Interference

"The modification in the distribution of intensity of light when two (or) more light rays are superimposed on one another is known as Interference".

When the resultant amplitude is equal to the sum of amplitudes of two waves then the interference is known as constructive ~~interference~~. Interference

When the resultant amplitude is equal to the difference of two amplitudes of ~~to~~ then the interference is known as destructive ~~interference~~ interference.

Superposition Principle:-

When two (or) more waves acting simultaneously at a point in the medium, the resultant displacement of the particle ~~at~~ is equal to the algebraic sum of the displacements of the particle due to individual waves in the absence of others.

If y_1 and y_2 be the displacements of the individual waves in the absence of the other. Then the resultant displacement is

$y = y_1 + y_2$ [waves are in same direction]

$y = y_1 - y_2$ [waves are in opposite direction]

Conditions for Interference:-

(1). Two sources should be monochromatic.

(2). Two sources should be coherent.

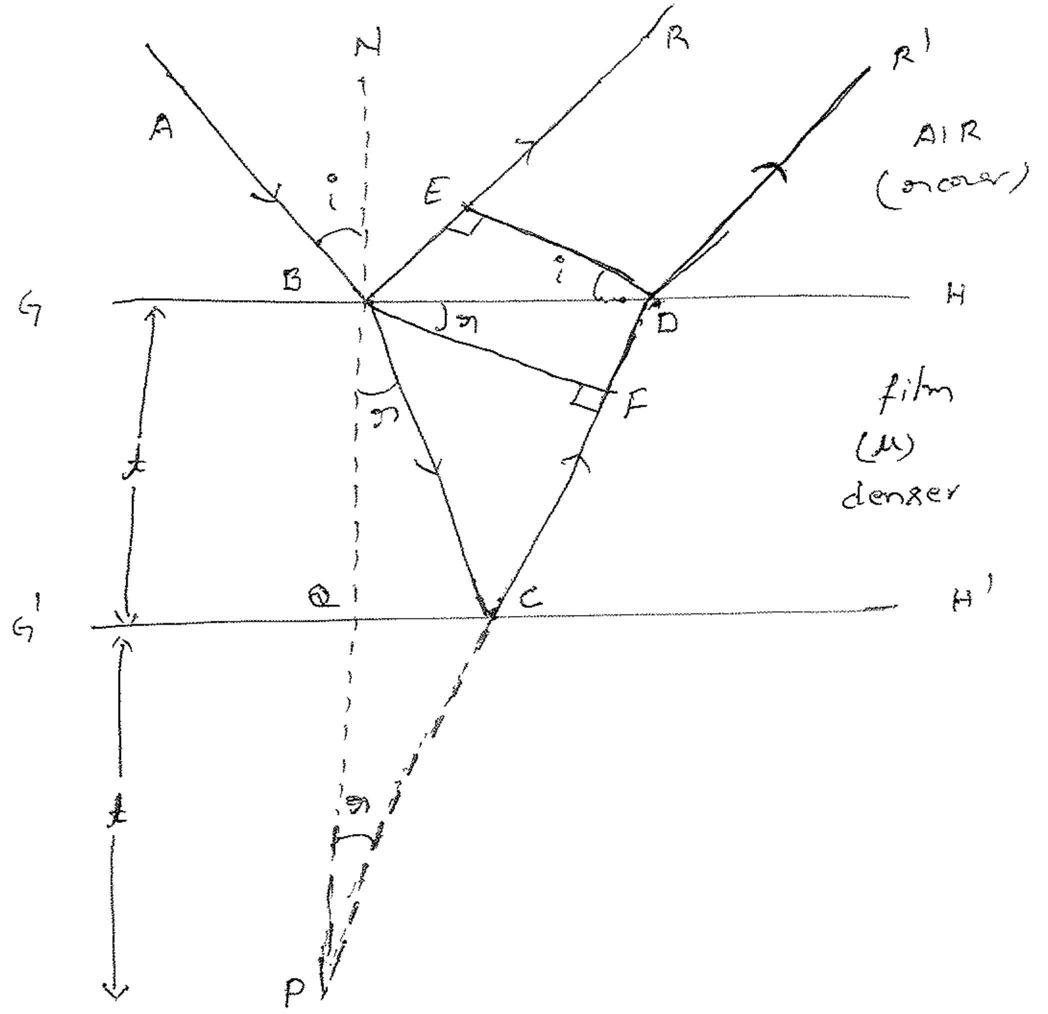
(3). The amplitudes of the two waves should be equal.

(4). The background of the screen should be dark.

(5). The distance between the ~~sour~~ slits should be small.

(6). The distance between the source and screen should be large.

Interference in thin films due to reflected light:-



Let GH and G'H' be the top and bottom layers of a transparent thin film of uniform thickness 't' and refractive index 'μ'. Suppose a light ray AB be incident on the top layer of the thin film ~~with~~ at an angle of incidence 'i'. It is partly reflected along BR and after one internal reflection from the bottom layer of the thin film reflected as DR' with an angle of refraction 'r'.

Now, the optical path travelled by the first reflected light ray is

$$\Delta_1 = AB + BR \quad \text{--- (1)}$$

The optical path travelled by the second reflected light ray is,

$$\Delta_2 = AB + \mu\, BC + \mu\, CD + DR' \quad \text{--- (2)}$$

The optical path difference b/w the two ray is

$$\Delta = \Delta_2 - \Delta_1$$

$$= \cancel{AB} + \mu(BC + CD) + DR' - \cancel{AB} - BR$$

$$= \mu(BC + CD) + DR' - BR \quad \text{--- (3)}$$

~~suppose~~ Draw a normal from D to BR and also another normal for B to CD. Further extend back the DC to meet the normal ~~BQ~~ NQ at P.

Then equation ~~eqn~~ 3 can be written as

$$\Delta = \mu(BC + CD) + DR' - (BE + ER)$$

$$= \mu(BC + CD) - BE \qquad [\because DR' = ER]$$

$$\therefore \Delta = \mu(BC + CD) - BE \quad \text{—— (4)}.$$

From Snell's law we know $\mu = \frac{\sin i}{\sin r}$

ΔBED ; $\sin i = BE/BD =$

ΔBFD ; $\sin r = FD/BD$

$$\Rightarrow \mu = \frac{\sin i}{\sin r} = \frac{BE/\cancel{BD}}{FD/\cancel{BD}} = \frac{BE}{FD}$$

$$\Rightarrow BE = \mu FD \quad \text{— (5)}$$

substitute eqn. 5 in eqn. 4 we get

$$\Delta = \mu(BC + CD) - \mu FD$$

$$= \mu(BC + CF + FD - FD)$$

$$= \mu(BC + CF)$$

$$= \mu(PC + CF) \qquad [\because BC = PC]$$

$$\Delta = \mu PF \quad \text{— (6)}$$

From ΔBPF, $\cos r = \frac{PF}{BP}$

$$\Rightarrow PF = BP\cos r$$

$$PF = 2t\cos r$$

$$\therefore \Delta = 2\mu t\cos r \quad \text{— (7)}.$$

But according to stoke's theorem, if a light ray is reflected back from a denser medium it will undergo a phase change of π (or) a path change of $\lambda/2$

$$\boxed{\therefore \Delta = 2\mu t\cos r \pm \lambda/2} \quad \text{— (8)}.$$

The above equation is also termed as "cosine law" in thin film. Condition for constructive interference (or maxima), is $\Delta = n\lambda$

$$\Rightarrow \boxed{2\mu t\cos r = (2n \pm 1)\lambda/2} \quad \text{— (9)}$$

where, $n = 0, 1, 2, 3 \ldots$ etc.

i.e. at this condition the thin film appear as bright.

Condition for destructive interference (or) minima is

$$\Delta = (2n+1)\lambda/2$$

$$\Rightarrow \quad 2\mu t \cos r \pm \lambda/2 = (2n+1)\lambda/2$$

$$\Rightarrow \quad \boxed{2\mu t \cos r = n\lambda} \quad \text{where } n = 1,2,3\ldots \quad — (10)$$

At this condition the film will appear as dark.

<u>Note</u> :-

(i) Similarly we can get the condition for the interference in thin films due to transmitted light as

$$\boxed{\Delta = 2\mu t \cos r} \quad — (11)$$

Then the condition for maxima will be

$$\Delta = n\lambda$$

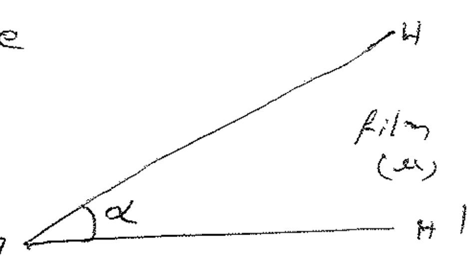
$$\Rightarrow \quad \boxed{2\mu t \cos r = n\lambda} \quad — (12)$$

The condition for minima will be

$$\Delta = (2n+1)\lambda/2$$

$$\Rightarrow \quad \boxed{2\mu t \cos r = (2n+1)\lambda/2} \quad — (13).$$

(ii). Suppose if we consider a wedge shaped thin film as shown in figure and adopting the same ray optics procedure, we will get the path difference between the two reflected light rays as



$$\Delta = 2\mu t \cos(r+\alpha) \pm \lambda/2$$

Where 'α' is the wedge angle.

The condition for maxima will be

$$2\mu t \cos(r+\alpha) = (2n\pm1)\lambda/2$$

The condition for minima will be

$$2\mu t \cos(r+\alpha) = n\lambda.$$

(iii). From the wedge shaped thin film we ~~can't~~ ~~the~~ can get fringe width i.e, the spaceing b/n bright (or) dark fringes is

$\cancel{\beta = \frac{\lambda L}{2t}}$ $\boxed{\beta = \frac{\lambda L}{2t}}$

Colours in thin films :-

Interference is responsible for the formation of colours in thin films like soap bubbles, water-oil films.

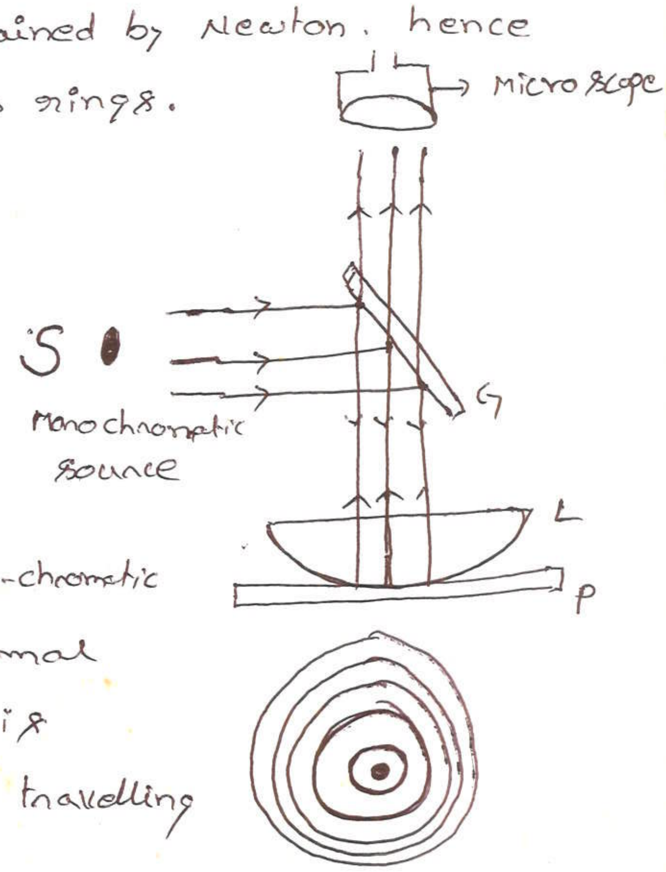
When a light is incident on a thin film it is split up by reflection at the top and bottom surfaces of the films. These reflected light rays are interfere and form different colours. From cosine law in thin films ($2\mu t\cos r \pm \lambda/2$), the condition for interference depends on thickness 't', wavelength 'λ', and angle of refraction 'r'. So, depending on our eye position i.e., for a particular value of 'r', which colour wavelength 'λ' satisfy the condition that colour will appear at that position. In this way for different eye positions i.e, different 'r' values different colours are appear as maximum.

Newtons Rings :-

When a plano-convex lens with its convex surface is placed on a glass plate, an air film of gradually increasing thickness is formed between them. If a monochromatic light is allowed to incident normally then alternate bright and dark concentric rings are observed. This phenomemon was first explained by Newton. hence these rings are called as Newton rings.

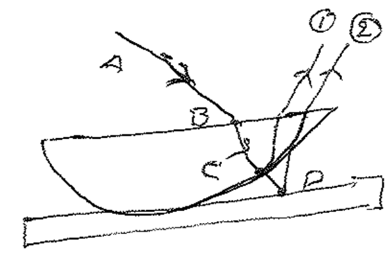
Experimental arrangment:-

A planoconvex lens 'L' is placed on a plane glass plate 'p' as shown in figure. Another glass plate 'G' which is kept at an angle 45° ~~with~~ to the incoming mono-chromatic light in order to create the normal incidence. The reflected light is observed normally by using a travelling microscope 'M'.



Newton Rings

When a monochromatic light ray (AB) from the source 'S' is incident on the arrangement normally we will get two reflected light rays, one from the convex surface of the lens (C) and another one from the top layer of the bottom glass plate (D) as shown in figure.



Now, these two reflected light rays from the air film will interfere each other and forms the Newton Rings.

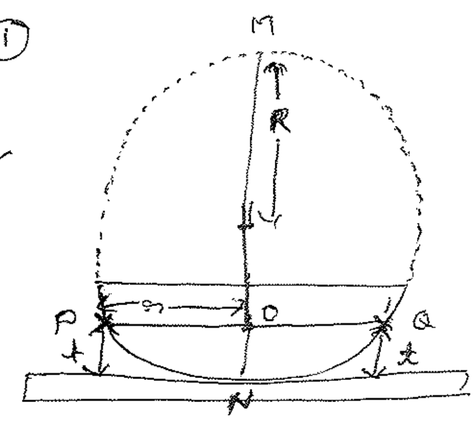
We know from the wedge shaped ~~air~~ film the path difference, $\Delta = 2\mu t \cos(r+\alpha) \pm \lambda/2$

Here, for air film $\mu = 1$ and for normal incidence $r \simeq 0$

~~for~~ large Radius of curvature $\alpha \to 0 \Rightarrow \cos(r+\alpha) = \cos 0 \to 1$

$\therefore \quad \Delta = 2 \cdot 1 \cdot t \cdot 1 + \lambda/2$

$\Rightarrow \quad \Delta = 2t + \lambda/2$ — ①



let us take the enlarge view of the lens, if 'r' is the radius of a ~~dark~~ ring for a particular thickness of the film 't'. Then from the property of chords

$ON \times OM = OP \times OQ$

$\Rightarrow t \times (2R - t) = r \times r$

$\Rightarrow r^2 = 2Rt - t^2$

$\Rightarrow r^2 \approx 2Rt \quad [\because R >> t]$

$\Rightarrow t = \dfrac{r^2}{2R}$ — ②

substitute eqn. ② in ① we will get

$\Delta = 2 \cdot \dfrac{r^2}{2R} + \lambda/2$

$\Rightarrow \Delta = \dfrac{r^2}{R} + \dfrac{\lambda}{2}$

$\Rightarrow \boxed{\Delta = \dfrac{D^2}{4R} + \lambda/2}$ — ③

Here D = Diameter of the ring $(r = D/2)$.

Bright ring :-

we know, the condition for the bright ring (Constructive)

path difference, $\Delta = n\lambda$

$$\Rightarrow \frac{D^2}{4R} + \lambda/2 = n\lambda \quad [\because \text{from eqn } ③]$$

$$\Rightarrow \frac{D^2}{4R} = (2n-1)\lambda/2$$

$$\Rightarrow D^2 = 2(2n-1)\lambda R$$

$$\Rightarrow D_n = \sqrt{(2n-1)}\sqrt{2\lambda R}$$

$$\Rightarrow D_n \propto \sqrt{(2n-1)} \quad — ④$$

$\therefore$ The diameter of the bright ring is proportional to the square root of odd natural numbers.

Dark ring :-

We know the condition for the dark ring (Destructive)

path difference, $\Delta = (2n+1)\lambda/2$

$$\Rightarrow \frac{D^2}{4R} + \lambda/2 = (2n+1)\lambda/2$$

$$\Rightarrow \frac{D^2}{4R} = n\lambda$$

$$\Rightarrow D^2 = 4Rn\lambda$$

$$\Rightarrow D^2 = 2n \cdot 2\lambda R$$

$$\Rightarrow D = \sqrt{2n}\sqrt{2\lambda R}$$

$$\Rightarrow D_n \propto \sqrt{2n} \quad — ⑤$$

$\therefore$ The diameter of the dark ring is proportional to square root of even natural numbers.

Determination of wavelength :-

From the dark ring condition, we can write

for n^{th} ring (dark) $D_n^2 = 4n\lambda R \quad — ⑥$

for m^{th} dark ring $D_m^2 = 4m\lambda R \quad — ⑦$

$\Rightarrow$ ⑦ - ⑥ we will get $D_m^2 - D_n^2 = 4\lambda R(m-n)$

$$\Rightarrow \boxed{\lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}} \quad — ⑧$$

By measuring the diameter of the m^{th} and n^{th} dark rings and by knowing the Radius of curvature 'R' value we can determine the wavelength of the light by using Newton Rings.

Determination of refractive index of a liquid by using Newton Rings :-

First of all perform the Newton rings experiment by using air film b/n the plano convex lens and glass plate. and measure the diameter of the m^{th} and n^{th} dark rings.

Theoretically i.e,

$$D_m^2 = 4m\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$D_m^2 - D_n^2 = 4\lambda R(m-n) \quad \text{--- (1)}$$

Now the liquid whose refractive index is to be determined is placed in between the lens and the glass plate. with out disturbing the arrangment. Then again measure the diameter of the same m^{th} and n^{th} dark rings

Theoretically Now,

$$(D_m')^2 = 4m\lambda R/\mu$$

$$(D_n')^2 = 4n\lambda R/\mu$$

$$\Rightarrow D_m'^2 - D_n'^2 = \frac{4\lambda R(m-n)}{\mu}$$

$$D_m'^2 - D_n'^2 = \frac{D_m^2 - D_n^2}{\mu}$$

$$\Rightarrow \boxed{\mu = \frac{D_m^2 - D_n^2}{(D_m')^2 - (D_n')^2}} \quad \text{--- (2)}$$

By using the above formula and on substituting the diameters of the m^{th} and n^{th} dark rings in air and in liquids the refractive index of the liquid will be measured easily.

Problems

① A parallel beam of light of wavelength $6000\,A^\circ$ is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction into the plate is 50°. Find the least thickness of the glass plate which will appear dark by reflection.

Ans:- $t = 311$ nm

~~the wavelength of~~

②. A soap film of refractive index 1.33 and thickness $5000\,A^\circ$ is exposed to white light. What wavelength in the visible region are reflected.

Ans:- $\lambda = 5320\,A^\circ$.

③. Newton's rings are observed in the reflected light of wavelength $5900\,A^\circ$. The diameter of 10^{th} dark ring is 0.5 cm. Find the radius of curvature of the lens used.

Ans:- $R = 1.059$ m

④. In Newton's rings experiment the diameter of 15^{th} ring was found to be 0.59 cm and that of 5^{th} ring 0.336 cm. The radius of curvature of the lens is 100 cm. Find the wavelength of the light

Ans:- $\lambda = 588$ nm

⑤. In Newton's rings experiment the diameter of the 12^{th} ring changes from 1.45 cm to 1.25 cm. when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid

Ans:- $\mu = 1.3456$

⑥. Calculate the thickness of the air film at 10^{th} dark ring in Newton's rings system viewed normally by a reflected light of wavelength 500 nm. The diameter of the 10^{th} dark ring is 2 cm.

Ans: $t = 2.5\,\mu m$

⑦. In a Newton's ring experiment the diameters of 5^{th} and 15^{th} dark rings are 0.336 cm and 0.59 cm respectively. If radius of curvature of plano-convex lens is 100 cm, find the wavelength of monochromatic light. What happens to ring diameter if air film is replaced with liquid of refractive index 1.33,

Ans: $\lambda = 5880\,A^\circ$; ring diameter decreases.