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Technological development in the Stirling cycle engines

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Abstract

The performance of Stirling engines meets the demands of the efficient use of energy and environmental security and therefore they are the subject of much current interest. Hence, the development and investigation of Stirling engine have come to the attention of many scientific institutes and commercial companies. The Stirling engine is both practically and theoretically a significant device, its practical virtue is simple, reliable and safe which was recognized for a full century following its invention by Robert Stirling in 1816. The engine operates on a closed thermodynamic cycle, which is reversible. Today Stirling cycle-based systems are in commercial use as a heat pump, cryogenic refrigeration and air liquefaction. As a prime mover, Stirling cycles remain the subject of research and development efforts. The objective of this paper is to provide fundamental information and present a detailed review of the past efforts taken for the development of the Stirling cycle engine and techniques used for engine analysis. A number of attempts have been made by researches to build and improve the performance of Stirling engines. It is seen that for successful operation of engine system with good efficiency a careful design of heat exchangers, proper selection of drive mechanism and engine configuration is essential. The study indicates that a Stirling cycle engine working with relatively low temperature with air of helium as working fluid is potentially attractive engines of the future, especially solar-powered low-temperature differential Stirling engines with vertical, double acting, and gamma configuration.
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Keywords: Stirling cycle engine; Heater; Cooler; Regenerator; Engine mechanism

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Nomenclature

a, b	convective heat transfer coefficient at particular surface
A_h	heater surface area
A_r	regenerator heat transfer area
B_N	Beale number
C_P	sp. heat capacity of working fluid at constant pressure
C_{PM}	sp. heat capacity of regenerator
C_V	sp. heat capacity of working fluid at constant volume
d_h	heater tube diameter
h	convective heat transfer coefficient
h_{bulk}	bulk heat transfer coefficient
h_h	convective heat transfer coefficient of heater
k	swept volume ratio
k_1 & k_2	ratio of regenerative process
k_h	thermal conductivity of heater tube material
l_h	heater length
L_R	regenerator length
M	mass of working fluid
m_c	mass of working fluid in compression space
m_{ck}	mass flow of working fluid between compression space and cooler
m_e	mass of working fluid in expansion space
m_f	fluid mass flow rate
M_{FR}	fluid mass resident in regenerator
m_h	mass of working fluid in heater
m_{he}	mass flow of working fluid between expansion space and heater
m_i	mass flow of working fluid to a Rallis generalized cell
m_k	mass of working fluid in cooler
M_M	mass of regenerator
m_o	mass flow of working fluid from a Rallis generalized cell
m_r	mass of working fluid in regenerator
n	mole number of working fluid
Nu	Nusselt number
p	instantaneous pressure
P_1, P_2, P_3, P_4	pressure of gas at silent pints on PV diagram
P_{AM}	arithmetic mean of pressure at regenerator end
P_i	engine power
p_{max}	maximum cycle pressure
p_{mean}	mean cycle pressure
Pr	Prandtl number
Q	heat transfer
Q_C	heat energy loss during compression
Q_h	heat energy supplied during heating
Q_k	heat energy loss during cooling
Q_R	rate of heat energy rejected
Q_s	rate of heat energy supplied
R	gas constant

Re	Reynolds number
r_v	volume/compression ratio
s	entropy
S	reduced dead volume
St	Stanton number
T	time coordinate
T_1, T_2, T_3, T_4	temperature of working fluid silent points on TS diagram
t_1	time associated with heat transfer at expansion space
t_2	time associated with heat transfer at compression space
T_C	sink temperature
T_c	temperature of working fluid in compression space
$T_{C\ av}$	average temperature of working fluid in compression space
T_{ck}	mean temperature of working fluid at cooler and compression space
T_D	temperature at dead space
T_E	highest temperature of working fluid
T_e	temperature at expansion side
$T_{E\ av}$	average temperature of working fluid in expansion space
T_e	temperature of working fluid in expansion space
T_F	fluid temperature
T_H	temperature at source side
T_h	temperature of working gas at heater
T_{he}	temperature of working fluid at heater and expansion space
T_i	inlet temperature to Urieli's generalized cell
T_k	temperature of working gas at coller
T_M	matrix temperature
T_{max}	maximum cycle temperature
T_{min}	minimum cycle temperature
T_o	outlet temperature at Urieli's generalized cell
v	velocity of gas
V_c	compression cylinder volume variation
V_d	dead space volume
V_e	expansion cylinder volume variation
V_r	regenerator volume
V_{sc}	compression space swept volume
V_{se}	expansion space swept volume
V_T	total volume
W_C	compression work
W_d	work done from engine
W_e	expansion work
W_S	west number
x	dead volume ratio
X	distance

Greek symbols

ρ	c/a

α	phase angle between expansion cylinder and compression cylinder.
ζ	dimensionless length
η_t	thermal efficiency
δ	b/a
θ	$\tan^{-1}((k \sin \alpha)/(k + k \cos \alpha))$
ϕ	Crank angle
γ	isentropic gas index
η	dimensionless time
δ	factor in Schmidt analysis
λ	flow periodic time
μ	gas viscosity
$\in_R$	regenerator effectiveness of efficiency
$\bigwedge$	reduced length
σ	matrix permeability
μ	fluid viscosity
ξ	ratio of reservoir temperature to hot temperature, T_1/T_3
τ	ratio of engine operating temperature
Γ'	ratio of sink to reservoir temperature
Π	reduced period

1. Introduction

The Stirling engine were invented in 1816 by Robert Stirling [1] in Scotland, some 80 years before the invention of diesel engine, and enjoyed substantial commercial success up to the early 1900s. A Stirling cycle machine is a device, which operates on a closed regenerative thermodynamic cycle, with cyclic compression and expansion of the working fluid at different temperature levels. The flow is controlled by volume changes so that there is a net conversion of heat to work or vice versa. The Stirling engines are frequently called by other names, including hot-air or hot-gas engines, or one of a number of designations reserved for particular engine arrangement. In the beginning of 19th century, due to the rapid development of internal combustion engines and electrical machine, further development of Stirling engines was severely hampered.

High heat efficiency, low noise operation and ability of Stirling engines to use many fuels meet the demand of the effective use of energy and environmental security [2]. Stirling engine-based units are considered best among the most effective low-power range solar thermal conversion units [3]. In order to analyze and to improve the performance of three main sub-systems of these units, namely the solar receiver, the thermodynamic gas circuit, and the drive mechanism, simulation codes are under development worldwide. In 1980, with fuel crises, Stirling engines become a viable proposition with rapid advances in material technology. This was the second stage of transformation for the Stirling engines.

This report provides a literature review on Stirling engine technological development. Number of research works on the Stirling engine is discussed. The aim of this review is to find a feasible solution, which may lead to a preliminary conceptual design of a workable Stirling engine.

Future needs of power plants and attractive properties of Stirling engine

Future needs and trends	Attractive Stirling properties
1. Depleting conventional fuels	• Multi-fuel capability • Low fuel consumption
2. Increasing fossil fuel cost	• High efficiency (utilization of non-fossil fuels)
3. Utilization of alternative fuels	• Multi-fuel capability (utilisation of non-fossil fuels)
4. Demand for low noise and less air-polluting prime mover	• Clean combustion • Low noise levels
5. Waste heat recovery [4]	• Low temperature operation

2. Thermodynamics of Stirling cycle engine

Robert Stirling invented the closed cycle regenerative engine and the regenerative heat exchanger. He builds an engine working on the closed thermodynamic cycle and operated. The engine and engine cycle invented by Robert Stirling represented on PV and TS diagram as shown in Fig. 1(a). The cycle consists of four processes namely isothermal compression and expansion and isentropic heat addition and rejection processes in the sequence as shown in Fig. 1. Consider a cylinder containing two opposed pistons with a regenerator between the pistons as shown in Fig. 1(b). The regenerator is like a thermal sponge alternatively absorbing and releasing heat, it is a matrix of finely divided metal in the form of wires or strips. The volume between regenerator and the right side piston is expansion volume and between regenerator and left side piston is compression volume. Expansion volume is maintained at high temperature and compression volume is maintained at low temperature. The temperature gradient of ($T_{max}-T_{min}$) between the ends of regenerator is maintained.

2.1. Engine cycle

To start with a cycle we assume that the compression space piston is at outer dead point (at extreme right side) and the expansion space piston is at inner dead point close to regenerator. All working fluid is in the cold compression space. The compression volume is at maximum and the pressure and temperature are at their minimum values represented by point 1 on PV and TS diagram. The four processes of the thermodynamic cycle are:

Process 1–2*, isothermal compression process*: During compression process from 1 to 2, compression piston moves towards regenerator while the expansion piston remains stationery. The working fluid is compressed in the compression space and the pressure increases from P_1 to P_2. The temperature is maintained constant due to heat flow from cold space to surrounding. Work is done on the working fluid equal in magnitude to the heat rejected from the cycle. There is no change in internal energy and there is a decrease in entropy. Isothermal compression of the working fluid involving heat transfer from

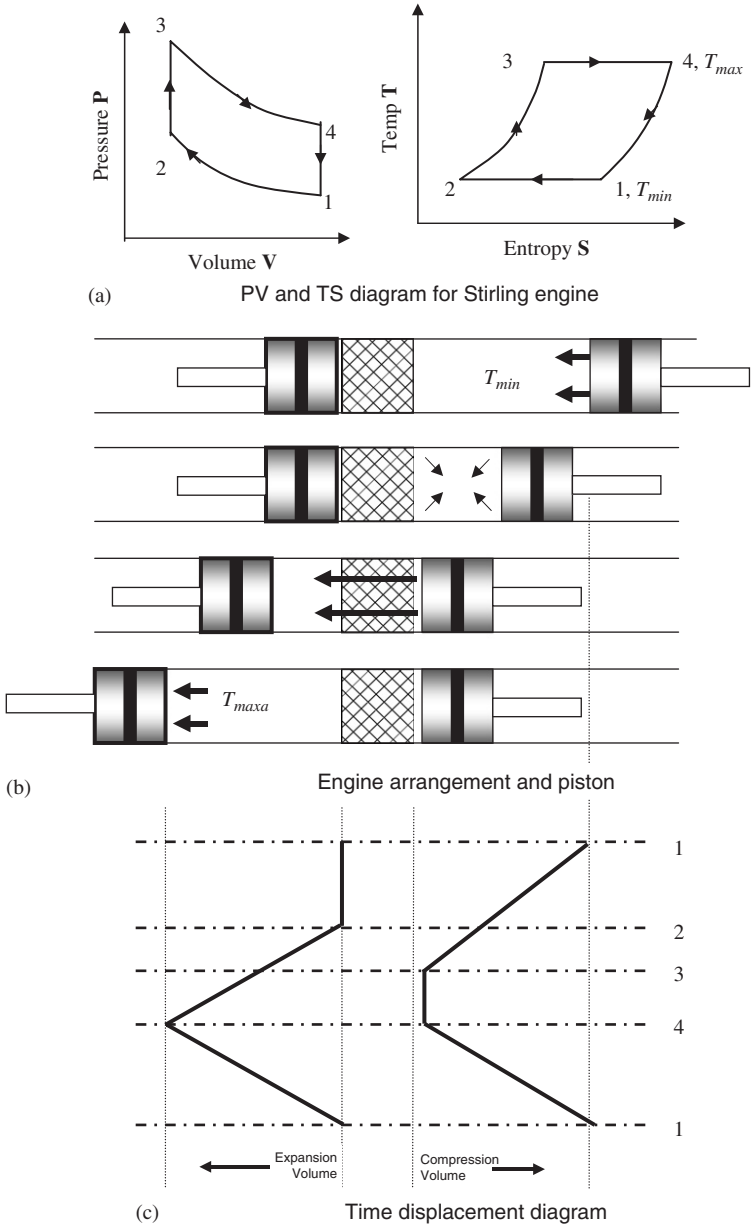

Fig. 1. The Stirling cycle.

working fluid to external dump at $T_{\min}$:

$$P_2 = \frac{P_1 V_1}{V_2} = P_1 r_v,$$

$$T_1 = T_2 = T_{\min},$$

Heat transfer Q = Work done W,

$$Q = W_c = P_1 V_1 \ln(1/r_v) = mRT_1 \ln(1/r_v),$$

$$\text{Change in entropy} = (s_2 - s_2) = R \ln\left(1/r_v\right).$$

Process 2–3, *constant volume regenerative transfer process*: In the process 2–3, both pistons move simultaneously, i.e. compression piston towards regenerator and expansion piston away from regenerator, so that the volume between pistons remains constant. The working fluid is transferred from compression volume to expansion volume through porous media regenerator. Temperature of working fluid increased from T_{min} to T_{max} by heat transfer from regenerator matrix to working fluid. The gradual increase in temperature of working fluid while passing through regenerator causes increase in pressure. No work is done and there is an increase in the entropy and internal energy of the working fluid.

Isochoric (const. volume) heat transfer to working fluid from the regenerator matrix:

$$P_3 = \frac{P_2 T_3}{T_2} = \frac{P_2}{\tau}; \quad V_3 = V_2.$$

If $\tau = (T_2/T_3)$ The temperature ratio, τ defined by Gustav Schmidt [5]:

$$\text{Heat transfer } Q = C_V(T_3 - T_2),$$

$$\text{Work done} = 0,$$

$$\text{Change in entropy} = (s_2 - s_2) = C_V \ln(1/\tau).$$

Process 3–4, *isothermal expansion process*: In the expansion process 3–4, the expansion piston continues to move away from the regenerator towards outer dead piston while compression piston remains stationery at inner dead point adjacent to regenerator. As the expansion proceeds, the pressure decreases as volume increases. The temperature maintained constant by adding heat to the system from external source at T_{max}. Work is done by the working fluid on piston equal in the magnitude to the heat supplied. There is no change in the internal energy, but an increase in the entropy of the working fluid.

$$P_4 = \frac{P_3 V_3}{V_4} = P_3(1/r_v); \quad T_4 = T_3 = T_{max},$$

$$\text{Heat transfer} = \text{Work done}, \quad Q = W = P_3 V_3 \ln r_v = mRT_3 \ln r_v,$$

$$\text{Change in entropy} = (s_3 - s_4) = R \ln r_v.$$

Process 4–1, *constant volume regenerative transfer process*: In the process 4–1 both pistons move simultaneously to transfer working fluid from expansion space to compression space through regenerator at constant volume. During the flow of working fluid through regenerator, the heat is transferred from the working fluid to the regenerator matrix reducing the temperature of working fluid to T_{min}. No work is done; there is a decrease in the internal energy and the entropy of the working fluid.

Isochoric (constant volume) heat rejection:

$$P_1 = \frac{P_4 T_4}{T_1} = P_1 \tau; \quad V_1 = V_4,$$

Heat transfer $Q = C_V(T_1 - T_4)$ and

Change in entropy $= (s_1 - s_4) = C_V \ln \tau$

and if

$$r_v = V_4/V_3 = V_1/V_2$$

the total heat supplied $= RT_3 \ln r_v$ and the total heat rejected $= RT_1 \ln r_v$.

Then the efficiency can be written as

$$\eta_t = \frac{mRT_3 \ln(r_v) - mRT_1 \ln(r_v)}{mRT_3 \ln(r_v)},$$

$$\eta_t = 1 - \frac{T_1}{T_3} = 1 - \frac{T_{min}}{T_{max}} = 1 - \tau. \quad (1)$$

The Stirling cycle is highly idealized thermodynamic cycle, which consists of two isothermal and two constant volume processes and the cycle is thermodynamically reversible. The first assumptions of isothermal working and heat exchange imply that the heat exchangers are required to be perfectly effective and to do so infinite rate of heat transfer is required between cylinder wall and working fluid. The second assumption requires zero heat transfer between walls and working fluid, both assumptions remains invalid in actual engine operation.

2.2. Isothermal analysis

To attain effective heat transfer Urieli, Berchowitz [6] have modified ideal engine as a five component serially connected model consisting, respectively, of compression space c, cooler k, regenerator r, heater h and expansion space e as shown in Fig. 2. The interconnecting duct to these volumes is added in to corresponding volumes for simplicity. The engine analysis is done for the heat transfer to working gas by evaluating the area enclosed in PV diagram. The analysis is started with an assumption that the total mass of working gas in the machine is constant.

$$M = m_c + m_k + m_r + m_h + m_e.$$

Considering pressure throughout the engine constant and temperature in compression space and cooler and expansion space and heater is constant, i.e. $T_c = T_k, T_h = T_e$.

Using ideal gas equation:

$$M = p/R\left(V_c/T_k + V_k/T_k + V_r/T_r + V_h/T_h + V_e/T_h\right).$$

The effective regenerator temperature T_r can be written as

$$T_r = (T_h - T_k)/\ln(T_h/T_k).$$

So engine cycle pressure,

$$p = \mathrm{MR}\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h/T_k)}{(T_h - T_k)} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right)^{-1}. \quad (2)$$

The equation given by Senft [7] for working gas pressure P as a function of volume variation V_c and V_e. Work is done on the surrounding by virtue of the varying volumes of the working space volume V_c and V_e.

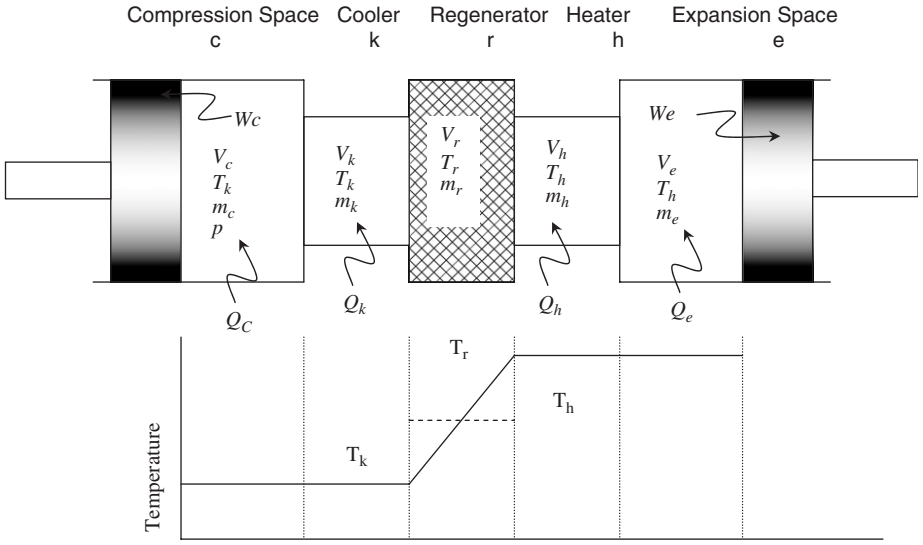

Fig 2. Ideal isothermal model.

Over a complete cycle the total work done by the engine is the algebraic sum of the work done by compression and expansion spaces.

$$W = W_c + W_e,$$

$$W = \oint p\,dV_c + \oint p\,dV_e,$$

$$W = \oint p\left(\frac{dV_c}{d\phi} + \frac{dV_e}{d\phi}\right)d\phi \quad \text{where } \phi \text{ is crank angle.} \tag{3}$$

2.3. Heat transfer in isothermal engine model

To investigate heat transfer to heater and cooler it is necessary to consider the energy equation for working gas. A generalized cell of working spaces modeled by Rallis [8] is as shown in Fig. 3, which may be either reduced to working space cell or a heat exchanger cell. The enthalpy is transferred in to the cell by means of mass flow m_i and temperature T_i and out of the cell by m_o and temperature T_o. The derivative operator is denoted by D and D_m refer to mass derivative (dm/dt)

The statement of energy equation for the working gas in the generalized cell is

{Rate of heat transfer into cell} + {Net enthalphy converted to cell}
= {Rate of work done on surrounding}
+ {Rate of increase of internal energy in the cell}.

Mathematically the statement becomes,

$$DQ + (C_P T_i m_i - C_P T_o m_o) = DW_d + + C_V D(mT), \tag{4}$$

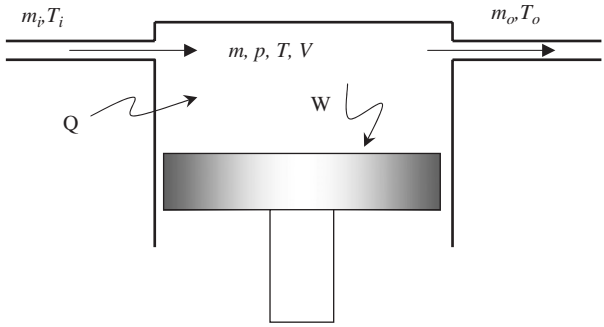

Fig 3. A generalized cell of working spaces modeled by C.J. Rallis.

where C_P and C_V are the specific heat capacities of the gas at constant pressure and constant volume, respectively. Eq. (4) is the well-known classical form of the energy equation for non-steady flow in which kinetic and potential energy terms have been neglected.

In the isothermal model for the compression and expansion spaces as well as for the heater and the cooler we have that $T_i = T_o = T$. Thus

$$DQ = C_P T(m_o - m_i) + C_V T Dm + DW_d.$$

From mass conservation considerations, the difference in the mass flow $(m_o - m_i)$ is simply the rate of mass accumulation within the cell and for working gas $R = C_P - C_V$. Then equation then simplifies to

$$DQ = RTDm + DW_d.$$

The net heat transferred to the working gas over the cycle is given by cyclic integration of the equation

$$Q = \oint DQ = RT \oint Dm + \oint DW_d. \tag{5}$$

2.4. Heat transfer in adiabatic engine model

In an ideal Stirling cycle engine isothermal model, compression and expansion spaces were maintained at isothermal (constant temperature) conditions. Rankine [9] has proved that neither the heating nor cooling takes place exactly at constant volumes or at constant temperature. This led to the paradoxical situation that neither the heater nor the cooler contributed any net heat transfer over the cycle and hence were redundant. All required heat transfer occurred across the boundaries of the isothermal working space. However, the practical requirements for effective heat exchange conflict with working space designed for expanding and compressing the working gas. Thus in real machines, the working space will tend to be adiabatic rather than isothermal. This implies that the net heat transfer over the cycle must be provided by the heat exchangers as shown in Fig. 4.

The Stirling cycle machines having non-isothermal working space were first analyzed by Finkelstein [10] in 1960 and this analysis represented the most significant development in

the century. His model assumed finite heat transfer in the working space by means of heat transfer coefficient. The resulting temperature variations in the gas in the working spaces led to temperature discontinuity at the working space interface. The theory presented by Finkelstein was further explored by Walker and Kahn [11] in 1965, with particular emphasis on the limiting case of adiabatic compression and expansion process. A study of the effect of four important design parameters temperature ratio, phase angle, swept volume ratio and dead volume ratio were made. More recently, the adiabatic analysis has been considered by Berchowitz [12].

Here Urieli's ideal adiabatic model is considered for analysis. The engine is configured as a five component, as configured earlier with perfect heat exchangers and regenerator. Thus the gas in cooler and heater is maintained at isothermal condition at temperature T_k and T_h, respectively. Both in the regenerator matrix and gas in the regenerator volume have the individual linear temperature distribution: the gas flowing through regenerator–cooler interface being at cooler temperature T_k, regenerator–heater interface at heater temperature T_h. The working spaces are, however, assumed to be adiabatic and thus the temperature T_c and T_e will vary over the cycle in accordance with adiabatic nature of these spaces as shown in Fig. 4. Energy equation applied to a generalized all can be written as

$$DQ + (C_P T_i m_i - C_P T_o m_o) = dW_d + C_V D(mT). \tag{6}$$

The equation of state is given by $PV = mRT$ and $C_P - C_V = R$.
Hence

$$C_P = \frac{R\gamma}{(\gamma - 1)}, \quad C_V = \frac{R}{(\gamma - 1)}, \quad \text{where } \gamma = C_P/C_V.$$

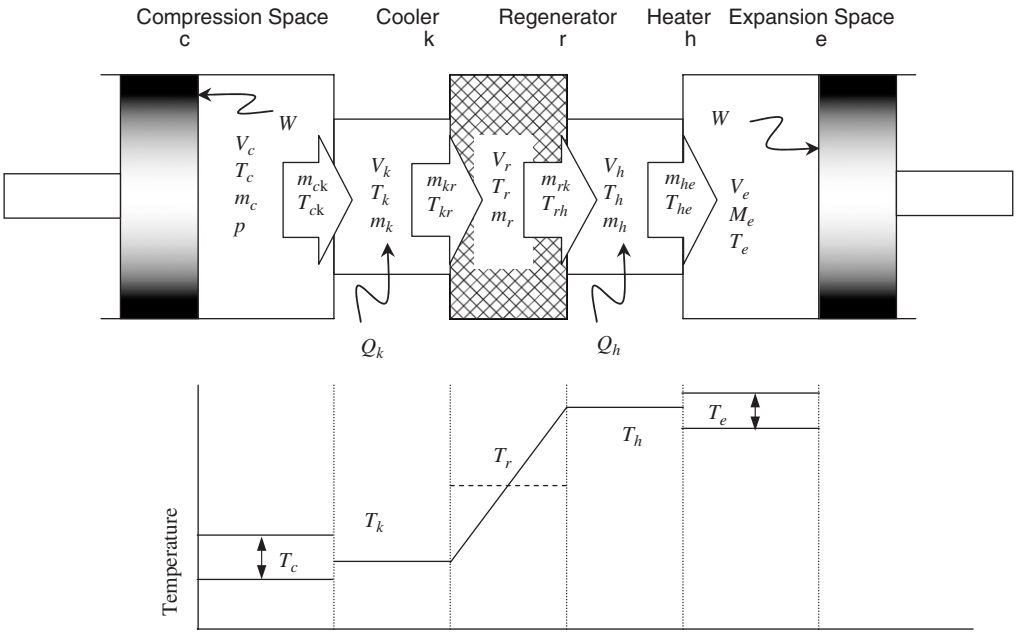

Fig 4. Ideal adiabatic model.

Taking logarithm of both sides of equation and differentiating we obtain differential form of the equation of state.

$$\frac{Dp}{p}+\frac{DV}{V}=\frac{Dm}{m}+\frac{DV}{V}.$$

The total mass of working fluid remains constant, so

$$m_c+m_k+m_r+m_h+m_e=M.$$

Differentiating the above equation

$$Dm_c+Dm_k+Dm_r+Dm_h+Dm_e=0.$$

Now for all heat exchanger cells, since the respective volumes and temperatures are constant, the differential equation of the equation of state reduces to

$$Dm/m=Dp/p. \tag{7}$$

Applying Eq. (7) to each of three heat exchangers and substituting in equation

$$Dm_c+Dm_e+Dp(m_k/p+m_r/p+m_h/p)=0.$$

Substituting the gas equation

$$Dm_c+Dm_e+(Dp/R)\left(\frac{V_k}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}\right)=0.$$

Appling energy Eq. (6) to the compression space we obtain

$$DQ_C-C_PT_{ck}m_{ck}=DW_C+C_VD(m_cT_c).$$

However, the compression space is adiabatic, $DQ_C=0$, further more the workdone $DW_C=pDV_c$, from continuity considerations the rate of accumulation of gas Dm_c is equal the mass inflow of gas given by m_{ck}

Thus the equation reduces to

$$C_PT_{ck}Dm_c=pDV_c+C_VD(m_cT_c). \tag{8}$$

Substituting the equation of state and associated ideal gas relations

$$Dm_c=(pDV_c+V_cD_p/\gamma)/(RT_{ck}). \tag{9}$$

Similarly for the expansion space

$$Dm_e=(pDV_e+V_eD_p/\gamma)/(RT_{he}). \tag{10}$$

Substituting equations and simplifying

$$Dp=\frac{-\gamma p((DV_c/T_{ck})+(DV_e/T_{he}))}{[V_c/T_{ck}+\gamma(V_k/R_k+V_r/T_r+V_h/T_h)+V_e/T_{he}]}. \tag{11}$$

We observe that Eqs. (9) and (11) are two simultaneous differential equations in the variables p and m_c. Once p and m_c are evaluated, the rest of the variables may be obtained by means of the equations of state and mass balance. Volume variations DV_c, DV_e, V_c and V_e are available analytically and all other parameters in Eqs. (9) and (11) are constant excluding T_{ck} and T_{he}. The interface temperatures T_{ck} and T_{he} are conditional on the direction of mass flow. In order to evaluate the mass flow (and thus the direction of mass

flow) we consider the continuity equation, given by

$$Dm = m_{\mathrm{i}} - m_{\mathrm{o}}.$$

Above equation is simply a statement that the rate of mass accumulation in a cell is equal to the net mass flow in to that cell, and is oblivious by inspection.

Successively applying above equation to each of the cells as shown in Fig. 4

$$\begin{aligned} m_{\mathrm{ck}} &= -Dm_{\mathrm{c}}, \\ m_{\mathrm{kr}} &= m_{\mathrm{ck}} - Dm_{\mathrm{k}}, \\ m_{\mathrm{rh}} &= m_{\mathrm{kr}} - Dm_{\mathrm{r}}, \\ m_{\mathrm{he}} &= m_{\mathrm{rh}} - Dm_{\mathrm{h}}. \end{aligned} \tag{12}$$

The total work done by the engine is the algebraic sum of the work done by expansion and compression spaces.

$$DW = pDV_{\mathrm{c}} + pDV_{\mathrm{e}}.$$

Considering the energy Eq. (6) substituting values of DW and mT in to Eq. (6) and simplifying we obtain a more convenient form of the energy equation

$$DQ + (C_{\mathrm{P}} T_{\mathrm{i}} m_{\mathrm{i}} - C_{\mathrm{P}} T_{\mathrm{o}} m_{\mathrm{o}}) = (C_{\mathrm{P}} pDV + C_{\mathrm{V}} VDp)/R. \tag{13}$$

In the heat exchanger spaces no work is done, since the respective volumes are constant. Thus applying Eq. (13) to the individual heat exchanger spaces we obtain

$$\begin{aligned} DQ_{\mathrm{k}} &= V_{\mathrm{k}} DpC_{\mathrm{V}}/R - C_{\mathrm{P}}(T_{\mathrm{ck}} m_{\mathrm{ck}} - T_{\mathrm{kr}} m_{\mathrm{kr}}), \\ DQ_{\mathrm{R}} &= V_{\mathrm{r}} DpC_{\mathrm{V}}/R - C_{\mathrm{P}}(T_{\mathrm{kr}} m_{\mathrm{kr}} - T_{\mathrm{rh}} m_{\mathrm{rh}}), \\ DQ_{\mathrm{h}} &= V_{\mathrm{h}} DpC_{\mathrm{V}}/R - C_{\mathrm{P}}(T_{\mathrm{rh}} m_{\mathrm{rh}} - T_{\mathrm{he}} m_{\mathrm{he}}). \end{aligned} \tag{14}$$

We note that since the heat exchangers are isothermal and the regenerator is ideal, we have by definition $T_{\mathrm{kr}} = T_{\mathrm{k}}$, $T_{\mathrm{rh}} = T_{\mathrm{h}}$. Mahkamov [13,14] is used computational fluid dynamic approach to investigate working process of an engine model made by Urieli [15]. This gives a close insight in the engine analysis than earlier one. A computer program to simulation of engine system is done by Organ [16], also Rallis [8] has compared ideal regenerative cycle having isothermal and adiabatic process of compression and expansion. The analysis shows that thermal efficiency is function of regenerative effectiveness and volume ratio.

2.5. Maximum theoretical obtainable efficiency of Stirling cycle engine

The actual Stirling cycle engine subjected to heat transfer, internal thermal losses and mechanical friction losses. To estimate these losses Senft [17] has defined some ratios of engine temperatures. The ratio of engine lower to higher operating temperature is defined as $\tau = T_{\mathrm{k}}/T_{\mathrm{E}}$, the ratio of sink to reservoir temperature is defined as $\Gamma = T_{\mathrm{C}}/T_{\mathrm{H}}$, the ratio of reservoir temperature to hot engine temperature is denoted by $\xi = T_{\mathrm{E}}/T_{\mathrm{H}}$ and $\delta = b/a$, $\rho = c/a$, where a and b are heat transfer coefficients.

So the cycle average power

$$P = Q_s - Q_R,$$
$$P = a(T_H - T_E) - b(T_C - T_k),$$
$$P = aT_H\{1 + \delta\Gamma - \xi - \xi\delta\tau\}. \tag{15}$$

As per second law thermal efficiency not to exceed Carnot cycle efficiency

$$\frac{Q_s - Q_R}{Q_s - Q_T} = \frac{P}{Q_s - Q_T} \leqslant 1 - \frac{T_C}{T_E} = 1 - \tau.$$

This condition can be expressed as

$$\xi\{(\delta + 1)\tau - \rho(1 - \tau)^2\} \geqslant \delta\Gamma + \tau. \tag{16}$$

Further for maximum power condition the equation can be written as

$$\xi = \frac{(\delta\Gamma + \tau)}{(\delta + 1)\tau - \rho(1 - \tau)^2}. \tag{17}$$

Substituting (17) into (15)
Max. Power P_i

$$P_i = \frac{aT_H\{\delta(\tau - \Gamma)(1 - \tau) - \rho(1 + \delta\Gamma)(1 - \tau)^2\}}{(\delta + 1)\tau - \rho(1 - \tau)^2}. \tag{18}$$

Above equation gives power output of reversible engine operating between temperature extremes. The P_i is the power produced by the working fluid before mechanical losses of the engine are deducted.

The domain of possible τ values for the function P_i ranges from τ_p to 1 where

$$\tau_p = \frac{\delta\Gamma + \rho(\delta\Gamma + 1)}{\delta + \rho(\delta\Gamma + 1)}.$$

Differentiating (18) yields following point where indicated power is maximum

$$\tau_M = \frac{\delta\rho(1 - \Gamma) + \sqrt{(1 + \delta)(\delta + \rho + \rho\delta)}(\rho + \rho\delta^2\Gamma^2 + \delta\Gamma(1 + \delta + 2\rho))}{\delta + \delta^2 + \rho + 2\delta\rho + \Gamma\delta^2\rho}. \tag{19}$$

Boer [18] has used linearized theory for maximum obtainable performance of Stirling engine. He showed that for a one-dimensional model of Stirling engine maximum obtainable values are independent on the value of regenerator conductance.

2.6. Schmidt's theory

After 55 years of Stirling cycle engine invention Schmidt [5] made analysis of engine cycle in 1871 called as classical analysis. The analysis made for three types of practical Stirling engine configuration wise alpha, beta and gamma. Schmidt obtained close theory, which provides sinusoidal volume variation of working space in the reciprocating engines. Theory retains the major assumptions of isothermal compression and expansion and of perfect regeneration. It, thus remains highly idealized, but is certainly more realistic than the ideal Stirling cycle. The engine configurations considered for classical analysis of operation of Stirling engine are shown in Figs. 5–7.

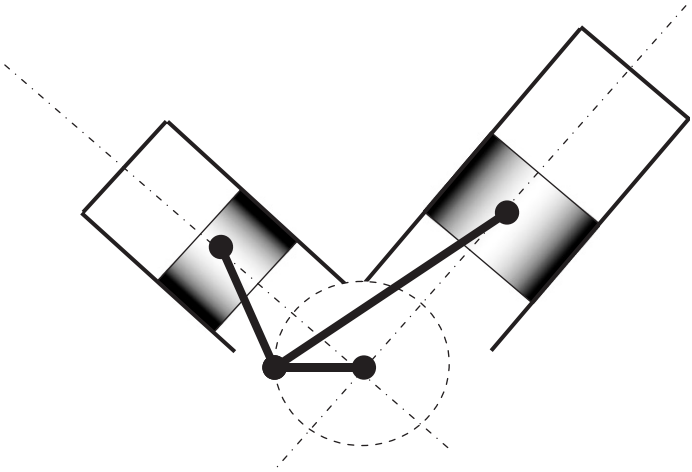

Fig. 5. Alpha configuration.

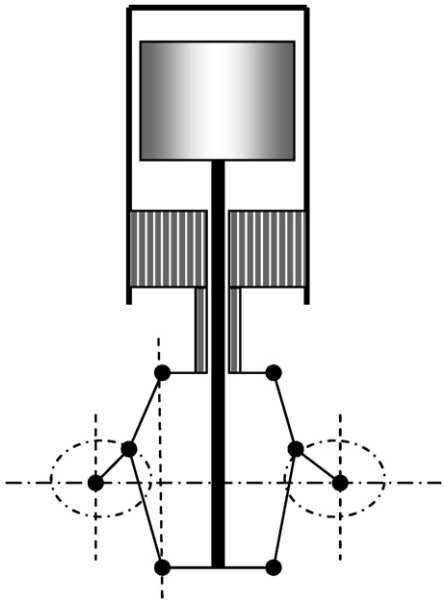

Fig. 6. Piston–displacer type—beta configuration.

Following are the principal assumptions of Schmidt's analysis of Stirling cycle engine:

1. All processes are reversible.
2. The regeneration process is perfect.
3. The working fluid obeys perfect gas law, $PV = mRT$.
4. The mass of air in the system remains constant, no leakage in the system.
5. The volume variation in the working space is sinusoidal.
6. There is no temperature gradient in the heat exchanger.

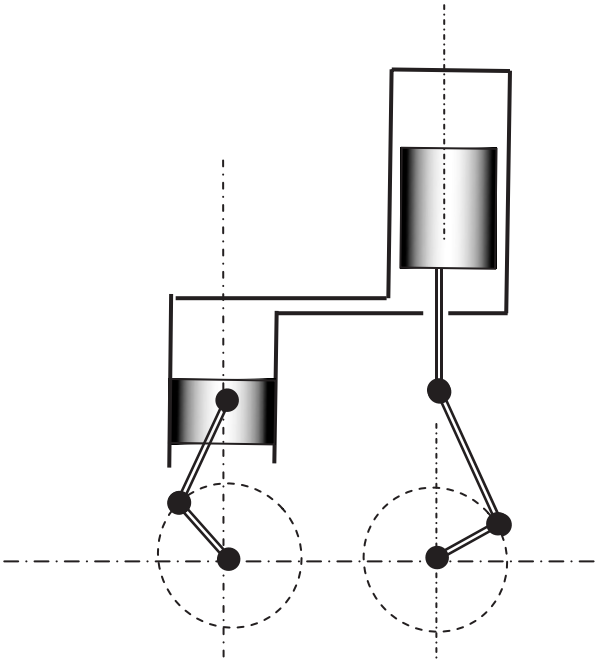

Fig. 7. Piston–displacer in separate cylinder—beta configuration.

7. The cylinder wall and piston temperature are constant.
8. The speed of the machine is constant.
9. Steady state conditions are established.
10. There are no flow losses and hence no spatial pressure loss.
11. No leakage of working gas.
12. The temperature in heater and expansion space is isothermal at T_E.
13. The temperature in cooler and compression space is isothermal at T_C.
14. The temperature in the dead space and hence in regenerator space is isothermal at T_D.

The classical analysis of operation of Stirling engine of three-engine configuration provided in Table 1. A reasonable level of caution in interpretation and predictions of the performance. Schmidt theory can be as a useful tool for engine design.

3. Actual Stirling cycle engine

The ideal thermodynamics of the Stirling engine have been examined, but the practical realities like adiabatic and or isothermal, internal heat exchangers, cylinder wall heat transfer and the general situations have not been accounted. The effects of the various practical factors that cause the actual engine cycle to deviate from the ideal case are required to be considered separately in order to highlight their influence. The basic four-path cycle will be used as the datum for illustrating the effects of practical factors. The working gas temperature through out the engine will have a tendency to be influenced in an adiabatic rather than an isothermal manner and this will influence the form of the PV

Table 1
Schmidt's analysis

Particulars	Alpha engine	Beta engine	Gamma engine
Expansion space volume variation V_e	$V_e = \frac{V_{se}}{2}(1+\cos\phi)$	$V_e = \frac{V_{se}}{2}(1+\cos\phi)$	$V_e = \frac{V_{se}}{2}(1+\cos\phi)$
Compression space volume variation V_c	$V_c = \frac{V_{SC}}{2}[1+\cos(\phi-\alpha)]$	$V_c = \frac{V_{SC}}{2}(1-\cos\phi) + \frac{V_{SP}}{2}[1+\cos(\phi-\alpha)] + V_{NO}$	$V_c = \frac{V_{se}}{2}(1-\cos\phi) + \frac{k_p V_{se}}{2}[1+\cos(\phi-\alpha)]$
Swept volume ratio k	$k = \frac{V_{SC}}{V_{se}}$	$k_p = \frac{V_{SP}}{V_{se}}$	$k_p = \frac{V_{SP}}{V_{se}}$
Compression space volume variation V_c	$V_c = \frac{kV_{SE}}{2}[1+\cos(\phi-\alpha)]$	$V_c = \frac{V_{se}}{2}[(1-\cos\phi)] + k_p[1+\cos(\phi-\alpha)] + V_{NO}$	
Mass of working gas (if mass of working fluid is constant and no flow losses) $M_T = m_e + m_c + M_D = \frac{p_E V_e}{RT_E} + \frac{p_C V_c}{RT_C} + \frac{p_D V_d}{RT_D}$	$M_T = \frac{pV_{se}}{2RT_C}\left\{\zeta(1+\cos\phi) + k[1+\cos(\phi-\alpha)] + \frac{4X\zeta}{\zeta+1}\right\}$	$M_T = \frac{pV_{se}}{2RT_C}\left\{\zeta(1+\cos\phi) + (1-\cos\phi) + k_p[1+\cos(\phi-\alpha)] - (1+k_p) + (k_p^2 - 2k_p\cos\alpha+1)^{1/2} + \frac{4X\zeta}{\zeta+1}\right\}$	$M_T = \frac{pV_{se}}{2RT_C}\left(\zeta(1+\cos\phi) + (1-\cos\phi) + k_p[1+\cos(\phi-\alpha)] + \frac{4X\zeta}{\zeta+1}\right)$
Ratio $\frac{C}{p}$	$\frac{C}{p} = \zeta(1+\cos\phi) + k[1+\cos(\phi-\alpha)] + \frac{4X\zeta}{\zeta+1}$	$\frac{C}{p} = \zeta + (k_p^2 - 2k_p\cos\alpha+1)^{1/2} + \frac{4X\zeta}{\zeta+1} + [k_p^2 + 2(\zeta-1)k_p\cos\alpha + (\zeta-1)^2]^{1/2}\cos(\phi-\theta)$	$\frac{C}{p} = \zeta(1+\cos\phi) + (1-\cos\phi) + k_p[1+\cos(\phi-\alpha)] + \frac{4X\zeta}{\zeta+1}$
Constant B	$B = (\zeta^2 + 2k\zeta\cos\alpha + k^2)^{1/2}$	$B = [k_p^2 + 2(\zeta-1)k_p\cos\alpha + (\zeta-1)^2]^{1/2}$	$B = [k_p^2 + 2k_p\cos\alpha(\zeta-1) + (\zeta-1)^2]^{1/2}$
Constant S	$S = \left(\zeta + k + \frac{4X\zeta}{\zeta+1}\right)$	$S = \zeta + (k_p^2 - 2k_p\cos\alpha+1)^{1/2} + \frac{4X\zeta}{\zeta+1}$	$S = \zeta + k_p + 1 + \frac{4X\zeta}{\zeta+1}$
Constant δ	$\delta = \frac{B}{S}$	$\delta = \frac{B}{S}$	$\delta = \frac{B}{S}$
Volume/compression ratio	$r_v = \frac{(1+k+2X) + (k^2+2k\cos\alpha+1)^{1/2}}{(1+k+2X) - (k^2+2k\cos\alpha+1)^{1/2}}$	$r_v = \frac{1+2X+(k_p^2-2k_p\cos\alpha+1)^{1/2}+k_p}{1+2X+(k_p^2+2k_p\cos\alpha+1)^{1/2}-k_p}$	$r_v = \frac{2(1+k_p+X)}{(2+2X)}$
Mass flow rate	$m_c = kV_{se}p_{max}(1-\delta) \times \frac{\left\{\begin{matrix}\delta[\sin(\phi-\theta)+\sin(\alpha-\theta)] \\ -\sin(\phi-\alpha)\end{matrix}\right\}\varpi}{2RT_C[1+\delta\cos(\phi-\theta)]^2}$	$m_c = \frac{p_{max}V_{se}(1-\delta)\varpi}{2RT_C[1+\delta\cos(\phi-\theta)]^2} \times\{\delta[k_p\sin(\alpha-\theta)+\sin\theta + (1+k_p+2k_o)\sin(\phi-\theta)] + k_p\sin(\alpha-\phi)+\sin\phi\}$	$m_c = \frac{p_{max}V_{se}(1-\delta)\varpi}{2RT_C[1+\delta\cos(\phi-\theta)]^2} \times\{\delta[k_p\sin(\alpha-\theta)+\sin\theta + (k_p+1)\sin(\phi-\theta)] + k_p\sin(\alpha-\phi)+\sin\phi\}$

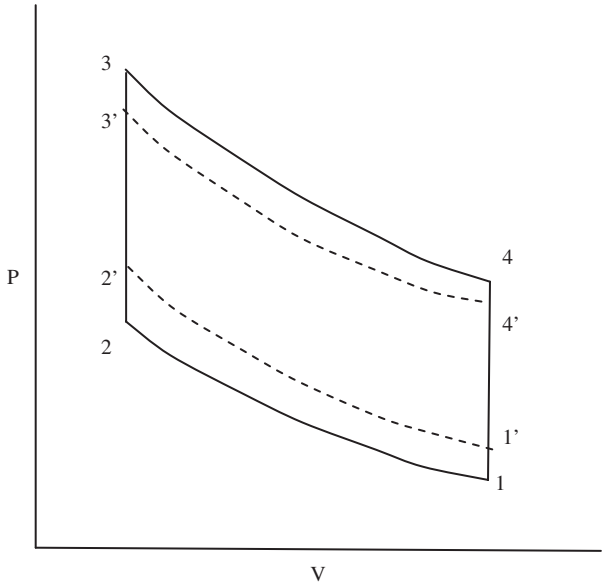

Fig 8. PV diagram of actual Stirling cycle.

diagram. The cylinder walls in fact will not provide a sufficient heat transfer medium to ensure that the cylinder gas temperature remains constant and even the use of tubular heat exchangers does not ensure that isothermal conditions prevail at the entry and exit area of regenerator. The deviation from the isothermal conditions is more pronounced on the hot side of the engine than the cold side as illustrated in Fig. 8 where the ideal cycle 1-2-3-4 is reduced to 1′-2′-3′-4′ because of heater and cooler heat transfer effects.

Kaushik and Kumar [19] have made thermodynamic evaluation of irreversible Ericsson and Stirling cycles. The analysis made for heat sink losses with engine power output on the basis of ideal gas and perfect regeneration. Shoureshi [20] has tried to optimize cooler, heater and regenerator based on Mach number, operating temperature ratio, and percentage of heat exchanger dead volume.

3.1. Heat transfer phenomenon in Stirling engine

Operation of Stirling engine is caused by variety of heat transfer modes. Costea and Feidt [21] have shown that the heat transfer coefficient varies linearly with respect to local temperature difference of the hot engine components. The regenerator must be able to deal with 4–5 times the heat load of the heater and if it is not capable of doing this then extra load will be imposed on the other heat exchangers. The regenerator may be as near as perfect as possible if the engine is to attain good efficiency values and this means that the gas must be delivered from the regenerator to the cold side of the engine at the lowest temperature and to the hot side at the highest temperature. If this temperature does not prevail then the temperature and thus the pressure of the cold gas will be too high whilst the pressure and temperature of the hot gas will be too low. Because of the regenerator ineffectiveness the gas enters the compression phase of the cycle at state 1′ rather than state

1 and the expansion phase at state 3′ rather than state 3. Chen et al. [22] have developed a combine model for analysis of engine performance with heat losses and imperfect regeneration for a solar-powered engine. The optimum operating temperature for engine operation at maximum power is derived with effect of regenerator imperfection. Wu et al. [23] established a relationship between net power output and efficiency for the Stirling engine with heat transfer and imperfect regeneration as

$$P_{\mathrm{i}} = \frac{\alpha\eta_{\mathrm{t}}}{\{1/(T_{\mathrm{H}} - T_{\mathrm{E\,av}}) + \frac{(1-\eta_{\mathrm{t}})(1-\mu)}{(1-\mu\eta_{\mathrm{t}}-\eta_{\mathrm{t}})T_{\mathrm{E\,av}}-(1-\mu\eta_{\mathrm{t}})T_{\mathrm{C}}} + A_{\mathrm{r}}\eta_{\mathrm{t}}\}}, \tag{20}$$

where

$$A = \alpha(k_1 + k_2)/[nC_{\mathrm{V}}(k-1)\ln r_{\mathrm{v}}],$$

$$\mu = \frac{x}{[(k-1)\ln r_{\mathrm{v}}]}.$$

3.2. Heat exchangers in Stirling engine

Heat exchangers are key components in the Stirling engine. There may be three or four heat exchangers in the Stirling engine system. These are illustrated in Fig. 9, which includes a heater, cooler, regenerator and pre-heater (may be optional). The heater transfers heat from external source to the engine working fluid contained within the engine working space. The cooler does just reverse, it absorbs heat from engine working fluid adjacent to compression space and rejects to atmosphere through coolant. The regenerator acts as a thermal sponge alternatively accepting heat from working fluid and rejecting heat back to working fluid. The heat flow in a Stirling engine is shown in Fig. 9. The Sankey plot for heat flow is first made by Zacharias [24] in 1971 and further work is done by Pertescu [25]

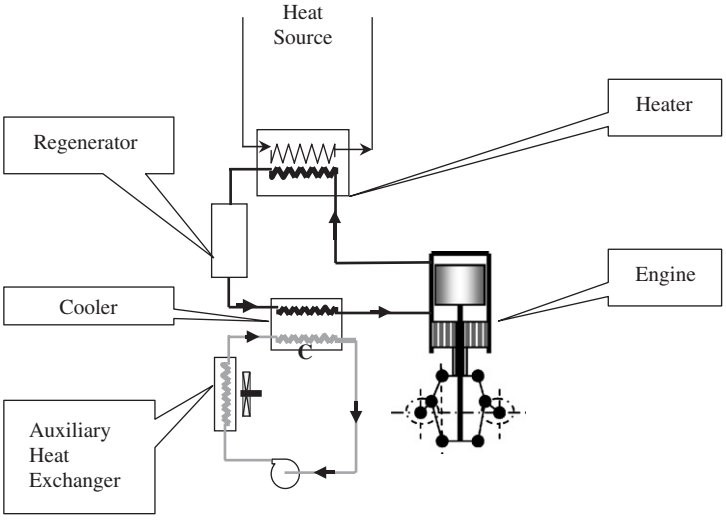

Fig. 9. Heat exchangers in Stirling engine.

which shows that the heat exchanger configuration is largely affected by amount of pumping losses [26].

3.2.1. Heater

The heat transfer phenomenon in heater is as follows:

(a) Convective heat transfer from external heating medium to walls of heater tubes or fins.
(b) Conductive heat transfer through outer tube wall surface to inner surface or to root.
(c) Convective heat transfer from internal wall of tube to the working fluid.

$$\text{The heat transferred } Q_{\mathrm{h}} = h_{\mathrm{h}} A_{\mathrm{h}} \Delta T_{\mathrm{h}}, \quad \text{where } A_{\mathrm{h}} = \pi d_{\mathrm{h}} n_{\mathrm{h}} l_{\mathrm{h}},$$

where d_{h} and n_{n} are diameter and number of heater tubes.

In the relatively simple case of steady turbulent flow analytical techniques are not available. The heat transfer coefficient must then be determined using the well-known Reynolds analogy in its original or in a modified form. This analogy relates heat transfer to fluid friction, using standard non-dimensional parameters. The Prandtl number Pr is a property of the working fluid and the Reynolds number Re is a property of flow. The Nusselt number Nu relates the heat transfer coefficient h, the length of the heater tube and the thermal conductivity of the working fluid k, namely:

$$Nu = \frac{hl}{k}.$$

Bergmann and Alberto [27] have redefined Nusselt number for in cylinder heat transfer coefficient:

$$Nu = \frac{hD}{k}.$$

There is no unique form of equation although some correlations seem better than others. The data tables and graphs developed by Walker [28] have provided the better correlation by replacing Nusselt number by Stanton number, St.

$$St = \frac{h}{C_{\mathrm{P}} \rho u},$$

where C_{P} is specific heat of working fluid, ρ is density of working fluid, u is flow velocity.

Reader [29] have designed and implemented a heater for the Stirling engine for underwater application at constant temperature operation.

The correlations between Nusselt, Reynolds and Prandtl Numbers taken as

$$Nu = f(Pr, Re).$$

So

$$(Nu) = 0.23(Re)^{0.33}.$$

This can be written as

$$(h_{\mathrm{h}} d_{\mathrm{h}} / k_{\mathrm{h}}) = 0.23(\rho_f v_f d_{\mathrm{h}} / \mu_f)^{0.3}. \tag{21}$$

and also

$$Nu = 0.053 Re_g^{0.8} Pr_g^{0.6}. \tag{22}$$

The heater is a difficult heat exchanger to design for the inner tube requirements and outer tube requirements. The design is also affected by the choice of heat source. The outer tube surface will usually experience a high temperature low-pressure steady flow environment. The inner tube surface will experience a high pressure, high temperature, very unsteady flow. The heat transfer coefficients will be significantly different at inside and outside and thus it will be almost evitable that surface area requirements will not be comparable with each other. There are two further constraints, one is the inner to outer diameter ratio will be determined by both pressure and thermal loading and second the optimum diameter ratio may not be in harmony with the surface area needs. All these factors may also be out of step with the frictional drag and dead space requirements.

The two parameters, which are of prime importance for the internal heater surfaces are the heat transfer coefficient and the friction factor. With knowledge of these two factors performance of heat exchanger can be assessed and the optimum dimensions of a proposed design can be formulated for given thermodynamic specifications. The rate at which the tube inner wall heat flux can be transferred to the gas depends upon the inner tube film coefficient of heat transfer, the mass flow rate and the specific heat of the gas.

The working fluid is mostly pressurized at high density and moving with high velocity so the internal heat transfer process is well developed. Similarly, most metals used for heater tube are good thermal conductors so a good heat transfer takes place by conductance with a small temperature difference. But the combustion system at atmospheric temperature limits heat transfer process due to low density and low velocity of products of combustion. Hence more focus is required on improvement on heat transfer process between hot combustion gases and heater tube.

3.2.2. Cooler

In principle, the Stirling engines may be air-cooled or water-cooled as like as IC engines. To reduce temperature of working fluid the cooling system of engine required to handle cooling load almost twice than of cooling load of conventional IC engine. As the coolant temperature increases there is considerable fall in thermal efficiency, so it is desirable to have coolant temperature at minimum possible value. The flow conditions are as similar as heater but at the lower temperature. Almost all engine designers have adopted water-cooling and the outer cooler tubes experience the same flow conditions as in conventional engine. The semi-empirical heat transfer coefficients for this condition are better documented, following a large number of investigations and many correlations available in the heat transfer literature NASA Lewis [30]. Following recommended expressions are satisfactory for cooler analysis, where h_t is the total heat transfer coefficient and h_w is the heat transfer coefficient of the water film.

$$h_t = \frac{h_w}{(1 + 0.882 h_w)}; \quad h_w = 0.35 Re^{0.55} Pr^{0.33} k_w d_o, \tag{23}$$

where k_w is the water thermal conductivity; d_o the outer diameter of water tubes.

3.2.3. Regenerator

Some of the heat supplied by external source to working fluid is converted in to useful work and while flowing out hot expanded gases from expansion space to the cooler the rest

of the heat is stored in a regenerator. After cooling in the cooler and compressing in compression space gases flows back to expansion space through regenerator. The stored heat in regenerator is given back to the working fluid during back-flow. This process is called as regeneration. And the efficiency of the Stirling cycle machine depends on the efficiency of the regeneration process or regenerator.

Ideal regeneration is achieved when the fluid entering and leaving the matrix does so one of the two constant temperatures, T_E at the expansion end and T_C at the compression end of the regenerator. This is possible only if operations are carried out infinitely slowly or the heat transfer coefficient or the area of heat transfer is infinite. This is also possible if the heat transfer capacity of fluid is zero or heat capacity of the matrix is infinite. The fundamental requirements of the regenerator are specified by the thermodynamic processes of the ideal cycle. In order to achieve maximum efficiency the heat rejected during the isochoric process 4–1 must be returned to the gas in the isochoric process 2–3. Ideally, the heat transfer is achieved reversibly in the regenerator. A linear temperature gradient from T_{max} to T_{min} is maintained along the length of the regenerator. The working fluid enters in the regenerator in thermodynamic state 4 and starts transferring its heat to the regenerator material and leaves at state 1. During this process temperature of each element of the regenerator is raised. After the compression process the working fluid now enters in the regenerator at state 2 (minimum cycle temperature) and flows back through regenerator receiving the heat stored in the regenerator by increasing its temperature from T_2 to T_3.

The regenerator of practical engine operates with conditions far away from those assumed for the ideal case. The temperature of working fluid entering the regenerator is not constant because the pressure, density and velocity of working fluid vary over a wide range. The effectiveness of the regeneration process largely depends upon the thermal capacity of the material also.

3.3. Regenerator analysis

There are various kinds of materials which can be used for the regenerator matrix, such as steel wool, steel felt, wire mesh, fine pipes, spring mesh, stacked screen, packed balls, metal foils and parallel plates etc. To achieve high heat storage capacity the matrices of above materials are used in current regenerators. To improve heat transfer coefficient and to establish the minimum temperature difference between matrix and the fluid it is necessary to expose the maximum surface area of matrix, therefore matrix should be finely divided [31].

Thus following are the desirable characteristics of regenerator matrix.

1. For maximum heat capacity—a large, solid matrix is required.
2. For minimum flow losses—a small, highly porous matrix is required.
3. For minimum dead space—a small, dense matrix is required.
4. For maximum heat transfer—a large, finely divided matrix is required.
5. For minimum contamination—a matrix with no obstruction is required.

Hence design of regenerator is a matter of optimization of regenerator volume for best values of above parameters.

3.3.1. Heat transfer and fluid friction in regenerator

For simplifying the analysis of regenerator following assumptions are made:

(a) The thermal conductivity of the matrix material of regenerator is constant.
(b) The specific heats of fluid and matrix do not change with temperature.
(c) The fluid flow and temperature that is constant over the flow section.
(d) The heat transfer coefficients and fluid velocities are constant with time and space.
(e) The rate of mass flow is constant.
(f) The pressure drop across the regenerator is negligible.
(g) The gas flow in duct is one-dimensional.
(h) The working gas is assumed to be a perfect gas.

The entire heat transfer process is carried out reversibly. The temperature differential between working fluid and the regenerator must be infinitesimal. To fulfill this requirement certain conditions have to be satisfied. First reversible process can assured if process is in thermodynamic equilibrium, i.e. regenerator system must pass through a series of equilibrium states. But this can never be achieved in practice hence every attempt must be made to satisfy the remaining condition, which can be readily identified by fundamental equation for the dynamic behavior of the regenerator.

So the transfer of heat from/to working fluid from/to the matrix on the passage of flow through regenerator is given by,

$$h_{\text{bulk}} \times A \times (T_{\text{M}} - T_{\text{F}}) = m_{\text{f}} \times C_{\text{PF}} \times L_{\text{R}} \times \frac{\delta Tf}{\delta x} + M_{\text{FR}} \times C_{\text{P}} \times \frac{\delta Tf}{\delta t}. \quad (23)'$$

The right-hand side of the equation represents heat energy available in the working fluid. To transfer the maximum heat energy from working fluid to regenerator the matrix fluid temperature difference $(T_{\text{M}}-T_{\text{F}})$ must be infinitesimal. Thus the bulk heat transfer coefficient h_{bulk} must be infinite to compensate for the small temperature differential. Once again this condition is not obtainable in practice. The only remaining parameter is the heat transfer area and thus this needs to be made infinite to enable the ideal condition to be approached, so every effort should be made to make surface area as large as possible.

The second Nusselt–Hausen equation describes the energy storage capacity of the regenerator as by Miyabe [32]. This is given by the equation:

$$h_{\text{bulk}} \times A \times (T_{\text{F}} - T_{\text{M}}) = M_{\text{M}} \times C_{\text{PM}} \times \frac{\delta T_{\text{M}}}{\delta t}. \quad (24)$$

The heat capacity of the regenerator C_{R} being represented by the term $M_{\text{M}*}C_{\text{PM}}$ is the product of mass and specific heat of regenerator. By taking in to account the first approximation equation can be written by equating Eqs. (23)′ and (24). Thus the regenerator matrix temperature

$$\Delta T_{\text{M}} \approx \frac{m_{\text{f}} \times C_{\text{PF}} \times \Delta T_{\text{F}}}{C_{\text{R}}}. \quad (25)$$

To maintain required temperature difference along the regenerator there should not be axial heat conduction and maximum conduction of heat should be normal to the flow of fluid. Engine on close cycle operations cannot be realized without an effective regenerative heat exchanger. The first step to define effectiveness of regenerator, which is denoted by $\in_{\text{R}}$

and is given by equation,

$$\in_{\mathrm{R}} = \frac{\text{heat actualy transfered}}{\text{heat available for transfer}}; \quad \in_{\mathrm{R}} = \frac{T_2 - T_1}{T_3 - T_1}. \tag{26}$$

To obtain an expression for effectiveness of regenerator and mode the regenerator behavior the Eqs. (23) and (24) are expressed in the form of dimensionless parameters η for time and ξ for distance such that,

$$\frac{\delta T_{\mathrm{F}}}{\delta \zeta} = T_{\mathrm{M}} - T_{\mathrm{F}}; \quad \frac{\delta T_{\mathrm{M}}}{\delta \eta} = T_{\mathrm{F}} - T_{\mathrm{M}},$$

where

$$\zeta = \frac{h_{\mathrm{bulk}} \times A}{m_{\mathrm{f}} \times C_{\mathrm{PF}} \times L_{\mathrm{R}}}$$

and

$$\eta = \frac{h_{\mathrm{bulk}} \times A}{M_{\mathrm{M}} \times C_{\mathrm{PM}}} \left(\chi - \frac{M_{\mathrm{FR}} \times x}{m_{\mathrm{f}} \times L_{\mathrm{R}}} \right)$$

Here, χ is the time period of the particular blow under condition. A further pair of dimensionless parameters for each blow period based on η and ς, which have became two of the main characteristic parameters of regenerator namely Λ, reduced length and Π, reduced period as proposed by Hausan [33] theory is given by

$$\Lambda = \frac{h_{\mathrm{bulk}} \times A}{m_{\mathrm{f}} \times C_{\mathrm{PF}}}$$

and

$$\Pi = \frac{h_{\mathrm{bulk}} \times A}{M_{\mathrm{M}} \times C_{\mathrm{PM}}} \left(\chi - \frac{M_{\mathrm{FR}}}{m_{\mathrm{f}}} \right).$$

These two factors contain the parameters, which need to be balanced to achieve perfect regeneration. Using these two factors effectiveness can be derived as

$$\in_{\mathrm{R}} = \frac{\Lambda}{\Pi} \tanh\left(\frac{\Pi}{\Lambda + 2} \right),$$

where Π factor is defined without consideration of gas storage within the regenerator, i.e. $M_{\mathrm{FR}} = 0$. This assumption is acceptable under certain flow condition. The term Λ/Π is a utilization factor, which is the ratio of the heat capacity of the working fluid for the blow period to the heat storage capacity of the matrix. If the regenerator is ideal then the overall cyclic heat transfer is zero. The performance of regenerator is the magnitude of cyclic heat transfer Q_{R} which is given by

$$Q_{\mathrm{R}} = \frac{\gamma R}{\gamma - 1} \oint (m_{\mathrm{r}} T_{\mathrm{C}} - m_{\mathrm{h}} T_{\mathrm{E}}) \delta t. \tag{27}$$

Considering the flow of gas through porous media the heat transfer can be correlated with the pressure drop using the Darcy law Isshiki et al. [34] as given below:

$$Q_{\mathrm{R}} = \frac{-A\sigma\gamma}{\mu L_{\mathrm{R}}(\gamma + 1)} \oint (\Delta P) P_{\mathrm{AM}} \, \mathrm{d}t. \tag{28}$$

Ercan Ataer [35] used Lagrangian approach for plotting engine gas temperature distribution in regenerator of Stirling engines. The effect of dead volume of engine on engine power output is an important issue. Fette [36] has estimated the effect of the volume ratio V_{max}/V_{min} on the efficiency of the regenerator in the Stirling engine [37,38].

4. Engine configuration

The elements of Stirling engine include two volumes at different temperatures connected to each other through a regenerative heat exchanger and auxiliary heat exchangers. To fulfill thermodynamic, gas dynamic and heat transfer requirements of the engine, these volumes are required to change periodically. The primary role of the drive mechanism must therefore be to reproduce these volumetric changes as precisely as possible. These elements can be combined to fulfill above requirements by wide range of mechanical arrangements. A comprehensive listing of possibilities of engine arrangement is given by Walker [39]. The drive mechanism is required to be considered when selecting an engine configuration because all mechanisms are not compatible with every engine arrangement. The basic engine parameters are also important which comprises the Beale number [40] namely engine speed, pressure and displacement. Gary Wood [41] of Sun-Power Corp. listed following parameters, which are required to be considered while selecting proper engine configuration. Senft [42] also mentioned that the optimum engine geometry will be based on the following engine parameters:

1. Engine cylinder layout/arrangement
2. Engine mechanism
3. Burner/heater type
4. Displacer and piston construction
5. Type and size of regenerator
6. Crankcase construction

The three levels of classifications are

(A) Mode of operation
   1. Single acting
   2. Double acting
   3. Single phase
   4. Multiphase
   5. Resonant
   6. Non-resonant

(B) Forms of cylinder coupling
   1. Alfa coupling
   2. Beta coupling
   3. Gamma coupling

(C) Forms of piston coupling
   (I) Rigid coupling
      1. Slider crank
      2. Rhombic drive

3. Swash plate
4. Crank rocker
5. Ross rocker

(II) Gas coupling
1. Free piston
2. Free displacer
3. Free cylinder

(III) Liquid coupling
1. Jet stream
2. Rocking beam
3. Pressure feedback

4.1. Mode of operation

The Stirling engines can be broadly classified in to two categories as single acting and double acting. The term single acting and double acting in Stirling engine technology is used to describe the mode of operation of particular engine. In single acting engines the working fluid is in contact with one side of the piston only. The fluid is shuttled between the two cylinders, from one cylinder to another cylinder. They may operate as single cylinder engines in multiple units arranged with a common crankcase and crankshaft. The first single acting engine is invented by Robert Stirling in 1815.

The double acting engine is developed by Babcock in 1885. The double acting engine uses both sides of the piston to move the fluid from one space to another. The double acting engine, which inherently has multiple working spaces, was believed to be complex for engine arrangement because ducting are required to connect working spaces with regenerator. Stirling engine using double acting principal are of necessary multi-cylinder engines, since a minimum of three pistons are required in order to obtain the appropriate difference between the expansion and compression processes. The most developed and manufactured Stirling engine of the double acting type are P′ series of United Stirling developed by Bratt [43]. A 40 kW four-cylinder double acting Stirling engine designed and tested by United Stirling. The design objectives of the engine were component development. Engine test were results with different working gas and different heater head temperatures. Finkelstein has described a number of new concepts and arrangements for multi-cylinder operation.

4.2. Forms of cylinder coupling

4.2.1. Alfa coupling

Alpha engines have two pistons in separate cylinders, which are connected in series by a heater, regenerator and cooler as shown in Fig. 5. The Alpha engine is conceptually the simplest Stirling engine configuration; however, it suffers from the disadvantage that both pistons need to have seals to contain the working gas. Ross [44] has been developing small air engines with extremely innovative Alpha designs, including the classical Ross–Yoke drive and more recently a balanced "Rocker-V" mechanism. The Alpha engine can also be compounded into a compact multiple cylinder configuration, enabling an extremely high

specific power output, as is required for automotive engine. In case of multi-cylinder arrangement, number of cylinders are interconnected, so that the expansion space of one cylinder is connected to the compression space of the adjacent cylinder via a series connected heater, regenerator and cooler. The pistons are typically driven by a swash plate, resulting in a pure sinusoidal reciprocating motion having a 90° phase difference between the adjacent pistons.

4.2.2. Beta coupling

Beta engines use displacer–piston arrangements as shown in Fig. 6. The engine construction is such that both displacer and piston are accommodated in same cylinder. The compression space in this form consists of the space swept by the underside of the displacer and the topside of the power piston. The piston and displacer may or may not physically touch but connected to crankshaft by separate linkage to maintain required phase angle.

4.2.3. Gamma coupling

Gamma engines use displacer–piston arrangements similar to Beta engine configuration with the displacer and the piston in separate cylinders as shown in Fig. 7. In this type of machine the compression space is split between two cylinders with an interconnecting transfer port. In between the passage from displacer cylinder and compression cylinder the cooler, heater and regenerator is connected serially. This arrangement provides advantage of simple crank mechanism.

4.3. Forms of piston coupling

4.3.1. Slider crank drive

The slider crank has been used from many years in IC engines and is extremely reliable, with a wealth of operating experience. It is being used extensively in double acting Stirling engines both with and without a crosshead. It has the advantage of reliability and ease of manufacture but has the disadvantage of being almost impossible to balance. The slider crank linkage do not provide an elegant solution to the drive mechanism problem when the piston and displacer are used in tandem in a cylinder, but it is the mechanism used in twin cylinder versions of the basic Stirling engine.

4.3.2. Rhombic drive

The rhombic drive Stirling engine is perhaps the most well known and certainly the most developed of all single cylinder Stirling engine. The evolution of the Stirling engine into a bigger power unit leads to the need to seal off the cylinder from the crankcase and thus avoid to pressurize the whole crankcase. The rhombic drive is developed by Philips [45] in 1950s. It also possessed the advantage of being dynamically balanced even for a single cylinder arrangement. Its main disadvantage is the complexity of unit. It has large number of moving parts, bearing surfaces and need of a matched pair of gear wheels for each assembly.

4.3.3. Swash plate

The swash plate has been used mainly on engines developed for the automobile purpose where space is at a premium. It is a system, which is dynamically balanced at a fixed swash

plate angle. Meijer [46] developed a method to vary swash plate angle during operation to control power but which experiences out of balance effect. The cylinders can be easily sealed off from the crankcase with good reliability in the sealing system for an application such as the automobiles, where large numbers of rapid cycles are required. Additionally, the swash plate can provide a way of controlling the engine power output by varying the angle of swash plate and thus altering the stroke of the engine.

4.3.3.1. Ross rocker. This mechanism was patented by Ross [47] and at present the mechanism is under investigation for use in Stirling engine at Cambridge University.

4.3.3.2. Ringbom type. This is a single cylinder type list complicated type drive arrangement. In the ringbom, the piston is mechanically linked to the crankshaft while the displacer is driven by the cyclic gas forces in a manner similar to the Beale [48] free piston engine.

4.3.4. Gas coupling

4.3.4.1. Free piston or cylinder or displacer engine. All the engines so far described have made use of drive mechanism in which the piston is connected to connecting rod by some form of mechanical linkage, which in turn is attached to a power output shaft. However, Stirling engine can operate without the piston being coupled mechanically. In this case, the power piston or displacer piston is described as being 'free'. The concept was first realized in practice by William Beale and, as a consequence, the term Beale free piston engine is often used to describe free piston Stirling engines.

The piston is free in the sense of not being coupled mechanically, but they are coupled gas dynamically. The configuration is identical to the piston–displacer form except that there is no mechanical crank mechanism and the cylinder is fully sealed at both ends, since no piston rod has to pass out of the system. The displacer–piston rod is hollow and open at one end so that the fluid inside the displacer is in continuous communication with the fluid in the bounce space which remains at the constant pressure at all the times. This space constitutes a fluid spring and, as we shall see, has a similar effect to the crankshaft in the conventional Stirling engine.

The FPSE are suitable for pumping water reciprocating pumps and linear alternator. Hsu [49] have shown that these engines can be used for generating power by using waste. A 1-m^2 hot end surface embedded inside the combustion chamber of the incinerator, the absorbed energy is enough to move a 2.5 kW Stirling generator set. The free piston Stirling engine/linear alternator system developed by George Dochat, which has proven to be reliable, stable and capable of power greater than 1 kW. Rifkin [50] FPSE designs have been described for pumping fluid, generating electricity and pumping heat. Extrapolation of the experiments results conducted by Shtrikman [51] shows that the linear generator is capable of performing within its performance goals. In each application the importance should be given to potential for attractive life cycle cost, long maintenance-free life, compact size and weight for kinematical Stirling engines. The problem incurred with these type engines is sealing and life of reciprocating components due to wear and tear. The test engine built by Berggren [52] shows that the engine performance degrades with increased wear of the seal. Shvangiradze and Shvangiradze [53] have recently tried a cam mechanism for conversion of reciprocating motion in to rotary motion for Stirling engines. The mechanism may prove its suitability for engine operation for variety of type of engine configurations.

4.4. Suitability of cylinder piston configuration

The analysis of heat engines need to concentrate on thermodynamic, heat transfer aspects of the engine and also the machine dynamics. The drive mechanism used for conventional engine is exclusively slider-crank mechanism, whereas for Stirling engines many forms of drive mechanisms are employed. Thus it is even more important with Stirling engines than the drive mechanism dynamics and kinematics to be given due regard. The study made by Shoureshi [54] has presented a general method of Stirling engine design optimization for any type of engine configuration under entire range of operating conditions. It is based and characterized by dimensionless group, in particular, by Mach number, Reynolds number, temperature ratio and percentage of dead volume. He found that Gamma type of the engine arrangement is the most suitable engine configuration as shown in Fig. 7. Suitability of engine configuration is defined by Gary Wood [55] which depends on mainly type of fuel, engine speed and power output of he engine. Senft [56] has developed a theory to establish relation of engine mechanical efficiency with cycle pressure and other engine parameters with wide variety of engines. This is useful for selection of proper engine mechanism to suit requirement.

5. Working fluids for Stirling engine

Any working fluid with high specific heat capacity may be used for Stirling cycle engine. With few exceptions the engines in 19th century used air as a working fluid. Most of them operated close to atmospheric pressure. Air was cheap, readily available. The working fluid in a Stirling engine should have following thermodynamic, heat transfer and gas dynamic properties.

1. High thermal conductivity
2. High specific heat capacity
3. Low viscosity
4. Low density

For better system performance in addition to above ease of availability, cost, safe operation, storage requirements are also important properties, which should not be neglected. The capability of working fluid in terms of specific heat capacity, thermal conductivity and density is defined by Martini [57] and Clarke [58] which is useful for preliminary selection of working fluid.

$$\text{Capability factor} = \frac{\text{thermal conductivity}}{\text{specific heat capacity} \times \text{density}}. \tag{29}$$

To determine the best working fluid the whole system performance with different working fluids can be analyzed. The experimental investigation of suitability is difficult and also expensive. Empirical equations derived by Beale [59] do not exist for working fluid assessment, probably because of lack of sufficient experimental data to enable any meaningful correlation to be formed. A simple approach suggested by Walker based on original steady flow analysis is useful for selection of fluid. By using Reynolds's analogy, a relationship between heat transfer and frictional drag in a flowing stream through duct for a system in terms of heat transfer ratio and temperature limits

Table 2
Relative heat transfer characteristics for various gases

Working fluid	Heat transfer	Capability factor
Air	1.0	1.0
Helium	1.42	0.83
Hydrogen	3.42	0.68
Water	1.95	0.39
Sodium–Potassium eutectic	32.62	1.32

Table 3
Relative performance of selected working fluid

Gas	Nominal molar mass (M kg/kmol)	Gas constant (R kJ/kg K)	Sp. heat		Sp. heat ratio (C_P/C_V)
			C_P (kJ/kg K)	C_V (kJ/kg K)	
H_2	2	4.12	14.20	10.08	1.41
He	4	2.08	5.19	3.11	1.67
Ne	20	0.415	1.03	0.62	1.66
N_2	28	0.297	1.04	0.74	1.4
CO	28	0.297	1.04	0.75	1.4
Air	29	0.287	1.01	0.72	1.4
O_2	32	0.260	0.92	0.66	1.4
Ar	40	0.208	0.52	0.31	1.67
CO_2	44	0.189	0.85	0.66	1.28

is derived. The relation is,

$$Q_{wf} \propto (\rho^2 C_P^2)^{0.5}. \tag{30}$$

It is required to simulate engine operation with different working fluids by the available equations so as to select best working fluid. In Table 2 various fluids are compared using Eqs. (29) and (30) at the average temperature and pressure of 800 k and 5 Mpa. It may be seen that no working fluid satisfies these two requirements except NaK eutectic. The feasibility of using this working fluid is currently under investigation at University of California, San Diego. Most of the physical properties involved vary with pressure and temperature and thus heat transfer Q_{wf} and capability factor should be determined under prevailing conditions. Although, NaK seems to be superior fluid for the immediate future.

As the data in Table 3 show, the predominant use of hydrogen and helium is not altogether justified. To improve the thermal efficiency of the engine Gu et al. [60] have suggested to use composite working fluid and also devised criteria to select working fluid.

6. Power and speed control of Stirling engines

Control systems are necessary to regulate the torque, power output and speed of the Stirling engine. In case of constant speed engines, engine speed is held constant with varying load condition in case of stationary-constant speed, fixed frequency electric power

generators. The power of Stirling engines can be controlled by altering the temperature, pressure, stroke, phase angle dead volume and speed or load. Each method has its own advantages and disadvantages. Following are some basic methods used for power control that can be used individually or in combination.

Working fluid temperature control
Mean pressure control (MPC)
Dead volume variation (DVV)
Phase angle variation
Effective stroke variation

6.1. Temperature control

Engine power regulation can be done by regulating engine maximum temperature, which is at heater tubes. Further it can be regulated by controlling fuel flow. As the load increases or decreases, the amount of fuel supplied to the burner should be properly adjusted so as to maintain the heater tube at limiting temperature. However, engine response to combustion process is not adequate because of type of fuel used for combustion. For liquid and gaseous fuels the flow can be easily regulated but for solid fuels it is difficult, so the temperature control method is not commonly used.

6.2. Mean pressure control

As discussed earlier the power output of Stirling engine is directly proportional to the mean cyclic pressure of the working fluid. Thus, if by some external means the pressure level can be altered, then the power developed can be adjusted. The simplest method would be to vent the engine for a power reduction and to supply working fluid for increasing power. In practice the required variation of mean cyclic pressure is not achieved quite so easily. There is a need for a gas storage vessel and a gas compressor to be built in the system. When an increase in power is required the working fluid can be fed in to engine from storage and to reduce power the working fluid has to be dumped to a reservoir. This quite simple conceptual system becomes more complex in practice, since the timing and degree of valve opening has to be carefully controlled and monitored.

6.3. Dead volume variation

Increase in dead volume within the Stirling engine system results in a loss of power but not necessarily a reduction in efficiency. By providing extra spaces the dead volume can be increased or decreased. Berrin Erbay [61] have analyzed the effect of dead volume variation on Stirling engine performance. The engine was operating in a closed regenerative thermodynamic cycle by using polytrophic processes for the power and displacement pistons.

6.4. Stroke variation

Power output in Stirling engines can be controlled by varying the effective stroke length of the engine reciprocating components. This system is both applicable for single and

double acting engines as well as free piston engines also, but it is a difficult task to achieve mechanical change in effective stroke to regulate power.

6.5. Phase angle variation

Variation in phase angle is one of the best possible ways for engine power control because power output of Stirling engine is function of phase angle in an approximately sinusoidal form. At zero phase angle, the volumes of expansion space and compression space varies exactly in phase as there is no cyclic flow of working fluid through system. The power control by varying phase angle is characterized by instant response and is a extremely convenient way to provide rapid engine power control. This system was first adopted by General Motors for their v eight engine. However, the power control by phase angle variation is not applicable to double acting engines.

7. Factors governing engine performance

The engine power output from engine specification can be derived by utilizing the Beale number and the West number. The Beale number is

$$B_{\mathrm{N}} = \frac{P_{\mathrm{i}}}{P_{\mathrm{M}} V_{\mathrm{se}} N}.$$

William Beale observed that in practice the maximum power output of well-developed engine is roughly proportional to pressure, volume and speed. One important factor that is not taken in to account by Beale correlation is the temperature at which the engine is operated. We have already discussed in Stirling engine simple analysis says that the increase in heater temperature will increase power for a fixed cooler temperature. Beale's empirical relation do not contain any temperature effects because most of the engines examined by him had the heater temperature above 650 °C. Iwamoto [62] shows that Beale number is about 0.15 in case of high temperature differential Stirling engine whose heater wall temperature is about 650 °C.

The effect of the temperature is considered by West [63]. The number is defined as

$$W_{\mathrm{S}} = \frac{P_{\mathrm{i}}}{P_{\mathrm{M}} V_{\mathrm{se}} N (T_{\mathrm{E}} - T_{\mathrm{C}}/T_{\mathrm{E}} + T_{\mathrm{C}})}.$$

It was found that West number is about 0.25 in case of 5–150 kW Stirling engines and is about 0.35 in case of smaller power engines. Patrescu [64] derived factor which affect engine performance based on first law of thermodynamics. The method used for the analysis is irreversible cycle with finite speed involves the direct integration of equations based on the first law for processes. A numerical program is written by Altman [65] is also useful for determining engine performance.

7.1. Regenerator effectiveness

The regenerator effectiveness increases with increase in reduced length and decreases in reduced period. The regenerator design must be such that the heat transfer coefficient and area of matrix of regenerator should be kept maximum possible by maintaining lowest

fluid flow rate. The flow in Stirling engine regenerator alternates in many rapid cycles so it is highly probable that only a portion of total gas charge passes through the matrix and some of the fluid may remain in the regenerator. This factor is called as regenerator hold-up. For reduced length less than 10, the effectiveness of regenerator is more for a regenerator with hold-up than the regenerator without hold-up.

7.2. Regenerator material

Choice of the materials for the regenerator matrix is a matter of concern as it influences the performance of the engine considerably. The efficiency and power output of the engine are function of engine speed for metallic and ceramic regenerator materials. Due to lower permeation rate of ceramics than metals it is found that efficiency and power output of the engine with ceramic-coated materials of regenerator is higher than the metallic regenerator under all speeds of investigation.

7.3. Working fluid and fluid leakage

In a practical engine some leakage of the working fluid-invariably a gas-is inevitable. The pressure within the space is usually higher than the idealized minimum cyclic pressure and this means that because of seal leakage paths the gas will flow out of the system at the high cyclic pressure but tend to flow back in to the system during the compression phase. Both effects reduce the work output of the cycle. The effect of pressure loss due to friction, finite speed and throttling process in the regenerator of the engine is presented by S. Pertescu [25].

7.4. Fluid friction

Fluid friction with regenerator mesh becomes more serious issue in Stirling cycle engine. The flow friction is mainly due to size, shape and density of wire mesh and properties of working fluid such as density and viscosity of working fluid. The flow friction consumes power from engine so net power output is reduced. The regenerator material required to selected and system should be designed such that it will cause minimum friction resistance. It is demonstrated that friction factor of simple stacked wire mesh become function of Reynolds number when the aperture size of wire mesh was selected as the representative length scale. Also friction and Nusselt number of porous media are similar to those of simple stacked wire mesh. This result suggests that friction factor decreases gradually with increases in value of Reynolds number. Isshiki [37] and Muralidhar [38] study shows that the there is considerable effect of flow resistance and heat transfer of regenerator wire meshes of Stirling engines in oscillatory flow.

8. Operational characteristics of the Stirling engine

8.1. Mean cycle pressure

The power output of Stirling cycle engines is found to in practice directly proportional to the mean cycle pressure. Berrin Erbay [66] have shown that to obtain high overall power levels and densities, pressures in the range of 10–20 Mpa are used. These high-pressure

values present particular problems with relation to working fluid contamination, stressing of heat exchanger and loading of drive mechanism. The Stirling cycle engines can achieve efficiencies of 65–70% of Carnot cycle efficiency with current technology. Banch Kongtragool's [67] engine powered by low temperature shown that the engine efficiency is relatively insensitive to speed effects provided that the heater tube temperature is maintained at a fixed value throughout the engine operating range and cooler temperature is not allowed to rise. The heater temperature should be kept as high as possible to attain high pressure.

8.2. Dead volumes

The un-swept volumes, called dead volumes in a Stirling engine, should be kept theoretically zero but in practice accounts for up to 50% of the total engine internal gas volume. Feng Wu [23] have made a criteria for optimization of dead volume, this amount of dead volume is required to accommodate the necessary heat exchangers and to allow sufficient heat transfer surfaces. Dead space reduces the power output of the engine but has conflicting effects on the efficiency, depending on the location of the dead space. Dead space can be altered during operation of the engine to control power output. Increasing the swept volume should increase the power output of the engine provided that pressure and temperature levels are maintained. No empirical correlation between power output and swept volume exists. To achieve a particular swept volume the bore to stroke ratio should be about two.

8.3. Thermal losses

As the engine speed increases the losses derived by K. Makhkamov becomes predominant factors, as they are proportional to the square of speed. To reduce these losses light molecular weight working fluids are to be used such as helium and Hydrogen. However, these gases are difficult to contain because of ability to diffuse with solid material. Imperfect regeneration is another important cause for poor engine performance.

8.4. Mechanical losses

The mechanical friction arises from piston rings, rubbing seals, bearings, oil pumping and other friction of parts. In comparison with conventional engine mechanical friction is less because of less number of moving parts.

9. Conclusions

The Stirling cycle engines have proven its multi-fuel capability to operate with any possible fuel source-liquid, gaseous or solid fuels with wide temperature range. This is an important feature of the engine that it can use abundant heat source from solar radiation, waste heat from industry, heat produced from agricultural waste and so many other low-temperature sources. This particular feature of the engine has keep Stirling engine in focus for design and development for better system efficiency where there is large scope.

Stirling engine system is most complicated thermo-mechanical system because of complications in mechanical arrangement caused by phase difference required between

compression and expansion spaces and presence of heat exchangers like-heater, cooler, regenerator and auxiliary heat exchangers along with a complicated power control system. The reliable and efficient operation of the engine is depends upon the dynamic behavior of engine mechanism and performance of all heat exchangers, which is interdependent. This difficult task to design a system where thermal, fluid and mechanical design considerations are required to be taken in to account jointly with system optimization.

The numerous investigations made by scientists and engineers since from invention of the engine have made a good base line information for designing engine system, but a more insight is essential to design systems together for thermo-fluid-mechanical approach. It is seen that for successful operation of such system a careful selection of drive mechanism and engine configuration is essential. An additional development is needed to produce a practical engine by selection of suitable configuration; adoption of good working fluid and development of better seal may make Stirling engine a real practical alternative for power generation.

The study indicates that a Stirling cycle engine working with relatively low temperature with air of helium as working fluid is potentially attractive engines of the future, especially solar-powered low-temperature differential Stirling engines with vertical, double acting, gamma configuration.

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