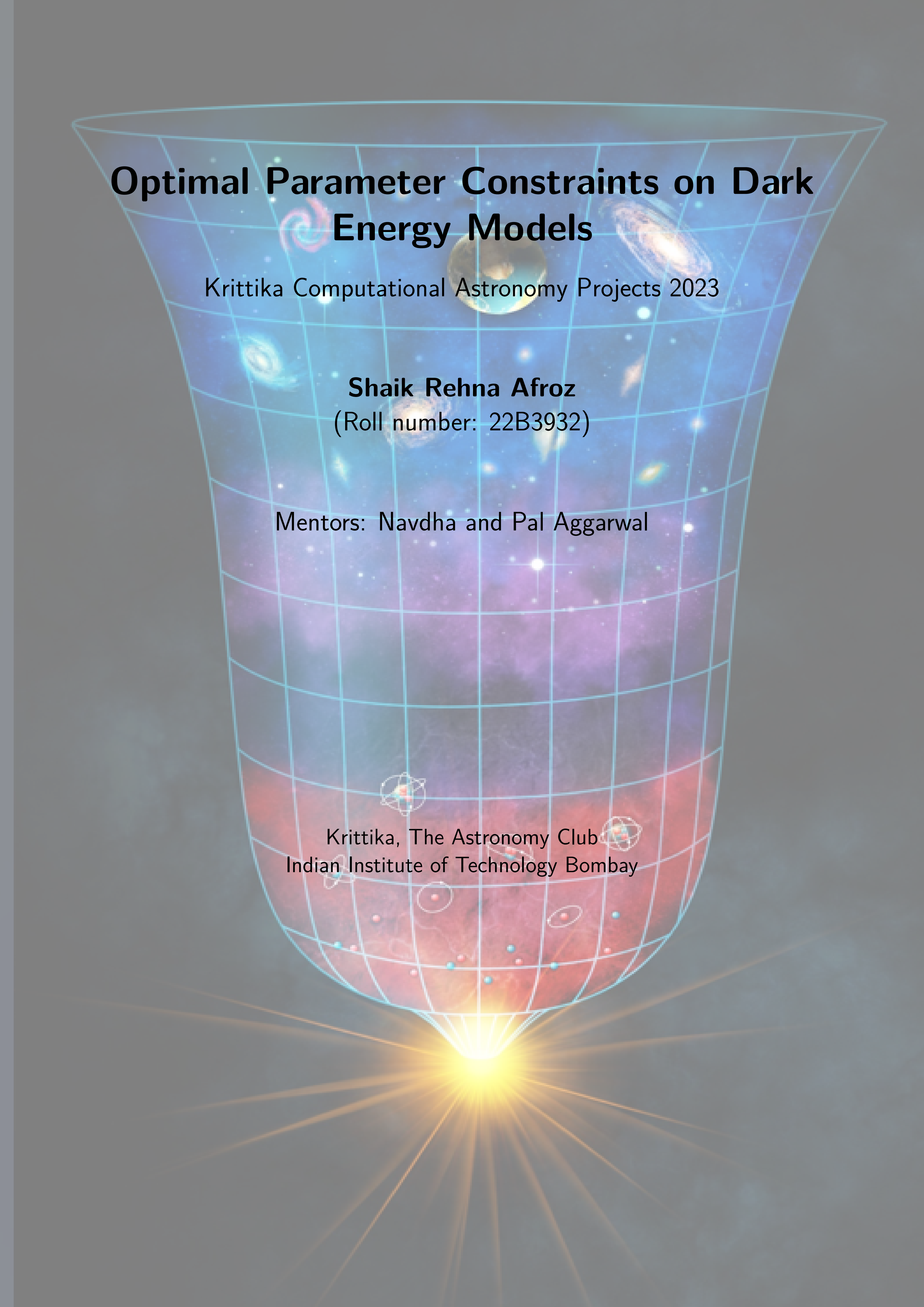

Optimal Parameter Constraints on Dark Energy Models

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Contents

1 Introduction

- The cosmic narrative, as described by the standard Big Bang Theory, attributes the universe's accelerated expansion to a cosmological constant known as dark energy, constituting a substantial 70% of the total energy budget of the universe.
- However, the precision of modern observational tools has ushered in an era of detailed scrutiny, revealing intriguing inconsistencies that challenge the robustness of this established model.
- These observational discrepancies have prompted a critical reexamination of our cosmological understanding, leading to the exploration of alternative models that incorporate non-zero spatial curvature and dynamic dark energy.

- In this context, this project endeavours to investigate and constrain four distinct cosmological models that diverge from the conventional paradigm.
- By using standardized, lower-redshift observations, this project aims to shed light on the details of cosmic evolution, offering insights into the nature of spatial curvature and the dynamical behaviour of dark energy.
- The foundation of the analysis rests on the combined use of Hubble parameter (H(z)) and Baryon Acoustic Oscillation (BAO) data, enabling the identification of the most likely cosmological scenarios.

2 Cosmological Models

2.1 ΛCDM model

- The Hubble parameter in the ΛCDM model (CDM - Cold Dark Matter) is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_\Lambda} \tag{1}$$

- In the above equation **Ho** is the Hubble constant and Ω_{m0}, Ω_{k0}, Ω_Λ are the present values of the non-relativistic matter density parameter, the spatial curvature energy density parameter, and the cosmological constant energy density parameter, respectively.
- Ω_{m0}, Ω_{k0}, Ω_Λ are related through the equation $\Omega_{m0} + \Omega_{k0} + \Omega_\Lambda = 1$
- In the **spatially non-flat ΛCDM** model, the conventional choice of free parameters is Ω_{m0}, Ω_{k0} and Ho, while in the **spatially-flat ΛCDM** model we use the same set of free parameters but now with $\Omega_{k0} = 0$.

2.2 XCDM model

- The Hubble parameter in the XCDM model is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{X0}(1+z)^{3(1+\omega_X)}} \tag{2}$$

- In the above equation **Ho** is the Hubble constant and Ω_{m0}, Ω_{k0}, Ω_{X0} are the present values of the non-relativistic matter density parameter, the spatial curvature energy density parameter, and the X-fluid dark energy density parameter, respectively.
- ω_X is the equation of state parameter of the X-fluid (the ratio of the pressure to the energy density).
- Ω_{m0}, Ω_{k0}, Ω_{X0} are related through the equation $\Omega_{m0} + \Omega_{k0} + \Omega_{X0} = 1$
- In the **spatially non-flat XCDM** model, the conventional choice of free parameters is Ω_{m0}, Ω_{k0}, ω_X and Ho, while in the **spatially-flat XCDM** model we use the same set of free parameters but now with Ω_{k0}=0.
- In the XCDM parameterization when ω_X = -1 the ΛCDM model is recovered.

2.3 ϕCDM model

- The Hubble parameter in the XCDM model is

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\phi}(z,\alpha)} \tag{3}$$

- In the above equation **Ho** is the Hubble constant and Ω_{m0}, Ω_{k0}, Ω_{ϕ} are the present values of the non-relativistic matter density parameter, the spatial curvature energy density parameter, and the scalar field dynamical dark energy density parameter, respectively.

- Ω_{m0}, Ω_{k0}, $\Omega_{\phi}(z,\alpha)$ are related through the equation $\Omega_{m0} + \Omega_{k0} + \Omega_{\phi}(z,\alpha) = 1$

- In the **spatially non-flat ϕCDM** model, the conventional choice of free parameters is Ω_{m0}, Ω_{k0}, α and Ho, while in the **spatially-flat ϕCDM** model we use the same set of free parameters but now with Ω_{k0}=0.

- In the ϕCDM parameterization when $\alpha = 0$ the ΛCDM model is recovered.

3 Data

- In this project a combination of H(z) and BAO is used.

3.1 H(z) data

- This data is crucial for understanding the evolution of the universe and can provide insights into the nature of dark energy, dark matter, and the overall dynamics of the cosmos.

- H(z) is Hubble parameter as a function of redshift.

- In this project 31 H(z) data measurements from [1] are used.These were determined using the cosmic chronometric technique.

- Table13 contains the H(z) data. H(z) and σ_H have units of $Kms^{-1}Mpc^{-1}$

3.2 BAO Data

- BAO typically stands for Baryon Acoustic Oscillations, a feature in the universe's large-scale structure. Baryon Acoustic Oscillations are fluctuations in the density of visible matter (baryons) that occurred shortly after the Big Bang. These fluctuations have left a signature in the distribution of galaxies and other cosmic structures we observe today.

- BAO data is often used in cosmology to study the universe's large-scale structure and measure the expansion rate. By analyzing the distribution of galaxies, astronomers can study the imprints of Baryon Acoustic Oscillations and use them as a cosmological ruler to probe the geometry and expansion history of the universe.

- In this project 11 BAO data measurements from [2] are used.

- Table14 contains the BAO data.

- In Table14 $D_M(r_{s,fid}/r_s)$,$D_V(r_{s,fid}/r_s), r_s$, and $r_{s,fid}$ have units of Mpc, while $H(z)(r_s/r_{s,fid})$ has units of Km$s^{-1}Mpc^{-1}$ and D_A/r_s is dimensionless.

$$r_s = \frac{55.154\exp\left[-72.3(\Omega_{\mathrm{vo}}h^2 + 0.0006)^2\right]}{(\Omega_{\mathrm{bo}}h^2)^{0.1280}(\Omega_{\mathrm{cbo}}h^2)^{0.25351}} \tag{4}$$

- Here $\Omega_{\mathrm{cbo}} = \Omega_{\mathrm{co}} + \Omega_{\mathrm{bo}} = \Omega_{\mathrm{mo}}$ - Ω_{vo} with Ω_{cbo}, Ω_{co},Ω_{bo} and Ω_{vo} = 0.0014 being the current values of the CDM + baryonic matter, CDM, baryonic matter, and neutrino energy density parameters, respectively, and the Hubble constant **H0 = 100h Kms^{-1}Mpc^{-1}**

- The subscript of 'o' on a given quantity denotes the current value of that quantity.

- Additionally,$\Omega_{\mathsf{bo}}h^2$ is slightly model dependent; the values of this parameter that are used in this project are from [3] and are given Table15
- BAO provides observers with a "standard ruler" which can be used to measure cosmological distances. These distances can be computed in a given cosmological model, so measurements of them can be used to constrain the parameters of the model in question.
- The transverse co-moving distance is

$$D_M(z) = \begin{cases} D_C(z) & \text{if } \Omega_{k0} = 0, \\ \frac{c}{H_0\sqrt{\Omega_{k0}}} \sinh\left(\frac{D_C H_0}{c}\sqrt{\Omega_{k0}}\right) & \text{if } \Omega_{k0} > 0, \\ \frac{c}{H_0\sqrt{|\Omega_{k0}|}} \sin\left(\frac{D_C H_0}{c}\sqrt{|\Omega_{k0}|}\right) & \text{if } \Omega_{k0} < 0. \end{cases}$$

- $D_H = \frac{c}{H(z)}$
- $D_C = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$,
- The volume-averaged angular diameter distance is $D_V(z) = [\frac{cz}{H_0}\frac{D_M^2(z)}{E(z)}]^{1/3}$
- The angular diameter distance at redshift (z) is $D_A(z) = \frac{D_M(z)}{1+z}$

4 Method

4.1 Chi-square Minimization

- In this project, the χ^2 statistic is used to find the best-fitting parameter values and limits for a given model. Most of the data points we use are uncorrelated. Most of the data we analyze are uncorrelated. For uncorrelated data points, χ^2 is given by

$$\chi^2(p) = \sum_{i=1}^{N} \frac{[A_{\mathsf{th}}(p; z_i) - A_{\mathsf{obs}}(z_i)]^2}{\sigma_i^2} \tag{5}$$

- Here p is the set of model parameters, for example, $p = (Ho, \Omega_{m0})$ in the flat ΛCDM model, z_i is the redshift at which the measured value is $A_{obs}(z_i)$ with one standard deviation uncertainty σ_i , and $A_{th}(p; z_i)$ is the predicted value computed in the model under consideration. The χ^2 expression in eq.(5) holds for the H(z) measurements listed in Table13 and the BAO measurements listed in the last five lines of Table14.
- The measurements in the first six lines of Table16 are correlated, in which case χ^2 is given by

$$\chi^2(p) = [A_{\mathsf{th}}(p) - A_{\mathsf{obs}}]^T C^{-1}[A_{\mathsf{th}}(p) - A_{obs}] \tag{6}$$

where C^{-1} is the inverse of the covariance matrix C.

$$C = \begin{bmatrix} 624.707 & 23.729 & 325.332 & 8.34963 & 157.386 & 3.57778 \\ 23.729 & 5.60873 & 11.6429 & 2.33996 & 6.39263 & 0.968056 \\ 325.332 & 11.6429 & 905.777 & 29.3392 & 515.271 & 14.1013 \\ 8.34963 & 2.33996 & 29.3392 & 5.42327 & 16.1422 & 2.85334 \\ 157.386 & 6.39263 & 515.271 & 16.1422 & 1375.12 & 40.4327 \\ 3.57778 & 0.968056 & 14.1013 & 2.85334 & 40.4327 & 6.25936 \end{bmatrix}$$

- For the QSO data the below formula is used:

$$\chi^2(p) = \sum_{i=1}^{N} \left[\frac{\theta_{\mathsf{th}}(p; z_i) - \theta_{\mathsf{obs}}(z_i)}{\sigma_i + 0.1\theta_{\mathsf{obs}}(z_i)}\right]^2 \tag{7}$$

Where $\theta_{\mathsf{th}}(p; z_i)$ is the model-predicted value of the angular size, $\theta_{\mathsf{obs}}(z_i)$ is the measured angular size at redshift z_i, and σ_i is the uncertainty on the measurement made at redshift z_i. The term is proportional to $\theta_{\mathsf{obs}}(z_i)$ in the denominator is added to σ_i to account for systematic uncertainties in the angular size measurements.

4.2 MCMC Analysis

- In this project **Markov Chain Monte Carlo (MCMC)** statistical technique is used to estimate the distribution of model parameters in a Bayesian framework. MCMC is particularly valuable when dealing with complex models or models for which the likelihood function is difficult to compute.
- **Bayesian Inference :** MCMC is often used in Bayesian statistical analysis. Bayesian inference involves updating our beliefs about a parameter based on new evidence or data. The updated belief is represented by a probability distribution, and MCMC helps in sampling from this distribution.
- **Markov Chain :** MCMC involves creating a Markov Chain, which is a sequence of random states where the probability of transitioning from one state to another depends only on the current state (Markov property). The chain is constructed in such a way that it eventually reaches a state where the distribution of interest is approximated.
- **Metropolis-Hastings Algorithm :** One of the common algorithms used in MCMC is the Metropolis-Hastings algorithm. It involves proposing new states based on the current state, accepting or rejecting the proposed state based on an acceptance criterion, and then updating the current state accordingly.
- **EMCEE** library in **Python** is used to implement Monte Carlo Markov Chain analysis for constraining cosmological parameters.

4.3 Chi-square minimization v/s MCMC

4.3.1 Chi-square Minimization:

- **Simplicity and Speed:** Chi-square minimization is relatively straightforward and computationally efficient. It is suitable for problems with simple models and well-defined parameter spaces.
- **Deterministic Optimization:** Chi-square minimization seeks to find the parameters that minimize the difference between observed and expected values.It provides a deterministic solution, and the final parameter values are chosen to best fit the data.
- **Assumptions:** Chi-square minimization assumes that the errors are normally distributed and that the model is correctly specified. It may not be robust in the presence of outliers or in cases where the model assumptions are violated.

4.3.2 MCMC Analysis:

- **Bayesian Inference:** MCMC is often employed in Bayesian statistical analysis, allowing for the estimation of the posterior distribution of parameters. It provides a full probabilistic description of the parameter uncertainties.
- **Dealing with Uncertainties:** MCMC naturally handles uncertainties in parameter estimation. It provides a range of plausible parameter values rather than a single-point estimate. This can be crucial when dealing with uncertain or incomplete information.
- **Incorporating Priors:** Bayesian MCMC allows the incorporation of prior information into the analysis. This is beneficial when we have prior knowledge about the parameters that we want to incorporate into the analysis.

4.3.3 WHY BOTH?

- **Validation and Robustness:** Using both methods allows us to cross-validate our results. If the results obtained from Chi-square minimization and MCMC analysis agree, it adds confidence to our findings.
- **Model Checking:** Different methods can help in checking the adequacy of our chosen model. Chi-square minimization might provide a best-fit solution, while MCMC allows us to explore the full parameter space and assess the credibility of different model scenarios.
- **Complementary Information:** Each method provides different types of information. Chi-square minimization gives a single best-fit solution, while MCMC provides a distribution of possible parameter values. Combining them can give a more comprehensive understanding of the problem.

5 Results

5.1 H(z) constraints

5.1.1 ΛCDM model

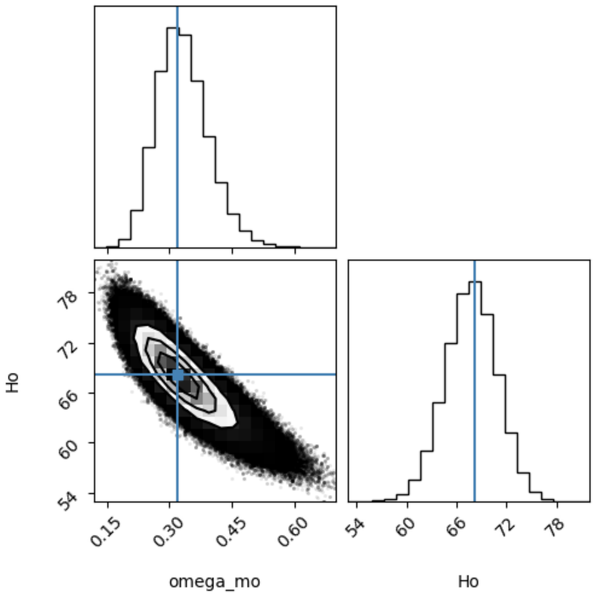
- **Flat ΛCDM model :** In the **spatially flat ΛCDM** model, the free parameters are Ω_{m0} and Ho.

Table 1: Chi-square minimization

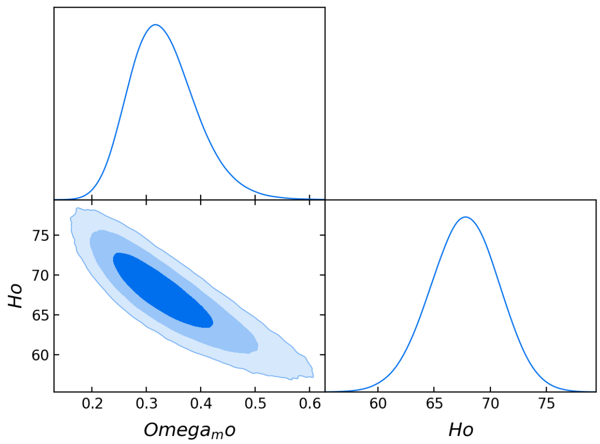
parameter	optimal value
Ω_{m0}	0.3195
Ho	68.1499

Table 2: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.3324
Ho	67.7187



(a) Corner Plot



(b) Contour Plot

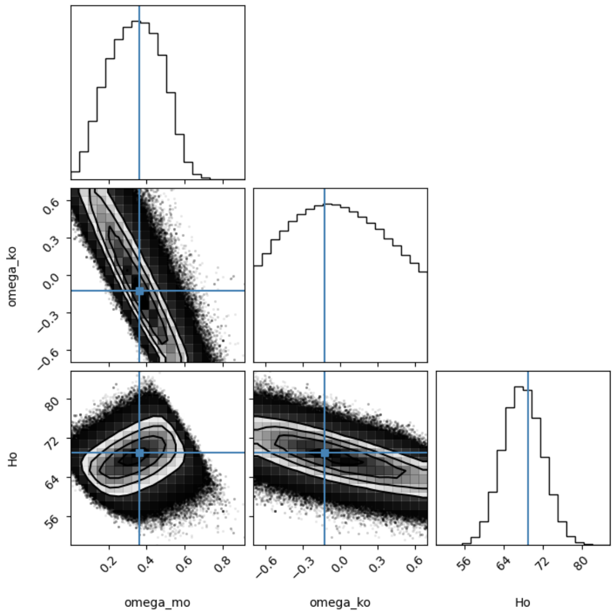
- **Non-Flat ΛCDM model :** In the **spatially non-flat ΛCDM** model, the free parameters are Ω_{m0} ,Ω_{k0} and Ho.

Table 3: Chi-square minimization

parameter	optimal value
Ω_{m0}	0.3597
Ω_{k0}	-0.1259
Ho	68.9987

Table 4: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.3393
Ω_{k0}	-0.0162
Ho	67.8891



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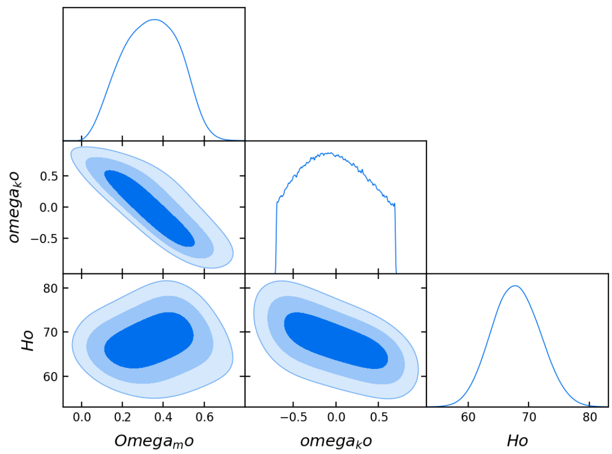


(b) Contour Plot

5.1.2 XCDM model

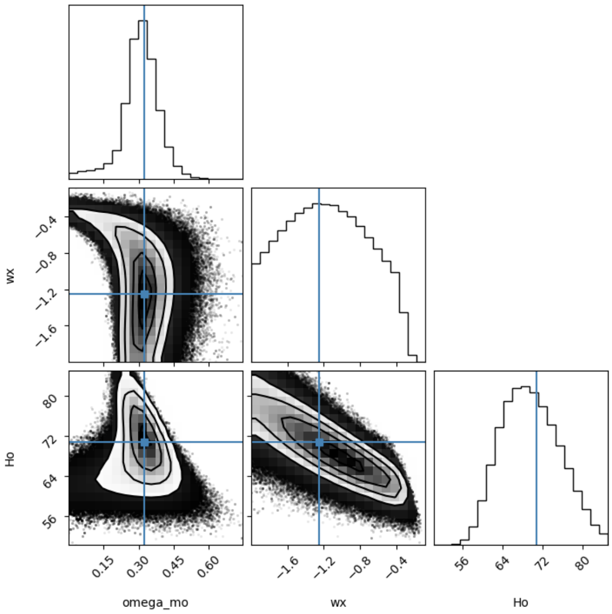
- **Flat XCDM model :** In the **spatially flat XCDM** model, the free parameters are Ω_{m0},Ω_X and Ho.

Table 5: Chi-square minimization

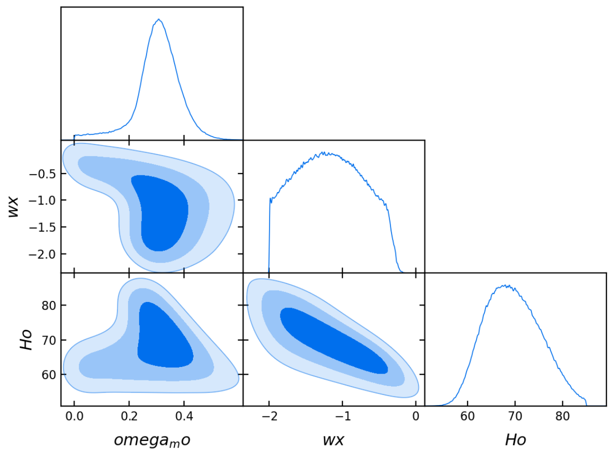
parameter	optimal value
Ω_{m0}	0.3223
ω_X	-1.2497
Ho	70.8207

Table 6: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.3049
ω_X	-1.1726
Ho	69.5819



(a) Corner Plot



(b) Contour Plot

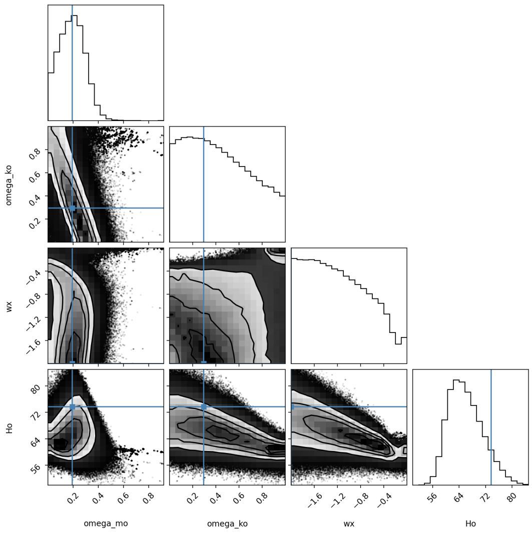
- **Non-Flat XCDM model :** In the **spatially non-flat XCDM** model, the free parameters are Ω_{m0}, Ω_{k0}, Ω_X and Ho.

Table 7: Chi-square minimization

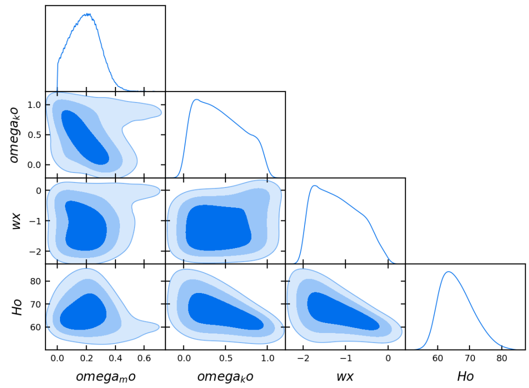
parameter	optimal value
Ω_{m0}	0.1943
Ω_{k0}	0.2940
ω_X	-2
Ho	73.7042

Table 8: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.1920
Ω_{k0}	0.4325
ω_X	-1.1886
Ho	66.0597



(a) Corner Plot



(b) Contour Plot

5.2 H(z) and BAO constraints

5.2.1 ΛCDM model

- **Flat ΛCDM model :** In the **spatially flat ΛCDM** model, the free parameters are Ω_{m0} and Ho.

Table 9: Chi-square minimization

parameter	optimal value
Ω_{m0}	0.0014
Ho	96.3655

Table 10: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.0014
Ho	75.9917

5.2.2 XCDM model

- **Flat XCDM model :** In the **spatially flat XCDM** model, the free parameters are Ω_{m0},Ω_X and Ho.

Table 11: Chi-square minimization

parameter	optimal value
Ω_{m0}	0.0014
ω_X	-0.8070
Ho	94.4642

Table 12: MCMC Analysis

parameter	optimal value
Ω_{m0}	0.0014
ω_X	-0.9641
Ho	74.0892

Table 13: H(z) data
[3]

z	H(z)	σ_H
0.07	69	19.6
0.09	69	12
0.12	68.6	26.2
0.17	83	8
0.179	75	4
0.199	75	5
0.20	72.9	29.6
0.27	77	14
0.28	88.8	36.6
0.352	83	14
0.3802	83	13.5
0.4	95	17
0.4004	77	10.2
0.4247	87.1	11.2
0.4497	92.8	12.9
0.47	89	50
0.4783	80.9	9
0.48	97	62
0.593	104	13
0.68	92	8
0.781	105	12
0.875	125	17
0.88	90	14
0.90	117	23
1.037	154	20
1.3	168	17
1.363	160	33.6
1.43	177	18
1.53	140	14
1.75	202	40
1.965	186.5	50.4

Table 14: BAO data
[3]

z	Measurement	Value
0.38	$D_M(r_{s,fid}/r_s)$	1512.39
0.38	H(z) $(r_s/r_{s,fid})$	81.2087
0.51	$D_M(r_{s,fid}/r_s)$	1975.22
0.51	H(z) $(r_s/r_{s,fid})$	90.9029
0.61	$D_M(r_{s,fid}/r_s)$	2306.68
0.61	H(z) $(r_s/r_{s,fid})$	98.9647
0.122	$D_V(r_{s,fid}/r_s)$	539 ± 17
0.81	D_A/r_s	10.75 ± 0.43
1.52	$D_V(r_{s,fid}/r_s)$	3843 ± 147
2.34	D_H/r_s	8.86
2.34	D_M/r_s	37.41

Table 15: Baryon densities for the models
[4]

Model	$\Omega_{bo}h^2$
Flat Λ CDM	0.02225
Nonflat Λ CDM	0.02305
Flat XCDM	0.02229
Nonflat XCDM	0.02305
Flat ϕ CDM	0.02221
Nonflat ϕ CDM	0.02303

[1, 4, 3, 2]

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[2] S. Cao, *Cosmological constraints from standardized non-CMB observations*. PhD thesis, Kansas State University, KS, 2023.

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