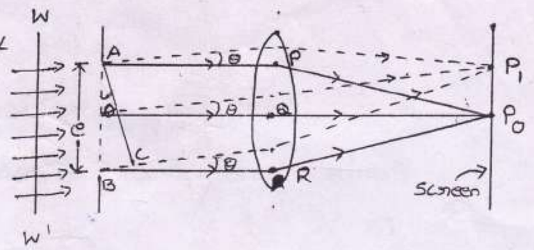
Fraunhofer diffraction at single slit :-



Let AB be a narrow slit of width "e" perpendicular to the plane of the paper and a plane wavefront ww' of monochromatic light of wavelength 'λ' propagating normal to the slit be incident on the slit. Let the diffracted light be focussed by means of a convex lens on a screen placed in the focal plane of the lens.

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According to Huygens-Fresnel, every point of the wavefront in the plane of the slit is a source of secondary spherical wavelets which spread out to the right in all directions. The secondary wavelets travelling normal to the slit are brought to focus at P_0 by the lens. Thus P_0 is a bright central image.

The secondary wavelets travelling at an angle 'θ' with the normal are focussed at a point P_1 on the screen. In order to find out intensity at P_1, draw a perpendicular AC on BR.

The path difference between secondary wavelets from A and B is, $= BC$

$= AB \sin\theta$

$\therefore$ path difference $= e \sin\theta$ — (1)

Now, the phase difference between the secondary wavelts from A and B is,

phase difference $= \frac{2\pi}{\lambda}$ (path difference)

$= \frac{2\pi}{\lambda} \times e \sin\theta$

$\therefore$ phase difference $= \frac{2\pi}{\lambda} \cdot e \sin\theta$ — (2)

Next

Let us consider the slit is divided into 'n' equ parts and the amplitude of the wave from each part is 'a'.

The phase difference between any two consecutive waves from these parts would be,

$\frac{1}{n}$ (Total phase difference)

$$= \frac{1}{n} \cdot \frac{2\pi}{\lambda} \cdot e\sin\theta$$

$$= d \text{ (say).} \quad \therefore d = \frac{1}{n} \cdot \frac{2\pi}{\lambda} \cdot e\sin\theta \quad \text{—— (3)}$$

Now, By using the vector addition of amplitude, the resultant amplitude R is given by,

$$R = a \frac{\sin nd/2}{\sin d/2}$$

$$= a \cdot \frac{\sin\left[\cancel{n} \cdot \frac{1}{\cancel{n}} \cdot \frac{\cancel{2}\pi}{\lambda} \cdot \frac{e\sin\theta}{\cancel{2}}\right]}{\sin\left[\frac{1}{n} \cdot \frac{\cancel{2}\pi}{\lambda} \cdot \frac{e\sin\theta}{\cancel{2}}\right]}$$

$$= a \cdot \frac{\sin\left(\frac{\pi}{\lambda} \cdot e\sin\theta\right)}{\sin\left(\frac{1}{n} \cdot \frac{\pi}{\lambda} \cdot e\sin\theta\right)}$$

$$= a \cdot \frac{\sin\alpha}{\sin\alpha/n} \qquad \text{Where, } \alpha = \frac{\pi}{\lambda} \cdot e\sin\theta$$

$$= a \cdot \frac{\sin\alpha}{\alpha/n} \qquad [\because \alpha/n \text{ very small}]$$

$$= na \cdot \frac{\sin\alpha}{\alpha}$$

$$R = A \cdot \frac{\sin\alpha}{\alpha} \quad \text{—— (4)}$$

Now, the intensity is given by,

$$I = R^2 = A^2 \cdot \frac{\sin^2\alpha}{\alpha^2} \quad \text{—— (5)}$$

<u>Principal maximum</u> :-

The resultant amplitude is,

$$R = A \frac{\sin\alpha}{\alpha}$$

$$= \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right]$$

If the negative terms vanish, the value of R will be maximum. i.e., $\alpha = 0$

$$\Rightarrow \frac{\pi}{\lambda} \cdot e\sin\theta = 0$$

$$\Rightarrow \sin\theta = 0$$

$$\Rightarrow \theta = 0 \quad \text{--- (6)}$$

Now, $R_{max} = A$

$$\Rightarrow I_{max} = R^2_{max} = A^2$$

The condition $\theta = 0$ means that the maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as principal maximum.

<u>Minimum intensity</u> :-

The intensity will be minimum when ~~$\sin\alpha = 0$~~.

$$\sin\alpha = 0$$

$$\Rightarrow \alpha = \pm\pi, \pm 2\pi - \cdots$$

$$\Rightarrow \alpha = \pm m\pi \quad \text{where, } m = 1, 2, 3, \cdots$$

$$\Rightarrow \frac{\pi}{\lambda} e\sin\theta = \pm m\pi$$

$$\Rightarrow e\sin\theta = \pm m\lambda \quad \text{--- (7)}$$

i.e., when the path difference is equal to integer multiples of wavelength. The value of $m = 0$ is not admissible, because for this value $\theta = 0$ and it leads to principal maximum.

Secondary maxima :-

In addition to principal maxima at $\alpha = 0$, there are weak secondary maxima between equally spaced minima. and these are obtained by differentiating I with respect to 'α' and equating to zero.

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left[\frac{A^2}{\alpha^2}\sin^2\alpha\right] = 0$$

$$A^2 \frac{d}{d\alpha}\left[\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 \cdot 2 \cdot \frac{\sin\alpha}{\alpha} \cdot \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\alpha = \tan\alpha \quad \text{—— (8)}$$

The values of 'α' satisfying above equation are,

$$\alpha = 0 \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2} \; \ldots$$

and more exactly

$$\alpha = 0 \pm 1.430\pi, \pm 2.462\pi \; \ldots$$

we know $\alpha = 0$ gives principal maximum.

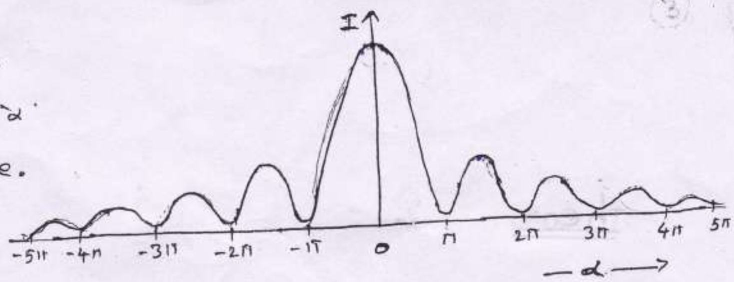
Substituting the various values of 'α' in equation (5) we ~~go~~ can get secondary maximas

$$I_0 = A^2 \quad \text{—— (9)}$$

$$I_1 = A^2 \cdot \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{A^2}{22} \quad \text{—— (10)}$$

$$I_2 = A^2 \cdot \left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{A^2}{62} \quad \ldots\ldots \text{—— (11)}$$

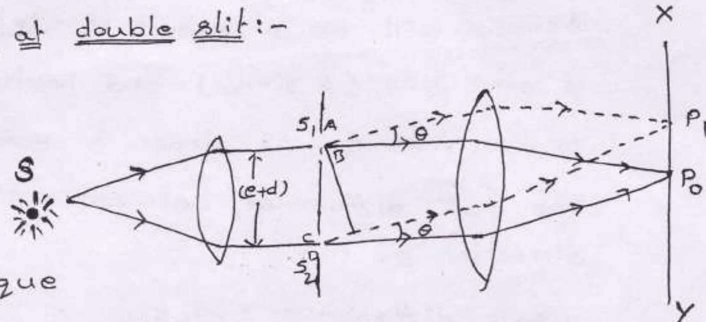
A graph showing the variation of intensity with 'α' is as shown in the figure.



From the graph, it is evident that most of the light is concentrated at the principal maxima, the subsidiary maximas of decreasing intensity on either sides and in between subsidiary maximas there are minimas.

Fraunhofer diffraction at double slit:-

Let AB and CD are two parallel slits of equal width 'e' and separated by an opaque distance 'd'.



The distance between the middle points of two slits is (e+d). Let a parallel beam of monochromatic light of wavelength λ be incident normally upon the two slits.

The light diffracted from these slits is focussed by a lens on the screen XY placed in the focal plane of lens. The diffraction at two slits is the combination of diffraction as well as interference.

When a plane wavefront is incident normally on the slits, the secondary wavelets travelling in the direction of incident light come to focus at P_0 while the secondary wavelets travelling in a direction making an angle θ with the incident direction come to focus at P_1.

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Theory:-

Simplicity we can consider the two slits as equivalent to two coherent sources S_1 and S_2 are arranged at-mid-points and each source ~~sti~~ sending a wavelet of amplitude $(A \frac{\sin\alpha}{\alpha})$ in a direction 'θ'.

Therefore, the resultant amplitude at a point P_1 on the screen will be a result of interference between two waves of amplitude $(A \sin\alpha/\alpha)$ and having a phase difference δ. To calculate δ, we draw a perpendicular S_1K on S_2K. The path difference between wavelets from S_1 and S_2 in the direction 'θ'.

$$\text{Path difference} = S_2K$$
$$= (e+d)\sin\theta \quad \text{--- (1)}$$

$$\therefore \text{ phase difference, } \delta = \frac{2\pi}{\lambda} \times (\text{Path difference})$$
$$= \frac{2\pi}{\lambda}(e+d)\sin\theta \quad \text{--- (2)}$$

The resultant amplitude R at P_1 can be obtained is,

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 [1+1+2\cos\delta]$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[1+\cos\delta]$$

$$= 2\left(\frac{A\sin\alpha}{\alpha}\right)^2 [\cancel{1} + 2\cos^2\delta/2 - \cancel{1}]$$

$$= 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2\delta/2$$

$$= 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right]$$

$$\therefore R^2 = 4A^2 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \quad \text{where } \beta = \pi/\lambda\,(e+d)\sin\theta$$
$$\text{--- (3)}$$

Therefore, the resultant intensity at P_0 is given by,

$$I = R^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \quad \text{--- (3)}$$

From equation ③ the intensity is depends upon the following two factors:

(i) on $A^2 \frac{\sin^2\alpha}{\alpha^2}$ which is the same as derived for a single Fraunhofer diffraction. Thus this gives the intensity distribution in the diffraction pattern due to the single slit.

(ii) ~~and~~ the ~~term~~ factor $\cos^2\beta$ gives the interference due to waves starting from two parallel slits.

The diffraction term $\sin^2\alpha/\alpha^2$ gives the central maximum in the direction $\theta = 0$. — (4)

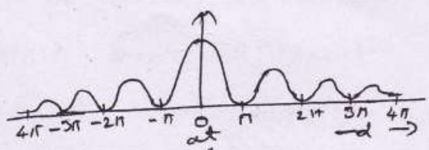
The minimas are obtained in the direction are.

$$\sin\alpha = 0 \quad \text{but } \alpha \neq 0.$$

$$\alpha = \pm m\pi, \quad m = 1, 2, 3 - - - - \quad — (5)$$

$$\frac{\pi}{\lambda} e \sin\theta = \pm m\pi$$

$$e\sin\theta = \pm m\lambda$$

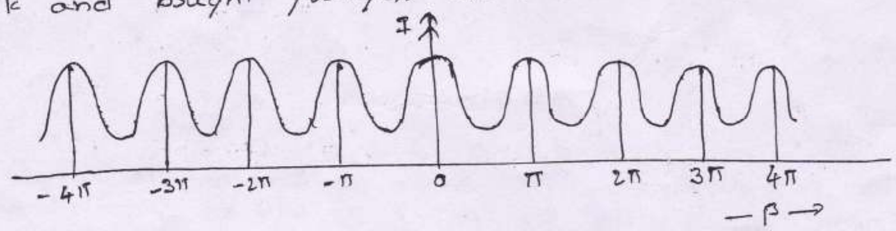


The position of secondary maximas are then

$$\alpha = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2 - - - - -$$

the intensity distribution on either side is as shown in fig.

The interference term $\cos^2\beta$ gives a set of equidistant dark and bright fringes as shown in fig.



The maxima are obtained in the direction given by.

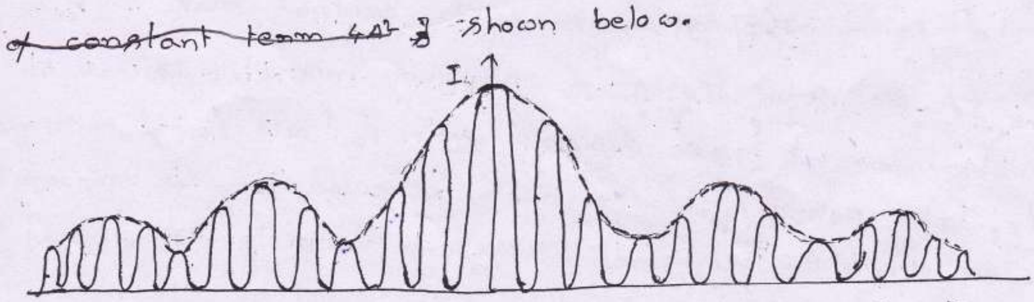
$$\cos^2\beta = 1$$

$$\beta = \pm n\pi \qquad n = 0, 1, 2, 3, - - - -$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda \quad — ⑥$$

The resultant diffraction pattern is ~~a plot of the product of constant term $4A^2$ &~~ shown below.



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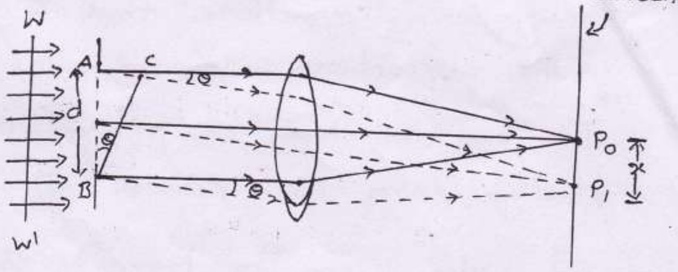
Fraunhofer diffraction at a circular aperture:-



Let AB be a circular aperture of diameter 'd'. A plane wavefront WW' of monochromatic light of wavelength λ propagating normally to the circular aperture incident on it. The diffracted beam is focussed on the screen by a convex lens.

The wavelets travelling along the normal to the circular aperture come to focus at P_0. Because all the normal wavelet travel the same distance before reaching the point P_0, therefore P_0 corresponds to the position of central maximum. And the secondary wavelets travelling in a direction inclined at an angle 'θ' with the normal are at a point 'P_1' on the screen. ~~Th.~~ Let $P_0P_1 = x$.

The path difference between the extreme waves from point A and B is, $= AC$

$$= AB \sin\theta$$

$$= d \sin\theta \quad \text{--- (1)}$$

The path difference is same as that of the single slit. So, from single slit, the point P_1 will be of minimum intensity if the path difference is an integral multiple of 'λ' and of maximum intensity if the path difference is odd multiple of $\lambda/2$ i.e.,

$$\left.\begin{aligned} d\sin\theta &= m\lambda \quad \text{for minima} \\ d\sin\theta &= (2m+1)\lambda/2 \quad \text{for maxima} \end{aligned}\right\} \text{--- (2)} \quad \text{where } m = 1, 2, 3, \ldots$$

The term $m=0$ corresponds to central max. at P_0.

If the point P_1 is of minimum intensity, then all the points which are at same distance from P_0 will be of minima intensity. The point P_0 traces out a circular ring of uniform illumination. Thus the diffraction pattern consists of a central bright disc, called the Airy's disc surrounded by alternate dark and bright rings called Airy's rings. The intensity of dark rings is zero and the bright rings decreases gradually.

The angular radius `θ' of the Airy's disc i.e., the angular separation between the centre of the bright disc and the first dark ring is given by,

$$\theta = \frac{1.22\lambda}{d} \quad \text{--- (3)}$$

where `d' is the diameter of the circular aperture.

If the collecting lens is very near to the circular aperture of the screen is at a large distance `x' from the lens, then.

$$\sin\theta = \theta = \frac{x}{f} \quad \text{--- (4)}$$

where f is focal length of the lens.

From the first secondary minimum.

$$d\sin\theta = 1.\lambda$$

$$\sin\theta = \frac{\lambda}{d}$$

$$\Rightarrow \theta = \frac{\lambda}{d} \quad \text{--- (5)}$$

From (4) & (5)

$$\frac{x}{f} = \frac{\lambda}{d}$$

$$\Rightarrow x = \frac{\lambda f}{d} \quad \text{--- (6)}$$

It was shown by Airy's the exact value of `x' is given by,

$$x = \frac{1.22\lambda f}{d} \quad \text{--- (7)}$$

Thus if the diameter of the aperture is large, the radius of the central disc is small.

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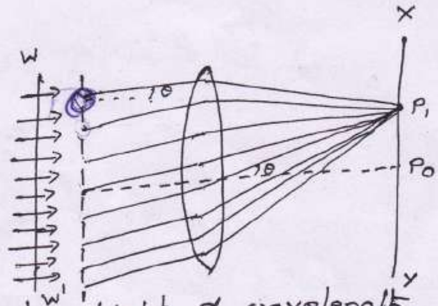
Fraunhofer diffraction at N parallel slits :-
(OR)
plane diffraction grating :-

Grating :- An arrangment consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction "grating."

Now a days grattings a constructed by ruling equidistant parallel lines on a transparent materials(glass). and the ruled lines are opaque to light with while the space b/n two lines is transparent to light and act as a slit. These type grattings are known as "plane transmission grating".

Theory:-



Let AB be a plane transmission grating and e, d are width of each slit and width of the each opaque part. Then (e+d) is known as grating element. Let a plane wavefront of mono-chromatic light of wavelength 'λ' is incident normally on the grating. By Huygen's principle each of the slit send secondary wavelets and the secondary wavelets travelling normal to the slits are focussed at P_0 and the point P_0 will be a central maximum. where as the secondary wavelets travelling at angle 'θ' with the incident light are focussed at a point P_1 on the screen. The intensity at P_1 may be considered as Fraunhofer's diffraction at single slit. The wavelets from each slit are having a amplitude $A \sin\alpha/\alpha$ where $\alpha = \frac{\pi}{\lambda} e \sin\theta$.

If there are 'N' slits then the path difference between any two consecutive slits is $(e+d)\sin\theta$. Then the corresponding phase difference is $(e+d)\sin\theta \times \frac{2\pi}{\lambda}$ and it is assumed as 2β.

$\therefore$ Phase difference, $2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta$

By the method of vector addition of amplitudes, the resultant amplitude,

$$R = a \frac{\sin n d/2}{\sin d/2} \quad \text{--- (2)}$$

In this case, ~~where~~ $a = A \sin\alpha/\alpha$

$n = N$

$d = 2\beta$.

$$\therefore \quad R = A \cdot \frac{\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta} \quad \text{--- (3)}$$

and the resultant intensity is given by,

$$I = R^2 = A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{--- (4)}$$

In the above equation the factor $\left(A\frac{\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to a single slit while the factor $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity as a combined effects of all the slits.

<u>Intensity distribution</u> :-

$\sin\beta \to$ mini $\quad \sin\beta \neq 0 \ldots$

(i) <u>principle maxima</u> :-

The intensity would be max. when. $\beta = \pm n\pi$ where $n = 0, 1, 2, 3, 4, \ldots$ but $\sin N\beta, \sin\beta = 0$. so, By applying L'Hospital's rule,

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$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos\beta}$$

$$= \pm N$$

$$\therefore \lim_{\beta \to \pm n\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2.$$

$\therefore$ Resultant Intensity, $I = \left(A \frac{\sin\alpha}{\alpha}\right)^2 \cdot N^2$. $\quad$ --- (5)

These max. intensities are called as principal maxima.

The maxima are obtained for

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda \quad \text{--- (6)}$$

where $n = 0, 1, 2, \ldots\ldots$

When $n=0$, then the maxima is zero order maxima, For $n=1,2,3 \ldots$ are known as first, second, third principal maxima. The sign $\pm$ shows that there are two principal maxima of same order lying on either side of zero order maximum.

(ii) **Minima :-**

A series of minimas are formed, when

$$\sin N\beta = 0 \quad \text{but } \sin\beta \neq 0$$

$$\Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$\Rightarrow N(e+d)\sin\theta = \pm m\lambda \quad \text{—— (7)}$$

where 'm' has all integers except $0, N, 2N \ldots nN$ because ~~the~~ ~~va~~ for these values $\sin\beta = 0$ and we get principal maxima. Therefore, $m = 1, 2, 3, \ldots (N-1)$ are minimas.

(iii) Secondary maxima :-

As there are $(N-1)$ minima between two adjacent principal maxima there must be $(N-2)$ other maxima b/n two principal maxima. These are known as secondary maxima. To find out these secondary maximas, we differentiate equation (4) with respect to β and equate to zero. ~~thus~~.

Thus $\frac{dI}{d\beta} = 0 \Rightarrow \left(A\frac{\sin\alpha}{\alpha}\right)^2 \cdot 2 \cdot \frac{\sin N\beta}{\sin\beta} \times \left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$

$$\Rightarrow N\cos N\beta \cdot \sin\beta - \sin N\beta\cos\beta = 0$$

$$\Rightarrow N\tan\beta = \tan N\beta \quad \text{—— (8)}$$

The roots for eqn. (8) are other than those for which $\beta = \pm n\pi$. As N increases the intensity of secondary maxima decreases and becomes negligible when N becomes large.

The variation of intensity due to the factors $\sin^2\alpha/\alpha^2$ and $\sin^2 N\beta/\sin^2\beta$ are shown below. The last graph represents variation of resultant intensity.

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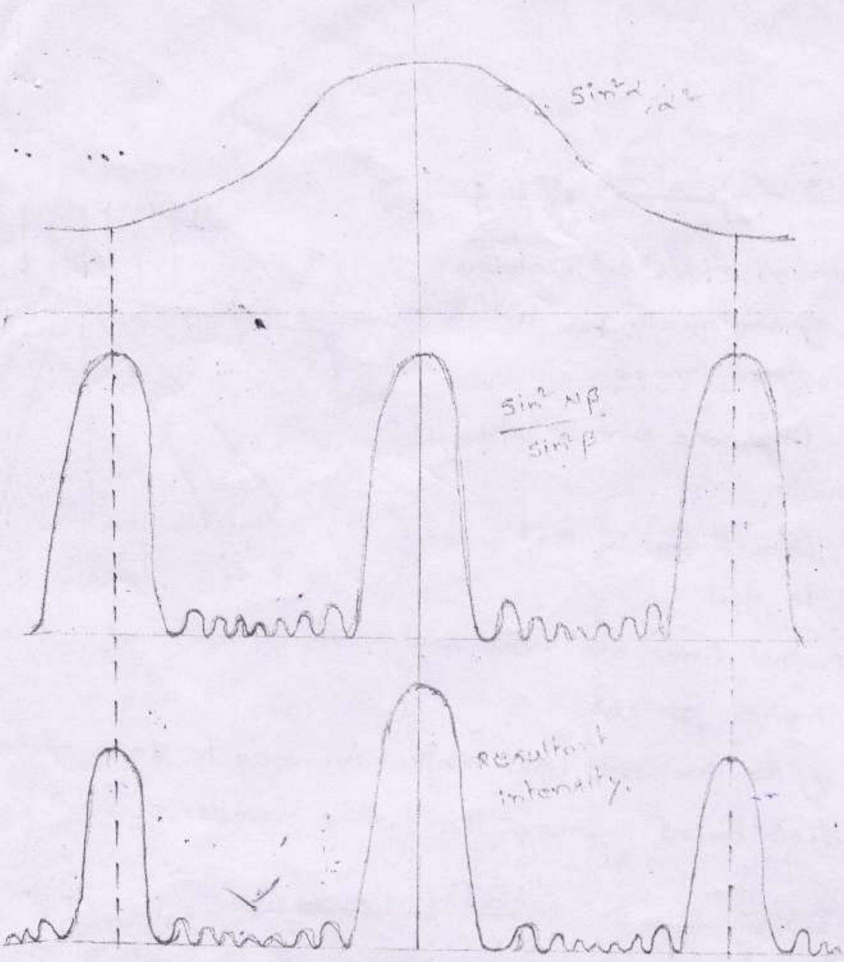

Grating Spectrum :-

We know the principal maxima is formed when,

$$(e+d)\sin\theta = \pm n\lambda \quad \text{— ①}$$

Where $(e+d)$ is the grating element, n the order of maxima and 'θ' is the angle of diffraction for the wavelength 'λ'.

From the above equation,

① For particular wavelength λ, the angle of diffraction 'θ' is different for principal maxima of different orders.

② For white light and for a particular order n, the light of different colours are diffracted in different directions. Longer the wavelength, greater is the angle of diffraction. So, the violet colour is the innermost one and red colour is the outermost one.

③ At centre $(n=0, \theta=0)$, gives maximum of all wavelengths and coincide all the wavelengths the central image is formed same colour as that of the ~~org~~ light source.

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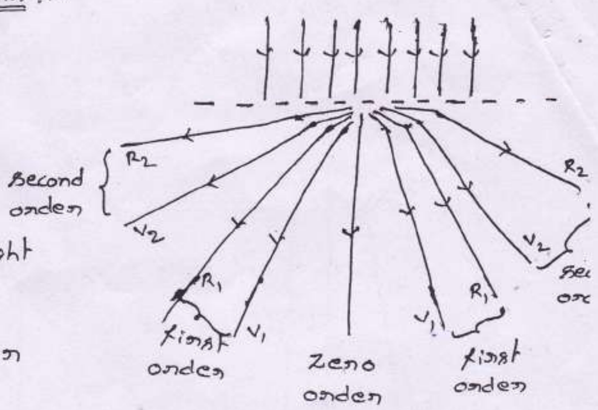
Characteristics of Grating spectrum :-



① Spectrum of different ~~colours~~ orders are situated symmetrically on both sides of zero order image.

② Spectral lines are almost straight and quite sharp.

③ Spectral colours are in the order from violet to red.

④ The spectral lines are more and more dispersed as we go ~~from~~ to higher orders.

⑤. Most of the incident light intensity goes to zero order and rest is distributed among the other orders.

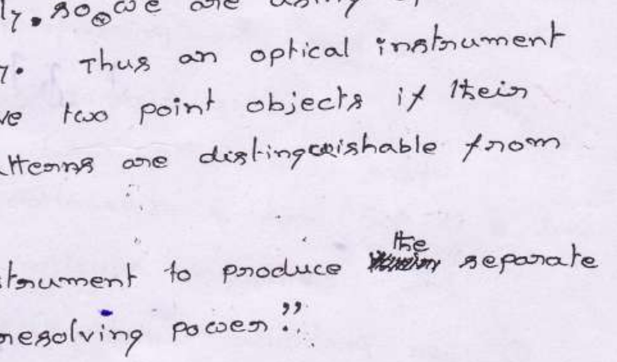
Resolving power of a optical instrument :-

When two objects are very near to each other or they are at very large distance from our eye, the eye may not able to see them as separately. so that's why we are using optical instruments to see them separately. Thus an optical instrument is said to be able to resolve two point objects if their corresponding diffraction patterns are distinguishable from each other.

"The ability of the instrument to produce the separate patterns of two point objects is known as a resolving power".

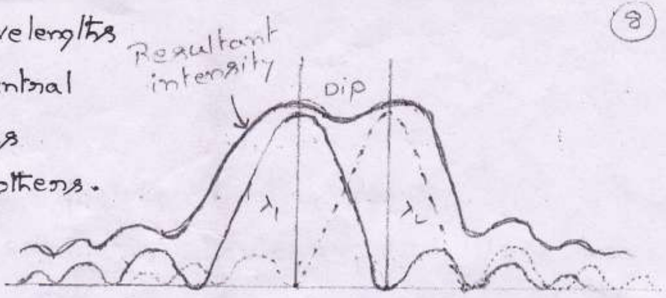
Rayleigh's criterion of resolution:-

"According to Rayleigh's criterion, two point sources are resolvable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa."

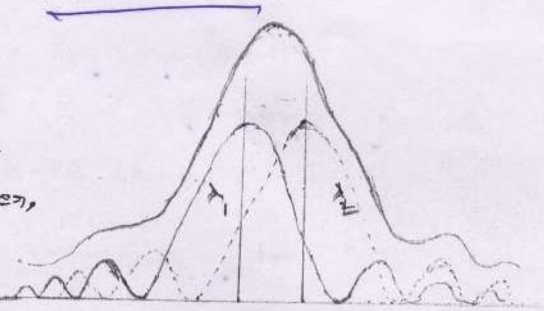


Let us consider λ_1 and λ_2 are the two wavelengths, the difference in wavelengths is such that their principal maxima are separately visible. i.e., There is a distinct point of zero intensity in between the two. Hence the two wavelengths are well resolved.

when the difference in wavelengths is smaller and such that the central maximum of wavelengths coincides with the first minimum of the others. The resultant intensity is shown by thick curve, the curve shows a dip in the middle of two central maxima, i.e., there is a decreate in intensity between the two central maxima indicating the presence of two wavelengths. Hence, they are just resolved.



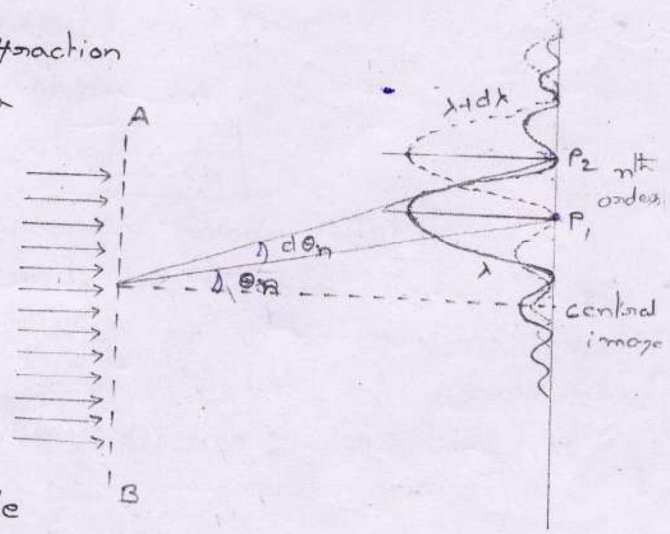
In the third case and the difference between wavelength is so small that central maxima corresponding to two wavelengths come still closer, In this case the resultant intensity is quite smooth without any dip giving impression there is only one wavelength source. Hence the two wavelengths are not resolved.



Resolving Power of a Grating :-

"The resolving power of a grating is defined as its capacity to form separate diffraction maxima of two wavelengths which are very close to each other." and it is measured by $\lambda/d\lambda$. where '$d\lambda$' is difference in wavelengths, λ is wavelength of either of them (or) mean wavelength.

Let AB be a plane diffraction grating having a N slits and with a grating element of $(e+d)$. Let a beam of light having two wavelengths λ and $\lambda+d\lambda$ incident on the grating. P_1 is the n^{th} principal maxima of wavelength 'λ' at an angle of diffraction θ_n. P_2 is the n^{th} principal maxima of wavelength $(\lambda+d\lambda)$ at an angle of diffraction $(\theta_n+d\theta_n)$.



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According to Rayleigh's criterion, the two wavelengths ~~are~~ will be resolved when the position of P_2 corresponds to first minima of P_1. i.e., the two lines will be resolved if the principal maxima of $(\lambda + d\lambda)$ in a direction $(\theta + d\theta_n)$ falls over the first minima of λ in the same direction $(\theta_n + d\theta_n)$.

The n^{th} principal maxima of ~~θ_n is given~~] λ in direction θ_n is given by

$$(e+d) \sin\theta = n\lambda \quad \text{--- (1)} \quad [\because n^{th} \text{ order}].$$

The first minima of λ in the direction $(\theta_n + d\theta_n)$ is given by

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda \quad \text{--- (2)}$$

The n^{th} principal maxima of $(\lambda + d\lambda)$ in the direction $(\theta_n + d\theta_n)$ is given by,

$$(e+d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \text{--- (3)}$$

$$(3) \times N \Rightarrow N(e+d)\sin(\theta_n + d\theta_n) = nN(\lambda + d\lambda) \quad \text{--- (4)}$$

From (2) & (4)

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nN d\lambda$$

$$\lambda = nN d\lambda$$

$$\boxed{\frac{\lambda}{d\lambda} = nN} \quad \text{--- (5)}$$

The above expression gives the resolving power of a grating and it is directly proportional to order of the spectrum and the total no. of (slits) lines on the grating surface.

From eqn. (1) $(e+d)\sin\theta_n = n\lambda$

$$\Rightarrow n = \frac{(e+d)\sin\theta_n}{\lambda} \quad \text{--- (6)}$$

$$\therefore \frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta_n}{\lambda} \quad \text{--- (7)}$$

This also an expression for resolving power of a grating.