**Stirling engine optimization**

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***Abstract***. This paper deals with the optimization of some design parameters of a Stirling engine. A lumped parameter model is used to describe the engine. The model is coupled to an optimizer based on genetic algorithms. After a preliminary analysis, a restricted set of parameters is chosen for the optimization process. Optimal values are hence identified. As a result, we provide a specific set of parameters inspired to a real application under investigation. The work also yields a practical example of the power of optimization algorithms for engineering design.

**Introduction**

Stirling engine, why are they studied in general and which is the present application.

Optimization, what are genetic algorithms and why are they used.

Specific implementation, the jMetal package and Octave/Matlab and similar softwares.

**Model**

We model the Stirling engine according to Urieli [urieli1]. The model code can be freely downloaded from Urieli’s website, where an extensive description of the model is also available. For the sake of completeness, the main equations governing the system are summarized in the Appendix.

The adopted model can be easily adapted to specific technological implementations. The main purpose of this work, however, is not to provide an improvement of the model itself, while to show how it can be coupled with an optimization algorithm in order to obtain a valuable and efficient design flow. We will provide the information for the implementation of an open source software tool freely available to researchers.

**Optimization parameter choice**

As the considered model contains several parameters, it is important to perform a preliminary analysis in order to identify a reasonable set of parameters to be adjusted in the optimization process. Many global parameters are indeed fixed by the needs of the specific application and are not worth to be included in the optimization. This is an important step in order to get en efficient computational process, as the complexity of the optimization problem significantly increase with the search space dimension.

Once the parameters which are worth to be optimized have been identified, it is necessary to provide proper bounds for their variation. This is again related to practical needs related to the considered application. In general, several competing effects can be identified, making difficult to understand the optimum conditions on a purely analytical basis, even for a relatively simplified theoretical model as the one considered here. Numerical optimization hence provides a valuable tool which can significantly speed up the design process. Moreover, numerical optimization can be applied even in the presence of more complex models, where the analytical understanding of the governing equations can be very difficult or where specific components in the model are simulated numerically as well.

From a general point of view, the Stirling engine is divided in five cells, i.e., two piston chambers (compression and expansion volumes), two heat exchangers (cooler and heater), and one regenerator. From general consideration, it turns out that the parameters which are most suited for numerical optimization are those related to the heat exchangers. Indeed, a typical design process can be the following:

* The approximate working temperatures are given by the considered application (ambient temperature for the cold end, maximum achievable temperature for the given heat source on the hot end, the latter source possibly being solar energy or any form of combustion energy).
* The power is chosen according to the desired application (typically a compromise between available heat and needed output).
* Some limiting size is chosen according to the space reasonably available for the chose application. [\*Consider including material costs in the optimizer?\*] This fixes the order of magnitude of the piston chambers. The total available volume combined to the required power also fixes the average operating pressure of the engine.
* Some operating frequency is chosen according to… [\*?\*]
* The regenerator is typically designed independently, as the engine efficiency is strongly dependent on the regenerator effectiveness, while the remaining parameters are weakly coupled to this component.
* The heat exchangers are optimized within the bounds given by the previous parameters.

On the basis of the above considerations, we decided to focus our optimization analysis only on the heat exchanger parameters. It is however important to point out that a more extended model (e.g., including material costs, piston friction losses and frequency effects) could easily benefit of a wider choice of optimization parameters. By presenting the workflow outlined in this paper, we hope to favor the implementation of more accurate and comprehensive open source models which could be coupled to an extended optimization process. A higher availability of reliable and efficient open source design tools for Stirling engines might foster the research and development in this field, possibly allowing to overcome the several difficulties which hindered the successful diffusion of this appealing technology so far.

**Heat exchanger parameters**

Heat exchangers can occur in several typical configurations, like shell and tubes heat exchangers or counter-flow heat exchangers. The specific type of heat exchanger adopted for the engine should be chosen according to the available heat source (or sink). The heat exchange with a burning flame will clearly require a different design than that with a hot fluid. The modeling of the heat exchanger in terms of specific parameters (e.g., tube shape and number) will hence be different depending on the application. In the present case we will refer to the case of straight tubes heat exchangers, a suitable configuration for our system, both on the cold (cooler) and hot (heater) side.

Given the general type of exchanger, the identification of suitable parameters can start. In the case of Stirling engines, some competing effects come into play, making the design a non-trivial process.

The primary requirement of a heat exchanger is clearly its effectiveness in transferring heat, which can be quantitatively related to its overall heat transfer coefficient. On the other side, pumping losses due to the wall-fluid friction typically increase when heat transfer properties are improved. Moreover, there is a strict relation between the engine power and the heat exchanger volume, as the larger is the volume not included in the piston chambers (i.e., the larger is the so-called dead volume with respect the so-called swept volume), the lower is the output power of the engine.

Such competing effects put in evidence the usefulness of an optimization process. The specific parameters chosen in our case are the number of straight pipes, , the pipe length, , and the pipe diameter, . Hence, three parameters for each heat exchanger will be available. We will use the suffix to denote the cooler parameters (, , ) and the suffix for the heater parameters (, , ). Some reasonable bounds to these parameters have to be decided before starting the optimizer, in order to avoid the random generation of unphysical solutions. In our case, heat exchangers with a hundred of pipes with a length of 20-30 cm and a diameter of a few mm were reasonable. We hence put our limits accordingly (see later).

**Genetic algorithms**

The used optimizer is a genetic algorithm implemented in the jMetal package, a freely available Java tool which can be coupled to Octave (or Matlab). [\* jmetal can be introduces in Result/Simulation methodology \*]

Genetic Algorithm (GA) is a meta-heuristic optimization algorithm inspired from nature. GA mimics the idea of natural selection and reproduction. For last several decades, GA has been applied to many real world complex problems.

Genetic algorithm seek for an optimal solution by starting from an initial “population” (where a single “individual” is identified by a given combination of the variable parameters) randomly chosen within the available space (i.e., within the given boundaries for the variable parameters). The population “evolves” through a series of “generations”. At each generation step, potential individual (called “parents”) are selected for mating from the present population and the parents are transformed through a series of operators (crossover and mutation) giving rise to new individuals (called “children”). The old parents and children are compete to each other to be selected for next generation. Selection of individuals will occur on the basis of a “fitness” function , which is a quantitative measure of the quality of the individuals. In the present case, the objective of the optimization is the maximization of the engine efficiency , so that the fitness function can be directly related to the latter parameter, in particular we chose and seek for the parameter combination corresponding to the minimum value of .

With a proper combination of genetic operators (where the probabilities of any transformation can be adjusted to speed up convergence), the available search space can be efficiently sampled, allowing for a quick identification of the optimal solutions, even in the presence of complex fitness landscapes (e.g., fitness functions which, within the search space, have several local minima where gradient-based minimization algorithms would remain stuck).

Constrains handing [cite] is one of the most important issue when it is require to work with real world problems. Because it is possible to get a very good solution but practically impossible to implement. That’s why on the starting of twenties century, the researchers develop techniques to adopt GA to handle constraints. The technique is applied mainly in two phases into the algorithm. First, when parents are selected for mating and second, when individuals are selected for next generation. An individual is better than the other in terms of constrains heading by the following criteria [cite].

1. A legal (the individual that do not violate any constraints) individual is always selected over an illegal individual.
2. When compare within two legal individuals, the individual with better fitness function is selected
3. When compare with two illegal individuals, the individual with smaller constraint violation is selected.

[\*some more may be added\*]

**Results**

To optimize the engine we take advantages of jMetal [cite], an object-oriented java based framework for solving optimization problems. We coupled jMetal with engine simulator/ numerical model (octave/Matlab script). A feedback process is developed between those two frameworks to help jMetal to optimize engine parameters to get maximum possible efficiency.

As the numerical model used for the engine takes a short time to run (of the order of a second), it is possible to perform several evaluations within the optimization algorithm. This makes it feasible to vary all the parameters together with a reasonable computation effort on a standard workstation. It is hence possible to perform a “blind” optimization of the system in order to get the highest possible efficiency (for the desired power, operating frequency, piston chamber volumes, etc.). It is worth pointing out that the optimizer was successful in finding the best solutions even without further settings. In order to show the reliability of the results and to provide more insight for their interpretation, we will however proceed by gradual steps, first presenting the optimization of the heater only, where some analytical considerations for a better understanding of the results will also be included, then showing the results for the optimization of the cooler only, and finally offering the outcomes for the coupled optimization of the two heat exchangers together.

**Engine parameters**

The reference engine parameters are as follows:

* Power, = 908 W
* Frequency, = 2 250 Hz (?)
* Hot temperature, = 300 °C (?)
* Cold temperature, = … °C
* Expansion swept volume, = 1.450e-004 m3
* Expansion clearance volume, = 0.090e-004 m3
* Compression swept volume, = 1.327e-004 m3
* Compression clearance volume, = 0.090e-004 m3
* Heat exchange pipe diameter, = 2.500e-3 m3
* Heat exchanger length , = 2.2500e-001 m3
* Heat Exchanger No of pipes , = 34
* Cooler pipes diameter, = 1.500e-3 m3
* Cooler piper length, = 1.700e-001 m3
* Cooler no of pipers , = 125
* Regenerator external tube diameter , = 7.00e-002 m3
* Regenerator internal tube diameter , = 7.00e-002 m3
* Regenerator length , = 4.200e-002 m3
* Regenerator no of tubes , = 1
* Operating gas type, = air
* Gas pressure , = 13500000.0
* cold sink temperature (K), = 313.0
* hot source temperature (K) , = 548.0

In the real engine, four Stirling cycles are combined together. By sharing pistons among consecutive cycles, an efficient setup is obtained. The total power of ther real engine is therefore about 3.5 kW.

The numerical optimization was applied to the parameters relative to the heater and the cooler, while leaving the remaining parameters unchanged. As mentioned before, initially the optimization will be presented for the heater alone (with reference parameters for the cooler), then for the cooler alone (with reference parameters for the heater), and then for the two heat exchangers together.

**Heater optimization**

As mentioned above, a heat exchanger with parallel circular pipes has been chosen. The variable parameters are hence the number of pipes , the pipe diameter , and the pipe length .

The chosen constraints are:

* 1 100
* 1 mm 5 mm
* 10 mm 50 mm
* 100

Such constraints are motivated by practical reasons connected with the chosen application. For example, due to the available spaces, it was not possible to increase the number of pipes or the pipe length beyond the above limits. On the other side, pipes with diameters below 1 mm or aspect ratios larger than 100 are difficult and expensive to realize. The upper limit for the pipe diameter, instead, could have been set larger, but it was clear from some preliminary analysis that small diameters were favored in the optimization process, so that we used a rather low bound to speed up convergence.

The optimal solution was found to reach the limiting values for both the number of pipes and the pipe aspect ratio. The obtained values are hence optimal only with the given constraints [\*Possible to use Linear Programming in this case?\*]. The specific values of pipe diameter and pipe length were hence mainly determined by the dead volume.

We provide a qualitative explanation of the results, showing how they can be interpreted in terms of the model. The following analytical arguments can be used for an initial guess of reasonable working parameters, though the actual calculation of the optimal values clearly requires the application of optimization algorithms to the numerical model. [\*Message: some analytical insight is available, it can be used to cross check that the numerical results are reasonable, but truly optimal results require the solution of the numerical model.\*]

There is a strict relationship between the chosen power and the dead volume of the engine. Such a relationship would be a one-to-one correspondence in the ideal model (i.e., for a given power, the dead volume is fixed). However, the inclusion of non-ideal effects (e.g., pumping losses) slightly modifies this relation (i.e., the dead volume depends not only on the output power, but also on other parameters). As a first approximation we can use the ideal result, which implies that the optimization is performed at constant dead volume (as the power is fixed). This yields the implicit constraint

Then, as we are only optimizing the heater, we can assume that the main parameter affecting the engine efficiency is the heater overall exchange coefficient . In fact, other effects (e.g., pumping losses) are expected to give a minor contribution to the global result. The consistency of this assumption can be validated a posteriori.

The overall heat exchange coefficient is given by

where is the heat transfer coefficient, is the interface surface, is the Nusselt number and is the fluid thermal conductivity. The proper evaluation of the Nusselt number in a non-stationary application as the present one is a non-trivial task. During the Stirling cycle the fluid flows through the pipes at varying velocity and in different directions, with a highly complex velocity pattern. A detailed calculation would involve different regimes (from laminar to turbulent) with a non-developed flow (the entrance length turns out to be typically larger than the pipe length itself). The simple assumption made in the considered model is that the Nusselt number can be calculated from the average Reynolds number = = , where is the fluid density, is the average velocity, is the mass flow rate, and is the fluid dynamic viscosity. The correlation formula adopted here is [urieli1,kays&london]

We are interested in the dependence of the above quantities on the variable parameters. One has , , and hence

This formula can be rewritten in several ways in terms of two variable parameters only, by exploiting the constant volume relation reported above. For example, since , one has , which makes evident that, in the absence of geometrical constraints on the pipe aspect ration, in order to maximize it is always convenient to maximize the pipe length and to minimize the pipe diameter .

The presence of practical constraints can however change the situation. As discussed above, in the present case during the optimization process the variables and do not reach their limits, due to the presence of the constraint on the aspect ratio . It is hence useful to rewrite in terms of and . First we observe that the constant volume relation implies and . Then we substitute in the expression for the overall heat exchange coefficient obtaining

This clearly shows that in order to maximize it is always convenient to increase and . There is hence a competition between the tendency to reach the available limits for the variable pairs and . It is non-trivial to understand from analytical considerations which constraints is met first by the optimizer, but the above considerations provide some insight for the interpretation of the numerical results.

The found relation also helps in explaining the optimizer behavior. Indeed, reaching convergence in terms of the number of pipes is significantly slower than reaching convergence in terms of the aspect ratio . This is well understood in terms of the exponents within the above relation: while is basically linear in , it has a quite weak dependence on . In other words, a given relative variation in affects (and hence the efficiency, which determines the fitness function) much more than the same relative variation in .

In the following we report some figures/tables concerning the numerical results.

First of all we provide a numerical confirmation of the constant volume assumption mentioned above. In order to test the validity of this approximation, we performed different simulations where the initial dead volume was allowed to vary with respect to the reference value. Then, the initial population was evolved by keeping only the configurations with the desired power [\*Describe better this constraint?\*]. As evident from the graph plotted in Figure 1, all the solutions evolve towards very close dead volume values, confirming the almost one-to-one relation between dead volume and power. [\*The figure is actually referred to usual optimization runs, so the selection process is not only related to constant power, but also to highest efficiency. The fact that the dead volume is the same for all the runs could in principle be due just to the fact that they all converge to the same configuration. It would hence be better to show the results of runs where the fitness function is something like . Then it could be interesting to explicitly show a plot of the dead volume as a function of the power, for given piston volumes, temperatures etc.\*]

Figure 1. Evolution of the engine dead volume during the selection process provided by the constant power constraint. [\*Check in the volume formula.\*]

The evolution of the ratio and of the number of pipes is reported in Figure 2. In both cases it is evident the tendency to reach the maximum possible limit [\*add limit line in the figure?\*]. It is also possible to observe the different convergence rate in the two cases: while the ratio basically reaches the limiting value after 10 [?] iterations, the number of pipes takes almost 20 [?] iterations to saturate. The different behavior can be explained with the different exponents in the relation for reported above. [\*Check/comment quality of runs used for the graphs: number of individuals etc.\*]

Figure 2. Evolution of the ratio and of the number of pipes during the genetic optimization process for different runs. Note the different convergence rate of the two graphs.

The optimization results are reported in Table 1 for different runs. [\*At the moment the table reports the data obtained with a population of 50 individuals after 30 generations. The power constraint was set to = 900 W 10 % so that = 900 W – 10 % = 810 W.\*] After the considered number of generations, convergence is basically reached in terms of the efficiency . [\*If we use these data we can comment that the first run is farther from convergence than the others. We can then provide average values and standard deviations or relative deviations among runs. For example the average optimized diameter is = 2.91 mm with a relative deviation (fluctuation from run to run) of about 1.5 %. It is also worth specifying the average result for the ratio (putting in evidence that it approaches the upper bound of the admitted range) and maybe the dead volume.\*]

Table 1. Optimization results for different runs.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run |  | [mm] | [mm] | [W] | [%] |
| 1 | 90 | 2.99 | 309 | 810 | 31.4 |
| 2 | 98 | 2.88 | 309 | 810 | 31.5 |
| 3 | 96 | 2.91 | 312 | 810 | 31.5 |
| 4 | 98 | 2.89 | 311 | 810 | 31.6 |
| 5 | 99 | 2.88 | 308 | 810 | 31.6 |

[\*Urieli files:

- pipefr.m: heat transfer coefficient , with ( is the Reynolds friction factor, while is the Fanning friction factor; note that the formula for is valid for 2100 < < ). Recall that: the Prandtl number is , the kinematic viscosity is , the thermal diffusivity is ; is the dynamic viscosity, is the thermal conductivity, is the density, is the specific heat at constant pressure. In conclusion Urieli assumes .\*]

**Cooler optimization**

Cooler

[\*Include a table of results, similar to that for the heater. Comment the results along the same lines.\*]

**Heater and cooler coupled optimization**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run |  | [mm] | [mm] |  | [mm] | [mm] | [W] | [%] |
| 1 | 100 | 2.43 | 262 | 196 | 1.14 | 156 | 905 | 30.2 |
| 2 | 100 | 1.92 | 207 | 137 | 1.65 | 224 | 905 | 29.9 |
| 3 | 87 | 2.31 | 249 | 190 | 1.33 | 181 | 905 | 30.1 |
| 4 | 64 | 2.72 | 285 | 194 | 1.19 | 162 | 905 | 29.9 |
| 5 | 99 | 2.29 | 247 | 148 | 1.38 | 188 | 905 | 30.1 |

Heater and cooler simultaneous optimization.

[\*Include a table of results, similar to that for the heater. Comment the results along the same lines.\*]

It can be observed that, differently from the case of the separate optimization of heater or cooler, in the case of the coupled optimization the best performances can be obtained with quite different configurations. Indeed, for different runs the algorithm can select significantly different values for the geometrical parameters of the exchangers. This is due to the fact that, while the total dead volume is basically the same for all the optimal configurations (due to the strong coupling between dead volume and power), the distribution of the dead volume between the heater and the cooler is practically equivalent. In other words, there is some freedom in the optimal design, as for a given total dead volume one can slightly favor the performances of one exchanger with respect to the other by keeping the same overall efficiency.

Concerning the relation between power (and hence dead volume) and efficiency, it is interesting to show explicitly some numerical results. We therefore performed a multi-objective (MO) optimization in terms of power and efficiency [\*Pareto frontier?\*].

Figure 3. Relation between power and efficiency as obtained by varying the parameters of the two exchanger around the reference values.

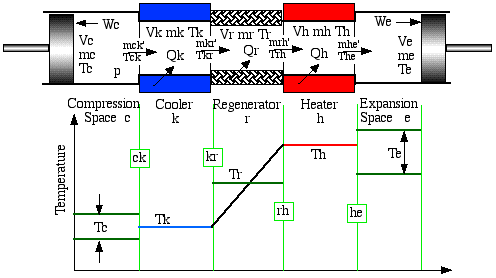
**Conclusions**

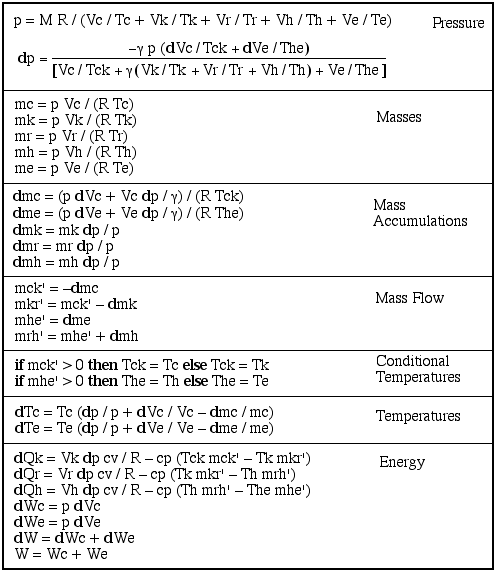
This work shows an example of an effective application of genetic algorithm optimization to a Stirling engine…

**Appendix**

Description of the Stirling engine model and corresponding equations.

Ideal adiabatic model [urieli1, thombare]:





Simple model:

* Regenerator losses
* Pumping losses
* Heat exchanger analysis

**References**

Books on Stirling engines.

Books on heat and mass transfer.

Papers on Stirling engine.

Books on genetic algorithms.

Papers on genetic algorithms.

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**Comments**

Points to be fixed:

* Check all parameter values