To find the values

a and b such that the probability of the random variable X taking a value between them is 0.99, we need to find the values that encompass the central 99% of the normal distribution. In other words, we want to find a and b such that:

P(a≤X≤b)=0.99

P(100−10×20≤X≤100+10×20)=0.99

P(80≤X≤120)=0.99

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

R1:(225−2× sqrt 64800,225+2×sqrt64800)

(315−2×sqrt85050,315+2×85050

R2:(315−2× 85050 ,315+2×85050)

1. Specify the 5th percentile of profit (in Rupees) for the company

The 5th percentile corresponds to a z-score of approximately -1.645 for a standard normal distribution. You can use this z-score to find the value for each division:

R1:225+(−1.645)× sqrt64800

R2:315+(−1.645)× sqrt85050

1. Which of the two divisions has a larger probability of making a loss in a given year?

To determine which division has a larger probability of making a loss, you need to compare the probabilities of having negative profits for each division. This involves finding the cumulative distribution function (CDF) values for zero profits for both distributions and comparing them. The division with a higher CDF at zero has a larger probability of making a loss.

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