Estimation of Correlated MIMO Channels using Partial Channel State Information and DPSS

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Estimation of Correlated MIMO Channels using Partial Channel State Information and DPSS

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***Abstract*—A novel approach is proposed for correlated multiple-input multiple-output (MIMO) channel estimation based on reduced-rank (RR) technique and partial channel state information (CSI). In contrast to previous proposals that used the channel correlation matrix (CCM) and its eigendecomposition, this paper shows that close linear minimum mean-square-error (LMMSE) performance can be achieved with the use of predefined bases derived from the knowledge of the maximum angular dispersion. A theoretical framework to synthesize a suitable set of bases is provided, from which discrete prolate spheroidal sequences (DPSSs) are identified as one of the appropriate predefined bases for spatial channel representation. The robustness of the proposed estimator allows changes in the propagation scenario to be managed according to the demands of realistic communications systems. The performance analysis of the channel estimator is shown and corroborated with simulation results.**

***Index Terms*—Generic bases, MIMO channel estimation, reduced-rank, spatial correlation.**

I. Introduction

When the MIMO communication concept was conceived, it opened a new area of research, as it foresaw the high increase in capacity over the same timespan and bandwidth of previous approaches [1]. However, the initial computed channel capacity was based on a channel model in which the set of all communication links, from any transmitter antenna to any receiver antenna, are all considered uncorrelated. This optimistic case is not commonly encountered in real life, as propagation campaigns have shown that multipath fading is commonly dispersive up to a maximum angle [2], yielding a MIMO channel with correlated coefficients [3].

Under these realistic conditions, MIMO channels can be represented using RR expressions, i.e., they can be decomposed through orthonormal basis expansion to provide an alternative representation [4]. This strategy can be found in digital beamforming [5], precoding design [6], and two remarkable papers on single-input single-output SISO and single-input multiple-output (SIMO) channel estimation [7], [8], respectively. Dimension reduction technique has also been applied for correlated MIMO channels in [9]. However, these approaches rely on the knowledge of eigenvalues and eigenvectors of the CCM (ECCM), which has several implications. First, knowledge of second-order channel statistics is assumed, in some cases on the transmitter side (optimal precoding), which is not realistic even for slow time-varying scenarios. Second, it is assumed that an eigendecomposition of the current CCM can be readily calculated online. When these two conditions are met, optimal techniques in the mean-square-error sense can be used. However, in spite of the knowledge of realistic statistical information, the computation of the eigenvectors might still be infeasible due to complexity, which increases with the number of antennas involved. Therefore, the investigation of suboptimal approaches is an important open research area with promising practical applications.

This paper addresses the topic of channel estimation based on RR representation constrained by partial knowledge of channel statistics. Related studies that consider complete knowledge of the CCM are [9] and [10] for LMMSE MIMO channel estimation and equalization, respectively. In [11] and [12], the channel estimation problem for spatially correlated fading MIMO systems is addressed from the optimal training design perspective. But again, they are based on the assumption that the receiver and the transmitter have knowledge of the second-order channel statistics.

In [13], partial channel state information is considered using only the main diagonal of the CCM. This result achieves better performance than LS, but it does not establish the performance with respect to LMMSE. Moreover, the results consider a particular channel scenario. Nevertheless, they show that a trade-off between channel estimation performance and required channel knowledge exists and can be exploited. In [14], a channel model expressed via orthogonal channel expansion and a diagonal matrix containing the mean angle of departure (AoD) is proposed. Hence, an iterative LS estimator is proposed by first estimating the cross weights of the modes in the orthogonal expansion, followed by the estimation of the matrix containing the AoD. The bases considered are not related to the CCM; thus, this information is not required. However, in its RR channel estimation, the number of required functions is not optimally derived, nor is the proposed estimator performance compared with an optimum estimator such as LMMSE. Furthermore, some of the results show worse performance than LS at low signal-to-noise ratio (SNR) levels. Finally, the optimality of the basis expansions utilized to model the channel is not considered.

This paper proposes an LMMSE channel estimator based on partial CSI that surpasses previous approaches. It achieves LMMSE performance without requiring the true CCM, being

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insensitive to (independent of) the shape of the power angular spectrum of the underlying propagation scenario.

A. Objectives and Contributions

The aim of this paper is to provide an RR LMMSE estimator for correlated MIMO fading channels constrained by partial knowledge of channel statistics. Three main contributions are provided to accomplish the goal: first, the optimum RR estimator is provided for narrowband MIMO spatially correlated channels; second, the performance of optimal RR LMMSE is closely approximated using any generic basis that spans the true CCM subspace with few elements; and third, such a generic basis can be synthesized by considering only the information of the maximum AoD. DPSS is shown to be one such suitable basis for CCM representation. In the sections below, this estimator is denoted as RR generic basis (RRGB).

B. Notation

Bold lower (upper) case letters are used to denote vector (matrices); $\mathbb{C}$ stands for the complex number field; $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote transpose, conjugate, and Hermitian operations, respectively; $\mathbf{I}_n$ denotes an $n \times n$ identity matrix; $|\cdot|$, $\|\cdot\|_F$ represent absolute value and Frobenius norm, respectively; $E\{\cdot\}$ is the expected value; $tr\{\cdot\}$ denotes trace of a matrix; and $\otimes$, $\lceil\cdot\rceil$ stand for Kronecker product and ceiling function, respectively. $\mathbb{C}\mathcal{N}(\mathbf{a}, \mathbf{\Sigma})$ denotes a multidimensional complex Gaussian distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{\Sigma}$. In addition, the main acronyms are: CCM (Channel Correlation Matrix); DPSS (Discrete Prolate Spheroidal Sequence); LMMSE (Linear Minimum Mean Square Error); LS (Least Squares); MIMO (Multiple-Input Multiple-Output); MSE (Mean Square Error); PAS (Power Azimuth Spectrum); RR (Reduced-Rank); RRGB (Reduced-Rank Generic Bases); ULA (Uniform Linear Array); UPAS (Uniform PAS).

C. Organization

Section II describes the MIMO system model and a review of LS and LMMSE channel estimators to obtain subsequently the optimal RR channel estimator performance. Section III is devoted to the channel estimator based on generic basis expansion. Section IV provides a synthesis algorithm to construct the required generic basis using partial CSI. Results are provided in Section V. Conclusions are given in Section VI.

II. Channel Estimation Based on Reduced-Rank Channel Expansion

A. System Model

Consider a wireless MIMO frequency-flat quasi-static block fading channel model, with n_T transmitting and n_R receiving antennas described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{n_T}$ is the transmitted vector comprising n_T elements denoted as $[x_1, x_2, \ldots, x_{n_T}]^T$ and $\mathbf{y} \in \mathbb{C}^{n_R}$ is the received vector. A random matrix $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is used that represents the correlated flat fading channel in a baseband equivalent Kronecker channel model [3] described by

$$\mathbf{H} = (\mathbf{R}_{Rx})^{1/2}\mathbf{H}_W(\mathbf{R}_{Tx})^{H/2} \tag{2}$$

where each element of $\mathbf{H}_W \in \mathbb{C}^{n_R \times n_T}$ is generated as an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variable $\mathbb{C}\mathcal{N}(0, 1)$. $\mathbf{R}_{Tx}$ is the $n_T \times n_T$ transmitting spatial correlation matrix, and $\mathbf{R}_{Rx}$ is the $n_R \times n_R$ receiving spatial correlation matrix. The derivation of these matrices from a propagation scenario is presented in Section IV. In this study, only correlation on the transmitter side is considered, in order to use similar assumptions as in [13], [14]. In addition, $\mathbf{n} \in \mathbb{C}^{n_R}$ is an additive white circularly symmetric complex Gaussian noise vector, with zero mean and variance $\sigma_n^2 = N_0/2$ for every real and imaginary dimension.

The block transmission scheme is assumed, where a "block" is defined as a single transmission burst. The channel matrix $\mathbf{H}$ is assumed to be constant for N channel uses and then changes to an independent realization for the next block. Here, N denotes the block length. For any $n_T \times N$ transmitted signal matrix $\mathbf{X}$, expression (1) can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{3}$$

where

$$\begin{aligned} \mathbf{Y} &\triangleq [\mathbf{y}_1\ \mathbf{y}_2 \cdots \mathbf{y}_N]\,; \\ \mathbf{X} &\triangleq [\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_N]\,; \\ \mathbf{N} &\triangleq [\mathbf{n}_1\ \mathbf{n}_2 \cdots \mathbf{n}_N]\,; \end{aligned} \tag{4}$$

are the matrices of the received signals, transmitted signals, and noise, respectively.

For channel estimation purposes using pilot-assisted transmission (PAT) [15] in a time division multiplexing (TDM) scheme, it is assumed that $N \geq n_T$ training signal vectors $\mathbf{x}_1, \ldots, \mathbf{x}_N$ are transmitted. The goal is to recover the channel matrix $\mathbf{H}$ based on the knowledge of $\mathbf{Y}$ and $\mathbf{X}$.

B. Least Square Channel Estimator

The channel estimation in its general form can be represented as $\hat{\mathbf{H}} = \mathbf{Y}\mathbf{Q}$. The least square (LS) estimator treats the channel estimation task as a deterministic optimization problem. Knowing $\mathbf{X}$ and the received signal $\mathbf{Y}$, the LS approach [16] of $\mathbf{H}$ is derived from (3) and is given by

$$\hat{\mathbf{H}}_{\text{LS}} = \mathbf{Y}\mathbf{Q}_{\text{LS}} \tag{5}$$

where $\mathbf{Q}_{\text{LS}} = \mathbf{X}^{\dagger} = \mathbf{X}^H(\mathbf{X}\mathbf{X}^H)^{-1}$ is the pseudoinverse of $\mathbf{X}$.

Constraining the transmitted power during the training interval to be

$$\|\mathbf{X}\|_F^2 = n_T N\,|P_x| = \mathcal{P} \tag{6}$$

and limiting the transmitted power per antenna, all the elements of the training matrix have the same magnitude. For example, if pilot symbols are taken from a PSK constellation, then $|P_x| = 1$.

From (5), the LS estimator requires the invertibility of $\mathbf{X}\mathbf{X}^H$. This condition is met if and only if $\mathbf{X}$ is full rank and $N \geq n_T$. This means that the number of pilots must not be smaller than the number of transmitting antennas. Ref. [13] shows that the optimal $\mathbf{X}$ that minimizes the channel estimation error and satisfies the condition in (6) is any training matrix that fulfills

$$\mathbf{X}\mathbf{X}^H = \frac{\mathcal{P}}{n_T}\mathbf{I} \tag{7}$$

where $\mathbf{I}$ is the $n_T \times n_T$ identity matrix. It follows that the LS channel estimate (5) can also be expressed as

$$\hat{\mathbf{H}}_{\mathrm{LS}} = \mathbf{H} + (n_T/\mathcal{P})\mathbf{N}\mathbf{X}^H. \tag{8}$$

The mean-square error (MSE) $\mathcal{J}(\mathbf{X})$ is in general dependent on the training matrix $\mathbf{X}$. For the LS case [13], it yields

$$\min_{\mathbf{X}} \mathcal{J}_{LS}(\mathbf{X}) = \frac{\sigma_n^2 n_T^2 n_R}{\mathcal{P}}. \tag{9}$$

C. Linear Minimum Mean-Square-Error Estimator

The LMMSE estimator is derived in [16] from the following optimization problem

$$\mathbf{Q}_{\mathrm{LMMSE}} = \arg\min_{\mathbf{Q}} \mathcal{E} \tag{10}$$

where $\mathcal{E} = E\left\{\left\|\mathbf{H} - \hat{\mathbf{H}}_{\mathrm{LMMSE}}\right\|_F^2\right\}$, and $\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}\mathbf{Q}_{\mathrm{LMMSE}}$. Then, using (3), the estimation error can be expressed as in [13] as

$$\begin{aligned}\mathcal{E} = tr\{\mathbf{R}\} - tr\{\mathbf{R}\mathbf{X}\mathbf{Q}\} - tr\{\mathbf{Q}^H\mathbf{X}^H\mathbf{R}\}\\ + tr\{\mathbf{Q}^H(\mathbf{X}^H\mathbf{R}\mathbf{X} + \sigma_n^2 n_R\mathbf{I})\mathbf{Q}\}\end{aligned} \tag{11}$$

where the one-side correlation is a semidefinite positive matrix denoted by $\mathbf{R}_{Tx} \triangleq E\{\mathbf{H}^T\mathbf{H}^*\}$ as defined in channel modeling area [17]. Hence, $\mathbf{R} = \mathbf{R}_{Tx}^* = E\{\mathbf{H}^H\mathbf{H}\}$.

Using $\partial\mathcal{E}/\partial\mathbf{Q} = \mathbf{0}$, it follows that

$$\mathbf{Q}_{\mathrm{LMMSE}} = (\mathbf{X}^H\mathbf{R}\mathbf{X} + \sigma_n^2 n_R\mathbf{I})^{-1}\mathbf{X}^H\mathbf{R}. \tag{12}$$

The LMMSE channel estimator can be formulated as

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}\mathbf{R}_{\mathbf{YY}}^{-1}\mathbf{R}_{\mathbf{HY}} \tag{13}$$

where $\mathbf{R}_{\mathbf{YY}} = (\mathbf{X}^H\mathbf{R}\mathbf{X} + \sigma_n^2 n_R\mathbf{I})$ and $\mathbf{R}_{\mathbf{HY}} = \mathbf{X}^H\mathbf{R}$.

The corresponding error covariance matrix is

$$\begin{aligned}\mathbf{C}_e &= E\left\{(\mathbf{H} - \hat{\mathbf{H}}_{\mathrm{LMMSE}})(\mathbf{H} - \hat{\mathbf{H}}_{\mathrm{LMMSE}})^H\right\}\\ &= \left(\mathbf{R}^{-1} + \frac{1}{\sigma_n^2 n_R}\mathbf{X}\mathbf{X}^H\right)^{-1}.\end{aligned} \tag{14}$$

Hence, the MMSE estimation error can be obtained by

$$\mathcal{J}_{\mathrm{LMMSE}}(\mathbf{X}) = tr\{\mathbf{C}_e\}. \tag{15}$$

This objective function can be minimized using the Lagrange multiplier method under constraint (6) to provide a solution for the optimal training matrix [13]. For comparison purposes, this paper performs orthogonal training and the transmit power constraint given by (6).

D. Reduced-Rank LMMSE Estimator

To the best of the authors' knowledge, the derivation of an optimal subspace estimator for spatially correlated channels is presented for the first time here. This estimator uses the principle of rank reduction [4] working with the correlation matrix of the MIMO channel. The low-rank estimator can be interpreted as first projecting the LS estimates onto a subspace and then performing the estimation. If the subspace has a small dimension and can describe the channel well for a particular SNR level, the complexity of the estimator will be low and will show a "good" performance in comparison with LMMSE estimators. Therefore, it can be viewed as a suboptimal or a simplified version of the LMMSE approach.

Departing from $\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}\mathbf{Q}_{\mathrm{LMMSE}}$ and using (12), the LMMSE estimator can be rewritten as

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}(\mathbf{X}^H\mathbf{R}\mathbf{X} + \sigma_n^2 n_R\mathbf{I})^{-1}\mathbf{X}^H\mathbf{R}. \tag{16}$$

Factorizing the last equation with $\mathbf{X}^H$ on the left side and $\mathbf{X}$ on the right side yields

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}[\mathbf{X}^H(\mathbf{R} + \sigma_n^2 n_R(\mathbf{X}\mathbf{X}^H)^{-1})\mathbf{X}]^{-1}\mathbf{X}^H\mathbf{R}. \tag{17}$$

Applying $\mathbf{X}^{-1} = \mathbf{X}^H\left(\mathbf{X}\mathbf{X}^H\right)^{-1}$ and after some algebraic manipulations, it follows that

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{Y}\mathbf{X}^H(\mathbf{X}\mathbf{X}^H)^{-1}(\mathbf{R} + \sigma_n^2 n_R(\mathbf{X}\mathbf{X}^H)^{-1})^{-1}\mathbf{R}. \tag{18}$$

Finally, with (5),

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{H}_{\mathrm{LS}}(\mathbf{R} + \sigma_n^2 n_R(\mathbf{X}\mathbf{X}^H)^{-1})^{-1}\mathbf{R}. \tag{19}$$

From (19), it can be inferred that the LMMSE is a postprocessing stage of the LS estimator. The eigendecomposition of $\mathbf{R}_{Tx}$ leads to

$$\mathbf{R}_{Tx} = \mathbf{U}\mathbf{A}\mathbf{U}^H \tag{20}$$

where matrix $\mathbf{U}$ is a unitary matrix containing the eigenvectors of $\mathbf{R}_{Tx}$, and matrix $\mathbf{A}$ is a diagonal matrix of non-negative eigenvalues expressed as

$$\mathbf{A} = diag[a_1, a_2, \cdots, a_{n_T}] \tag{21}$$

with $a_1 \geq a_2 \geq \cdots \geq a_{n_T}$.

Likewise, using (5), an estimate $\hat{\mathbf{R}} = E\{\mathbf{H}_{\mathrm{LS}}^H\mathbf{H}_{\mathrm{LS}}\}$ can be computed as

$$\hat{\mathbf{R}} = \mathbf{R} + \sigma_n^2 n_R(\mathbf{X}\mathbf{X}^H)^{-1} \tag{22}$$

which allows $\hat{\mathbf{H}}_{\mathrm{LMMSE}}$ to be represented as

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{H}_{\mathrm{LS}}(\hat{\mathbf{R}})^{-1}\mathbf{R}. \tag{23}$$

Using $\mathbf{R} = \mathbf{U}^*\mathbf{A}\mathbf{U}^T$ and the Cholesky decomposition $\hat{\mathbf{R}}^{-1} = \hat{\mathbf{R}}^{-1/2}\hat{\mathbf{R}}^{-H/2}$, then

$$\hat{\mathbf{R}}^{-1/2} = \mathbf{U}^*\left(\mathbf{A} + \sigma_n^2 n_R(\mathbf{X}\mathbf{X}^H)^{-1}\right)^{-1/2}\mathbf{U}^T. \tag{24}$$

From (23) and (22) with $\mathbf{R} = \mathbf{U}^*\mathbf{A}\mathbf{U}^T$, $\hat{\mathbf{R}}^{-1} = \hat{\mathbf{R}}^{-1/2}\hat{\mathbf{R}}^{-H/2}$, the $\hat{\mathbf{H}}_{\mathrm{LMMSE}}$ can be rewritten as

$$\hat{\mathbf{H}}_{\mathrm{LMMSE}} = \mathbf{H}_{\mathrm{LS}}\mathbf{F} \tag{25}$$

where

$$\mathbf{F} = \hat{\mathbf{R}}^{-1/2}\mathbf{U}^*\left(\mathbf{A} + \sigma_n^2 n_R (\mathbf{X}\mathbf{X}^H)^{-1}\right)^{-1/2}\mathbf{U}^T\mathbf{U}^*\mathbf{A}\mathbf{U}^T$$
$$= \hat{\mathbf{R}}^{-1/2}\mathbf{U}^*\left(\mathbf{A} + \sigma_n^2 n_R (\mathbf{X}\mathbf{X}^H)^{-1}\right)^{-1/2}\mathbf{A}\mathbf{U}^T \quad (26)$$

with $\mathbf{U}^T\mathbf{U}^* = \mathbf{I}$. By replacing $\hat{\mathbf{R}}^{-1/2}$ in (26) with (24) and after performing some algebraic manipulations, it follows that

$$\mathbf{F} = \mathbf{U}^*\left(\mathbf{A} + \sigma_n^2 n_R (\mathbf{X}\mathbf{X}^H)^{-1}\right)^{-1}\mathbf{A}\mathbf{U}^T = \mathbf{U}^*\Phi\mathbf{U}^T \quad (27)$$

where Φ is a diagonal matrix expressed as

$$\Phi = diag\left[\frac{a_1}{a_1 + g}, \cdots, \frac{a_{n_T}}{a_{n_T} + g}\right] \quad (28)$$

with $g = \frac{\sigma_n^2 n_T n_R}{\mathcal{P}}$ and $\left(\mathbf{X}\mathbf{X}^H\right)^{-1} = \frac{n_T}{\mathcal{P}}\mathbf{I}$ from (7).

Consider the optimal low-rank r-modeling that is achieved by the eigendecomposition of a correlation matrix [4]. It states that if the tail-end eigenvalues $a_{r+1}, \ldots, a_{n_T}$ of the CCM (in this case the one-side correlation) are very small, the MSE yielded by the low-rank approximation is less than that produced by working with the original information without any approximation (i.e., with LS criteria). This is because noise is present in all dimensions with the same variance, but the signal to be estimated is not. Therefore, by leaving out those dimensions where this signal does not have significant energy, large amounts of noise energy can be excluded, enhancing the SNR.

The essence of the optimal low-rank modeling or dimensionality reduction applied to a particular problem becomes established by the bias-variance trade-off [10], [18], [19], where the optimal dimension r is chosen according to

$$r = \underset{r\in\{1\ldots n_T\}}{\arg\min}\left(\frac{1}{n_T}\sum_{i=r+1}^{n_T} a_i + \frac{r}{n_T}\sigma_n^2\right). \quad (29)$$

From (25) and (27), the best rank-estimator excludes the contribution of the eigenvectors with small eigenvalues. Thus

$$\hat{\mathbf{H}}^r_{\text{LMMSE}} = \mathbf{H}_{\text{LS}}\mathbf{F}_r \quad (30)$$

where $\mathbf{F}_r = \mathbf{U}^*\begin{bmatrix}\Phi_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{U}^T$, and Φ_r is the $r \times r$ upper left corner of Φ in (28).

To get an expression for the MSE for the rank-r approximation, the estimation error can be computed as

$$\mathcal{J}^r_{LMMSE}(\mathbf{X}) = E\left\{\left\|\mathbf{H} - \hat{\mathbf{H}}^r_{\text{LMMSE}}\right\|_F^2\right\} \quad (31)$$

With the use of (30), (5), and (3), an alternative representation of (31) is found to be

$$\mathcal{J}^r_{LMMSE}(\mathbf{X}) = E\left\{\left\|\mathbf{H}(\mathbf{I} - \mathbf{F}_r) - \mathbf{N}\mathbf{X}^\dagger\mathbf{F}_r\right\|_F^2\right\}. \quad (32)$$

Recalling that $\|\mathbf{H}\|_F^2 = tr\left\{\mathbf{H}^H\mathbf{H}\right\}$, (32) yields

$$\mathcal{J}^r_{LMMSE}(\mathbf{X}) =$$
$$tr\left[E\left\{(\mathbf{I} - \mathbf{F}_r)^H\mathbf{H}^H\mathbf{H}(\mathbf{I} - \mathbf{F}_r) + \mathbf{F}_r^H\left(\mathbf{X}^\dagger\right)^H\mathbf{N}^H\mathbf{N}\mathbf{X}^\dagger\mathbf{F}_r\right\}\right]. \quad (33)$$

It follows that

$$\mathcal{J}^r_{LMMSE}(\mathbf{X}) = tr\left[(\mathbf{I} - \mathbf{F}_r)^H\mathbf{R}(\mathbf{I} - \mathbf{F}_r) + \mathbf{F}_r^H(\mathbf{X}^\dagger)^H\mathbf{R}_N\mathbf{X}^\dagger\mathbf{F}_r\right]$$
$$= tr\left[\mathbf{U}^*\left(\mathbf{I} - \begin{bmatrix}\Phi_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\mathbf{A}\left(\mathbf{I} - \begin{bmatrix}\Phi_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\mathbf{U}^T\right]$$
$$+ tr\left[\mathbf{F}_r^H(\mathbf{X}^\dagger)^H\mathbf{R}_N\mathbf{X}^\dagger\mathbf{F}_r\right] \quad (34)$$

where $\mathbf{R}_N$ denotes the noise correlation matrix. Expressing (34) in summation notation, the final form of the RR LMMSE estimator error when using the transmitting power constraint (6) and orthogonal training is then

$$\mathcal{J}^r_{LMMSE} = \sum_{i=1}^{r} a_i(1 - \phi_i)^2 + \sum_{i=r+1}^{n_T} a_i + \sigma_n^2 n_R tr\left[\mathbf{F}_r^H\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\mathbf{F}_r\right]$$
$$= \sum_{i=1}^{r}\left(a_i(1 - \phi_i)^2 + \frac{\sigma_n^2 n_T n_R}{\mathcal{P}}\phi_i^2\right) + \sum_{i=r+1}^{n_T} a_i\ . \quad (35)$$

III. Reduced-Rank Channel Estimation Using Generic Bases

In mobile wireless links, the channel statistics depend on particular environments and could change with time. Moreover, the CCM computing induces a correlation mismatch. Thus, it is helpful (or useful) to design a "robust" estimator [20] in the sense that it should work in different scenarios while managing correlation mismatch (spatial domain in this case) between true and approximated CCMs.

The proposed RRGB robust estimator is based on the RR LMMSE channel estimator developed in the previous Section, but employs a predefined basis $\mathbf{V}$. Assuming now that an alternative (predefined) CCM denoted as $\mathbf{B}_{Tx}$ is provided, then the channel estimate is computed using (30), employing the principal eigenvectors and eigenvalues of $\mathbf{B}_{Tx}$ instead of the eigenvectors and eigenvalues of $\mathbf{R}_{Tx}$. To be more specific, $\mathbf{F}_r$ is obtained by first expressing $\mathbf{B}_{Tx} = \mathbf{V}\mathbf{D}\mathbf{V}^H$, where $\mathbf{V}$ and $\mathbf{D}$ are the eigenvector and the eigenvalue matrices of $\mathbf{B}_{Tx}$, and then using $\mathbf{V}$ instead of $\mathbf{U}$ and $\mathbf{D}$ instead of $\mathbf{A}$. Hence Φ_r contains the first r eigenvalues of $\mathbf{D}$. A theoretical synthesis framework for providing the alternative $\mathbf{B}_{Tx}$ is developed in Section IV.

In the following section, the performance of RRGB is analyzed using generic bases and their required properties are established. Using the eigenvectors of $\mathbf{B}_{Tx}$ and the channel correlation $\mathbf{R}_{Tx}$, the following quadratic form is proposed

$$\bar{\mathbf{A}} = \mathbf{V}^H\mathbf{R}_{Tx}\mathbf{V}. \quad (36)$$

It is important to highlight that if there is no spatial correlation mismatch, then $\mathbf{B}_{Tx} = \mathbf{R}_{Tx}$ and consequently $\mathbf{V} = \mathbf{U}$ and $\bar{\mathbf{A}} = \mathbf{A}$. In general, the matrix $\bar{\mathbf{A}}$ will not be a diagonal matrix, but rather have the following properties:

a.) $tr(\bar{\mathbf{A}}) = tr(\mathbf{A})$.

b.) If $\mathbf{A}$ is represented by its spectral form, then it can be confirmed that $\mathbf{A}$ and $\bar{\mathbf{A}}$ have the same eigenvalues with different eigenvectors.

From (26), $\mathbf{F}_r$ can be computed using $\mathbf{B}_{Tx}$ and the low-rank modeling. The theoretical MSE can be computed by substituting the value of $\bar{\mathbf{A}}$ obtained from (36) for (35), yielding

$$\mathcal{J}^{r,\mathbf{B}_{Tx}}_{LMMSE} = \sum_{i=1}^{r}\left(\bar{a}_i(1 - \phi_i)^2 + \frac{\sigma_n^2 n_T n_R}{\mathcal{P}}\phi_i^2\right) + \sum_{i=r+1}^{n_T}\bar{a}_i \quad (37)$$

where $\bar{a}_1 \geq \bar{a}_2 \geq \cdots \geq \bar{a}_{n_T}$ are the diagonal elements of the matrix $\mathbf{A}$.

Assuming medium-to-high SNR levels for a particular value of r, the MSE achieved using $\mathbf{B}_{Tx}$ in (37), instead of using $\mathbf{R}_{Tx}$ in (35), leads to an error between these two approximations,

$$\mathcal{E}_r = \mathcal{J}^r_{LMMSE} - \mathcal{J}^{r,\mathbf{B}_{Tx}}_{LMMSE} = \sum_{i=r+1}^{n_T} a_i - \sum_{i=r+1}^{n_T} \bar{a}_i. \qquad (38)$$

From (38), it can be inferred that the optimal choice of $\mathbf{B}_{Tx}$ is $\mathbf{R}_{Tx}$, or in other words, the optimal basis corresponds to the ECCM. However, for the purpose of constructing a robust estimator, this information must be precluded, i.e., the approach uses a basis that is able to span $\mathbf{R}_{Tx}$ as in (36), as well as deal with other CCMs coming from different propagation scenarios.

Recalling (38), if it is allowed to utilize a predefined set of bases, that contains the energy of the channel realizations in a reduced dimension, then, it is clear that a performance quite similar to LMMSE can be achieved.

The conditions required for the bases to provide a near-LMMSE performance can now be defined:

a) They should be the basis for several CCMs derived from several propagation scenarios; therefore, the set of eigenvectors is precluded.

b) They should account for most of the CCMs' energy in a reduced dimension, allowing for truncated representation.

c) They must be able to rely on partial CSI.

It will subsequently be shown that by taking the knowledge of maximum AoD as the CSI available, such "generic" bases can be synthesized.

IV. Bases For CCM Representation

A. Synthesis of Generic Bases

Consider a MIMO system composed of n_R receiver antennas and n_T transmitter antennas. The relation that connects channel realizations in the angular-domain with the channel realizations of the MIMO system having a uniform linear array (ULA) as the antenna topology and considering only azimuth propagation scenarios (no elevation angle) is given by

$$\mathbf{H} = \sum_{m=1}^{M} \alpha_m \mathbf{v}_{Rx,m} \mathbf{v}^T_{Tx,m} \qquad (39)$$

where α_m is the weight of the m^{th} path modeled as a complex random Gaussian variable with zero mean. $\mathbf{v}_{Rx,m}$ and $\mathbf{v}_{Tx,m}$ correspond to the array manifold vector (AMV) on the Rx and Tx side, respectively, for the m^{th} path, defined as:

$$\mathbf{v}_{Tx,m} = \left[1, e^{k_0 d_{Tx} \sin(\varphi_{Tx,m})}, \ldots, e^{k_0 d_{Tx}(N-1)\sin(\varphi_{Tx,m})}\right]^T \qquad (40)$$

$$\mathbf{v}_{Rx,m} = \left[1, e^{k_0 d_{Rx} \sin(\varphi_{Rx,m})}, \ldots, e^{k_0 d_{Rx}(N-1)\sin(\varphi_{Rx,m})}\right]^T \qquad (41)$$

where $d_{Rx(Tx)}$ is the spacing between array elements in the $Rx(Tx)$ array. $k_0 = 2\pi/\lambda$ is the free-space wavenumber of a single wave front, λ is the wavelength of the transmitted signal, and φ_{Rx}, φ_{Tx} denote the angle of arrival (AoA) and the angle of departure (AoD), respectively. M is the number of involved multipath links that could be infinite in a diffuse scattering environment. CCM at Tx can be obtained from $\mathbf{R}_{Tx} = E\{\mathbf{H}^T\mathbf{H}^*\}$ and (39) as follows:

$$\begin{aligned}\mathbf{R}_{Tx} &= E\left\{\left(\sum_{m=1}^{M} \alpha_m \mathbf{v}_{Rx,m}\mathbf{v}^T_{Tx,m}\right)^T \left(\sum_{m'=1}^{M} \alpha_{m'} \mathbf{v}_{Rx,m'}\mathbf{v}^T_{Tx,m'}\right)^*\right\} \\ &= E\left\{\left(\sum_{m=1}^{M} \alpha_m \mathbf{v}_{Tx,m}\mathbf{v}^T_{Rx,m}\right) \sum_{m'=1}^{M} \alpha^*_{m'} \mathbf{v}^*_{Rx,m'}\mathbf{v}^H_{Tx,m'}\right\} \\ &= n_R \sum_{m=1}^{M} \sigma^2_m \mathbf{v}_{Tx,m}\mathbf{v}^H_{Tx,m} \qquad (42)\end{aligned}$$

where an uncorrelated scattering scenario has been considered with $\sigma^2_m = E\{\alpha_m\alpha^*_m\}$, for $m = m'$ and $\sigma^2_m = 0$ for $m \neq m'$. From (42) it can be inferred that $\mathbf{R}_{Tx}$ is nothing but a weighted sum of the outer products of the Tx AMVs. The weights σ^2_m are related to the m^{th} AoD. The power azimuth spectrum (PAS) on the Tx side is defined as [21]:

$$P(\varphi_{Tx}) = \sum_{m=1}^{M} \sigma^2_m \delta(\varphi_{Tx} - \varphi_{Tx,m}) \qquad (43)$$

i.e., the PAS is the energy distribution of paths departing from the transmitter side along the AoD. Therefore, once some scenario has been given in terms of a PAS, its associated CCM can be computed using (42) and (40). In the case of a diffuse PAS, M can be infinite and this sum becomes an integral. However, this approach is appropriate if M is large. The goal is to provide a set of "generic" bases that permit the spectral representation of true CCM as established in (36), irrespective of the particular shape of a given PAS. The result being sought can be defined in terms of the following theorem:

Theorem 1: Let $\mathbf{q}_m = \mathbf{v}_{Tx,m}$ and $\mathbf{R}_{Tx,1}$, $\mathbf{R}_{Tx,2}$ be represented as $\mathbf{R}_{Tx,1} = \sum_{m=1}^{M} \sigma^2_m \mathbf{Q}_m$, $\sigma^2_m > 0$, $\mathbf{R}_{Tx,2} = \sum_{m=1}^{M} \eta^2_m \mathbf{Q}_m$, $\eta^2_m \geq 0$ where $\mathbf{Q}_m = \mathbf{q}_m\mathbf{q}^H_m$ denotes the outer product of a vector $\mathbf{q}_m$. Then, it holds that $\text{Image}\{\mathbf{R}_{Tx,2}\} \subset \text{Image}\{\mathbf{R}_{Tx,1}\}$.

Proof of this theorem is given in [22]. This Theorem has been adapted from the Theorem provided in [22] for SISO channel estimation. $\mathbf{R}_{Tx,2}$ is defined, as in *Theorem 1*, as the correlation function $\mathbf{R}_{Tx}$ of the target channel to be approximated using predetermined "generic" bases. It follows that an infinite set of predetermined PASs are able to fulfill *Theorem 1*. Any of these PASs can be utilized to generate a corresponding CCM $\mathbf{R}_{Tx,1}$, whose eigenvectors span either the channel realizations of the target channel or its CCM.

A particular PAS selection to generate $\mathbf{R}_{Tx,1}$ that fulfills *Theorem 1* is discussed in the following subsection. Next, the dimensionality of random processes in the spatial domain is discussed, as well as its connection with the applicability of RR channel estimation.

Define $\varphi^{\min}_{Tx} = \min|\varphi_{Tx}|$ and $\varphi^{\max}_{Tx} = \max|\varphi_{Tx}|$ as the minimum and maximum AoD, respectively, where the PAS magnitude is zero outside of $\left[\varphi^{\min}_{Tx}, \varphi^{\max}_{Tx}\right]$. Considering that the ULA for the Tx side is located along the $\vec{\mathbf{y}}$ axis, it follows that the minimum and the maximum wavenumbers correspond to $k_{\min} = k_0 \sin\left(\varphi^{\min}_{Tx}\right)$ and $k_{\max} = k_0 \sin\left(\varphi^{\max}_{Tx}\right)$, respectively [21].

For a random process limited to a wavenumber $k_{\max}$ and observed along a limited physical size A_a (in this case, the antenna array aperture measured in wavelengths), its dimensionality Dim is

$$\mathrm{Dim} = \left\lceil 2\left(\left(1/2\pi\right) k_{\max} A_a\right)\right\rceil + 1 \tag{44}$$

at the spatial domain irrespective of the sampling interval (antenna location in the ULA). Note that (44) is the adaptation of the well-known dimensionality theorem of a band-limited time series observed within a temporal window [23]. Therefore, when Dim is less than the number of antennas, subspace algorithms (in this case channel estimation) are applicable.

B. DPSS for CCM representation

Among infinite alternative PAS shapes that fulfill *Theorem 1*, the selection of a particular shape is proposed, and it requires only the knowledge of the angular range (minimum and maximum AoD):

$$\mathrm{PAS}\left(\varphi_{Tx}\right) = \zeta \left|\cos\left(\varphi_{Tx}\right)\right|, \quad \varphi_{Tx}^{\min} \leq \varphi_{Tx} < \varphi_{Tx}^{\max} \tag{45}$$

where the parameter ζ is adjusted to satisfy the following normalization condition (a lossless channel):

$$\int_{-\pi}^{\pi} \mathrm{PAS}(\varphi_{Tx}) d\varphi_{Tx} = 1 \tag{46}$$

which leads to $\zeta = 1/\left(\sin\varphi_{Tx}^{\max} - \sin\varphi_{Tx}^{\min}\right)$.

The corresponding wavenumber spectrum (WNS) denoted by $S(k)$ of (45) can be obtained using the Gans mapping [21] which is defined as

$$S(k) = \frac{2\pi}{\sqrt{k_0^2 - k^2}} \left[\mathrm{PAS}\left(\varphi_R + \cos^{-1}(k/k_0)\right) + \mathrm{PAS}\left(\varphi_R - \cos^{-1}(k/k_0)\right)\right], \quad |k| \leq k_0 \tag{47}$$

where φ_R is the azimuth orientation angle of reference (ULA orientation). Assuming $\varphi_R = \pi/2$,

$$S(k) = \frac{2\pi}{\sqrt{k_0^2 - k^2}} \zeta \left|-\sin\left(\cos^{-1}(k/k_0)\right)\right|. \tag{48}$$

Using the identity $\sin\left(\cos^{-1}(a)\right) = \sqrt{1-(a)^2}$, (48) gives the rectangular WNS:

$$\begin{aligned} S(k) &= \frac{2\pi}{\sqrt{k_0^2 - k^2}} \zeta \left|-\sqrt{1-(k/k_0)^2}\right| = \frac{2\pi\zeta}{k_0} \\ &= \frac{2\pi}{k_0\left(\sin\varphi_{Tx}^{\max} - \sin\varphi_{Tx}^{\min}\right)}, \qquad k_{\min} \leq k \leq k_{\max} \end{aligned} \tag{49}$$

The CCM associated with this spectrum is related to the well-known DPSS set. To elucidate this, consider first that $\left|\varphi_{Tx}^{\min}\right| = \varphi_{Tx}^{\max}$, i.e., a symmetrical interval. From (49), it follows that

$$S(k) = \frac{2\pi}{k_0\left(\sin\varphi_{Tx}^{\max} - \sin\varphi_{Tx}^{\min}\right)} = \frac{2\pi}{2k_{\max}}, \quad |k| \leq k_{\max}. \tag{50}$$

Its corresponding continuous autocorrelation function $R(r)$ is calculated from the inverse Fourier transform of $S(k)$, considering the wavenumber as angular frequency. Then

$$R(r) = \mathfrak{I}^{-1}\left(S(k)\right) = \frac{\sin\left(k_{\max}\tau\right)}{k_{\max}\tau} \tag{51}$$

with $\tau \in \mathbb{R}$. By sampling (51) at the antenna locations, $\tau = nd_{Tx}$ with $n \in \mathbb{N}$. This result then yields the following discrete correlation function of the stationary spatial process

$$R(nd_{Tx}) = \frac{\sin\left(k_{\max} n d_{Tx}\right)}{k_{\max} n d_{Tx}}. \tag{52}$$

Expressing this function in its general (two-dimensional) form and selecting only a set of $n_{Tx} \times n_{Tx}$ elements yields a CCM that specifies a windowed random discrete process:

$$\left[\mathbf{R}_{Tx,1}\right]_{i,j} = \frac{\sin\left(k_{\max} d_{Tx}(i-j)\right)}{k_{\max} d_{Tx}(i-j)}, \quad i,j = 0,1,\ldots,n_{Tx}-1 \tag{53}$$

which is readily recognized as the DPSS kernel from [23].

Note that with the previous kernel and $\mathbf{B}_{Tx} = \mathbf{R}_{Tx,1}$, the eigenvectors associated with $\mathbf{B}_{Tx}$ can be utilized for the RR generic channel estimator developed in Section III.

It is worthwhile to highlight the following remarks regarding the basis thus synthesized:

a) The CCM defined by (53) requires only the knowledge of $\varphi_{Tx}^{\max}$.

b) According to *Theorem 1*, eigenvectors of (53) will be adequate to represent other CCMs, regardless of the particular shape of the PAS considered.

c) For a given antenna aperture, (44) readily provides the spatial channel dimensionality; therefore, it provides a priori knowledge for subspace approaches.

d) The bases' properties for channel estimation using generic bases, as stated in Section III, are fulfilled.

When the angular range of the proposed PAS in (45) is nonsymmetrical, it yields a nonsymmetrical flat WNS, whose autocorrelation function corresponds to a modulated DPSS kernel [24]. However, the properties and results presented here are maintained.

It should be pointed out that the use of the DPSS basis for spatial channel representation was discussed in [25]; however, this paper has introduced both the PAS that generates this basis and its suitability for MIMO channel estimation. Furthermore, this paper has shown that the DPSS basis is just one of an infinite set of solutions for RR CCM representation. The requirements and conditions for synthesizing other bases have been also provided.

V. Simulation Results

This section presents simulation results to demonstrate the performance of the proposed channel estimator using reduced-rank channel representation. The parameters of the MIMO scenario being considered are as follows: An 8×8 MIMO system working in a frequency-flat fading scenario is used. The ULA for transmission is composed of 8 antennas and the spacing between each pair of array elements is $\lambda/2$ ($A_a = 3.5\lambda$). The ULA on the receiver side is composed of 8 antennas and the spacing between each pair of antennas is 4λ. It is assumed that the environment surrounding the receiver side is rich scattering with negligible spatial correlation fading (i.e., $\mathbf{R}_{Rx} = \mathbf{I}_8$). The transmission frequency is fixed at 2GHz. On the transmitter side, two propagation scenarios are considered for comparison; both are limited to a maximum dispersion at an angle of ±20 degrees and specified by two different functions denoted as

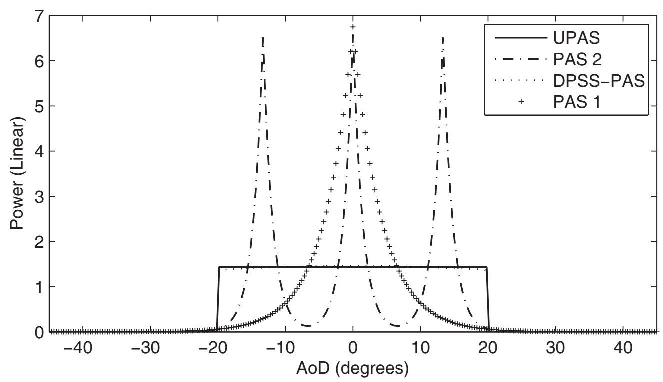

Fig. 1. Power Azimuth Spectrum corresponding to PAS1 and PAS2 Propagation scenarios, and the alternative PASs (DPSS-PAS and UPAS) used for bases construction.

PAS1 and PAS2, which represent realistic scenarios from [2], [26], and are defined as follows. PAS1 is

$$\mathrm{PAS1}(\varphi_{Tx}) = \frac{e^{-\left|\sqrt{2}\varphi_{Tx}\right|/\sigma_L}}{\sqrt{2}\sigma_L(1 - e^{-\sqrt{2}\pi/\sigma_L})} \tag{54}$$

where $\varphi_{Tx} \in [-\pi, \pi]$ and $\sigma_L = A_s \frac{\pi}{180}$ is the standard deviation of the PAS (which corresponds to the numerical value of angular spread (AS) with $A_s = 6°$) adjusted to $\varphi_{Tx}^{\max}$. Likewise, PAS2 is

$$\mathrm{PAS2}(\varphi_{Tx}) = \zeta \frac{1}{3} \sum_{p=-1}^{1} \frac{e^{-\left|\varphi_{Tx} - \frac{2p}{3}\varphi_{Tx}^{\max}\right|/\sigma_L}}{2\sigma_L(1 - e^{-\pi/\sigma_L})} \tag{55}$$

where $\varphi_{Tx} \in [-\pi, \pi]$, $\sigma_L = .0253$, and ζ is adjusted to satisfy $\int_{-\pi}^{\pi} \mathrm{PAS2}(\varphi_{Tx}) d\varphi_{Tx} = 1$.

Fig. 1 shows the plots corresponding to PAS1 and PAS2. In addition, it plots the uniform PAS (UPAS) and DPSS-PAS defined in (45) that will be utilized as generic PAS for RR channel estimation. All PAS shapes are restricted to the same maximum AoD.

From (42), the corresponding CCM1 and CCM2 for PAS1 and PAS2, respectively, were obtained. A resolution of 180/500 degrees per sample was considered (in this case, $M = 111$ paths in (42)). Once the $\mathbf{R}_{Rx}$ is obtained and the $\mathbf{R}_{Tx}$ is considered as either CCM1 or CCM2, channel realizations are obtained using the Kronecker model with (2).

Now the generic bases for channel representation can be obtained. The chosen values of carrier frequency and maximum AoD yield $k_0 = 41.8879$ and $k_{\max} = 14.3265$. The spatial channel dimensionality can be calculated using (44), then Dim=4. This dimensionality value is below the number of antennas for transmission, allowing subspace algorithms. According to the results in Section IV, one of the generic basis solutions is given by the DPSS.

From (53) and the parameters being considered, the DPSS kernel is $[\mathbf{R}_{Tx,1}]_{i,j} = \frac{\sin(1.0745(i-j))}{1.0745(i-j)}, \quad i, j = 0, 1, \ldots, 7$. For comparison purposes, a second generic basis that also fulfills *Theorem 1* has been considered. It is given by the UPAS depicted in Fig. 1. Its associated CCM, denoted as CCMU, is computed by (42), approximating UPAS as a discrete PAS shape with the same resolution ($M = 111$) that was used for the construction of CCM1 and CCM2.

TABLE I
DIAGONALIZATION OF THE CCM FOR PAS1 AND PAS2 BY ORTHOGONAL TRANSFORMATIONS

Vector	ECCM1	ECCMU-CCM1	DPSS-CCM1	ECCM2	ECCMU-CCM2	DPSS-CCM2
1	5.8862	5.1790	5.2465	2.8877	2.8854	2.8835
2	1.5738	1.4648	1.4696	2.8656	2.8587	2.8569
3	0.4128	1.1070	1.0400	2.0934	2.0789	2.0814
4	0.0997	0.2083	0.2035	0.1379	0.1442	0.1460
5	0.0224	0.0351	0.0346	0.0146	0.0316	0.0310
6	0.0044	0.0047	0.0047	0.0006	0.0013	0.0013
7	0.0007	0.0009	0.0009	0.0000	0.0000	0.0000
8	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Trace	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000

Table I shows the normalized power of the eigenvalues obtained from the eigenvectors of CCM1 and CCM2, denoted as ECCM1 and ECCM2, respectively. In addition, it shows the corresponding values of the quadratic form of the eigenvectors of CCMU (ECCMU) and DPSS, over CCM1 and CCM2 where the trace of each matrix has been normalized to 8.

From Table I and according to the dimensionality value calculated, it can be seen that on average, more than 99% of the total energy is accumulated within the four principal DPSS and ECCMU. This corroborates the present paper's claim that not only can RR channel estimation be performed, but also the same set of vectors can be used to approximate any shape of PAS, restricted only to the maximum dispersion in angle of the PAS to be approximated. It means DPSS can span both CMM1 and CCM2, as well as any other CCM obtained from a PAS with the same restricted maximum AoD. Moreover, DPSS is just one of an infinite set of alternatives, as shown in Section IV, from which ECCMU is one more valid generic basis.

It can be noted from Table I that the energy associated with each vector in ECCMU and DPSS is practically the same. This is because the corresponding PASs, from which they are obtained, are very similar, as can be noted from their PAS shapes depicted in Fig. 1.

Computer simulations have been run to verify and extend the analytical results shown in Sections II and III. The channel estimation task is performed using PAT with LS, LMMSE, and RRGB estimators, with the MSE as a figure of merit as in [27], but for MIMO single-carrier systems. Each simulation point was obtained by averaging over 1000 independent Monte Carlo simulation runs. These numerical results assume orthogonal training using Hadamard sequences with length $N = n_T$. The channel matrix and the receiver noise are assumed to be circular complex Gaussian random variables with unit variance.

Fig. 2 shows the MSE performance of the RRGB channel estimator for the propagation scenario when PAS1 is used. The RR algorithm employs different bases, i.e., ECCM1, ECCMU, and DPSS, with the best rank r approximation.

It can be seen from Fig. 2 that LS and LMMSE results represent the boundaries. Also, the theoretical and simulated RRGB(ECCM1) performance matches perfectly. As seen in Section III, the RRGB(ECCM1) estimator, which uses eigenvectors of CCM1, closely approaches the LMMSE estimator while avoiding the matrix inverse calculation, resulting in a computational complexity reduction. However, it requires

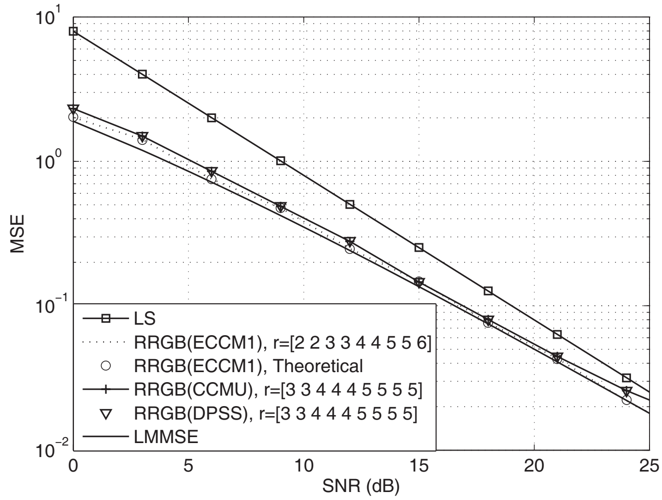

Fig. 2. MSE of MIMO channel estimate corresponding to the propagation MIMO scenario with PAS1. The notation RRGB(basis) indicates the RRGB channel estimator using a particular basis.

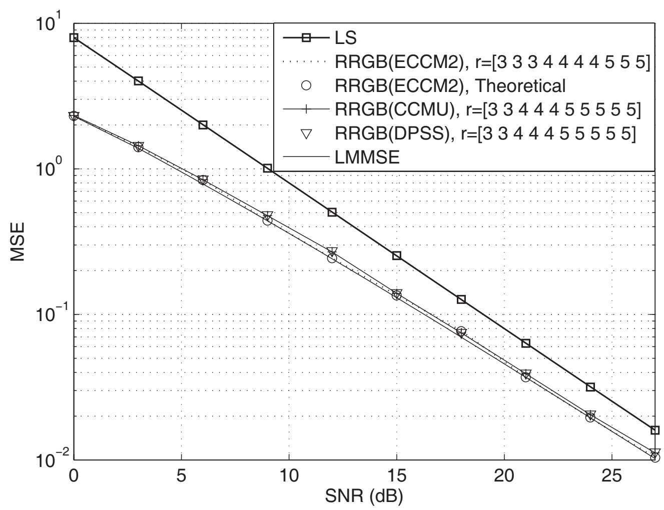

Fig. 3. MSE of MIMO channel estimate corresponding to the propagation MIMO scenario with PAS2. The notation RRGB(basis) indicates the RRGB channel estimator using a particular basis.

the knowledge of CCM1 and the performance of an online eigendecomposition. On the other hand, the proposed RRGB approach (based on DPSS and ECCMU), which requires only the maximum AoD, achieves practically the same performance as RRGB(ECCM1). This is an important result, as it seems that optimal estimation performance is closely approximated with less complexity using partial CSI. In fact, this approach avoids the estimation and inversion of the CCM, which is required in the direct implementation of the LMMSE approach. The same performance is achieved for CCM2 as shown in Fig. 3. Furthermore, the same predefined set of DPSS vectors was utilized to estimate channel realizations for both scenarios. This confirms the robustness of the proposed approach, in the sense that it is independent of the PAS shape of the scenario being considered.

VI. Conclusions

A novel algorithm has been introduced for estimating correlated MIMO channels based on partial CSI. The proposed approach requires neither an online decomposition nor the knowledge of second-order statistics, while achieving a performance very close to LMMSE. It is based on the use of generic bases derived from partial knowledge of the propagation scenario being considered, particularly the maximum AoD. The framework presented here shows how to synthesize an infinite set of bases suitable for channel expansion, from which DPSS is proposed as one of the possible generic bases. The concepts of spatial channel dimensionality and RR channel representation, by means of generic bases as discussed in this paper, can be applied to other areas such as channel modeling, digital beamforming, and space-time precoding.

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