Inferential Statistics

GITAA
Transforming careers

Purposes of Statistics

- To Describe a situation/phenomena/process
  - Descriptive Statistics
- To Infer conclusions about a population from samples of data
  - Inferential Statistics
- To Predict the value a variable would take, if we know certain other variables related to the situation/phenomena/process
  - Predictive analysis

Purposes of Statistics

- To Infer conclusions about a population from samples of data
  - Inferential Statistics

Motivation

- A new mobile model claims a battery life of 96 hours – How do we test it?
- A car dealer claims that men in the age group of 30 to 35, who visit the showroom with their family are the ones to pay attention to.
- What we have is a Claim/Hypothesis – and we need to decide whether we reject or not reject the hypothesis.

Concept of Sampling Distribution

- A new mobile model claims a battery life of 96 hours – How do we test it?

- It might be difficult to check every piece (N), and therefore we will take a sample of "n" from the N pieces to do the test.

- We find out the battery life of "n" batteries, and take an average and see how close the average is to "96 Hours"

- What we are now interested is not in individual values of the test data (Battery Life in Hours), but the average of all test data

Concept of Sampling Distribution

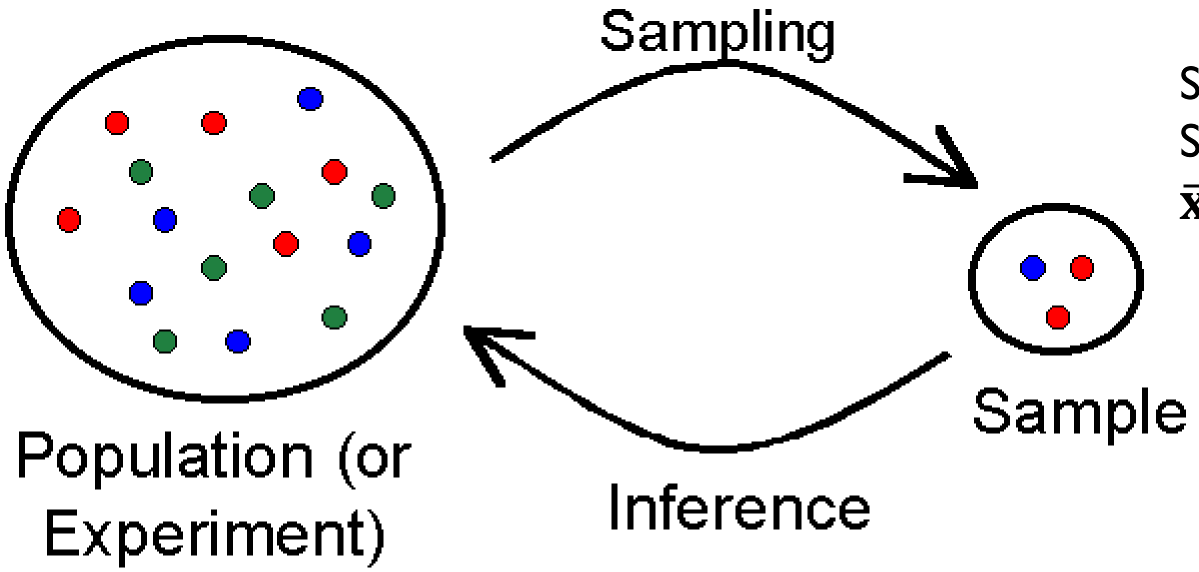
- What we are now interested is not in individual values of the test data (Battery Life in Hours), but the average of all test data

- We are therefore now interested in knowing how will the averages be distributed if we take many samples of "n" from the population of N Batteries

  – Sampling Distribution of Mean

Population and Samples

Population
Parameters
μ, π, σ^2



Sample
Statistics
$\bar{x}$, $\hat{p}$, s^2

Method of Sampling, and Size of Sample are important to inference

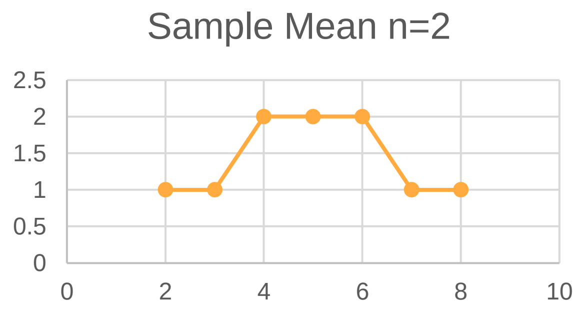
The sampling distribution enables us to understand the relationship between the test statistics, and the population parameter values

Example of Sampling Distribution (Mean)

- **Population (N=5) 1 3 5 7 9 : Mean μ = 5 : Variance σ^2 = 8**
- **Let's say, to Estimate the population mean we take a sample of 2 (n=2)**
- Possible samples of 2 are

Samples of 2			Sample Mean $\bar{x}$,
	1	3	2
	1	5	3
	1	7	4
	1	9	5
	3	5	4
	3	7	5
	3	9	6
	5	7	6
	5	9	7
	7	9	8

Mean (Sample Mean)	5
Variance (Sample Mean)	3

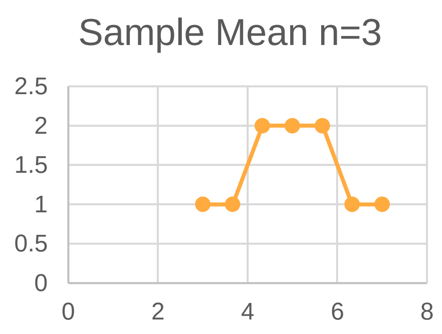


Example of Sampling Distribution

- **Population (N=5) 1 3 5 7 9 : Mean $\mu = 5$: Variance $\sigma^2 = 8$**
- **Let's say, to Estimate we take a sample of 3 (n=3)**
- Possible samples of 3 are

Samples of 3				**Sample Mean $\bar{x}$**
	1	3	5	3.00
	1	3	7	3.67
	1	3	9	4.33
	1	5	7	4.33
	1	5	9	5.00
	1	7	9	5.67
	3	5	7	5.00
	3	5	9	5.67
	3	7	9	6.33
	5	7	9	7.00

Mean (Sample Mean)	5
Variance (Sample Mean)	1.33



Example of Sampling Distribution

- **Population (N=5) 1 3 5 7 9 : Mean μ = 5 : Variance σ^2 = 8**

For n = 2

Mean (Sample Mean)	5
Variance (Sample Mean)	3

For n = 3

Mean (Sample Mean)	5
Variance (Sample Mean)	1.33

The relationship between **Mean($\bar{x}$)** and **μ** is: **Mean($\bar{x}$) = μ**

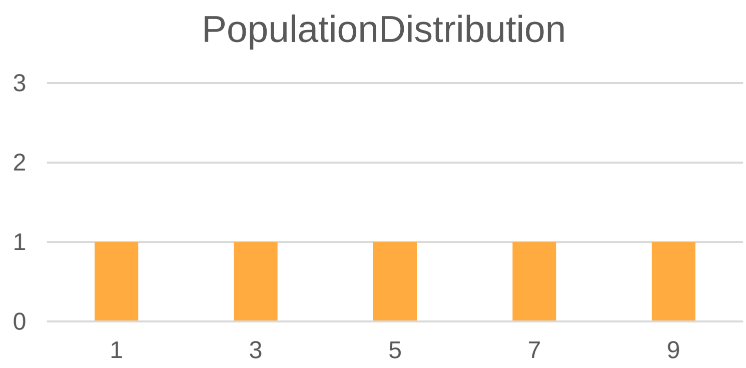
The relationship between **Variance ($\bar{x}$)** and **σ^2** is: **Variance** ($\bar{x}$) $= \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$

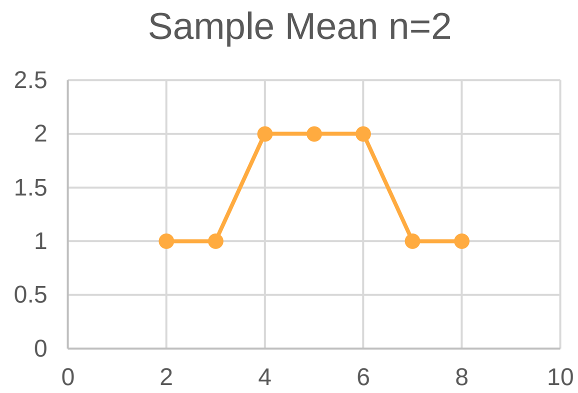
When "N" is Large the term $\frac{N-n}{N-1}$ will approach 1: **Variance** ($\bar{x}$) $= \frac{\sigma^2}{n}$

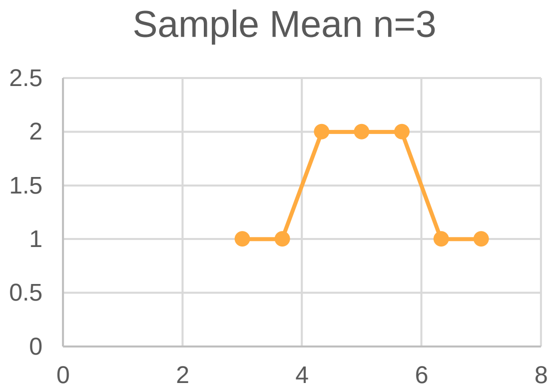
Example of Sampling Distribution

When "N" is Large the term $\frac{N-n}{N-1}$ will approach 1:

Variance $(\bar{x},) = \dfrac{\sigma^2}{n}$

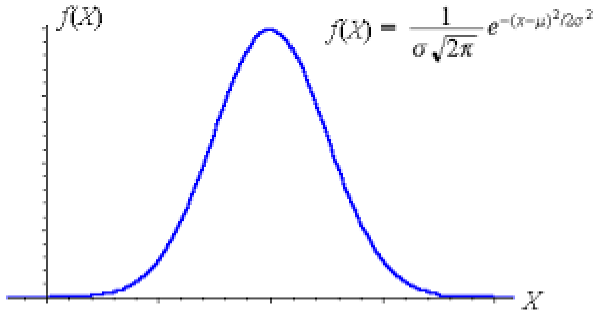


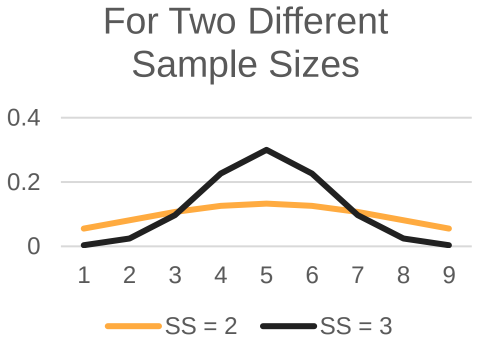




As "n" increases to a large number (>30), the sampling distribution is well approximated by a normal curve, with a mean of **μ,** and a variance of $\dfrac{\boldsymbol{\sigma}^2}{n}$

-Central Limit Theorem





More on CLT

- For large n, the sampling distribution is approximately normal, irrespective of whether the population distribution is normal or not.

- This is useful in practice, since in many real life cases, we do not know, what the population distribution of a parameter is.

Implication of CLT

- If the sampling distribution of sample mean $\bar{x}$ can be approximated by a normal distribution, then, there is a 95 % probability that the sample statistic $\bar{x}$ will fall within 2 standard errors of the population mean **μ,** and a 99% probability that it will fall within 3 standard errors of the population mean **μ.**

- $\overline{\boldsymbol{X}} = \boldsymbol{\mu} \pm 1.96 * \frac{\sigma}{\sqrt{n}}$ (With a 95% probability)

- $\overline{\boldsymbol{X}} = \boldsymbol{\mu} \pm 2.58 * \frac{\sigma}{\sqrt{n}}$ (With a 99% probability)

Sampling Distribution - Proportions

- **Toss of a coin: Pr(H) = p = 0.5 : Pr(T) = q = 0.5**
- **Let's say we toss the coin 5 times (n=5), Can we estimate the number of times heads will turn up?**

Number of Heads	Freq	Prob
0	1	0.03125
1	5	0.15625
2	10	0.3125
3	10	0.3125
4	5	0.15625
5	1	0.03125

The probabilities can be computed by the binomial distribution function formula

$$P_n(x) = C(n,x)p^x q^{n-x} = \frac{n!}{x!\,(n-x)!}p^x q^{n-x}$$

The binomial distribution function can also be well approximated by a normal curve, with a mean of **$\mu = np$,** and a variance of **$\sigma^2 = npq$**

Sampling Distribution - Proportions

Tossing a fair coin 50 times

# of Heads, X	Pr(X), n=50, p=0.5
0	8.88178E-16
1	4.44089E-14
2	1.08802E-12
3	1.74083E-11
4	2.04547E-10
5	1.88184E-09
6	1.41138E-08
7	8.87152E-08
8	4.76844E-07
9	2.22527E-06
10	9.12362E-06
11	3.31768E-05
12	0.000107825
13	0.000315179
14	0.000832974
15	0.001999138
16	0.004373115
17	0.00874623
18	0.016034755

Pr(X) =0 to 18 = 0.032

# of Heads, X	Pr(X), n=50, p=0.5
19	0.027005903
20	0.041859149
21	0.059798785
22	0.078825671
23	0.095961686
24	0.107956897
25	0.112275173
26	0.107956897
27	0.095961686
28	0.078825671
29	0.059798785
30	0.041859149
31	0.027005903

Pr(X) =19 to 31 = 0.935

A fair coin will give you 19 to 31 heads out of 50 tosses, 93.5 % of times you do this experiment

# of Heads, X	Pr(X), n=50, p=0.5
32	0.016034755
33	0.00874623
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41	2.22527E-06
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44	1.41138E-08
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48	1.08802E-12
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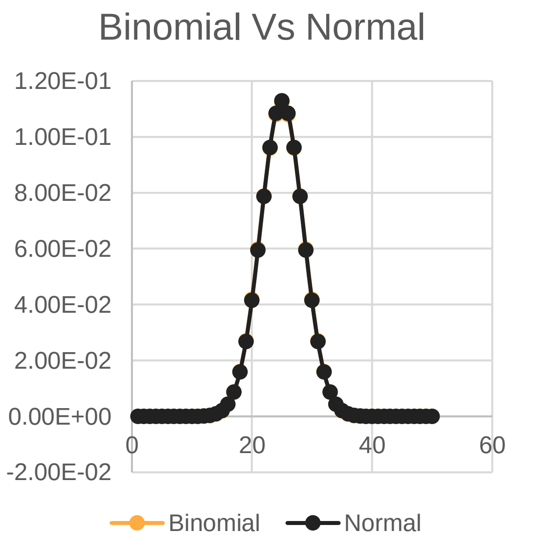
50	8.88178E-16

Pr(X) =32 to 50 = 0.032



Using μ = **np,** and σ^2 **=npq**

Confidence Interval

Point and Range Estimates

$\overline{X}$ = μ ± 1.96* $\frac{\sigma}{\sqrt{n}}$ (With a 95% probability) – From CLT

- The Range estimate of **μ** = $\overline{X}$ ± z* $\frac{\sigma}{\sqrt{n}}$ (or) **μ** = $\overline{X}$ ± z* $\frac{s}{\sqrt{n}}$
- The Range estimate of π = $\hat{p}$ ± z*$\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$
- The larger the "n", the narrower the range of estimate
- The larger the confidence level, the broader the range of estimate
- The Larger the variability within the data, the larger the range of estimate

Confidence Level	Z - Value
90%	1.645
95%	1.96
99%	2.576

Confidence Interval - Example

- A new mobile model claims a battery life of 96 hours – How do we test it?
- If our sample mean is 94 hours and sample standard deviation is 2 hours (On a sample size of 100 mobiles), our range estimate (95% confidence interval) for **μ** is

- $\boldsymbol{\mu} = \mathbf{94} \pm 1.96 * \frac{2}{\sqrt{\mathbf{100}}} = 94 \pm 0.392$ Hrs = 93.608 to 94.392 Hrs

Hypothesis Testing

Hypothesis Testing- Concept

- What is a Hypothesis?
- A Proposition made as a basis for reasoning, without any assumption of its truth

Hypothesis Testing: Components

- A statement of a hypothesis – Usually expressed as Null and alternate hypothesis
- A α value – referred to as Significance Level
- A test Statistic
- A p value

Hypothesis Testing – H. α, β, and p

Your Product development claims to have created a superior version to the existing product
Modifications to the existing production facilities to produce the new product would cost USD 20 Million.
The potential increase in profits could be up to USD 40 Million if the new product is superior
You decided to test prototypes with customers

H_0 : The New product is as good as the old product, H_1 : The New Product is Superior

Data		**Reality**	
		New Product Not Superior	New Product is Superior
Suggestion of Evidence	New Product is not Superior	Correct Decision	Wrong Decision (Type II Error) β
	New Product is Superior	Wrong Decision (Type I Error) α	Correct Decision

Hypothesis Testing – H, α, β, and p

H_0 :The New product is as good as the old product,
H_1 :The New Product is Superior

Your Call		**Reality**	
		New Product Not Superior	New Product is Superior
Suggestion of Evidence	New Product is not Superior	Correct Decision	Wrong Decision (Type II Error) β
	New Product is Superior	Wrong Decision (Type I Error) α	Correct Decision

α is the maximum level of uncertainty that one is willing to concede, in the conclusion that the new product is superior(i.e. the null hypothesis can be rejected), when the new product is not superior

α , and β are inversely related, and you can only optimize, between the two.

The p value is a computed level of uncertainty (from the data) about the evidence supporting the alternate hypothesis

The Values of α , and β are really a function of the consequences of the decisions

Hypothesis Testing – H, α, β, and p

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		New Product Not Superior	New Product is Superior
Suggestion of Evidence	New Product is not Superior	Correct Decision	Wrong Decision (Type II Error) β
	New Product is Superior	Wrong Decision (Type I Error) α	Correct Decision

You Can fix your α, understanding the consequences in terms of costs. Your own risk appetite is the only unknown.

Hypothesis Testing – H, α, β, and p

- You can fix α , because you are hypothesizing a given value for H_0
- β comes into play only H_0 is not true, but you still do not know the true value of the parameter to compute it. You only know what it is not.
- α is normally fixed at one of the following values:
  - 0.01 – If you seek very strong evidence
  - 0.05 – If you seek fairly strong evidence
  - 0.10 – If you are OK with a weak evidence
- If p (Computed Uncertainty) is lower than α (the maximum permissible uncertainty), you reject H_0

Understanding α and β – Coin Toss Experiment

# of Heads, X	Pr(X), n=50, p=0.5
0	8.88178E-16
1	4.44089E-14
2	1.08802E-12
3	1.74083E-11
4	2.04547E-10
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Pr(X) =19 to 31 = 0.935

A fair coin will give you 19 to 31 heads out of 50 tosses, 93.5 % of times you do this experiment

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46	2.04547E-10
47	1.74083E-11
48	1.08802E-12
49	4.44089E-14
50	8.88178E-16

Pr(X) =32 to 50 = 0.032

If you set your acceptable range of heads as 19 to 31, to conclude that the coin is fair → you are setting your α at 0.065

Understanding α and β – Coin Toss Experiment

# of Heads, X	Pr(X), n=50, p=0.4
19	0.110863116
20	0.114558553
21	0.109103384
22	0.095878731
23	0.077814622
24	0.058360967
25	0.040463604
26	0.025938207
27	0.01537079
28	0.008417337
29	0.004257044
30	0.001986621
31	0.00085446

Pr(X) =19 to 31 = 0.664

An unfair coin Pr(H) = 0.4 will give you 19 to 31 heads out of 50 tosses, 66.4 % of times you do this experiment

# of Heads, X	Pr(X), n=50, p=0.1
19	1.16005E-07
20	1.99786E-08
21	3.17121E-09
22	4.6447E-10
23	6.28269E-11
24	7.85336E-12
25	9.07499E-13
26	9.6955E-14
27	9.57581E-15
28	8.73982E-16
29	7.3669E-17
30	5.72981E-18
31	4.10739E-19

Pr(X) =19 to 31 = 0

A very unfair coin Pr(H) = 0.1 will give you 19 to 31 heads out of 50 tosses, 0 % of times you do this experiment

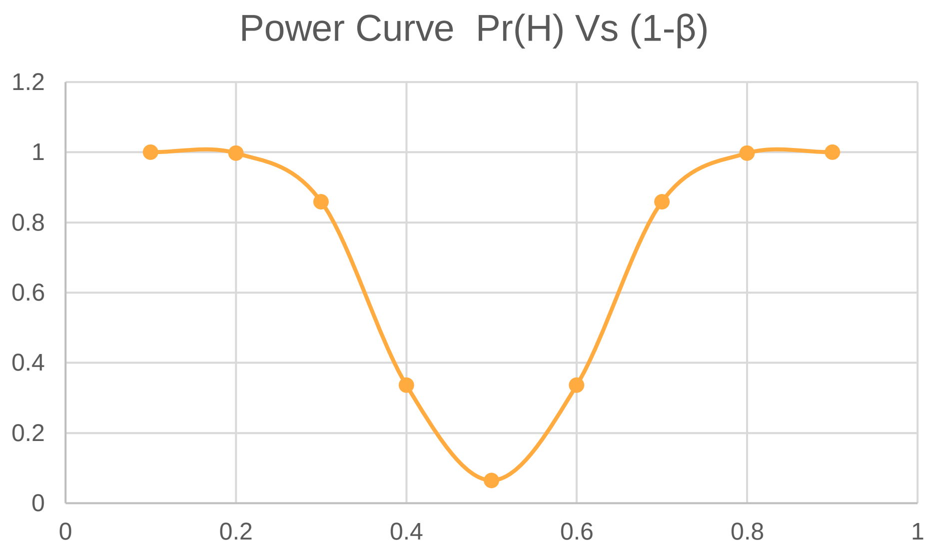
If you set your range as 19 to 31, you are ending with a β of 0.664 if Pr(H) = 0.4

If you set your range as 19 to 31, you are ending with a β of 0 if Pr(H) = 0.1

β, and 1- β

The value of β will vary depending on the true value of the parameter. The term 1- β is called the power of the test. For the given α value of 0.065, the β values for different alternate reality is in the table given below.

pr(H)	β	1-β
0.1	0	1
0.2	0.003	0.997
0.3	0.141	0.859
0.4	0.664	0.3361
0.6	0.664	0.3361
0.7	0.141	0.859
0.8	0.003	0.997
0.9	0	1



Higher the power of the test, the better it is.

Improving power levels can be done either through increasing α or increasing the sample size/trials

The Test Statistic

- This will vary depending on the test, but it will typically be the difference observed in the sample/s divided by a standard error. In this class we will see z, t, χ^2, and F test statistics.

- The choice of test is a function of
  - What is tested?, and
  - Assumed Distribution of the population on what is tested, and
  - Number of samples involved

The Test Statistic

- What is tested?: The most commonly tested items are "Test of Differences"
- Is the proportion equal to a particular value, or is it different?
- Are two proportions from the study relating to two different sub-groups, equal or different?
- Is the average from the study, equal to a particular value, or is it different?
- Are two averages from the study, relating to two different sub-groups equal or different?

The Test Statistic

- Assumed Distribution of the population on what is tested

- **Parametric** statistical **test** is one that makes assumptions about the parameters (defining properties) of the population distribution(s) from which one's data are drawn – General assumptions are the sample is drawn from a population, wherein the parameter is Normally distributed.

The Test statistic

- Number of samples involved
- One Sample

Example: To test whether the proportion of cars that have a damage is is < 25%, we will consider all the data as one Sample

- Two Samples – Independent

Example: To test whether the proportion of cars with damage is related to transmission type we will consider the data for two transmission types, as two samples, independent of each other, i.e. there is no overlap of cars between the two groups

The Test Statistic

- Two Samples – Paired

Example: To test whether the proportion of subscription to phone services and internet services are similar in our population , we will consider all the data as two paired samples. i.e. The data comes from the same respondents

Hypotheses Testing Procedure

- Identify the parameter of interest (mean, proportion, variance, regression coefficient) which you wish to test
- Construct the null and alternative hypotheses
- Compute a test statistic which is a function of the sample set of observations
- Derive the distribution of the test statistic under the null hypothesis assumption
- Choose a test criterion (threshold) against which the test statistic is compared to reject/not reject the null hypothesis

Types of Tests

Sample	Data	Purpose	Hypothesis	Test/s
One	Categorical (Nominal/Ordinal)	Test proportion	$H_0 : \pi = K$	Binomial Test **z -Test**
One	Continuous (Interval/Ratio)	Test Mean	$H_0 : \mu = K$	One sample t-test

Sample	Data	Purpose	Hypothesis	Test/s
Two Independent	Categorical (Nominal/Ordinal)	Test proportion	$H_0 : \pi_1 = \pi_2$	**z -Test**
Two - Independent	Continuous (Interval/Ratio)	Test Mean	$H_0 : \mu_1 = \mu_2$	t- Test (with F-Test)

Sample	Data	Purpose	Hypothesis	Test/s
Two - Paired	Categorical (Nominal/Ordinal)	Test proportion	$H_0 : \pi_1 = \pi_2$	McNemar Test
Two - Paired	Continuous (Interval/Ratio)	Test Mean	$H_0 : \mu_1 = \mu_2$	t- Test

Examples of Hypotheses Tests

Data for Hypotheses Tests

Price Quoted	Vehicle Type	Year Of Registration	Gearbox	PowerPS	kilometers Run	Fuel Type	Brand	Any Damage
4450	limousine	2003	manual	150	150000	diesel	bmw	
13299	suv	2005	manual	163	150000	diesel	volvo	no
3200	bus	2003	manual	101	150000	diesel	volkswagen	
4500	small car	2006	manual	86	60000	petrol	seat	no
18750	suv	2008	automatic	185	150000	diesel	volvo	no
988	limousine	1995	manual	90	150000	petrol	volkswagen	no
400	station wagon	1996	manual	0	150000	petrol	opel	
1399	coupe	1997	manual	136	150000	petrol	mercedes_benz	no
4680	station wagon	2005	manual	122	150000	petrol	opel	no
8340	limousine	2005	automatic	140	125000	diesel	skoda	no
1870	limousine	2001	manual	82	150000	petrol	mercedes_benz	no
2500	coupe	2001	manual	105	125000	petrol	opel	no
990	small car	2001	manual	68	150000	petrol	toyota	no
3000		2016	manual	116	150000	petrol	bmw	yes
2200	limousine	2003	manual	83	125000	petrol	opel	no
1600	station wagon	1999	manual	140	150000	diesel	volvo	no
1300	suv	1993	manual	101	150000	diesel	nissan	yes
4100	others	2002	manual	125	150000	diesel	others	
600	small car	1998	manual	75	150000	petrol	volkswagen	yes
698		2017		0	150000		volkswagen	
680	limousine	1992	manual	88	150000	petrol	mazda	no

One sample test for mean

One Sample Test of Mean

Q: Three Years back, the average price of a used car was $ 6000. Has it Changed now?

Hypotheses Testing Steps	
Hypotheses	H_0 : μ = 6000 H_A : μ ≠ 6000
Sample Statistics	$\bar{x}$ “s” being used as an estimator of “σ”
Test Statistics	**“t” value**
Max Uncertainty α	0.05
Computed Uncertainty p	0.4
Decision on H_0	Do Not Reject Null Hypothesis Conclude μ = 6000

Sample Sizes(s)	**Sample Statistic(s)**	**Test**	**Test Statistic**	**p-value**
1000	Mean=$ 6118.34	t Test	0.81	0.4

One sample test for proportion

One Sample Test of Proportion

**Q: Three Years back, % of used car with automatic transmission were 23%
Has it Changed now?**

Hypotheses Testing Steps	
Hypotheses	H_0 : π = 0.23 H_A : $\pi \neq$ 0.23
Sample Statistics	$\hat{p}$ "H_0" is used to compute "σ"
Test Statistics	**"z" value**
Max Uncertainty α	0.05
Computed Uncertainty p	0.22
Decision on H_0	Do Not Reject Null Hypothesis Conclude $\pi = 0.23$

Sample Sizes(s)	Sample Statistic(s)	Test	Test Statistic	p-value
1000	Prop of automatically geared=0.214	z Test	-1.23	0.22

Two sample test for mean

Two Sample Test of Means

Q: Is the mean price of cars that have run 30000 - 60000 KM, the same as that for cars that have run 70000 - 90000 KM?

Hypotheses Testing Steps	
Hypotheses	H_0 : $\mu_1 = \mu_2$ H_A : $\mu_1 \neq \mu_2$
Sample Statistics	$\overline{x}$ "s" being used as an estimator of "σ"
Test Statistics	**Depends on whether the two groups have equal or unequal variance**
Max Uncertainty α	
Computed Uncertainty p	
Decision on H_0	

Two Sample Test of Means

Q: We first need to test whether the variance in price of cars that have run 30000 - 60000 KM, the same as the variance in price of cars that have run 70000 - 90000 KM?

Hypotheses Testing Steps	
Hypotheses	$H_0 : \sigma_1^2 = \sigma_2^2$ $H_A : \sigma_1^2 \neq \sigma_2^2$
Sample Statistics	s_1^2, s_2^2
Test Statistics	**F - Value**
Max Uncertainty α	0.05
Computed Uncertainty p	0.00
Decision on H_0	Reject H_0 $\sigma_1^2 \neq \sigma_2^2$

Sample Sizes(s)	Sample Statistic(s)	Test	Test Statistic	P-value
500, 500	Variances • 30000-60000=155442577.95 • 70000-90000=86753098.35	F Test	0.56	$5.05*10^{-11}$

Two Sample Test of Means

Q: Is the mean price of cars that have run 30000 - 60000 KM, the same as that for cars that have run 70000 - 90000 KM?

Hypotheses Testing Steps	
Hypotheses	H_0 : $\mu_1 = \mu_2$ H_A : $\mu_1 \neq \mu_2$
Sample Statistics	$\overline{x}$ "s" being used as an estimator of "σ"
Test Statistics	**Welch "t" statistics for unequal variances**
Max Uncertainty α	0.05
Computed Uncertainty p	0.00
Decision on H_0	Reject H_0 $\mu_1 \neq \mu_2$

Sample Sizes(s)	Sample Statistic(s)	Test	Test Statistic	p-value
500, 500	Mean • 30000-60000= $14515.68 • 70000-90000= $ 9450.59	Welch t - Test	-7.28	$7.26*10^{-13}$

Two sample test for proportion

Two Sample Test of Proportion

Q: Are the proportion petrol cars in two different time periods (2009 – 2013, and 2014 – 2018, different?

Hypotheses Testing Steps	
Hypotheses	$H_0 : \pi_1 = \pi_2$ $H_A : \pi_1 \neq \pi_2$
Sample Statistics	$\hat{p}$ Pooled estimate of $\hat{p}$ used to estimate "σ"
Test Statistics	**"z" value**
Max Uncertainty α	0.05
Computed Uncertainty p	0.59
Decision on H_0	Do Not Reject Null Hypothesis Conclude $\pi_1 = \pi_2$

Sample Sizes(s)	Sample Statistic(s)	Test	Test Statistic	P-value
1000	Prop of fuelType=Petrol for • 2009-2013=0.506 • 2014-2018=0.494	Z Test	-0.54	0.59


operation == "MIRROR_X":
mirror_mod.use_x = True
mirror_mod.use_y = False
mirror_mod.use_z = False
operation == "MIRROR_Y":
mirror_mod.use_x = False
mirror_mod.use_y = True
mirror_mod.use_z = False
operation == "MIRROR_Z":
mirror_mod.use_x = False
mirror_mod.use_y = False
mirror_mod.use_z = True
selection at the end -add
_ob.select= 1
ier_ob.select=1
ntext.scene.objects.active
("Selected" + str(modifier
mirror_ob.select = 0
bpy.context.selected_ob
data.objects[one.name].se
int("please select exactly

THANK YOU