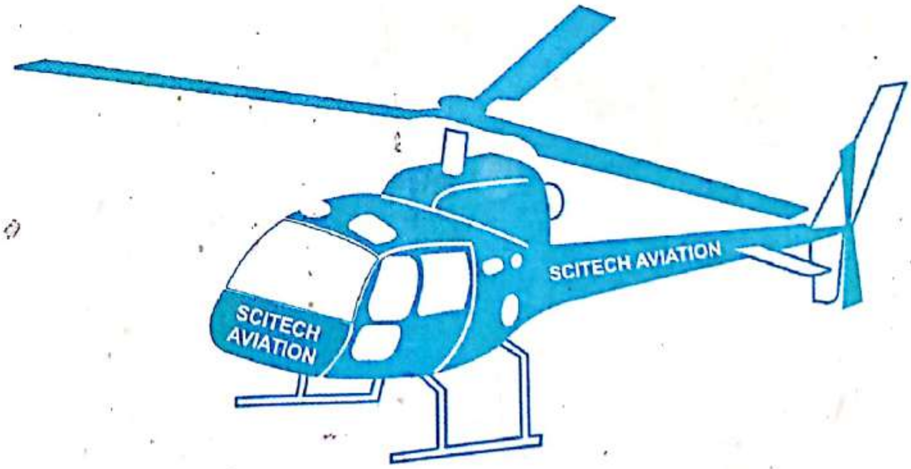
"All children are artists. The problem is how to remain an artist once he grows up" - *Pablo Picasso*

K.L. Narayana ~ P. Kannaiah

ENGINEERING DRAWING

Second Edition



Knowledge for Ages

SCITECH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

B.Tech-CSE – II Sem	**L**	**T**	**P/D**	**C**
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(20A03101T) ENGINEERING DRAWING
(Common to All Branches of Engineering)

Course Objectives:

- Bring awareness that Engineering Drawing is the Language of Engineers.
- Familiarize how industry communicates technical information.
- Teach the practices for accuracy and clarity in presenting the technical information.
- Develop the engineering imagination essential for successful design.

Unit: I

Introduction to Engineering Drawing: Principles of Engineering Drawing and its significance-Conventions in drawing-lettering - BIS conventions.

a)Conic sections including the rectangular hyperbola- general method only,
b) Cycloid, epicycloids and hypocycloid c) Involutes

Learning Outcomes:

At the end of this unit the student will be able to

- Understand the significance of engineering drawing
- Know the conventions used in the engineering drawing
- Identify the curves obtained in different conic sections
- Draw different curves such as cycloid, involute and hyperbola

Unit: II

Projection of points, lines and planes: Projection of points in any quadrant, lines inclined to one or both planes, finding true lengths, angle made by line. Projections of regular plane surfaces.

Learning Outcomes:

At the end of this unit the student will be able to

- Understand the meaning of projection
- Know how to draw the projections of points, lines
- Differentiate between projected length and true length
- Find the true length of the lines

Unit: III

Projections of solids: Projections of regular solids inclined to one or both planes by rotational or auxiliary views method.

Learning Outcomes:

At the end of this unit the student will be able to

- Understand the procedure to draw projection of solids
- Differentiate between rotational method and auxillary view method.
- Draw the projection of solid inclined to one plain
- Draw the projection of solids inclined to both the plains

Unit: IV

Sections of solids: Section planes and sectional view of right regular solids- prism, cylinder, pyramid and cone. True shapes of the sections.

Learning Outcomes:

At the end of this unit the student will be able to

- Understand different sectional views of regular solids
- Obtain the true shapes of the sections of prism
- Draw the sectional views of prism, cylinder, pyramid and cone

Unit: V

Development of surfaces: Development of surfaces of right regular solids-prism, cylinder, pyramid, cone and their sectional parts.

Learning Outcomes:

At the end of this unit the student will be able to

- Understand the meaning of development of surfaces
- Draw the development of regular solids such as prism, cylinder, pyramid and cone
- Obtain the development of sectional parts of regular shapes

Text Books:

1. K.L.Narayana & P.Kannaiah, Engineering Drawing, 3/e, Scitech Publishers, Chennai, 2012.
2. N.D.Bhatt, Engineering Drawing, 53/e, Charotar Publishers, 2016.

Reference Books:

1. Dhanajay A Jolhe, Engineering Drawing, Tata McGraw-Hill, Copy Right, 2009
2. Venugopal, Engineering Drawing and Graphics, 3/e, New Age Publishers, 2000
3. Shah and Rana, Engineering Drawing, 2/e, Pearson Education, 2009
4. K.C.John, Engineering Graphics, 2/e, PHI, 2013
5. Basant Agarwal & C.M.Agarwal, Engineering Drawing, Tata McGraw-Hill, Copy Right, 2008.

Course Outcomes:

After completing the course, the student will be able to

- Draw various curves applied in engineering. (12)
- Show projections of solids and sections graphically. (12)
- Draw the development of surfaces of solids. (13)

Additional Sources

Youtube: http-sewor,Carleton.cag,kardos/88403/drawings.html conic sections-online, red woods.edu

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR
B.Tech-CSE – II Sem

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(20A03101P) ENGINEERING GRAPHICS LAB
(Common to All Branches of Engineering)

Course Objectives:

- Instruct the utility of drafting & modeling packages in orthographic and isometric drawings.
- Train the usage of 2D and 3D modeling.
- Instruct graphical representation of machine components.

Computer Aided Drafting:

Introduction to AutoCAD: Basic drawing and editing commands: line, circle, rectangle, erase, view, undo, redo, snap, object editing, moving, copying, rotating, scaling, mirroring, layers, templates, polylines, trimming, extending, stretching, fillets, arrays, dimensions.

Dimensioning principles and conventional representations.

Orthographic Projections: Systems of projections, conventions and application to orthographic projections - simple objects.

Isometric Projections: Principles of isometric projection- Isometric scale; Isometric views: lines, planes, simple solids.

Text Books:

1. K. Venugopal, V.Prabhu Raja, Engineering Drawing + Auto Cad, New Age International Publishers.
2. Kulkarni D.M, AP Rastogi and AK Sarkar, Engineering Graphics with Auto Cad, PHI Learning, Eastern Economy editions.

Reference Books:

1. T. Jayapoovan, Engineering Graphics using Auto Cad, Vikas Publishing House
2. K.L.Narayana & P.Kannaiah, Engineering Drawing, 3/e, Scitech Publishers, Chennai, 2012.
3. Linkan Sagar, BPB Publications, Auto Cad 2018 Training Guide.
4. K.C.John, Engineering Graphics, 2/e, PHI, 2013
5. Basant Agarwal & C.M.Agarwal, Engineering Drawing, Tata McGraw-Hill, Copy Right, 2008.

Course Outcomes:

After completing the course, the student will be able to

- Use computers as a drafting tool. (L2)
- Draw isometric and orthographic drawings using CAD packages. (L3)

Additional Sources

1. Youtube: http-sewor,Carleton.cag, kardos/88403/drawings.html conic sections-online, red woods.edu

CHAPTER - 1

ENGINEERING DRAWING AND DRAUGHTING TOOLS

1.1 ENGINEERING DRAWING

A drawing prepared by an engineer, for an engineering purpose is known as an engineering drawing. It is the graphic representation of physical objects and their relationships. It is prepared, based on certain basic principles, symbolic representations, standard conventions, notations, etc. It is the only universal means of communication used by engineers, a language of ever increasing value.

1.1.1 Importance of engineering drawing

Engineering drawing is a two dimensional representation of a three-dimensional object. It is the graphic language, from which a trained person can visualize the object. As an engineering drawing displays a precise picture of the object to be produced, it conveys the same picture to every trained eye. Drawings prepared in one country may be utilized in any other country, irrespective of the language spoken there. Hence, engineering drawing is called the universal language of engineers.

Knowledge in engineering drawing is equally essential for the persons holding responsible positions in engineering field. An engineer without adequate knowledge of this language is considered to be professionally illiterate.

1.1.2 Role of drawing in engineering education

The ability to read drawings is the most important requirement of all technical people in engineering profession. The potentialities of drawing as an engineer's language may be made use of as a tool for imparting knowledge and providing information on various aspects of engineering.

The classification of engineering drawings include: Building drawing, machine drawing, electrical drawing, etc.

While teaching majority of subjects; figures or sketches of related objects, machines or systems are made use of, to explain the principles of operation, relation between the parts, etc. Unless the figures are presented, following the norms of draughting practice, the required information cannot be fully conveyed. Hence, the knowledge in engineering drawing is useful in understanding the other subjects as well.

1.1.3 Scope of the subject

The subject matter presented here relates to basic engineering drawing. It mainly deals with geometrical drawing. It is the art of representation of geometrical objects on a drawing sheet and is the foundation of all engineering drawings.

Plane geometrical drawing deals with the representation of objects having two dimensions. Solid geometrical drawing deals with the representation of objects having three dimensions.

1.2 DRAUGHTING TOOLS

The drawing instruments or draughting tools are used to produce drawings quickly and more accurately. To obtain satisfactory results in the form of accurate drawings, the draughting tools used must be of high quality. The students are advised to procure quality draughting tools, which will facilitate to increase efficiency in their draughting work.

The present chapter deals with description of draughting tools used by professional draughtsmen and their methods of use. The following is the list of a majority of draughting tools used by professional draughtsmen:.

1. Drawing board
2. Mini-draughter
3. Instrument box, containing the following:
   - (i) Compass
   - (ii) Bow-compass
   - (iii) Spring bow-compass
   - (iv) Divider
   - (v) Bow-divider
   - (vi) Bow-pen
   - (vii) Inking pen
4. 30°-60° and 45°-45° set-squares
5. Protractor
6. Set of scales
7. French curves
8. Flexible curve
9. Templates
10. Drawing sheet
11. Paper fasteners
12. Pencils
13. Eraser
14. Erasing shield

15. Draughting brush
16. Drawing ink
17. Tracing paper
18. Lettering pens

1.3 DRAWING BOARD

Drawing boards are usually made of well-seasoned soft wood. To prevent warping, narrow strips of wood are glued together. Prevention of warping, will also be aided by two battens cleated at the bottom side of the board. In addition, the battens help to give rigidity to the board and raise it above the surface of the drawing table (Fig. 1.1). One edge (width) of the drawing board is provided with a working edge, made of hard and durable wood. The working edge is required while using T-square in draughting work.

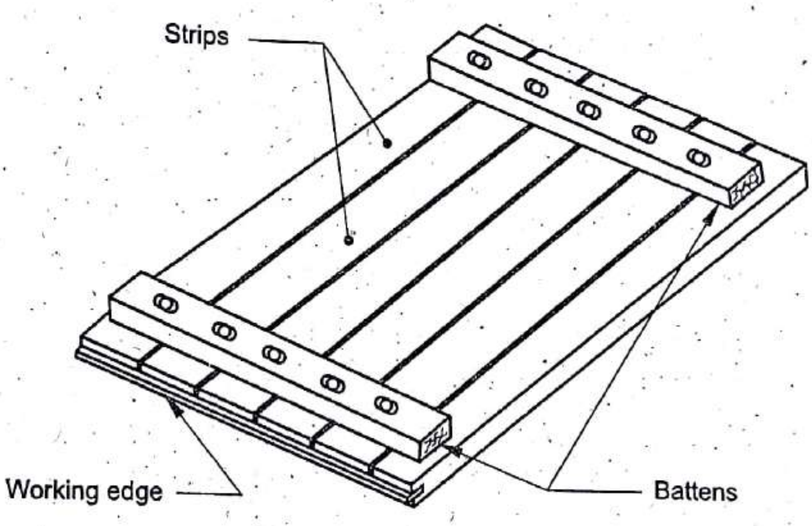


Fig. 1.1 Drawing board

The size of the drawing board will depend upon the size of the drawing paper used. Bureau of Indian Standards (BIS) recommends the sizes for the drawing board, as given in Table 1.1.

Table 1.1 Sizes of drawing board

Designation	Size (mm)
B0	1250 × 900
B1	900 × 650
B2	650 × 500
B3	500 × 350

1.3.1 Care of the drawing board

1. Handle the drawing board carefully so that no dents or holes are made on its surface.

2. Check the working edge at regular intervals and correct it whenever it is found defective. This is required only when T-square is used, instead of mini-draughter, for draughting work.
3. Fasten a sheet of paper on the board to keep its surface clean.

1.4 MINI-DRAUGHTER

Today in many drawing offices, a unit known as mini-draughter, similar to the one shown in Fig. 1.2 is used. It is designed to combine the functions of T-square, set-squares, protractor and scales. This unit, when used, can result in savings of approximately 35% of time in machine drawing and 50% of time in structural drawing work. It consists of an angle, formed by two arms with scales marked and set exactly at right angle to each other. The angle is removable and hence, a variety of scales may be used.

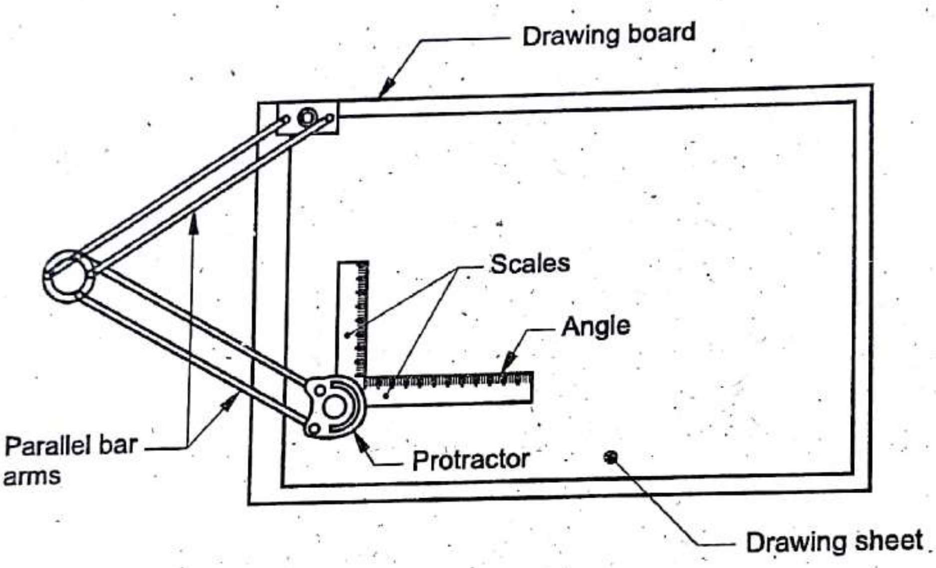


Fig. 1.2 Mini-draughter

In the normal position, one of the two arms is horizontal and the other vertical. The arms may also be set and clamped at any desired angle by means of an adjusting head, which has a protractor (with vernier attachment in some cases). The angle of 90° between the arms of course, remains unaltered. When the angle formed by the two arms is moved over the drawing sheet, the design of the unit permits the arms to move only into parallel positions.

1.4.1 Use and care of the unit

Before commencing drawing work, the mini-draughter must be so positioned that the edge of the horizontal arm is in-line with the horizontal edge of the drawing sheet, and at the same time, the protractor head should read 0°. Later, whenever work is re-started on the incomplete drawing (sheet), the edge of the horizontal arm should be made to coincide with any horizontal line already drawn, simultaneously ensuring 0° setting of the protractor head.

CHAPTER - 2

PRINCIPLES OF GRAPHICS

2.1 INTRODUCTION

Any language to be communicative, should follow certain rules so that it conveys the same meaning to every one. Similarly, draughting practice must follow certain rules, if it is to serve as a means of communication. For this purpose, Bureau of Indian Standards (BIS), adopting the International Standards on code of practice for drawing, has formulated certain rules and published through BIS, SP: 46-2003, titled, "Engineering Drawing practices for schools and colleges". These are included in this chapter and followed throughout the text.

The other foreign standards are DIN (Germany), ANSI (USA) and BS (UK).

2.2 DRAWING SHEET

2.2.1 Drawing sheet sizes

In metric units, drawing sheet sizes are based on A0 size with an area of 1 square metre, having a length to width ratio of $\sqrt{2}:1$. The area of the succeeding smaller size is half of the preceding one; the length to width ratio being constant (Fig.2.1). The preferred standard sizes for drawing sheets as specified by Bureau of Indian Standards are given in article 1.10.

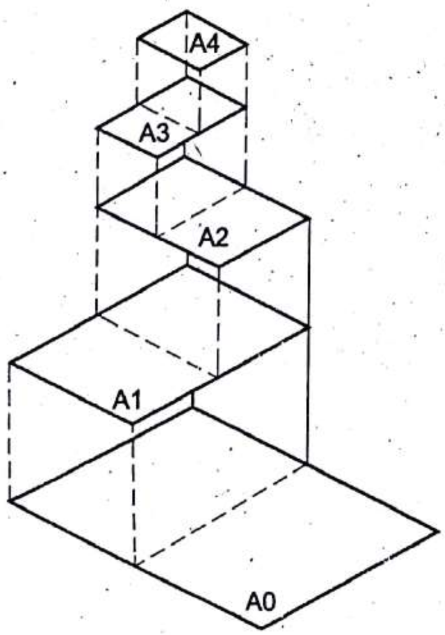


Fig. 2.1 Drawing sheet formats

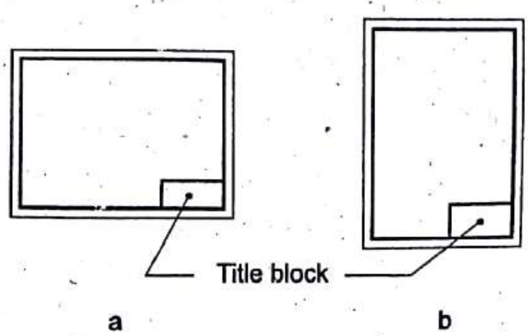


Fig. 2.2 Location of title block

2.2.3.4 *Grid reference system (Zoning)* The grid reference system is recommended for all sizes, in order to permit easy location on the drawing, any details, additions, modifications, etc. The number of divisions should be divisible by two and be chosen in relation to the complexity of the drawing. It is recommended that the length of any side of the grid should not be less than 25 mm and not more than 75 mm. The rectangles of the grid should be referenced by means of capital letters along one edge and numerals along the other edge. The numbering direction may start at the corner, opposite to the title block and be repeated on the opposite sides.

2.2.3.5 *Trimming marks* Trimming marks are provided in the borders at the four corners of the sheet, in order to facilitate trimming. These marks may be in the form of right-angled isosceles triangles or two short strokes at each corner (Fig. 2.4).

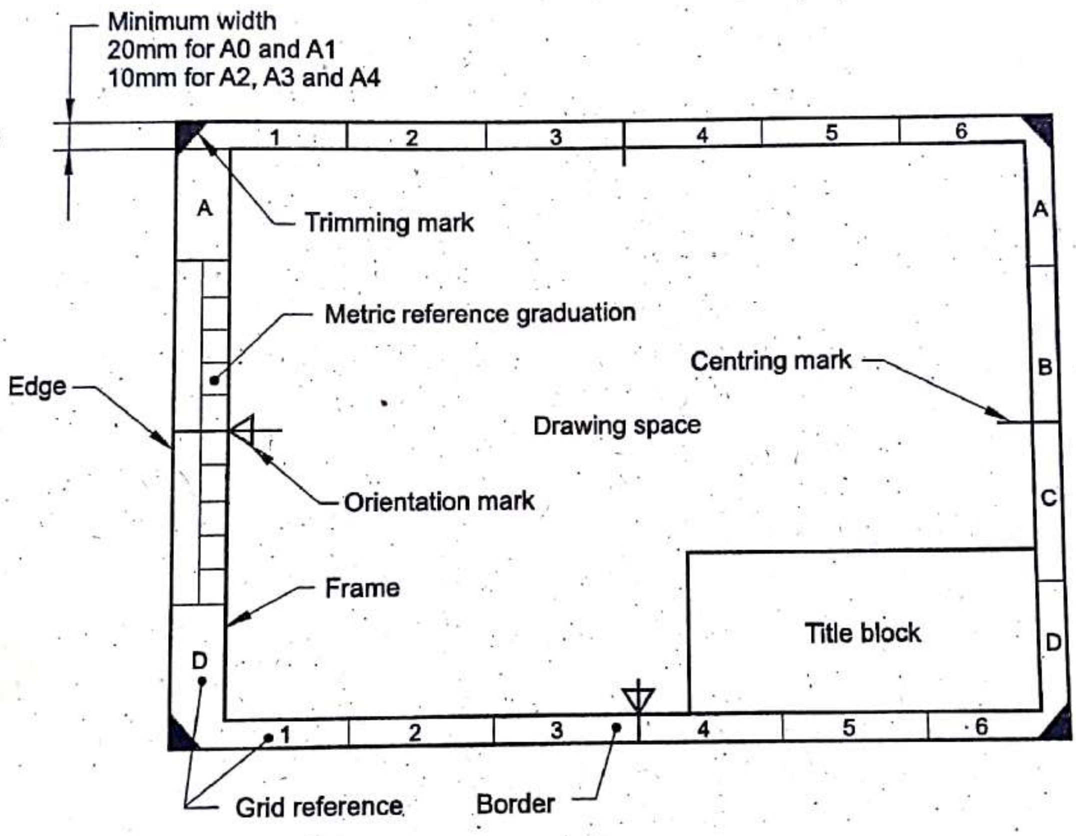


Fig. 2.4 Drawing sheet layout

2.3 FOLDING OF DRAWING SHEETS

There are two methods of folding drawing sheets. In the first method (Fig. 2.5), the drawing sheets are so folded that they can be filed or bound. In the second method (Fig. 2.6), the sheets are folded so that, they can be kept individually in a filing cabinet.

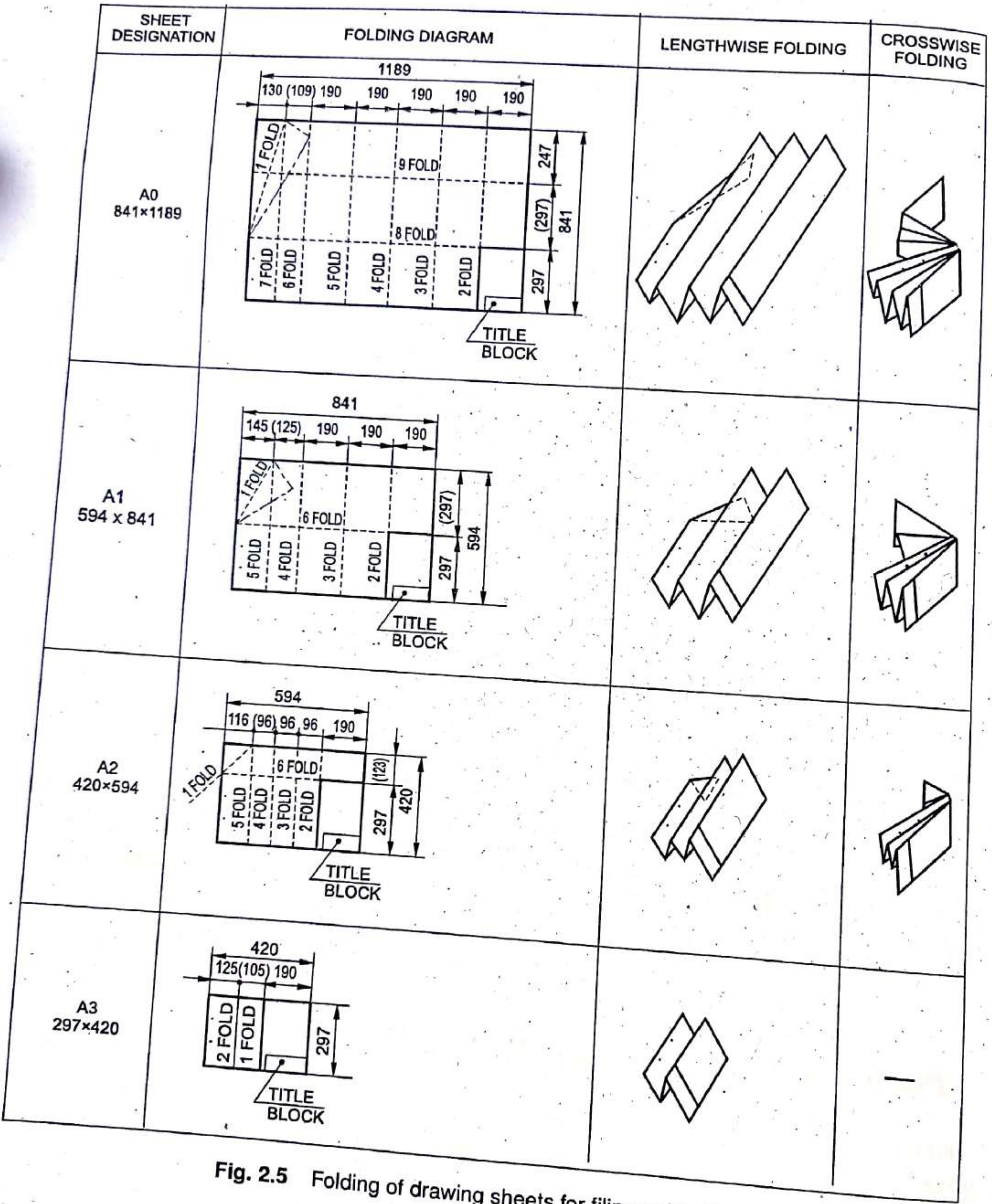


Fig. 2.5 Folding of drawing sheets for filing or binding

2.4 LINES

The various types of lines used in drawing, form the alphabet of the draughting language. Just like the letters of the alphabet, the various lines differ in appearance, thickness and construction. For better understanding of the drawing, the contrast between the various lines must be good. Table 2.1 shows the various types of lines with the recommended applications. Figure 2.7 shows a drawing, indicating the typical applications of different types of lines.

Table 2.1 Types of lines and their applications

Line	Description	General applications
A	Continuous thick	A1 Visible outlines and edges
B	Continuous thin (straight or curved)	B1 Imaginary lines of intersection B2 Dimension lines B3 Projection lines B4 Leader lines B5 Hatching lines B6 Outlines of revolved sections B7 Short centre lines
C	Continuous thin, free-hand	C1 Lines of partial views and sections
D	Continuous thin with zig-zags	D1 Line (see Fig. 2.7)
F	Dashed thin	F1 Hidden outlines and edges
G	Chain thin	G1 Centre lines G2 Lines of symmetry G3 Trajectories
H	Chain thin, thick at ends and changes of direction	H1 Cutting planes
K	Chain thin, double dashed	K1 Outlines of adjacent parts K2 Alternate and extreme positions of movable parts

NOTE

(i) Centre lines should project only a short distance beyond the outline to which they refer. But, in case of necessity such as for dimensioning, the centre line may further be extended.

(ii) The dotted lines, representing the invisible features, must be shown only when their presence aids in the interpretation of the drawing. Otherwise they may be omitted, as their presence only causes confusion. However, until perfection is achieved, the student is advised to follow the principles of graphics in toto.

2.6.4 Hatching lines

The sectioned portions in sectional views are represented by thin hatching lines, drawn usually at an angle of 45° to the major outline of the drawing. The spacing of the hatching lines should, as far as possible be uniform to give a better appearance to the drawing. The exact distance between the lines will depend upon the size of the drawing. Figure 2.14 shows the preferred hatching angles to be used, depending upon the orientation of the outline of the drawing.

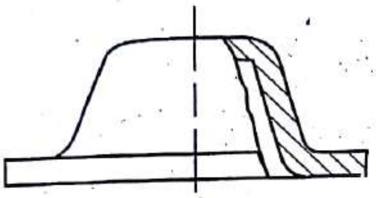


Fig. 2.13 Local section

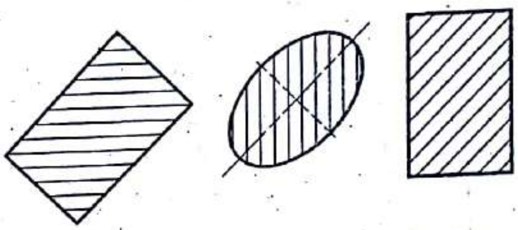


Fig. 2.14 Preferred hatching angles

Separated areas of a section of a single component must be hatched in the same direction. But two components, adjacent to each other in a section must be represented, by hatching lines in opposite directions, as shown in Fig. 2.15.

In case of large areas under section, the hatching lines may be limited to a small zone along the contour, as shown in Fig. 2.16.

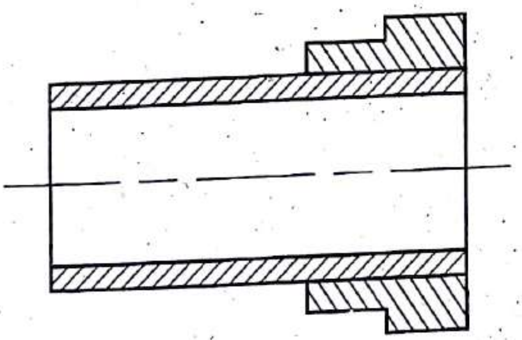


Fig. 2.15 Hatching of adjacent components

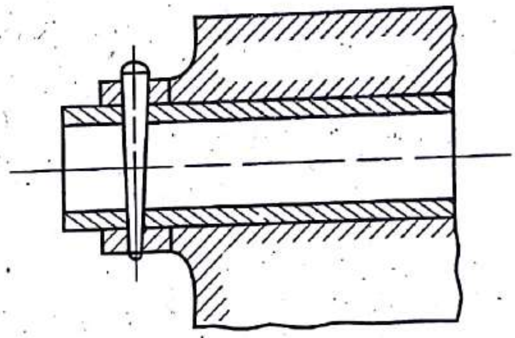


Fig. 2.16 Hatching of large areas

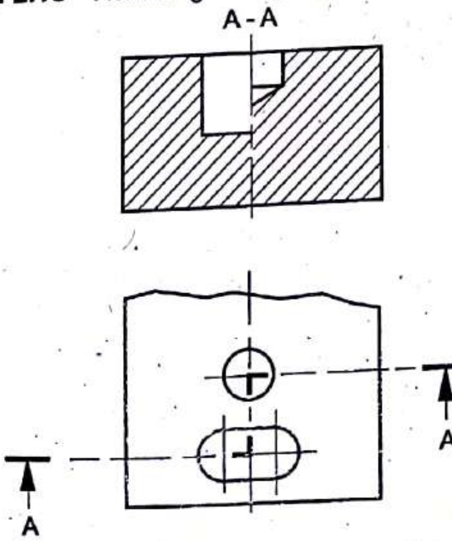


Fig. 2.17 Sectioning along two parallel planes

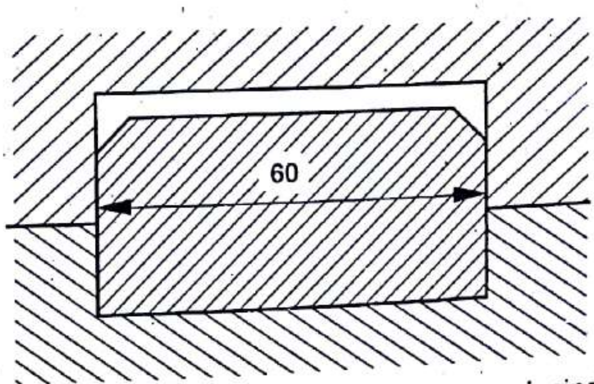


Fig. 2.18 Hatching interrupted for dimensioning

When a component undergoes sectioning along two parallel planes, the hatching lines in the sectional view must be spaced equally, but must be off-set along the dividing line between the sections, as shown in Fig. 2.17.

When a dimension is required to be placed in the sectioned zone, the hatching must be locally removed to place the dimension, as shown in Fig. 2.18.

2.7 DIMENSIONS

An object can be represented by a number of views. The entire drawing, consisting of number of views is called a working drawing, provided it gives complete information necessary for the trades person to produce the object. The working drawing must then consists of the views necessary to describe the shape of the object, the dimensions required for the trades person and other specifications such as material, quality, types of machining operations, etc. Any later information is normally provided in the form of a note either on the drawing or in the title block.

Dimensions are indicated on the drawing to define geometric characteristics such as lengths, diameters, radii, angles and locations.

2.7.1 Elements of dimensioning

The elements of dimensioning include the projection line, dimension line, leader line, dimension line termination, the origin indication and the dimension itself. The various elements of dimensioning are shown in Fig. 2.19.

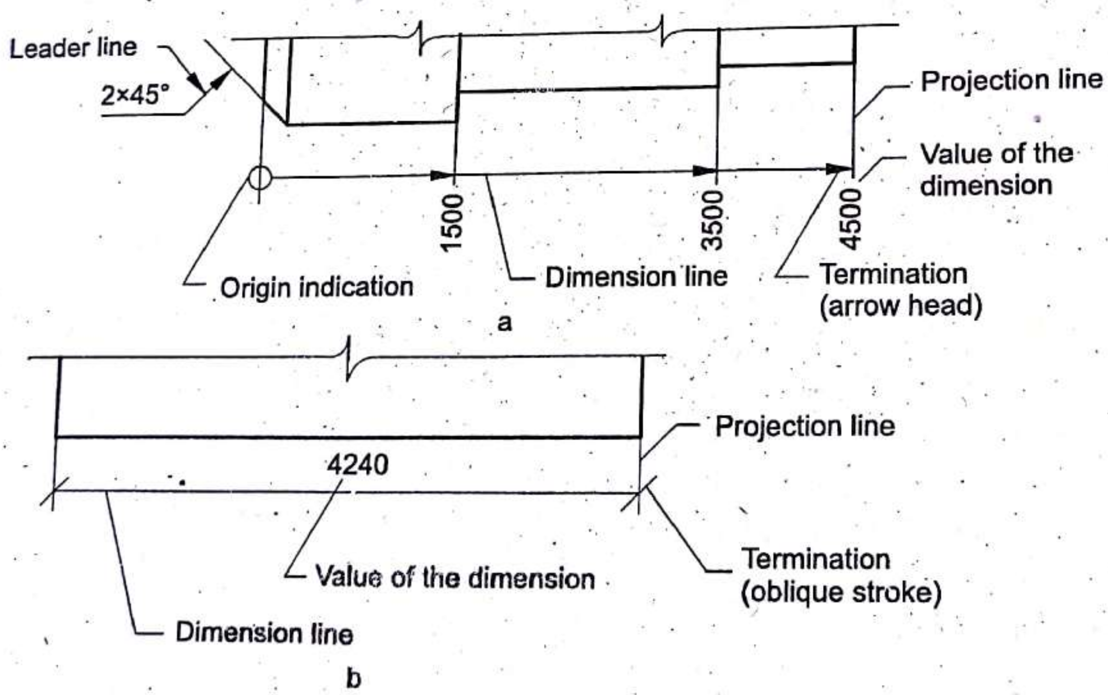


Fig. 2.19 Elements of dimensioning

2.7.2 Principles of dimensioning

The following are some of the principles to be applied while dimensioning:

1. Any dimension given, must be clear and permit only one interpretation.

2. Dimensions indicated in one view need not be repeated in another view, except for the purpose of identification, clarity or both.

3. Dimensions shall be placed on the view, where the shape is best shown (Fig. 2.20).

4. As far as possible, dimensions should be placed outside the view, as shown in Fig. 2.21.

5. Dimensions should not be placed very near to the parts being dimensioned, as shown in Fig. 2.22.

6. Dimensions should be marked from visible outlines rather than from hidden lines, as shown in Fig. 2.23.

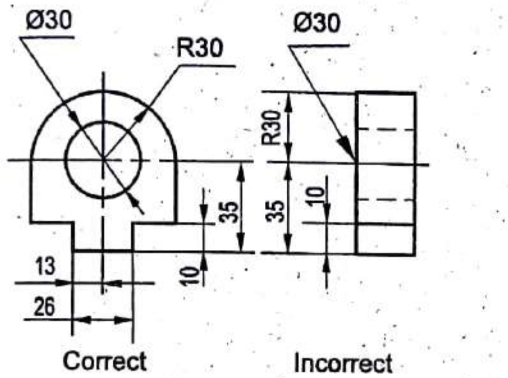


Fig. 2.20 Dimensioning where the shape is best shown

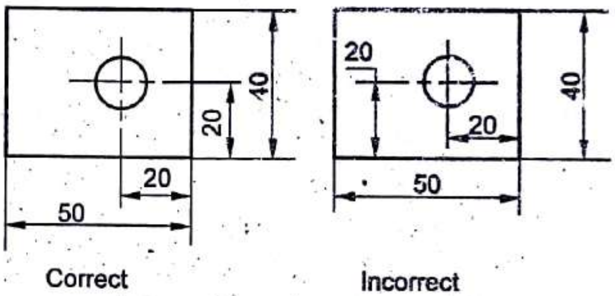


Fig. 2.21 Placing dimensions outside the view

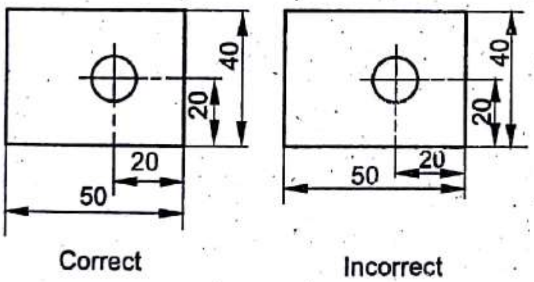


Fig. 2.22 Spacing of dimension lines

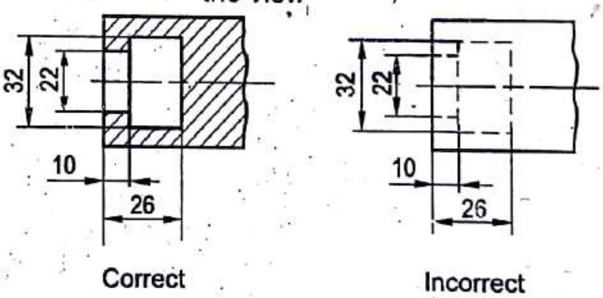


Fig. 2.23 Placing dimensions from the visible outlines

7. Dimensions should be marked from a base line or centre line of a hole or cylindrical part or finished surfaces, etc. Dimensioning to a centre line should be avoided except when the centre line passes through the centre of a hole or a cylindrical part, as shown in Fig.2.24.

8. An axis or a contour line should never be used as a dimension line but may be used as a projection line, as shown in Fig. 2.25.

9. The crossing of dimension lines should be avoided as far as possible. Intersecting projection and dimension lines should also be avoided (Fig. 2.26a). However, when it is unavoidable, the lines should not be shown with a break. Dimension line should not be used as an extension line (Fig. 2.26b).

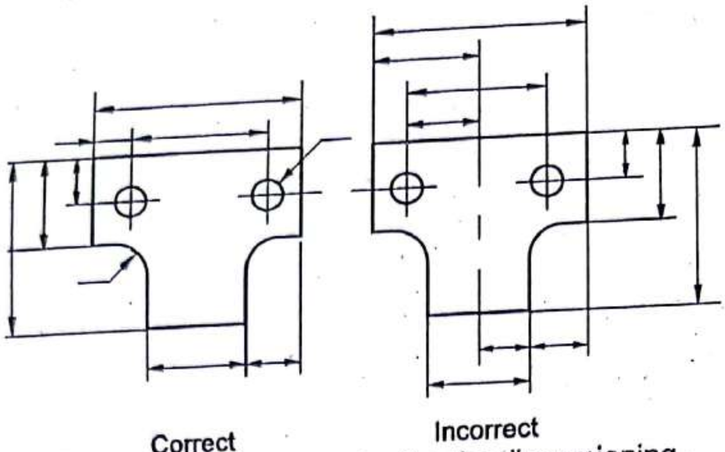


Fig. 2.24 Avoiding centre line for dimensioning

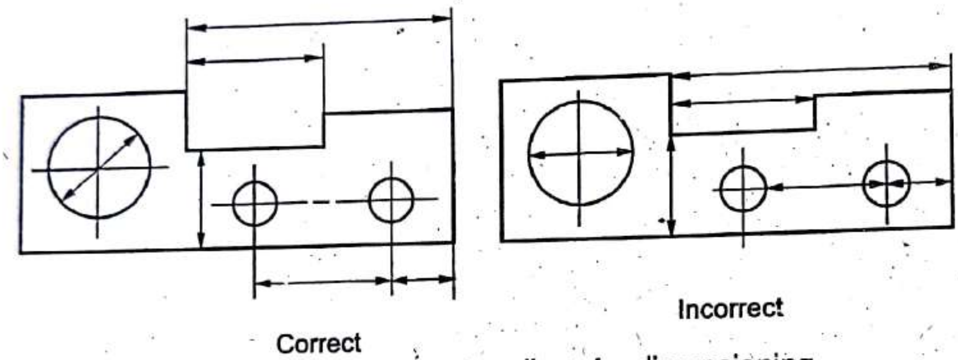


Fig. 2.25 Avoiding contour lines for dimensioning

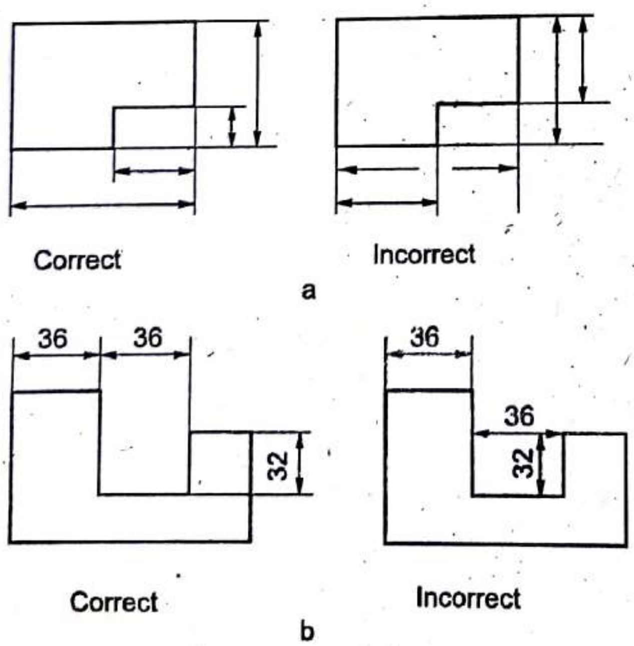


Fig. 2.26 Avoiding crossing of dimension lines

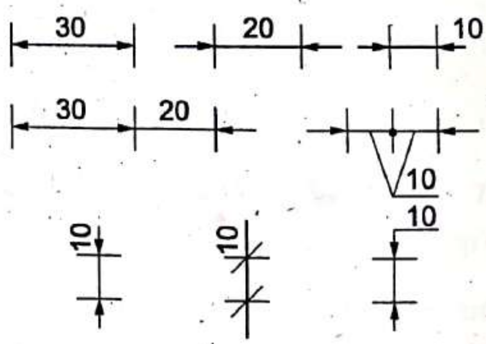


Fig. 2.27 Different methods of dimensioning a length

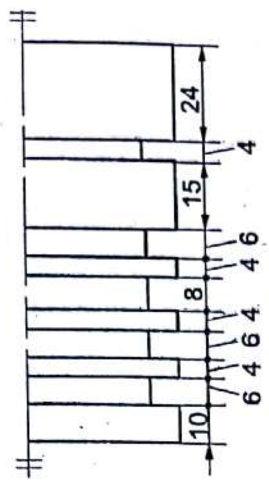


Fig. 2.28 Use of dots for dimensioning

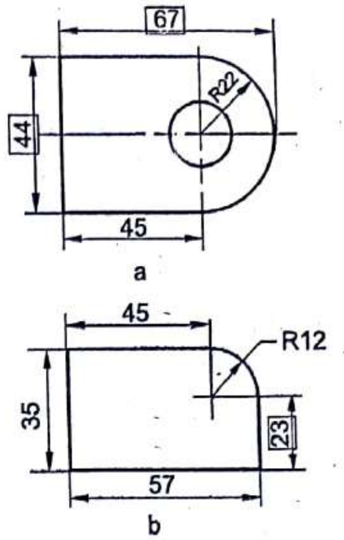


Fig. 2.29 Indication of only necessary dimensions

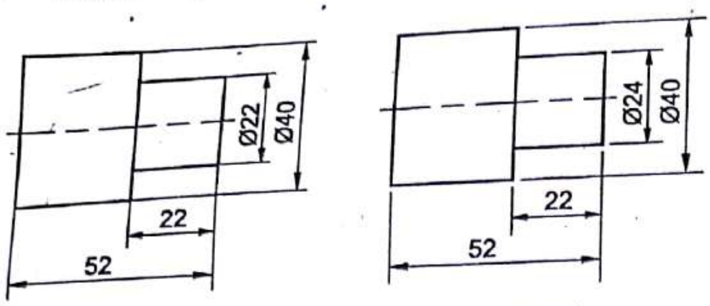
10. Dimensions should be expressed in one unit only, preferably in millimetres. The unit "mm" can then be dropped at the end of each dimension, by adding a note separately that all dimensions are in millimetres.
11. When the space for dimensioning is insufficient, the arrow heads may be reversed, as shown in Fig. 2.27 or adjacent arrow heads may be replaced by a dot, as shown in Fig. 2.28.
12. No more dimensions than are necessary to describe a part shall be shown on a drawing. No feature of a part shall be defined by more than one dimension in any one direction (Fig.2.29). For example, in Fig. 2.29, the dimensions shown in boxes are redundant.

NOTE The unit "mm" is dropped both in dimensioning and in the script throughout the book.

2.7.3 Execution of dimensions

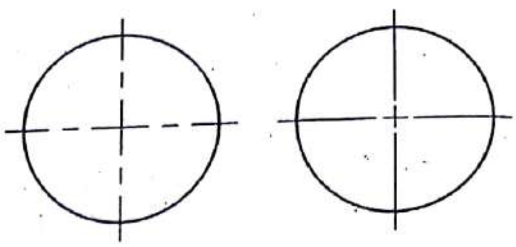
The following are the points to be observed while executing dimensions:

1. Projection lines should be drawn from the visible features of the object, without a gap and extend slightly beyond the dimension line (Fig.2.30).
2. Crossing of centre lines should be done by a long dash and not a short dash (Fig.2.31).
3. A dimension line should be shown unbroken, even where the feature to which it refers is shown broken (Fig. 2.32).
4. Projection lines must be drawn perpendicular to the outline of the feature to be dimensioned. In some cases such as on tapered features, these may be drawn obliquely but parallel to each other, as shown in Fig.2.33.



Correct Incorrect

Fig. 2.30 Avoiding gap between feature and projection line



Correct Incorrect

Fig. 2.31 Crossing of centre lines

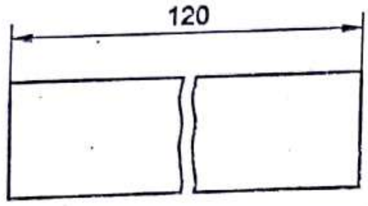


Fig. 2.32 Unbroken dimension line

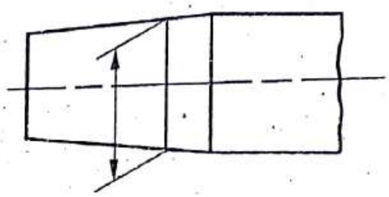


Fig. 2.33 Projection lines on a tapered feature

a b c

Fig. 2.34 Termination of leader lines

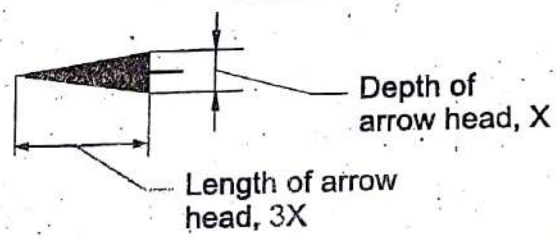


Fig. 2.35 Proportions of an arrow head

5. A leader line is a line referring to a feature (dimension, object, outline, etc.). It is drawn at an angle greater than 30°. Leader lines (Fig.2.34) should terminate :

   (i) with a dot, if they end within the outlines of an object.

   (ii) with an arrow head, if they end on the outline of an object.

   (iii) without dot or arrow head, if they end on a dimension line.

6. For arrow heads used at the ends of dimension and leader lines, the length may be taken as three times the depth, and the space is filled. The size of the arrow head should be proportionate to the size of the drawing (Fig.2.35).

7. When several arcs are dimensioned, it is preferable that separate leaders be used rather than extending the leaders (Fig.2.36).

8. Dimension lines should show distinct termination in the form of an arrow head or oblique stroke or where applicable, an origin indication (Fig.2.37).

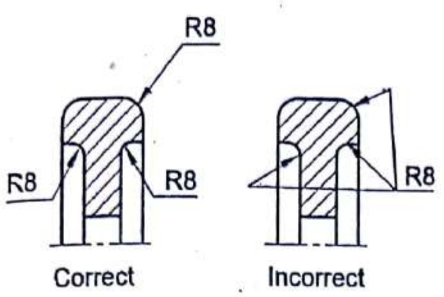


Fig. 2.36 Dimensioning several arcs

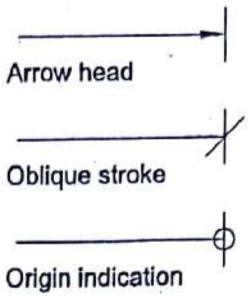


Fig. 2.37 Terminations of dimension lines

2.7.4 Placing of dimensions

Dimensions may be placed according to either of the following recommended systems:

2.7.4.1 *Aligned system* In an aligned system, all the dimensions are placed above the dimension lines such that, they may be read either from the bottom or from the right hand side of the drawing, as shown in Fig.2.38. Dimensions on oblique dimension lines shall be oriented as shown in Fig.2.39 and except where unavoidable, they should not be placed in the 30° zone. Angular dimensions may be oriented as shown in Fig.2.40.

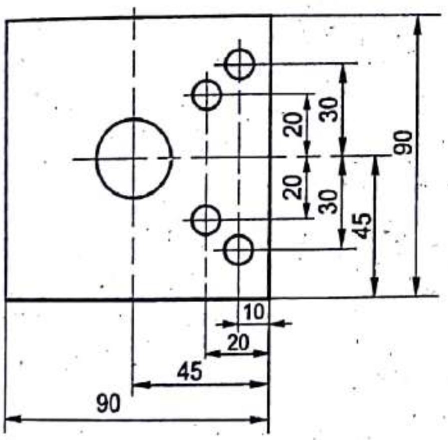


Fig. 2.38 Dimensioning-Aligned system

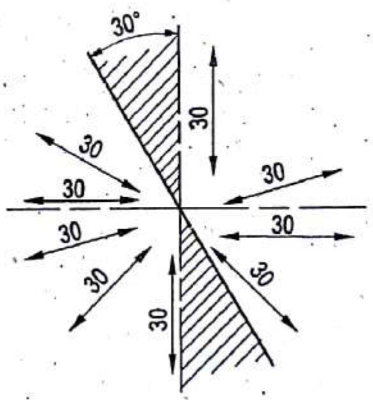


Fig. 2.39 Dimensioning on oblique dimension lines

2.7.4.2 *Uni-directional system* In uni-directional system, all the dimensions are placed in one direction such that they may be read from the bottom of the drawing only. Also, in this system, the non-horizontal dimension lines are interrupted, preferably in the middle, for insertion of the dimension, as shown in Fig.2.41. This system is useful for very big drawings, where it is inconvenient to read the dimensions from two sides. Angular dimensions may be oriented as shown in Fig. 2.42.

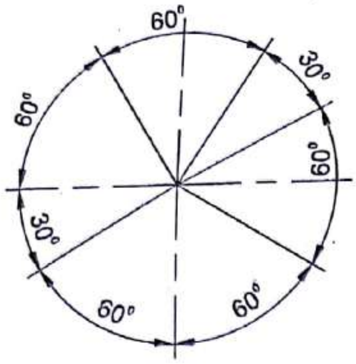


Fig. 2.40 Angular dimensions-Aligned system

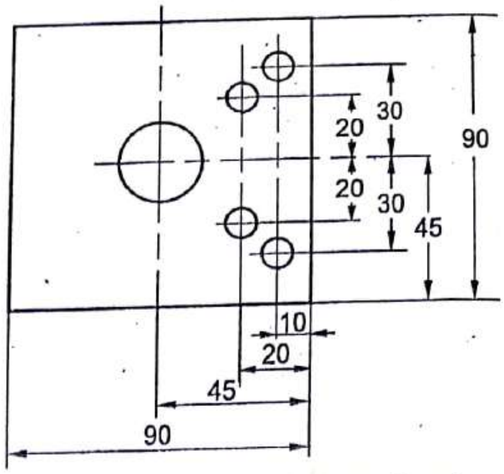


Fig. 2.41 Dimensioning-Uni-directional system

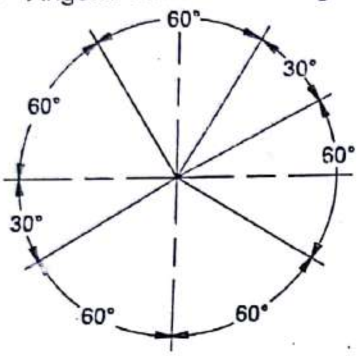


Fig. 2.42 Angular dimensions-Uni-directional system

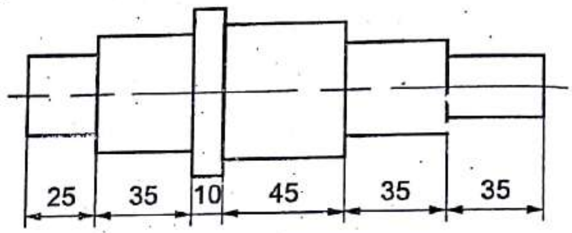


Fig. 2.43 Chain dimensioning

2.7.5 Arrangement of dimensions

2.7.5.1 *Chain dimensions* Chain dimensioning should be used only where the possible accumulation of tolerances does not endanger the functional requirement of the object, as shown in Fig. 2.43.

2.7.5.2 *Parallel dimensions* Parallel dimensioning is followed when a number of dimensions have a common datum feature, as shown in Fig. 2.44.

2.7.5.3 *Combined dimensions* In combined dimensioning, both the chain and parallel dimensions are followed, as shown in Fig. 2.45.

2.7.6 Dimensioning of common features

The following indications are used with dimensions for shape identification and to improve the drawing interpretation (Fig.2.46). Dimensioning of holes and circles may be made as shown in Fig.2.47. Figure 2.48 shows the method of dimensioning an arc, when the centre falls beyond the limits of the space permitted.

CHAPTER - 3

GEOMETRICAL CONSTRUCTIONS

3.1 INTRODUCTION

There are a number of geometrical constructions with which a draughtsman or an engineer should be familiar, as they frequently occur in Engineering Drawing. The methods presented in this chapter are actually applications of the principles of plane geometry. Since the subject of plane geometry is a pre-requisite for a course in "Engineering Drawing," the mathematical proofs are omitted here.

3.2 SIMPLE CONSTRUCTIONS

3.2.1 To bisect a given arc

Construction (Fig. 3.1)

1. Draw the given arc AB.
2. With centre A and radius equal to more than half the chord length AB, draw arcs on either side of AB.
3. With centre B and the same radius, draw arcs intersecting the above arcs at C and D.
4. Draw a line through C and D to intersect the given arc at E.

   The point E bisects the given arc.

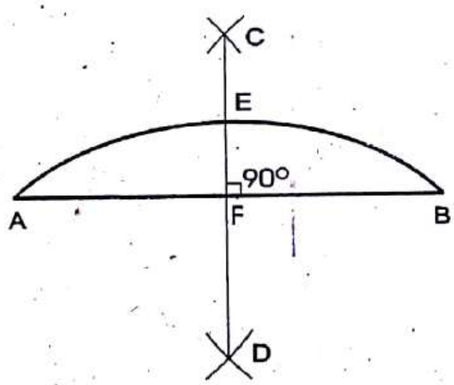


Fig. 3.1 Bisection of an arc

NOTE 1. The above procedure may be followed to bisect the given line AB.

2. The point F bisects the line AB and the line CD is called the perpendicular bisector of the line AB.

3.2.2 To divide a given line into a specified number of parts, say six

(May/June 2010, JNTU)

Construction (Fig. 3.2)

1. Draw the given line AB.
2. Through A, draw a line AC, making any convenient angle with AB.

3. Set the compass to any convenient length and lay-off eight equal divisions 1′, 2′, —— 6′, starting from A.
4. Join 6′, B.
5. Draw lines through 5′, 4′, 3′, etc., and parallel to 6′ B, to meet the line AB at 5, 4, 3, etc.

   The points 1, 2, 3, etc., divide the line AB into six equal parts.

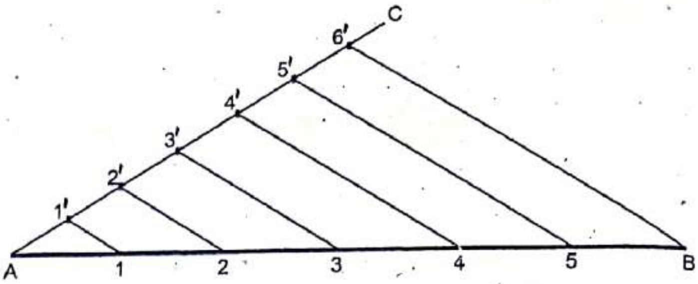


Fig. 3.2 Division of a line into a number of equal parts

3.2.3 To divide a given line into unequal parts by proportioning

Construction (Fig. 3.3)

1. Draw the given line AB.
2. Construct a square/rectangle ABCD, choosing the side AD as desired.
3. Draw diagonals AC and DB and locate the mid - point E at the intersection of the diagonals.
4. Through E, draw a perpendicular to AB, meeting at the mid-point F of the line AB.
5. Join D,F and locate the intersection point G on the line AC.
6. The line through G and perpendicular to AB intersects it at H (AH=1/3 AB).
7. Similarly, locate the sub-divisions 1/4, 1/5, 1/6, etc.

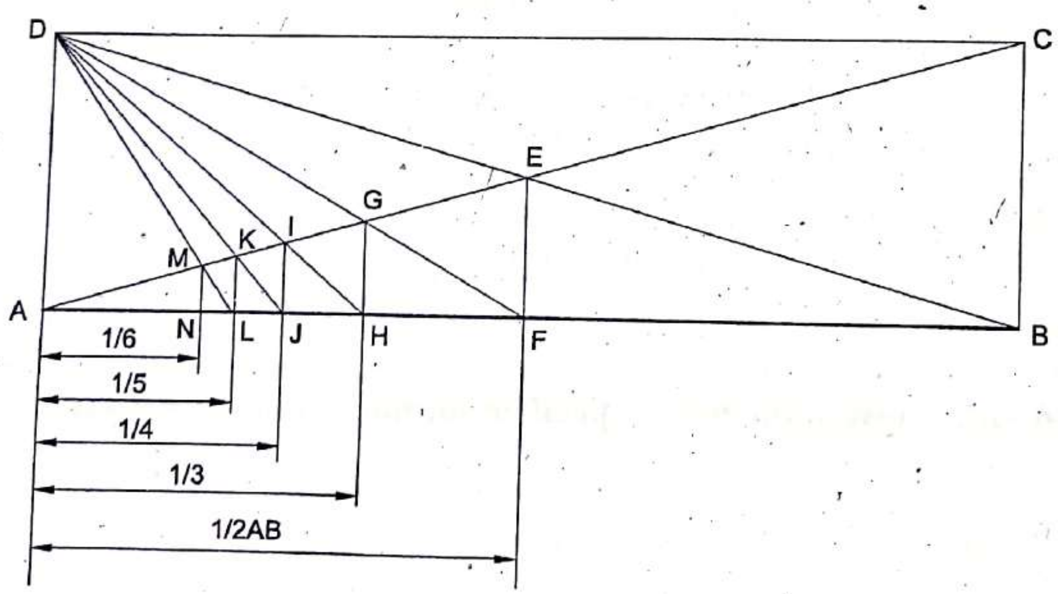


Fig. 3.3 Proportionate division of a line

3.2.4 To bisect a given angle

Case I When the vertex of the angle is accessible

Construction (Fig. 3.4)

1. Draw the lines AB and AC, making the given angle.
2. With centre A and any convenient radius R_1, draw an arc intersecting the sides of the angle at D and E.
3. With centres D and E and radius larger than half the chord length DE, draw arcs intersecting at F.
4. Join A, F.
   Now, $\angle$ BAF = $\angle$ FAC.

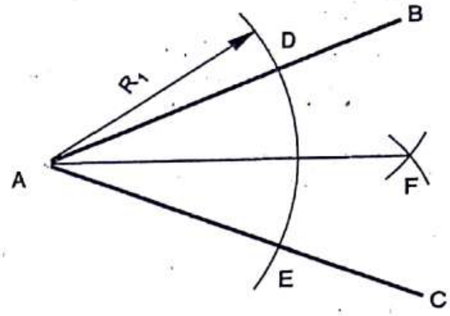


Fig. 3.4 Bisection of an angle

NOTE This procedure may be repeated to divide the given angle into 4, 8, 16, etc., equal parts.

Case II When the vertex of the angle is in-accessible

Construction (Fig. 3.5)

1. Draw the lines BC and DE, inclined at the given angle.
2. Draw a line FG parallel to BC, at any suitable distance x.
3. Draw a line FH parallel to DE, at the distance x.
4. Bisect the $\angle$GFH, following the method proposed in the preceding case.

   The line FI bisects the angle between the lines BC and DE.

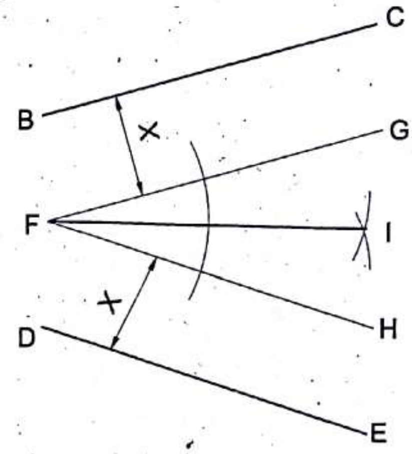


Fig. 3.5 Bisection of an angle with in-accessible vertex

3.2.5 To locate the centre of a given arc

Construction (Fig. 3.6)

1. Draw the given arc, AB.
2. Draw any two chords CD and EF to the given arc AB.
3. Draw perpendicular bisectors to CD and EF, intersecting each other at O.

   Point O is the required centre.

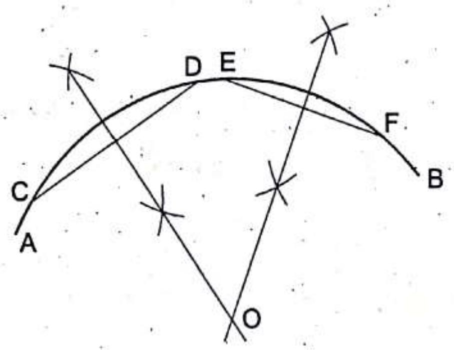


Fig. 3.6 Location of the centre of an arc

NOTE The above procedure may be followed to locate the centre of a circle.

3.2.6 To find the approximate length of the circumference of a given circle

Construction (Fig. 3.7)

1. With centre O, draw the given circle.
2. Draw vertical diameter through O, meeting the circle at A.

3. Draw the horizontal diameter through O, meeting the circle at D.
4. Through A, draw a line AB at right angles to AO.
5. Along the line AB, set-off distance AC equal to three times the diameter of the circle.
6. With centre D on the horizontal diameter and radius equal to the radius of the circle, draw an arc intersecting the circumference of the circle at E.
7. Draw the line EF, parallel to AB.
8. Join F, C.

The length FC is approximately equal to the circumference of the circle.

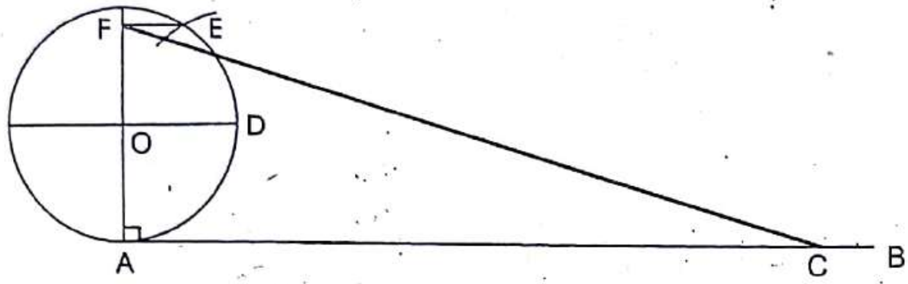


Fig. 3.7 Determination of the circumference of a circle

3.2.7 To draw an arc, passing through three points, not in a straight line

Construction (Fig. 3.8)

1. Locate the given points A, B and C.
2. Draw lines AB and BC.
3. Draw perpendicular bisectors of the lines AB and BC, intersecting at O.
4. With centre O and radius OA (=OB =OC), draw an arc.

The arc passes through the points A, B and C.

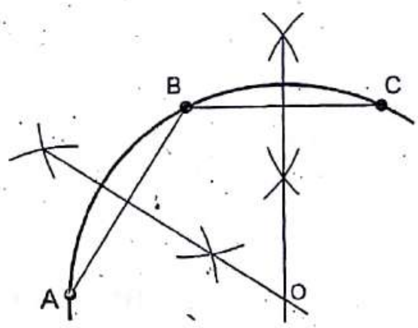


Fig. 3.8 Drawing an arc through three points

3.3 POLYGONS

A polygon is a plane figure, bounded by straight edges. When all the edges are of equal length, the polygon is said to be a regular polygon.

3.3.1 To construct a regular polygon of any number of sides N, given the length of its side

Method I

Construction (Fig. 3.9)

1. Draw a line AB equal to the length of the side.
2. With centre A and radius AB, draw a semi-circle.
3. Divide the semi-circle into the same number of equal parts, as the number of sides N, say 6.

4. Draw radial lines through 2, 3, 4, etc., (second division point 2 will always be a vertex of the polygon).
5. With centre B and radius equal to the side, draw an arc intersecting the radial line through 5 at C.
6. Repeat the procedure till the point on the radial line through 3 is obtained.

   The figure obtained by joining the points A, B, C, etc., is the required polygon.

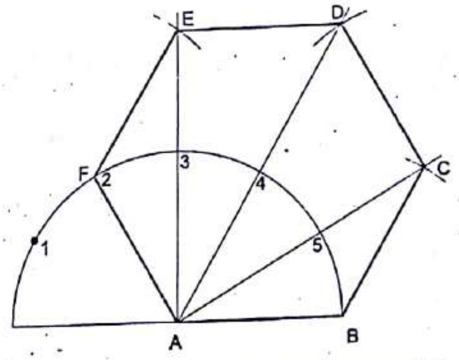


Fig. 3.9 Construction of a regular polygon-Method I

Method II

Construction(Fig. 3.10)

1. Follow the steps 1 to 4 as above.
2. Draw perpendicular bisectors of lines 2A and AB, intersecting at O.
3. With centre O and radius OA, draw a circle passing through the points 2 and B.
4. Locate the corners C, D, etc., of the polygon, where the circle meets the radial lines.

   The figure obtained by joining the points A, B, C, etc., is the required polygon.

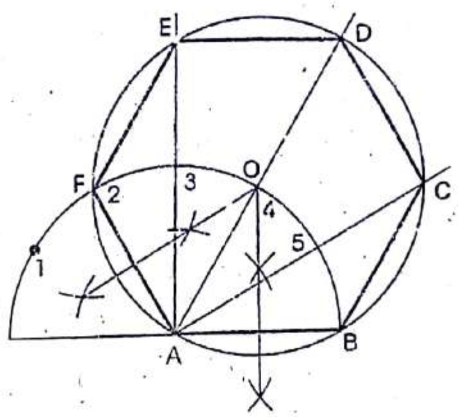


Fig. 3.10 Construction of a regular polygon-Method II

Method III

Construction (Fig. 3.11)

1. Draw a line AB equal to the given length of side.
2. At B, erect a perpendicular BC of length AB.
3. Join A, C.
4. With centre B and radius BA, draw the arc AC.
5. Draw a perpendicular bisector to AB, intersecting the line AC at 4 and the arc AC at 6.
6. Locate the mid -point of the line 4-6 and number it as 5.
7. Along the bisector, locate the points 7, 8, --- N, such that the distances 4-5 = 5-6 = 6-7, etc.
8. A pentagon of side equal to AB can be inscribed in a circle drawn with centre 5 and radius 5A.

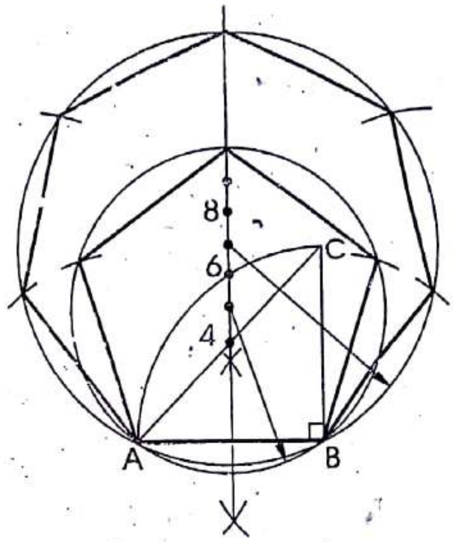


Fig. 3.11 Construction of a regular polygon-Method III

9. A heptagon of side equal to AB can be inscribed in a circle drawn with centre 7 and radius 7A.

   A polygon of any number of sides, N can be inscribed in a circle drawn with centre N and radius NA.

3.4 SPECIAL METHODS

3.4.1 To construct a pentagon, given the length of side

Construction (Fig. 3.12)

1. Draw a line AB, equal to the given length of side.
2. Bisect AB at P.
3. At B, erect a perpendicular BQ, equal in length to AB.
4. With centre P and radius PQ, draw an arc intersecting AB produced at R (AR is equal to the diagonal length of the pentagon).
5. With centres A and B and radii equal to AR and AB respectively, draw arcs intersecting at C.
6. With centres A and B and radii equal to AB and AR respectively, draw arcs intersecting at E.
7. With centres A and B and radius AR, draw arcs intersecting at D.

The figure obtained by joining the points B, C, D, E and A is the required pentagon.

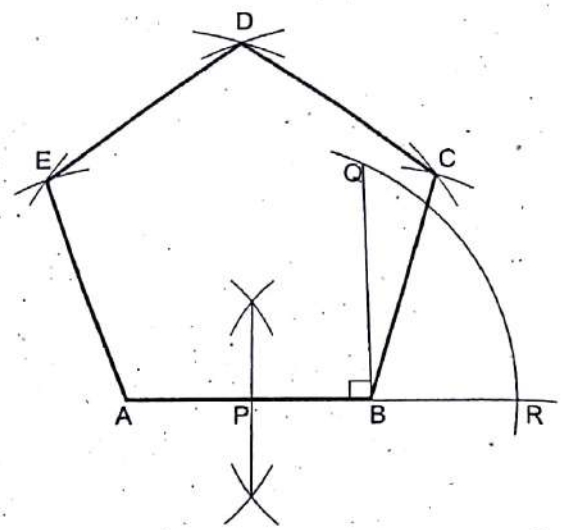


Fig. 3.12 Construction of a pentagon

NOTE The lines joining the alternate corners of a pentagon are called diagonals.

3.4.2 To construct a hexagon, given length of side

Construction (Figs. 3.13a and b)

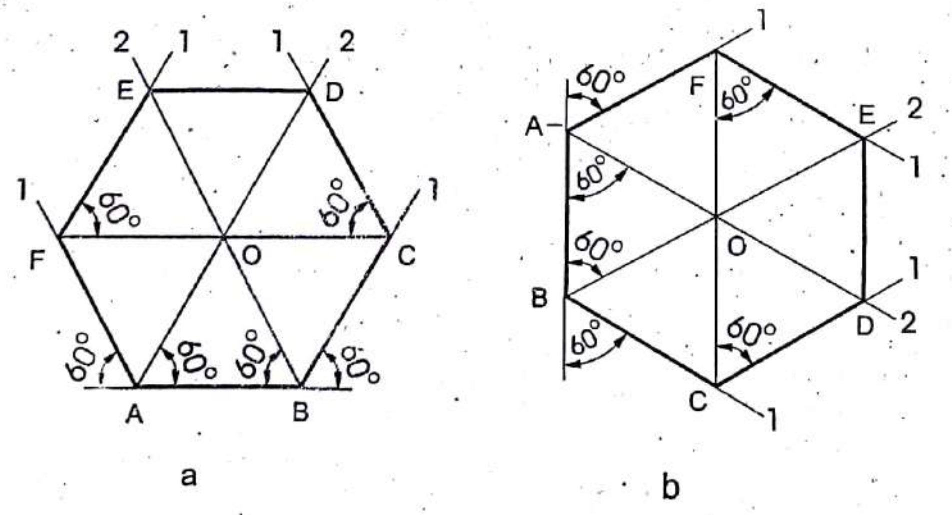


Fig. 3.13 Construction of a hexagon-given a side

$$\text{Scale} = \frac{20\text{ mm}}{1\text{ m}} = \frac{20\text{ mm}}{1000\text{ mm}} = \frac{1}{50} \text{ or } 1:50$$

If a maximum length of 10 m is required to be measured by the scale, the length of the scale is obtained by multiplying the scale factor with the maximum length required. Thus,

$$\text{The length of the scale} = \frac{1 \times 10,000}{50} = 200\text{ mm}$$

4.2.2 Metric measurements

The relation between the various units in SI system are given below:

10 mm = 1 cm (centimetre)
10 cm = 1 dm (decimetre)
10 dm = 1 m (metre)
10 m = 1 dam (decametre)
10 dam = 1 hm (hectometre)
10 hm = 1 km (kilometre)
1 hectare = 10,000 square metres

4.3 PLAIN SCALES

A line, suitably divided into equal parts (primary divisions) is called a plain scale. Generally, the first part is sub-divided into smaller parts (secondary divisions). Thus, a plain scale is used to represent either two units or a unit and its fraction such as km and hm; m and dm, etc.

Problem 1 *Construct a scale of 1: 8 to show decimetres and centimetres and to read upto 1m. Show a length of 7.6 dm on it.*

Construction (Fig. 4.1)

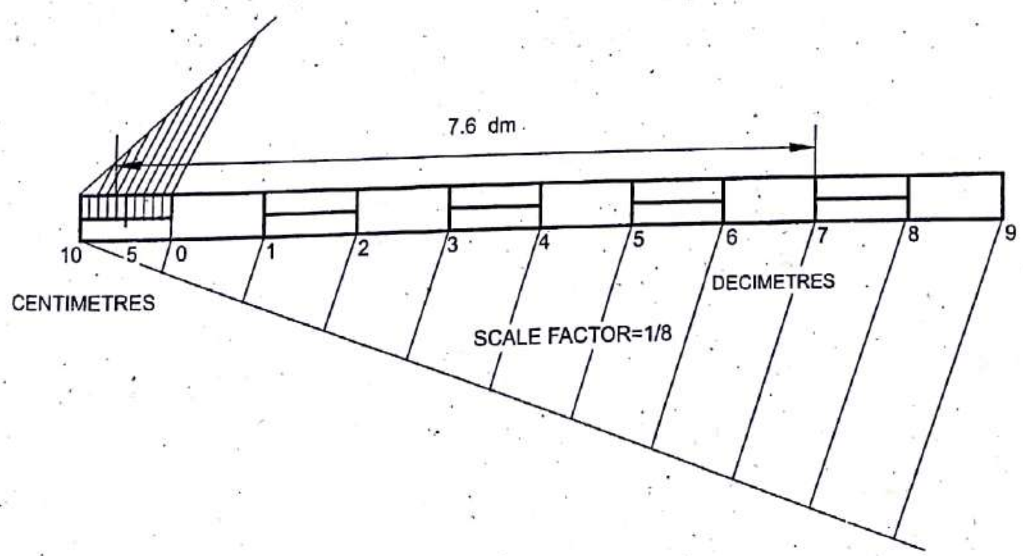


Fig. 4.1

CHAPTER - 5

CURVES USED IN ENGINEERING PRACTICE

5.1 INTRODUCTION

In engineering practice, the profiles of some of the objects contain regular curved features. Some are obtained as intersections, when a plane passes through a cone and some are obtained by tracing the locus of a point moving according to the mathematical relationship, applicable to that particular curve. The following types of curves are considered in this chapter:

Conic sections,
Cycloidal curves,
Involutes, and
Helices

5.2 CONIC SECTIONS

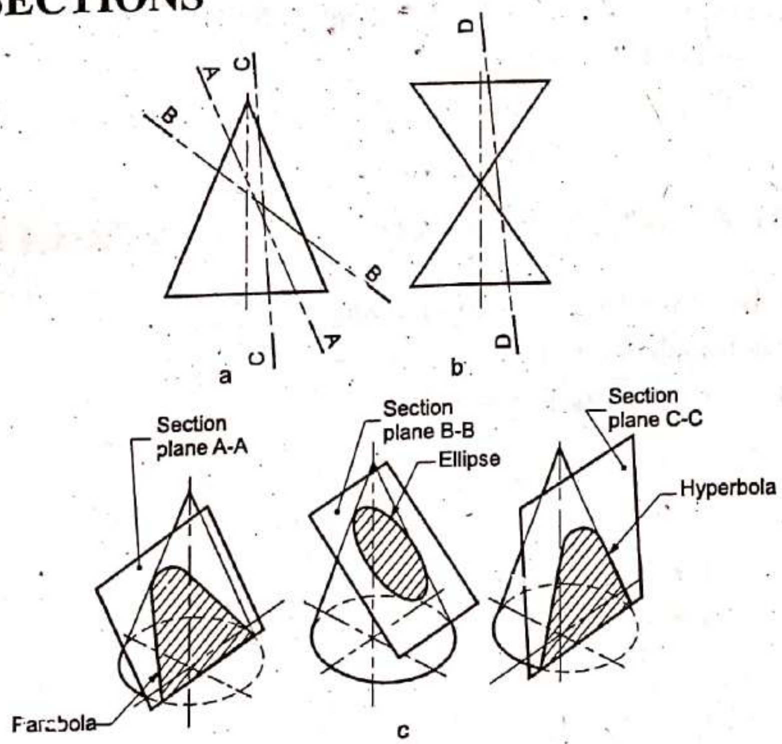


Fig. 5.1 Sections of a cone

The conic sections are the intersections of a right regular cone, by a cutting plane in different positions, relative to the axis of the cone. Considering the apex angle of the cone as 2θ, and the inclination of the cutting plane as α, the following are the possible conic sections (Fig. 5.1a).

5.2.1 Parabola

If the cutting plane angle α is equal to θ, i.e., when the section plane A-A is parallel to a generator of the cone, the curve of intersection is a parabola, which is not a closed curve. The size of the parabola depends upon the distance of the section plane from the generator of the cone.

5.2.2 Ellipse

When a cone is cut by a section plane B-B at an angle α, which is more than half of the apex angle θ and less than 90°, the curve of intersection is an ellipse. The size of the ellipse depends upon the angle α and the distance of the section plane from the apex of the cone. Further, it may be noted that only when the section plane cuts all the generators of the cone, the elliptical section formed is a closed curve.

5.2.3 Hyperbola

If the angle α is less than θ (section plane C-C), the curve of intersection is a hyperbola. The curve of intersection is hyperbola, even if $\alpha = 0$, i.e., section plane parallel to the axis, provided the section plane is not passing through the apex of the cone. If the section plane passes through the apex, the section produced is an isosceles triangle.

If a double cone is cut by a section plane D-D, on one side of the common axis (Fig. 5.1b), the curves of intersection result in two branches of hyperbola. However, if $\alpha=0$, the two branches of the curve will be symmetric in form.

Figure 5.1c shows the different conic sections, produced by the section planes A-A, B-B and C-C of Fig. 5.1a.

5.3 CONSTRUCTION OF CONICS - ECCENTRICITY METHOD

A conic section may be defined as the locus of a point moving in a plane such that, the ratio of its distance from a fixed point to a fixed straight line is always a constant.

The fixed point is called the focus and the fixed straight line, the directrix.

$$\text{The ratio, } \frac{\text{distance of the point from the focus}}{\text{distance of the point from directrix}}$$

is known as eccentricity, e. The value of e is less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola (Fig. 5. 2).

The line passing through the focus and perpendicular to the directrix is called the axis. The point at which the conic section intersects the axis is called its vertex.

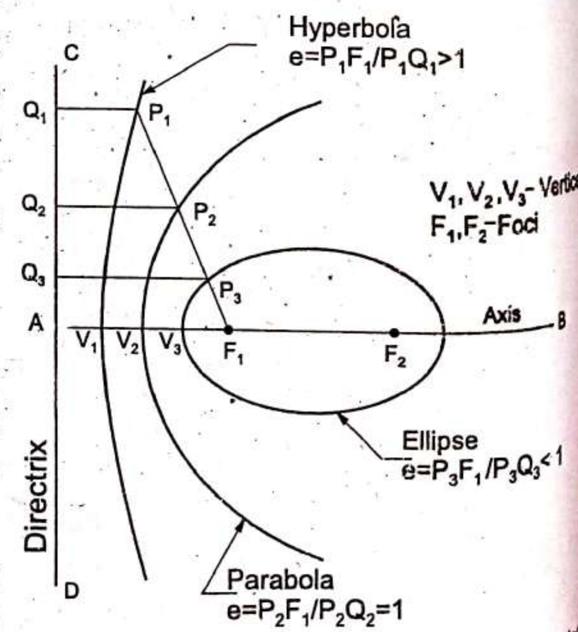


Fig. 5.2 Eccentricity values for different conics

Problem 1 *Construct a parabola, with the distance of the focus from the directrix as 50. Also, draw normal and tangent to the curve, at a point 40 from the directrix. (May/June 2010,2012, JNTU)*

NOTE A parabola is a curve traced by a point, moving such that, at any position, its distance from a fixed point (focus) is always equal to its distance from a fixed straight line (directrix).

Construction (Fig. 5.3)

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark the focus F on the axis such that, AF = 50.
3. Locate the vertex V on AB such that, AV = VF = 25.
4. Draw a line VE, perpendicular to AB such that, VE = VF.
5. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF}{VA} = 1$, the eccentricity.
6. Locate a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equi-distant.

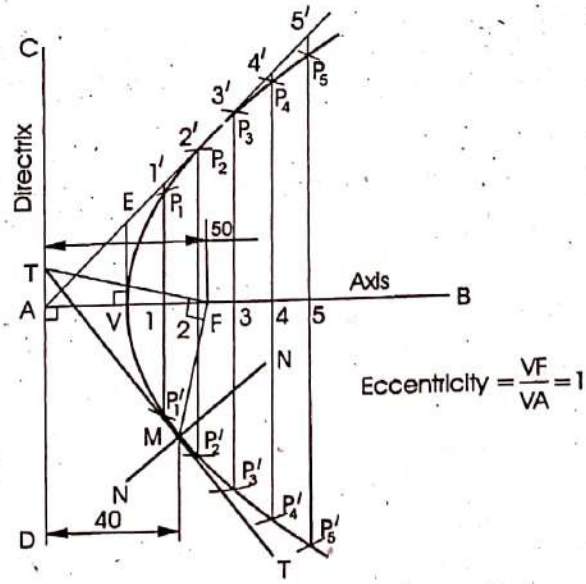


Fig. 5.3 Construction of parabola-Eccentricity method

7. Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at 1′, 2′, 3′, etc.
8. With centre F and radius 1-1′, draw arcs intersecting the line through 1 at P_1 and P_1'. P_1 and P_1' are the points on the parabola, because, the distance of P_1 (P_1') from F is 1-1′ and from CD, it is A-1 and,

$$\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = 1$$

Similarly, locate the points P_2, P_2' ; P_3', P_3; etc., on either side of the axis.

9. Join the points by a smooth curve, forming the required parabola.

To draw tangent and normal to the parabola, locate the point M, which is at 40 from the directrix. Then, join M to F and draw a line through F, perpendicular to MF, meeting the directrix at T. The line joining T and M and extended (T-T) is the tangent and a line N-N, through M and perpendicular to T-T is the normal to the curve.

Problem 2 *Construct an ellipse, with distance of the focus from the directrix as 50 and eccentricity as 2/3. Also, draw normal and tangent to the curve at a point 40 from the directrix.*

(May/June 2008, 2010,2011, JNTU)

Construction (Fig. 5.4)

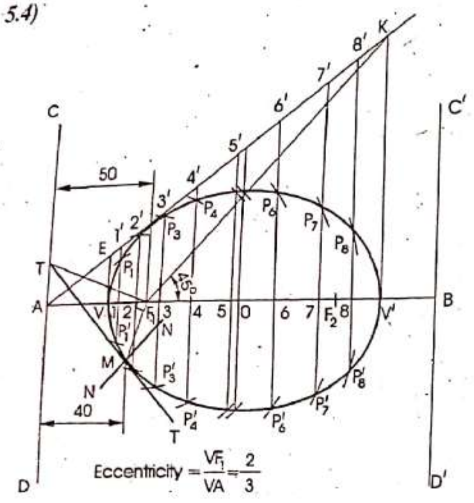


Fig. 5.4 Construction of ellipse-Eccentricity method

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark focus F_1 on the axis such that, $AF_1 = 50$.
3. Divide AF_1 into 5 equal parts.
4. Locate the vertex V on the third division point from A.
5. Draw a line VE, perpendicular to AB such that, $VE = VF_1$.
6. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$, the eccentricity.
7. Mark a number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equidistant.
8. Through the points 1, 2, 3. etc., draw lines perpendicular to the axis and to meet the line AE extended at 1′, 2′, 3′, etc.
9. With centre F_1 and radius 1-1′, draw arcs intersecting the line through 1 at P_1 and P_1'. P_1 and P_1' are the points on the ellipse, because the distance of P_1 from F_1 is 1-1′ and from CD, it is A-1 and, $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF_1}{VA} = \frac{2}{3}$, the eccentricity.
10. Similarly, locate the points P_2, P_2'; P_3, P_3'; etc., on either side of the axis.
11. Join the points by a smooth curve, forming the required ellipse.

NOTE (i) The ellipse is a closed curve and has two foci, two directrices and two vertices. To locate the other vertex V′, draw a line at 45° to the axis passing through F_1 and intersecting AE produced at K. A vertical line drawn from K meets the axis at V′.

(ii) To draw tangent and normal to the ellipse, the construction, similar to the one given for parabola may be followed.

(iii) The second focus F_2 may be located such that, $V'F_2 = VF_1$.

Problem 3 *Construct a hyperbola, with the distance between the focus and the directrix as 50 and eccentricity as 3/2. Also, draw normal and tangent to the curve at a point 30 from the directrix.* (Aug/Sep 2008, JNTU)

Construction (Fig. 5.5)

1. Draw the axis AB and the directrix CD, at right angles to each other.
2. Mark focus F on the axis such that, $AF = 50$.
3. Divide AF into 5 equal parts.
4. Locate the vertex V on the second division point from A.
5. Draw a line VE, perpendicular to AB such that, $VE = VF$.
6. Join A, E and extend. By construction, $\frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$, the eccentricity.
7. Repeat steps 6 to 8 of Construction: Fig. 5.3. P_1 and P_1' are the points on the hyperbola, because the distance of P_1 from F is 1-1′ and from CD, it is A-1, and $\frac{1-1'}{A-1} = \frac{VE}{VA} = \frac{VF}{VA} = \frac{3}{2}$, the eccentricity.

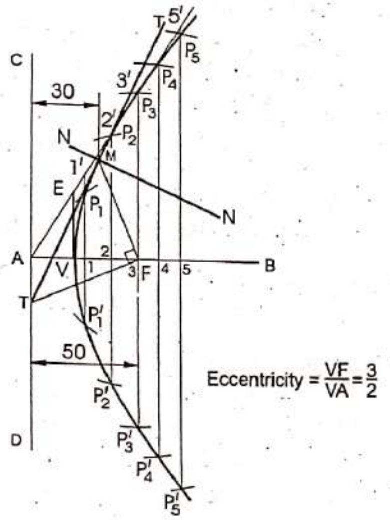


Fig. 5.5 Construction of hyperbola-Eccentricity method

8. Similarly, locate the points P_2, P_2'; P_3, P_3'; etc., on either side of the axis.
9. Join the points by a smooth curve, forming the required hyperbola.

NOTE (i) A hyperbola is an open curve and has one focus and one directrix.

(ii) To draw tangent and normal to the hyperbola, the construction similar to the one given for parabola may be followed.

5.4 CONSTRUCTION OF CONICS - OTHER METHODS

5.4.1 Parabola

Problem 4 *Construct a parabola, with the length of base as 60 and axis 30 long. Also, draw tangent to the curve at a point 25 from the base.*

Tangent method

Construction (Fig. 5.6)

1. Draw the base AB (= 60) and axis CD (=30) such that CD is a perpendicular bi-sector to AB.
2. Produce CD to E such that DE = CD.
3. Join E, A and E, B. These are the tangents to the parabola at A and B.
4. Divide AE and BE into the same number of equal parts and number the points as shown.
5. Join 1, 1′ ; 2, 2′ ; 3, 3′ ; etc., forming the tangents to the required parabola.

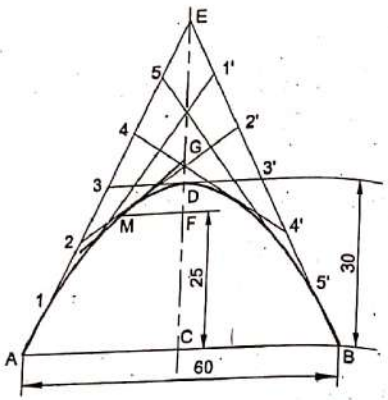


Fig. 5.6 Construction of parabola-Tangent method

A smooth curve passing through A, D and B and tangential to the above lines is the required parabola.

To draw tangent to the parabola, locate the point M, which is at 25 from the base. Then, draw a horizontal through M, meeting the axis at F. Mark G on the extension of the axis such that, DG = FD. Join G, M and extend, forming the tangent to the curve at M.

NOTE A stone thrown at an angle, traverses the path of a parabola; the distance covered by the stone being equal to the base of the parabola.

Problem 5 *Construct a parabola with base 60 and length of the axis 40. Draw tangent to the curve at a point 20 from the base. Also, locate the focus and directrix to the parabola.*

(May/June 2010, JNTU)

Rectangle method

Construction (Fig. 5.7)

1. Draw the base AB (=60) and axis CD (=40) such that, CD is a perpendicular bisector to AB.
2. Construct the rectangle ABKL, passing through D.
3. Divide AC and AL into the same number of equal parts and number the points as shown.
4. Join 1, 2 and 3 to D.
5. Through 1′, 2′ and 3′, draw lines parallel to the axis; intersecting the lines 1-D, 2-D and 3-D at P_1, P_2 and P_3 respectively.

6. Obtain the points P_1', P_2' and P_3', which are symmetrically placed to P_1, P_2 and P_3, with respect to the axis CD.

Join the points by a smooth curve, forming the required parabola.

To draw a tangent to the curve

(i) Locate the given point M on the curve, which is at 20 from the base.

(ii) Draw a horizontal line through M, meeting the axis at O.

(iii) Locate the point G on the axis such that GD = OD.

(iv) Join G, M and extend, forming the required tangent.

To locate the focus and directrix

(i) Draw a perpendicular bisector to the tangent GM, intersecting the axis at F.

(ii) Mark the point H on the axis such that HD = FD.

(iii) Draw a line PQ, perpendicular to the axis and passing through H

F is the focus and PQ, the directrix of the given parabola.

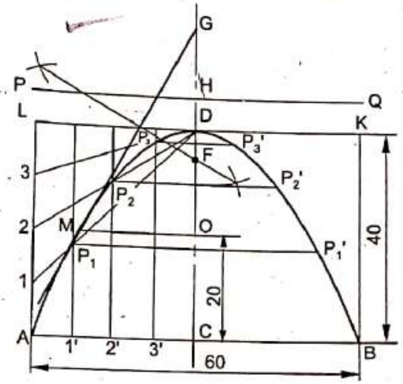


Fig. 5.7 Construction of parabola-Rectangle method

Problem 6 *Construct a parabola in a parallelogram of sides 100 x 60 and with an included angle of 75°.*

Parallelogram method

Figure 5.8 shows the construction of a parabola, in the parallelogram ABKL, following the method similar to Construction: Fig. 5.7.

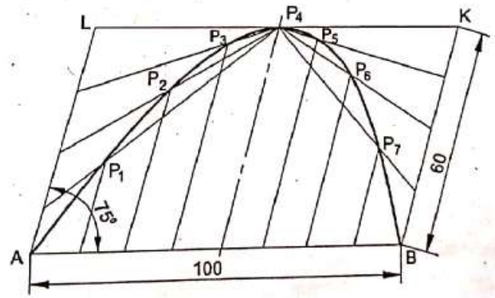


Fig. 5.8 Construction of parabola-Parallelogram method

Problem 7 *Determine the axis of the parabola, shown in Fig. 5.9.*

Construction (Fig. 5.9)

1. Draw any two parallel chords AB and CD, to the given parabola, separated by any suitable distance.
2. Locate the mid-points E and F of the chords AB and CD respectively.
3. Draw a line GH, passing through E and F. The line GH is parallel to the axis.
4. Draw a chord IJ, perpendicular to GH.
5. Locate the mid-point K of the chord IJ and through it, draw a line LM, parallel to GH.

The line LM is the required axis to the curve.

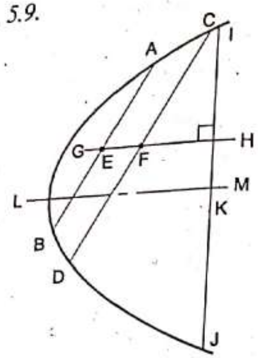


Fig. 5.9 Determination of the axis of a parabola

Problem 8 *Determine the focus and directrix of the parabola, shown in Fig. 5.10.*

Construction (Fig. 5.10)

1. Locate any point M on the given parabola.
2. Draw the line MA, perpendicular to the axis.
3. Mark the point B on the axis such that, BV = VA.
4. Join B, M and extend.
5. Draw the perpendicular bisector EF to BM, intersecting the axis at F, the focus.
6. Mark the point O on the axis such that, OV = VF.
7. Through O, draw the line CD perpendicular to the axis.

CD is the directrix of the given parabola.

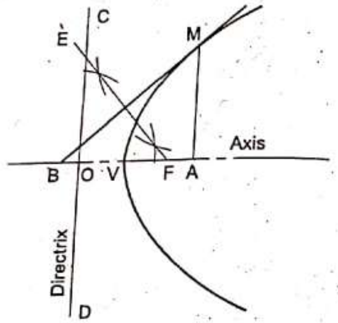


Fig. 5.10 Determination of the focus and directrix of a parabola

Problem 9 *Draw normal and tangent to the given parabola (Fig. 5.11) at any point on it, given the focus and directrix.*

Construction (Fig. 5.11)

1. Locate a point M on the curve.
2. Join M,F and draw MP, perpendicular to the directrix CD.
3. Bisect ∠ PMF.

The bisector TM produced is the required tangent.

4. Draw the line MN, perpendicular to MT, forming the required normal.

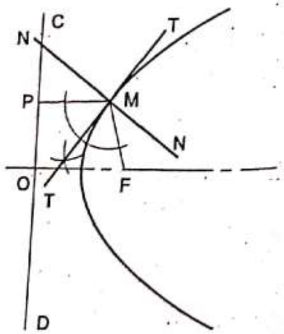


Fig. 5.11 Drawing normal and tangent to a parabola

Problem 10 *A stone is thrown from a building of 7m height and at its highest flight, the stone just crosses a palm tree of 14m height. Trace the path of the stone, if the distance between the building and the tree is 3.5m.*

Construction (Fig. 5.12)

1. Draw lines AB and OT, representing the building and palm tree respectively; 3.5m apart and above the ground line GL.
2. Draw a horizontal line through B; intersecting OT at C.
3. Locate D on BC extended such that, CD = BC and complete the rectangle BDEF.
4. Inscribe the parabola in the rectangle BDEF, by rectangle method.
5. Draw the path of the stone till it reaches the ground (H); extending the principle of rectangle method.

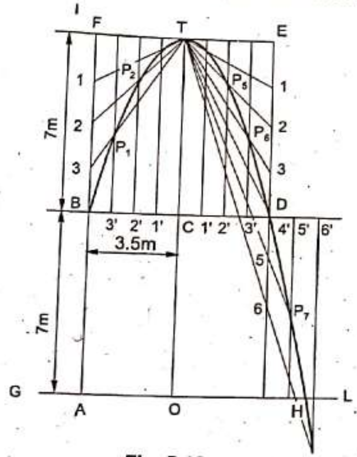


Fig. 5.12

Problem 11 *Construct a right angled triangle EFG such that FG = 40 and FE = 20 and ∠ EFG = 90°. The point G is on the parabola, whose focus is the point F on the triangle. If FE is a part of parabola's axis, draw the parabola and determine the double ordinate at a distance of 80 from its directrix.*

Construction (Fig. 5.13)

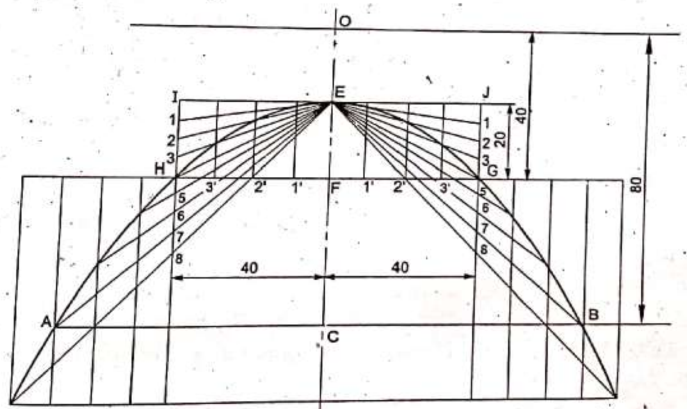


Fig. 5.13

1. Construct the given right-angled triangle EFG.
2. Draw the axis, by extending EF, on either side.

3. Locate the point O on the axis such that, FO = FG.
4. Through O, draw a line, perpendicular to the axis; forming the directrix of the parabola.

NOTE $\dfrac{\text{Distance of the point G from focus F}}{\text{Distance of the point G form the directrix}} = 1$

$\therefore$ FG = FO

5. Complete the rectangle GHIJ, passing through the points F and E; keeping FH = FG.
6. Inscribe the parabola in the rectangle, by rectangle method.
7. Extend the parabola, beyond the points H and G, by rectangle method, as shown.
8. Locate the point C on the axis such that, OC = 80.
9. Through C, draw a line perpendicular to the axis; meeting the parabola at A and B.

The length AB is equal to the length of the double ordinate of the parabola.

Problem 12 *A jet of water is issuing through an orifice of 50 diameter fitted to a vertical side of a tank. The centre of the orifice is 1.5 m above the ground and centre of the jet touches the ground at a distance of 2.5 m from the orifice. Draw the locus of the centre of the jet, which is just issuing from the orifice, till it reaches the ground. Name the curve.* (take 1m = 4cm)

Construction (Fig. 5.14)

1. Draw the ground line GL and locate the point A, the centre of the orifice, on the vertical line through a point D on GL such that DA = 1.5 m.
2. Locate the point C, the point where the jet touches the ground on GL such that DC = 2.5 m.
3. Complete the rectangle ABCD.
4. Inscribe the parabolic curve through A and C in the rectangle; following the rectangle method (Refer Construction: Fig 5.7).

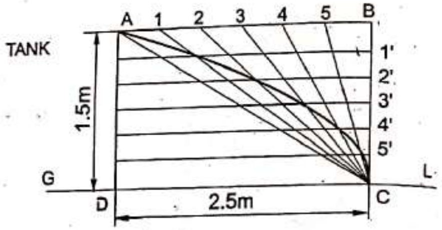


Fig. 5.14

5.4.2 Ellipse

The ellipse is a curve traced by a point, moving such that, the sum of its distances from two points, known as foci, is constant and equal to the major axis.

The ellipse, shown in Fig. 5.15, has two foci F_1 and F_2. The line passing through the two foci and terminated by the curve, AB is called the major axis. The line bisecting the major axis at right angle and terminated by the curve, CD is called the minor axis. The foci are at equi-distant from centre O.

The points P, C, Q, etc., are on the curve. By definition,

$$PF_1 + PF_2 = CF_1 + CF_2 = QF_1 + QF_2 = AB$$

$$\therefore \quad CF_1 = CF_2 = \frac{1}{2} AB$$

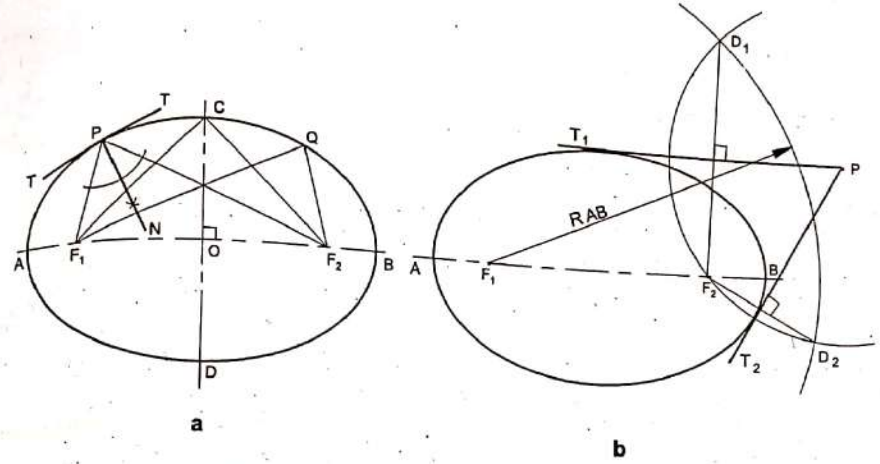


Fig. 5.15 Drawing tangent and normal to an ellipse

Hence, the distance from an end of minor axis to the focus is equal to half of the major axis.

NOTE Out of the three details, major axis, minor axis and foci; if any two are given, the third one may be determined by using the above property of the ellipse.

Problem 13 *Draw tangent and normal to the given ellipse, through a given point P on it (Fig. 5.15a).*

Construction (Fig. 5.15a)

1. Join the foci F_1, F_2 with the given point P.
2. Bisect $\angle F_1PF_2$.

The bisector PN is the required normal and the line TT, passing through P and perpendicular to PN, is the tangent to the ellipse.

To draw tangents to an ellipse from a point outside it.

Construction (Fig. 5.15b)

1. Locate the given point P outside the ellipse.
2. With P as centre and PF2 as radius, draw an arc.
3. With F_1 as centre and major axis as radius, draw an arc intersecting the previously drawn arc at D_1 and D_2.
4. Join D_1, F_2 and D_2, F_2.
5. The lines drawn perpendicular to D_1F_2 and D_2F_2 and passing through the point P are the required tangents to the ellipse from the outside point P.

Problem 14 *The major and minor axes of an ellipse are 120 and 80. Draw an ellipse.* (May/June 2010, May 2012, JNTU)

I Foci or arcs of circles method

Construction (Fig. 5.16)

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. With centre C (or D) and radius OA (=OB), draw arcs intersecting the major axis at F_1 and F_2, the foci.
3. Mark a number of points 1, 2, 3, etc., between F_1 and O, which need not be equi-distant.
4. With centres F_1 and F_2 and radii A-1 and B-1 respectively, draw arcs intersecting at points P_1 and P_1'.

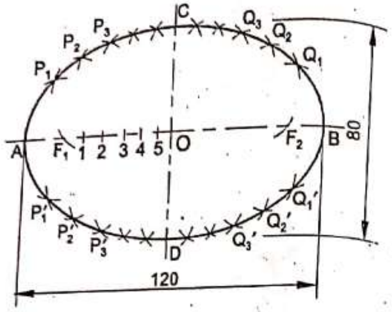


Fig. 5.16 Construction of an ellipse-Foci/arcs of circles method

5. With centres F_1 and F_2 and radii B-1 and A-1 respectively, draw arcs intersecting at points Q_1 and Q_1'.
6. Repeat the steps 4 and 5 with the remaining points 2, 3, 4, etc., and obtain additional points on the curve.

A smooth curve through all the points is the required ellipse.

II Oblong method

Construction (Fig. 5.17)

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. Draw the rectangle KLMN, passing through D, B, C and A.
3. Divide AO and AN, into the same number of equal parts and number the points as shown.
4. Join C with the points $1'$, $2'$ and $3'$.
5. Join D with 1, 2, 3 and extend till they meet the above lines, i.e., C-$1'$, C-$2'$ and C-$3'$ respectively at P_1, P_2 and P_3.

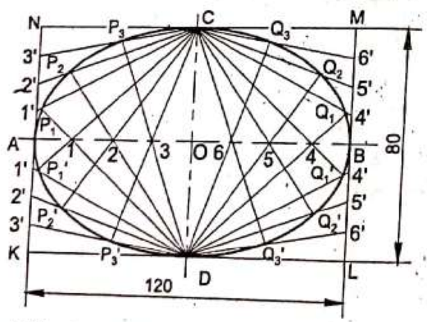


Fig. 5.17 Construction of an ellipse-Oblong method

6. Repeat steps 3 to 5 and obtain the points in the remaining quadrants.

A smooth curve through all the points is the required ellipse.

NOTE If it is required to inscribe an ellipse in a rectangle 120× 80, then the major and minor axes of the ellipse are equal to 120 and 80 respectively.

III Concentric circles method

Construction (Fig. 5.18)

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. With centre O and major and minor axes as diameters, draw two concentric circles.
3. Divide both the circles into the same number of equal parts, say 12 by radial lines.
4. Considering radial line O-$1'$ -1, draw a horizontal line from $1'$ and a vertical line from 1, intersecting at P_1.
5. Repeat the construction through all the points and obtain P_2, P_3, etc.

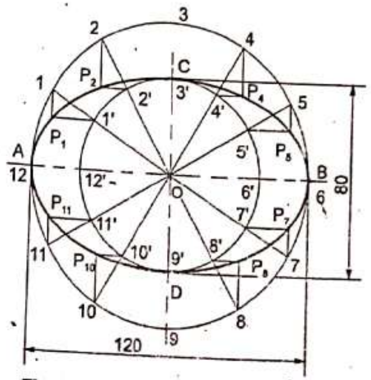


Fig. 5.18 Construction of an ellipse-Concentric circles method

A smooth curve through the points A, P_1, P_2, etc., is the required ellipse

IV Four centre method

Construction (Fig. 5.19)

1. Draw the major (AB = 120) and minor (CD = 80) axes and locate the centre O.
2. With centre O and radius OA, draw the arc AE.
3. With centre C and radius CE, draw an arc meeting the line AC at F.
4. Draw perpendicular bisector of AF, meeting AB at K and CD extended at G.
5. Locate point L on AB such that, OL = OK and H on DC extended such that, HC = GD.

The points K, L, G and H are the four centres that may be used to draw the ellipse.

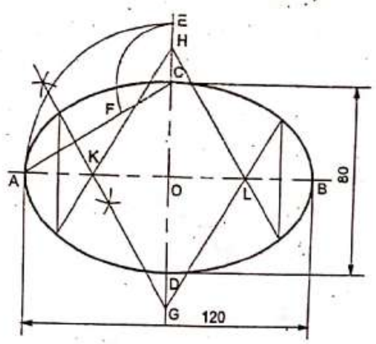


Fig. 5.19 Construction of an ellipse-Four centre method

6. With G and H as centres and radius CG, draw two arcs.
7. With K and L as centres and radius KA, draw two arcs.

The four arcs meet tangentially, forming the required ellipse.

V Parallelogram method

A parallelogram has sides 100 and 80, at an included angle of 70°. Inscribe an ellipse in the parallelogram. Find the major and minor axes of the curve. (May/June 2010, JNTU)

Construction (Fig. 5.20)

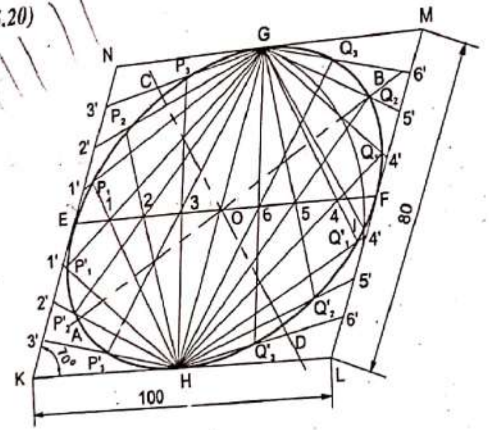


Fig. 5.20 Construction of an ellipse-Parallelogram method

1. Draw the parallelogram KLMN of given sides and angle. The two axes EF and GH are called the conjugate axes (diameters).
2. Divide EO and EN into the same number of equal parts and number the division points as shown.
3. Join G with $1'$, $2'$ and $3'$.
4. Join H with 1, 2, 3 and extend till these meet the lines $G1'$, $G2'$ and $G3'$ at P_1, P_2 and P_3 respectively.
5. Repeat steps 2 to 4 and obtain the points in the remaining quadrants.

A smooth curve through all the points is the required ellipse.

To find the major and minor axes, with centre O and radius OG, draw an arc meeting the ellipse at I. Join G, I and then draw a line CD through O and parallel to GI. The line CD is the minor axis and a line AB drawn through O and perpendicular to CD is the major axis.

VI Ellipse through three points, not in a straight line

Construction (Fig. 5.20)

1. Locate the given points E, G and F.
2. Join E and F and locate its mid-point O.
3. Draw GO and extend it to H such that, OH = GO.
4. Draw the parallelogram KLMN through the points E, G, F and H.
5. Follow the steps given for parallelogram method and obtain the points on the curve.

A smooth curve through the points is the required ellipse.

NOTE (i) A number of ellipses may be drawn, passing through 3 given points, not in a straight line.

(ii) EF is taken as a diameter, to draw the unique ellipse passing through the 3 given points.

(iii) Any line passing through the centre of an ellipse and bound by the curve is a diameter of the ellipse. If the tangents drawn to the ellipse at the ends of a diameter are parallel to another diameter, these diameters are called conjugate diameters. The given ellipse may have unlimited pairs of conjugate diameters. The major and minor axes form one such pair with an included angle of 90°.

VII Circle method

Two conjugate diameters EF and GH of an ellipse are 75 and 50 long, with an included angle of 60° between the two. Draw an ellipse, passing through the points E, G, F and H.

Construction (Fig. 5.21)

(i) Draw the conjugate diameters EF and GH, at the given included angle and bisecting at O.

(ii) With centre O and diameter EF, draw a circle and divide it into any number of parts, say 16.

(iii) Draw the lines perpendicular to EF and passing through the above division points; meeting EF at $1'$, $2'$, $3'$, etc.

(iv) Join 4, G.

(v) Through 1, 2, 3, etc., draw lines parallel to 4-G.

(vi) Through $1'$, $2'$, $3'$, etc., draw lines parallel to GH; intersecting the above lines at P_1, P_2, P_3, etc.

A smooth curve through the points E, P_1, P_2, P_3, etc., is the required ellipse.

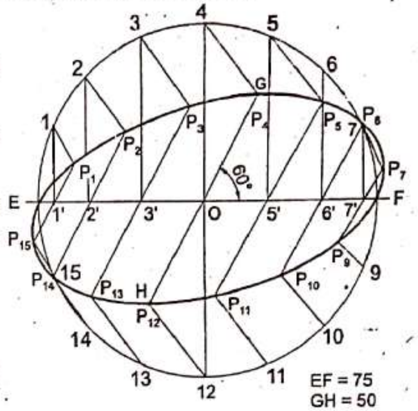


Fig. 5.21 Construction of an ellipse-circle method

Problem 15 *Two fixed points A and B are 100 apart. Trace the complete path of a point P moving (in the same plane as that of A and B) in such a way that the sum of the distances from A and B is always equal to 125. Name the curve. Draw another curve parallel to and 25 away from this curve.* (May /June 2008, 2010, JNTU)

Construction (Fig. 5.22)

NOTE The path of the point P is an ellipse and the major axis is 125 (properties of ellipse).

(i) Draw a line CD of length 125 (major axis), locate centre O and the fixed points (foci) A and B symmetrically such that, AB is 100.

(ii) Mark a number of points 1, 2, 3, etc., between A and O which need not be equidistant.

(iii) With centres A and B and radii C-1 and D-1 respectively, draw arcs intersecting at points P_1 and P_1'.

(iv) With centres A and B and radii D-1 and C-1 respectively, draw arcs intersecting at points P_8 and P_8'.

(v) Repeat steps 3 and 4 with the remaining points 2, 3, etc., and obtain additional points on the curve.

A smooth curve through all these points is the required curve, the ellipse.

To draw a parallel curve

(vi) Choose a number of points on the ellipse as centres and draw arcs of circles with radius 25.

A smooth curve, drawn tangential to these arcs is the required parallel curve.

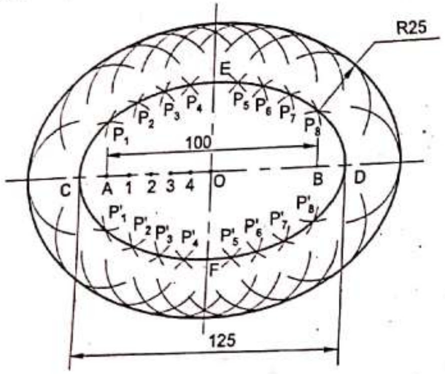


Fig. 5.22 Ellipse

Problem 16 *Construct an ellipse with the following data:*

Distance between the directrices = 200

Distance between the vertices = 150

Determine the eccentricity and the minor axis.

Construction (Fig. 5.23)

(i) Draw a line AB of 200 long and locate $V_1 V_2 = 150$ centrally to AB. Let F_1 and F_2 are the two foci of the ellipse. The eccentricity may be obtained from:

$$\frac{V_1F_1}{V_1A} = \frac{V_1F_2}{V_1B}$$

$$\therefore \quad \frac{V_1F_1 + V_1F_2}{V_1A + V_1B} = \frac{V_1V_2}{AB} = \frac{150}{200} = \frac{3}{4}$$

(ii) Locate F_1 and F_2 such that $= \dfrac{V_1F_1}{V_1A} = \dfrac{3}{4}$

$\therefore V_1 F_1 = (3/4) \times 25 = 18.75 = V_2 F_2$

(iii) Draw the minor axis CD such that, $CF_1 = V_1O$.

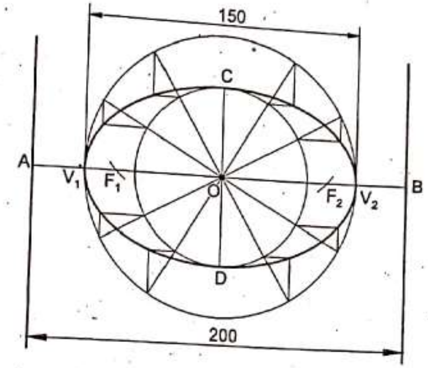


Fig. 5.23

The ellipse may now be constructed by following any one of the methods of construction discussed above. The ellipse shown in Fig. 5.23, is obtained by following the concentric circles method.

Problem 17 *Figure 5.24 shows an ellipse with major axis 120 and minor axis 80. Determine the eccentricity and the distance between the directrices.*

Eccentricity, $e = \dfrac{V_1F_1}{V_1A} = \dfrac{V_1F_2}{V_1B}$

$$\therefore \quad = \frac{V_1F_2 - V_1F_1}{V_1B - V_1A} = \frac{F_1F_2}{V_1V_2}$$

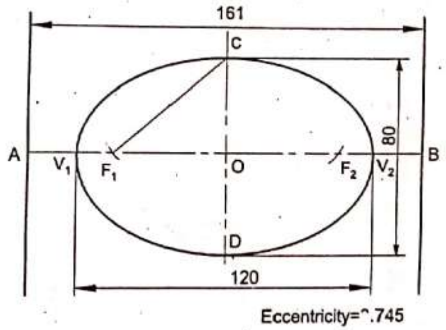


Fig. 5.24

From the triangle F_1 CO,

OC = 40 (half the minor axis)

$F_1C = 60$ (half the major axis)

Thus, $F_1O = \sqrt{(60^2 - 40^2)} = 44.7$

Hence, $F_1 F_2 = 2F_1O = 89.4$

On substitution, e = 89.4/120 = 0.745

From the preceding problem, eccentricity, $e = \dfrac{V_1V_2}{AB}$

Hence, AB, the distance between the directrices $= V_1V_2/e = 161$

5.4.3 Hyperbola

A hyperbola is a curve generated by a point moving such that, the difference of its distances from two fixed points, called the foci, is always constant and equal to the distance between the vertices of the two branches of hyperbola. This distance is also known as the major axis of the hyperbola. Referring to Fig. 5. 25,

$P_1F_2 \sim P_1F_1 = V_1V_2$, and

$P_2F_1 \sim P_2F_2 = V_1V_2$

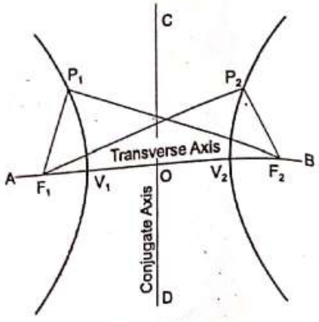


Fig. 5.25

NOTE The axes AB and CD are respectively known as transverse and conjugate axes of the hyperbola. The curve has two branches, which are symmetric about the conjugate axis.

Problem 18 *Construct a hyperbola with its foci, 70 apart and the major axis 45. Draw tangent to the curve at a point 20 from the focus. Also, determine the eccentricity of the curve.*

Construction (Fig. 5.26)

(i) Draw the axis AB and locate a point O on it.

(ii) Locate the foci F_1, F_2 (F_1 F_2 = 70) and the vertices V_1, V_2 (V_1V_2 = 45) on AB which are symmetric about O.

(iii) Mark a number of points 1, 2, 3, etc., on AB, to the right of F_2, which need not be equidistant.

(iv) With centre F_1 and radius V_1-1, draw arcs on either side of the transverse axis.

(v) With centre F_2 and radius V_2-1, draw arcs intersecting the above arcs at P_1 and P_1'.

(vi) With centre F_2 and radius V_1-1, draw arcs on either side of the transverse axis.

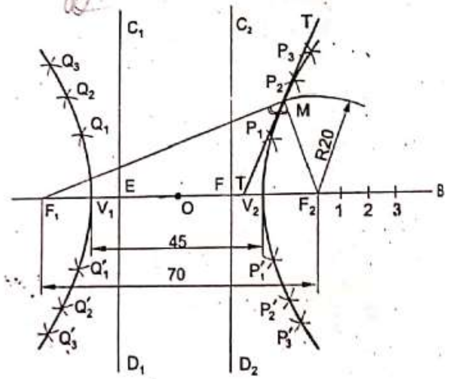


Fig. 5.26

(vii) With centre F_1 and radius V_2-1, draw arcs intersecting the above arcs at Q_1 and Q_1'.

(viii) Repeat steps 4 to 7 and obtain the points P_2, P_2' ; etc., and Q_2, Q_2' ; etc.

Join the points in the order and obtain the two branches of the hyperbola.

To draw a tangent to the hyperbola, locate the point M, which is at 20 from the focus, say F_2. Then, join M to the foci F_1 and F_2. Draw a line TT, bisecting $\angle F_1MF_2$ forming the required tangent.

To locate the directrices

(i) Determine the eccentricity, $e = \dfrac{OF_1}{OV_1} = \dfrac{OF_2}{OV_2} = \dfrac{35}{22.5} = 1.56$

(ii) Locate the points E and F on the transverse axis such that, $\dfrac{OV_1}{OE} = \dfrac{OV_2}{OF} = e$

$\therefore$ $OE = OV_1/e = 22.5/1.56 = 14.4 = OF$

Draw lines through the points E and F, perpendicular to the transverse axis; representing the directrices to the two branches of the hyperbola.

Problem 19 *Draw a hyperbola with half the major axis as 40, the abscissa 50 and double ordinate 140.*

Construction (Fig. 5.27)

(i) Locate the points O, V and E on a horizontal line such that, OV = 40 (half the major axis) and VE = 50 (abscissa).

(ii) Through E, draw a perpendicular and mark on it, the points B and C such that, BE = EC = 70.

(iii) Draw the rectangle ABCD, passing through V, as shown.

(iv) Divide EC and DC into the same number of equal parts, say 4 and name the parts as shown.

(v) Join O, 1; O, 2 and O, 3.

(vi) Join V, 1′; V, 2′ and V, 3′; intersecting the above lines at P_1, P_2 and P_3.

(vii) Follow the steps 4 to 6 and locate the points in the other half of the rectangle.

Join the points in the order and obtain the hyperbola.

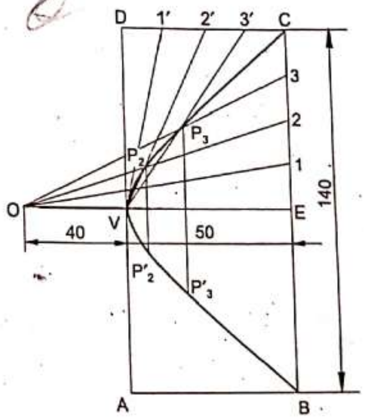


Fig. 5.27

Rectangular hyperbola

It is a curve generated by a point which moves in such a way that the product of its distances from two fixed straight lines, the asymptotes at right angles to each other, is a constant.

Problem 20 *Construct a rectangular hyperbola, when a point P on it is at distances of 18 and 34 from two asymptotes. Also, draw a tangent to the curve at a point 20 from an asymptote.*

(May/June 2010, JNTU)

Construction (Fig. 5.28)

(i) Draw the asymptotes OA and OB at right angles to each other and locate the given point P.

(ii) Draw the lines CD and EF; passing through P and parallel to OA and OB respectively.

(iii) Locate a number of points 1,2,3, etc., along the line CD, which need not be equi-distant.

(iv) Join 1, 2, 3, etc., to O and extend if necessary, till these lines meet the line EF at points 1′, 2′, 3′, etc.

(v) Draw lines through 1, 2, 3, etc., parallel to EF and through 1′, 2′, 3′ etc., parallel to CD, to intersect at P_1, P_2, P_3, etc.

A smooth curve passing through these points is the required rectangular hyperbola.

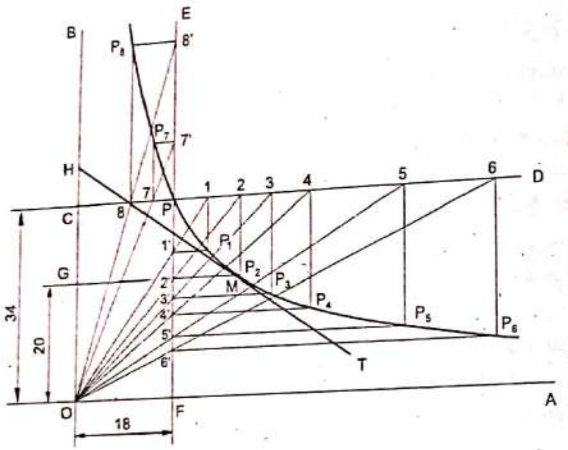


Fig. 5.28 Construction of rectangular hyperbola

To draw tangent to the curve, locate the point M on the curve by drawing a line GM, parallel to OA and at a distance 20 from it. Then, locate the point H on OB such that GH = OG. The line HT passing through M is the required tangent to the curve.

NOTE If the eccentricity for a hyperbola is $\sqrt{2}$, the asymptotes will be at right angles to each other and the hyperbola is known as a rectangular hyperbola.

Problem 21 *The asymptotes of a hyperbola are inclined at 70° to each other. Construct the curve when a point P on it is at distances of 20 and 30 from the two asymptotes.*

(May/June 2010, JNTU)

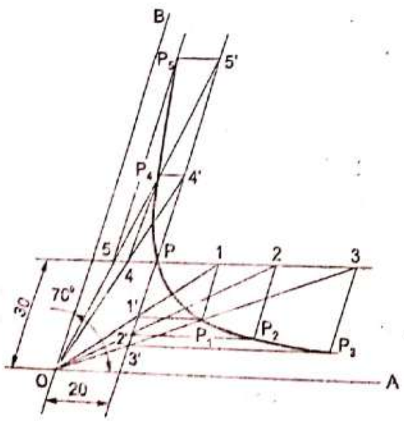


Fig. 5.29 Construction of hyperbola-Given asymptotes and a point on it

A hyperbola passing through any given point, located between the two asymptotes, making any angle other than 90°, may also be constructed (Fig. 5.29); following the method similar to construction (Fig. 5.28)

5.5 CYCLOIDAL CURVES

Cycloidal curves are generated by a point on the circumference of a circle, when it rolls without slipping along a straight or curved path. The rolling circle is called the generating circle and the fixed straight line / circle is called the directing line/ circle respectively.

5.5.1 Cycloid

A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls along a straight line without slipping (Fig.5.30). Obviously, the size of the curve depends upon the diameter of the generating circle.

Problem 22 *Construct a cycloid, given the diameter of the generating circle as 40. Draw tangent to the curve at a point on it, 35 from the line.* (May/June 2008, 2010, May 2012, JNTU)

Construction (Fig. 5.30)

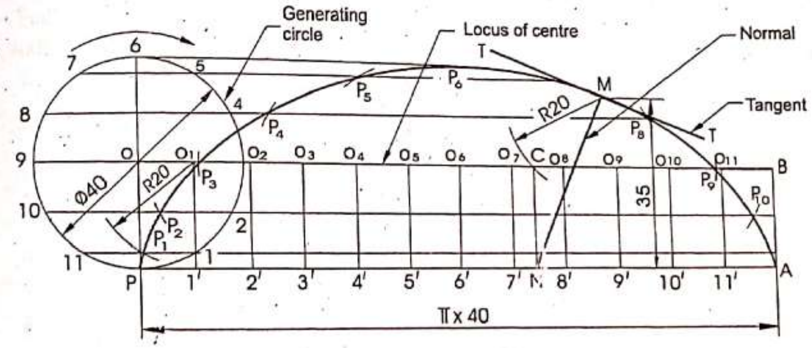


Fig. 5.30 Cycloid

(i) With centre O and radius 20, draw the generating circle.

(ii) Locate the initial position of the generating point P on the circumference of the circle.

(iii) Draw a line PA, tangential and equal to the circumference of the circle.

(iv) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(v) Draw the line OB, parallel and equal to PA, which is the locus of the centre of the generating circle.

(vi) Erect perpendiculars at 1′, 2′, etc., to meet the line OB at O_1, O_2, etc.

(vii) Through the points 1, 2, 3, etc., draw lines parallel to PA.

(viii) With O_1 as centre and radius 20, draw an arc intersecting the line through 1 at P_1.

P_1 is the position of the point P, when the centre of the generating circle moves to O_1.

(ix) With O_2 as centre and radius 20, draw an arc intersecting the line through 2 at P_2.

(x) Similarly, locate the points P_3, P_4, etc.

A smooth curve passing through these points is the required cycloid.

To draw the tangent and normal

(i) Locate the point M on the curve, which is at 35 from the directing line.

(ii) With M as centre and radius 20, draw an arc intersecting the locus of the centre (OB) at C.

(iii) Through C, draw a line perpendicular to the directing line PA, meeting it at N (the point of contact of the generating circle, when its centre moves to C).

The line joining the points M and N is the required normal and a line T-T perpendicular to it and passing through M is the tangent to the cycloid.

Problem 23 *A circle of 50 diameter rolls on a horizontal line for half a revolution clock-wise and then on a line inclined at 60° to the horizontal for another half, clock-wise. Draw the curve traced by a point P on the circumference of the circle, taking the top-most point on the rolling circle as the initial position of the generating point.* (Aug/Sep 2008, JNTU)

Construction (Fig. 5.31)

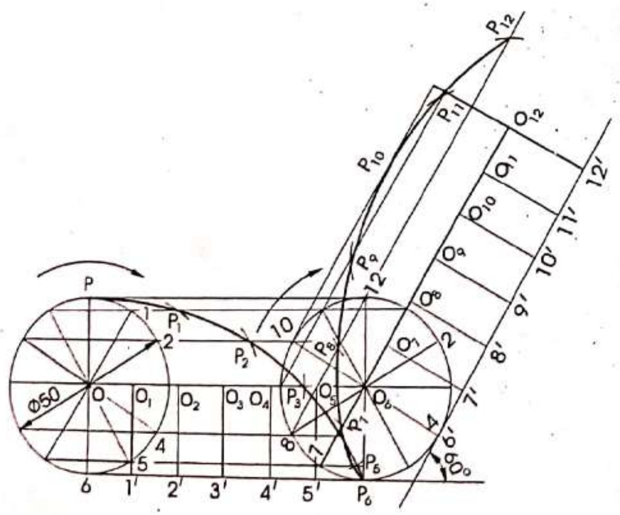


Fig. 5.31 Cycloid

(i) With centre O and radius 25, draw the given circle.

(ii) Divide the circle into a number of equal parts, say 12.

(iii) Mark the division points on the circle and draw a tangent (directing line) passing through 6.

(iv) Mark half of the circumference along the tangent and divide it into 6 equal parts.

(v) Locate the initial position of the generating point P.

(vi) Draw a smooth curve, representing the path of P, as the circle rolls along the directing line for half a revolution, following the Construction: Fig. 5.30.

(vii) With O_6 as centre, draw the circle, representing the position of the rolling circle after half revolution.

(viii) Draw a line tangential to the circle, making 60° with the directing line. This is the directing line for the next half revolution of the generating circle.

(ix) Trace the path of P for the next half revolution of the generating circle, as shown.

Problem 24 *ABC is an equilateral triangle of side 70. Trace the loci of vertices A, B and C, when the circle circumscribing ABC, rolls without slipping, along a fixed straight line, for one complete revolution.*

Construction (Fig. 5.32)

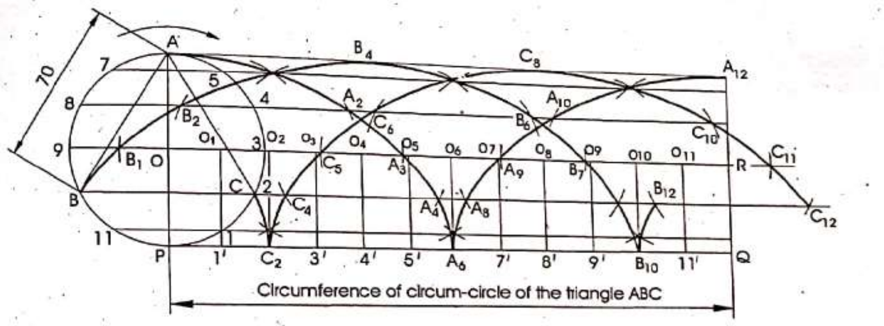


Fig. 5.32

(i) Draw the equilateral triangle ABC and draw its circum-circle.

(ii) Divide the circle into a number of equal parts such that the corners (vertices) of the triangle A, B and C coincide with the division points.

(iii) Draw the line PQ, tangential and equal to the circumference of the circle.

(iv) Divide the line PQ into the same number of equal parts, as that of the circle.

(v) Draw the line OR, parallel and equal to PQ, which is the locus of the centre of the (generating) circle.

(vi) Erect perpendiculars at 1′, 2′, etc., to meet the line OR at O_1, O_2, etc.

(vii) Follow the steps 7 to 10 of Construction: Fig. 5.30 suitably and obtain the loci of vertices A, B and C.

5.5.2 Epi-cycloid

An epi-cycloid is a curve traced by a point on the circumference of a circle, which rolls without slipping on another circle (directing circle) outside it.

Problem 25 *Draw an epi-cycloid of a circle of 40 diameter, which rolls on another circle of 120 diameter for one revolution clock-wise. Draw a tangent and a normal to it at a point 90 from the centre of the directing circle.* (Aug/Sep 2011, JNTU)

Construction (Fig. 5.33)

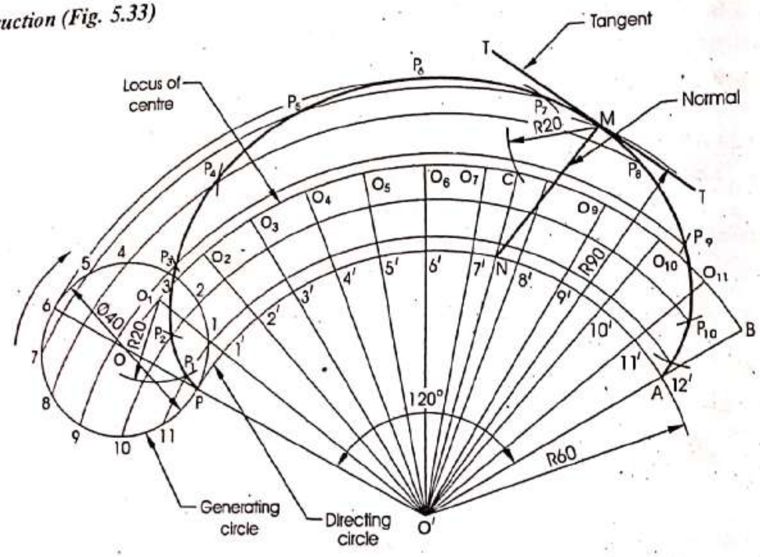


Fig. 5.33 Epi-cycloid

(i) Draw a part of the directing circle with O′ as centre and radius 60.

(ii) Draw any radial line O′ P and extend it.

(iii) Locate the point O on the above line such that, OP = 20, the radius of the generating circle.

(iv) With O as centre and radius 20, draw the generating circle.

(v) Locate the point A on the directing circle such that, the arc length PA is equal to the circumference of the generating circle.

The point A is obtained by setting ∠ PO′ A = 360° × 20/60 = 120°.

(vi) With centre O′ and radius O′O, draw an arc intersecting the line O′ A produced at B.

The arc OB is the locus of the centre of the generating circle.

(vii) Divide the generating circle and the arc PA into the same number of equal parts and number them as shown.

(viii) Join O′ ,1′ ; O′ , 2′ ; etc., and extend, meeting the arc OB at O_1, O_2, etc.

(ix) Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O′ as centre.

(x) With centre O_1 and radius 20, draw an arc intersecting the arc through 1 at P_1.

(xi) In a similar manner, obtain points P_2, P_3, etc.

A smooth curve through these points is the required epi-cycloid.

To draw the tangent and normal

(i) Locate the point M on the curve, which is at 90 from the centre of the directing circle.

(ii) With M as centre and radius 20, draw an arc intersecting the locus of the centre of the generating circle at C.

(iii) Join C to O′ , intersecting the directing circle at N. The line joining N to M is the required normal and a line T-T, perpendicular to it and passing through M is the required tangent.

NOTE When the diameters of the generating circle and directing circle are equal, the epi-cycloid traced is called a cardioid, as shown in Fig. 5.34.

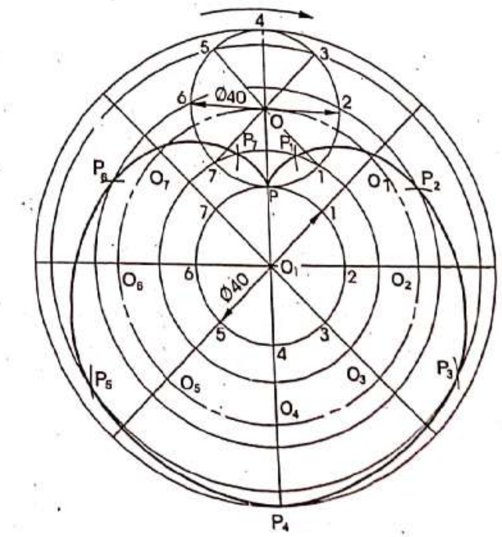


Fig. 5.34 Cardioid

Problem 26 *A circle of 50 diameter rolls without slipping on the outside of another circle of diameter 150. Show the path of a point on the periphery of the (generating) rolling circle, diametrically opposite to the initial point of contact between the circles.* (Aug/Sep 2011, JNTU)

Construction (Fig. 5.35)

(i) Draw a part of the directing circle with O′ as centre and radius 75.

(ii) Draw any radial line O′ A and extend it.

(iii) Locate the point O on the above line such that, OA = 25.

(iv) With O as centre and radius 25 (=OA), draw the generating circle.

(v) Locate the point B on the directing circle such that, the arc length AB is equal to the circumference of the generating circle.

The point B is obtained, by setting $\angle AO'B = 360° \times \dfrac{25}{75} = 120°$

(vi) With O′ as center and O′O as radius, draw an arc intersecting the line O′ B extended at C. The arc OC is the locus of the centre of the generating circle.

(vii) Divide the generating circle and the arc AB into the same number of equal parts and number them as shown.

(viii) Join O′ , 1′ ; O′ , 2′ ; etc; and extend; meeting the arc OC at O_1, O_2, etc.

(ix) Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O′ as centre. Locate the point P on the generating circle, which is lying diametrically opposite to the initial point of contact between the two circles.

(x) With O as centre and radius 25, draw an arc intersecting the arc through 1 at P_1.

(xi) In a similar manner, obtain points P_2, P_3 etc.

A smooth curve through P, P_1, P_2, etc., is the required path of the point P, the epi-cycloid.

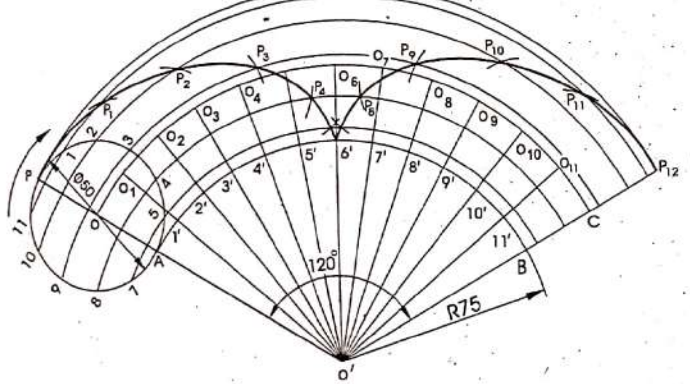


Fig. 5.35 Epi-cycloid

5.5.3 Hypo-cycloid

A hypo-cycloid is a curve traced by a point on the circumference of a generating circle, which rolls without slipping on another circle (directing circle), inside it.

Problem 27 *Draw, a hypo-cycloid of a circle of 40 diameter which rolls inside another circle of 160 diameter, for one revolution counter clock-wise. Draw a tangent and a normal to it at a point 65 from the centre of the directing circle.* (Aug/Sep 2008, JNTU)

A procedure similar to the above (Fig. 5.35), may be followed for constructing the hypo-cycloid (Fig. 5.36), keeping in view that the generating circle rolls inside the directing circle.

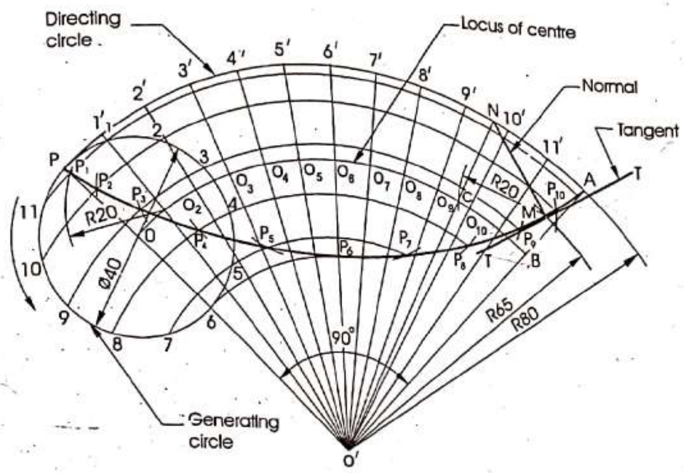


Fig. 5.36 Hypo-cycloid

The method of drawing the tangent and normal to the hypo-cycloid is also similar to the one that is followed for the epi-cycloid.

Problem 28 *Show by means of a drawing that when the diameter of the directing circle is twice that of the generating circle, the hypo-cycloid is a straight line. Take the diameter of the generating circle equal to 40.* (June 2008, JNTU)

Construction (Fig. 5.37)

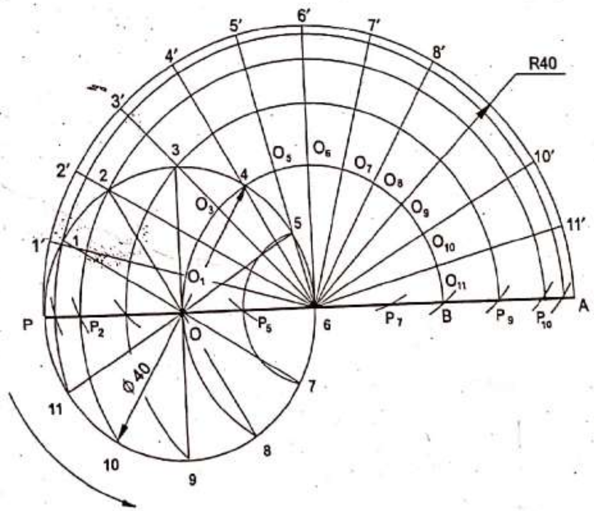


Fig. 5.37 Hypo-cycloid-straight line

Construct the hypo-cycloid, following the procedure under Construction: Fig. 5.36.

It may be noted that when the diameter of the generating circle is equal to the radius of the directing circle, the hypo-cycloid traced is a straight line.

5.6 INVOLUTE

An involute is a curve traced by a point, on a perfectly flexible thread, while unwinding from a circle or a polygon; the thread being kept tight.

Problem 29 *Draw the involute of an equilateral triangle of side 20 and draw a normal and a tangent at a distance 60 from the centre of the triangle.* (May/June 2010,JNTU)

***Construction** (Fig 5.38)*

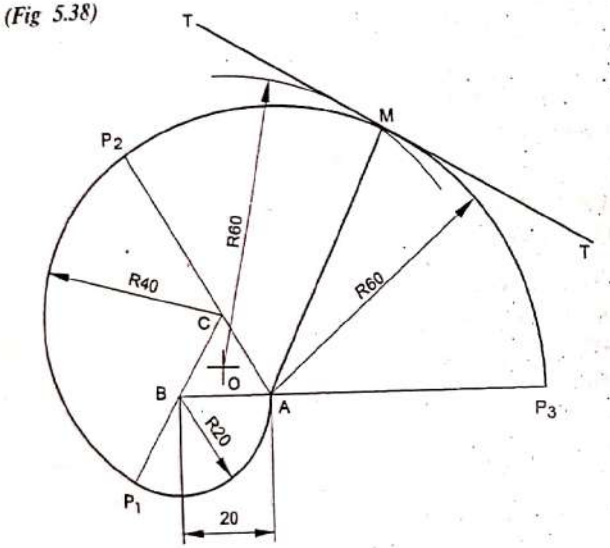


Fig. 5.38 Involute of a triangle

(i) Draw the given triangle ABC of side 20 and locate its centre O.

(ii) Assuming A as the starting point; with B as centre and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 (= 2× 20), draw an arc intersecting the line AC extended at P_2.

(iv) With centre A and radius AP_2 (=3×20), draw an arc intersecting the line BA produced at P_3.

The curve through A, P_1, P_2, P_3 is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre (centre of the triangle) and radius 60, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which A is centre.

(ii) Join A, M; forming the normal to the curve.

(iii) A line T-T, perpendicular to AM at M is the required tangent.

Problem 30 *Draw the involute of a regular hexagon of side 20. Draw a tangent and a normal to the curve at a distance 100 from the centre of the hexagon.*

***Construction** (Fig. 5.39)*

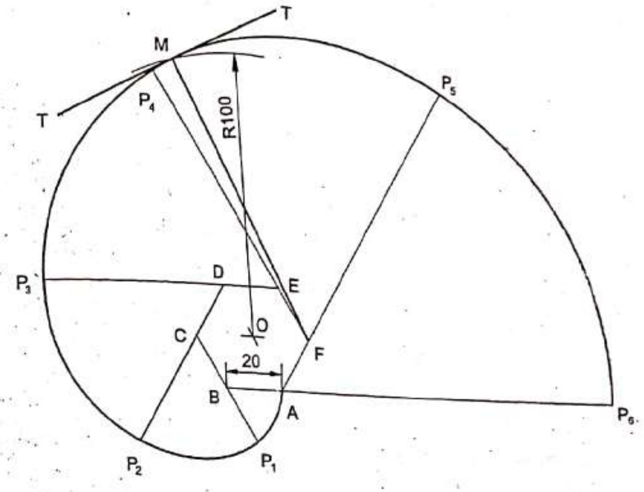


Fig. 5.39 Involute of a hexagon

(i) Draw the hexagon ABCDEF of side 20 and locate its centre O.

Assuming that the thread is unwound from A in the clock-wise direction, the starting point for the involute is A,

(ii) With centre B and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 (=2 ×20), draw an arc intersecting the line DC extended at P_2.

(iv) In a similar way, obtain the other points P_3, P_4, etc.

A smooth curve through the above points is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre and radius 100, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which F is the centre.

(ii) Join F, M; forming normal to the curve.

(iii) A line T-T, perpendicular to FM at M is the required tangent.

Problem 31 *Draw the involute of a circle of 40 diameter. Also, draw a tangent and a normal to the curve at a point 95 from the centre of the circle.* (May 2012, JNTU)

Construction (Fig. 5.40)

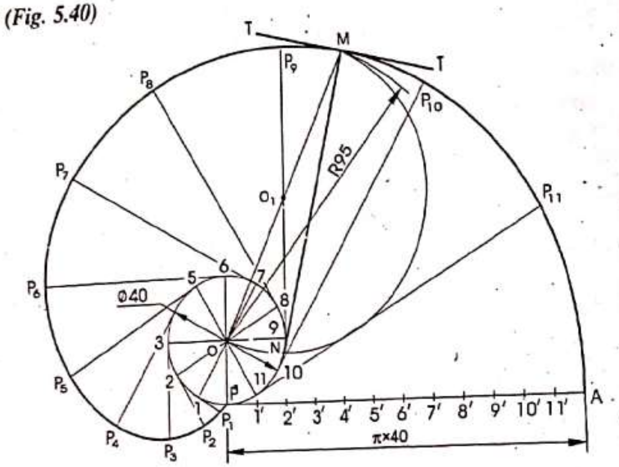


Fig. 5.40 Involute of a circle

(i) With centre O and diameter 40, draw the given circle.

(ii) Assuming P as the starting point, draw a line PA, tangent to the circle and equal to the circumference of the circle.

(iii) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(iv) Draw a tangent to the circle at the point 1 and locate P_1 on it such that, $1P_1 = P1'$.

(v) Draw a tangent to the circle at point 2 and locate P_2 on it such that, $2P_2 = P2'$.

(vi) Locate other points P_3, P_4, etc., in a similar way.

A smooth curve through these points is the required involute.

From the construction, it is obvious that a tangent to the circle is normal to the involute. So, to draw the tangent and normal,

(i) Locate the point M on the curve, which is at 95 from the centre of the circle.

(ii) Join M, O and locate its mid-point O_1.

(iii) With centre O_1 and radius O_1 M, draw a semi-circle intersecting the given circle at N.

(iv) Join N, M; forming the normal to the curve and a line T-T, perpendicular to NM at M is the tangent to the curve

Problem 32 *A thread of length 165 is wound round a circle of 40 diameter. Trace the path of end point of the thread.*

(May/June 2011,2012, JNTU)

Construction (Fig. 5.41)

(i) With centre O and radius 20, draw the given circle.

(ii) From point A on the circle, draw a line AP, tangential to the circle and equal to 165, the length of the thread.

(iii) Divide the circle into 12 equal parts and mark the chord lengths along the line AP.

(iv) Draw tangents to the circle at points 1, 2, etc.

(v) Along the tangent through 1, mark P_1 such that, $1P_1 = P1'$.

(vi) Along the tangent through 2, mark P_2 such that, $2P_2 = P2'$.

(vii) In a similar way, locate the points P_3, P_4, etc.

A smooth curve through these points is the required path.

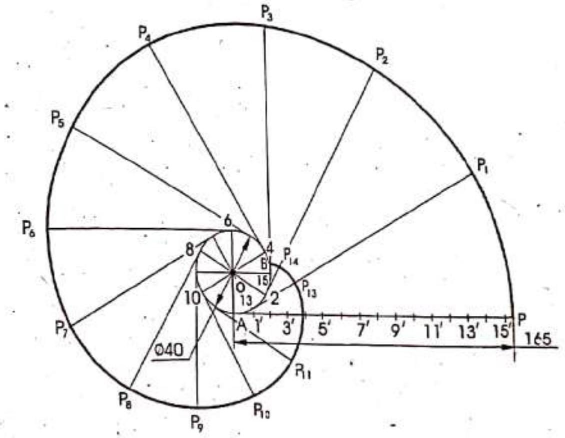


Fig. 5.41 Path of end point of thread, wound round a circle

NOTE (i) The points P_{13}, P_{14} and P_{15} are located along the tangents through 1, 2 and 3.

(ii) The point B on the circle is located such that, the chord length 15-B = 15′ -P.

Problem 33 *A line AC of 150 long, is tangential to a circle of diameter 60. Trace the paths of A and C, when the line AC rolls on the circle without slipping.*

Construction (Fig. 5.42)

(i) Draw a circle of diameter 60.

(ii) Draw the tangent AC to the circle at A, of length 150.

(iii) Divide the circle into a number of equal parts, say 12 and number them as shown.

(iv) Mark 1′, 2′, 3′, etc., on AC such that, A-1′ = 1′ -2′ = 2′ -3′, etc., = $1/12^{th}$ circumference of the circle.

(v) When the line AC rolls on the circle and 1′ coincides with 1, locate the positions of A and C; such that $1\text{-}A_1$ = A-1′ and $1\text{-}C_1$ = 1′- C.

(vi) Similarly, locate the end points for different positions of the line, as it rolls on the circle. Join A, A_1, A_2, A_3, etc., and C, C_1, C_2, C_3, etc., representing the paths of A and C.

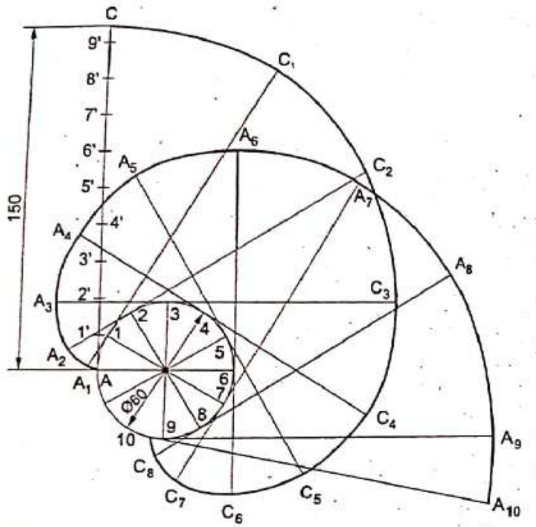


Fig. 5.42 Paths of end points of a line rolling on a circle

Problem 34 *A disc in the form of a square of 35 side is surmounted by semi-circles on the two opposite sides. Draw the path of the end of the string, unwound from the circumference of the disc.*

Construction (Fig. 5.43)

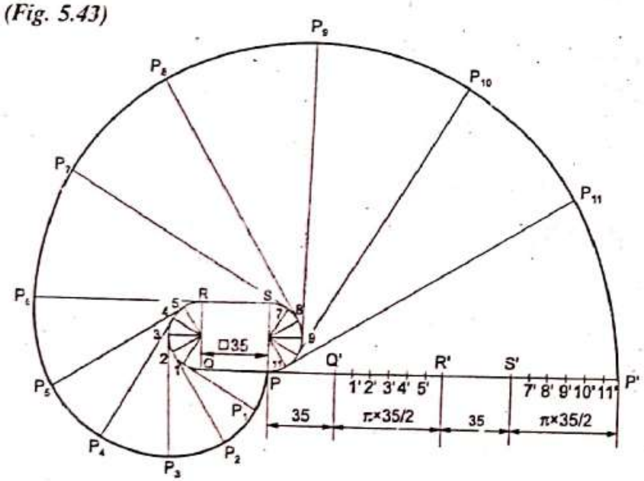


Fig. 5.43 Path of end point of a string unwound from a disc

(i) Draw the square PQRS of side 35, surmounted by two semi-circles on the two opposite sides.

(ii) Divide the semi-circles into six equal parts and number them as shown.

(iii) Extend QP to P′ such that, $PP' = 2 \times 35 + \pi \times 35$ and mark the division points as shown.

(iv) Draw tangents to the semi-circles at 1, 2, – – – – R, 7, 8, – – – – 11.

(v) Assuming that the string is unwound from P, locate P_1 along the tangent at 1 such that, $1P_1 = P1'$.

(vi) Locate P_2 along the tangent at 2 such that, $2P_2 = P2'$ and so on.

(vii) Join the points, P, P_1, P_2, etc., by a smooth curve forming the path of the end of the string.

5.7 HELICAL SURFACES

Helix is a curve, generated by a point moving on the surface of a cylinder or cone in a circular direction. The point moves at a constant angular velocity and with a simultaneous uniform rate of advance in axial direction; the ratio of the two movements being constant. The amount of the axial advance for one revolution is called the pitch or lead of the helix.

Threaded elements containing helical surfaces may be classified under the following two groups, depending upon their use.

(i) Fasteners, for holding various parts in a structure, viz., bolts, studs, cap screws, machine screws and set-screws.

(ii) Elements, which transform the rotary motion into translatory motion, viz., screw conveyor, lifting screw, wheel puller, lead screw, cylindrical cam, propeller blade, helical chute, etc.

5.8 SINGLE HELIX

Single helix is a curve, generated by a single moving point on the surface of a cylinder or cone in a circumferential direction.

Problem 35 *Draw a helix of pitch equal to 45, upon a cylinder of 40 diameter and develop the surface of the cylinder along with the helix. Assume the starting point P to be on the left extreme horizontal centre line in the top view.*

HINT Assume the generating point P to move upwards and in anti-clockwise direction.

Construction (Fig. 5.44)

(i) Draw the projections of the cylinder.

(ii) Divide the base and pitch of the helix into the same number of equal parts, say 12.

(iii) Draw the generators in the front view, corresponding to the division points in the top view of the cylinder.

(iv) Mark P_1, the position of the point P at the intersection point between generator 1 and the horizontal line drawn through the first division point of the pitch (as the point moves by $1/12^{th}$ of revolution from P to 1 around the cylinder, it advances along the axis of the cylinder by $1/12^{th}$ of the pitch and occupies the position P_1).

(v) Locate the other points P_2, P_3, etc., in a similar manner.

A smooth curve through the points, P, P_1, P_2, etc., is the required helix.

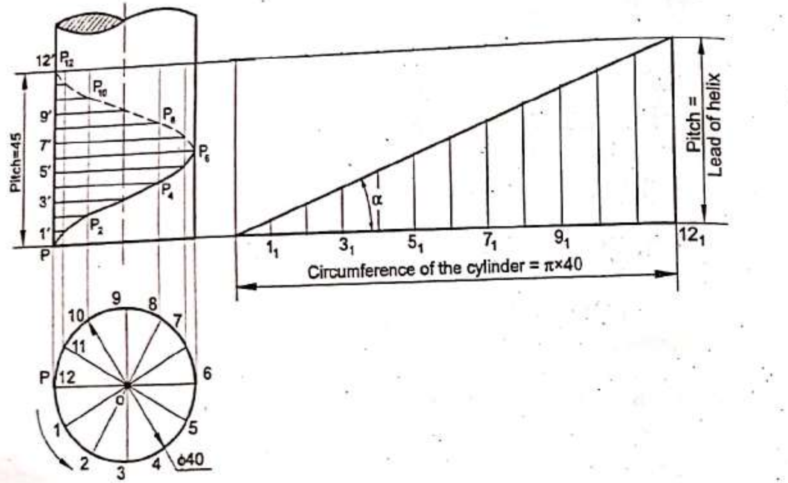


Fig. 5.44 Right hand helix on a cylinder

NOTE (i) The helix can be a left hand or right hand one. The helix obtained in Fig. 5.44 is a right hand one. If nothing is specified about the type of the helix, it is the usual practice to consider it as a right hand one.

(ii) The helix corresponding to one revolution of the moving point is known as one convolution of the helix.

(iii) The portion of the curve joining P_6, P_7, – – – – P_{12} is invisible as it lies on the rear side of the cylinder.

(iv) The distance, the generating point advances along the axis, for one convolution of the helix, is called the lead. For single helix, the lead and pitch are equal.

Figure 5.44 also shows the development of the cylinder along with the helix. The angle α is known as the helix angle. The relation between the helix angle and the lead of the helix is given by,

$$\tan\alpha = \frac{\text{lead of the helix}}{\text{circumference of the cylinder}}$$

Figure 5.45 shows a left hand helix on a cylinder of 40 diameter with 45 pitch. Here, the generating point P moves upwards and in clock-wise direction.

Problem 36 *Draw a helix of one convolution upon a cone, with diameter of base 40, axis 50 long and pitch 40. Assume the starting point P to be on the left extreme horizontal centre line in the top view. Also, develop the surface of the cone and show the helix on it.*

HINT The pitch of the conical helix is measured parallel to the axis of the cone.

Construction (Fig. 5.46)

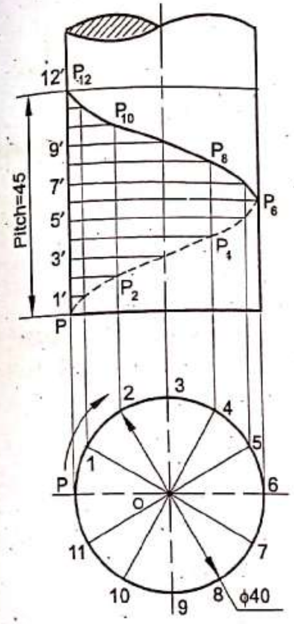


Fig.5.45 Left hand helix on a cylinder

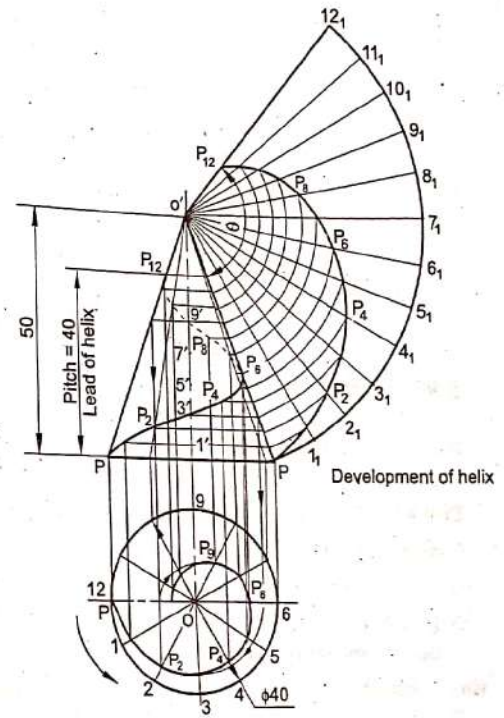


Fig.5.46 Right hand helix on a cone

(i) Draw the projections of the cone.

(ii) Divide the pitch (=lead) along the axis of the cone and the base of the cone into the same number of equal parts, say 12.

(iii) Draw the generators in the front view, corresponding to the division points in the top view.

(iv) Locate the points of intersection P_1, P_2, etc., between the division lines of the pitch and generators.

(v) Join the points, P, P_1, P_2, etc., by a smooth curve, forming the required conical helix. The part of the curve between the points P_6 to P_{12} is invisible, as the points lie on the rear side of the cone.

CHAPTER - 6

ORTHOGRAPHIC PROJECTIONS

6.1 INTRODUCTION

Any object has three dimensions, viz., length, width and thickness. A projection is defined as a representation of an object on a two dimensional plane. The following are the elements to be considered while obtaining a projection:

1. The object,
2. The plane of projection,
3. The point of sight, and
4. The rays of sight.

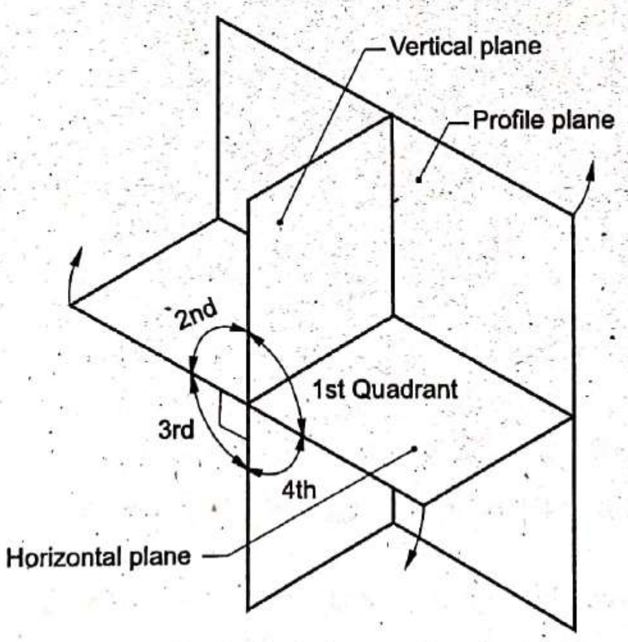


Fig. 6.1 Principal planes of projection

A projection may be obtained by viewing the object from the point of sight and tracing in correct sequence, the points of intersection between the rays of sight and the plane to which the object is projected. A projection is called an orthographic projection, when the point of sight is imagined to be located at infinity so that the rays of sight are parallel to each other and intersect the plane of projection at right angle to it.

The principles of orthographic projection may be followed in four different angles or systems: first, second, third and fourth angle projections. Figure 6.1 shows the three planes used in orthographic projections in which all the four quadrants are marked. A projection is said to be the first, second, third or fourth angle when the object is imagined to be either in the first, second, third or fourth quadrant respectively. However, only two systems, viz., the first and third angle projections are being followed.

The Bureau of Indian Standards though recommends first angle projection, this chapter deals with the principles involved in both the systems of projection so that the student is in a better position to adopt to any system in his career that is demanded of him.

6.2 PRINCIPLES OF THE TWO SYSTEMS OF PROJECTION

6.2.1 First angle projection

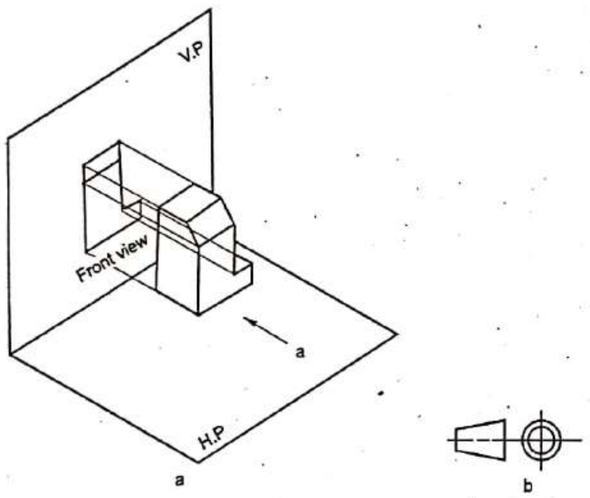


Fig. 6.2 Method of obtaining front view-First angle projection

In first angle projection, the object is imagined to be positioned in the first quadrant. The front view of the object is obtained by looking at the object from the right side of the quadrant and tracing, in correct sequence, the points of intersection between the projection plane and the lines of sight extended. In this case, the object will be in-between the observer and the plane of projection (V.P). Here, the object is imagined to be transparent and the projection lines are extended from various points of the object to intersect the plane of projection. Thus, in first angle projection, any view is so placed that it represents the side of the object away from it.

Figure 6.2a shows the method of obtaining the front view of an object in first angle projection and Fig. 6.2b, the symbol used for first angle projection.

6.2.2 Third angle projection

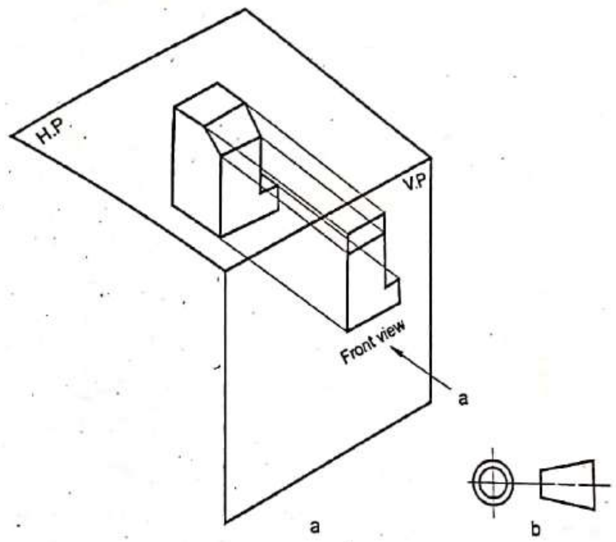


Fig. 6.3 Method of obtaining front view - Third angle projection

In third angle projection, the object is imagined to be positioned in the third quadrant. The front view of the object is obtained by looking at the object from the right side of the quadrant and tracing in correct sequence, the points of intersection between the projection plane and the lines of sight reaching the object. In this case, the plane of projection (V.P) is in-between the object and observer and so, the projection plane is imagined to be transparent and the rays of sight pass through it and reach the object. Thus, in third angle projection, any view is so placed that it represents the side of the object nearer to it.

Figure 6.3a shows the method of obtaining the front view of an object in third angle projection and Fig. 6.3b, the symbol used for third angle projection.

6.3 METHODS OF OBTAINING ORTHOGRAPHIC VIEWS

6.3.1 Front view

The front view of an object is defined as the view that is obtained as projection on the vertical plane, by looking the object normal to its front surface. It is the usual practice to position the object such that, its front view reveals most of the important features. Figures 6.2 and 6.3 reveal the methods of obtaining the front view in first and third angle projections respectively.

6.3.2 Top view

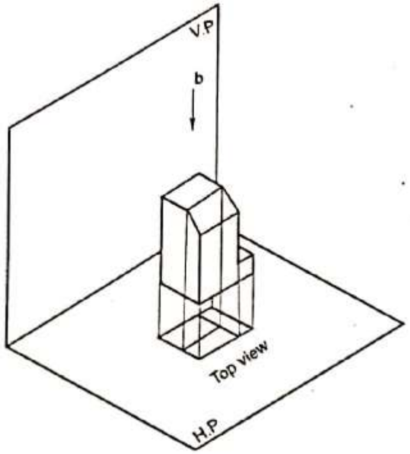


Fig. 6.4 Method of obtaining top view - First angle projection

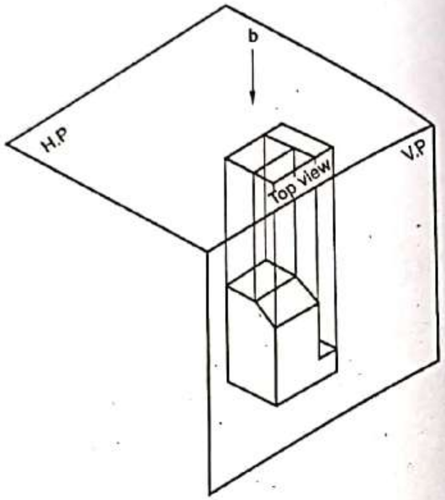


Fig. 6.5 Method of obtaining top view - Third angle projection

The top view of an object is defined as the view that is obtained as projection on the horizontal plane, by looking the object normal to its top surface. Figure 6.4 shows the method of obtaining the top view of the object considered above, in first angle projection. Figure 6.5 shows the method of obtaining the top view of the same object in third angle projection.

6.3.3 Side view

The side view of an object is defined as the view that is obtained as projection on the profile plane, by looking the object, normal to its side surface. As there are two sides for an object, viz., left side and right side; two possible side views, viz., the left side and right side views may be obtained for any object.

In the first angle projection, a left side view is obtained on a profile plane by placing it to the right side of the object. Figure 6.6 shows the method of obtaining a left side view of an object in first angle projection.

Figure 6.7 shows the method of obtaining a left side view in third angle projection. It may be noted that in third angle projection, a left side view is obtained by placing the profile plane to the left side of the object.

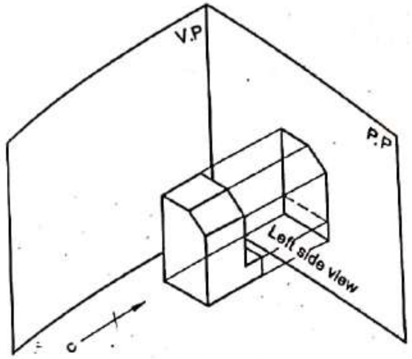


Fig. 6.6 Method of obtaining left side view - First angle projection

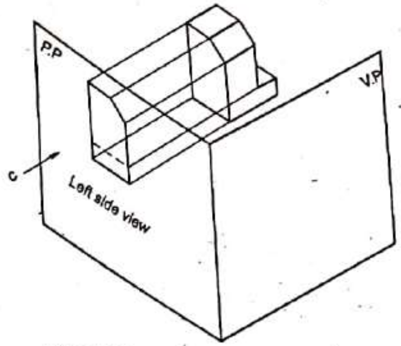


Fig.6.7 Method of obtaining left side view - Third angle projection

6.4 PRESENTATION OF VIEWS

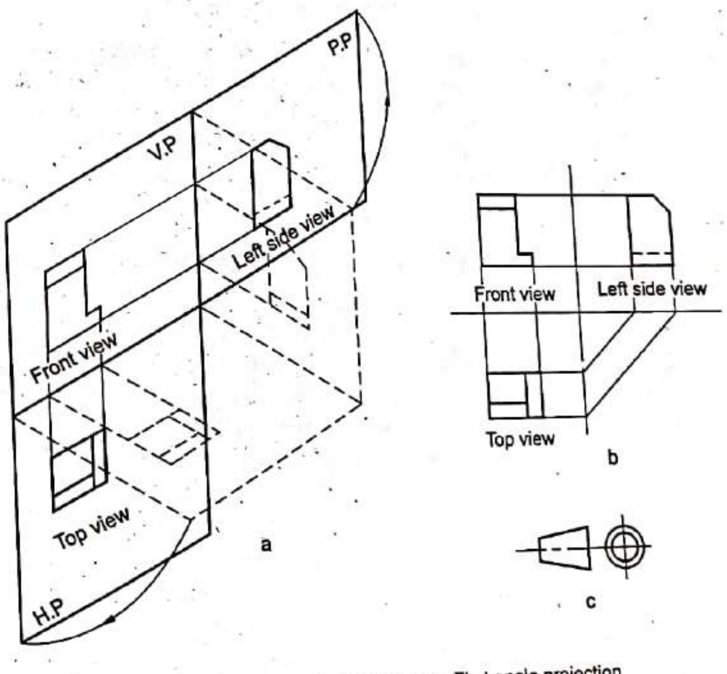


Fig. 6.8 Presentation of views - First angle projection

CHAPTER - 7

PROJECTIONS OF POINTS

7.1 INTRODUCTION

Projections of points, lines and planes must be studied in order to understand the projections of solids, because it could be said that a solid consists of a number of planes, a plane consists of a number of lines and a line consists of a number of points.

A solid may be generated by a plane moving in space (Fig.7.1a), a plane may be generated by a straight line moving in space (Fig.7.1b) and a straight line in-turn, may be generated by a point moving in space (Fig.7.1c).

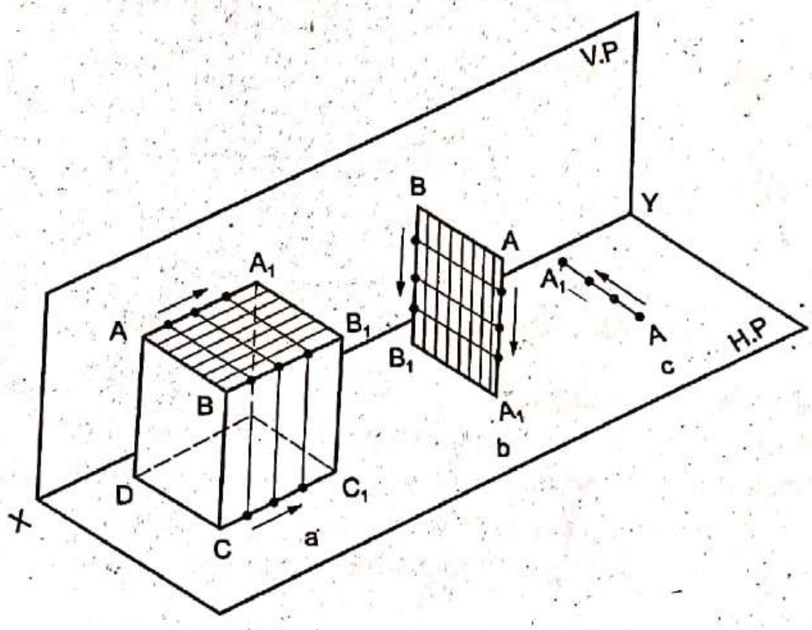


Fig. 7.1

To understand the principles of orthographic projections of solids, it is necessary to study first the principles involved in the projections of the points, lines and planes. This chapter deals with the principles involved in the orthographic projections of points.

7.2 TWO VIEW PROJECTIONS OF POINTS

In space, one can identify three mutually perpendicular planes. A point in space may be thought of lying either in one of the four quadrants formed by the planes or on one of the three planes. This article deals with the two view projections of points in space.

7.2.1 Projections of a point situated in first quadrant

Problem 1 *A point A is 20 above H.P and 30 in front of V.P. Draw the projections of the point.* (May/June 2008, JNTU)

Figure 7.2a shows the position of the point A in the first quadrant. When the point is viewed in the direction a, the front view a′ is obtained at the intersection point between the ray of sight through A and V.P. When the point is viewed in the direction b, the top view a is obtained at the intersection point between the ray of sight and H.P.

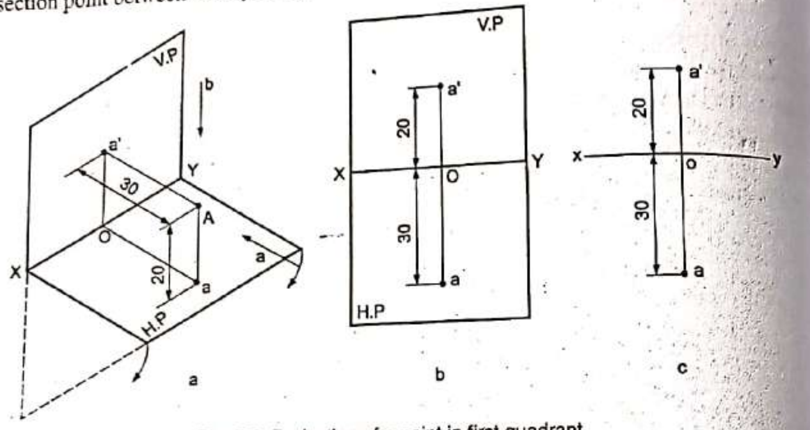


Fig. 7.2 Projection of a point in first quadrant

For presenting the views on a plane sheet, the H.P is rotated till it comes in-line with V.P. In general, as stated in the previous chapter, irrespective of the location of the point, the plane(s) must be rotated such that the first quadrant always opens-out. Figure 7.2b shows the relative positions of the views or projections, as otherwise called, along with the planes of projection. Figure 7.2c shows only the relative positions of the views, as it is customary not to show the planes of projection.

The following may be noted from the study of Fig. 7.2:

1. The line XY is the intersection line between H.P and V.P (refer Figs. 7.2a and b). In Fig. 7.2c the line is represented by xy, which is known as the base or reference line. Actually, XY is the line about which the rotation of the plane (H.P) is made.
2. It is customary to use capital letters to specify the position of the points in space and lower case letters for their projections. As an example, for the point A in space, the front view, top view and side view are represented by a′ , a and a″ respectively.
3. The ray of sight passing through the point A meets the corresponding plane of projection at right angles to it. This line is known as a projection line.
4. The line joining a and a′, intersects the reference line xy at right angles at o. This line is known as a projector.

5. The front view a′ is above xy and top view a is below xy.
6. The distance a′ o is equal to the distance of the point from H.P.
7. The distance ao is equal to the distance of the point from V.P.

7.2.2 Projections of a point situated in second quadrant

Problem 2 *A point B is 20 above H.P and 30 behind V.P. Draw its projections.*

Figure 7.3a shows the position of the point B in the second quadrant. When the point is viewed in the direction **a** and assuming V.P to be transparent, the front view b′ is obtained at the point of intersection between the ray of sight (towards the point) and V.P. Similarly, the top view b is obtained on H.P by viewing the point in the direction **b**. Here, it is assumed that V.P is transparent.

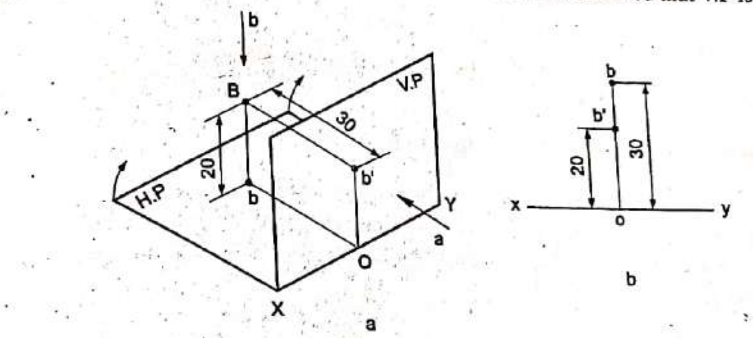


Fig. 7.3 Projection of a point in second quadrant

Figure 7.3b shows the relative positions of the views. These are obtained by rotating the H.P till it coincides with V.P. It may be noted that both the front and top views are above the reference line xy.

7.2.3 Projections of a point situated in third quadrant

Problem 3 *A point C is 20 below H.P and 30 behind V.P. Draw its projections.* (May/June 2008, JNTU)

Figure 7.4a shows the position of the point C in the third quadrant. c′ and c are the front and top views obtained on V.P and H.P, by viewing the point in the directions **a** and **b** respectively. Here, it is assumed that H.P and V.P are transparent.

Figure 7.4b shows the relative positions of the views. These are obtained by rotating the H.P till it comes in-line with V.P. It may be noted that the front view c′ is below xy and the top view c is above xy.

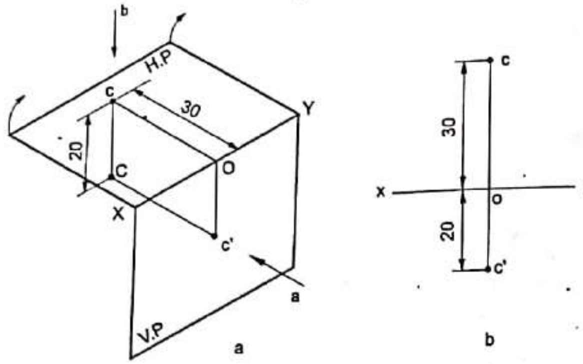


Fig. 7.4 Projections of a point in third quadrant

7.2.4 Projections of a point situated in fourth quadrant

Problem 4 *A point D is 20 below H.P and 30 in front of V.P. Draw its projections.*

Figure 7.5a shows the position of the point D in fourth quadrant. d′ and d are the front and top views obtained on V.P and H.P, by viewing the point in the directions **a** and **b** respectively. Here, it is assumed that H.P is transparent.

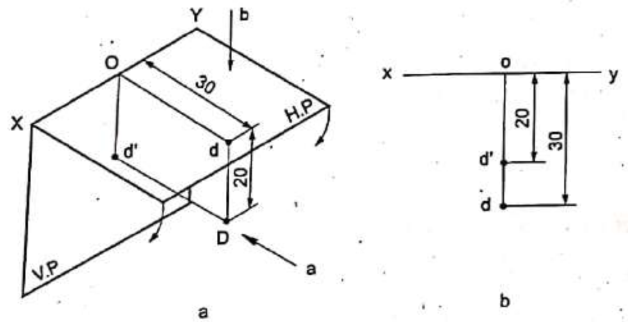


Fig. 7.5 Projection of a point in fourth quadrant

Figure 7.5b shows the relative positions of the views. These are obtained by rotating H.P till it coincides with V.P. It may be noted that both the front and top views are situated below the reference line xy.

7.2.5 Projections of a point situated on H.P and in front of V.P

Problem 5 *A point E is on H.P and 30 in front of V.P. Draw its projections.*

7.2.6 Projections of a point situated on V.P and above H.P

Problem 6 A point F is on V.P and 20 above H.P. Draw its projections.

7.2.7 Projections of a point situated on both H.P and V.P

Problem 7 A point G is lying on both H.P and V.P. Draw its projections.

All the above three cases are shown in Fig. 7.6a. Figure 7.6b shows the relative positions of the views for each case. The following may be noted from the Fig. 7.6b:

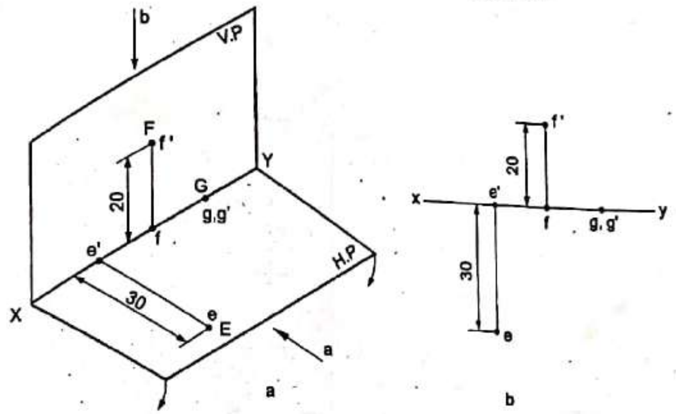


Fig. 7.6

1. When a point lies on H.P, its front view will lie on xy.
2. When a point lies on V.P, its top view will lie on xy.
3. When a point lies on both H.P and V.P, its front and top views lie on xy.

NOTE The student is advised to understand the principles of projection for the cases when (i) a point lies on H.P but behind V.P and (ii) on V.P but below H.P.

7.3 THREE VIEW PROJECTIONS OF POINTS

In Chapter 6, it is mentioned that, when the two views of an object are not sufficient to describe the shape completely, then it is necessary to go for a third view, preferably a side view.

Generally, for obtaining projections of a solid, the solid is imagined to be either in the first or third quadrant. The projections thus obtained are respectively known as the first and third angle projections. It is not advisable to imagine the solid to be placed in the second and fourth quadrants because in both the cases the front and top views will lie on one side of xy. This is not convenient for understanding the views.

pg- 8.1

CHAPTER - 8

PROJECTIONS OF STRAIGHT LINES

8.1 INTRODUCTION

In Chapter 7, it is stated that a straight line may be generated by a point moving in one direction. A straight line may also be defined as the shortest distance between two points. A straight line may be located in space either by specifying the location of the end points or by specifying the location of one end point and the direction.

8.2 TWO VIEW PROJECTIONS OF STRAIGHT LINES

The following are the possible positions of a straight line, with respect to the planes of projection:

1. Parallel to both the planes
2. Perpendicular to one plane
3. Inclined to one plane and parallel to the other
4. Inclined to both the planes
5. Contained by a plane, perpendicular to both the principal planes

8.2.1 Straight line parallel to both the planes

When a line is parallel to any plane, its projection on that plane is a straight line of the same length. This is because, in orthographic projections, the line is imagined to be viewed from infinity. Hence, the rays of sight are parallel to each other. When they pass through the end points of the straight line, meet the plane of projection at two points, the distance between them is equal to the length of the projected line itself.

Problem 1 *A line AB of 50 long, is parallel to both H.P and V.P. The line is 40 above H.P and 30 in front of V.P. Draw the projections of the line.*

Figure 8.1a shows the position of the line AB in the first quadrant. The points a′, b′ on V.P and a, b on H.P are the front and top views of the ends A and B of the line AB. The lines a′ b′ and ab are the front and top views of the line AB respectively.

Figure 8.1b shows the relative positions of the views along with the planes, after rotating H.P, till it is in-line with V.P. Figure 8.1c shows the relative positions of the views only.

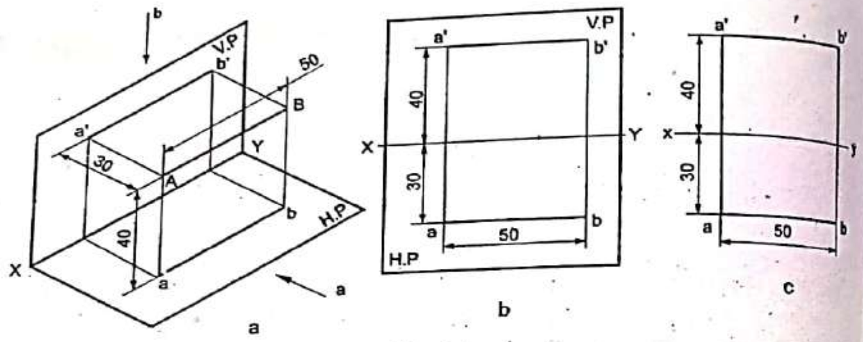


Fig. 8.1

Construction (Fig. 8.1c)

1. Draw the reference line xy and draw a projector at any convenient point on it.
2. Locate the projections of the end point A of the line a′ , at 40 above xy and a, at 30 below
3. Through a′ and a, draw lines a′ b′ and ab, parallel to xy and of length 50.
4. Join the points b′ and b by a projector.

a′ b′ and a b are the projections of the line AB.

8.2.2 Straight line perpendicular to one plane

When a line is perpendicular to one of the planes, it is evident that it is parallel to the other p[lane]. Further, when a line is parallel to a plane, the length of the projection on that plane is equal t[o the] true length of the line.

Problem 2 *A line AB of 25 long, is perpendicular to H.P and parallel to V.P. The end poin[ts A] and B of the line are 35 and 10 above H.P respectively. The line is 20 in front of V.P. Dra[w the] projections of the line.*

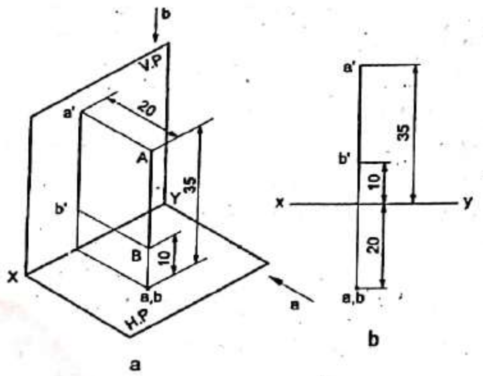


Fig. 8.2

Figure 8.2a shows the position of the line AB in the first quadrant. As the line is parallel to V.P, the length of the front view is equal to the true length of the line and the top view appears as a point.

Construction (Fig. 8.2b)

1. Draw the front view a′ b′ , a line perpendicular to xy such that, a′ and b′ are 35 and 10 above xy respectively.
2. Locate the top view of the line a, b; a point 20 below xy.

Problem 3 *A line AB of 25 long, is perpendicular to V.P and parallel to H.P. The end points A and B of the line are 10 and 35 in front of V.P respectively. The line is 20 above H.P. Draw its projections.*

Figure 8.3a shows the position of the line AB in the first quadrant. As the line is parallel to H.P, the length of the top view is equal to the true length of the line and the front view appears as a point.

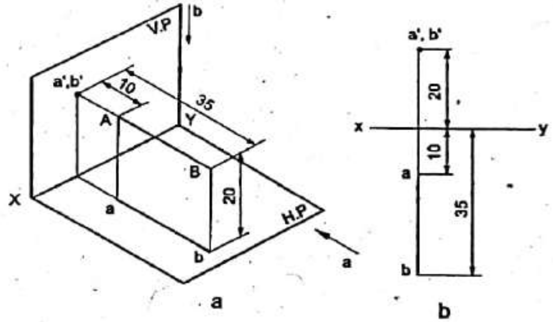


Fig. 8.3

Construction (Fig. 8.3b)

1. Draw the top view of the line ab, a line perpendicular to xy such that, a and b are 10 and 35 below xy respectively.
2. Locate the front view of the line a′, b′ ; a point 20 above xy.

NOTE In both the above cases, the projectors drawn, connecting both the views, intersect the reference line xy at right angle.

8.2.3 Straight line inclined to one plane and parallel to the other

The problems of this nature are normally solved in two stages. In the first stage, the line is assumed to be parallel to both the planes and projections are drawn. In the second stage, the line is rotated to make the required angle and the final views are obtained.

Problem 4 *A line AB is 30 long and inclined at 30° to H.P and parallel to V.P. The end A of the line is 15 above H.P and 20 in front of V.P. Draw the projections of the line.*

Figure 8.4a shows the position of the line in the first quadrant, along with the views obtained by projection on H.P and V.P.

Construction (Fig. 8.4b)

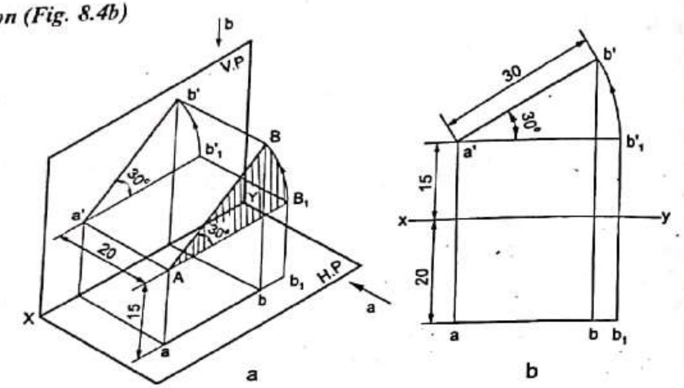


Fig. 8.4

Stage I Assume that the line is parallel to both H.P and V.P.

1. Draw the projections ab_1 and $a'b'_1$ of the line.

Stage II Rotate the line such that it makes the given angle with H.P.

2. Rotate the line $a'b_1'$ by 30°, to the position $a'b'$.
3. Drop a projector from b' till it meets the line ab_1 at b.

$a'b'$ and ab are the required projections.

NOTE (i) The distance 20 of the point B from V.P remains unchanged, irrespective of the angle of inclination of the line with H.P. The actual position of b on ab_1 depends upon the angle θ. In other words, the line ab_1 may be termed as the locus of the top view of the end point B.

(ii) It may be noted that the length $a'b'$ is the true length of the given line AB and the inclination of the front view with xy is the true inclination of the line with H.P.

Problem 5 *A line AB is 30 long and inclined at 30° to V.P and parallel to H.P. The end A of the line is 15 above H.P and 20 in front of V.P. Draw its projections.*

Figure 8.5a shows the position of the line in the first quadrant, along with the views obtained by projection.

Construction (Fig. 8.5b)

1. Draw the projections $a'b_1'$ and ab_1 for the line, assuming that it is parallel to both H.P and V.P.
2. Rotate the line ab_1 by 30°, to the position ab.
3. Drop a projector from b till it meets the line $a'b_1'$ at b'.

$a'b'$ and ab are the required projections.

NOTE

1. The line $a'b_1'$ is the locus of the front view of the end point B.
2. The length ab is the true length of the given line and the inclination of the top view with xy is the true inclination of the line with V.P.

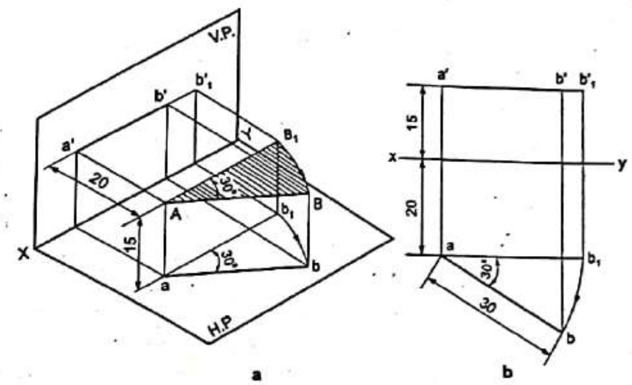


Fig. 8.5

8.2.4 Straight line inclined to both the planes

A line inclined to both H.P and V.P is called an oblique line. The methods discussed in the above two problems may be combined to draw the projections of oblique lines.

Problem 6 *A line AB of 100 length, is inclined at an angle of 30° to H.P and 45° to V.P. The point A is 15 above H.P and 20 in front of V.P. Draw the projections of the line.*

(May/June 2008, JNTU)

Construction (Fig. 8.6)

Stage I Assume that the line is inclined at 30° to H.P and parallel to V.P

1. Draw the projections $a'b_1'$ and ab_1 of the line $AB_1 = AB$ (Figs. 8.6a,b).

Keeping the inclination 30° constant, rotate the line AB_1 to AB, till it is inclined at 45° to V.P. This process of rotation does not change the length of the top view ab_1 and the distance of the point B_1 (=B) from H.P. Hence, (i) the length of ab_1 is the final length of the top view and (ii) the line f-f, parallel to xy and passing through b_1' is the locus of the front view of the end point B.

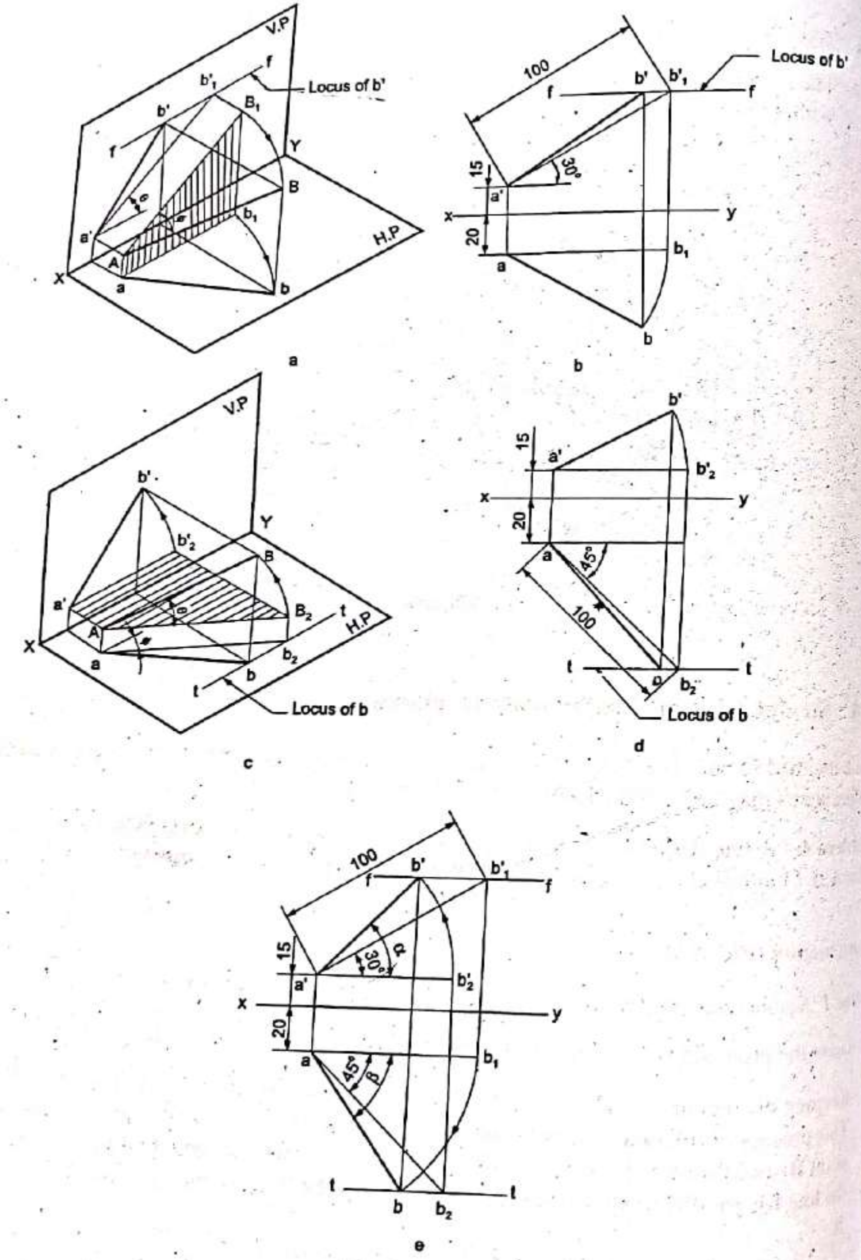


Fig. 8.6

Stage II *Assume that the line is inclined at 45° to V.P and parallel to H.P.*

2. Draw the projections ab_2 and $a'b_2'$ of the line $AB_2 = AB$ (Figs. 8.6c,d).

Extending the discussion on the preceding stage to the present one, the following may be concluded:

(i) The length $a'b_2'$ is the final length of the front view and (ii) the line t-t, parallel to xy and passing through b_2 is the locus of the top view of the end point B.

Stage III *Combine stages I and II*

3. Obtain the final projections by combining the results from stages I and II, as detailed below (Fig. 8.6e):

   (i) Draw the projections $a'b_1'$ (=AB) at 30° and ab_2 (=AB) at 45° with xy, after locating the projections of A.
   (ii) Obtain the projections $a'b_2'$ and ab_1, parallel to xy.
   (iii) Draw the lines, f-f and t-t, the loci, parallel to xy and passing through b_1' and b_2 respectively.
   (iv) With centre a' and radius $a'b_2'$, draw an arc meeting f-f at b'.
   (v) With centre a and radius ab_1, draw an arc meeting t-t at b.
   (vi) Join a', b' and a, b; forming the required projections.

The following may be noted from Fig. 8.6e:

1. The points b' and b lie on a single projector.
2. The projections a'b' and ab make angles α and β with xy, which are respectively greater than 30°(θ) and 45° (ϕ). The angles α and β are known as the apparent angles.

8.2.5 Straight line contained by a plane (profile plane) perpendicular to both H.P and V.P

When a line is contained by a plane which is perpendicular to both H.P and V.P; the sum of the inclinations θ and ϕ with H.P and V.P is equal to 90°.

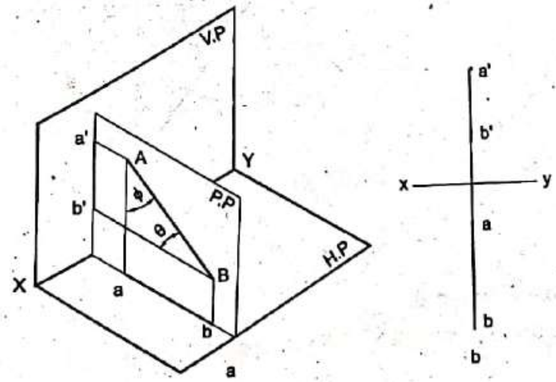


Fig. 8.7

Figure 8.7a shows the position of the line along with the views obtained by projection on H.P and V.P. Figure 8.7b shows the relative positions of the front and top views of the line. It may be noted that both the front and top views, i.e., a′ b′ and ab lie on a single projector and are shorter than the true length of the line.

8.2.6 True length and true inclinations

When projections of a line are given, its true length and true inclinations with H.P and V.P are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane. Following are the methods employed for the purpose:

Method I Rotating line method In this, each view is made parallel to the reference line and the other view is projected from it. This is the exact reversal of the procedure adopted in article 8.2.4.

Method II Trapezoidal method In this, the line is rotated about its projections till it lies in H.P or V.P.

Method III Auxiliary plane method In this, the views are projected on auxiliary planes, parallel to each view (Refer Ch.10).

Problem 7 *Figure 8.8a shows the projections of a line AB. Determine the true length of the line and its inclinations with H.P and V.P.*

Method I

Construction (Fig. 8.8b)

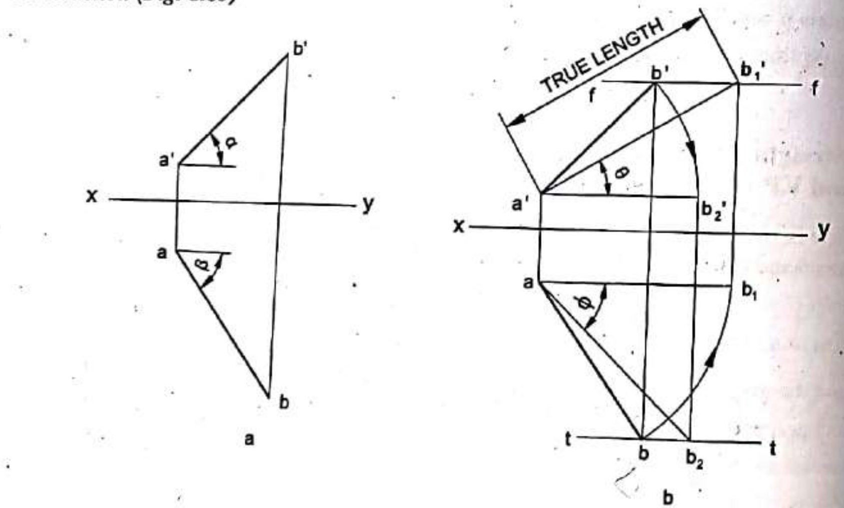


Fig. 8.8

1. Draw the given projections a′ b′ and ab.
2. Draw f-f and t-t, the loci passing through b′ and b and parallel to xy.

3. Rotate a′ b′ to a′ b_2', parallel to xy.
4. Draw a projector through b_2' to meet the line t-t at b_2.
5. Rotate ab to ab_1, parallel to xy.
6. Draw a projector through b_1 to meet the line f-f at b_1'.
7. Join a′, b_1' and a, b_2.
8. Measure and mark angles θ and ϕ.

The length a′ b_1' (=ab_2) is the true length of the line and the angles θ and ϕ are the true inclinations of the line with H.P and V.P respectively.

Method II Trapezoidal method

Principle of the method (Figs. 8.9 a, b)

Figures 8.9a and b show the position of the line AB in the first quadrant along with the projections a′ b′ and ab of the line. By geometry, the following may be observed:

1. In the trapezoid ABb′ a′, a′ A and b′ B are perpendicular to a′ b′ and are equal to ao_1 and bo_2 (the distances of a and b from xy in the top view).
2. The angle between AB and a′ b′ is the true angle of inclination of AB with V.P, ϕ.
3. In the trapezoid ABba, aA and bB are perpendicular to ab and are equal to $a'o_1$ and $b'o_2$ (the distances of a′ and b′ from XY in the front view).

The angle between AB and ab is the true angle of inclination of AB with H.P, θ.

The true length of the line and the true inclinations with the H.P and V.P may be obtained as follows:

(i) Rotate the trapezoid ABb′a′ about a′ b′ till the line AB coincides with V.P at A_1B_1.
(ii) Rotate the trapezoid ABba about ab till the line AB coincides with H.P at A_2B_2.
(iii) The lengths A_1B_1 and A_2B_2 are equal to AB, the true length of the given line.

The angles between the lines A_2B_2, ab and A_1B_1, a′ b′ are respectively equal to θ and ϕ, the true inclinations with H.P and V.P respectively.

Construction (Fig. 8.9c)

1. Draw the given projections a′ b′, ab and locate o_1, o_2.
2. Erect perpendiculars aA_2 and bB_2 to the line ab, equal to o_1 a′ and o_2b′ respectively.
3. Join the points A_2, B_2 and measure the angle θ.
4. Erect perpendiculars $a'A_1$ and b′ B_1 to the line a′ b′, equal to o_1a and o_2b respectively.
5. Join the points A_1, B_1 and measure the angle ϕ.

∴ A_1B_1 and A_2B_2 represent the true length of the given line, while the angles θ and ϕ are the true inclinations of the line with H.P and V.P.

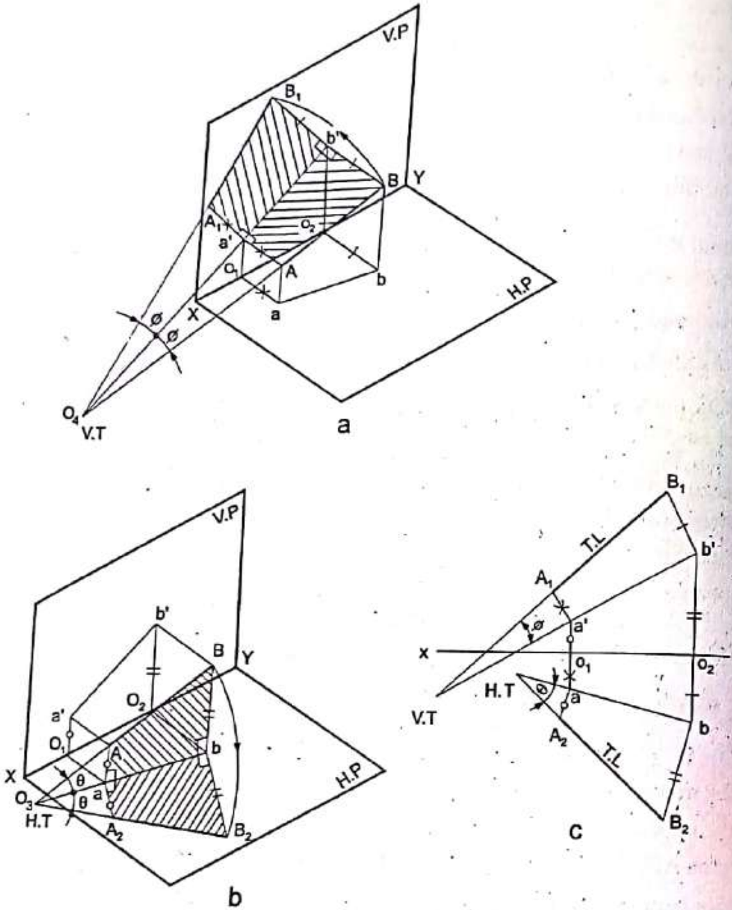


Fig. 8.9

8.3 TRACES OF A LINE

The trace of a line is the point of intersection between the given line or its extension and the plane of projection. When the line (or its extension) meets H.P, the intersection point is known as horizontal trace or H.T and when it meets V.P, it is known as vertical trace or V.T.

8.3.1 Straight line parallel to both H.P and V.P

When a straight line is parallel to both H.P and V.P, the line will not meet the planes of projection, even when it is extended. Therefore, there are no traces for the line (Fig.8.10).

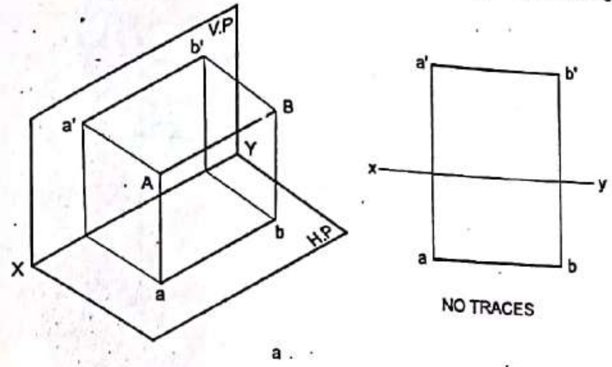


Fig. 8.10

8.3.2 Straight line perpendicular to H.P and parallel to V.P

In this case, H.T of the line will coincide with the top view of the line and there is no V.T, as the line is parallel to V.P (Fig. 8.11).

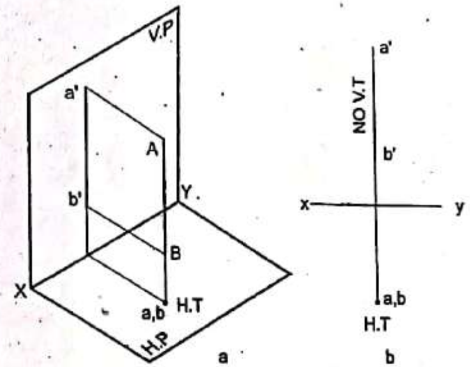


Fig. 8.11

8.3.3 Straight line perpendicular to V.P and parallel to H.P

In this case, V.T of the line will coincide with the front view of the line and there is no H.T, as the line is parallel to H.P (Fig. 8.12).

8.3.4 Straight line inclined to H.P and parallel to V.P

Referring to Fig. 8.13a, the line BA when extended will meet H.P at H.T. The following may be observed from the figure:

(i) When b' a' is extended, it meets the reference line xy at h.

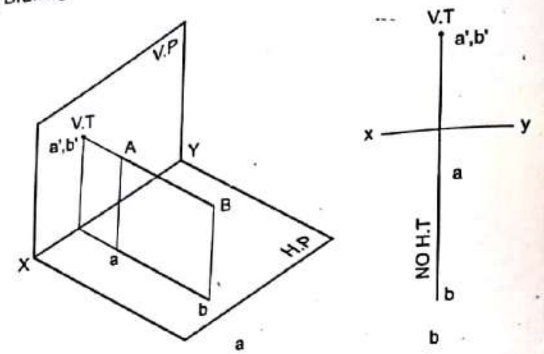


Fig. 8.12

(ii) The points h and H.T lie on a projector.

(iii) The H.T lies on the line ba extended.

(iv) There is no V.T, as the line is parallel to V.P.

Construction (Fig. 8.13b)

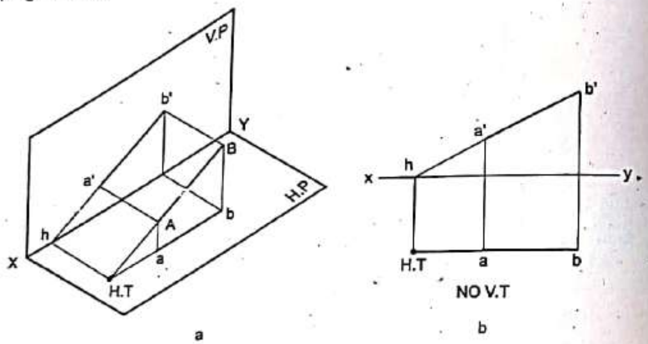


Fig. 8.13

1. Draw the projections a′ b′ and ab.
2. Locate the point h at the intersection between the reference line xy and b′ a′ (or its extension).
3. Locate H.T at the intersection between a projector drawn from h and the top view ba (or its extension).

8.3.5 Straight line inclined to V.P and parallel to H.P

Figure 8.14a shows the position of a line AB and the location of V.T. The following may be observed from the figure:

(i) When the top view ab is extended if necessary, it meets the reference line XY at v.

(ii) The points v and V.T lie on a projector.

(iii) The V.T lies on the line a′ b′ (or its extension).

(iv) There is no H.T, as the line is parallel to H.P.

Construction (Fig. 8.14b)

1. Draw the projections a′ b′ and ab.
2. Locate the point v at the intersection between the reference line xy and ba (or its extension).
3. Locate V.T at the intersection between a projector drawn from v and the front view b′ a′ (or its extension).

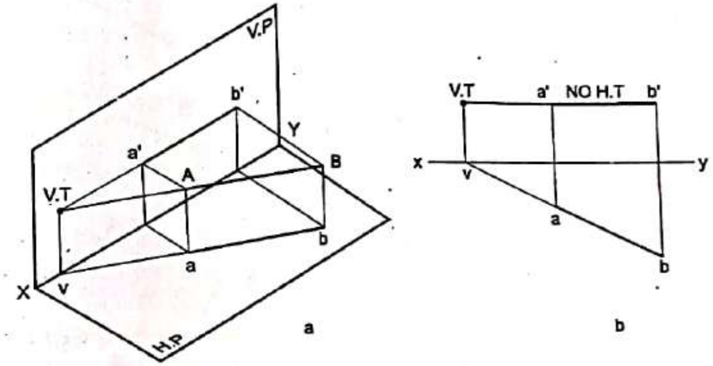


Fig. 8.14

8.3.6 Straight line inclined to both H.P and V.P

Method I Figure 8.15a shows the position of the line AB and its traces marked, using the methods presented in the sections 8.3.4 and 8.3.5.

Construction (Fig. 8.15b)

1. Draw the projections a′ b′ and ab.
2. Extend the line b′ a′ to meet xy at h.
3. Draw the projector from h to intersect the line ba extended at H.T.
4. Extend the line ba to meet xy at v.
5. Draw a projector from v to intersect the line b′ a′ extended at V.T.

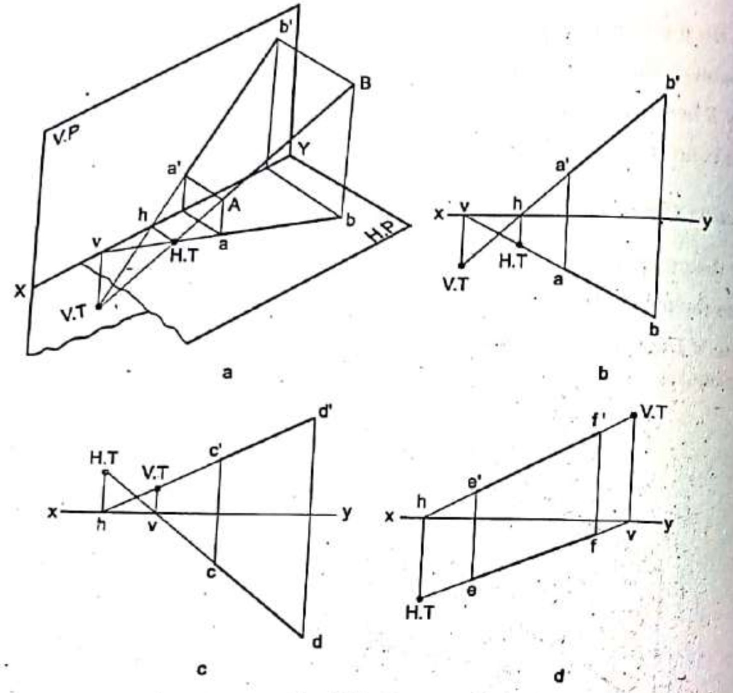


Fig. 8.15

NOTE (i) In general, once the position of the line is fixed, the position of the traces remain unchanged even if the length of the line is altered.

(ii) The relative positions of the traces H.T and V.T, to the reference line xy depend upon the orientation of the line with respect to H.P and V.P. Figure 8.15c shows the position of the views of a line CD and its traces marked and Fig. 8.15d, that of a line EF.

Method II It may be observed from Figs. 8.9a and b, that the rotation of the line AB about ab and a′b′ has actually taken place about o_3 and o_4 respectively as centres. Also, it may be observed that the point o_3 lies on H.P and o_4 on V.P. As per the definition of the traces of a line, these points are respectively known as H.T and V.T.

Construction (Fig. 8.9c)

1. Draw the projections of the line AB.
2. Draw the lines A_1B_1 and A_2B_2 as described in article 8.2.6 (Method II).
3. Extend the lines b′ a′ and B_1A_1 and locate the point of intersection, the V.T of the line.
4. Extend the lines ba and B_2A_2 and locate the point of intersection, the H.T of the line.

8.3.6.1 *Special cases of traces*

Case I A line AB is inclined to both H.P and V.P and has its end A in H.P and end B in V.P. Figure 8.16a shows the projections of a line. From the figure, it is clear that H.T of the line coincides with a, the top view of A and V.T coincides with b′, the front view of B.

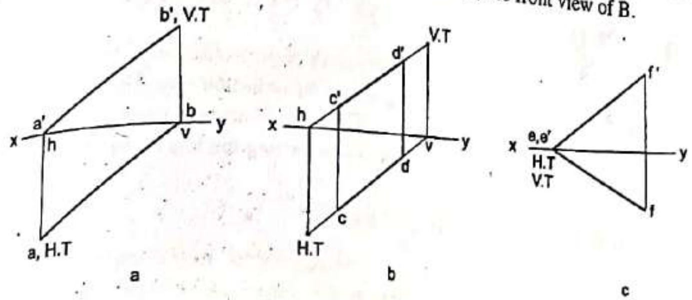


Fig. 8.16

Case II Same as Case I, but the line AB is shortened from both its ends and occupying the position CD.

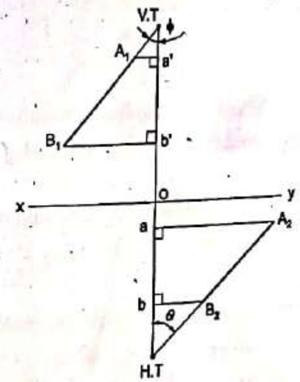
Figure 8.16b shows the projections of the line along with the traces located. From the figure, it is clear that the traces of the line CD are still the same as those for Case I.

Case III A line EF is inclined to both H.P and V.P and has its end E on both H.P and V.P.

Figure 8.16c shows the projections of the line. From the figure, it is clear that H.T and V.T of the line coincide with e and e′, the projections of the end point E.

Hence, it may be concluded that when a line has an end on a plane, its trace on that plane coincides with the projection of that end on that plane.

8.3.7 Straight line contained by a profile plane



When a line is contained by a profile plane, its projections lie on a single projector. Therefore, the method I of the preceding article cannot be used to locate the traces. However, the method II of the preceding article may be used as shown in Fig. 8.17.

8.4 THREE VIEW PROJECTIONS OF STRAIGHT LINES

Projections of a straight line on the two principal planes of projection, viz., H.P and V.P, result in the top and front views respectively. The line also may be projected on to the profile plane. In the first angle projection, if a left profile plane is considered; the view obtained on it is known as the right side view. A left side view may be obtained by projecting the line on to a right profile plane.

8.4.1 Straight line inclined to both H.P and V.P

Problem 8 *A line AB of 100 length is inclined at an angle of 30° to H.P and 45° to V.P. The point A is 15 above H.P, 20 in front of V.P and 120 from right profile plane (RPP). Draw (i) front view, (ii) top view and (iii) left side view of the line AB.*

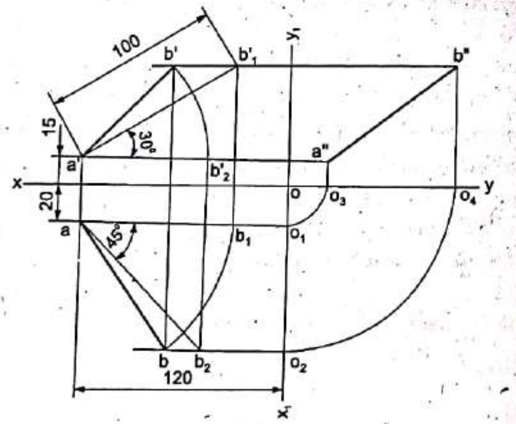


Fig. 8.18

Construction (Fig. 8.18)

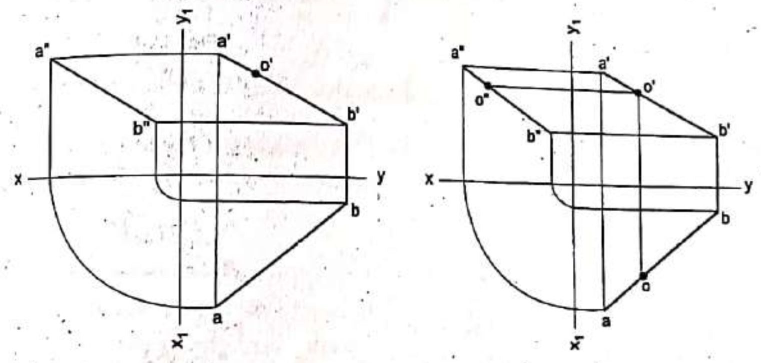
1. Draw the front and top views, a′ b′ and ab of the line AB, following the Construction: Fig. 8.6.
2. Draw the line x_1y_1 at right angles to xy, intersecting at o and at 120 from the projector containing a′, a.
3. Draw the projectors through a and b, meeting x_1 y_1 at o_1 and o_2.
4. With centre o and radii oo_1 and oo_2, draw arcs meeting xy at o_3 and o_4.
5. Through o_3 and o_4, draw projectors.
6. Through a′ and b′, draw projectors meeting the above projectors at a″and b″ respectively.
7. Join a″, b″.

a′ b′, ab and a″ b″ are the three views of the given line.

8.5 LOCATION OF A POINT ON A LINE

A line is defined as the locus of a moving point. In other words, it may be said that a line is composed of infinite number of points. It is often required to specify the location of specific points on lines. From the projections of straight lines, it is already observed that the end points of a line are located on the projectors, which are perpendicular to the line xy.

Problem 9 *Figure 8.19a shows (i) front view, (ii) top view and (iii) right side view of a line AB and the front view o′ of a point O on a′ b′. Locate the top and right side views of the point O on the top and right side views of the line.*



a b

Fig. 8.19

Construction (Fig. 8.19b)

1. Draw the three views of the line and locate the point o′ on a′ b′.
2. Draw the projectors through o′ to meet ab at o and a″ b″ at o″.

It may be noted that the points o′, o and o″ divide the lines a′ b′, ab and a″ b″ respectively in the same ratio, i.e., $\frac{a'o'}{a'b'} = \frac{ao}{ab} = \frac{a''o''}{a''b''}$

From the above, it is evident that if a point is located at the middle of a line, it will appear at the mid-point in all the projections.

8.6 EXAMPLES

Problem 10 *A line AB of 70 long, is parallel to and 25 in front of V.P. Its one end is on H.P while the other is 40 above H.P. Draw the projections of the line and determine its inclination with H.P.*

CHAPTER - 9

PROJECTIONS OF PLANES

9.1 INTRODUCTION

In structural design, it is necessary to know the representation of plane surfaces of different shapes in all the possible orientations. The treatment presented here is based on the principles dealt with, in the preceding two chapters.

Plane surfaces have two dimensions, viz., length and breadth; the third dimension, the thickness being zero. Plane surfaces may be considered of infinite sizes. However, for convenience, segments of planes only are considered in the treatment presented here. Planes are represented in space by the following:

a - three points, not on a straight line

b - two intersecting lines

c - a line and a point

d - two parallel lines

The front and top views for the above cases are shown in Fig. 9.1.

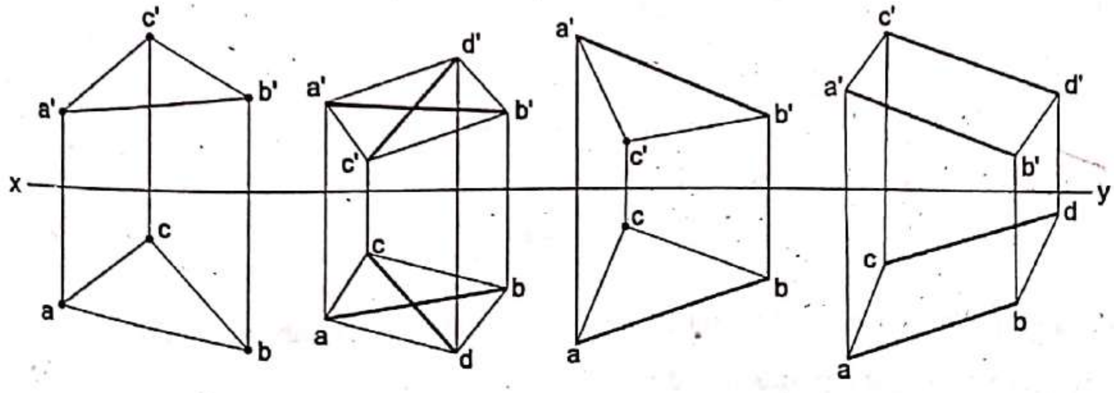


Fig. 9.1

9.2 TWO VIEW PROJECTIONS

The following are the possible orientations of the planes, with respect to the principal planes of projection:

1. Plane parallel to one of the principal planes and perpendicular to the other
2. Plane inclined to one of the principal planes and perpendicular to the other
3. Plane perpendicular to both the principal planes
4. Plane inclined to both the principal planes

9.2.1 Plane parallel to one of the principal planes and perpendicular to the other

9. 2. 1. 1 *Plane parallel to H.P and perpendicular to V.P*

Problem 1 *A square plane ABCD of side 30, is parallel to H.P and 20 away from it. Draw the projections of the plane, when two of its sides are (i) parallel to V.P and (ii) inclined at 30° to V.P.*

Figure 9.2a shows the first quadrant with the plane ABCD, such that two of its sides are parallel to V.P. As the plane is parallel to H.P, its projection on H.P, viz., the top view reveals the true shape of the plane. The front view appears as a straight line, parallel to xy.

Construction (Fig. 9.2b)

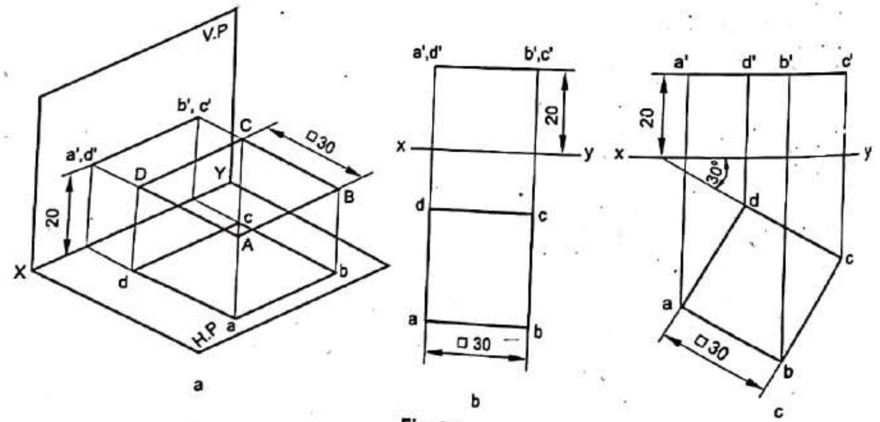


Fig. 9.2

1. Draw a square abcd of side 30 such that, one of its sides, say dc (ab) is parallel to xy.
2. Draw projectors from the points d and c.
3. Locate the point a′ (d′) at 20 above xy.
4. Draw a line through a′ and parallel to xy, intersecting the projector through c at b′ (c′).

a′ b′ c′ d′ and abcd are the required projections.

Figure 9.2c shows the projections of the plane, when two of its sides are inclined at 30° to V.P.

NOTE When the inclination ϕ of the side dc with xy is 45°, then all the sides of the plane are said to be equally inclined to V.P.

9. 2. 1. 2 *Plane parallel to V.P and perpendicular to H.P*

Problem 2 *An equilateral triangular plane ABC of side 40, has its plane parallel to V.P and 20 away from it. Draw the projections of the plane when one of its sides is (i) perpendicular to H.P, (ii) parallel to H.P and (iii) inclined to H.P at an angle of 45°.* (Aug/Sep 2008, 2010, JNTU)

Figure 9.3a shows the first quadrant with the plane ABC such that, one of its sides is perpendicular to H.P. As the plane is parallel to V.P, its projection on V.P, i.e., the front view appears in its true shape. The top view appears as a straight line, parallel to xy.

Construction (Fig. 9.3b)

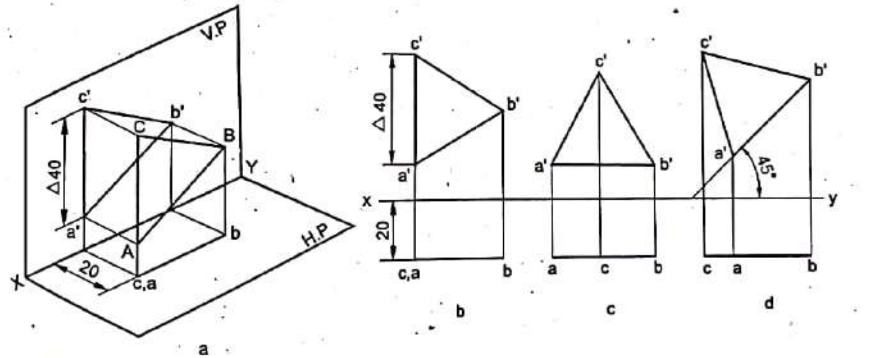


Fig. 9.3

1. Draw an equilateral triangle a′ b′ c′ of side 40 such that, one side, say a′ c′ is perpendicular to xy.
2. Draw projectors from the points a′ and b′.
3. Locate the point c (a) at 20 below xy.
4. Draw a line through c and parallel to xy, intersecting the projector through b′ at b.

a′ b′ c′ and abc are the required projections.

Figure 9.3c shows the projections of the plane, when one of its sides ABis parallel to H.P. Figure 9.3d shows the projections of the plane, when one of its sides AB is inclined at 45° to H.P.

NOTE The symbol □ preceding the dimension, represents the side of a square. Similarly, the symbolΔ is used to represent the side of an equilateral triangle

9.2.2 Plane inclined to one of the principal planes and perpendicular to the other

9.2.2.1 *Plane inclined to H.P and perpendicular to V.P* The problems of this nature, as explained in the preceding chapter; are normally solved in two stages. In the first stage, the plane is assumed to be parallel to that plane to which it is actually inclined and projections are drawn. In the second stage, the plane is rotated till it makes the required angle with the plane and then the final views are obtained.

Problem 3 *Draw the projections of a regular pentagon of 25 side with its surface making an angle of 45° with H.P. One of the sides of the pentagon is parallel to H.P and 15 away from it.*

(Aug/Sep 2008, 2010, JNTU)

Figure 9.4a shows the first quadrant with the plane in it, depicting the two stages of obtaining the projections.

Construction (Fig. 9.4b)

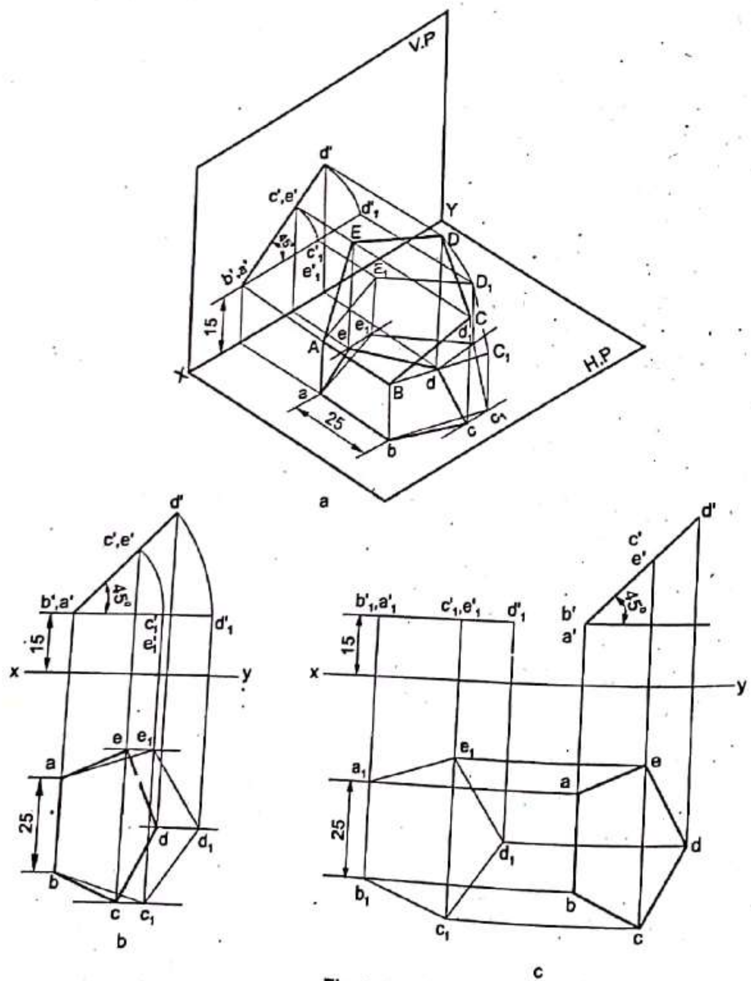


Fig. 9.4

Stage I Assume that the plane is parallel to H.P and perpendicular to V.P.

1. Draw the top view $abc_1d_1e_1$, at any convenient location below xy, keeping the side ab perpendicular to xy.

The plane appears in its true shape in the top view, as it is parallel to H.P.

2. Draw projectors through a, e_1 and d_1.
3. Locate the point b′ (a′) at 15 above xy, on the projector through a.
4. Draw a line through b′ and parallel to xy, intersecting the above projectors at c_1' (e_1') and d_1' respectively, forming the front view.

The front view appears as a line, parallel to xy, as the plane is parallel to H.P and perpendicular to V.P.

Stage II Rotate the plane till it makes the given angle 45° with H.P.

5. With b′ (a′) as centre, rotate the front view through 45° to xy, to the position a′ (b′) c′ (e′) d′, forming the final front view of the plane.
6. Through c_1, d_1 and e_1, draw lines parallel to xy, representing the loci of the top views of the corners C, D and E of the plane.
7. Through c′ (e′) and d′, draw projectors meeting the above loci at c, e and d.
8. Join b, c; c, d; d, e and e, a, forming the final top view of the plane.

Figure 9.4c shows the method of drawing the projections, in which the construction pertaining to stage II is shown separately. While drawing the projections of planes, the following may be observed:

(i) If a plane has an edge parallel to H.P, that edge should be kept perpendicular to V.P. Similarly, if the edge of a plane is parallel to V.P, that edge should be kept perpendicular to H.P.

(ii) If a plane has a corner on H.P, the sides containing that corner should be equally inclined to V.P, whereas, if the corner is on V.P, the sides containing the corner should be kept equally inclined to H.P.

9.2.2.2 *Plane inclined to V.P and perpendicular to H.P*

Problem 4 *A regular hexagonal plane of 30 side has a corner at 20 from V.P and 50 from H.P. Its surface is inclined at 45° to V.P and perpendicular to H.P. Draw the projections of the plane.*

(May/June 2008, JNTU)

Figure 9.5a shows the first quadrant with the plane in it, depicting the two stages of obtaining the projections.

Construction (Fig. 9.5b)

Stage I Assume that the plane is parallel to V.P and perpendicular to H.P.

1. Draw the front view $a'\ b_1'\ c_1'\ d_1'\ e_1'\ f_1'$ such that, the corner a′ is 50 above xy.

The plane appears in its true shape in the front view, as it is parallel to V.P.

2. Draw projectors through a′, b_1', c_1' and d_1'.

3. Locate the point a at 20 below xy.
4. Draw a line through a and parallel to xy, intersecting the above projectors at f_1 (b_1), e_1 (c_1) and d_1 respectively; forming the top view.

The top view appears as a line parallel to xy, as the plane is parallel to V.P and perpendicular to H.P.

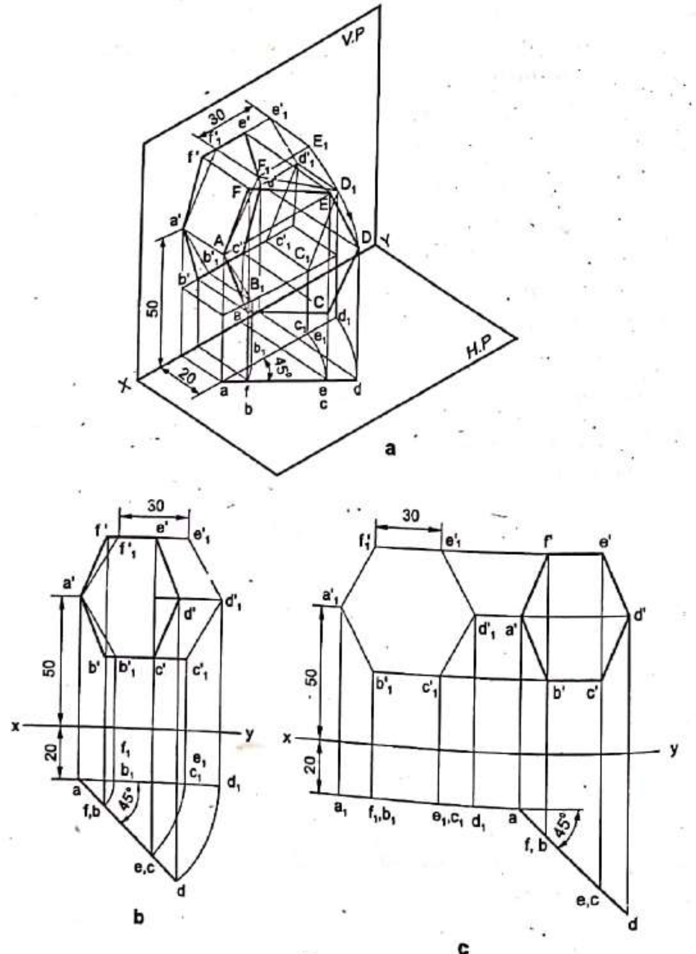


Fig. 9.5

Stage II *Rotate the plane till it makes the given angle 45° with V.P.*

NOTE As the plane is to be rotated about the corner A; to make it inclined to V.P, the sides containing the corner A, i.e., AB and AF should be equally inclined to H.P.

5. With a as centre, rotate the top view through 45° to xy, to the position af (b) e (c) d; forming the final top view of the plane.
6. Through b_1' (c_1'), d_1' and e_1' (f_1'), draw lines parallel to xy, representing the loci of the front views of the corners B (C), D and E (F) of the plane.
7. Through f (b), e(c) and d, draw projectors, meeting the above loci at f′, b′, e′, c′ and d′.
8. Join a′, b′; b′, c′; c′, d′; d′, e′; e′, f′; and f′, a′, forming the final front view of the plane.

Figure 9.5c shows the method of drawing the projections, in which the construction with respect to stage II is shown separately.

9.2.3 Plane perpendicular to both H.P and V.P

When a plane is positioned such that it is perpendicular to both the principal planes, then it is said to be lying parallel to the profile plane. In this case, the side view appears in its true shape and both the front and top views appear as straight lines on a single projector.

Problem 5 *A rectangular plane of 50 × 25 size is perpendicular to both H.P and V.P. The longer edges are parallel to H.P and the nearest one is 20 above it. The shorter edge, nearer to V.P is 15 from it. The plane is 50 from the profile plane. Draw the projections of the plane.*

Figure 9.6a shows the first quadrant, with the rectangular plane positioned in it. The figure also shows the method of obtaining the three views on the respective planes.

Construction (Fig. 9.6b)

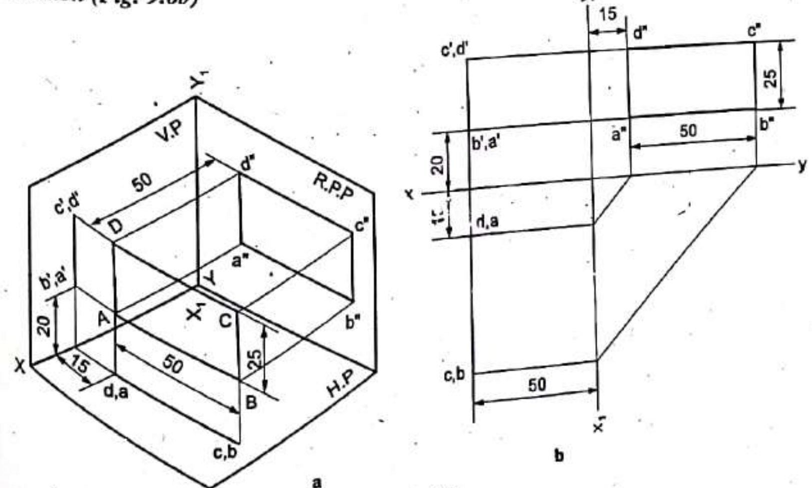


Fig. 9.6

1. Draw the reference lines xy and x_1 y_1.
2. Draw the side view a″ b″ c″ d″ such that, the longer edge a″ b″ is 20 above xy and the shorter edge a″ d″ is 15 from x_1 y_1.

3. Draw a projector at 50 from $x_1 y_1$.
4. Obtain the front and top views of the plane, from the side view, by projection

9.2.4 Oblique plane (plane inclined to both H.P and V.P)

The projections of an oblique plane are obtained in three stages. While drawing the projections of oblique planes, the following may be observed:

1. The surface of the plane should be considered parallel to the principal plane, to which it is actually inclined.
2. The edge should be considered perpendicular to the principal plane, to which it is inclined.
3. The surface of the plane is tilted through the required angle in the II stage, and
4. The edge is tilted through the required angle in the III stage, for obtaining the final projections.

Problem 6 *A rectangular plane of size 60 × 30 has its shorter side on H.P and inclined at 30° to V.P. Draw the projections of the plane, if its surface is inclined at 45° to H.P.*

Construction (Fig. 9.7)

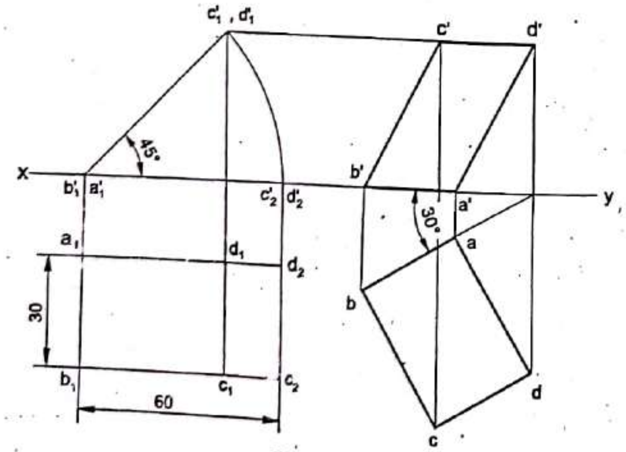


Fig. 9.7

***Stage I** Assume that the plane is lying in H.P and the shorter edge of it is perpendicular to V.P.*

1. Draw the top view a_1 b_1 c_2 d_2; representing the true shape of the plane, at any convenient position below xy.
2. Project the front view b_1' (a_1') c_2' (d_2'), which is a straight line, coinciding with xy

***Stage II** Tilt the plane such that it makes an angle of 45° with H.P.*

3. Rotate the front view about b_1' (a_1'), to the position b_1' (a_1') c_1' (d_1') such that, it makes 45° with xy.
4. Obtain the top view a_1 b_1 c_1 d_1, by projection.

***Stage III** Rotate the plane such that its shorter edge AB makes 30° with V.P.*

5. Redraw the top view (abcd) such that, the side ab makes 30° with xy. abcd is the final top view of the plane.
6. Obtain the final front view a′ b′ c′ d′, by projection.

NOTE When the plane is tilted such that its shorter edge makes the given angle with V.P, the shape of the top view and the distances of the corners of the plane from H.P are not altered.

9.3 TRACES OF PLANES

A plane extended if necessary, will meet the principal planes of projection along the lines known as traces. The intersection of a plane with H.P is called the horizontal trace, H.T and with V.P, the vertical trace, V.T. Normally, planes are represented by their traces.

Figure 9.8 shows the quadrant with the following types of planes situated in it:

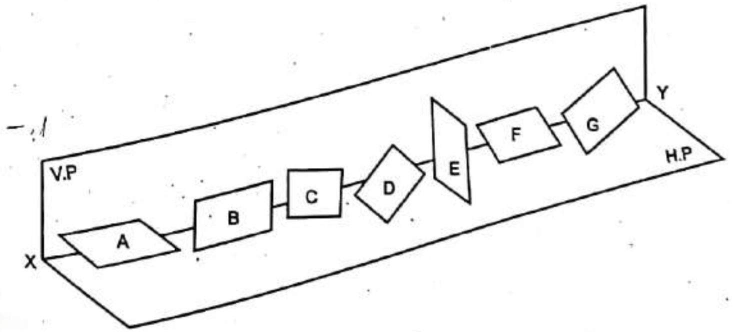


Fig. 9.8

A - Plane is parallel to H.P and perpendicular to V.P. It has only the V.T.
B - Plane is parallel to V.P and perpendicular to H.P. It has only the H.T.
C - Plane is perpendicular to H.P and inclined to V.P by an angle ϕ. Its V.T is perpendicular to xy and its H.T is inclined to xy by an angle ϕ.
D - Plane is perpendicular to V.P and inclined to H.P by an angle θ. Its H.T is perpendicular to xy and its V.T is inclined to xy by an angle θ.
E - Plane is perpendicular to both H.P and V.P. It has both H.T and V.T, at right angles to xy.
F - Plane is inclined to both H.P and V.P, but parallel to xy. It has both H.T and V.T, which are parallel to xy.
G - Plane is inclined to both H.P and V.P but not parallel to xy. It has both H.T and V.T, which are inclined to xy.

Figure 9.9 shows the relative positions of the views and the traces for the planes A to F and Fig. 9.10 shows the same for the plane G. The method of obtaining the traces is similar to that of the lines.

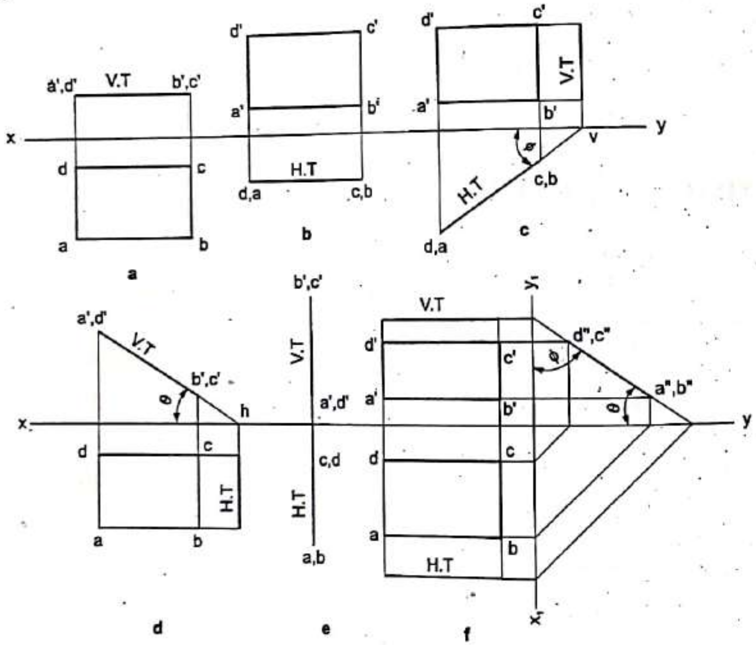


Fig. 9.9

Problem 7 *Figure 9.10 shows the projections of an oblique plane ABCD of type G. Locate the traces of the plane.*

Construction (Fig. 9.10)

1. Draw the given projections of the plane.
2. Extend b′ a′ and c′ d′ to meet xy at h_1 and h_2 respectively.
3. Draw projectors through h_1 and h_2.
4. Extend ba and cd to meet the above projectors at H_1 and H_2 respectively.
5. Join H_1, H_2
6. Extend ba and cd to intersect xy at v_1 and v_2 respectively.
7. Draw projectors through v_1 and v_2.
8. Extend b′ a′ and c′ d′ to intersect the above projectors at V_1 and V_2 respectively.
9. Join V_1, V_2.

$H_1 H_2$ and $V_1 V_2$ are the required traces.

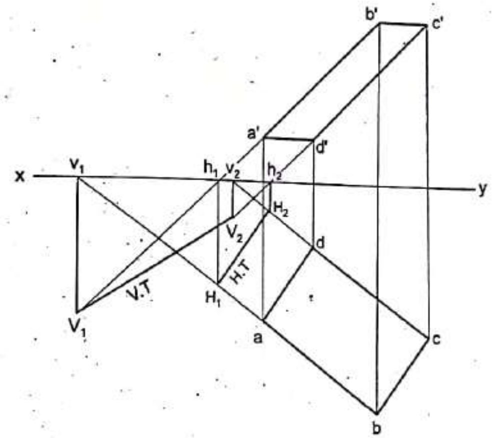


Fig. 9.10

9.4 EXAMPLES

Problem 8 *A rectangle ABCD of size 40 × 25, has the corner A, 10 above H.P and 15 in front of V.P. All the sides of the rectangle are equally inclined to H.P and parallel to V.P. Draw its projections and locate its traces.*

Construction (Fig. 9.11)

1. Draw the front view a′ b′ c′ d′ (rectangle of size 40 × 25), with the corner a′ at 10 above xy and all the sides are inclined at 45° to xy.
2. Draw the projectors through the corners of the front view and obtain the top view, a line, parallel to xy and 15 below it.

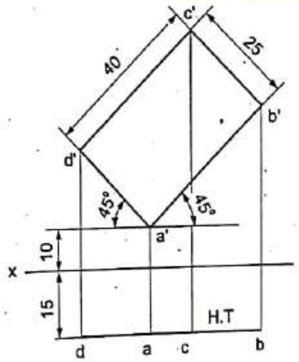


Fig. 9.11

For the given plane, there is no V.T. The top view is its H.T and also called the edge view of the plane.

Problem 9 *A regular pentagon of 25 side is parallel to H.P and perpendicular to V.P. The plane is 15 above H.P and an edge of it lies on V.P. Draw the projections and show its traces.*

Construction (Fig. 9.12)

1. Draw the top view abcde (pentagon of side 25) so that, an edge (de) coincides with xy.
2. Draw the projectors through the corners of the top view and obtain the front view, a line, parallel to xy and 15 above it.

For the given plane, there is no H.T. The front view is its V.T and also called the edge view of the plane.

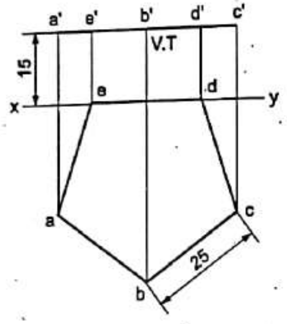


Fig. 9.12

Problem 10 *A regular hexagon of 25 side has its one edge on H.P. The surface of the plane is perpendicular to V.P and inclined at 40° to H.P. Draw the three views of the plane and locate its traces.*

Construction (Fig. 9.13)

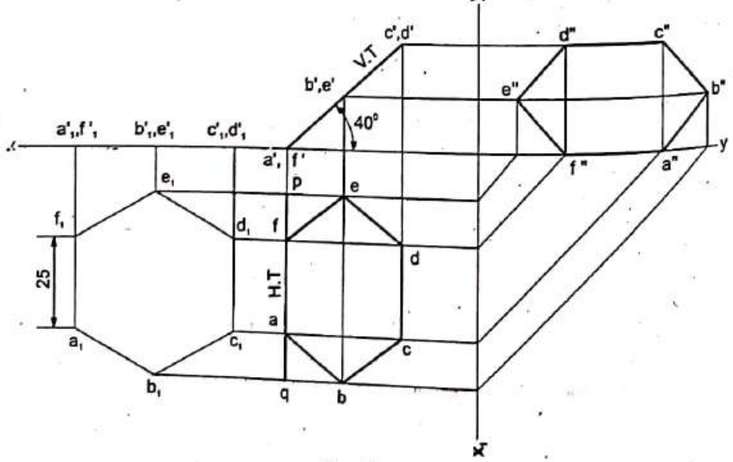


Fig. 9.13

1. Draw the projections of the plane, assuming it to be on H.P and one edge perpendicular to V.P.
2. Redraw the front view such that, it makes 40° with xy and one side of it, a′ (f′) lies on xy (final front view).
3. Obtain the final top view, by projection.
4. Draw the reference line $x_1 y_1$, perpendicular to xy and obtain the side view, by projection.

The side view obtained is the left side view of the plane. The V.T coincides with the front view and the line pq represents the H.T of the plane.

Problem 11 *Draw the projections of a circle of 60 diameter, resting on V.P on a point on the circumference. The plane is inclined at 45° to V.P and perpendicular to H.P. The centre of the plane is 40 above H.P. Also locate its traces.* (May/June 2008, JNTU)

Construction (Fig. 9.14)

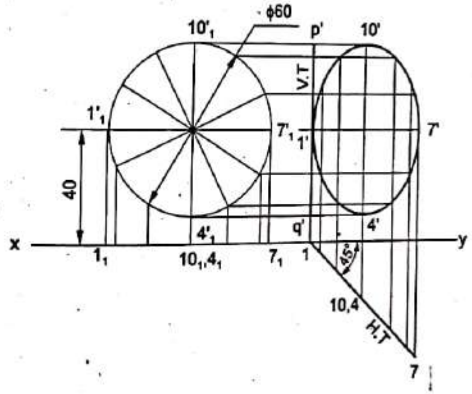


Fig. 9.14

1. Draw the projections of the circle, assuming it to be lying on V.P and with its centre at 40 above H.P.
2. Divide the circumference of the circle (front view) into some equal parts, say 12.
3. Transfer the division points on to the top view.
4. Redraw the top view such that, it makes an angle of 45° with xy and one end of it lies on xy. This forms the final top view.
5. Obtain the final front view, by projection.

The H.T coincides with the top view and the line p′ q′ represents the V.T of the plane.

Problem 12 *A composite plane ABCD consists of a square of 60 side, with an additional semi-circle constructed on CD as diameter. Draw the projections of the plane, when the side AB is vertical and the plane makes an angle of 45° with V.P.*

Construction (Fig. 9.15)

1. Draw the projections of the plane, assuming it to be parallel to V.P and perpendicular to H.P; keeping the edge AB vertical.
2. Rotate the top view till it makes 45° with xy. This is the final top view.
3. Obtain the final front view, by projection.

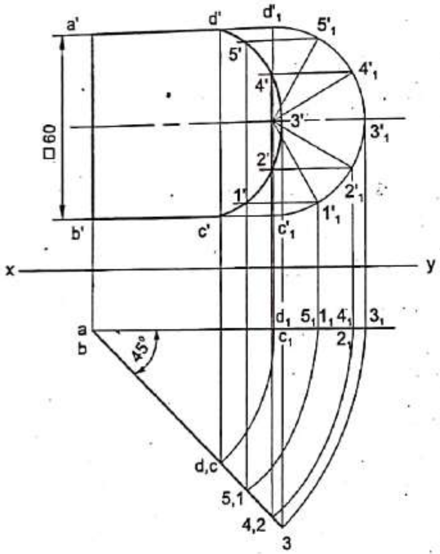


Fig. 9.15

Problem 13 *A circular plate of 60 diameter, has a hexagonal hole of 20 side, centrally punched. Draw the projections of the plate, resting on H.P on a point, with its surface inclined at 30° to H.P. Any two parallel sides of the hexagonal hole are perpendicular to V.P. Draw the projections of the plate.*

(Aug/Sep 2008, JNTU)

Construction (Fig. 9.16)

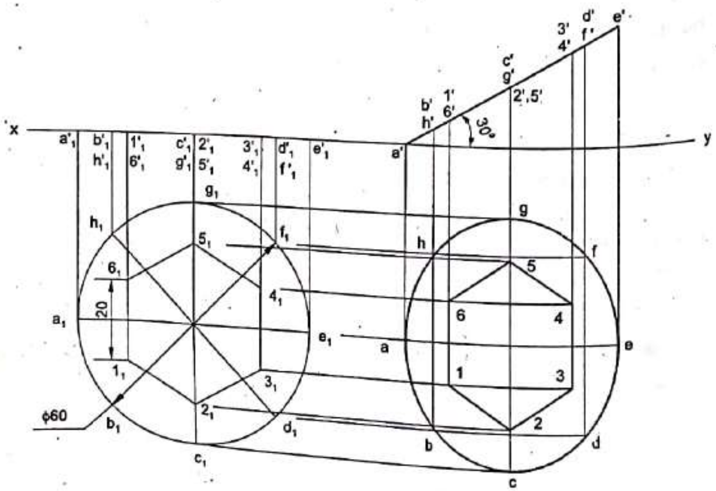


Fig. 9.16

1. Draw the projections of the plate with a hexagonal hole; assuming it to be resting on H.P such that, two parallel sides of the hexagonal hole are perpendicular to V.P.
2. Divide the circumference of the circle (top view) into some equal parts, say 8.
3. Transfer the division points and the points pertaining to the corners of the hexagon, to the front view.
4. Redraw the front view such that, it makes an angle of 30° with xy and one end of it lies on xy.
5. Obtain the final front view, by projection.

Problem 14 *The top view of a plane object is a regular hexagon of side 40, with a central hole of 30 diameter and with two sides of the hexagon parallel to xy, when the surface of the object is inclined at 45° to H.P and with a corner on H.P. Determine the true shape of the object.*

Construction (Fig. 9.17)

1. Draw the top view of the object, a hexagon of side 40 with a central hole of diameter 30 such that, two sides of the hexagon are parallel to xy.
2. Draw the front view, which is an edge view of the plane such that, it is inclined at 45° with xy and one end of it is on xy.
3. Divide the circle into, say 8 equal parts and locate the corresponding points in the front view.
4. Redraw the front view such that, it coincides with xy.
5. Obtain the true shape of the plane, by projection.

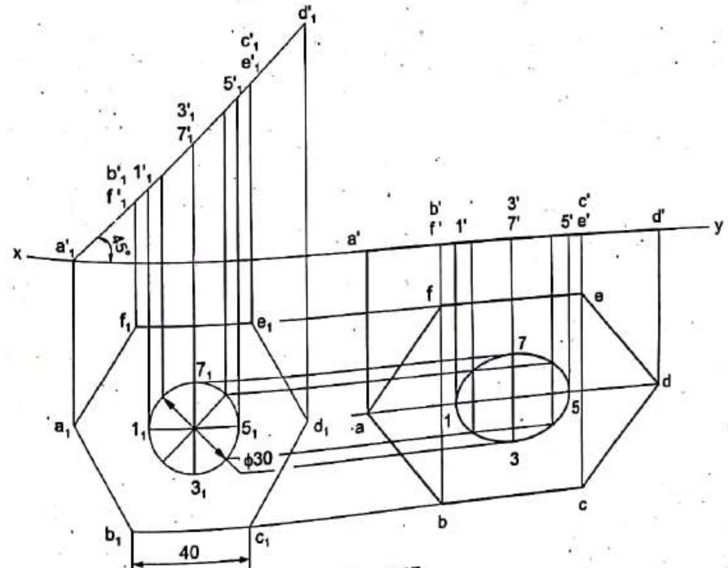


Fig. 9.17

Problem 15 *Construct an equilateral triangle abc of side 50 with ab perpendicular to xy. abc represents the top view of triangle ABC. Points B and C are 30 above H.P while point A is 45 above H.P. Draw the front view and find the true shape of the triangle ABC.*

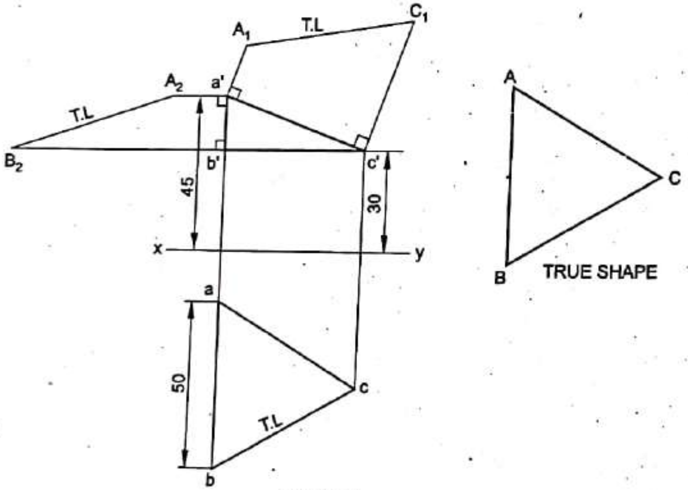


Fig. 9.18

Construction (Fig. 9.18)

1. Draw the equilateral triangle abc, keeping AB perpendicular to xy; representing the top view of the triangle ABC.
2. Project and obtain the front view a′ b′ c′ such that, the points b′ c′ are at 30 above xy and a′ at 45 above xy.
3. The length of the top view bc represents the true length of the edge BC of the triangle ABC; since the front view b′ c′ is parallel to xy.
4. Determine the true lengths of the edges AB (=$A_2 B_2$) and AC (=$A_1 C_1$) by using the trapezoidal method (refer Construction: Fig. 8.9c).
5. Using the true lengths of the edges, draw the true shape of the triangle ABC, as shown.

Problem 16 *A hexagon of 30 side has one of its corners on H.P. The plane of the hexagon is perpendicular to both H.P and V.P. The longest diagonal passing through the corner on H.P is perpendicular to H.P. Draw its projections.*

Construction (Fig. 9.19)

1. Draw the side view of the hexagon, keeping the corner A on H.P and the diagonal AD, perpendicular to it.

2. Draw the projectors from the side view, and obtain the front view such that, it is perpendicular to xy.
3. Obtain the top view, by projection.

NOTE The H.T coincides with the top view and the V.T, with the front view.

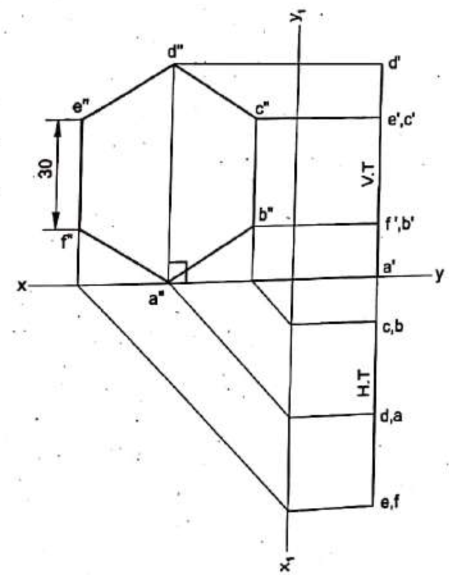


Fig. 9.19

Problem 17 *The front view of a plane object is an equilateral triangle of side 60, with one side inclined at 45° to xy. The true shape of the plane is an isosceles triangle of base 60 and altitude 80. Draw the projections of the plane.* (June 2008, JNTU)

Construction (Fig. 9.20)

1. Draw the front view a′ b′ c′, an equilateral triangle of side 60, with the side (base) a′ b′ making an angle of 45° with xy.
2. Project a′ b′ and obtain ab, the top view of the base, parallel to xy, as a′ b′ represents the true length of the base.
3. Rotate the altitude d′ c′ about d′, to the position d′ c_1', parallel to xy.
4. Draw a projector through c_1'.
5. With centre d and radius 80 (true length of the altitude), draw an arc intersecting the above projector at c_1.

6. Through c_1, draw t-t, parallel to xy, representing the locus of top view of C.
7. Draw a projector through c′, meeting t-t at c.
8. Join a, c and c, b.

   a′ b′ c′ and abc are the required projections.

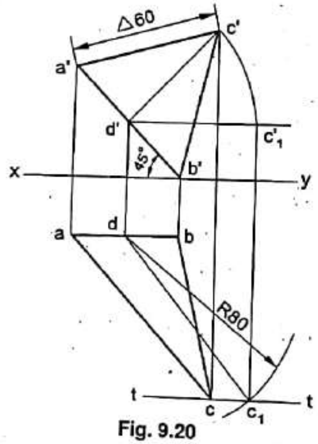


Fig. 9.20

Problem 18 *Draw the projections of a rhombus, having diagonals 120 and 60 long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to H.P.*

Construction (Fig. 9.21)

Stage I *Assume that the plane is on H.P, with the smaller diagonal perpendicular to V.P.*

1. Draw the top view (true shape of the rhombus) such that, the smaller diagonal is perpendicular to xy.
2. Obtain the front view (edge view) of the plane, coinciding with xy.

Stage II *Rotate the plane such that, the surface (longer diagonal) makes 30° with H.P.*

3. Rotate the front view about a_1' such that, the longer diagonal makes 30° with xy.
4. Obtain the top view by projection.
5. Redraw the top view such that, the smaller diagonal bd is parallel to V.P (xy). This is the final top view.
6. Obtain the final front view, by projection.

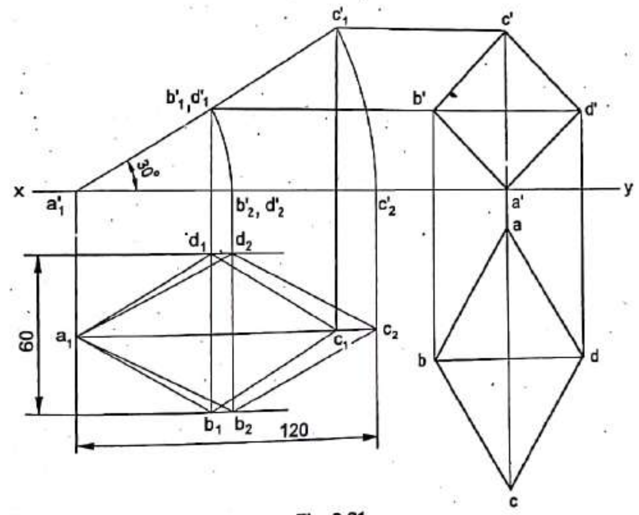


Fig. 9.21

Problem 19 *A square lamina ABCD of 30 side, rests on the corner C such that, the diagonal AC appears at 30° to V.P. in the top view. The two sides BC and CD, containing the corner C make equal inclinations with H.P. The surface of the lamina makes 45° with H.P. Draw its projections.*

Construction (Fig. 9.22)

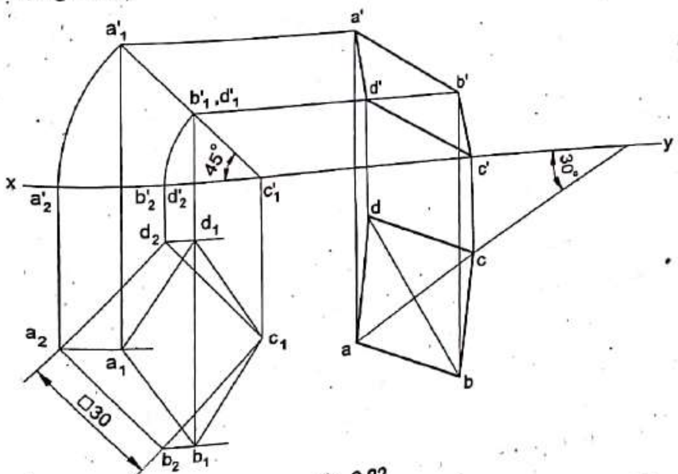


Fig. 9.22

1. Draw the projections of the square lamina ABCD, assuming it to be lying on H.P and the sides are equally inclined to V.P.

2. Rotate the front view about c_1' through 45°.
3. Obtain the second top view, by projection.
4. Redraw the above view such that, ca makes 30° with xy. This is the final top view.
5. Obtain the final front view, by projection.

Problem 20 *A thin 30° - 60° set-square has its longest edge (diagonal) on H.P and inclined at 30° to V.P. Its surface makes an angle of 45° with H.P. Draw the projections, choosing suitable size for the set-square.* (June 2009, JNTU)

Construction (Fig. 9.23)

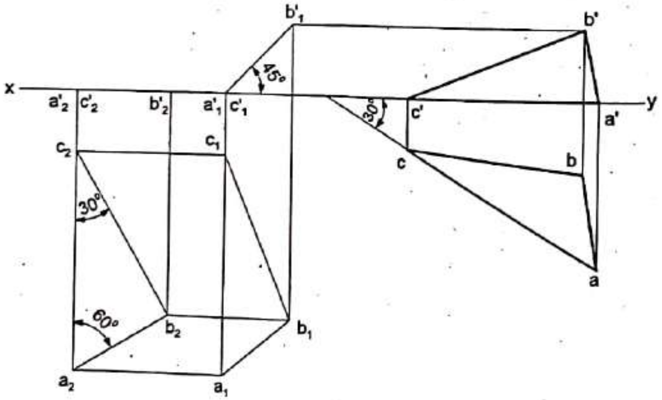


Fig. 9.23

Stage I *Assume that the set-square is on H.P, with its diagonal perpendicular to V.P.*

1. Draw the projections of the set-square, choosing suitable size.

Stage II *Rotate the plane such that, the surface makes 45° with H.P.*

2. Redraw the front view such that, it is inclined at 45° with xy; keeping one end a_1' (c_1') on xy.
3. Obtain the top view, by projection.

Stage III *Rotate the plane such that, its diagonal makes an angle of 30° with V.P.*

4. Redraw the top view such that, the diagonal ac makes 30° with xy. This forms the final top view.
5. Obtain the final front view, by projection.

Problem 21 *A regular pentagon of 30 side, is resting on one of its edges on H.P, which is inclined at 45° to V.P. Its surface is inclined at 30° to H.P. Draw its projections.* (Aug/Sep 2008, JNTU)

Construction (Fig. 9.24)

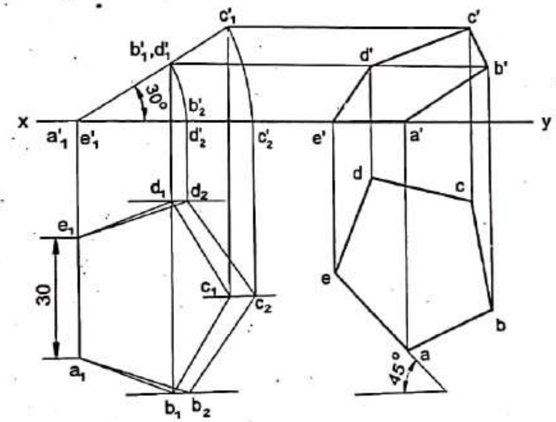


Fig. 9.24

1. Draw the projections of the pentagon, assuming it to be lying on H.P, with one of its edges AE perpendicular to V.P.
2. Rotate the front view, about a_1' (e_1'), till it makes 30° with xy.
3. Obtain the second top view, by projection.
4. Redraw the above top view such that, the edge ae makes an angle of 45° with xy. This is the final top view.
5. Obtain the final front view, by projection.

Problem 22 *A rhombus has its diagonals 100 and 60 long. Draw the projections of the rhombus, when it is so placed that its top view appears to be a square of diagonal 60 long and the vertical plane through the longer diagonal makes 30° with V.P.*

Construction (Fig. 9.25)

1. Draw the projections of the rhombus, assuming it to be lying on H.P, with the shorter diagonal perpendicular to V.P.
2. Rotate the front view about a_1' such that, the projected length of the longer diagonal in the top view is 60 and then complete the second top view.
3. Redraw the above top view such that, the top view of the longer diagonal ac makes 30° with xy. This is the final top view.
4. Obtain the final front view, by projection.

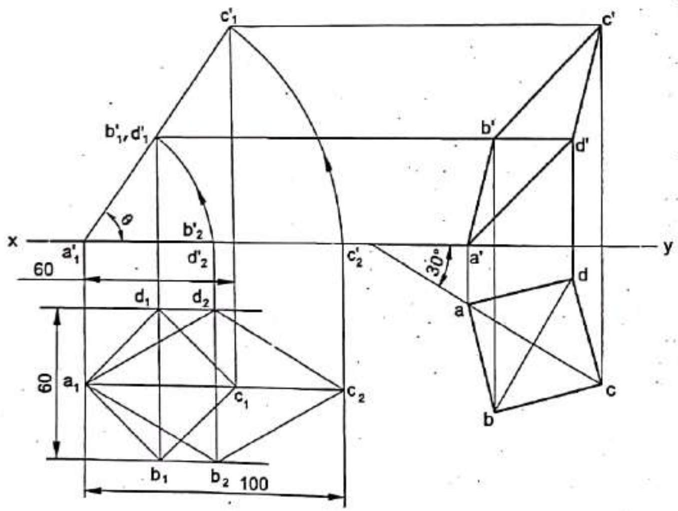


Fig. 9.25

Problem 23 *A plate is of the shape of an isosceles triangle of base 40 and altitude 60. Draw the projections of the plate, when it is placed such that, the front view appears as an equilateral triangle of side 40 and one of the edges of the plate make 45° with H.P.*

Construction (Fig. 9.26)

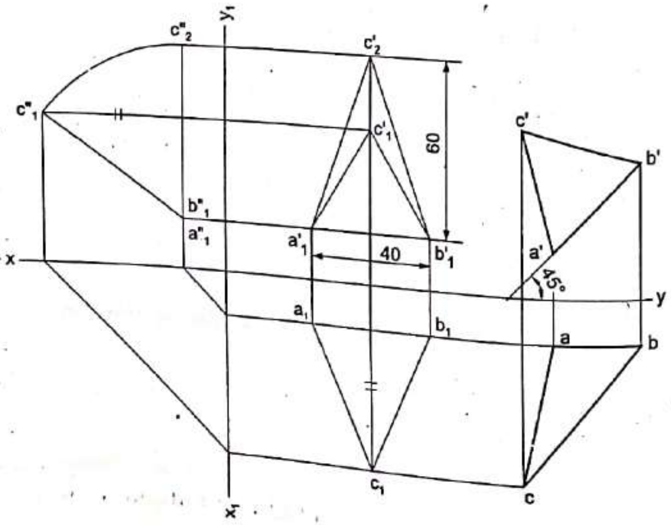


Fig. 9.26

1. Draw the front view of the plate, assuming it to be parallel to V.P, with the base parallel to H.P.
2. Obtain the side view.
3. Rotate the side view about its end b_1'' (a_1''), till the front view becomes an equilateral triangle of side 40.
4. Obtain the top view, by projection.
5. Redraw the above front view such that, the edge a′ b′ makes an angle of 45° with xy. This is the final front view.
6. Obtain the final top view, by projection.

Problem 24 *A regular pentagonal lamina of 25 side, is resting on H.P on one of its sides, while the opposite corner touches V.P. Draw the projections of the lamina, when it makes an angle of 60° with H.P.*

Construction (Fig. 9.27)

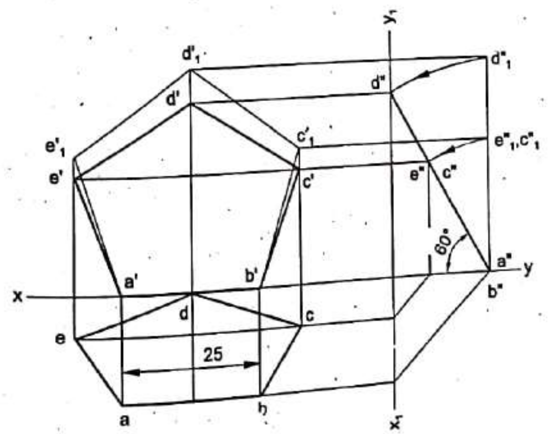


Fig. 9.27

1. Draw the front and side views of the plane, assuming it to be parallel to V.P and resting on an edge on H.P.
2. Tilt the side view about a″ (b″), till it makes 60° with xy, ensuring that the point d″ touches $x_1 y_1$.
3. Obtain the final front and top views, by projection.

Problem 25 *A pentagonal plane of side 30 is resting on an edge on H.P and parallel to V.P. The corner, opposite to the resting edge, is on V.P and 25 above H.P. Draw the projections of the plane and determine its inclinations with the planes of projection and also determine the distance between the resting edge and V.P.*

Construction (Fig. 9.28)

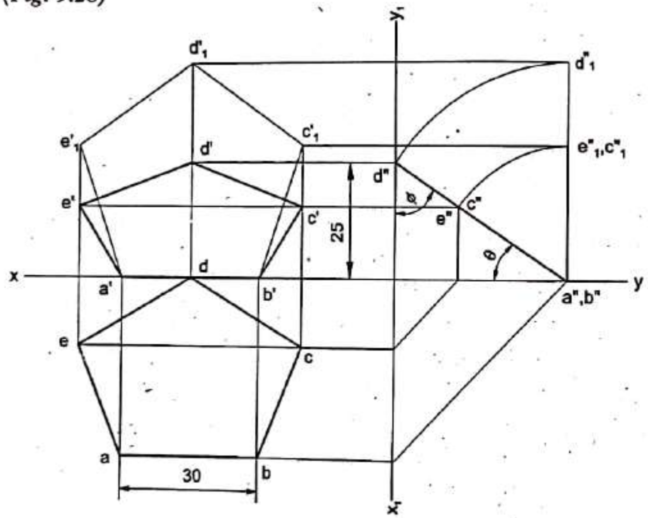


Fig. 9.28

1: Draw the front and side views of the plane, assuming it to be parallel to V.P and resting on an edge on H.P.

2. Rotate the side view about a″ (b″), till the point d_1'' moves to the position d″, which is 25 above xy. This is the final end view of the plane.

3. Draw the reference line x_1y_1, passing through d″ and perpendicular to xy.

4. Obtain the final front and top views, by projection.

The distance of a″ (b″) from x_1y_1 is the distance between the resting edge and V.P.

Problem 26 *ABCDE is a regular pentagonal plate of 40 side and has its corner A on H.P. The plate is inclined to H.P such that, the top view lengths of edges AB and AE are each 35. The side CD is parallel to both the reference planes. Draw the projections of the plate and find its inclination with H.P.*

Construction (Fig. 9.29)

1. Draw the projections of the pentagonal plate, assuming it to be lying on H.P; with the sides AB and AE equally inclined to V.P.

2. Draw the lines t_1 - t_1 and t_2 -t_2, which are parallel to xy and passing through e_2 and b_2 respectively. t_1 - t_1 and t_2 - t_2 represent the loci of top views of the edges E and B respectively.

3. Draw the second top views of the edges, i.e., a_1e_1 and a_1b_1 such that, they are of lengths 35 and the points e_1 and b_1 lie on t_1 - t_1 and t_2 - t_2 respectively.

4. Draw a projector passing through b_1 and e_1.

5. With a_1' as centre and radius a_1' b_2' (e_2'), draw an arc intersecting the above projector at b_1' (e_1').

6. Join a_1', b_1' (e_1') and extend.

7. With a_1' as centre and radius a_1' c_2' (d_2'), draw an arc intersecting the above extended line at c_1'(d_1'); forming the second front view.

8. Complete the second top view, by projection.

9. Redraw the above top view such that, the top view cd of the edge CD is parallel to xy. This is the final top view.

10. Obtain the final front view, by projection.

Angle θ represents the inclination of the plate with H.P.

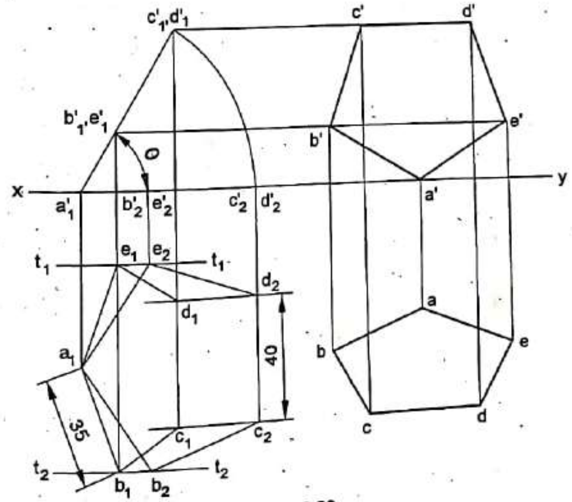


Fig. 9.29

Problem 27 *Top view of a plate, the surface of which is perpendicular to V.P and inclined at 60° to H.P, is a regular hexagon of side 50, with an edge perpendicular to xy. (i) Find the true shape of the plate and (ii) draw the projections of the plate, when the edge whose top view was perpendicular to xy earlier, becomes parallel to V.P; while the surface of the plate is still inclined at 60° to H.P.*

Construction (Fig. 9.30)

1. Draw the top view of the plate; a hexagon of side 50 such that, a side of it is perpendicular to xy.

2. Draw the front view which is an edge view of the plate such that, it is inclined at 60° with xy and one end of it is on xy.

3. Rotate the above front view till it coincides with xy.
4. Obtain the true shape of the plate $a_1b_1c_2d_2e_2f_2a_1$, by projection.
5. Redraw the initial top view such that, the top view ab of the edge AB is parallel to xy. This is the final top view.
6. Obtain the final front view, by projection.

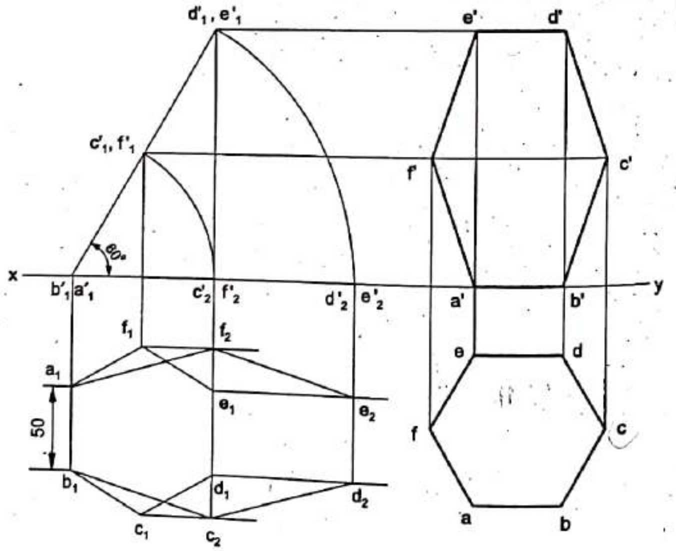


Fig. 9.30

Problem 28 *A semi-circular plate of 80 diameter, has its straight edge on V.P and inclined at 30° to H.P, while the surface of the plate is inclined at 45° to V.P. Draw the projections of the plate.*

(June 2008, Aug/Sep 2011 JNTU)

Construction (Fig. 9.31)

1. Draw the projections of the plate, assuming it to be lying on V.P, with the straight edge perpendicular to H.P.
2. Redraw the top view such that, it makes 45° with xy and one end of it g_1 (a_1), coincides with xy.
3. Obtain the second front view, by projection.
4. Redraw the above front view such that, the straight edge makes 30° with xy. This is the final front view.
5. Obtain the final top view, by projection.

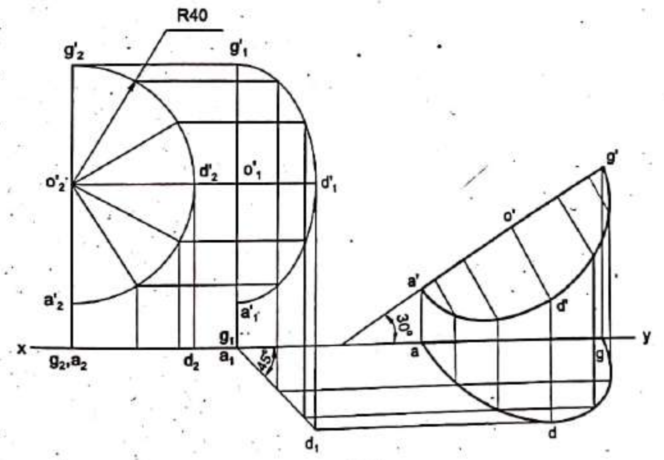


Fig. 9.31

Problem 29 *A composite plate of negligible thickness is made up of a rectangle 60 × 40 and a semi-circle on one of its longer sides. Draw its projections when the longer side lying on H.P and inclined at 45° to V.P; the surface of the plate making 30° angle with H.P.*

(May/June 2008, JNTU)

Construction (Fig. 9.32)

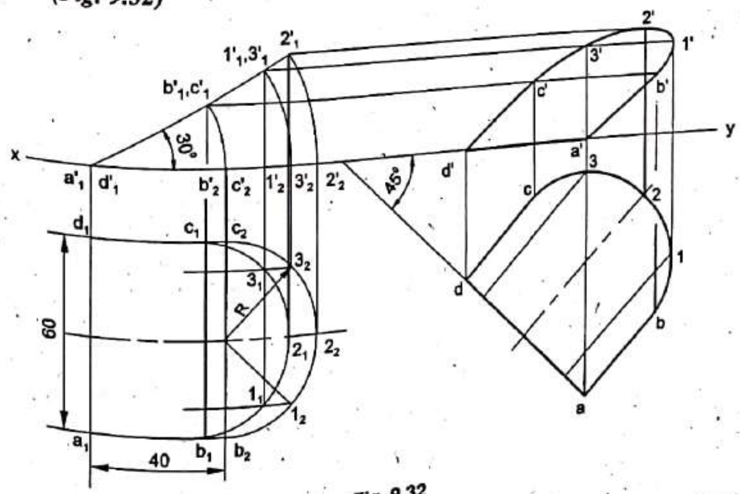


Fig. 9.32

1. Draw the projections of the composite plate, assuming it to be lying on H.P, with the longer edge perpendicular to V.P.
2. Rotate the front view about $a_1'(d_1')$, till it makes 30° with xy.

3. Obtain the second top view, by projection.
4. Redraw the above top view such that,, the longer edge makes 45° with xy. This is the final top view.
5. Obtain the final front view by projection.

Problem 30 *A regular hexagonal plane of 45 side has a corner on H.P, and its surface is inclined at 45° to H.P. Draw the projections, when the diagonal through the corner, which is on H.P, makes 30° with V.P.*

Construction (Fig. 9.33)

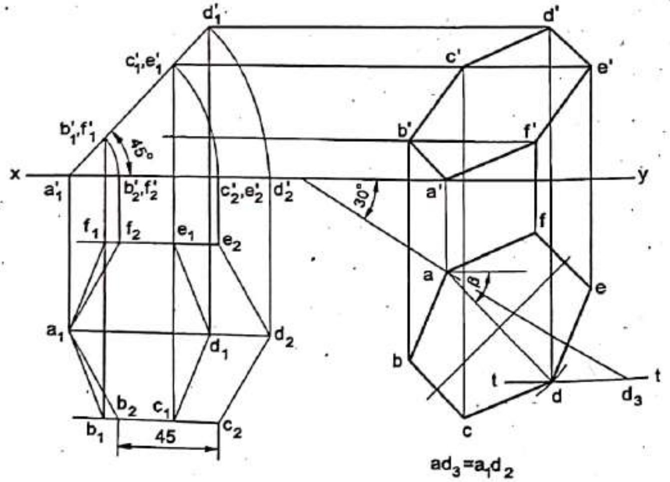


Fig. 9.33

1. Draw the projections of the plane, assuming it lying on H.P, with the diagonal AD parallel to V.P.
2. Rotate the front view about the corner a_1', till it makes 45° with xy.
3. Obtain the second top view, by projection.
4. Determine the apparent angle of inclination β, which the diagonal AD makes with V.P.

NOTE *The diagonal AD is inclined at an angle of 45° with H.P and when it also makes an angle ϕ (30°) with V.P, the top view of AD will be inclined at an apparent angle β with xy, which is larger than ϕ.*

To determine the apparent angle of inclination

(i) Draw a line making ϕ (30°) with xy, from any convenient point on xy.
(ii) Mark the true length of the diagonal AD ($=ad_3$) along the line and draw the locus t-t of d_3, parallel to xy.

(iii) With a as centre and top view length of AD ($=a_1d_1$) as radius, draw an arc intersecting the locus at d.
(iv) Join a, d.
The angle ad makes with xy, is the apparent angle β.

5. Redraw the above top view such that, the diagonal a d makes β with xy.
6. Obtain the final front view, by projection.

Problem 31 *Draw a rhombus of diagonals 100 and 60 long, with the longer diagonal horizontal. The figure is the top view of a square lamina of 100 long diagonal, with a corner on H.P. Draw its front view and determine the angle, its surface makes with H.P.*

Construction (Fig. 9.34)

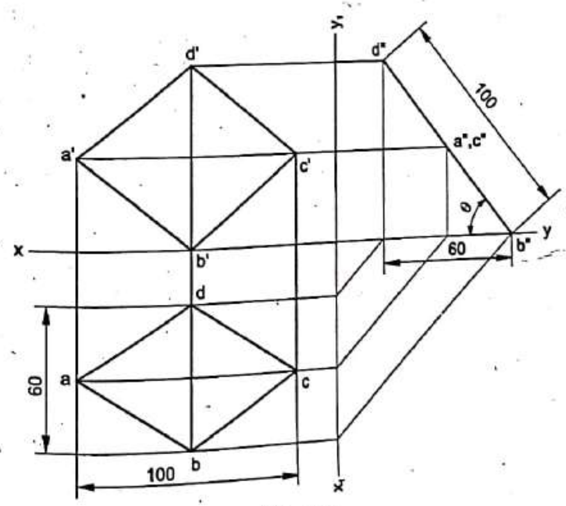


Fig. 9.34

1. Assuming that the lamina is perpendicular to P.P, with a corner on H.P, draw the side view such that, the projected length of its diagonal on H.P is 60 long.
2. Obtain front and top views, by projection.

The inclination of the side view with xy, i.e., θ, is the true angle of inclination of the surface with H.P.

Problem 32 *Draw the projections of a circle of 75 diameter having the end A of the diameter AB in H.P, and the end B in V.P and the surface inclined at 30° to H.P and 60° to V.P.* (June 2008, JNTU)

Construction (Fig. 9.35)

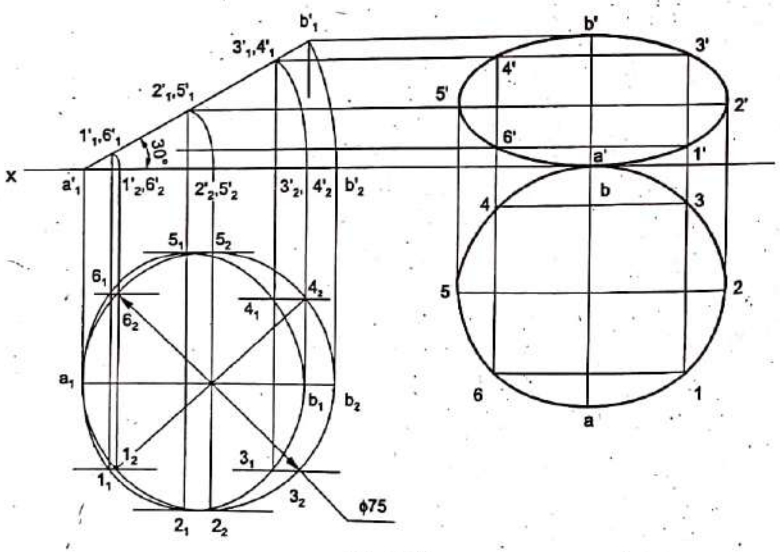


Fig. 9.35

1. Draw the projections of the circle, assuming it to be lying on H.P and with the diameter AB, parallel to V.P.
2. Divide the circumference of the circle (top view) into some equal parts, say 8.
3. Transfer the division points on to the front view.
4. Rotate the above front view about a_1' till it makes 30° with xy.
5. Obtain the top view, by projection.
6. Redraw the above top view such that, the top view of the diameter, ab is perpendicular to xy and b' coincides with xy. This is the final top view.
7. Obtain the final front view, by projection.

Problem 33 *A circle of 40 diameter, is resting on H.P on a point, with its surface inclined at 30° to H.P. Draw the projections of the circle, when (i) the top view of the diameter, through the resting point, makes an angle of 45° with xy and (ii) the diameter passing through the resting point makes an angle of 45° with V.P.* (Aug/Sep 2008, JNTU)

Construction (Fig. 9.36)

1. Draw the projections of the circle, assuming it to be lying on H.P.
2. Redraw the front view, making 30° with xy and one end of it is lying on xy.
3. Obtain the second top view, by projection (Fig. 9.36a).

Case I

4. Redraw the second top view such that, the top view of the diameter a_1g_1(=ag) makes 45° with xy. This is the final top view.
5. Obtain the final front view, by projection (Fig. 9.36b).

Case II

6. Determine the apparent angle of inclination β, which the diameter AB makes with V.P.
7. Redraw the second top view such that, the top view of the diameter, i.e., a_1 g_1 (=ag) makes β with xy. This is the final top view.
8. Obtain the final front view, by projection (Fig. 9.36c).

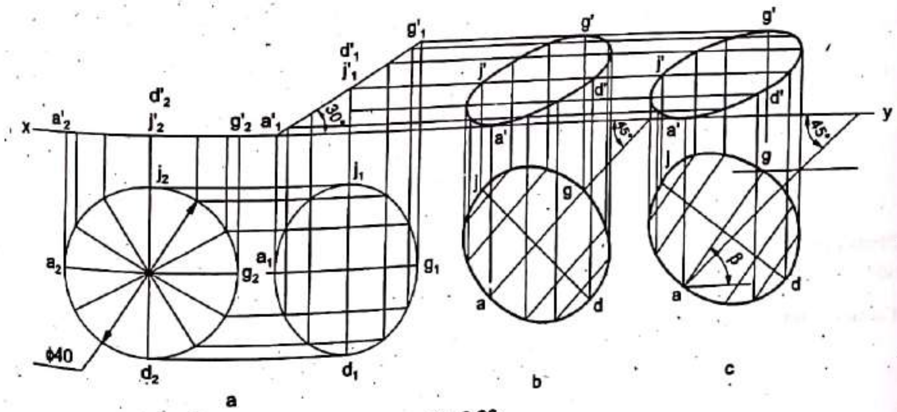


Fig. 9.36

Problem 34 *A circular plate of 50 diameter, appears as an ellipse in the front view, having its major axis 50 long and minor axis 30 long. Draw the top view, when the major axis of the ellipse is horizontal.* (May/June 2008, JNTU)

Construction (Fig. 9.37)

1. Draw the projections of the plate, assuming it to be parallel to V.P.
2. Locate $1_1'$ and $5_1'$ on the horizontal line through $1_2'$ ($5_2'$) such that, $1_1'5_1' = 30$, the minor axis of the ellipse.
3. Draw a projector through $5_1'$.
4. Locate the top view 1_1, by projection.
5. With 1_1 as centre and radius 1_25_2, draw an arc; intersecting the projector through $5_1'$ at 5_1.
6. Join 1_1, 5_1 forming the second top view. Locate the points 2_1,3_1, etc., on this view.
7. Obtain the second front view, by projection. Obviously, the shape of this view is an ellipse with major axis equal to 50.
8. Redraw the final front view such that, the major axis $3'$-$7'$ is horizontal (parallel to xy).

9. Obtain the final top view, by projection.

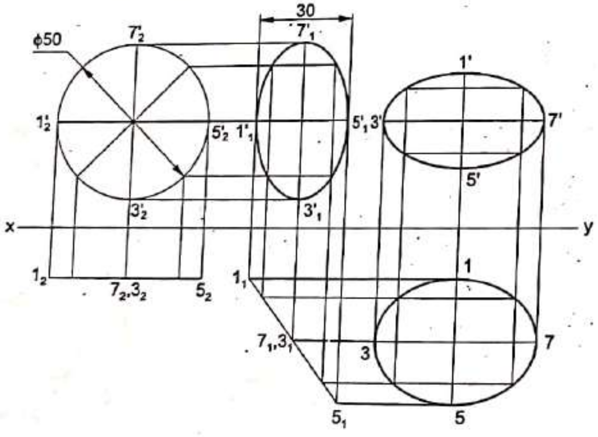


Fig. 9.37

Problem 35 *An equilateral triangle ABC of side 75 long, has its side AB on V.P and inclined at 60° to H.P. Its plane makes an angle of 45° with V.P. Draw its projections.*

Construction (Fig. 9.38)

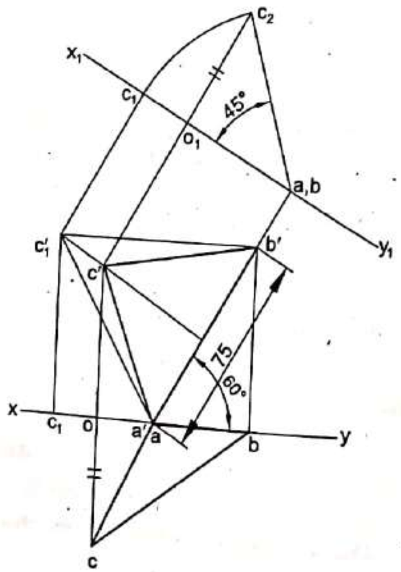


Fig. 9.38

1. Draw the reference line xy. Assuming that the plane is lying on V.P with the side AB making 60° with H.P and the end A is on H.P, draw the front view $a'b'c_1'$.
2. Locate a and b on xy as AB lies on V.P.
3. Draw the reference line x_1y_1, corresponding to an AIP, and perpendicular to a′ b′ .
4. Project and obtain the edge view of the plane c_1a (b), by projection.
5. Rotate the edge view about a(b) to a(b)c_2 such that, it makes an angle of 45° with x_1y_1.
6. Through c_2, draw a projector meeting the median of the triangle through c_1' at c′.
7. Locate c on the projector through c′ such that, $oc = o_1c_2$.
8. Join a′, c′ and b′ , c′ .
9. Join a, b; b, c and a, c.

a′ b′ c′ and abc are the required projections.

Problem 36 *The top view abc of a triangle ABC is an equilateral triangle of side 50; ab being inclined at 45° to xy. The point A is on V.P and 35 above H.P and the points B and C are on H.P. Draw the projections of the triangle and determine the true shape.*

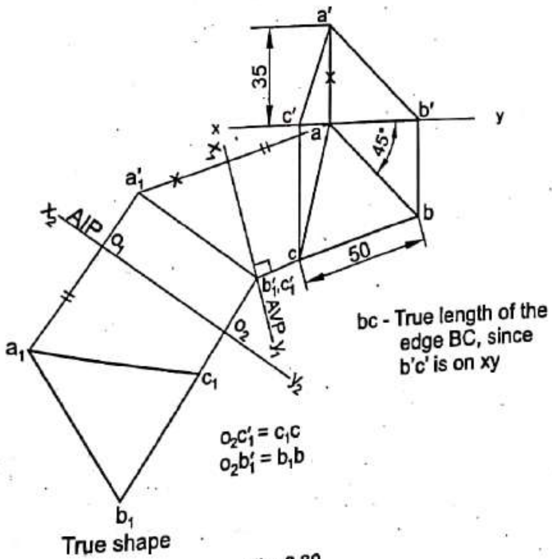


Fig. 9.39

Construction (Fig. 9.39)

1. Draw the reference line xy and complete the top view abc satisfying the given conditions.
2. Locate a′ at 35 above xy and on the projector through a and complete the front view, satisfying the other given conditions.

3. Draw the reference line x_1y_1, corresponding to an AVP, and perpendicular to cb (since cb represents the true length of the edge CB of the plane).
4. Project and obtain the edge view (auxiliary front view) $a'_1b'_1$ (c'_1) of the plane.
5. Draw the reference line x_2y_2, corresponding to an AIP, and parallel to the above edge view.
6. Project and obtain the normal view $a_1b_1c_1$ (auxiliary top view), representing the true shape of the triangle.

EXERCISES

9.1 A square lamina of 40 side is perpendicular to H.P. One of its sides is 20 above H.P and 15 in front of V.P. Draw its projections. (May/June 2008, JNTU)

9.2 An equilateral triangular lamina of side 30 is parallel to H.P and perpendicular to V.P. One of its sides is 20 in front of V.P and 30 above H.P. Draw its projections. (June 2009, JNTU)

9.3 A pentagonal plate of 35 side is perpendicular to V.P and parallel to H.P. One of its edges is perpendicular to V.P. Draw its projections. (May/June 2008, 2010, JNTU)

9.4 An equilateral triangle of 50 side, has its plane parallel to H.P and 30 away from it. Draw the projections when one of its sides is (i) perpendicular to V.P, (ii) parallel to V.P and (iii) inclined to V.P at an angle of 45°. Locate its traces. (May/June 2011, May 2012, JNTU)

9.5 A triangular lamina of 50 side is standing on one of its sides, which is inclined at 45° to V.P, and the surface of the lamina is making an angle of 30° to H.P. Draw the projections. (May/June 2011, May 2012, JNTU)

9.6 An isosceles triangle ABC of base 60 and altitude 75, has its base AC in H.P and inclined at 30° to V.P. The corners A and B are in the V.P. Draw its projections. (June 2009, JNTU)

9.7 A rectangle ABCD of size 60 × 40, has a corner on H.P and 20 away from V.P. All the sides of the rectangle are equally inclined to H.P and parallel to V.P. Draw its projections and locate its traces. (May/June 2008, JNTU)

9.8 A thin rectangular plate of 60 × 40 size, has its shorter edge on H.P and inclined at 30° to V.P. Draw the projections of the plate, when its top view is a square of 40 side.

9.9 A thin rectangle ABCD of size 60 × 40 has a corner on V.P and 20 away from H.P. The front view of the plate is a square of 40 side. The smaller edge of the plate through the corner (which is on V.P) is inclined at 30° to H.P and parallel to V.P. Draw the projections of the plate.

9.10 A square of 30 side has one side on H.P. Its plane is inclined at 60° to H.P and perpendicular to V.P. Draw its projections and locate its traces. (May 2012, JNTU)

9.11 A square ABCD of 50 side, has its corner A in the H.P, its diagonal AC inclined at 30° to the H.P and the diagonal BD inclined at 45° to the V.P and parallel to H.P. Draw its projections. (May 2012, JNTU)

9.12 A pentagon of 30 side has one corner on V. P. Its plane is inclined at 65° to V.P and perpendicular to H.P. Draw its projections. (May/June 2008, JNTU)

9.13 A square lamina ABCD of 30 side rests on one of its corners on the ground. Its plane is inclined at 35° with H.P and diagonal DB inclined at 65° to V.P and parallel to H.P. Draw its projections. (Aug/Sep 2008, JNTU)

9.14 Draw the projections of a regular pentagon of 40 side having its surface inclined at 30° to V.P and the side on which it rests on V.P, makes an angle of 60° with H.P. (Aug/Sep 2008, May 2012, JNTU)

9.15 A regular pentagon of length of 30 side has one of its corners on V.P and its surface is inclined at 60° to V.P. The edge, opposite to the corner on V.P, makes an angle of 45° with H.P. Draw the projections of the plane. (May/June 2011, May 2012, JNTU)

9.16 Draw the projections of a regular hexagon of 25 side having one of its sides in H.P and inclined at 60° to V.P and its surface making an angle of 45° with H.P. (June 2008, JNTU)

9.17 A circular plate is parallel to H.P. Its radius is 30 and the centre is 50 above H.P and 40 in front of V.P. Draw its projections. (May 2012, JNTU)

9.18 Draw the projections of a circle diameter of 5 cm having its plane vertical and inclined at 30° to V.P. Its center is 3cm above H.P and 2cm in front of V.P. (May/June 2011, JNTU)

9.19 Draw the projections of a regular pentagon of 50 side having its surface inclined at 45° with H.P. A side of the pentagon lies on H.P, inclined at 30° to V.P.

9.20 A circular plate of negligible thickness and 60 diameter appears as an ellipse in the top view, having its major axis 60 and minor axis 30. Draw its projections and find the inclination of the plate with H.P. (June 2009, JNTU)

9.21 A circular plate of 60 diameter has a hexagonal hole of 20 side, centrally punched. Draw the projections of the plate, resting on H.P on a point, with its surface inclined at 30° to H.P. Any two parallel sides of the hexagonal hole are perpendicular to V.P. Draw the projections of the plate. (Aug/Sep 2008, JNTU)

9.22 A circular plane of 60 diameter rests on V.P on a point A on its circumference. Its plane is inclined at 45° to V.P. Draw the projections of the plane when (i) the front view of the diameter AB makes 30° with H.P and (ii) the diameter AB itself makes 30° with H.P. (Aug/Sep 2008, JNTU)

9.23 Draw the projections of a circle of 50 diameter resting in H.P on a point A on the circumference, its plane is inclined at 45° to H.P and

(i) The top view of the diameter AB making 30° angle with the V.P.

(ii) The diameter AB making 30° angle with the V.P.

(May/June 2011, May 2012, JNTU)

9.24 Draw the projections of a circle of 60 diameter, resting on the ground on a point A on the circumference, when its plane inclined at 45° to H.P and the top view of the diameter AB making 30° angle with the V.P. (June 2008, JNTU)

9.25 A thin semi-circular plate of 70 diameter, has its straight edge in H.P and inclined at 45° to V.P; while the surface of the plate is inclined at 30° to H.P. The end A of the diameter AB is nearer to the V.P and is at a distance 25 from it. Draw the projections of the plate.

(June 2009, JNTU)

9.26 Draw the projections of a circle of 75 diameter having the end A of the diameter AB on H.P and the end B on V.P, and the surface inclined at 30° to H.P and 60° to V.P.

(June 2008, JNTU)

9.27 The top view of a plate, the surface of which is perpendicular to V.P and inclined at 60° to H.P is a circle of 60 diameter. Draw its three views.

9.28 Top view of a plate, the surface of which is perpendicular to V.P and inclined at 60° to H.P, is a regular pentagon of side 50, with one edge perpendicular to xy. (a) Find the true shape of the plate and (b) draw the projections of the plate, when the edge whose top view was perpendicular to xy earlier, becomes parallel to V.P; while the surface of the plate is still inclined at 60° to H.P.

9.29 A regular hexagonal plate of 35 side has one corner touching V.P and another opposite corner touching H.P. The plate is inclined at 60° to H.P and 30° to V.P. Draw the projections of the plate, neglecting the thickness of it.

9.30 A regular hexagon of 40 side has a corner on H.P. Its surface is inclined at 45° to H.P and the top view of the longest diagonal through the corner on which it rests, makes an angle of 60° with V.P. Draw its projections. (May/June 2011, JNTU)

9.31 A regular hexagonal plane of 50 side, has a corner on V.P and its surface is inclined at 45° to V.P. Draw the projections when (a) the front view of the diagonal through the corner which is on V.P, makes 30° with H.P and (b) the diagonal itself makes 30° with H.P.

9.32 A regular hexagonal plane of 45 side has a corner on H.P, with its surface inclined at 45° to H.P. Draw its projections when (i) the top view of the diagonal through the resting corner makes 30° with V.P and (ii) the diagonal itself makes 30° with V.P.

9.33 A pentagonal plane with a 30 side lies on the GP with an edge parallel to and 20 behind the PP. The station point is 50 in front of PP, 65 above GP and lies in a CP which is at a distance of 40 towards right of the centre of the object. Draw its perspective view.

(May/June 2011, JNTU)

9.34 A pentagonal plane with a 30 side stands vertically on the GP on an edge and a corner touching the PP. The surface of the plane makes an angle of 30° with the PP. The station point is 60 in front of PP, 75 above GP and lies in a CP which is at a distance of 40 towards right of the centre of the plane. Draw its perspective view.

9.35 A hexagonal plane with a 40 side has a centrally cut square with a 30 side such that a side of the hole and a side of the hexagon are parallel to PP. It lies on the GP with a nearer edge of the hexagon 10 behind the PP. The station point is 50 in front of PP, 70 above GP and lies in a CP which is at a distance of 40 towards right of the centre of the plane. Draw its perspective view.

9.36 Draw the projections of a rhombus having diagonals 120 and 50 long; the smaller diagonal being parallel to both the principal planes, while the other is inclined at 30° to H.P.

(Aug/Sep, 2011, JNTU)

9.37 A thin 45° - 45° set-square, has its longest edge (250 long) on V.P and inclined at 30° to H.P. Its surface makes an angle of 45° with V.P. Draw its projections.

9.38 An equilateral triangular lamina of side 50 is perpendicular to both the planes. Draw its projections. (June 2009, JNTU)

9.39 A circular plate of 50 diameter is perpendicular to both planes. Its centre if 60 above H.P and 50 in front of V.P. Draw the projections of the plate. (June 2009, JNTU)

9.40 A regular pentagon of 30 side is resting on one of its edges on H.P, which is inclined at 45° to V.P. Its surface is inclined at 30° to H.P. Draw its projections. (June 2009, JNTU)

9.41 Draw the projections of a regular pentagon of 20 side with its surface making an angle of 45° with H.P. One of the sides of the pentagon is parallel to H.P and 15 away from it.

(June 2009, JNTU)

9.42 A thin 30° - 60° set square has its longest edge in the V.P and inclines at 30° to H.P. Its surface makes an angle of 45° with the V.P. Draw its projections. (June 2009, JNTU)

9.43 A hexagonal plane with distance between parallel sides 50 is resting on a side on H.P and the opposite side on V.P at a height of 30 from H.P. Draw the projections of the plane.

REVIEW QUESTIONS

9.1 How do you specify a plane in space?

9.2 Name the possible orientations of the planes, with respect to the principal planes of projection.

9.3 What is a trace of a plane?

9.4 What is an oblique plane?

9.5 When the traces of an oblique plane will be parallel to xy?

9.6 What is an edge view of a plane?

9.7 When both the views of a plane are straight lines?

9.8 Explain the three stages of obtaining projections of an oblique plane.

OBJECTIVE QUESTIONS

9.1 Plane surfaces have *one dimension/two dimensions*.

9.2 When a plane is perpendicular to a reference plane, its projection on that plane is a ———.

9.3 When the surface of a plane is perpendicular to both H.P and V.P; the front and top views are straight lines. (True/False)

CHAPTER - 11

PROJECTIONS OF SOLIDS

11.1 INTRODUCTION

A solid is a three dimensional object having length, breadth and thickness. In engineering practice, one often comes across solids bounded by simple or complex geometric surfaces. To represent a solid in orthographic projections, the number and types of views necessary will depend upon the type of solid and its orientation with respect to the principal planes of projection. The applications of auxiliary planes are also considered here.

11.2 TYPES OF SOLIDS

Solids may be classified as (i) polyhedra and (ii) solids of revolution.

11.2.1 Polyhedra

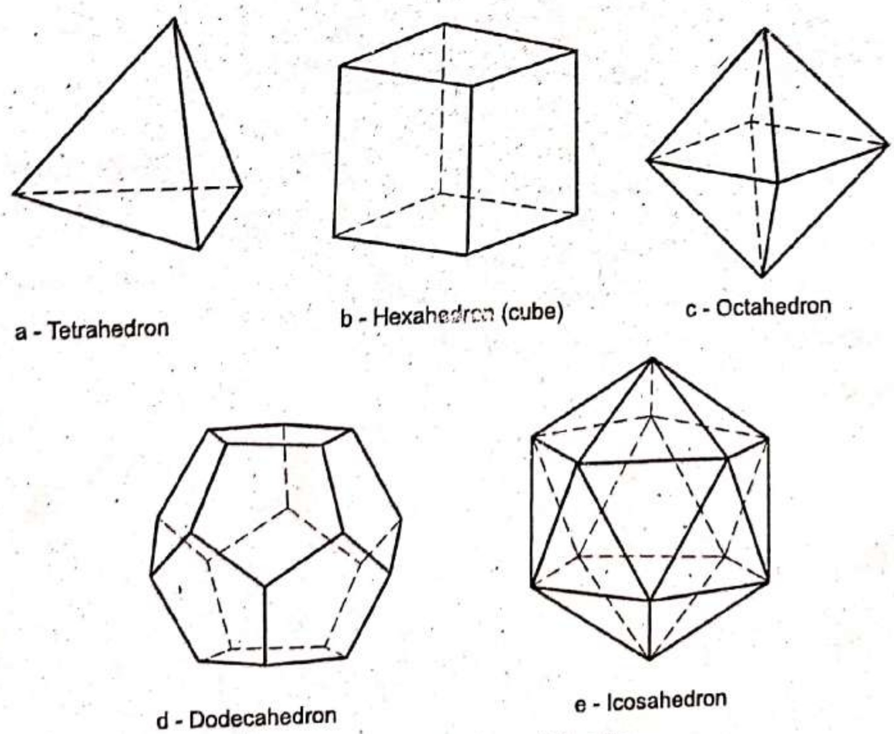


Fig. 11.1 Regular polyhedra

A polyhedron is defined as a solid bounded by plane surfaces called faces. There are two types of polyhedra: (i) Regular and (ii) irregular or oblique polyhedra.

11. 2. 1. 1 *Regular polyhedra*

A regular polyhedron is a solid bounded by plane surfaces, which are equal and regular. The examples are: (i) Tetrahedron, (ii) hexahedron, (iii) octahedron, (iv) dodecahedron and (v) icosahedron.

Tetrahedron It has four equal faces, each an equilateral triangle (Fig. 11.1a).
Hexahedron or cube It has six equal faces, each a square (Fig. 11.1b).
Octahedron It has eight equal faces, each an equilateral triangle (Fig. 11.1c).
Dodecahedron It has twelve equal faces, each a regular pentagon (Fig. 11.1d).
Icosahedron It has twenty equal faces, each an equilateral triangle (Fig. 11.1e).

There are two more categories of polyhedra, namely (i) prisms and (ii) pyramids.

Prism A prism is a polyhedron having two equal ends or bases, parallel to each other. The two bases are joined by faces, which are rectangles (Fig.11.2). The imaginary line joining the centres of the bases is called the axis of the solid.

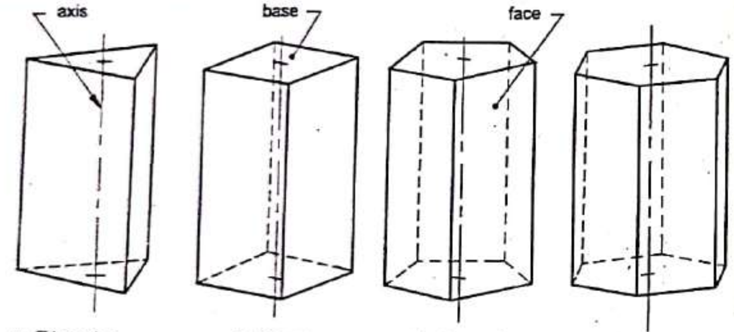


Fig. 11.2 Regular prisms

Pyramid A pyramid is a polyhedron having one base and a number of isosceles triangular faces, meeting at a point called the vertex or apex (Fig.11.3). The imaginary line joining the centre of the base and apex is called the axis of the solid.

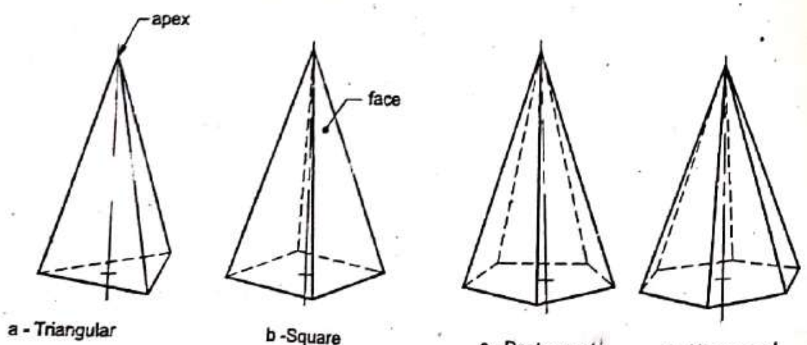


Fig. 11.3 Regular pyramids

A pyramid or prism is said to be regular when the axis is perpendicular to the base. Both pyramids and prisms are named according to the shape of the base, viz., triangular pyramid/prism, pentagonal pyramid/prism and so on.

11.2.2 Solids of revolution

Solids of revolution are obtained or generated by rotating a plane figure about one of its edges. Examples are: (i) cylinder, (ii) cone and (iii) sphere. Solids of revolution, viz., cylinders and cones, may also be classified as: (i) regular and (ii) oblique.

11. 2. 2. 1 *Regular solids of revolution*

Cylinder A cylinder is generated by rotating a rectangle about one of its edges. The lateral surface is connected at its either end by two circular bases (Fig. 11.4).

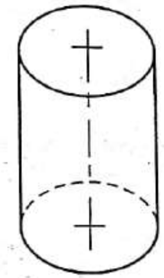


Fig. 11.4 Cylinder

Cone A cone is generated by rotating a right angled triangle about one of its perpendicular sides. The lateral surface of the cone is connected by a circular base (Fig.11.5).

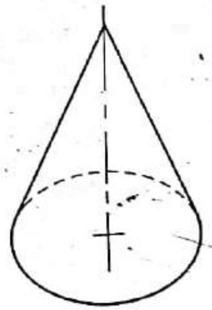


Fig. 11.5 Cone

A cylinder or a cone is said to be regular when the axis is perpendicular to the base. The lines drawn on the surface of a cylinder and parallel to the axis, are known as generators. The length of a generator is equal to the height of the cylinder. A line drawn from the vertex to any point on the base of a cone is also known as a generator, whose length is equal to the slant height of the cone.

Sphere A sphere is also a solid of revolution generated by rotating a semi-circle about its diameter (Fig. 11.6). The mid-point of the diameter is the centre of the sphere. All points on the surface of a sphere are equidistant from the centre.

Cylinders and cones may also be termed as solids with single curved surfaces, whereas a sphere has a double curved surface.

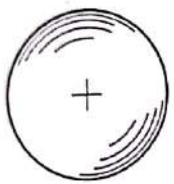


Fig. 11.6 Sphere

11.3 TWO-VIEW DRAWINGS

The position of a solid in space may be specified by the location of either the axis, edges, diagonals or surfaces, with the principal planes of projection. The following are some of the positions of solids:

(i) Axis perpendicular to one of the principal planes, (ii) Axis inclined to one of the principal planes and parallel to the other and (iii) Axis inclined to both the principal planes.

11.3.1 Axis perpendicular to one of the principal planes

When the axis of a solid is perpendicular to one of the principal planes, it is parallel to the other. Also, when the axis of a solid is perpendicular to any plane, the projection on that plane, will show the true shape and size of its base and the other projection will reveal the true length of the solid. So, when the axis is perpendicular to H.P, the top view must be drawn first and the front view is then projected from it. When the axis is perpendicular to V.P, the front view must be drawn first and then the top view is projected from it.

Problem 1 *Draw the projections of a cylinder of base 30 diameter and axis 50 long, when it is resting on H.P on one of its bases.* (Aug/Sep 2008, JNTU)

Construction (Fig. 11.7)

1. Draw a circle of 30 diameter, representing the top view of the cylinder.
2. Project the front view, which is a rectangle of height 50.

   The bottom base in the front view, coincides with xy, as the solid is resting on H.P on one of its bases.

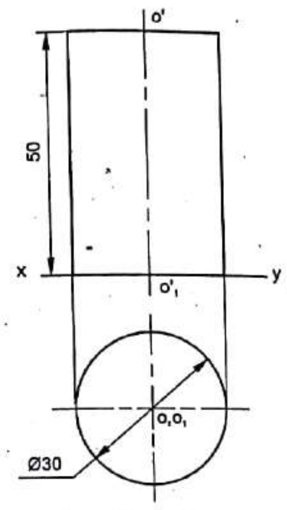


Fig. 11.7

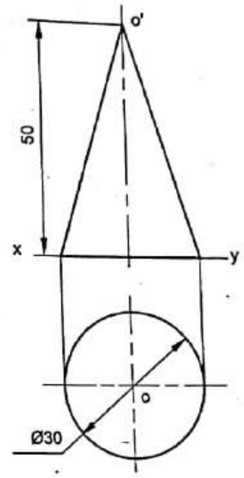


Fig. 11.8

Problem 2 *Draw the projections of a cone of base 30 diameter and axis 50 long, when it is resting on H.P on its base.* (Aug/Sep 2008, 2010, JNTU)

Construction (Fig. 11.8)

1. With centre o, draw a circle of 30 diameter; representing the top view of the cone.
2. Through o, draw a projector and locate the apex o′, at 50 above xy.
3. Obtain the front view which is a triangle, making the base coinciding with xy.

Problem 3 *A cube of 40 side is resting with a face on H.P such that, the vertical faces are equally inclined to V.P. Draw, its projections.* (May/June 2008, 2010, JNTU)

Construction (Fig. 11.9a)

1. Draw a square abcd of side 40 and with its sides making 45° with xy. This represents the top view of the cube.
2. Project the front view, keeping the height equal to 40.

It may be noted from the front view that the front view d′4′ of the invisible edge D4 coincides with the front view b′2′ of the visible edge B2.

Figure 11.9b shows the projections of the cube, when one of its vertical faces is inclined at 30° to V.P. It may be noted that the edge D4 in the front view (d′4′) is invisible and hence, represented by dotted line.

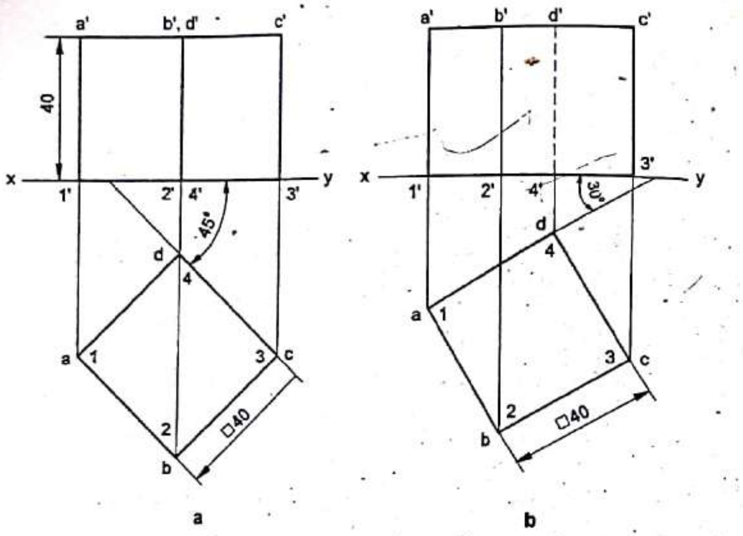


Fig. 11.9

Problem 4 *A triangular prism of base 30 side and axis 50 long, is resting on H.P on one of its bases, with a face perpendicular to V.P. Draw the projections of the solid.*

(May/June 2008, JNTU)

Construction (Fig. 11.10a)

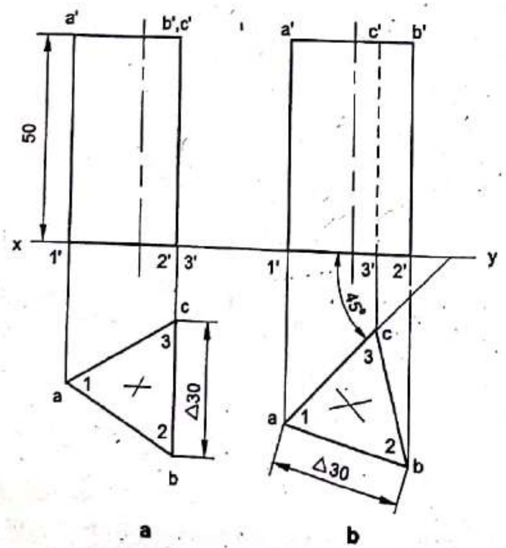


Fig. 11.10

1. Draw the top view, an equilateral triangle of 30 side such that, one edge of it is perpendicular to xy.
2. Project the front view, keeping its height equal to 50 and the bottom edge coinciding with xy.

Figure 11.10b shows the projections of the above solid, when one of its faces is inclined at 45° with V.P.

Problem 5 *A pentagonal pyramid of base 25 side and axis 60 long, is resting on an edge of the base on H.P. Draw the projections of the pyramid, when its axis is perpendicular to V.P and the base is at 15 from V.P.*

Construction (Fig. 11.11a)

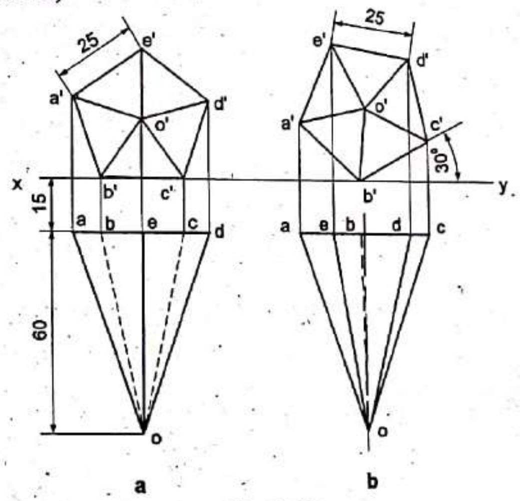


Fig. 11.11

1. Draw the front view of the base a'b'c'd'e', a pentagon of side 25, with the side b'c' coinciding with xy.
2. Project the top view such that, the top view of the base abcde is at 15 from xy and the apex o is at 60 from abcde.

NOTE For the given position of the solid, when it is seen from above, the slant edges OB and OC are invisible. Hence, the lines, ob and oc are represented by dotted lines in the top view.

Figure 11.11b shows the projections of the pentagonal pyramid, when it is resting on H.P on a base corner, with an edge of the base containing that corner, making 30° with H.P.

11.3.2 Axis inclined to one of the principal planes and parallel to the other

When the axis of the solid is inclined to any principal plane, the final projections are drawn in two stages. In the first stage, the axis of the solid is assumed to be perpendicular to the principal plane, to which it is actually inclined and the views are drawn. In the second stage, one of the views is redrawn to suit the given condition and the other view is projected from it. This method is known as change of position method.

In the second stage, instead of reconstructing one of the views as mentioned above, an auxiliary plane is imagined satisfying the given condition and the other view is projected on it. This method is known as change of reference line method. This is advantageous compared to the former one, as this avoids redrawing one of the views in the second stage. This advantage may be appreciated with respect to the solids having curved surfaces or too many edges.

While drawing the projections of solids, the following must be observed:

1. If a solid has an edge of its base parallel to H.P or on H.P, that edge should be kept perpendicular to V.P. If the edge of the base is parallel to V.P or on V.P, that edge should be kept perpendicular to H.P.
2. If a solid has a corner of its base on H.P, the sides of the base containing that corner should be kept equally inclined to V.P; if the corner of the base is on V.P, the sides of the base containing that corner should be kept equally inclined to H.P.

Problem 6 *Draw the projections of a hexagonal prism of base 25 side and axis 60 long, when it is resting on one of its corners of the base on H.P. The axis of the solid is inclined at 45° to H.P. Follow the change of position method.*

(May/June 2008, JNTU)

Construction (Fig. 11.12)

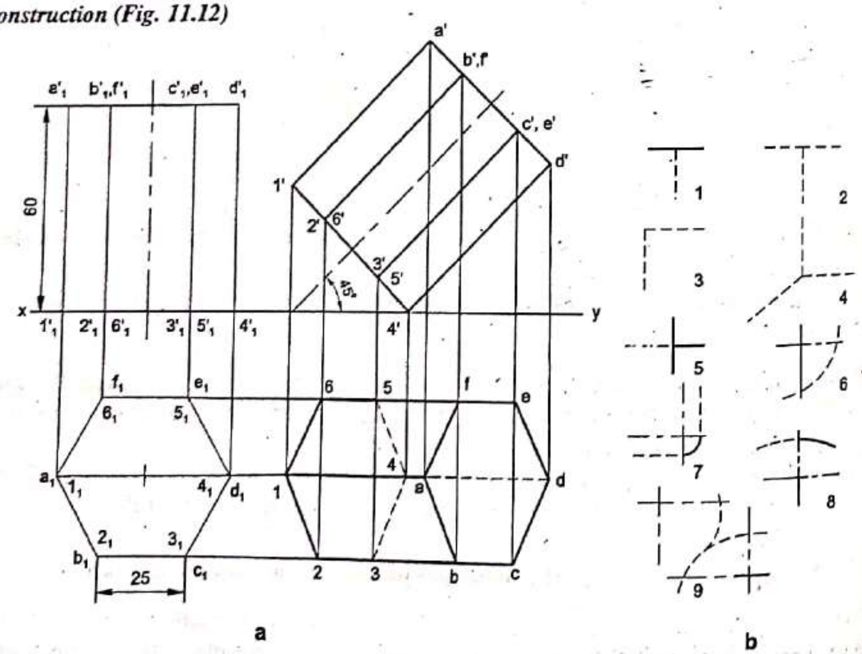


Fig. 11.12

Stage I

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, keeping a side of the base parallel to V.P.

The orientation chosen will permit the prism to be lying on one of its corners of the base on H.P, when it is tilted so as to make the axis inclined to H.P.

Stage II

2. Redraw the front view such that, its axis makes 45° with xy. This forms the final front view.
3. Obtain the final top view, by projection and by following the rules of visibility

For example, intersecting point between the horizontal projector through a_1 and vertical projector through a′ locates a in the final top view. Similarly, locate all the other points in the final top view.

In drawing the final views of the solids inclined to one or both the principal planes, the following rules of visibility and sequence may be observed:

1. Draw the lines of the edges of the visible base.

NOTE In the first angle projection, the base which is away from xy in one view will be fully visible in the other view.

2. Draw the lines corresponding to the longer edges of the solid.

NOTE The lines that pass through the visible base are invisible.

3. Draw the lines corresponding to the edges of the other base.

NOTE (i) It must be kept in mind that the lines corresponding to the boundary of the view must be visible and hence must be represented by thick lines.
(ii) When two lines representing the edges cross each other; one of them must be invisible and therefore, must be represented by dotted lines.
(iii) Various conditions arising in the representation of the invisible features are standardised in the method of showing them. These are illustrated in the Fig. 11.12b.

1. An invisible line intersects a visible line with a dash in contact.
2. An invisible line intersects another invisible line at the crossing point of two dashes in contact.
3. Invisible lines meeting at a corner have two dashes meeting at the corner.
4. Three invisible lines meeting at a corner, have three dashes intersecting at the corner.
5. When an invisible line shown as a continuation of a visible line, the invisible line begins with a space.
6. Invisible arcs begin with a dash.
7. When arcs are too small, they may be made continuous.

8. When an invisible arc is a continuation of a visible arc, the invisible arc begins with a space.
9. When two invisible arcs meet, the intersection at the point of tangency is located with a dash on each arc.

Problem 7 *Draw the projections of a pentagonal pyramid of side of base 30 and altitude 65, when (i) one of its sloping edges is vertical and (ii) one of its triangular faces is perpendicular to H.P. Follow the change of reference line (auxiliary plane) method.*

Construction (Fig. 11.13)

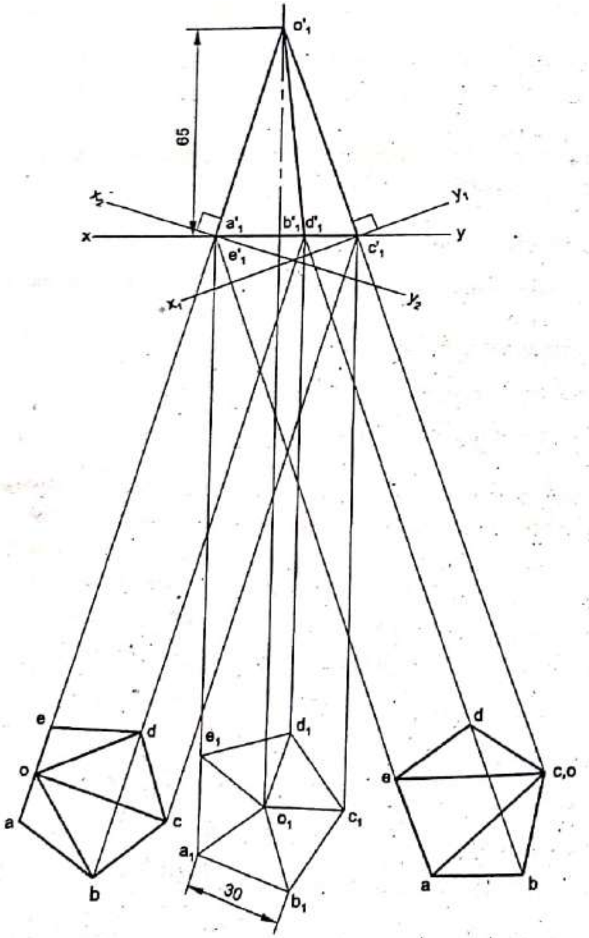


Fig. 11.13

Stage I

1. Draw the projections of the solid, assuming that it is resting on its base on H.P, keeping an edge of the base perpendicular to V.P.

Stage II One of the sloping edges vertical

2. Draw the reference line x_1y_1, representing auxiliary inclined plane; passing through c_1' in the front view and perpendicular to c_1' o_1'.
3. Draw projectors from all the points in the front view and perpendicular to x_1y_1.
4. On the above projectors, mark points, keeping the distance of each point from x_1y_1 equal to its distance from xy in the top view and obtain the auxiliary top view, by joining the points.

The front view and the auxiliary top view with respect to x_1y_1 are the required views.

Stage III One of the triangular faces perpendicular to H.P

2. Draw the reference line x_2y_2, representing the auxiliary inclined plane; passing through a_1' (e_1') in the front view and perpendicular to a_1' o_1'.
3. Obtain the auxiliary top view, by projection.

The front view and the auxiliary top view with respect to x_2y_2 are the required views.

Problem 8 *Draw the projections of a pentagonal prism of base 25 side and axis 50 long, when it is resting on one of its rectangular faces on H.P. The axis of the solid is inclined at 45° to V.P. Follow the change of position method.* (May/June 2008, JNTU)

Construction (Fig. 11.14)

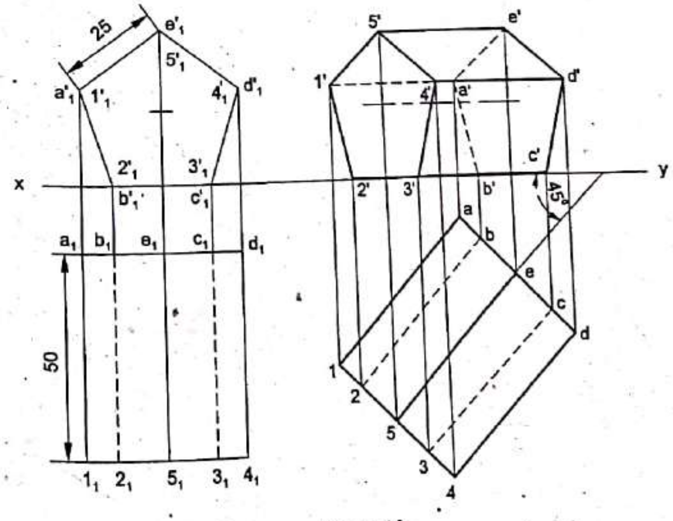


Fig.11.14

Stage I

1. Draw the projections of the solid, assuming that it is resting on one of its faces on H.P, with its axis perpendicular to V.P.

Stage II

2. Redraw the top view such that, the axis makes 45° with xy; forming the final top view.
3. Obtain the final front view, by projection.

11.3.3 Axis inclined to both the principal planes

A solid is said to be inclined to both the planes when, (i) the axis is inclined to both the planes or (ii) the axis is inclined to one plane and an edge of the base is inclined to the other. In all such cases, the final projections are obtained in three stages.

Stage I Assume that the axis is perpendicular to one of the planes and draw the projections.

Stage II Redraw one of the views, by making the axis inclined to one of the planes and project the other view from it.

Stage III Redraw one of the views obtained in stage II, satisfying the remaining condition and project the other view from it.

Stages II and III may also be drawn by the use of auxiliary plane method.

Problem 9 *An equilateral triangular prism of side of base 25 and axis 50 long, is resting on an edge of its base on H.P. The face containing that edge is inclined at 30° to H.P. Draw the projections of the prism, when the edge on which the prism rests, is inclined at 60° with V.P. Follow the change of position method.* (Aug/Sep 2008, JNTU)

Construction (Fig. 11.15)

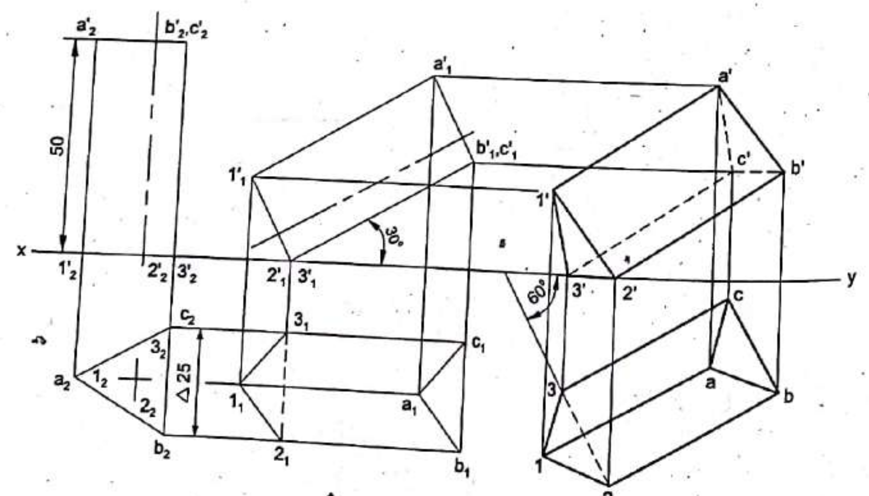


Fig.11.15

Stage I Assume that the solid is resting on its base on H.P, with one edge perpendicular to V.P.

1. Draw the projections of the solid.

Stage II Tilt the solid about the edge, which is perpendicular to V.P such that, the face containing the edge makes 30° to H.P.

2. Redraw the front view such that, the front view of the face 23BC is inclined at 30° to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.

Stage III Rotate the solid, till the edge on which it rests is inclined at 60° to V.P.

4. Redraw the above top view such that, the top view of the edge 2-3 is inclined at 60° to xy. This forms the final top view.
5. Obtain the final front view, by projection.

Problem 10 *Draw the projections of a square prism, side of base 30 and axis 60 long, resting with one of the edges of its base on H.P. Its axis is inclined at 30° to H.P and the top view of the axis at 45° to xy line.*

Construction (Fig. 11.16)

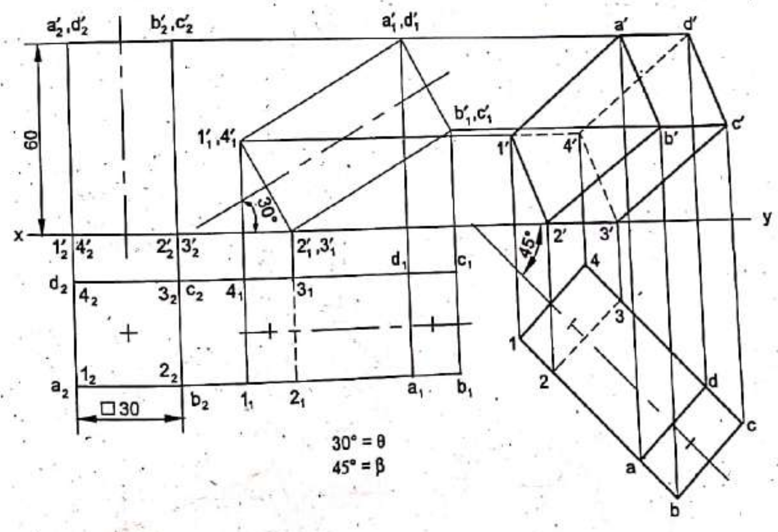


Fig.11.16

1. Draw the projections of the prism, assuming that it is resting on its base on H.P, with one edge perpendicular to V.P.
2. Redraw the front view such that, the axis is inclined at 30° to xy and the front view of the edge 2-3 lying on xy.
3. Obtain the top view, by projection.

4. Redraw the above top view such that, the axis is inclined at 45° to xy. This forms the final top view.
5. Obtain the final front view, by projection.

Problem 11 *A pentagonal pyramid of edge of base 25 and height 60, is resting on a corner of its base on H.P and the slant edge containing that corner is inclined at 45° with H.P. Draw the projections of the solid, when its axis makes an angle of 30° with V.P. Follow the auxiliary plane method.*

Construction (Fig. 11.17)

Stage I *Assume that the solid is resting on its base on H.P, with an edge of the base perpendicular to V.P.*

1. Draw the projections of the solid.

Stage II *Tilt the solid till it rests on a corner of the base on H.P and the slant edge through the resting corner is inclined at 45° with H.P.*

2. Draw the reference line x_1y_1; representing the auxiliary inclined plane, passing through c_2' and making an angle of 45° with $c_2'\ o_2'$.
3. Obtain the auxiliary top view (final top view), by projection.

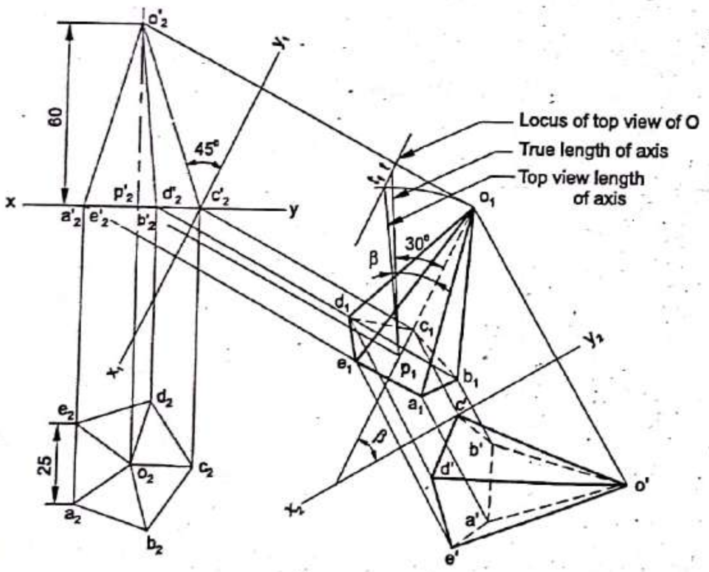


Fig.11.17

Stage III *Rotate the solid till the axis of the solid makes 30° with V.P.*

4. Determine the apparent angle of inclination β ; the axis making with the reference line.

5. Draw the reference line x_2y_2; representing the auxiliary vertical plane, making the angle β with the axis in the top view.
6. Obtain the final front view, by projection.

NOTE To determine the apparent angle β (ref. auxiliary top view),

(i) Through p_1, draw the line p_1t, equal to the true length of the axis of the solid and making an angle of 30° with p_1o_1.

(ii) Through t, draw a line parallel to p_1o_1; representing the locus of the apex in the top view.

(iii) With p_1 as centre and p_1o_1 as radius, draw an arc; intersecting the above locus at t_1.

$\angle o_1p_1t_1$ is the apparent angle.

11.4 THREE VIEW DRAWINGS

The number of views required to describe the shape of any object will depend upon the complexity of it. In some cases, three views are required to describe the shape of an object. To understand the principles of projection better, the students are advised to practice the addition of the side view to all the cases of projection of solids, dealt with under the preceding section. However, one case is considered here, where the solution starts with the side view.

Problem 12 *A triangular prism of side of base 30 and axis 50 long, is resting on a longer edge on H.P and a face containing that edge is inclined at 45° with H.P. Draw the projections of the solid, when its axis is parallel to both H.P and V.P.*

Construction (Fig. 11.18)

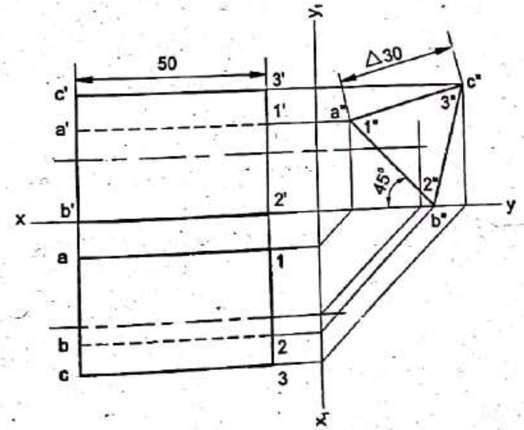


Fig.11.18

1. Draw the reference lines xy and x_1y_1.
2. Draw the side view a″ b″ c″ such that, the corner b″ (2″) is on xy and the side of the face 1AB2 is inclined at 45° with xy.
3. Obtain the front and top views, by projection; keeping the length of the axis as 50.

11.5 PROJECTIONS OF SPHERES

The projection of a sphere in any position and on any plane is always a circle of diameter equal to the diameter of the sphere itself. Figure 11.19 shows the projections of a sphere, lying on H.P.

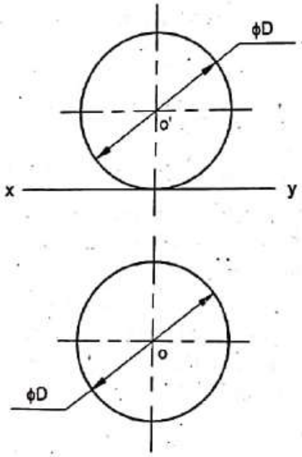


Fig.11.19

Problem 13 *Draw the projections of two equal spheres of 15 radius, resting on H.P and in contact with each other so that the line joining their centres is inclined at (i) 30° to V.P and (ii) 45° to H.P.*

Construction (Fig. 11.20a)

1. With centers a′ and a and radius 15, draw circles representing the projections of a sphere, say A.
2. Locate the centre b of the other sphere B in the top view such that, the line ab joining the centres, makes an angle of 30° with xy.
3. Locate the centre b′ in the front view, by projection.

   The line a′ b′ is parallel to xy, as the spheres are equal in size and are lying on H.P.
4. Draw the projections of the sphere B.

NOTE (i) The two circles, representing the top views of the two spheres are fully visible.
(ii) In the front view, sphere B is fully visible and sphere A is partially visible.

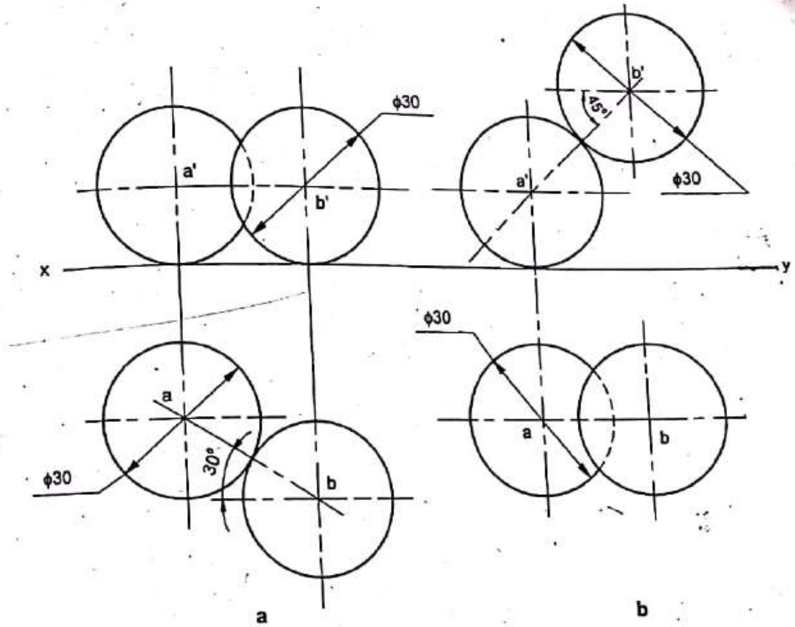


Fig.11.20

Figure 11.20b shows the projections of the spheres, when the line joining their centres is inclined at 45° to H.P.

Problem 14 *Three equal spheres of 40 diameter are lying on H.P such that, each touches the other two and the line joining the centres of two spheres is parallel to V.P. A fourth sphere of diameter 60 is placed on top of the three spheres. Draw the projections of the arrangement.*

Construction (Fig. 11.21)

All the three spheres are equal in size and are resting on H.P. Hence, the line joining the centres will be parallel to xy in the front view. In the top view, the centres will be lying at the corners of an equilateral triangle of side equal to 40 (twice the radius of the sphere).

1. In the top view, locate the centres of spheres a, b and c at the corners of an equilateral triangle abc of 40 side, with one side, say ab parallel to xy.
2. In the front view, locate the centres a′, b′ and c′, by projection and on a line parallel to and 20 above xy.
3. With centres a, b, c, a′, b′ and c′ and radius 20, draw circles, forming the projections.

   When the fourth sphere is placed on top of the three spheres, its centre d will lie above the centre of the triangle abc, while a, b, c and d form the corners of a triangular pyramid.

To locate d′

(i) Rotate dc parallel to xy to dc_1.
(ii) Through c_1, draw a projector meeting the line of centres in the front view at c_1'.
(iii) With c_1' as centre and radius 50 = (20+30), draw an arc intersecting the projector through d at d'.
(iv) With d' and d as centres and radius 30, draw circles.
(v) Follow the rules of visibility and complete the projections.

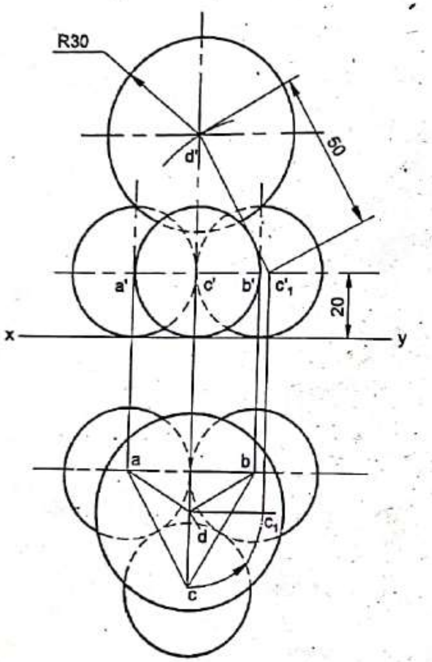


Fig.11.21

11.5.1 Projections of unequal spheres

When two unequal spheres are on H.P and in contact with each other, the length of the line joining the centres will be seen in its true length in the front view, if the line is parallel to V.P. In the top view, the length of the line will be shorter but remains constant, even when it is inclined to V.P.

Problem 15 *Three spheres A, B and C of 30, 50 and 70 diameters are placed on H.P such that, each one touches the other two. Draw the projections of the arrangement, when the line joining the centres of the spheres B and C is parallel to V.P.* (June 2009, JNTU)

Construction (Fig. 11.22)

1. Draw the projections of the spheres B and C, touching each other such that, the line joining their centres is parallel to V.P.

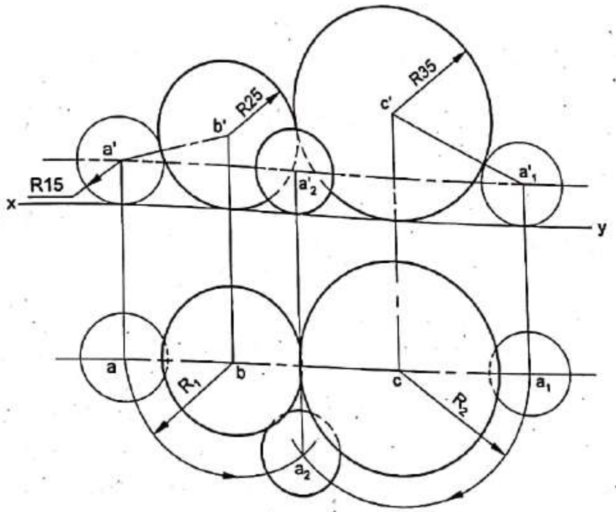


Fig.11.22

2. Draw the projections of the spheres A-B and C-A, touching each other, assuming that the lines joining their centres are parallel to V.P. Determine the top view lengths of ba and ca_1. The line joining a' and a_1' is the locus of the centre of the sphere A in the front view.
3. With b as centre and radius ba, draw an arc.
4. With c as centre and radius ca_1, draw an arc; intersecting the above arc at a_2.
5. With a_2 as centre, draw the top view of the sphere A.
6. Draw a projector through a_2; meeting the locus at a_2'.

   a_2' is the centre of the sphere A in the front view.
7. With a_2' as centre, draw the front view of the sphere A.

11.6 EXAMPLES

Problem 16 *A cube of 50 side, has one face on V.P and an adjacent face inclined at 35° to H.P; the lower edge of the later face being on H.P. Draw its projections.*

Construction (Fig. 11.23)

1. Draw the projections of the cube, assuming it to be lying on a face on V.P and the adjacent lower face on H.P.
2. Redraw the front view such that, the face lying on H.P is inclined at 35° to xy and the front view a' e' of the lower edge of the inclined face is on xy.

   This is the final front view.
3. Obtain the final top view, by projection.

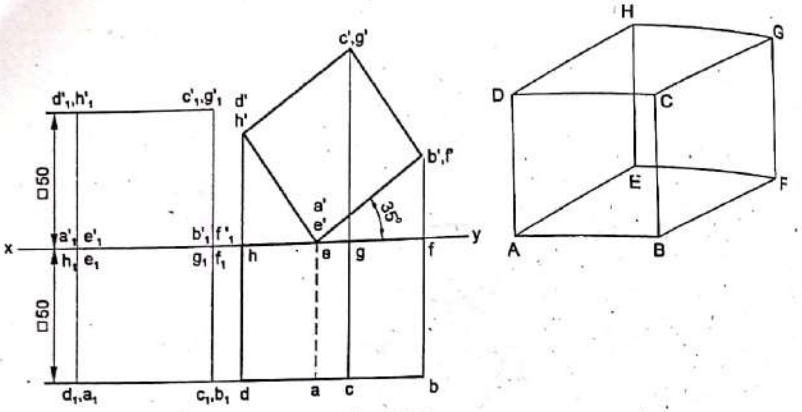


Fig.11.23

Problem 17 *A triangular pyramid of base 30 side and axis 50 long, is resting on H.P on its base, with a face perpendicular to V.P. Draw the projections of the pyramid.*

Construction (Fig. 11.24a)

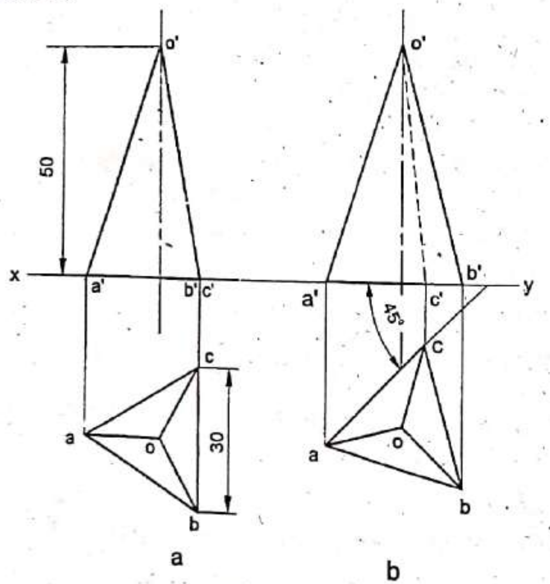


Fig.11.24

1. Draw the top view of the pyramid; keeping one edge of the base perpendicular to xy.

   It may be noted that triangle abc represents the top view and the lines oa, ob and oc meeting at the centre of the triangle, represent the slant edges of the pyramid.

2. Obtain the front view such that, the base a′ b′ c′ lies on xy and the apex o′ is at 50 from xy.

Figure 11.24b shows the projections of the pyramid, when an edge of the base is inclined at 45° to V.P.

Problem 18 *A tetrahedron of 40 long edges, is resting on H.P on one of the faces, with an edge of that face parallel to V.P. Draw the projections of the solid.*

Construction (Fig 11.25)

1. Draw an equilateral triangle abc of side 40, with one side, say ac parallel to xy. Locate its centre o and join it with the corners; forming the top view of the tetrahedron.
2. Project and obtain the front view a′ b′ c′ of the face ABC, coinciding with xy.
3. Through o, draw a projector.

To locate o′ in the front view

(i) With o as centre, rotate, say oc to oc_1, parallel to xy.

(ii) Project c_1 to c_1' on xy.

(iii) With c_1' as centre and radius 40, draw an arc; intersecting the projector though o at o′.

(iv) Join o′, a′; o′, b′ and o′, c′; forming the front view.

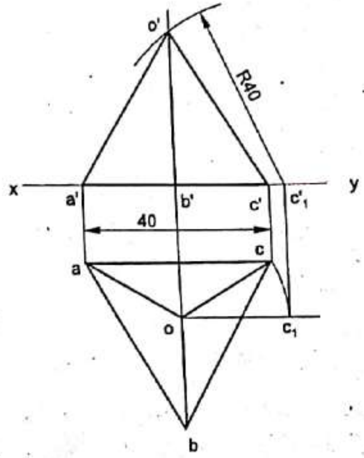


Fig.11.25

Problem 19 *An octahedron of side 40, is resting with one of its corners on H.P. The axis passing through this corner is vertical and one of its horizontal edges is parallel to and 15 in front of V.P. Draw the projections of the solid.*

Construction (Fig. 11.26)

1. Draw a square abcd of side 40, with one side, say cd parallel to and 15 below xy. Locate its centre o_1 (o_2) and join it with the corners, forming the top view of the solid.
2. Through o_1 (o_2), draw a projector and locate o_2' on xy.

   The lengths, ac and bd represent the true lengths of the diagonals AC and BD, as these diagonals are parallel to H.P. Hence,
3. Mark o_1' on the projector through o_1 (o_2) such that $o_1'\ o_2' = ac = bd$.
4. Through the mid-point of $o_1'\ o_2'$, draw a horizontal line.
5. Locate the points a′(d′) and b′ (c′), by projection.
6. Join a′(d′) and b′(c′) with o_1' and o_2', forming the front view.

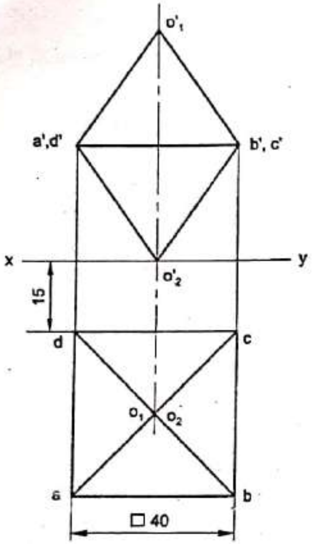
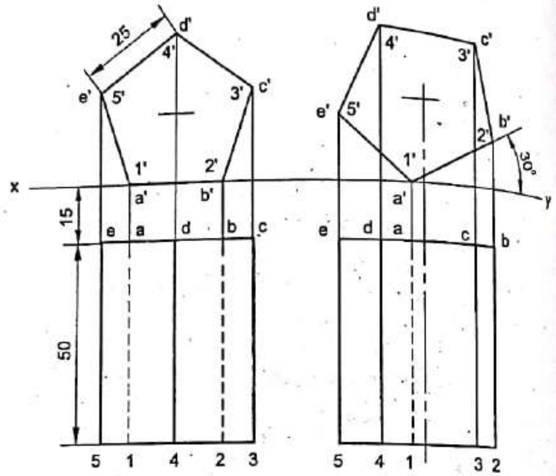


Fig.11.26



a b

Fig.11.27

Problem 20 *A pentagonal prism of side of base 25 and axis 50 long, is resting on one of its faces on H.P, with the axis perpendicular to V.P and a base 15 away from V.P. Draw its projections.*

Construction (Fig. 11.27a)

1. Draw the front view a′ b′ c′ d′ e′, a pentagon of side 25, keeping the edge a′ b′ on xy.
2. Project the top view such that, the base abcde is at 15 from xy and of length 50.

Figure 11.27b shows the projections of the prism, when it is resting on a lateral edge on H.P and a rectangular face containing that edge is inclined at 30° to H.P.

Problem 21 *A square pyramid of side of base 40 and altitude 70, lies with all the edges of the base equally inclined to H.P and the axis parallel to and 50 from both H.P and V.P. Draw its projections.*

Construction (Fig. 11.28)

1. Draw the side view of the pyramid such that, the base edges are equally inclined to H.P.
2. Draw the reference lines xy and x_1y_1 such that, they are at 50 from the centre, o″ of the side view.
3. Obtain the front and top views, by projection; keeping the length of the axis as 70.

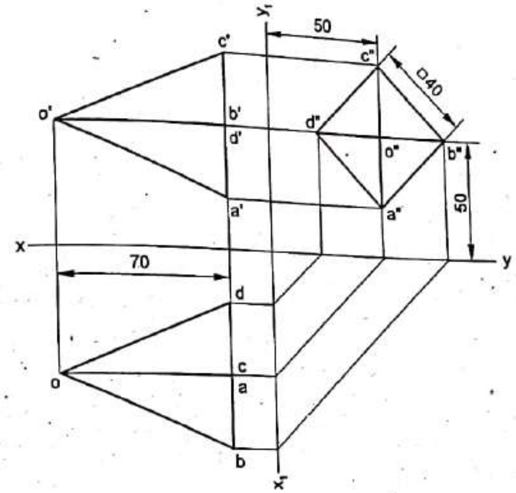


Fig.11.28

Problem 22 *Draw the projections of a cylinder of 40 diameter and axis 60 long, when it is lying on H.P, with its axis inclined at 45° to H.P and parallel to V.P. Follow the change of position method.* (May/June 2010, May 2012 JNTU)

Construction (Fig. 11.29)

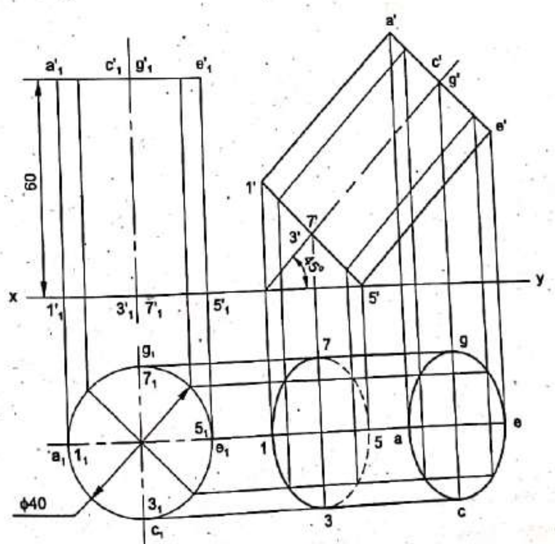


Fig.11.29

1. Draw the projections of the cylinder; assuming that it is lying on its base on H.P.
2. Divide the circle (top view) into a number of equal parts, say 8 and draw the corresponding generators in the front view.
3. Redraw the front view such that, it rests on a point of its base on xy and the axis is inclined at 45° to xy.
4. Obtain the final top view, by projection.

NOTE (i) In the final top view, the axis is parallel to xy.
(ii) The students are advised to divide the circle into twelve equal parts, so that more number of points may be obtained to draw smooth curves in the final top view.
(iii) The generators are only imaginary lines and so they must be represented by thin lines.

Problem 23 *Draw the projections of a cone of diameter of base 40 and axis 60 long, when it is lying on a point of the base on H.P, with its axis inclined at 45° to H.P and parallel to V.P. Follow the auxiliary plane method.* (May/June 2010, JNTU)

Construction (Fig. 11.30)

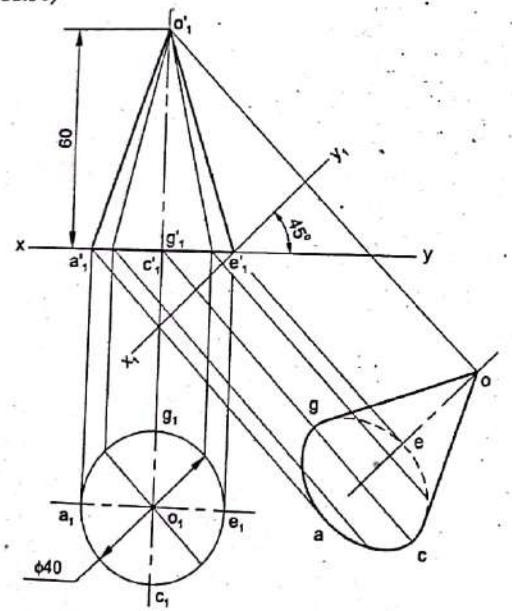


Fig.11.30

1. Draw the projections of the cone, assuming that it is resting on its base on H.P.
2. Divide the circle (top view) into a number of equal parts and draw the corresponding generators in the front view.

3. Draw the reference line x_1y_1, representing the auxiliary inclined plane and passing through the lowest point e_1' of the extreme generator and making an angle of 45° with the axis.
4. Obtain the final (auxiliary) top view, by projection.

Problem 24 *Draw the projections of a hexagonal pyramid, with side of base 30 and axis 70 long, which is resting with a slant face on H.P such that, the axis is parallel to V.P. Follow the change of position method.* (May/June 2010, JNTU)

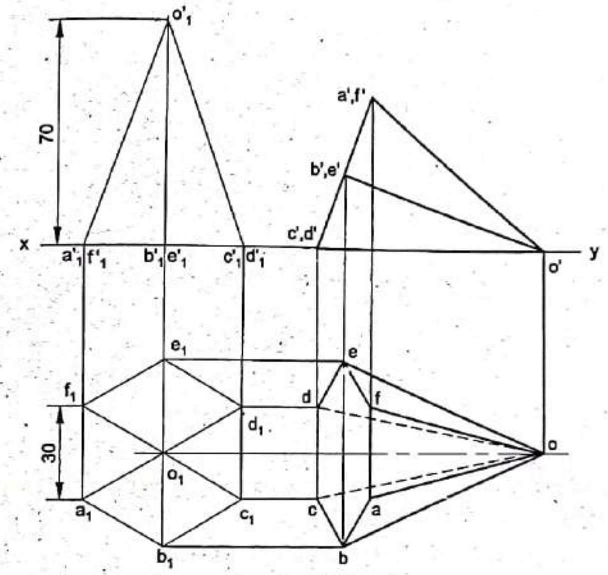


Fig.11.31

Construction (Fig. 11.31)

1. Draw the projections of the pyramid, assuming that it is resting on its base on H.P, with two edges of the base perpendicular to V.P.
2. Redraw the front view such that, the line o′ c′ (d′), representing front view of the slant face OCD, coincides with xy.
3. Obtain the final top view, by projection.

Problem 25 *A square pyramid of side of base 30 and axis 50 long, is freely suspended from a corner of its base. Draw the projections. Follow the auxiliary plane method.*

HINT When a solid is freely suspended from a corner, the imaginary line passing through that corner and the centre of gravity of the solid will be vertical. It is obvious that the centre of gravity of a prism and a cylinder will lie at the mid-point of the axis. However, for a pyramid and a cone, it lies at 1/4th of the length of the axis from the base.

CHAPTER - 12

SECTIONS OF SOLIDS

12.1 INTRODUCTION

The conventional orthographic views, if selected and drawn properly, may reveal sufficient information about the shape and size of the object. However, the conventional views may consist of too many hidden lines for complicated objects, which make the interpretation difficult. To overcome this, it is customary to imagine the object, cut by a section plane. The portion of the object between the observer and the section plane is assumed to be removed. The projection of the remaining solid is known as a sectional view. The actual sectioned portion of the view is shown by cross-hatched lines.

12.2 POSITION OF SECTION PLANES

The shape of the section obtained or revealed will depend upon the orientation of the solid and the section plane, with respect to the principal planes of projection. The following are some of the positions of the section planes:

1. Section plane parallel to H.P
2. Section plane parallel to V.P
3. Section plane inclined to H.P and perpendicular to V.P
4. Section plane inclined to V.P and perpendicular to H.P
5. Section plane perpendicular to both H.P and V.P
6. Section plane inclined to both H.P and V.P

Section planes are usually represented by their traces. The types of solids that are dealt with here are: (i) Polyhedra and (ii) solids of revolution.

12.2.1 True shape of a section

The projection of the section on a plane parallel to the section plane, will appear in its true shape of the section. Thus, when the section plane is parallel to H.P, the true shape of the section will be seen in the sectional top view. When it is parallel to V.P, the true shape of the section will appear in the sectional front view.

When the section plane is inclined, the section has to be projected on an auxiliary plane, parallel to the cutting plane, to obtain its true shape. However, when the section plane is perpendicular to both H.P and V.P, the sectional side view shows the true shape of the section.

12.2.2 Section

When the section plane is inclined to H.P and perpendicular to V.P, the section plane in the front view coincides with the V.T of the section plane. The section in the top view is known as apparent section and does not represent the true shape. Similarly, when the section plane is inclined to V.P and perpendicular to H.P, the section plane in the top view coincides with the H.T of the section plane and the front view shows the apparent section of the solid. The apparent section may be obtained by:

1. Locating the intersection points between the trace of the cutting plane and either the sides or generators of solid, and
2. Locating these points in the other view and joining them by straight lines or curved lines in the correct sequence.

12.3 SECTIONS OF POLYHEDRA AND SOLIDS OF REVOLUTION

When a section plane passes through any polyhedron, the intersection of the section plane with the surfaces of the solid consists of a number of straight lines. Hence, the sectioned portion in a projected view is a plane figure bounded by straight lines.

When a section plane passes through the lateral surface of the solids of revolution, the intersection is a smooth curve. If the section plane passes through the base, that portion of the boundary will be a straight line.

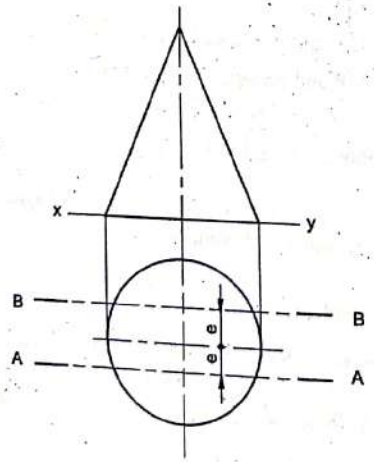


Fig. 12.1

Figure 12.1 shows the method of indicating the trace of a cutting plane on the projection of a solid. In general, the cutting plane is represented by a chain dotted line. However, it should be noted that only the longer dash of the chain dotted line should cross the boundary of the projection; followed by a short dash and finally terminated by a thick longer dash.

Further, referring Fig. 12.1; given the off-set (e) of the cutting plane from the axis of the solid, it may be seen that there are two possible positions A-A and B-B for its location. However, it is customary to select the position such that, only a minor portion of the solid is removed. Thus, referring Fig. 12.1, the cutting plane along the line A-A is preferred.

12.3.1 Section plane parallel to H.P

When a section plane parallel to H.P, passes through any polyhedron resting on H.P, the sectioned portion will appear in its true shape in the top view. In the front view, it will appear as a straight line, parallel to xy and coincides with V.T of the section plane. When a section plane parallel to H.P, passes through any solid of revolution, resting on a base on H.P, the section is a circle and it also represents the true shape of the section.

NOTE When a cone or pyramid is cut by a section plane, parallel to the base; the retained portion of the solid is called the frustum.

(4)

Problem 1 *A cube of 40 edge, is resting on H.P on one of its edges, with a face parallel to V.P. One of the faces containing the resting edge is inclined at 30° to H.P. The solid is cut by a section plane, parallel to H.P and 10 above the axis. Draw the projections of the remaining solid.*

base

HINT As the section plane is parallel to H.P, it is perpendicular to V.P. Hence, the section plane is represented by its trace (V.T) in the front view.

Construction (Fig. 12.2)

1. Draw the projections of the cube, satisfying the given conditions.
2. Draw the V.T of the section plane in the front view, at a height of 10 above o′.
3. Locate the intersection points 1′, 2′, 3′ and 4′, between the V.T and edges of the cube.
4. Project and locate these points, on the corresponding edges in the top view.
5. Join the points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.

NOTE
1. The points 1′, 2′, 3′ and 4′ are on the edges a′ d′, d′ c′, r′ s′ and q′ p′ respectively.
2. The shape and size of the sectioned portion will depend upon the position of the section plane.
3. The section also represents the true shape.

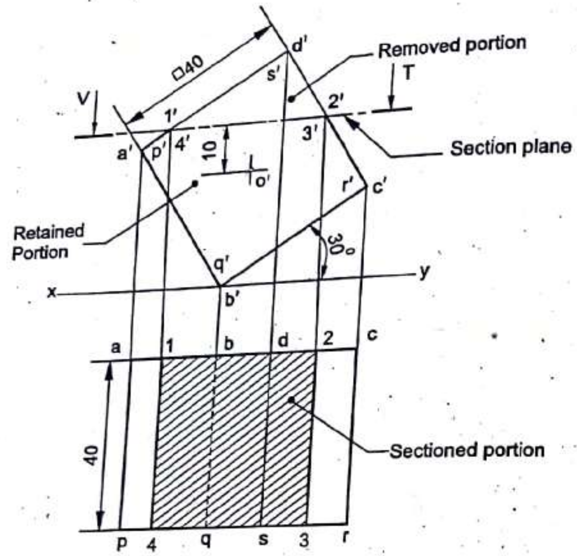


Fig. 12.2

Problem 2 *A pentagonal pyramid with side of base 30 and axis 60 long, is resting with its base on H.P and one of the edges of its base is perpendicular to V.P. It is cut by a section plane, parallel to H.P and passing through the axis at a point 35 above the base. Draw the projections of the remaining solid.*

Construction (Fig. 12.3)

1. Draw the projections of the pyramid, keeping one edge of the base perpendicular to V.P.
2. Draw the V.T of section plane, at a height of 35 above xy.
3. Locate the intersection points 1′, 2′, etc., between the trace and slant edges of the pyramid.
4. Project and locate the corresponding points in the top view, on the respective slant edges.
5. Join these points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.

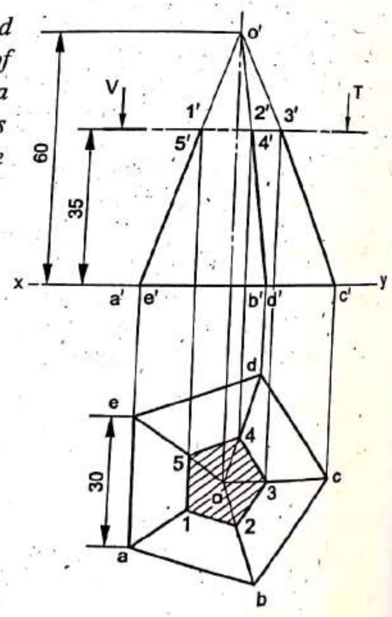


Fig. 12.3

NOTE As the section plane is parallel to the base of the pyramid, the sectioned portion will appear in its true shape as a pentagon, the size of which depends upon the location of the cutting plane.

Problem 3 *A triangular prism of base 30 side and axis 50 long, is lying on H.P on one of its rectangular faces, with its axis inclined at 30° to V.P. It is cut by a section plane, parallel to H.P and at a distance of 12 above H.P. Draw the front and sectional top view.*

Construction (Fig. 12.4)

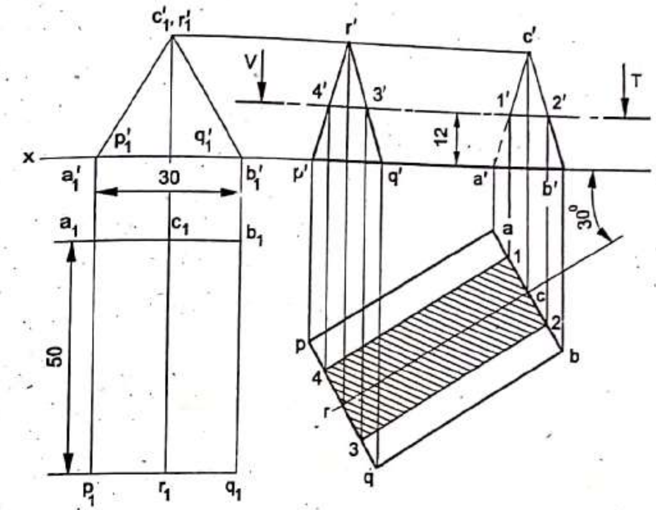


Fig. 12.4

1. Draw the projections of the prism, satisfying the given conditions.
2. Locate the V.T of section plane, at a height of 12 from xy.
3. Locate the intersection points 1′, 2′, 3′ and 4′, between the V.T and edges of the solid, a′ c′, c′ b′, q′ r′ and r′ p′ respectively.
4. Project and locate these points on the corresponding edges in the top view.
5. Join these points in the order by straight lines and complete the sectional top view by cross-hatching the sectioned portion.

Problem 4 *A cone with base 60 diameter and axis 75 long, is resting on its base on H.P. It is cut by a section plane parallel to H.P and passing through the mid-point of the axis. Draw the projections of the cut solid*

Construction (Fig. 12.5)

1. Draw the projections of the cone.
2. Draw the V.T of section plane, passing through the mid-point m′ of the axis.
3. Locate the intersection points 1′ and 2′, between the V.T and extreme generators of the cone.
4. With centre o and diameter 1′ -2′, draw a circle in the top view and cross-hatch it; completing the sectional top view.

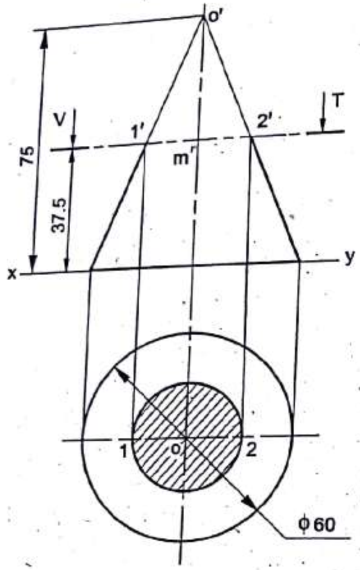


Fig. 12.5

Problem 5 *Figure 12.6a shows the projections of a cone, with the front view p′ of a point P and top view q of a point Q. Locate the top view of the point P and front view of the point Q.*

Construction (Fig. 12.6b and c)

To locate the top view of P

Method I

1. Through p′, draw a line 1′ -1′, parallel to the base.
2. With centre o and diameter equal to 1′ -1′, draw a circle in the top view.
3. Through p′, draw a projector intersecting the circle at p and p_1.

p is the top view of the visible point of p′ and p_1 is the top view of another point, lying on the rear side of the cone and coinciding with p′ in the front view.

Method II

1. Through p′, draw a generator.
2. Project and obtain the corresponding division lines in the top view.
3. Through p′, draw a projector; meeting the above lines at p_1 and p.

To locate the front view of Q

Method I

1. With centre o and radius oq, draw a circle; meeting the horizontal axis at 2-2.

2. Through 2, draw a projector; meeting the extreme generator at 2′.
3. Through 2′, draw the line 2′ -2′, parallel to the base.
4. Through q, draw a projector; meeting 2′ -2′ at q′, which is the required front view of the point Q.

Method II

1. Join o, q and extend, forming the division line in the top view.
2. Draw the corresponding generator in the front view.
3. Through q, draw a projector meeting the above generator at q′.

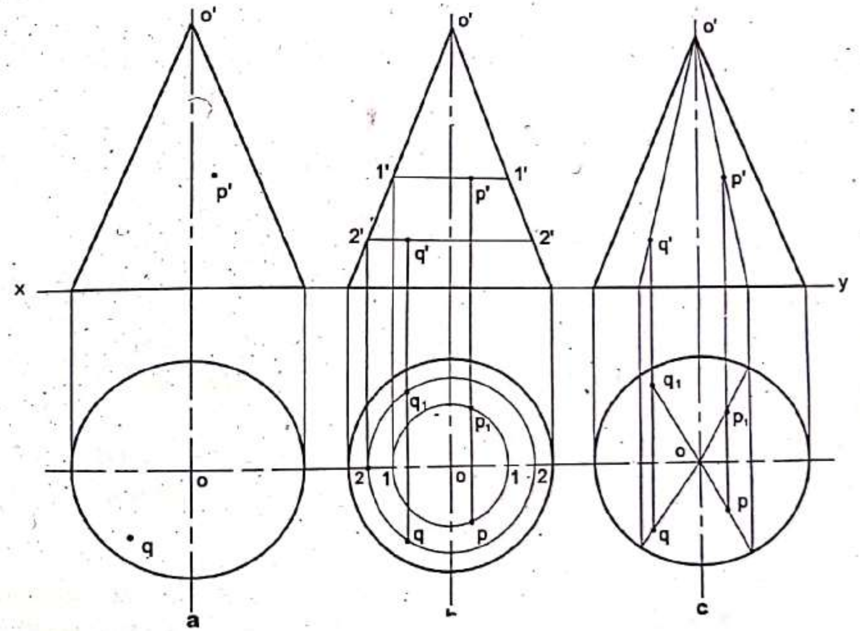


Fig. 12.6

12.3.2 Section plane parallel to V.P

When a section plane passing through a solid is parallel to V.P, the sectioned portion will appear in its true shape in the front view. In the top view, it will appear as a straight line, parallel to xy and coincides with its H.T. When the section plane passes through a solid of revolution, the section produced depends upon the type of solid. In the case of a cylinder, the section is a rectangle; in the case of a cone, when the section plane passes through the apex, it is a triangle; otherwise, it is a hyperbola and in the case of a sphere, it is a circle.

Problem 6 *A cylinder of 40 diameter and axis 55 long, stands vertically with its base on H.P. It is cut by a section plane, parallel to V.P and passes at a distance of 10 from the axis. Draw the projections of the retained solid.*

Construction (Fig. 12.7)

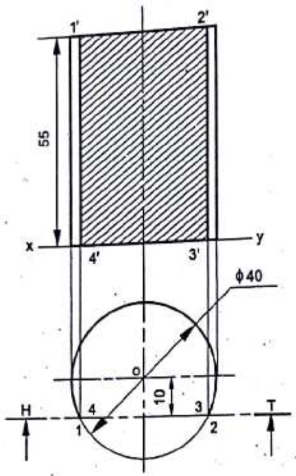


Fig. 12.7

1. Draw the projections of the cylinder.
2. Draw the H.T of section plane, which lies at a distance of 10 from the axis.
3. Locate the points of intersection 1, 2, 3 and 4, between the H.T and bases of the cylinder (points 1 and 2 lie on the top base and 3 and 4 on the bottom base).
4. Project and locate the points 1′, 2′, 3′ and 4′, on the corresponding bases in the front view.
5. Join these points in the order by straight lines and complete the sectional front view, by cross-hatching the sectioned portion.

NOTE When a cylinder is cut by a section plane, parallel to its axis, the sectioned portion is a rectangle, the length being equal to the length of the axis but the width depends upon the position of the cutting plane from the axis.

Problem 7 *A tetrahedron of side 50, is resting on H.P on one of its faces, with an edge of it parallel to V.P and away from it. It is cut by a section plane parallel to V.P and at a distance of 10 from the apex. Draw the projections of the retained solid.*

Construction (Fig. 12.8)

1. Draw the projections of the tetrahedron.
2. Draw the H.T of section plane, at a distance of 10 from o.
3. Locate the points of intersection 1, 2, 3, 4, between the H.T and edges of the solid in a sequence.

For the given position of the solid, points 1 and 4 are on the edges of the base AC and BC respectively and the points 2 and 3 are on the slant edges OA and OB respectively.

4. Project and locate these points on the corresponding edges in the front view.
5. Join the points in the order by straight lines and complete the sectional front view, by cross-hatching the sectioned portion.

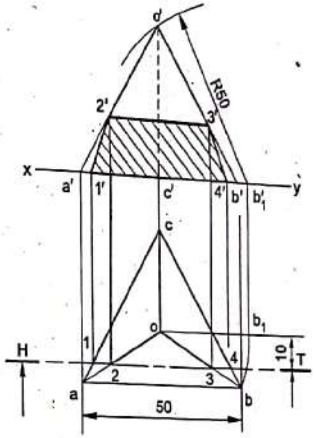


Fig. 12.8

Problem 8 *A pentagonal pyramid of side of base 35 and axis 60 long, stands with its base on H.P such that, one of the base edges is perpendicular to V.P. A section plane parallel to V.P, cuts the solid at a distance of 15 from the corner of the base which is nearer to the observer. Draw the top and sectional front views of the cut solid.*

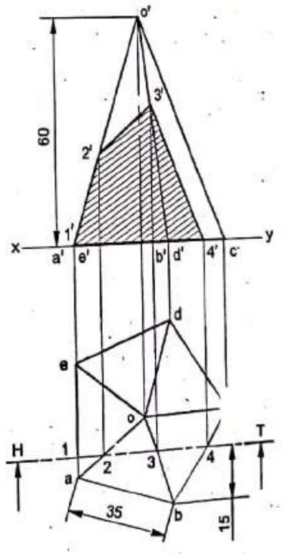


Fig. 12.9

Construction (Fig. 12.9)

1. Draw the projections of the pyramid.
2. Draw the H.T of section plane, at a distance of 15 from b.
3. Repeat steps 3 to 5 of Construction: Fig. 12.8 and complete the sectional front view.

12.3.3 Section plane inclined to H.P and perpendicular to V.P

When a section plane passing through a solid is inclined to H.P and perpendicular to V.P, its V.T is inclined to the reference line xy. The sectioned portion in the top view does not reveal the true shape. In all such cases, the true shape of the section may be obtained on an auxiliary plane (A.I.P), parallel to the section plane.

In general, the shape and size of the true shape of the section depends upon the orientation of the solid and the position of the section plane. The position of the section plane will be specified by the inclination of it, with the principal planes of projection and the point in the solid through which it passes.

NOTE When a solid is cut by a section plane, inclined to the base, the retained portion is called the truncated solid.

Problem 9 *A cube of side 40, is resting on H.P on one of its faces, with a vertical face inclined at 45° to H.P and passing through the axis at 8 from the top surface. Draw the projections of the solid and also show the true shape of the section.*

Construction (Fig. 12.10)

1. Draw the projections of the cube.
2. Draw the V.T of section plane, inclined at 45° to xy and passing through a point at 8 from the top end of the axis.
3. Locate the points of intersection 1′, 2′, 3′ and 4′, between the vertical trace and the edges of the cube.
4. Repeat steps 4 and 5 of Construction: Fig. 12.2 and complete the sectional top view.

To obtain the true shape of the section:

(i) Draw the reference line x_1y_1, parallel to the V.T of section plane.

(ii) Project the points 1′, 2′, 3′ and 4′, through x_1y_1 and obtain the true shape of the section.

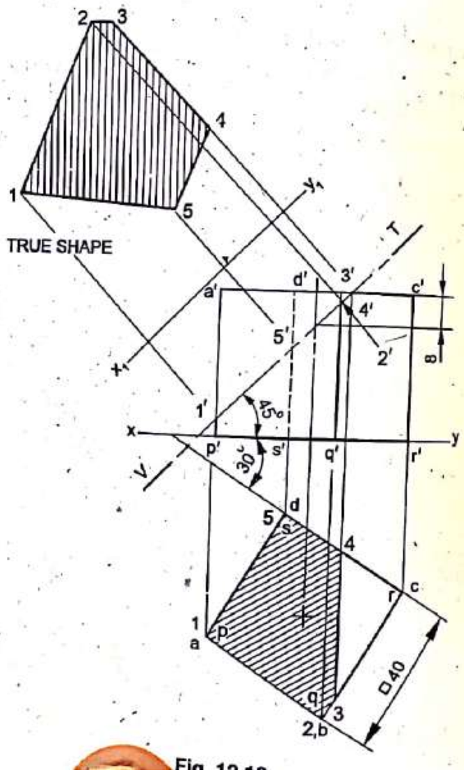


Fig. 12.10

Problem 10 *A pentagonal prism of edge of base 30 and axis 60 long, is resting on one of its faces on H.P. The axis of the prism is parallel to both H.P and V.P. It is cut by a section plane, inclined at 45° to H.P and passing through the axis at 10 from one base. Draw the projections and show the true shape of the section.*

Construction (Fig. 12.11)

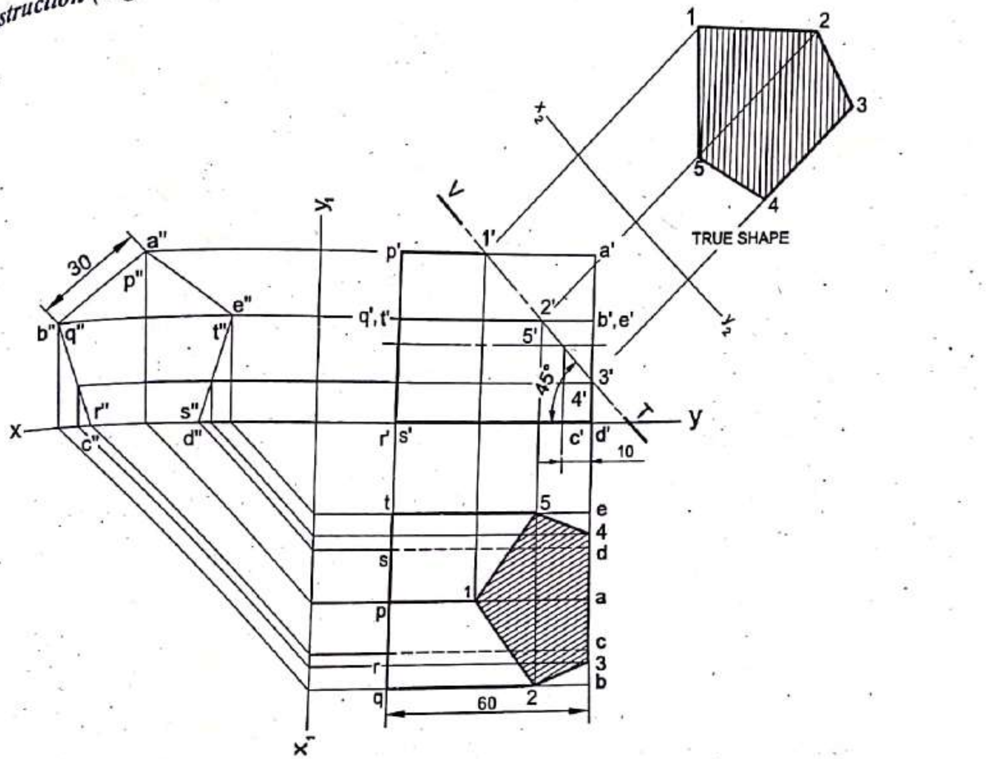


Fig. 12.11

1. Draw the projections of the prism.
2. Draw the V.T of section plane, inclined at 45° to xy and passing through the axis at 10 from one of the bases.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and edges of the prism
4. Repeat steps 4 to 6 of Construction: Fig. 12.10 suitably and obtain the sectional top view and true shape of the section.

Problem 11 *A hexagonal pyramid of side of base 30 and axis 60 long, is resting on its base on H.P, with an edge of the base perpendicular to V.P. It is cut by a section plane, inclined at 30° to H.P and passing through the axis at 20 from the base. Draw the three views of the solid and obtain the true shape of the section*

Construction (Fig. 12.12)

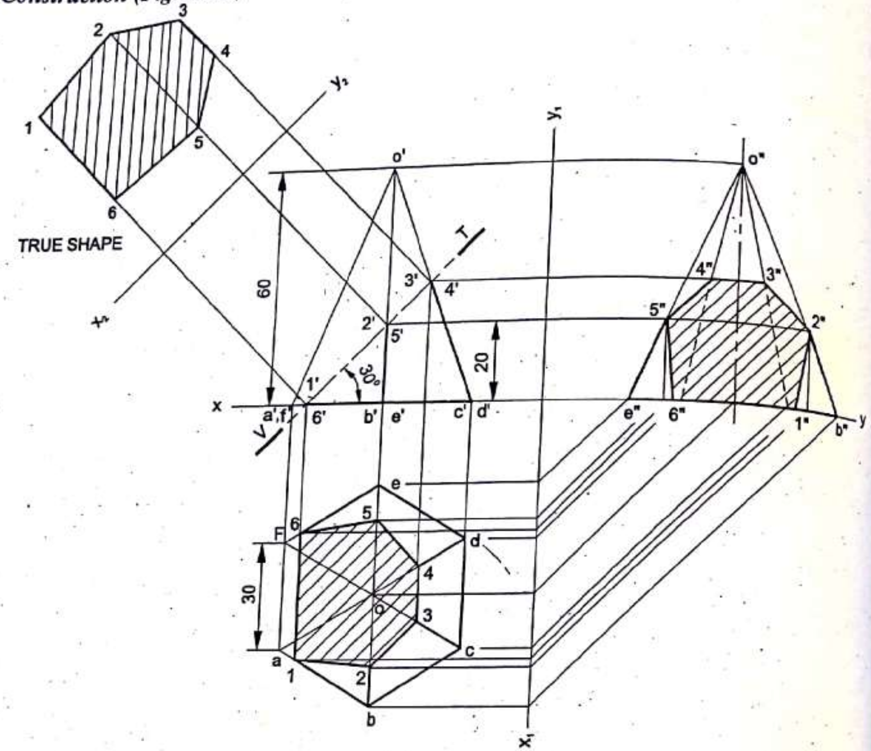


Fig. 12.12

1. Draw the projections of the pyramid.
2. Draw the V.T of section plane, inclined at 30° to xy and passing through a point on the axis at 20 from the base.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and edges of the pyramid.

It may be noted that the points 1′ and 6′ lie on the edges of the base, whereas the remaining points lie on the slant edges of the solid.

4. Project the points 1′ and 6′ on to the top view and other points on to the side view.
5. Transfer the points 2″, 3″, etc., to the top view by projection and complete the sectional top view.
6. Transfer the points 1 and 6 to the side view by projection and complete the sectional side view.

Problem 12 *A cylinder of 45 diameter and 70 long, is resting on one of its bases on H.P. It is cut by a section plane, inclined at 60° with H.P and passing through a point on the axis at 15 from one end. Draw the three views of the solid and also obtain the true shape of the section.*

Construction (Fig. 12.13)

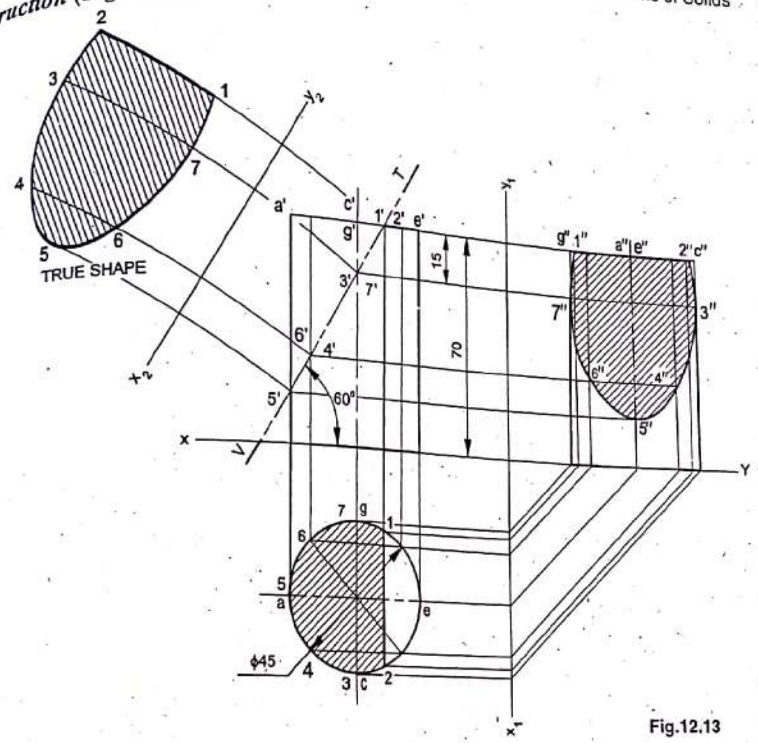


Fig.12.13

1. Draw the projections of the cylinder.
2. Divide the base (top view) into a number of equal parts, say 8 and draw the corresponding generators in the front view.
3. Draw the V.T of section plane, inclined at 60° to xy and passing through a point on the axis at 15 from its top end.
4. Locate the points of intersection 1′, 2′, etc., between the V.T and base and generators of the cylinder.
5. Project and locate the corresponding points 1, 2, etc., in the top view.
6. Join the points in the order and complete the sectional top view and cross-hatch the sectioned portion.
7. Obtain the true shape of the section and sectional left side view, by suitably following the principle of Construction: Fig. 12.12.

NOTE (i) The boundary of the intersection is a straight line, when the section plane passes through the base.

(ii) The remaining part of the boundary, corresponding to the plane, passing through the curved surface of the solid is a curve

Problem 13 *A cone with diameter of base 50 and axis 60 long, is resting on its base on H.P. It is cut by a section plane inclined at 45° to H.P and passing through the axis at a point 35 above H.P. Draw the projections of the cut solid.*

Construction (Fig. 12.14)

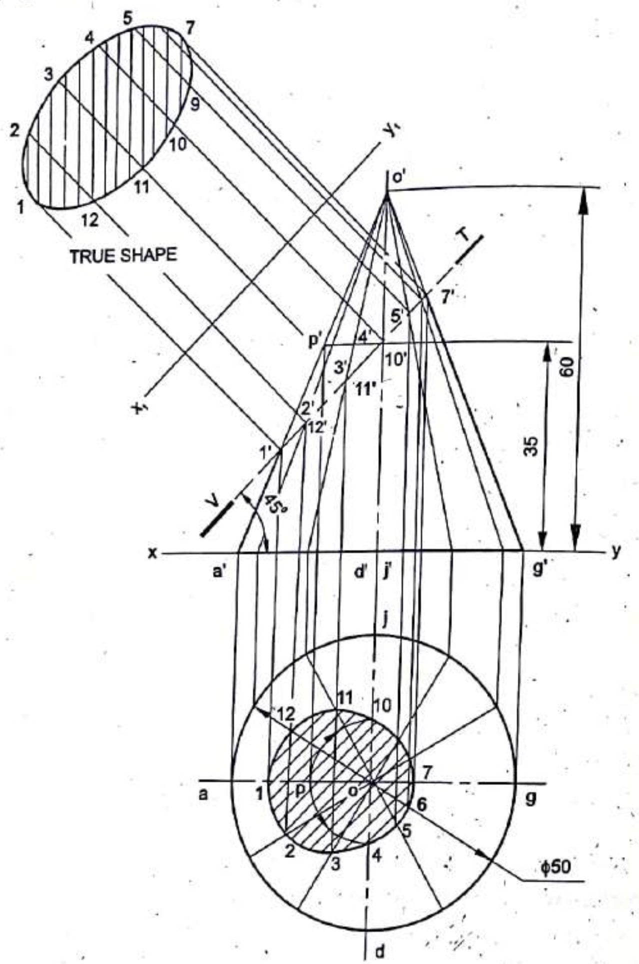


Fig. 12.14a

1. Draw the projections of the cone.
2. Draw the V.T of section plane, inclined at 45° to xy and passing through a point on the axis at 35 from the base.

The two methods to locate the intersection points in the top view are: (i) Generator method and (ii) cutting plane method.

Generator method (Fig. 12.14a)

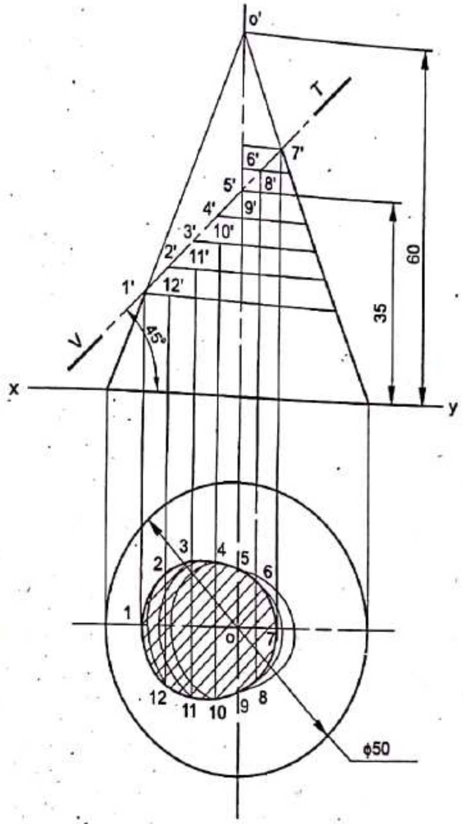


Fig. 12.14b

1. Divide the circle (top view) into say, 12 equal parts.
2. Locate the corresponding generators in the front view.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and generators.
4. Obtain the corresponding points in the top view, by projection.
5. Join these points by a smooth curve and obtain the sectional top view.
6. Obtain the true shape of the section by projecting on an A.I.P, parallel to the V.T.

NOTE *To locate the points 4 and 10 in the top view*

(i) Through 4′ (10′), draw a line parallel to the base; meeting the extreme generator at p′

(ii) Through p′, draw a projector; meeting the horizontal line ag in the top view, at p.

(iii) With centre o and radius op, draw an arc; meeting the division lines od and oj at 4 and 10 respectively.

Cutting plane method (Fig. 12.14b)

1. Locate the points 1′ and 7′ at which the V.T cuts the extreme generators.
2. Select a number of horizontal cutting planes, say 5 between 1′ and 7′, which need not be equidistant.
3. Locate the points of intersection between the V.T and cutting planes 2′, 3′, 4′, etc.
4. Draw the circles, produced by these section planes in the top view and transfer the above intersection points on to the respective circles.
5. Join these points in the order by a smooth curve and obtain sectional top view.

NOTE When a cone, resting on its base on H.P, is cut by a section plane, inclined to H.P such that, it passes through the extreme generators; the true shape of the section is an ellipse. If the length of the major axis is given; the V.T of section plane can be drawn through the front view of the cone such that, the distance between the points of intersection with the extreme generators is equal to the length of the major axis. Obviously, given the length of the major axis, there will be number of possible positions for location of the V.T.

12.3.4 Section plane inclined to V.P and perpendicular to H.P

When a section plane passing through the solid is inclined to V.P and perpendicular to H.P, its H.T is inclined to xy. The true shape of the section may be obtained on an A.V.P, parallel to the given section plane.

Problem 14 *A hexagonal prism of side of base 25 and axis 60 long, is resting on its base on H.P such that, an edge of the base is parallel to V.P. It is cut by a section plane, inclined at 45° to V.P and 10 away from the axis. Draw the projections of the solid. Also obtain an auxiliary front view, showing the true shape of the section.*

Construction (Fig. 12.15)

1. Draw the projections of the prism.
2. Draw the H.T of the section plane, inclined at 45° to xy and 10 away from axis.
3. Locate the points of intersection 1, 2, 3 and 4 between the H.T and edges of the base.
4. Project and locate the corresponding points 1′, 2′, 3′ and 4′ in the front view.
5. Join the points in the order by straight lines and obtain the section.
6. Draw the reference line x_1y_1, parallel to the H.T of section plane and obtain the auxiliary front view for the retained portion of the solid, which also shows the true shape of the section.

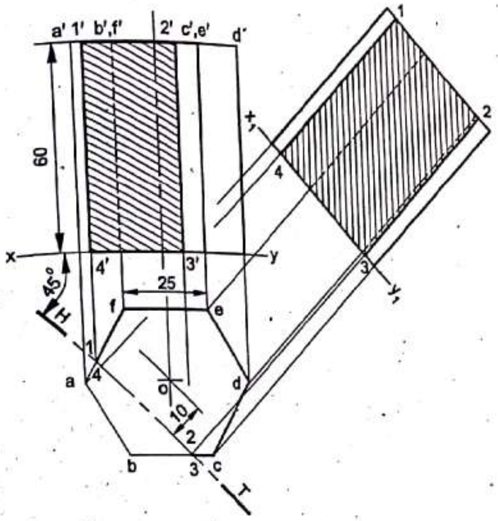


Fig. 12.15

Problem 15 *A pentagonal pyramid with edge of base 25 and axis 65 long, is resting on H.P on its base with an edge nearer to the observer, parallel to V.P. It is cut by a section plane, inclined at 60° to V.P and at a distance of 6 from the axis. Draw the projections and obtain the true shape of the section.*

Construction (Fig. 12.16)

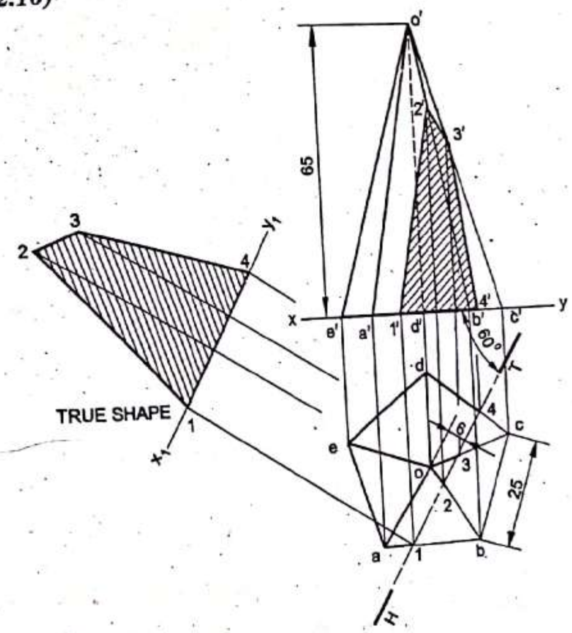


Fig. 12.16

1. Draw the projections of the pyramid.
2. Draw the H.T of section plane, inclined at 60° to xy and 6 away from axis.
3. Locate the points of intersection between the H.T and base (1 and 4) and the slant edges (2 and 3) of the pyramid.
4. Project and obtain the corresponding points in the front view.
5. Join these points in the order by straight lines and obtain the section.
6. Obtain the true shape of the section, considering an auxiliary vertical plane, parallel to the H.T of section plane.

Problem 16 *A cone of base 60 diameter and axis 70 long, is resting on its base on H.P. It is cut by a section plane, inclined at 60° to V.P and 10 away from its axis. Draw the projections of the cut solid and obtain the true shape of the section.*

Construction (Fig. 12.17)

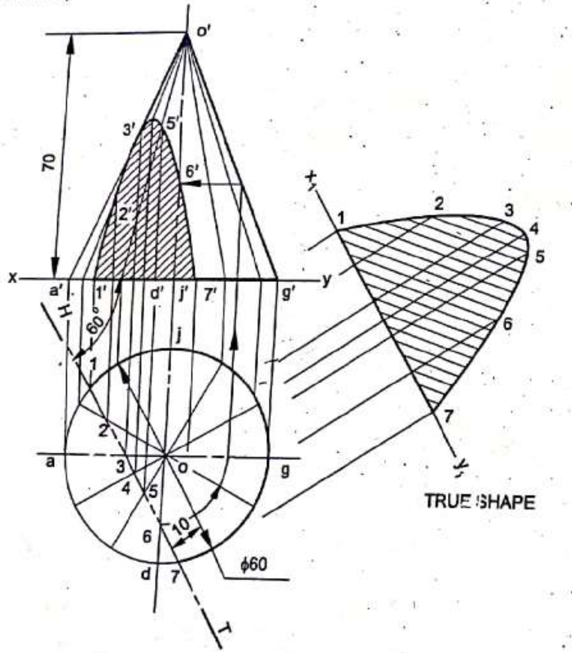


Fig. 12.17

1. Draw the projections of the cone.
2. Draw the H.T of section plane, inclined at 60° to xy and 10 away from axis.
3. Locate the points of intersection between the H.T and base of the cone (1and 7) and the generators (2, 3,– – –– 6).

4. Repeat steps 4 to 6 of Construction: Fig. 12.16 suitably, and obtain the sectional front view and the true shape of the section.

12.3.5 Section plane perpendicular to both H.P and V.P

When a section plane, perpendicular to both H.P and V.P, passes through a solid resting on H.P, the section appears as a straight line, coinciding with the traces of the section plane both in front and top views. However, in the side view, the section appears in its true shape.

Problem 17 *A hexagonal pyramid of side of base 25 and axis 60 long, is resting on its base on H.P, with an edge of the base parallel to V.P. A section plane perpendicular to both H.P and V.P cuts the solid, 5 away from the axis. Draw the sectional side view of the cut solid.*

Construction (Fig. 12.18)

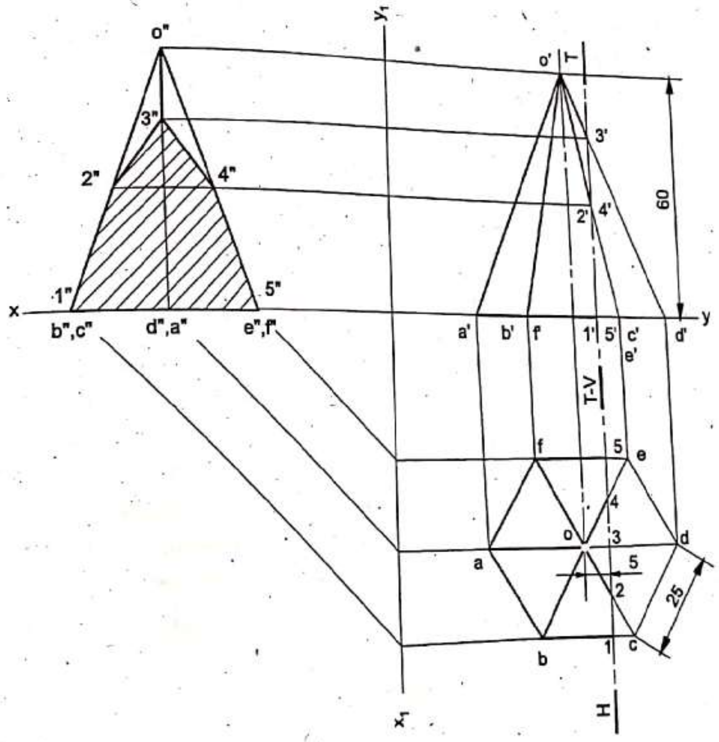


Fig. 12.18

1. Draw the three views of the pyramid, satisfying the given conditions.
2. Draw the H.T and V.T of section plane, perpendicular to xy and 5 away from the axis.
3. Locate the points of intersection between the traces and base and slant edges of the pyramid, in both the projections.

4. Project and locate the corresponding points in the left side view.
5. Join the points in the order by straight lines and obtain the section.

Problem 18 *A cone of base 40 diameter and axis 50 long, is resting on its base on H.P. It is cut by a section plane perpendicular to both H.P and V.P and 5 away from the axis. Draw the sectional side view of the cut solid.*

Construction (Fig. 12.19)

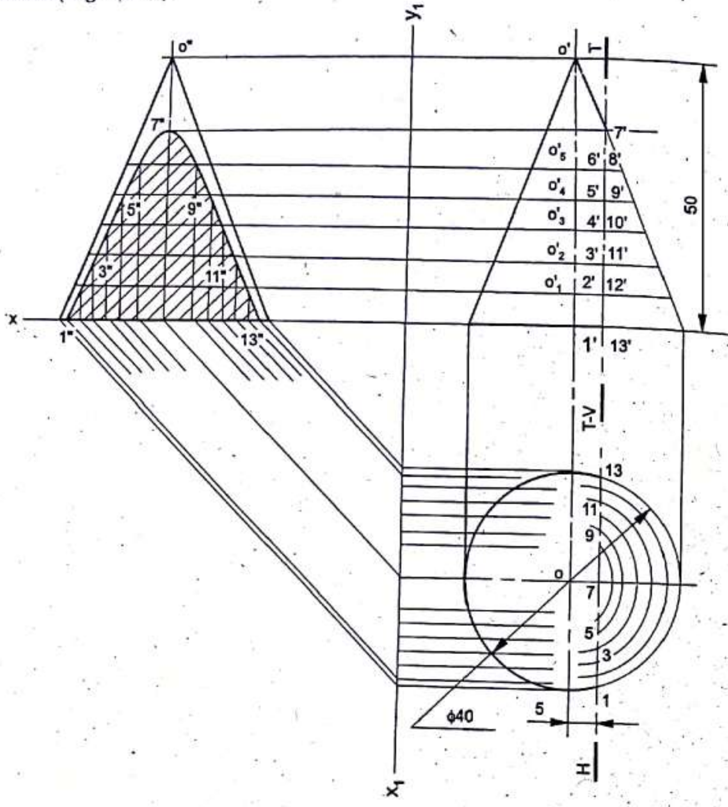


Fig. 12.19

1. Draw the three views of the cone.
2. Draw both the traces of the section plane, perpendicular to xy and 5 away from the axis.
3. Identify a number of cutting planes, perpendicular to the axis, between $1'$ and $7'$ (need not be equidistant).
4. Draw the circular sections produced in the top view by the cutting planes and locate the intersection points with the H.T of section plane.

5. Project and obtain these intersection points in the right side view.
6. Join the points by a smooth curve and complete the section by cross-hatching.

12.4 EXAMPLES

Problem 19 *A cube of 60 edge, stands vertically on H.P such that, its vertical faces are equally inclined to V.P. A section plane, perpendicular to V.P and inclined to H.P, cuts the solid in such a way that the true shape of the section is an equilateral triangle of 60 side. Draw the projections and true shape of the section and determine the inclination of the section plane with H.P.*

Construction (Fig. 12.20)

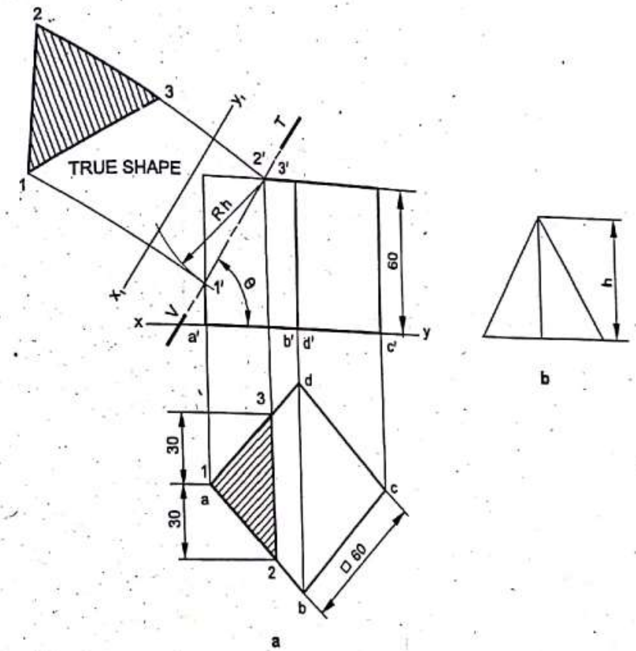


Fig. 12.20

1. Draw the projections of the cube.
2. Locate the points 2 and 3 on ab and ad in the top view such that, the line 2-3 is parallel to the diagonal bd and of length 60, the side of the equilateral triangle.
3. Through 2 (3), draw a projector meeting the top face of the cube in the front view at $2'$ $(3')$.
4. With $2'$ $(3')$ as centre and radius equal to the altitude of the equilateral triangle **h**, draw an arc; meeting the extreme vertical edge at the left at $1'$.
5. Draw the V.T of section plane passing through $1'$ and $2'$ $(3')$.

The inclination of the V.T with xy is equal to the inclination of the section plane with H.P.

6. Cross-hatch the sectioned portion in the top view and complete the sectional top view.
7. Draw the reference line x_1y_1, parallel to the V.T of section plane and obtain the true shape of the section, by projection.

Problem 20 *A cube of 50 edge, rests on one face on H.P, with its vertical faces equally inclined to V.P. It is cut by a section plane, perpendicular to V.P, producing a large rhombus. Draw the projections, true shape of the section and determine the inclination of the section plane with H.P.*

Construction (Fig. 12.21)

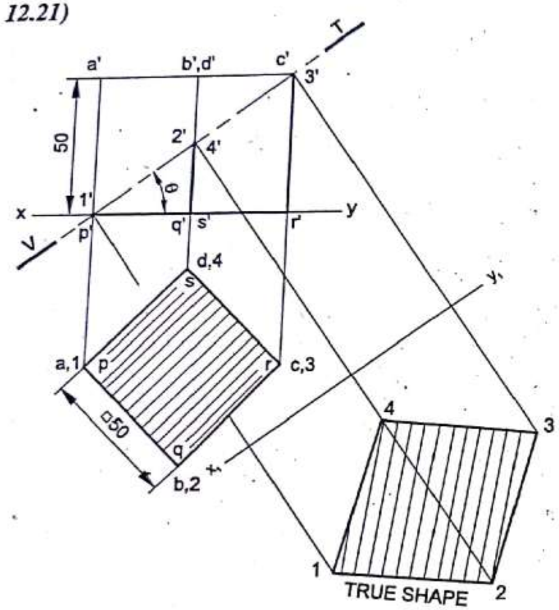


Fig. 12.21

1. Draw the projections of the cube, satisfying the given conditions.

NOTE When the V.T of a section plane is inclined to xy and passes through the extreme vertical edges of the front view, the true shape produced is a rhombus. So, to produce a largest possible rhombus, the section plane should pass through the diagonally opposite corners of the front view.

2. Draw the V.T of section plane such that, it passes through the diagonally opposite corners, p′ and c′ in the front view.
3. Locate the points of intersection 1′, 2′, 3′ and 4′ between the V.T and edges of the cube.
4. Cross-hatch the complete top view, as the entire area comes under sectioned zone.
5. Measure the angle θ which is the inclination of the section plane with H.P.
6. Draw the reference line x_1y_1, parallel to the V.T of section plane and obtain the true shape of the section, by projection.

Problem 21 *A hexagonal prism of side of base 30 and length of axis 75, is resting on a corner of its base on H.P, with the longer edge containing that corner, inclined to H.P at 30°. It is cut by a section plane parallel to H.P and passing through the mid-point of the axis. Draw the front and sectional top views of the solid.*

Construction (Fig. 12. 22)

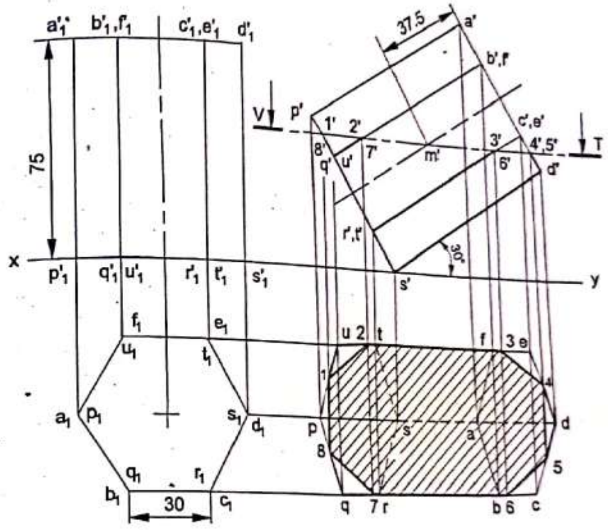


Fig. 12.22

1. Draw the projections of the prism, satisfying the given conditions.
2. Draw the V.T of section plane, parallel to xy and passing through the mid-point m′ of the axis.
3. Repeat steps 3 to 5 of Construction: Fig. 12.4 suitably and obtain the sectional top view.

Problem 22 *A pentagonal pyramid of side of base 25 and 50 height, rests on a triangular face on H.P, with its axis parallel to V.P. It is cut by a horizontal section plane, bisecting the axis. Draw the projections of the retained solid.*

Construction (Fig. 12. 23)

1. Draw the projections of the pyramid, satisfying the given conditions.
2. Repeat steps 2 to 5 of Construction: Fig. 12.4 suitably and obtain the sectional top view.

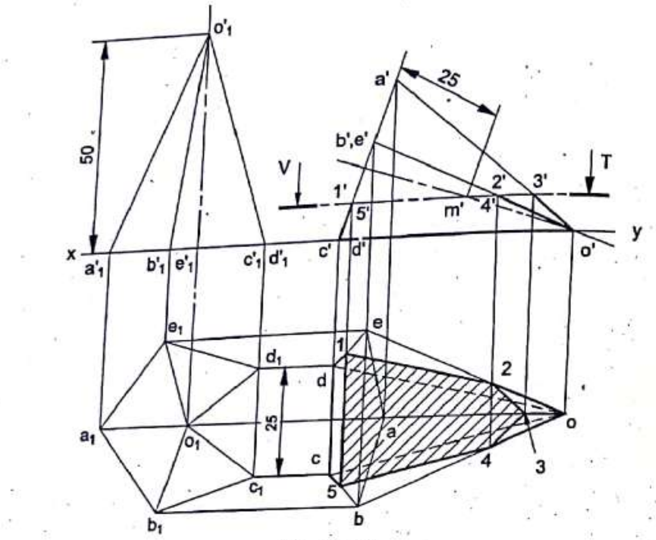


Fig. 12.23

Problem 23 *A cone of base 40 diameter and axis 55 long, lies on one of its generators on H.P, with its axis parallel to V.P. A horizontal section plane, bisects the axis of the cone. Draw the projections of the retained portion of the solid.*

Construction (Fig. 12. 24)

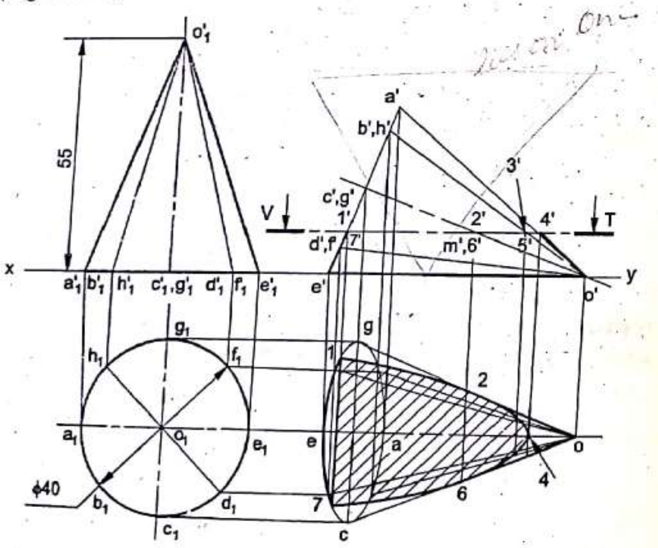


Fig. 12.24

1. Draw the projections of the cone, satisfying the given conditions.
2. Repeat steps 2 to 5 of Construction: Fig. 12.14a suitably and obtain the sectional top view.

Problem 24 *A cylinder of 50 diameter and axis 70 long, lies on H.P on one of its generators such that, the axis is inclined at 45° to V.P. A section plane parallel to V.P, passes through the farthest point of the visible base. Draw the projections of the cut solid.*

Construction (Fig. 12. 25)

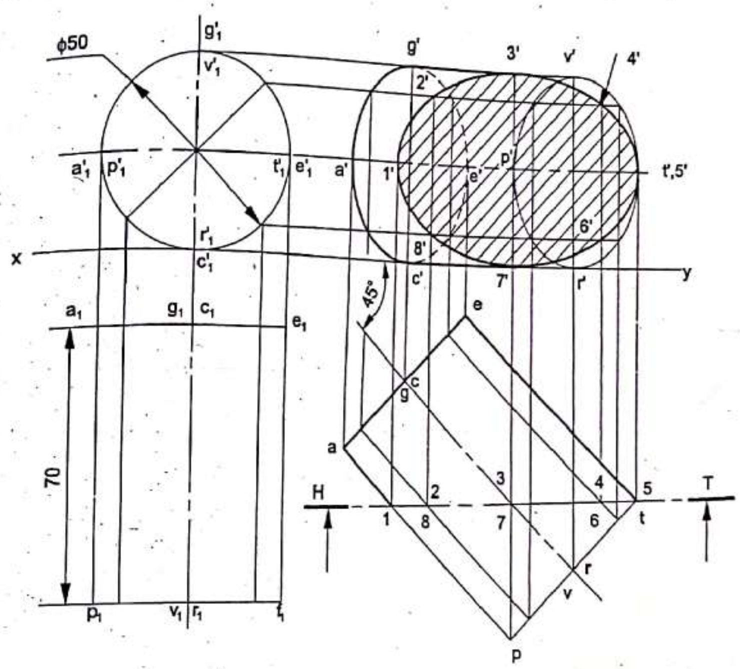


Fig. 12.25

1. Draw the projections of the cylinder, satisfying the given conditions.
2. Draw the H.T of section plane, parallel to xy and passing through the farthest point of the visible base.
3. Locate the points of intersection 1, 2, etc., between the H.T and generators.
4. Transfer the above points to the front view and obtain the sectional front view by joining them with a smooth curve.

NOTE The sectioned portion obtained in the front view is in its true shape and it is an ellipse.

Problem 25 *A cone of base 50 diameter and axis 60 long, stands with its base on H.P. A section plane parallel to V.P, cuts the solid at 12 from the axis. Draw the projections of the sectioned solid.*

Construction (Fig. 12.26)

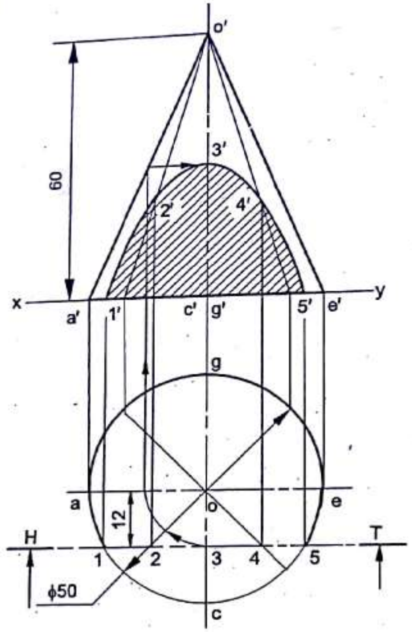


Fig. 12.26

1. Draw the projections of the cone.
2. Draw the H.T of section plane, parallel to xy and at a distance of 12 from the axis.
3. Repeat steps 3 to 5 of Construction: Fig. 12. 8 and obtain the sectional front view.

NOTE The sectioned portion obtained in the front view is in its true shape and the name of the curve is hyperbola.

Problem 26 *A square pyramid of base side 25 and height 40 rests on H.P with its base edges equally inclined to V.P. It is cut by a plane perpendicular to V.P and inclined at 30° to H.P, meeting the axis at 21 from the base. Draw the sectional top view and true shape of the section.*

Construction (Fig. 12. 27)

1. Draw the projections of the pyramid.
2. Draw the V.T of the section plane, inclined at 30° to xy and passing through a point on the axis at 21 from the base.
3. Locate the points of intersection 1′, 2′, 3′, etc., between the V.T and edges of the pyramid.
4. Project and locate these points, on the corresponding edges in the top view.

5. Join the points in the order by straight lines and complete the sectional top view, by cross-hatching the sectioned portion.
6. Consider a reference line x_1y_1, parallel to the V.T of the section plane and obtain the true shape of the section, by projection.

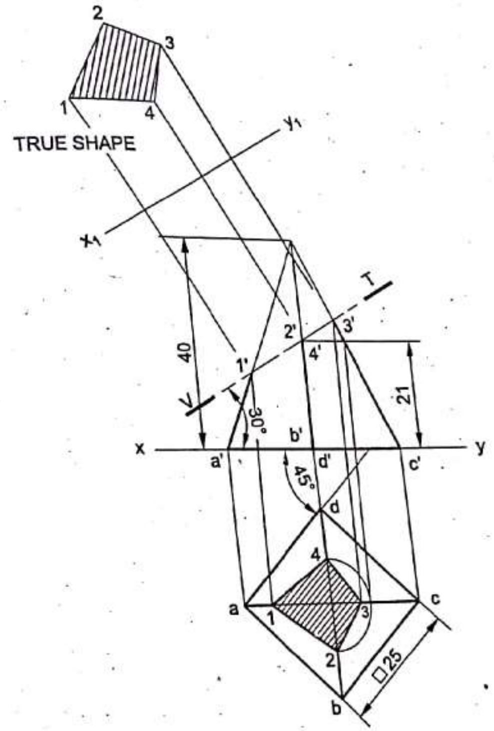


Fig. 12.27

Problem 27 *A square pyramid of base 40 side and axis 70 long, rests with its base on H.P, with all the edges of the base equally inclined to V.P. It is cut by a section plane inclined at 60° to H.P and passing through a point on the axis at 30 from the apex. Draw the three views of the cut solid.*

Construction (Fig. 12.28)

1. Draw the projections of the pyramid.
2. Draw the V.T of section plane, passing through the axis, at a distance of 30 from the apex and inclined at 60° to xy.
3. Repeat steps 4 to 6 of construction: Fig. 12.12 and obtain sectional top and side views.
4. Project the front and top views and obtain the sectional left side view.

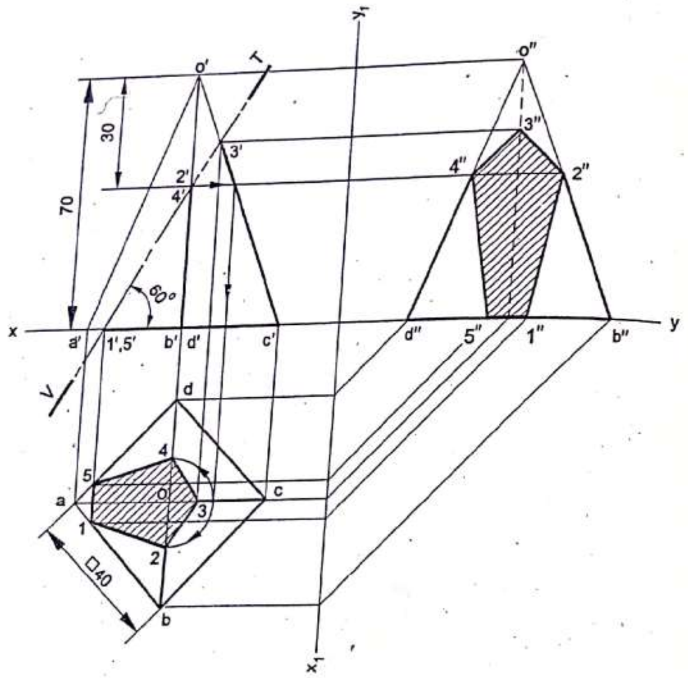


Fig. 12.28

Problem 28 *A cylinder of 50 diameter and 70 long, is lying on H.P on one of its generators such that, the axis is parallel to both H.P and V.P. A section plane inclined at 30° to H.P, passes through the mid-point of the axis. Draw the projections of the solid and show the true shape of the section.*

Construction (Fig. 12.29)

1. Draw the projections of the cylinder, along with the generators.
2. Draw the V.T of section plane, passing through the mid-point of the axis and making 30° with xy.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and bases and generators of the solid.
4. Project and locate the corresponding points 1, 2, etc., in the top view.
5. Join the points in the order and obtain the sectional top view.
6. Obtain the true shape of the section on an A.I.P, parallel to the V.T.

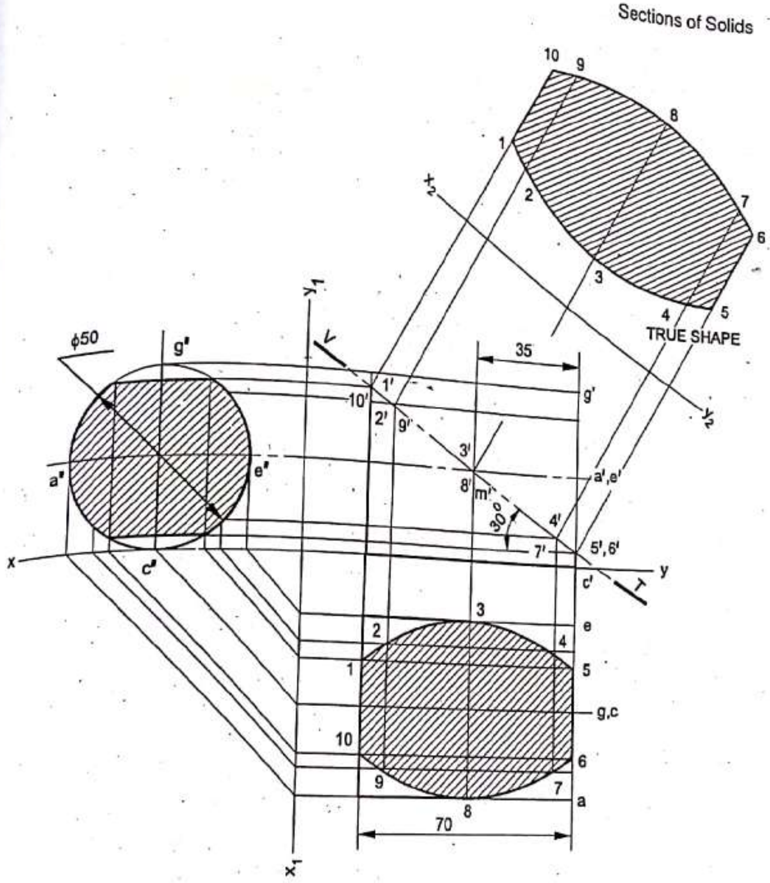


Fig. 12.29

Problem 29 *A cone of base 50 diameter and 60 height, is resting on its base on H.P. It is cut by a section plane such that, the true shape produced is a parabola of base 40. Locate the V.T of the section plane and draw the projections of the solid.*

HINT When a cone is cut by a section plane, parallel to an extreme generator, the section produced is a parabola, the size of which depends upon the position of the section plane.

Construction (Fig. 12.30)

1. Draw the projections of the cone.
2. In the top view, locate the chord 1-11 of length 40 and perpendicular to xy.
3. Through 1 (11), draw a projector, meeting xy at 1′ (11′).

4. Through 1′, draw the V.T. parallel to the extreme generator.

5. Repeat steps 3 to 6 of Construction: Fig. 12.14a suitably and obtain sectional top view and the true shape of the section.

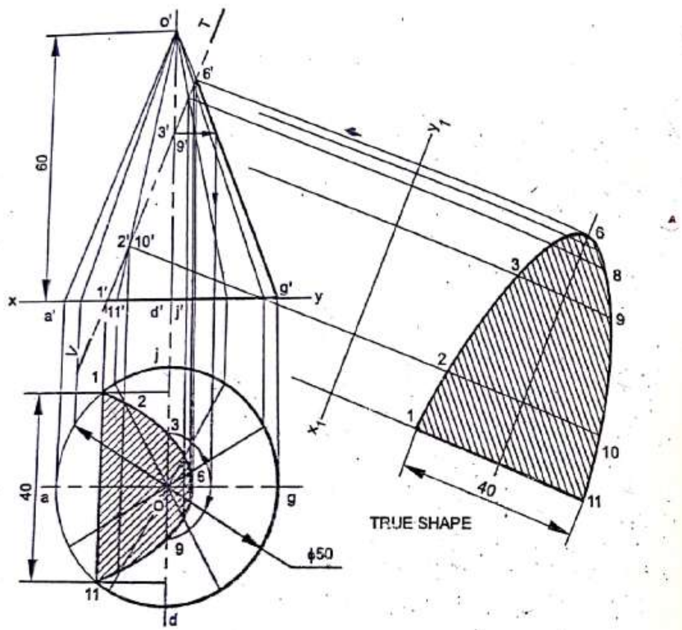


Fig. 12.30

Problem 30 *A hexagonal prism with side of base 30 and axis 70 long, rests on a corner of its base on H.P, with the axis inclined to H.P at 30° and parallel to V.P. It is cut by a vertical section plane, inclined at 30° to V.P and passing through the axis at 20 from one end. Draw the projections of the solid.*

Construction (Fig. 12. 31)

1. Draw the projections of the prism.

2. Draw the H.T of section plane, passing through a point on the axis at 20 from one end and inclined at 30° to xy.

3. Repeat steps 3 to 5 of Construction: Fig. 12.15 suitably and obtain the sectional front view.

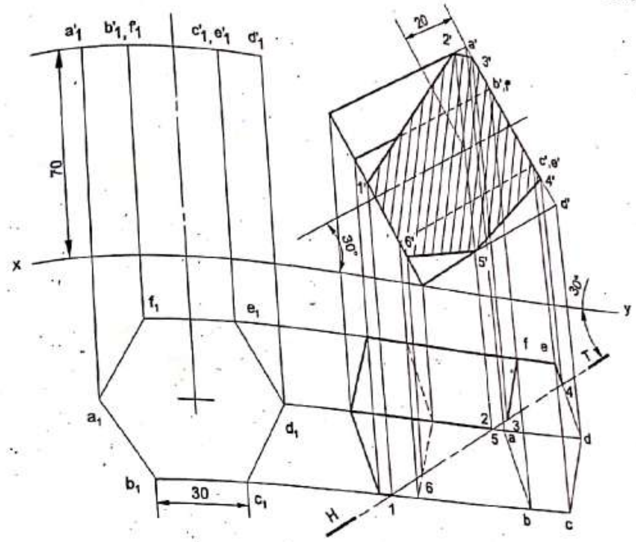


Fig. 12.31

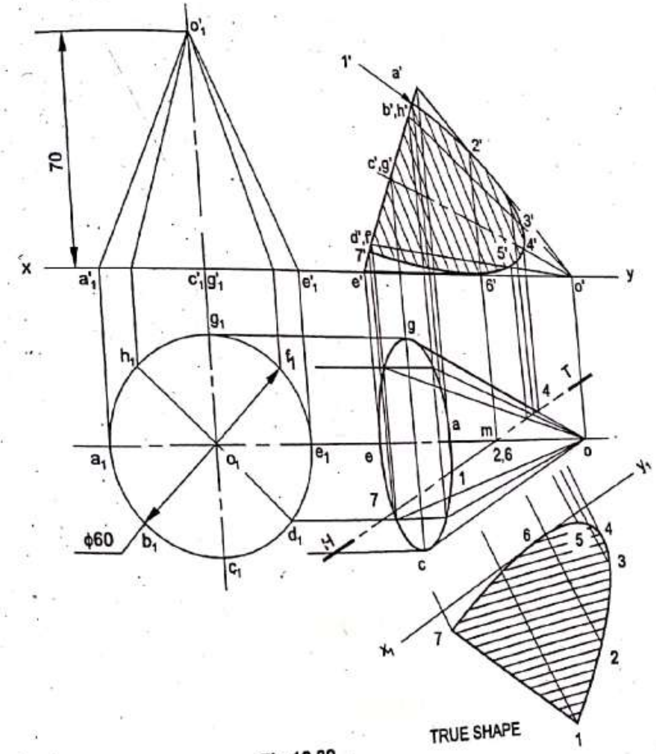


Fig.12.32

Problem 31 *A cone of base 60 diameter and axis 70 long, is lying on one of its generators on H.P. with its axis parallel to V.P. A vertical section plane, parallel to the extreme generator, bisects the axis. Draw the projections of the cut solid and show the true shape of the section.*

Construction (Fig. 12. 32)

1. Draw the projections of the cone along with the generators.
2. Draw the H.T of section plane, passing through the mid-point of the axis and parallel to the extreme generator in the top view such that, only a minor portion of the solid is cut and removed.
3. Repeat steps 3 and 4 of Construction: Fig. 12.17 suitably and obtain the sectional front view and the true shape of the section.

Problem 32 *A sphere of 60 diameter rests on H.P. It is cut by a section plane perpendicular to V.P., inclined at 45° to H.P. and at a distance 15 from its centre. Draw the sectional top view and true shape of the section.*

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Construction (Fig. 12. 33)

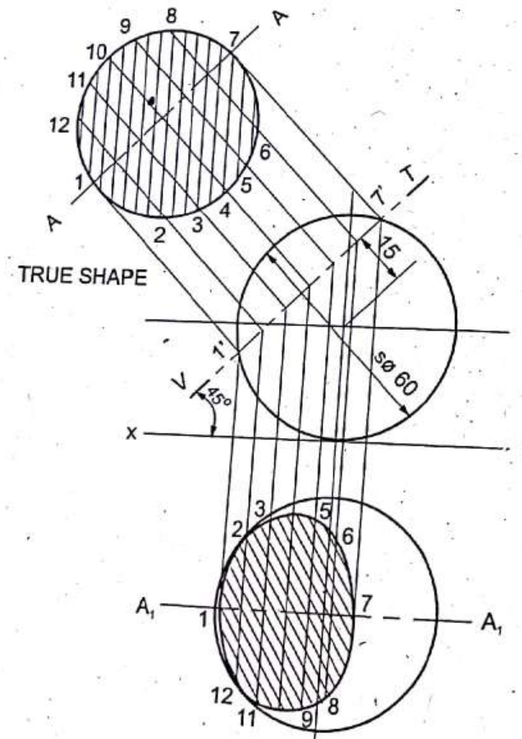


Fig. 12.33

1. Draw the projections of the sphere.
2. Draw the V.T of section plane, inclined at 45° to xy and at 15 from the centre.
3. Draw the true shape of the section, which is a circle with 1′-7′ as diameter.
4. Transfer the points, which are symmetrically placed about A-A, on the true shape, to the top view, symmetrically to A_1-A_1.
5. Join the points in the order by a smooth curve and complete the sectional top view.

Problem 33 *A hollow sphere of 40 internal diameter and 60 external diameter, is cut by a section plane, inclined at 45° to V.P and at a distance of 10 from its centre. Draw the projections of the sphere.*

Construction (Fig. 12. 34)

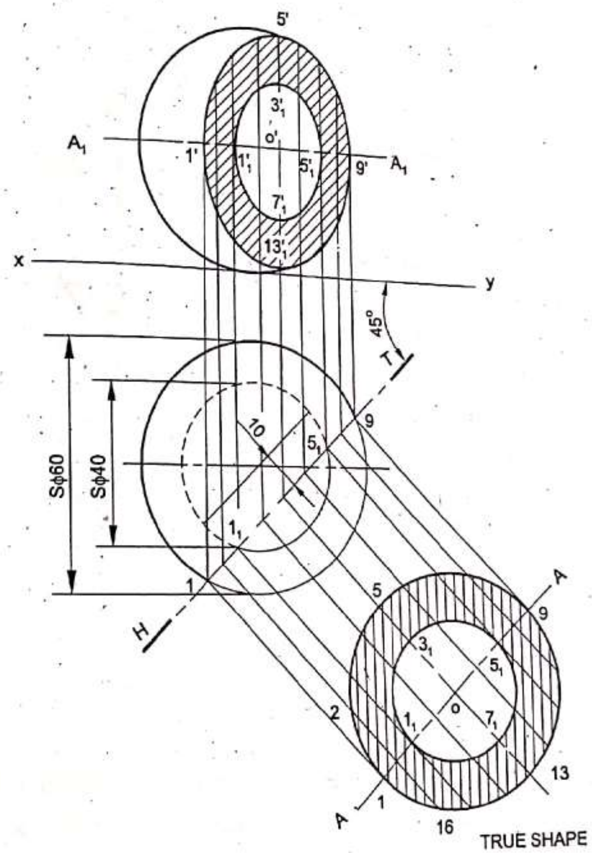


Fig. 12.34

1. Draw the projections of the hollow sphere.
2. Draw the H.T of section plane, inclined at 45° to xy and at 10 from the centre.

3. Draw the true shape of the section, consisting of two concentric circles with 1_1-5_1 and 1-9_1 as diameters.
4. Transfer the points, which are symmetrically placed about A-A on the true shape, to the front view, which are symmetric to A_1-A_1.
5. Join the points in the order by smooth curves and complete the sectional front view.

Problem 34 *A cube of 60 long edges, is resting on one of its edges on H.P such that, a face containing that edge is inclined at 45° to H.P. It is cut by a section plane, perpendicular to H.P such that, the true shape of the section produced is a regular hexagon. Determine the inclination of the cutting plane with V.P. and draw the projections of the solid and show the true shape of the section.*

Construction (Fig. 12.35)

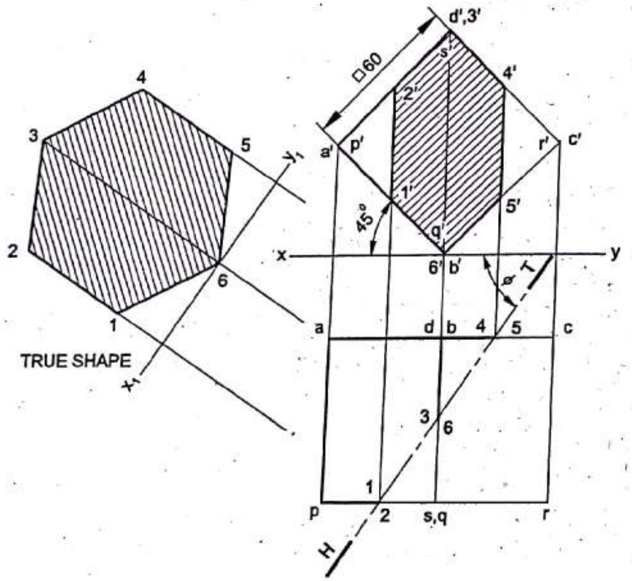


Fig. 12.35

1. Draw the projections of the cube.

   HINT (i) A section plane perpendicular to H.P and inclined to V.P, is to be assumed passing through the mid-point of the axis and both the bases, to produce a hexagonal section.

   (ii) The distance between the corners of a hexagon is twice the side of the hexagon.
2. Locate the H.T of section plane such that, it passes through the mid-points of bc and pq in the top view. The angle ϕ is the inclination of the section plane with V.P.
3. Project the points of intersection between the H.T and edges of the cube and obtain the sectional front view.

4. Consider a reference line x_1y_1, parallel to the H.T of section plane and obtain the true shape of the section, by projection.

Problem 35 *A square prism of base 50 side and axis 100 long, stands with its base on H.P such that, all the faces are equally inclined to V.P. It is cut by a section plane, perpendicular to V.P such that, the true shape of the section is a rhombus of longer diagonal 90. Find the inclination of the section plane with H.P. Draw the projections of the solid.*

Construction (Fig. 12.36)

1. Draw the projections of the prism.

   HINT For the true shape of the section to be a rhombus, the section plane should pass through the extreme lateral edges in the front view
2. Locate the V.T of section plane such that, it passes through the extreme lateral edges of the prism in the front view and the length of the intercept is equal to 90. The angle θ is the inclination of the section plane with H.P.
3. Consider a reference line x_1y_1, parallel to the V.T of section plane and obtain the true shape of the section, by projection.

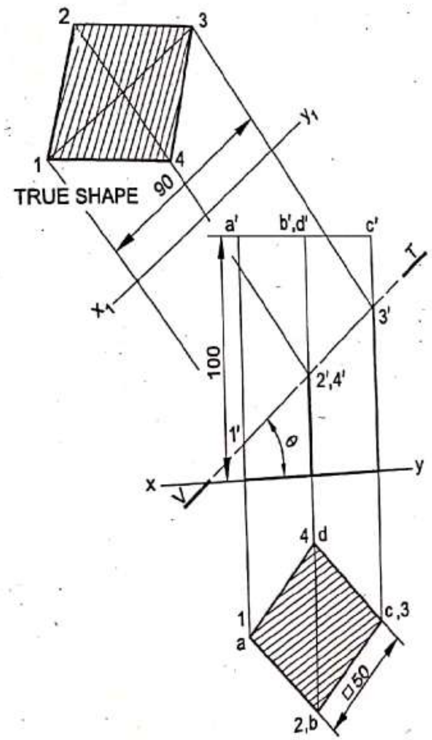


Fig. 12.36

CHAPTER - 13

DEVELOPMENT OF SURFACES

13.1 INTRODUCTION

The complete surface of an object, laid-out on a plane is called the development of the surface or flat pattern of the object. The development of geometrical surfaces is important in the fabrication of not only small, simple shapes made of thin sheet metal, but also, sophisticated pieces of hardware such as space capsules. In actual practice, bend allowances have to be made in the layout of the pattern to ensure proper fabrication. These allowances depend upon the degree of bend, thickness of the metal and type of metal being used. However, these allowances are not considered in presenting the subject here, as it falls outside the scope of the book.

In making the development of a geometrical surface, the opening should be determined first. Every line used in making the development must represent the true length of that line on the actual surface.

Developments are made possible, with the application of basic graphic and geometric principles, in conjunction with mathematics. Since, different shapes must be joined together in many instances, principles of intersections are closely related to developments. If a development problem is resolved into basic geometric elements, the solution will be simpler.

13.2 CLASSIFICATION OF OBJECTS

In general, objects are bounded by geometric surfaces. These may be classified as:

1. Solids bounded by plane surfaces - cube, prism, pyramid, etc.
2. Solids bounded by single curved surfaces - cylinder, cone, etc.
3. Solids bounded by double curved surfaces - sphere, paraboloid, etc.
4. Solids bounded by warped surfaces - ellipsoid, hyperboloid, etc.

The first two types of solids can be developed accurately, whereas the last two can only be developed approximately, by dividing them into a number of parts.

13.3 METHODS OF DEVELOPMENT

Solids bounded by plane surfaces and single curved surfaces can be developed by: (i) Parallel line development method, based on stretch-out line principle; used for prisms and cylinders, (ii) radial line development method, making use of true length of slant edge or generator; used for pyramids and cones, (iii) triangulation method normally used for developing the transition pieces - connecting ducts, pipes, openings and similar objects with various sizes and shapes and (iv) approximate method, used to develop the objects with double curved or warped surfaces such as sphere, paraboloid, ellipsoid, hyperboloid, etc.

Only the lateral surfaces are generally developed and shown as presented here, omitting the bases or ends of solids.

13.3.1 Parallel line development

The surfaces of right prisms, cylinders and also oblique prisms and cylinders may be developed by this method.

Problem 1 *A square prism of side of base 40 and axis 80 long, is resting on its base on H.P such that, a rectangular face of it is parallel to V.P. Draw the development of the prism.*

Construction (Fig. 13.1)

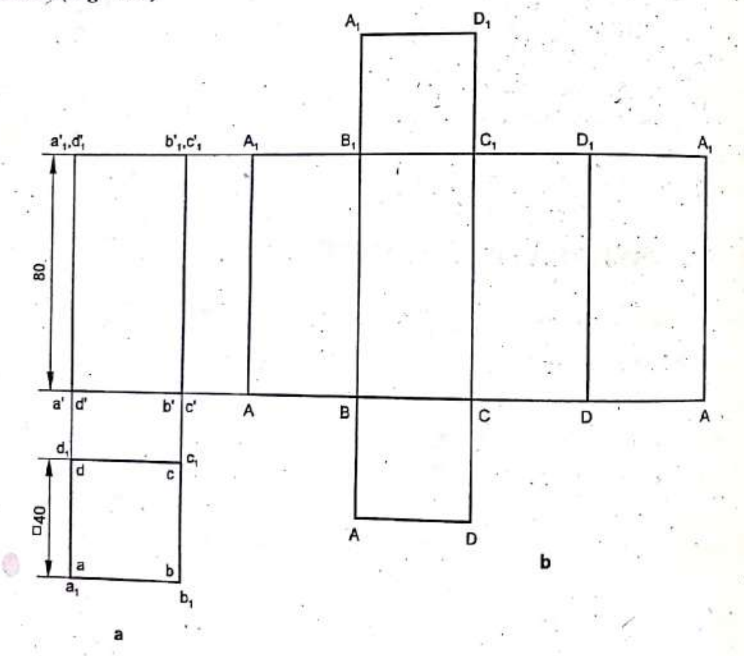


Fig.13.1

1. Draw the projections of the prism.
2. Draw the stretch-out line AA and mark - off the sides of the base along this line in succession, i.e., AB, BC, CD and DA.
3. Erect perpendiculars through these points and mark the edges AA_1, BB_1, etc.
4. Add the bases ABCD and $A_1B_1C_1D_1$ suitably.

NOTE (i) Stretch-out line is drawn in-line with the base in the front view, to complete the development quickly.

(ii) All the lines on the development should represent the true lengths.

Problem 2 *A hexagonal prism of side of base 30 and axis 75 long, is resting on its base on H.P such that, a rectangular face is parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at 30° to H.P. The section plane is passing through the top end of an extreme lateral edge of the prism. Draw the development of the lateral surface of the cut prism.*

Construction (Fig. 13.2)

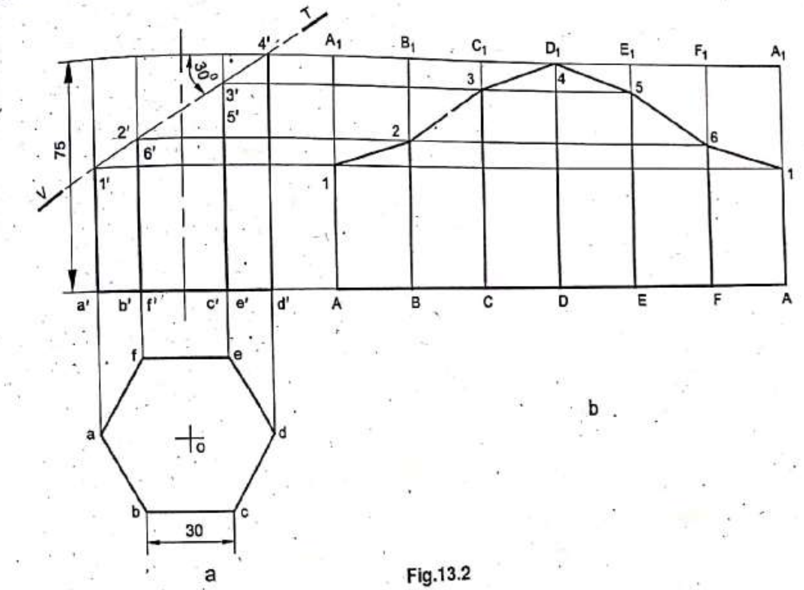


Fig.13.2

1. Draw the projections of the prism.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development $AA_1 - A_1A$ of the complete prism, following the stretch-out line principle.
4. Locate the points of intersection 1′, 2′, etc., between the V.T and edges of the prism.

5. Draw horizontal lines through $1'$, $2'$, etc., and obtain 1, 2, etc., on the corresponding edges in the development.
6. Join the points 1, 2, etc., by straight lines and darken the sides, corresponding to the retained portion of the solid.

NOTE It is the usual practice to cut open (for the development) the surface of the solid at the shortest edge/length.

Problem 3 *A cube of 50 edge, is resting on a face on H.P such that, a vertical face is inclined at 30° to V.P. It is cut by a section plane perpendicular to V.P and inclined to H.P at 30° and passing through a point at 12 from the top end of the axis. Develop the lateral surface of the lower portion of the cube.*

Construction (Fig. 13.3)

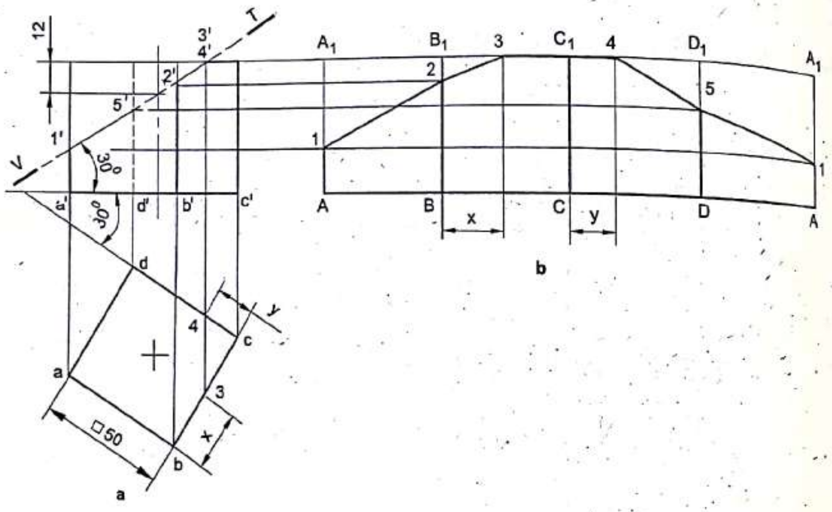


Fig.13.3

1. Draw the projections of the cube.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development $AA_1 - A_1A$ of the complete cube, following the stretch-out line principle.
4. Repeat steps 4 to 6 of Construction: Fig. 13.2 and obtain the development of the cut solid.

NOTE 1. The points $3'$ and $4'$ are on the top surface of the cube. To locate the corresponding points in the development:

(i) Draw a projector through $3'$ ($4'$), meeting bc at 3 and cd at 4 in the top view.

(ii) Mark the points 3 and 4 in the development such that, $B_13 = b3 = x$ and $C_14 = c4 = y$.

2. The sectioned portion in the top view is not cross-hatched as it is made use of only for locating certain points in the development.

Problem 4 *A cylinder of diameter of base 40 and axis 55 long, is resting on its base on H.P. It is cut by a section plane, perpendicular to V.P and inclined at 45° to H.P. The section plane is passing through the top end of an extreme generator of the cylinder. Draw the development of the lateral surface of the cut cylinder.*

Construction (Fig. 13.4)

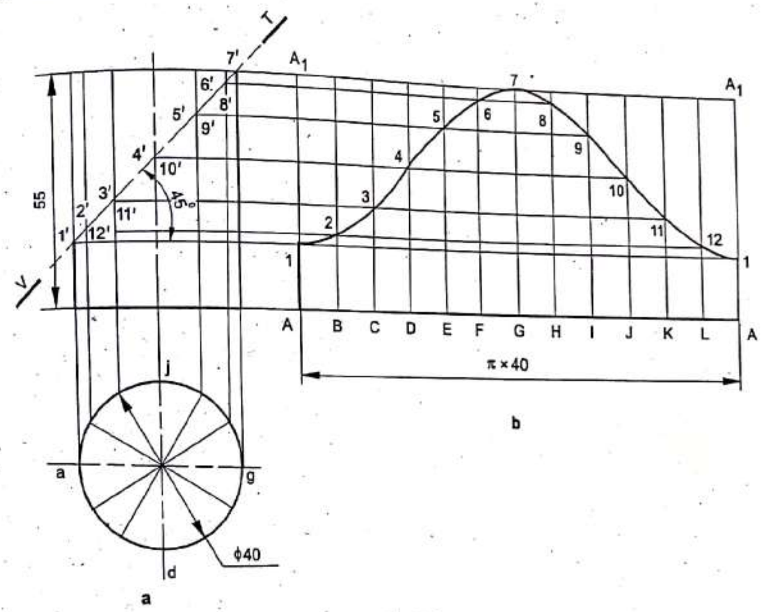


Fig.13.4

1. Draw the projections of the cylinder.
2. Divide the circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the V.T of section plane, satisfying the given conditions.
4. Draw the stretch-out line AA, equal to the circumference of the base of the cylinder.
5. Divide the stretch-out line AA, into the same number of equal parts as that of the base circle/set-off chord lengths by a divider and locate the generators through the division points B, C, D, etc.
6. Locate the points of intersection $1'$, $2'$, etc., between the V.T and generators.

7. Transfer these intersection points to the corresponding generators in the development; by projection.
8. Join the points 1, 2, etc., by a smooth curve and obtain the development.

NOTE (i) $AA_1 - A_1A$ represents the development of the complete cylinder.

(ii) Only one half of the development may be shown when a solid is symmetric about an axis.

(iii) The generators should not be drawn thick, since they do not represent the folding edges.

Problem 5 *Figure 13.5a shows the projections of a cut cylinder. Draw the development of the lateral surface of the cut cylinder.*

Construction (Fig. 13.5)

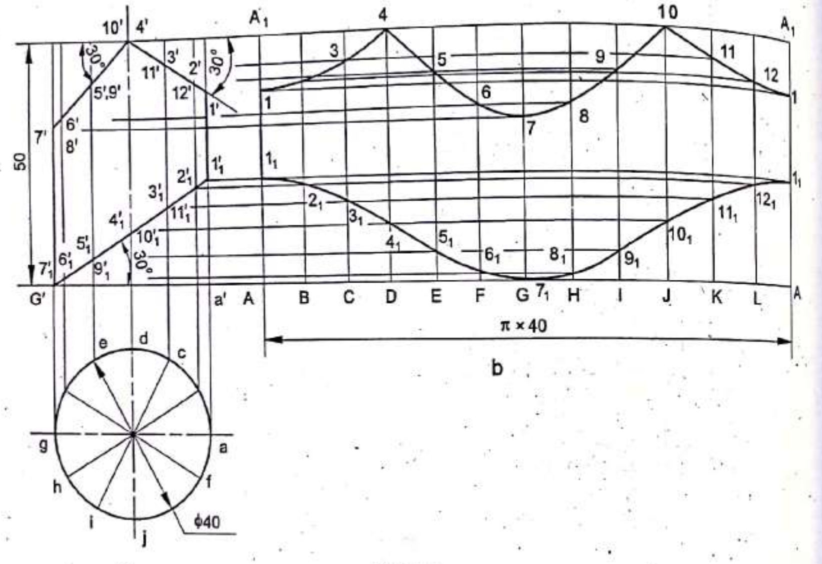


Fig.13.5

1. Divide the base circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
2. Locate the points of intersection between the cut edges and generators.
3. Draw the stretch-out line AA, equal to the circumference of the base and complete the development of the complete cylinder.
4. Locate the generators in the development.
5. Repeat steps 7 and 8 of Construction: Fig. 13.4 suitably and obtain the development of the cut cylinder.

13.3.2 Radial line development

The lateral surfaces of right pyramids and cones may be developed by this method.

Problem 6 *A pentagonal pyramid of side of base 30 and axis 60 long, is resting on its base on H.P. with an edge of the base parallel to V.P. Draw the development of the lateral surface of the pyramid.*

HINT (i) The development of a pyramid consists of a number of equal isosceles triangles. The base of the triangle is equal to the edge of the base and the sides equal to the slant height of the pyramid respectively.

(ii) The true length of the slant edge may be measured from the front view, by making its top view parallel to xy.

Construction (Fig. 13.6)

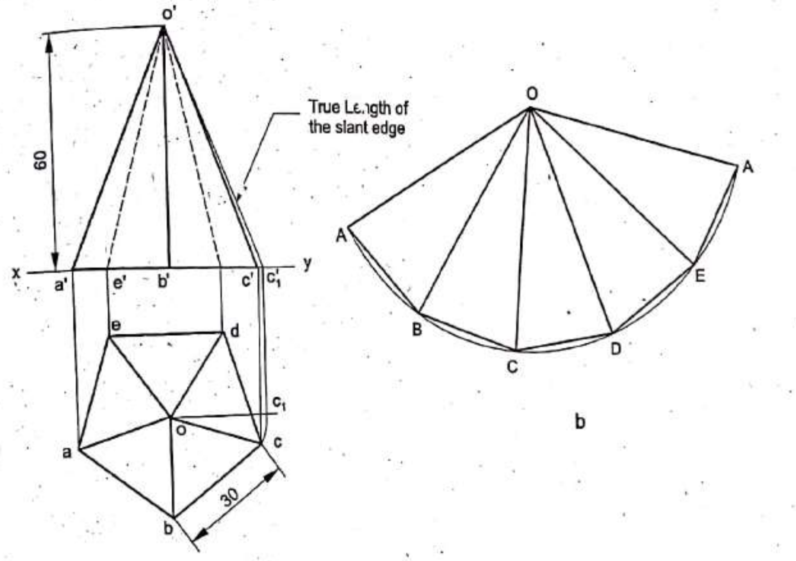


Fig.13.6

1. Draw the projections of the pentagonal pyramid.
2. Determine the true length of the slant edge OC of the pyramid as shown.
3. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
4. With radius equal to the side of the base, step-off five divisions on the above arc.
5. Join the above division points in the order and also with the centre of the arc.

The figure thus formed is the development of the lateral surface of the pyramid.

5

Problem 7 *A square pyramid, with side of base 30 and axis 50 long, is resting on its base on H.P, with an edge of the base parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at 45° to H.P. The section plane is passing through the mid-point of the axis. Draw the development of the surface of the cut pyramid.*

Construction (Fig. 13.7)

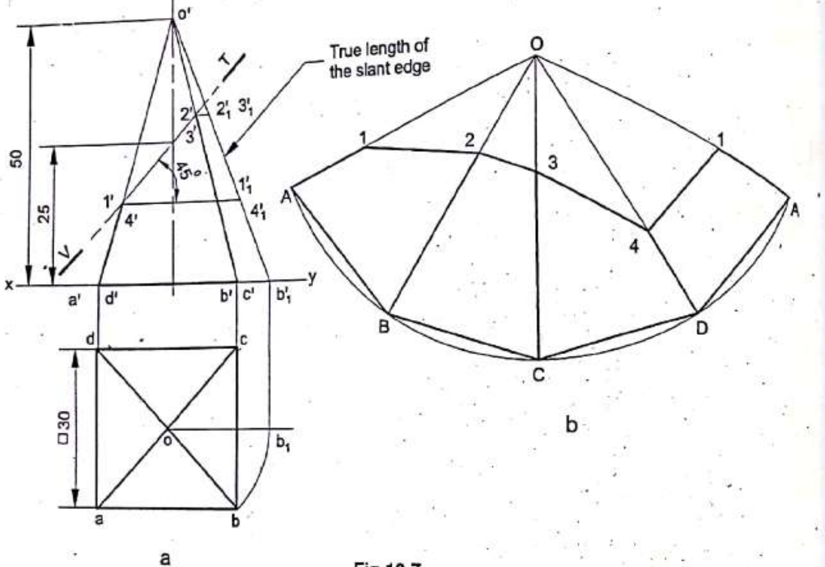


Fig.13.7

1. Draw the projections of the given pyramid.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Locate the points of intersection 1′, 2′, etc., between the V.T and slant edges of the pyramid.
4. Determine the true length of the slant edge of the pyramid.
5. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of the circle.
6. Follow steps 4 and 5 of Construction: Fig.13.6 and obtain the full development of the pyramid.
7. Obtain the true lengths of the slant edges of the cut pyramid, by projecting on to the true length line.
8. Transfer the above true lengths to the corresponding edges in the development.
9. Join the points 1, 2, etc., by straight lines and obtain the development of the cut pyramid.

Problem 8 *A cone of diameter of base 50 and axis 60 long, is resting on its base on H.P. Draw the projections of the cone and show on it, the shortest path traced by a point, starting from a point on the circumference of the base of the cone, moving around it and reaching the same point.*

HINT The solution to this problem makes use of development of lateral surface of the cone. The development of the lateral surface of the cone is a sector of a circle, the radius and length of the arc of which are equal to the slant height and circumference of the base circle of the cone respectively.

Construction (Fig. 13.8)

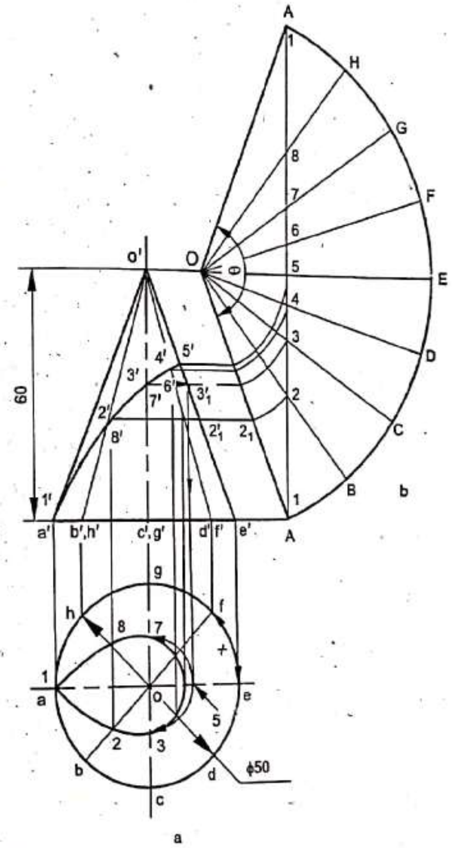


Fig.13.8

1. Draw the projections of the cone.
2. Divide the circle (top view) into, say 8 equal parts and locate the corresponding generators in the front view.
3. With any point O as centre and radius equal to the true length of the slant height, draw a sector of a circle, representing the development of the cone. The length of the arc should be equal to the circumference of the base circle.

This may be determined by two ways.

The angle subtended by the arc at O is given by,

$$\theta = 360 \times \frac{\text{radius of the base circle}}{\text{slant length of the cone}}$$

Divide the arc into 8 equal parts and draw the corresponding generators on it.

OR

Along the arc, starting from any point, say A, step-off with a divider, 8 equal divisions, each equal to the arc length of one division (x) and locate the corresponding generators on it.

4. Join A - A on the development, representing the shortest path and intersecting the generators at 1, 2, etc.
5. Transfer the above points on to both the projections.

   To transfer the point, say 2 on the development to the front and top views

   (i) Transfer the true length O2 to the extreme generator o′e′, in the front view.
   (ii) Transfer the above point to the generator o′b′, by drawing a horizontal line.
   (iii) Locate the top view of the above point by projection, on the corresponding generator.
6. Join the points 1′, 2′, etc., and 1, 2, etc., by smooth curves, representing the paths traced by the point in the front and top views respectively.

6

Problem 9 *A cone of base 50 diameter and axis 60 long, is resting on its base on H.P. It is cut by a section plane perpendicular to V.P and parallel to an extreme generator and passing through a point on the axis at a distance of 20 from the apex. Draw the development of the retained solid.*

Construction (Fig. 13.9)

1. Draw the projections of the cone.
2. Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in the front view.
3. Draw the development of the complete cone, following Construction: Fig. 13. 8.
4. Draw the V.T of section plane, satisfying the given conditions.
5. Locate the points of intersection between the V.T and generators and base of the cone.
6. Determine the true lengths of o′2′, o′3′, etc., by drawing horizontal lines to the extreme generator.
7. Transfer these true lengths to the development.
8. Join the points 1, 2, etc., by a smooth curve and obtain the required development.

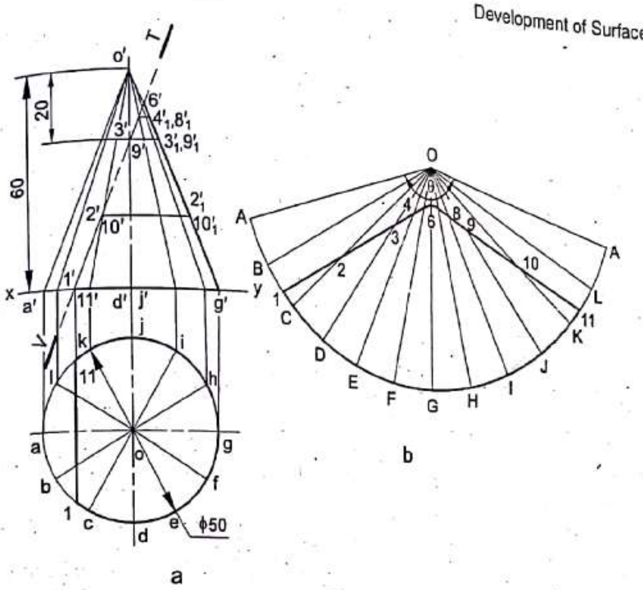


Fig. 13.9

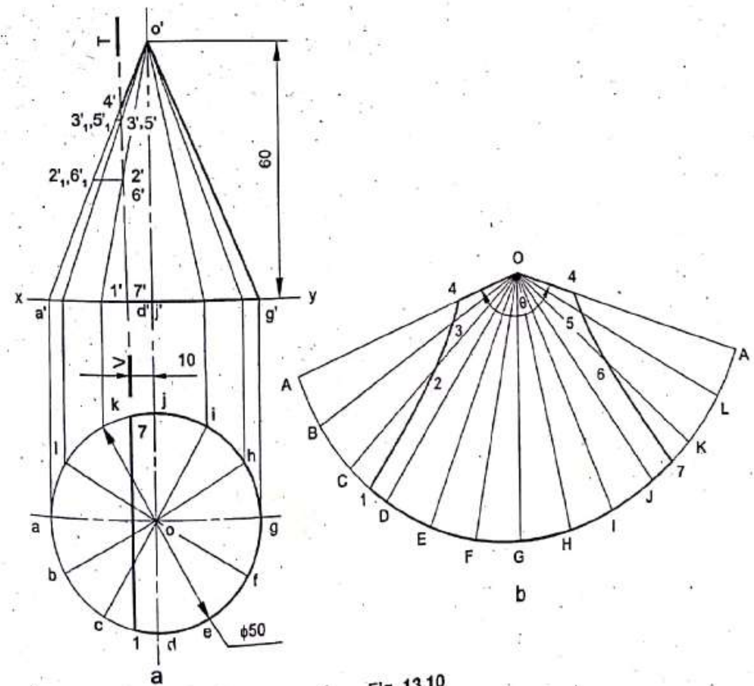


Fig. 13.10

Problem 10 *A cone of base 50 diameter and axis 60 long, is resting on its base on H.P. A section plane perpendicular to H.P. and V.P. cuts the cone at a distance of 10 from the axis. Draw the development of the cut solid.*

Construction (Fig. 13. 10)

Draw the development, following the method of Construction: Fig. 13.9.

13.4 DEVELOPMENT OF SPHERICAL SURFACES

A sphere has a double curved surface and hence, it cannot be developed as a single piece. However, a sphere may be developed approximately by dividing its surface into a number of segments. Developing the surface of the sphere by dividing it into a number of cylindrical segments, is called "poly-cylindrical or Gore or Lune method". A Gore or Lune is the portion between two planes, which contain the axis of the sphere. Developing the surface of a sphere by dividing it into a number of conical segments is called "Poly-conic or Zone" method. A zone is a portion of the sphere, enclosed between two planes perpendicular to the axis.

Problem 11 *Draw the development of a sphere of 50 diameter.*

Construction (Fig. 13. 11) Gore (Lune) method

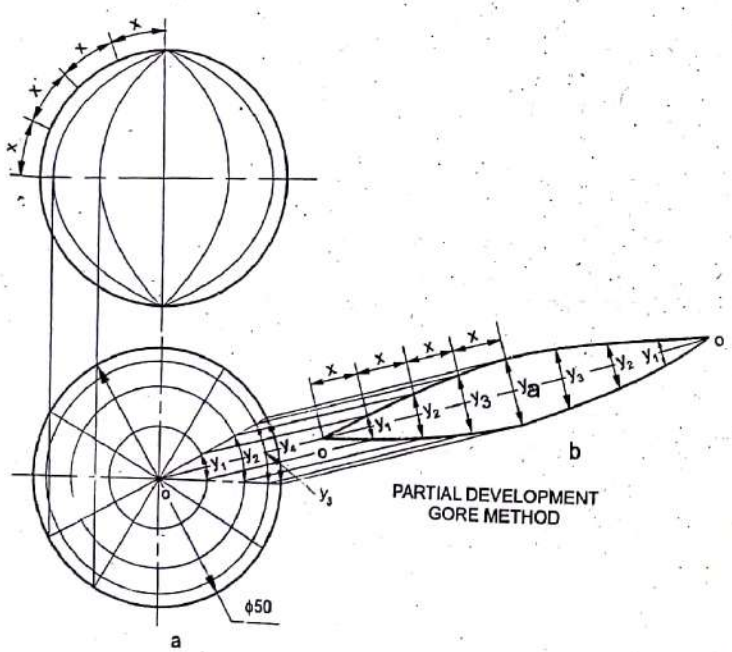


Fig. 13.11

1. Draw the projections of the sphere.
2. Draw six equally spaced imaginary vertical cutting planes passing through the pole (centre) in the top view.
3. Draw a series of parallel lines on the front view, so that they establish equal arcs x along the circumference. These lines divide the sphere into a number of slices, say 8 as shown.
4. Project the true size of the Gore, from the top view, keeping its length equal to 8x.

NOTE (i) Drawing one section (Gore or Lune) is sufficient, as this may be used as a pattern for the remaining parts of the surface.

(ii) In the present case, the sphere consists of 12 such parts when fully developed.

Construction (Fig. 13. 12) Zone method

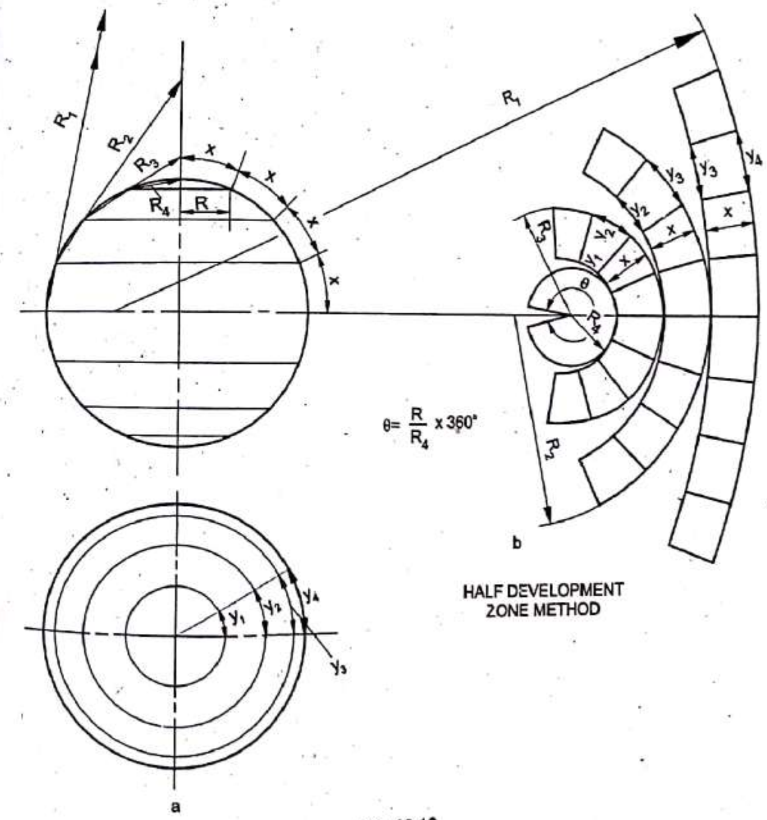


Fig. 13.12

1. Draw the projections of the sphere.
2. Draw a series of parallel lines on the front view, so that they establish arcs x along the circumference. These lines divide the sphere into a number of slices, say 8 as shown.

NOTE The slices are assumed to represent a series of truncated cones for which, one parallel serves as a base of the cone and the other as the truncated top.

3. Determine the slant lengths of the cones, corresponding to the truncated portions, i.e., R_1, R_2, R_3 and R_4.
4. Draw the development of the truncated cones, with radii equal to the slant lengths, the width being x. Figure 13.12 shows the development of one-half of the sphere.

13.5 EXAMPLES

Problem 12 *Figure13.13a shows the projections of a hexagonal prism, cut by two section planes as indicated. Draw the development of the retained portion of the prism.*

Construction (Fig. 13.13b)

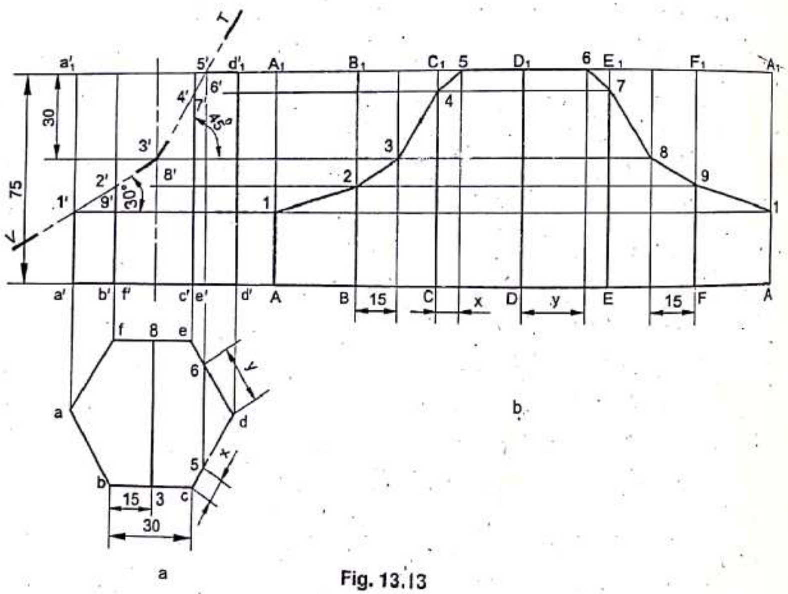


Fig. 13.13

1. Draw the development AA_1-A_1A of the complete prism, following the stretch-out line principle.
2. Locate the points of intersection 1′, 2′, etc., between the V.T of cutting plane and edges of the prism.

3. Transfer the intersection points to the development by horizontal projectors, except 5′ and 6′, to the corresponding edges in the development.
4. Locate the points 5 and 6 such that $C_1\,5 = c5 = x$ and $D_1\,6 = d6 = y$.
5. Join these points by straight lines and obtain the development of the cut prism, by darkening the edges.

Problem 13 *A cube of 50 edge, stands on one of its faces on H.P. with the vertical faces equally inclined to V.P. A hole of 35 diameter is drilled centrally through the cube such that, the axis of the hole is perpendicular to V.P. Draw the development of the cube.*

Construction (Fig. 13.14)

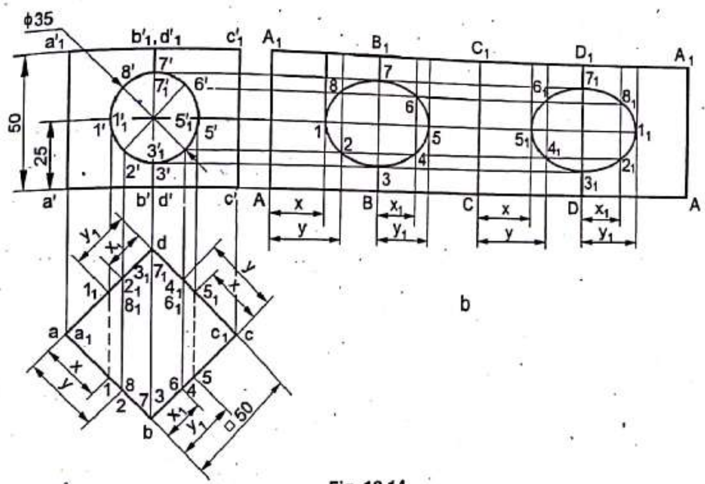


Fig. 13.14

a

1. Draw the projections of the cube.
2. Draw the development AA_1 - A_1A of the cube, following the stretch-out line principle.
3. Divide the circle in the front view, into equal number of parts, say 8 and obtain the corresponding generators in the top view.
4. Locate the points of intersection between the above generators and edges of the cube in the top view.
5. Draw horizontal projectors from $1'$ $(1_1')$, $2'$ $(2_1')$, etc., and locate the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., on the corresponding generators in the development.
6. Join the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., by smooth curves; representing the two openings of the hole in the development. Darken the edges corresponding to the retained portion of the cube.

Problem 14 *Figure13.15a shows the projections of a pentagonal prism, with a partial hole as indicated. Draw the development of the prism.*

Construction (Fig. 13.15b)

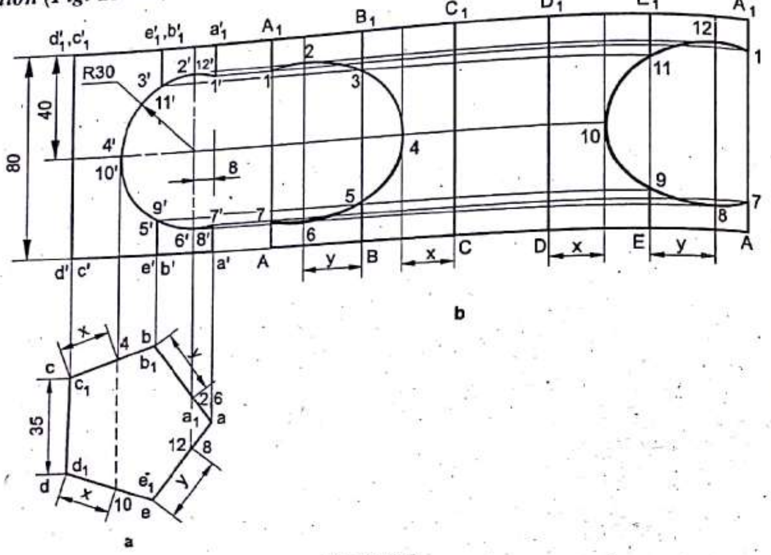


Fig. 13.15

1. Draw the development AA_1 - A_1A of the complete prism, following the stretch-out line principle.
2. Locate the points of intersection between the hole and edges of the prism.
3. Transfer the above points on to the development, by projection.
4. To get a good curve, consider some more generators, passing through the critical points of the hole, such as 2′ (12′), 4′ (10′) and 6′ (8′).
5. Locate the points 2 (12), 4 (10) and 6 (8) on the development, measuring the distances from the edges in the top view.

A smooth curve through the points 1, 2, 3, etc., is the shape of the hole in the development.

Problem 15 *A hexagonal prism of 20 side of base and 50 height, rests on a base on H.P. with a vertical face parallel to V.P. A circular hole of 35 diameter, is drilled through the prism such that, the axis of the hole bisects the axis of the prism and is perpendicular to V.P. Draw the development of the prism.*

Construction (Fig.13.16)

1. Draw the projections of the prism with the hole.
2. Draw the development AA_1-A_1A of the complete prism, following the stretch-out line principle.

3. Divide the circle in the front view into a number of parts such that, certain points lie on the longer-edges of the prism. Also, locate the transition points 1′, 5′ and $1_1'$, $5_1'$, and obtain the corresponding points in the top view.
4. Draw horizontal lines from 1′ ($1_1'$), 2′ ($2_1'$), etc., and locate the points 1,2,3, etc., and 1_1,2_1,3_1, etc., on the corresponding edges (lines) in the development.
5. Join the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., by smooth curves, representing the two openings of the hole in the development.
6. Darken the edges corresponding to the retained portion of the prism and complete the development.

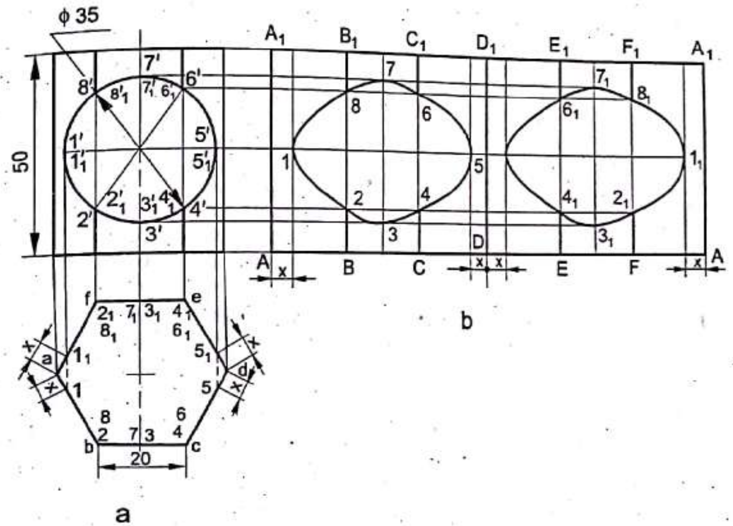


Fig. 13.16

Problem 16 *Draw the development of the lateral surface of the cut cylinder, shown in Fig. 13.17a.*

Construction (Fig. 13.17b)

Follow the principle of Construction: Fig.13.9 suitably and obtain the development as shown in Fig.13.17b.

Problem 17 *Draw the development of the lateral surface of the retained portion of the cone, shown in Fig. 13.18a.*

Construction (Fig. 13.18b)

1. Locate a number of generators in the top view which will cover the cut portion in the front view.
2. Project and obtain the corresponding generators in the front view.

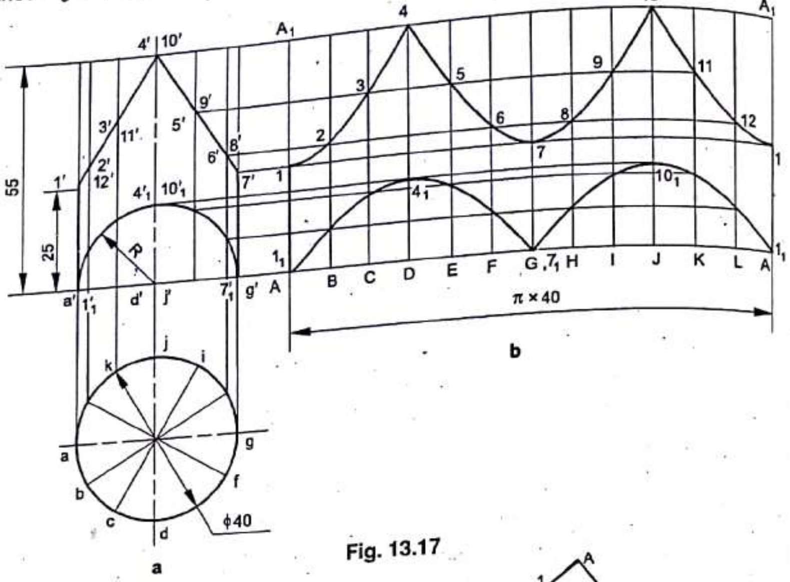


Fig. 13.17

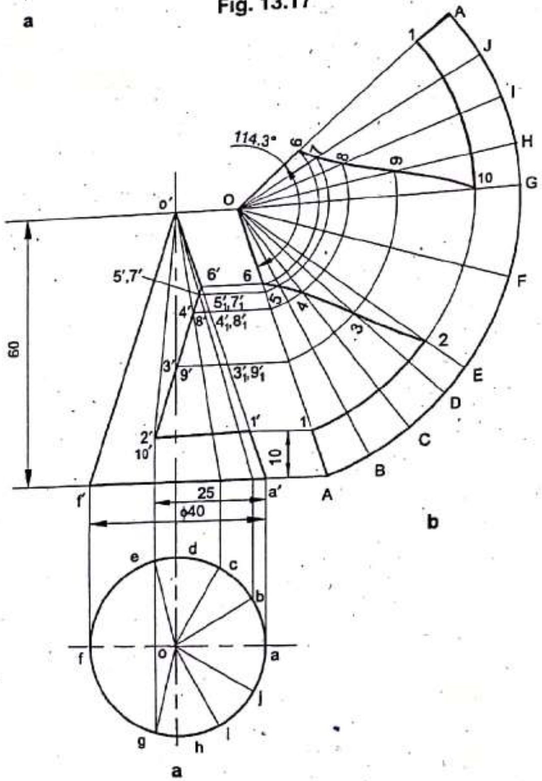


Fig. 13.18

3. Draw the development of the complete cone, following Construction: Fig. 13.9.
4. Locate the points of intersection between the cut portion and the generators, 1′, 2′, ------9′ in the front view.
5. Determine the true lengths of o′2′, o′3′, etc., by drawing horizontal lines to the extreme generator.
6. Transfer these true lengths on to the corresponding generators on the development.
7. Join the points 1, 2, etc., by smooth curves as shown and obtain the required development of the cut cone.

Problem 18 *A cylinder of base 80 diameter and axis 110 long, is resting on its base on H.P. It has a circular hole of 60 diameter, drilled through centrally such that, the axis of the hole is perpendicular to V.P. and bisects the axis of the cylinder at right angles. Develop the lateral surface of the cylinder.*

Construction (Fig. 13.19)

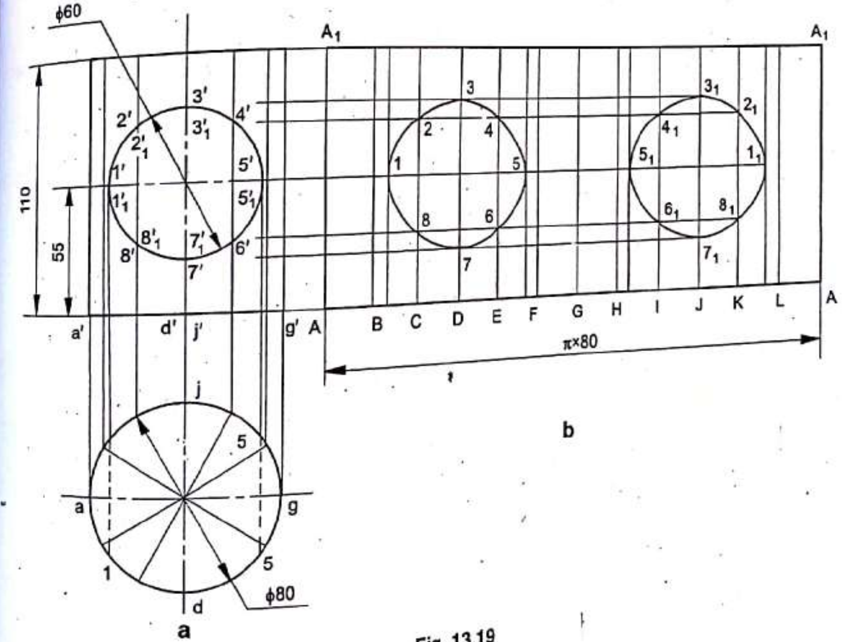


Fig. 13.19

1. Draw the projections of the cylinder, with the hole through it.
2. Divide the circle (top view of the cylinder) into a number of equal parts, say 12 and draw the corresponding generators in the front view.
3. Obtain the complete development AA_1-A_1A of the cylinder and draw the generators on it.

4. Determine the points of intersection 1′, 2′, etc., and $1_1'$, $2_1'$, etc., between the hole and generators in the front view.
5. Transfer these points on to the development by projection, including the transition points 1′ ($1_1'$) and 5′ ($5_1'$).
6. Join the points 1, 2, etc., and 1_1, 2_1, etc., by smooth curves and obtain the two openings in the development.

Problem 19 *Draw the development of a cylinder of 50 diameter and 75 height, containing a square hole of 25 side. The sides of the hole are equally inclined to the base and the axis of the hole bisects the axis of the cylinder.*

Construction (Fig.13.20)

1. Draw the projections of the cylinder, with the hole.
2. Draw number of generators in the front view, passing through the chosen points 1′ ($1_1'$), 2′ ($2_1'$), etc., on the edges of the hole.
3. Locate the corresponding generators in the top view.
4. Obtain the development AA_1 - A_1A of the complete cylinder and draw the chosen generators on it.
5. Transfer the points 1′,2′,3′, etc., and $1_1'$ $2_1'$, $3_1'$, etc., to the development, by projection.
6. Join the points 1, 2, etc., and 1_1, 2_1, etc., suitably and obtain the two openings in the development.

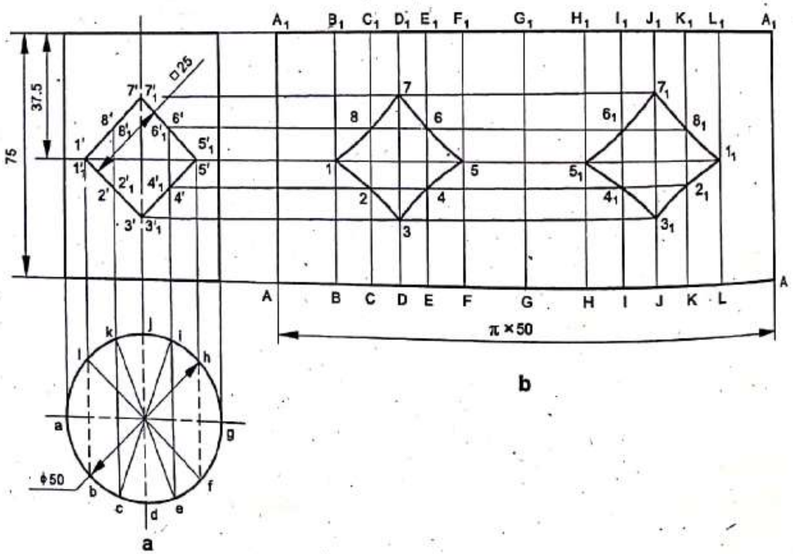


Fig. 13.20

Problem 20 *Draw the development of the cut hexagonal pyramid, shown in Fig.13.21a.*

Construction (Fig. 13.21)

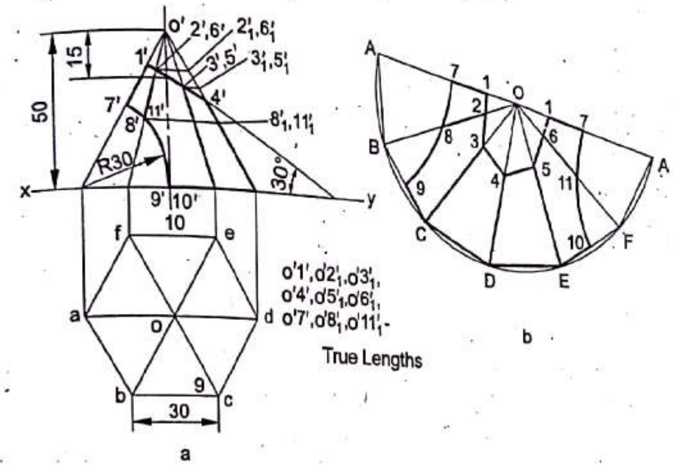


Fig. 13.21

1. Locate the points of intersection 1′, 2′, 3′, etc., between the cut sections and the edges of the pyramid.
2. Determine the true distances of the points 2′, 3′, 5′, 6′, 8′ and 11′ from o′ in the front view by projecting on to the true length line. o′1′, o′7′ and o′4′ are the true distances, as the points 1′, 7′ and 4′ lie on the true length lines. The points 9 and 10 lie at the mid-points of the edges BC and EF respectively.
3. With any point O as centre and radius equal to the true length of the slant edge, draw an arc of a circle.
4. Follow steps 4 and 5 of Construction: Fig. 13.6 suitably and obtain the full development of the pyramid.
5. Using the true lengths, transfer the above points on to the corresponding edges in the development.
6. Join the points in the order and obtain the development of the cut pyramid.

Problem 21 *Figure 13.22a shows the projections of a pentagonal pyramid with a square hole through it. Draw the development of the pyramid.*

Construction (Fig. 13.22)

1. Locate the points of intersection between the edges of the hole and the slant edges of the pyramid in the front view.

2. Draw the full development of the pyramid, following the radial line development method.
3. Transfer the above points on to the corresponding lines on the development (ref. Construction: Fig.13. 7).
4. Draw the generators through the critical points $1'$ ($6'$) and $3'$ ($10'$).
5. Transfer the above points, after locating the corresponding generators on the development.
6. Join the points in the order by straight lines, obtaining the two openings in the development.

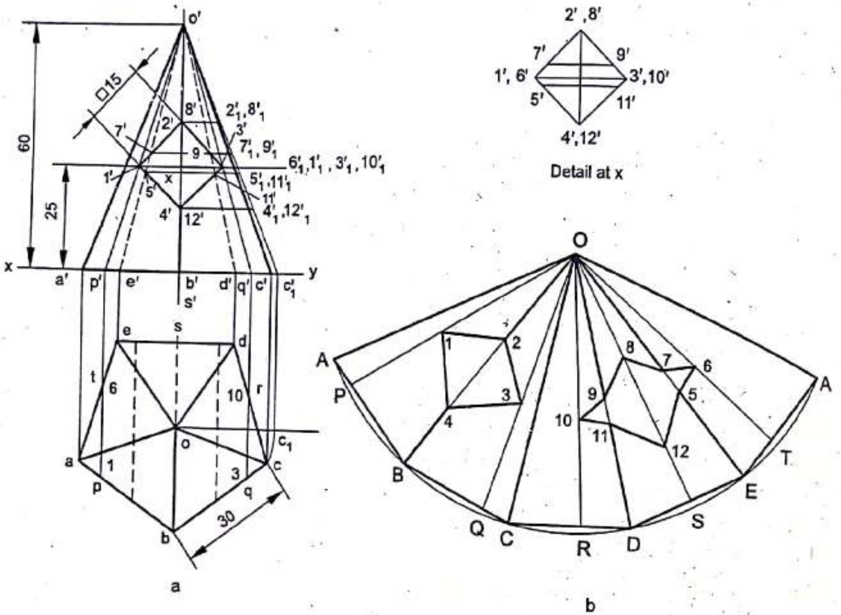


Fig. 13.22

Problem 22 *A hexagonal pyramid of side of base 30 and length of axis 80, rests on its base on H.P. An ant, initially situated at the left extreme corner of the base, moves on the surface of the pyramid and reaches the mid-point on the right extreme edge (slant edge). Find the shortest path of travel of the ant. Show the path in front and top views.*

Construction (Fig.13.23)

1. Draw the projections of the pyramid and its development.
2. Draw the path of the ant A-3 in the development; the point 3 being the mid-point of OD.
3. Locate the points 1 and 2 in the development, on the edges OB and OC.
4. Transfer the points 1,2 and 3 on to both the projections ($o'3' = O3$, $o'2_1' = O2$ and $o' 1_1' = O1$).
5. Join the points in the order by straight lines, in both the projections.

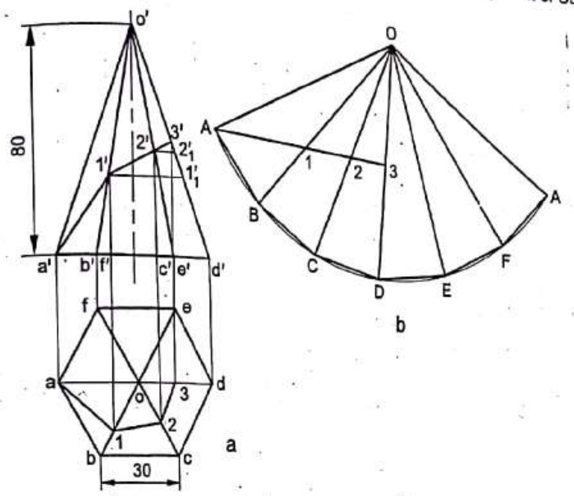


Fig. 13.23

Problem 23 *A flume in the form of a frustum of a pyramid, has the base 6m square, top 2m square and altitude 12m. A lightening conductor is taken from the mid-point of one of the top edges to the mid-point of the opposite edge in the base. Determine the shortest length and draw the projections of its path.*

Construction (Fig.13.24)

1. Draw the projections of the frustum of the pyramid and its development.
2. Locate the position of the conductor in the development such that, its ends 1 and 4 lie at the mid-points of the bottom (BC) and top (D_1A_1) edges of two alternate faces.
3. Locate the points 2 and 3 on the edges C-C_1 and D-D_1 respectively, in the development.
4. Transfer the points 1, 2, 3 and 4 on to both the projections (O-$2 = o'2_1'$, O-$3 = o'3_1'$).
5. Join the points in the order by straight lines, in both the projections.

Problem 24 *Figure13.25a shows the projections of a cone, cut by two section planes, as indicated by the traces. Draw the development of the cut cone.*

Construction (Fig. 13.25)

Follow the principle of Construction: Fig. 13.18 and obtain the development as shown in Fig.13.25b.

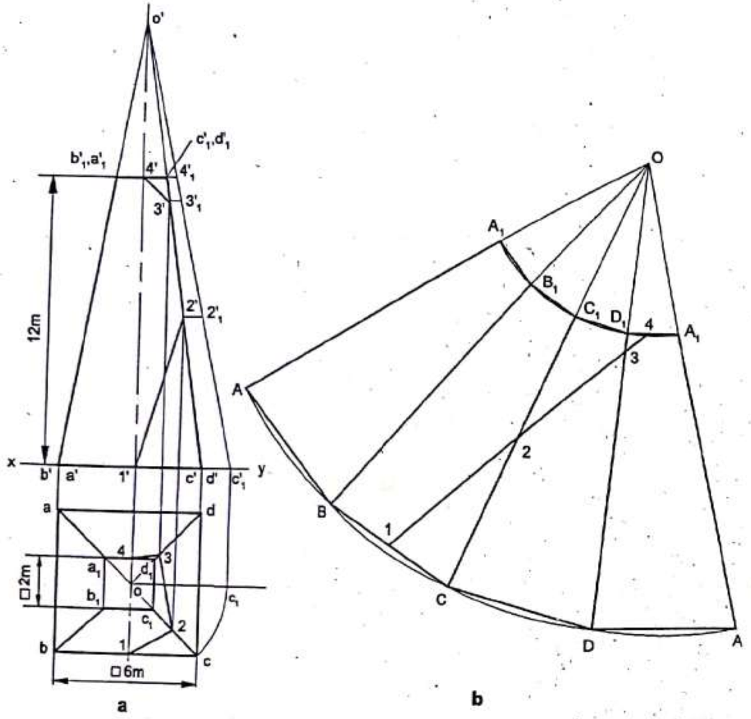


Fig. 13.24

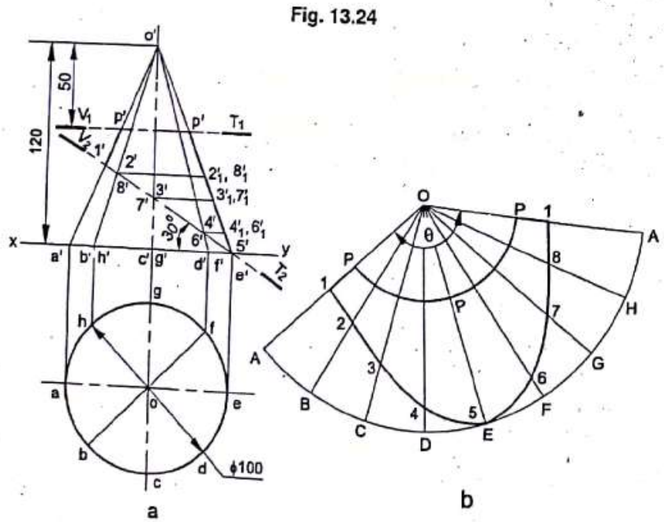


Fig. 13.25

Problem 25 *A cone of base diameter 80 and axis height 80, rests on H.P on its base. A square hole of side 40 is cut horizontally through the cone such that, the axis of the hole and the cone intersect at a height of 16 from the base. If the sides of the hole are equally inclined to H.P, draw the development of the lateral surface of the cone.*

Construction (Fig. 13.26)

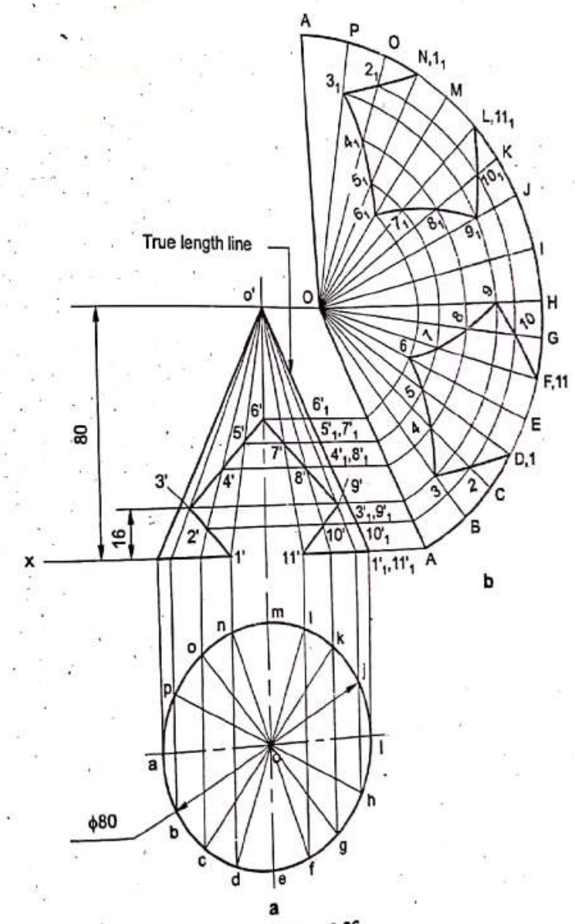


Fig. 13.26

1. Draw the projections of the cone, with the square hole through it.
2. Locate a number of points 1′, 2′, 3′, etc., on the edges of the hole in the front view and draw generators through them.

3. Locate these generators in the top view.
4. Draw horizontal lines from $1'$, $2'$, $3'$, etc., to the true length line and determine the distances of these points from o′.
5. Draw the development of the cone, by radial line development method.
6. Locate the generators in the development which are present in the top view.
7. Locate the points 1, 2, 3, etc., and 1_1, 2_1, 3_1, etc., in the development, on the corresponding generators, using the information from step 4.
8. Join the points by smooth curves and obtain the two openings in the development.

Problem 26 *In a semi-circular plate of 120 diameter, a largest circular hole is made. The plate is folded to form a cone. Draw the two views of the cone.*

Construction (Fig. 13.27)

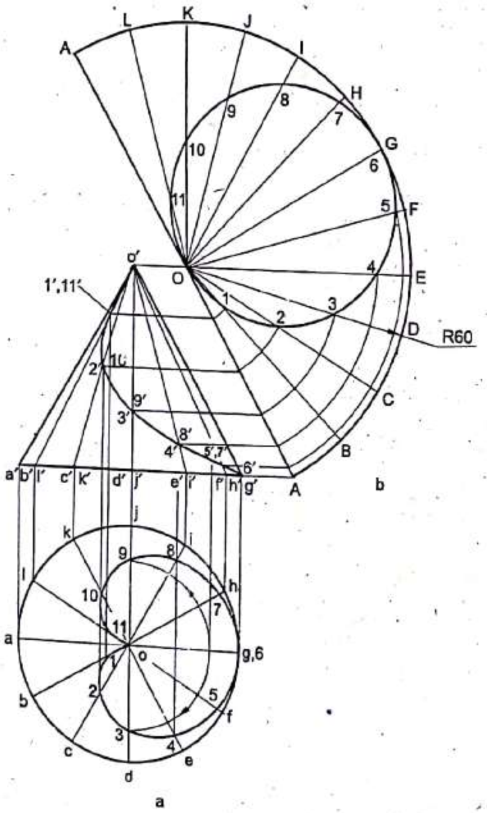


Fig. 13.27

NOTE The slant height s of the cone, formed after folding the plate is 60 and the radius r of the base of the cone is calculated from,

$$\theta = 180^\circ = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

$$= 360^\circ \times \frac{r}{s} = \frac{360^\circ \times r}{60}$$

$$\therefore \quad r = 30 \text{ mm}$$

1. Draw the projections of the cone, with the radius of the base 30 and slant height 60.
2. Divide the circle (top view) into, say 12 equal parts and locate the corresponding generators in both the views.
3. Draw the given semi-circular plate with the hole; representing the development of the cut cone.
4. Locate the generators on the development.
5. Locate the points of intersection between the generators and the circular hole.
6. Transfer the above points to both the projections (Refer Construction: Fig. 13.8).

Join the points in the order by smooth curves, forming the projections of the cone with the cut.

Problem 27 *In a semi-circular plate of 120 diameter, a largest hole is made. The plate is folded to form a cone. Draw the two views of the cone, when the hole is "square" in form.*

NOTE The slant height, s of the cone, formed after folding the plate is 60 and the radius, r of the base is obtained from,

$$\theta = 180^\circ = 360^\circ \times \frac{\text{radius of the base circle}}{\text{slant height}}$$

$$= 360^\circ \times \frac{r}{s} = \frac{360^\circ \times r}{60}$$

$$\therefore \quad r = 30 \text{ mm}$$

Construction (Fig. 13.28)

1. Draw the projections of the cone, with the radius of the base 30 and slant height 60.
2. Draw the given semi-circular plate with the hole; representing the development of the cone.
3. Draw a number of generators, passing through the hole in the development.
4. Locate the above generators; first in the top view and then in the front view.
5. Locate the points of intersection 1, 2, - - - - - - - -7, between the edges of the hole and the generators.
6. Transfer the above points to both the projections (Refer Construction: Fig. 13.8).
7. Join the points in the order suitably, forming the projections of the cone with the cut.

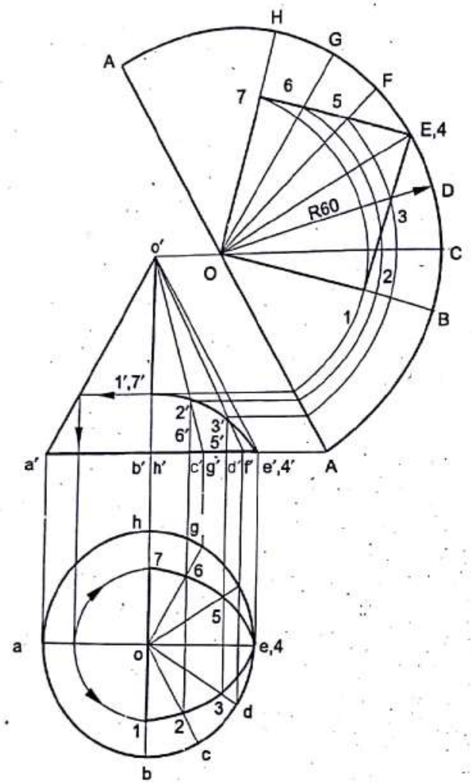


Fig. 13.28

Problem 28 *A pipe of 40 diameter and 85 long (along the axis) is welded to the vertical side of a tank. Show the development of the pipe if it makes an angle 60° with the side to which it is welded. The other end of the pipe makes an angle 30° with its own axis. Neglect the thickness of the pipe.* (June 2009, JNTU)

Construction (Fig. 13.29)

1. Draw the front view of the pipe and the tank assembly, satisfying the given conditions.
2. Draw the edge view of the pipe (circle) and divide it into 8 equal parts and locate the corresponding generators in the front view of the pipe.
3. Draw the stretch-out line A-A, equal to the circumference of the pipe and complete the development of the complete uncut pipe.
4. Locate the generators on the development.
5. Locate the points of intersection between the generators and the cut portions of the pipe ends, $1'$, $2'$, etc., and $1_1'$, $2_1'$, etc.

6. Transfer these intersection points on to the corresponding generators in the development by projection.
7. Join the points 1, 2, etc., and 1_1, 2_1, etc., by smooth curves and obtain the development.

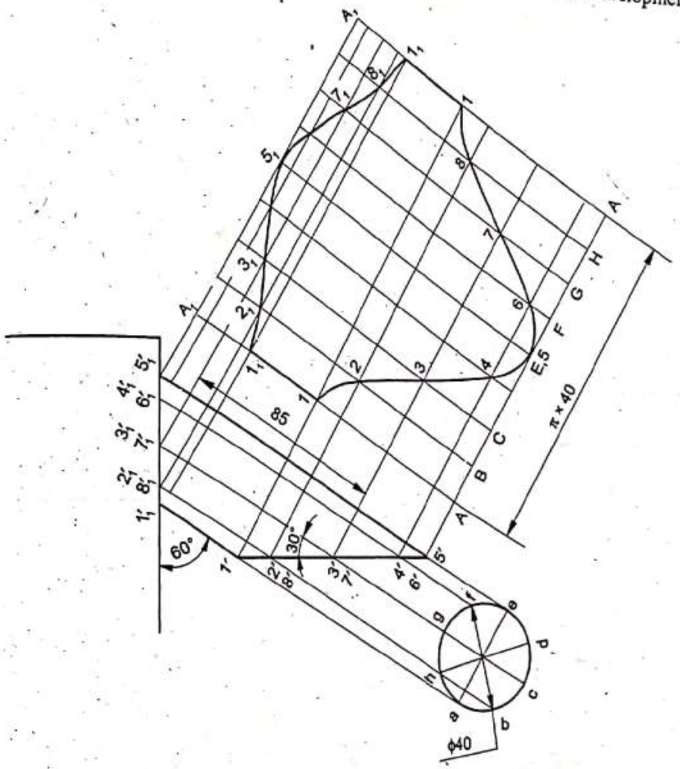


Fig. 13.29

Problem 29 *Draw the development of the lateral surface of the oil can, shown in Fig. 13.30a.* The oil can consists of three parts P, Q and R; P and Q being parts of a cone and R, a cylindrical part. Figure 13.30b shows the construction for obtaining the developments of the three parts P, Q and R; which is self explanatory.

Problem 30 *Draw the development of the tray; the orthographic views of which are given in Fig. 13.31a.*

Construction (Fig. 13.31b)

NOTE (i) The base edges AB and CD and top edges EF and GH are parallel to both H.P and V.P. Hence, the lengths of the edges a'b' (= ab), c'd' (= cd) and e'f' (= ef) and g'h' (= gh) represent the true lengths.

AUXILIARY PROJECTION

10.1 INTRODUCTION

Three views, viz., front, top and side views of an object are sometimes not sufficient to reveal complete information regarding the size and shape of the object. To serve the purpose, additional views known as auxiliary views, projected on auxiliary planes may be used. Auxiliary plane is a plane, perpendicular to one of the principal planes of projection and inclined to the other. Auxiliary views may also be used to determine the true length of a line and the true shape of plane surfaces.

10.2 TYPES OF AUXILIARY PLANES AND VIEWS

Two types of auxiliary planes, viz., (i) auxiliary vertical plane and (ii) auxiliary inclined plane are made use of, to obtain the auxiliary views.

10.2.1 Auxiliary vertical plane (A.V.P)

It is a plane perpendicular to H.P and inclined to V.P. The projection on to an A.V.P is called an auxiliary front view.

When the inclination of an A.V.P to V.P is 90°, it then becomes a profile plane. The projection on the profile plane gives the auxiliary front view, known as the side view of the object. Projections of points, lines and planes on the profile planes are already dealt with under three view drawings.

10.2.2 Auxiliary inclined plane (A.I.P)

It is a plane inclined to H.P and perpendicular to V.P. The projection on an A.I.P is called an auxiliary top view.

For showing the relative positions of the views, the auxiliary plane should always be rotated about the plane to which it is perpendicular.

10.3 PROJECTIONS OF POINTS

10.3.1 Projections of a point on an A.V.P

Problem 1 *A point A is 25 above H.P and 15 in front of V.P. Draw the front and top views of the point. Also, obtain the auxiliary front view of the point on a plane, which makes an angle of 60° with V.P and perpendicular to H.P.*

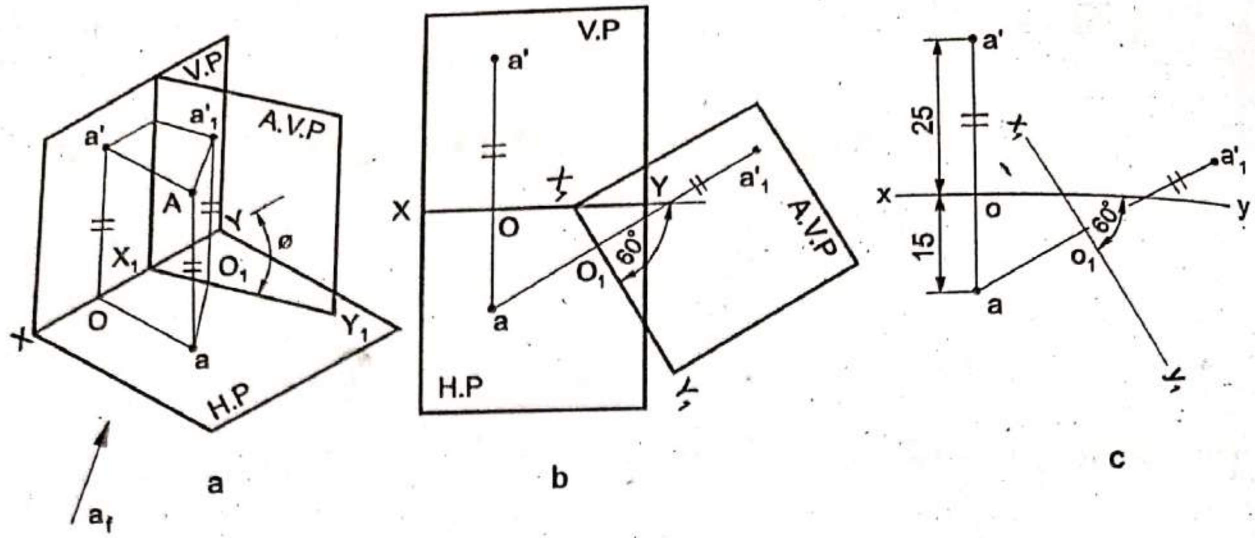


Fig. 10.1

Figure 10.1a shows the quadrant with the point A marked in it. The auxiliary front view is obtained by viewing the point in the direction $\mathbf{a_f}$ such that the ray of sight passing through A, meets the A.V.P perpendicularly. Figure 10.1b shows the three planes opened out with the views marked. X_1 Y_1 is the reference line between H.P and A.V.P. Figure 10.1c shows the relative positions of the views, with the planes removed.

From the geometry of the Figs. 10.1a and b; the following may be observed:

(i) The distance of the auxiliary front view from X_1 Y_1 is equal to the distance of the front view from XY, which in turn is the distance of the point A from H.P.

(ii) The line X_1 Y_1 is inclined to XY by an angle ϕ, which is the true angle of inclination of A.V.P with V. P.

(iii) The top view and auxiliary front view lie on a single projector, which is perpendicular to X_1 Y_1.

Construction (Fig. 10.1c)

1. Draw the reference line xy and locate the front and top views of the point a′, a and mark o, the point of intersection between the projector a′ a and xy.
2. Draw the reference line x_1 y_1, at any convenient location, making an angle of 60° with xy.
3. Through a, draw a line perpendicular to x_1 y_1, intersecting it at o_1.
4. Locate the auxiliary front view a_1' such that, o_1 $a_1' = oa'$.

   a′, a are the front and top views and a_1' is the auxiliary front view of the point.

   It may be noted that there are four possible positions for the line x_1 y_1 relative to xy.

10.3.2 Projections of a point on an A.I.P

Problem 2 *A point B is 25 above H.P and 15 in front of V.P. Draw the front and top views of the point. Also, obtain the auxiliary top view of the point on a plane, which makes an angle of 45° with H.P and perpendicular to V.P.*

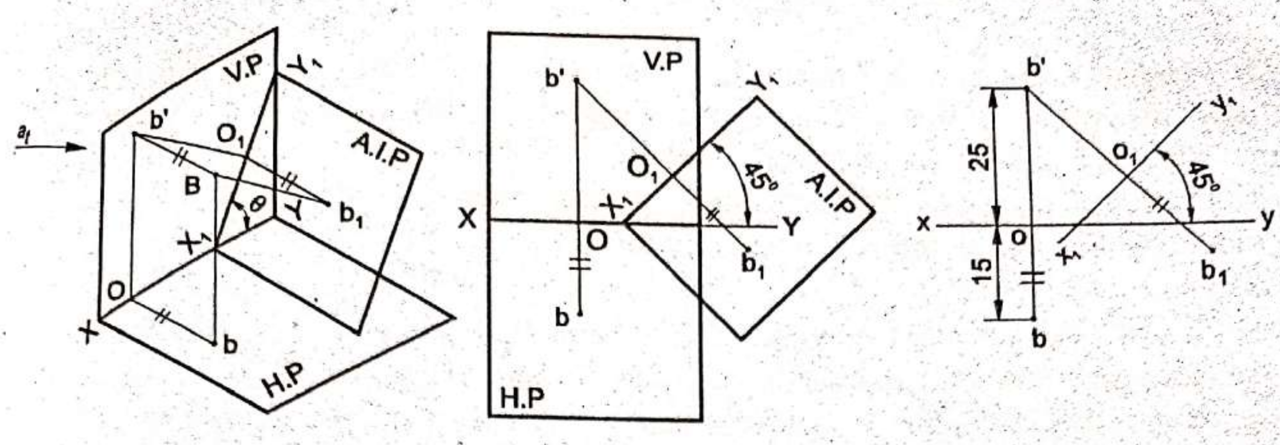


Fig. 10.2

Figure 10.2a shows the quadrant with the point B marked in it. The auxiliary top view is obtained by viewing the point in the direction a_t such that, the ray of sight passing through B, meets A.I.P perpendicularly. Figure 10.2b shows the relative positions of the three planes after opening out. Here, $X_1\ Y_1$ is the reference line between V.P and A.I.P. Figure 10.2c shows the relative positions of the views, with the planes removed.

From geometry of the Figs. 10.2a and b, the following may be observed:

(i) The distance of the auxiliary top view from $X_1\ Y_1$ is equal to the distance of the top view from XY, which in-turn is the distance of the point B from V.P.

(ii) The line $X_1\ Y_1$ is inclined to XY by an angle θ, equal to the true angle of inclination of A.I.P with H.P.

(iii) The front view and auxiliary top view lie on a single projector, which is perpendicular to $X_1\ Y_1$.

Construction (Fig. 10.2c)

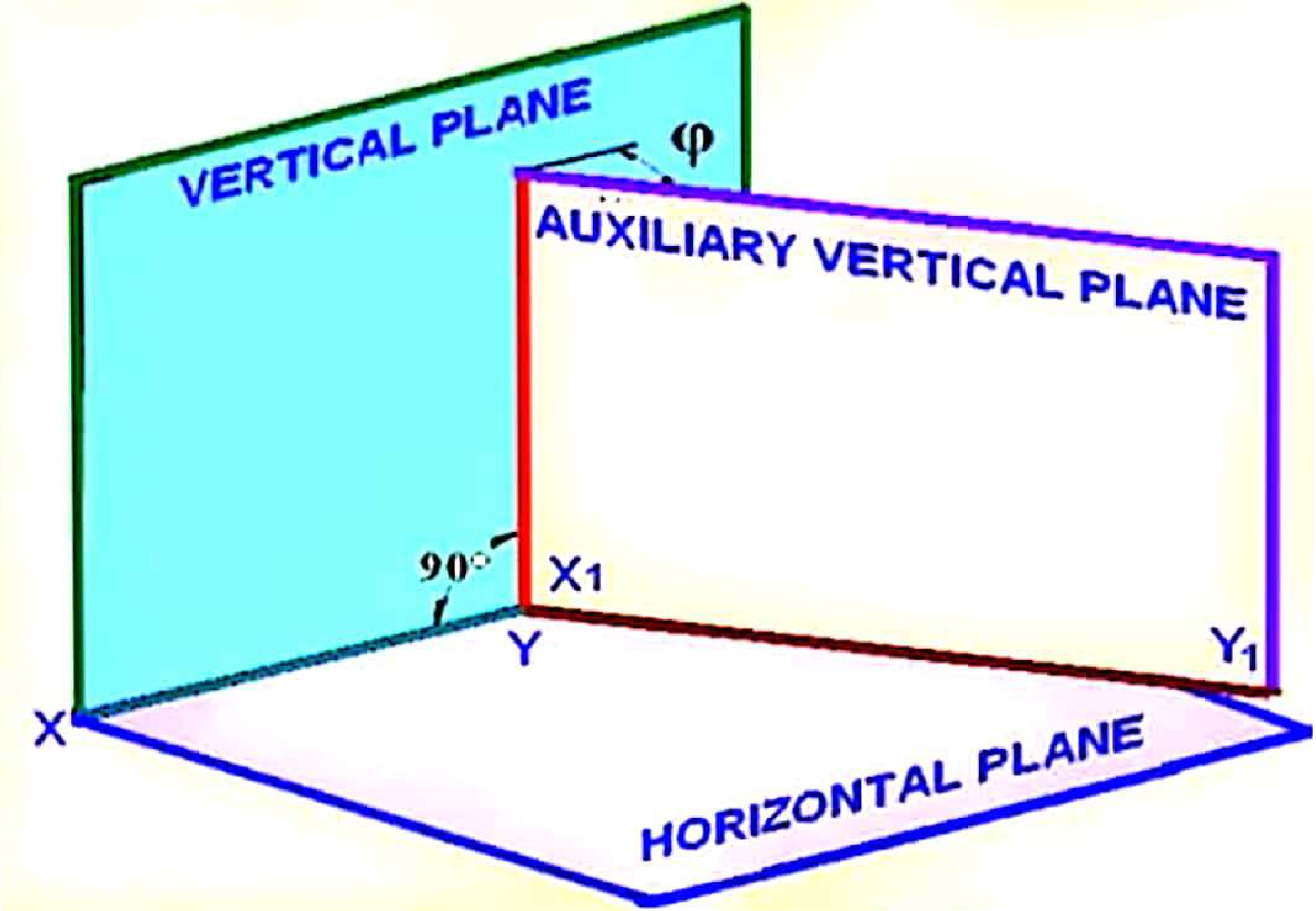
1. Draw the reference line xy and locate the front and top views b′, b of the point and mark o, the point of intersection between the projector b′ b and xy.
2. Draw the reference line $x_1\ y_1$ at any convenient location, making an angle of 45° with xy.
3. Through b′, draw a line perpendicular to $x_1\ y_1$; intersecting it at o_1.
4. Locate the auxiliary top view b_1 such that, $o_1\ b_1 = ob$.

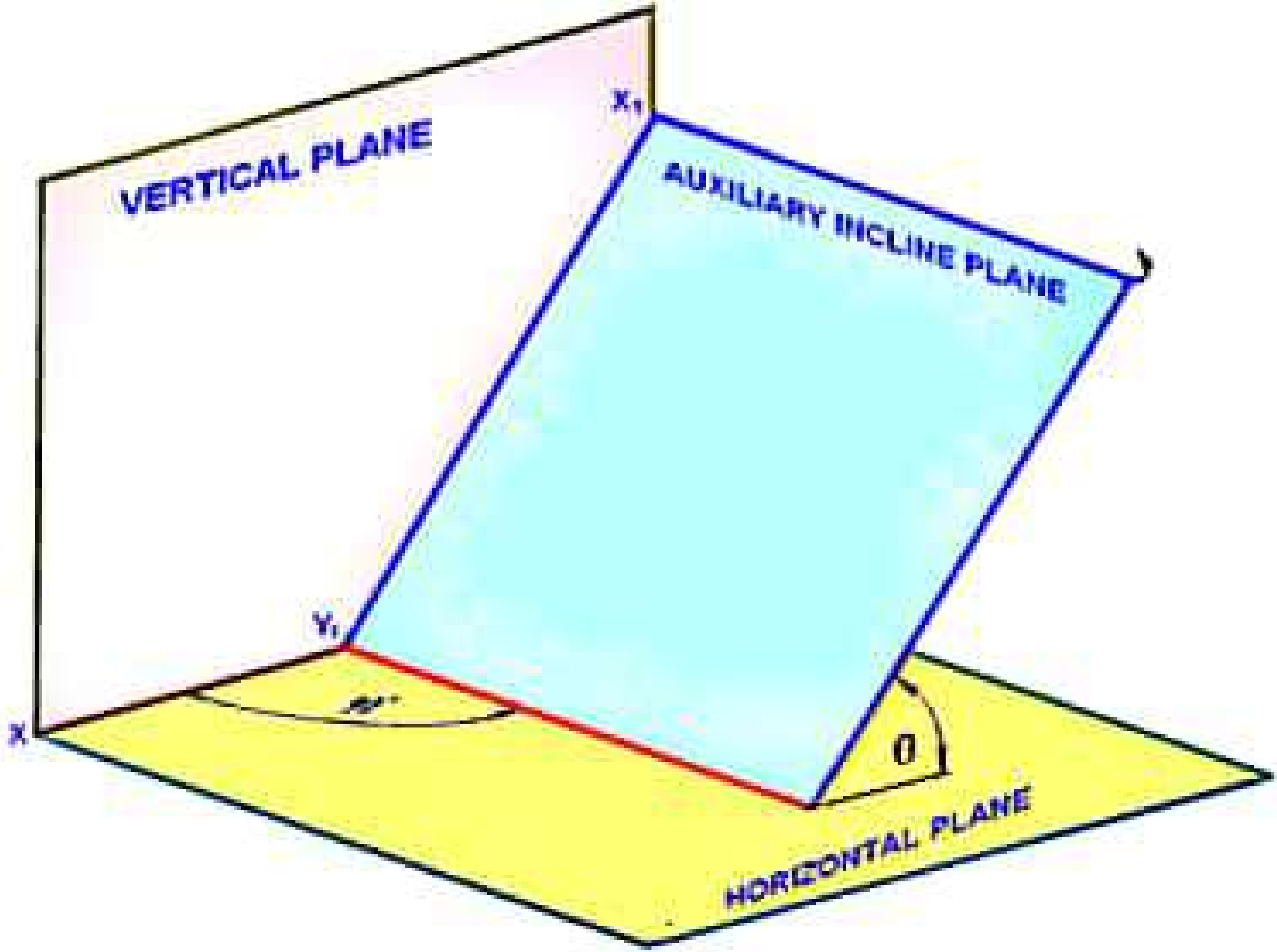
b′, b are the front and top views and b_1 is the auxiliary top view of the point.

10.4 PROJECTIONS OF STRAIGHT LINES

Projections of straight lines on the auxiliary planes, may be used to obtain the following:

1. The true lengths of lines,
2. The point or edge views of lines, and
3. The conventional projections.


VERTICAL PLANE
φ
AUXILIARY VERTICAL PLANE
90°
X1
Y
Y1
X
HORIZONTAL PLANE



VERTICAL PLANE
X1
AUXILIARY INCLINE PLANE
Y1
X
θ
HORIZONTAL PLANE