Channel Estimation for OFDM Systems with Transmitter Diversity in Mobile Wireless Channels

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***Abstract*— Transmitter diversity is an effective technique to improve wireless communication performance. In this paper, we investigate transmitter diversity using space-time coding for orthogonal frequency division multiplexing (OFDM) systems in high-speed wireless data applications. We develop channel parameter estimation approaches, which are crucial for the decoding of the space-time codes, and we derive the MSE bounds of the estimators. The overall receiver performance using such a transmitter diversity scheme is demonstrated by extensive computer simulations. For an OFDM system with two transmitter antennas and two receiver antennas with transmission efficiency as high as 1.475 bits/s/Hz, the required signal-to-noise ratio is only about 7 dB for a 1% bit error rate and 9 dB for a 10% word error rate assuming channels with two-ray, typical urban, and hilly terrain delay profiles, and a 40-Hz Doppler frequency. In summary, with the proposed channel estimator, combining OFDM with transmitter diversity using space-time coding is a promising technique for highly efficient data transmission over mobile wireless channels.**

I. Introduction

In orthogonal frequency division multiplexing (OFDM) [1]–[8], the entire channel is divided into many narrow parallel subchannels, thereby increasing the symbol duration and reducing or eliminating the intersymbol interference (ISI) caused by the multipath environments. On the other hand, since the dispersive property of wireless channels causes frequency selective fading, there is higher error probability for those subchannels in deep fades. Hence, techniques such as error correction code and diversity have to be used to compensate for the frequency selectivity. In this paper, we investigate transmitter diversity using space-time coding for OFDM systems.

Transmitter diversity is an effective technique for combating fading in mobile wireless communications, especially when receiver diversity is expensive or impractical. Many researchers have studied transmitter diversity for wireless systems. As indicated in [9] and the references therein, transmitter diversity may be based on linear transforms. The performance gain of linear transform-based diversity with ideal maximum-likelihood sequence estimation (MLSE) and an arbitrary number of transmitter antennas is investigated and compared with receiver diversity in [10]. Some asymptotic properties of linear transform-based transmitter diversity are derived in [11]. Transmitter diversity with elaborate selection of the signal constellations has been proposed in [12]–[15]. Transmitter diversity combined with Reed–Solomon code has been proposed for clustered OFDM in [2]. More recently, space-time coding [16] has been developed for high data-rate wireless communications. Space-time coding is characterized by high code efficiency and good performance; hence, it is a promising technique to improve the efficiency and performance of OFDM systems. In [17], space-time coding with OFDM has been studied. However, decoding of space-time codes requires channel state information, which is usually difficult to obtain, especially for time-variant dispersive fading channels. The work in [17] assumes ideal channel state information. This paper focuses on parameter estimation for transmitter diversity using space-time coding in OFDM systems.

Channel parameter estimation has been successfully used to improve the performance of OFDM systems. For systems without cochannel interference, with estimated channel parameters [4], [5], [7], [18], coherent demodulation can be used instead of differential demodulation and can achieve a 3–4 dB signal-to-noise ratio (SNR) gain. Moreover, for systems with receiver diversity, maximal ratio diversity combining (MR-DC), which is equivalent to minimum mean-square error diversity combining (MMSE-DC) in the absence of cochannel interference, can be achieved directly using the estimated channel parameters. For systems with cochannel interference, the coefficients for the MMSE-DC must be calculated from estimated channel parameters and the instantaneous correlation of the signals from each receiver [19]. However, there is no literature on parameter estimation for OFDM systems with transmitter diversity.

For OFDM systems with transmitter diversity using space-time coding, two or more different signals are transmitted from different antennas simultaneously. The received signal is the superposition of these signals, usually with equal average power. If the channel parameters corresponding to each transmitter and receiver antenna pair are estimated by the approach developed previously in [18], the signals from other transmitter antenna(s) will become interference. The signal-to-interference ratio will always be less than 0 dB, and the MSE of the estimation will be very large. Hence, novel parameter estimation approaches are desired for transmitter diversity using space–time coding.

In this paper, we first derive an approach exploiting the delay profile characteristics of typical mobile wireless chan-

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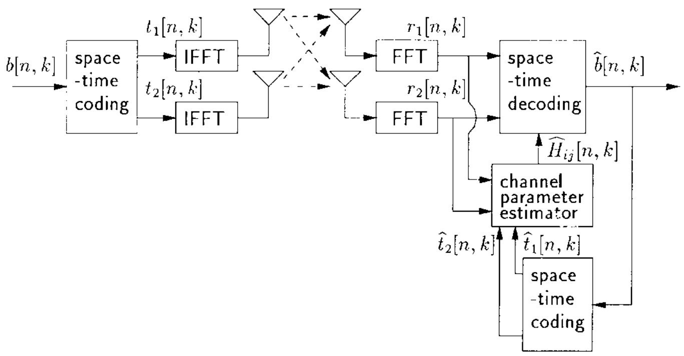

Fig. 1. OFDM system with transmitter diversity using space-time code.

nels. Based on this fundamental idea, we develop a simplified approach, which automatically estimates the channel delay profile and reduces the computation complexity using this information. As demonstrated by computer simulations, with estimated channel parameters for decoding of space-time codes, the OFDM system with two transmitter and two receiver antennas can transmit data at an efficiency of 1.475 bits/s/Hz while the required SNR is 7 dB for a 1% bit error rate (BER) and 9 dB for a 10% word error rate (WER).

The rest of this paper is organized as follows. Section II describes transmitter diversity using space-time coding for OFDM systems and briefly introduces the statistics of mobile wireless channels. Section III derives a basic and simplified approach to parameter estimation for OFDM systems with transmitter diversity. Section IV analyzes the performance bounds of the proposed estimation approaches and discusses the training block design and identifiability conditions. Finally, Section V presents computer simulation results to demonstrate the effectiveness of transmitter diversity for OFDM systems in various mobile wireless environments.

II. Transmitter Diversity for OFDM Systems in Mobile Wireless Channels

A. Transmitter Diversity Using Space-Time Coding

An OFDM system with two-branch transmitter diversity using a space-time code is shown in Fig. 1. At a transmission time n, a binary data block $\{b[n, k]\text{: } k = 0, 1, \ldots\}$ is coded into two different signals, $\{t_i[n, k]\text{: } k = 0, 1, \ldots\}$, for $i = 1, 2$. Each of these signals forms an OFDM block. The two transmitter antennas simultaneously transmit OFDM signals modulated by $t_i[n, k]$ for $i = 1, 2$. Hence, discrete Fourier transform (DFT) of the received signal at each receiver antenna is the superposition of two distorted transmitted signals, which can be expressed as

$$r_j[n, k] = \sum_{i=1}^{2} H_{ij}[n, k] t_i[n, k] + w_j[n, k] \tag{1}$$

where $H_{ij}[n, k]$ is the channel frequency response for the kth tone at time n, corresponding to the ith transmitter antenna and the jth receiver antenna, whose characteristics will be discussed in Section II-B; $w_j[n, k]$ denotes the additive complex Gaussian noise, with zero mean and variance σ_n^2, on the jth receiver antenna, which is uncorrelated for different n's, k's, or j's. Note that, for systems with transmitter diversity, the average SNR ratio at the receiver is defined as

$$\text{SNR} = \frac{E\{|H_{1j}[n,k]|^2 + |H_{2j}[n,k]|^2\}}{\sigma_n^2} \tag{2}$$

that is, both $H_{1j}[n, k]t_1[n, k]$ and $H_{2j}[n, k]t_2[n, k]$ are regarded as the desired signals. $E\{*\}$ denotes the ensemble average.

The decoding of a space-time code [16] uses the Viterbi decoding algorithm with the metrics

$$||\mathbf{r}[n, k] - \hat{\mathbf{H}}[n, k]\hat{\mathbf{t}}[n, k]||^2 \tag{3}$$

where $\| * \|$ denotes Euclidean norm, $\mathbf{r}[n, k]$ and $\hat{\mathbf{H}}[n, k]$ are the received signal vector and the estimated channel parameter matrix, respectively, defined as

$$\mathbf{r}[n, k] \triangleq \begin{pmatrix} r_1[n, k] \\ r_2[n, k] \end{pmatrix}$$

$$\hat{\mathbf{H}}[n, k] \triangleq \begin{pmatrix} \hat{H}_{11}[n, k] & \hat{H}_{21}[n, k] \\ \hat{H}_{12}[n, k] & \hat{H}_{22}[n, k] \end{pmatrix}$$

and $\hat{\mathbf{t}}[n, k]$ is the estimated signal vector, defined as

$$\hat{\mathbf{t}}[n, k] \triangleq \begin{pmatrix} \hat{t}_1[n, k] \\ \hat{t}_2[n, k] \end{pmatrix}.$$

Hence, accurate estimation of channel parameters is the key to the decoding of the space-time codes. In Section III, we describe parameter estimation approaches, which exploit the delay profile characteristics of mobile wireless channels. In the following, we provide useful representations of these characteristics.

B. Characteristics of Mobile Wireless Channels

The complex baseband representation of the mobile wireless channel impulse response can be described by [20]

$$h(t, \tau) = \sum_k \gamma_k(t) c(\tau - \tau_k) \tag{4}$$

where τ_k is the delay of the kth path, $\gamma_k(t)$ is the corresponding complex amplitude, and $c(t)$ is the shaping pulse, the frequency response of which is usually a square-root raised-cosine Nyquist filter. Hence, the frequency response at time t is

$$\begin{aligned} H(t, f) &\triangleq \int_{-\infty}^{+\infty} h(t, \tau) e^{-j2\pi f \tau}\, d\tau \\ &= C(f) \sum_k \gamma_k(t) e^{-j2\pi f \tau_k} \end{aligned} \tag{5}$$

with

$$C(f) \triangleq \int_{-\infty}^{+\infty} c(\tau) e^{-j2\pi f \tau}\, d\tau.$$

Due to the motion of the vehicle, $\gamma_k(t)$'s are modeled to be wide-sense stationary (WSS), narrowband complex Gaussian processes, which are independent for different paths. Furthermore, $\gamma_k(t)$'s for all k have the same normalized time correlation function and different average powers σ_k^2. It has been demonstrated in [18] that the correlation function of $H(t, f)$ can be separated into the multiplication of a time-domain correlation and a frequency-domain correlation, which makes it possible to exploit the correlation in both domains separately.

For OFDM systems with proper cyclic extension and sample timing, it has been shown in [4] and [18] that, with tolerable leakage, the channel frequency response can be expressed as

$$H[n, k] \triangleq H(nT_f, k\Delta f) = \sum_{l=0}^{K_o-1} h[n, l]W_K^{kl} \qquad (6)$$

where $h[n, l] \triangleq h(nT_f, kt_s)$, $W_K = \exp(-j(2\pi/K))$, K is the number of tones of an OFDM block, T_f and Δf are the block length and tone spacing of the OFDM system, respectively, and t_s is the sample interval of the system that relates to Δf by $t_s = 1/\Delta f$. In (6), $h[n, l]$'s, for $l = 0, 1, \ldots, K_o-1$, are WSS, narrowband complex Gaussian processes. The average power of $h[n, l]$ and index K_o ($\ll K$) depend on the delay profiles and dispersion of the wireless channels.

The two-ray [21], typical urban (TU), and hilly terrain (HT) [20], [22] models are three commonly used delay profiles. The average power and delay of each ray for TU and HT delay profiles are shown in Fig. 2(a) and (b), respectively. When comparing the performance of OFDM systems under different delay profiles, we assume the same delay spread, defined as

$$\tau_d \triangleq \sqrt{\sum_k \frac{\sigma_k^2}{\sum_l \sigma_l^2}\tau_k^2 - \left(\sum_k \frac{\sigma_k^2}{\sum_l \sigma_l^2}\tau_k\right)^2}.$$

For the two-ray profile with equal average power on each ray, the delay spread is $t_d/2$ ($t_d = \tau_1 - \tau_0$), i.e., a half of the delay difference between the two rays. The delay spreads for TU and HT delay profiles in Fig. 2 are 1.06 and 5.04 μs, respectively.

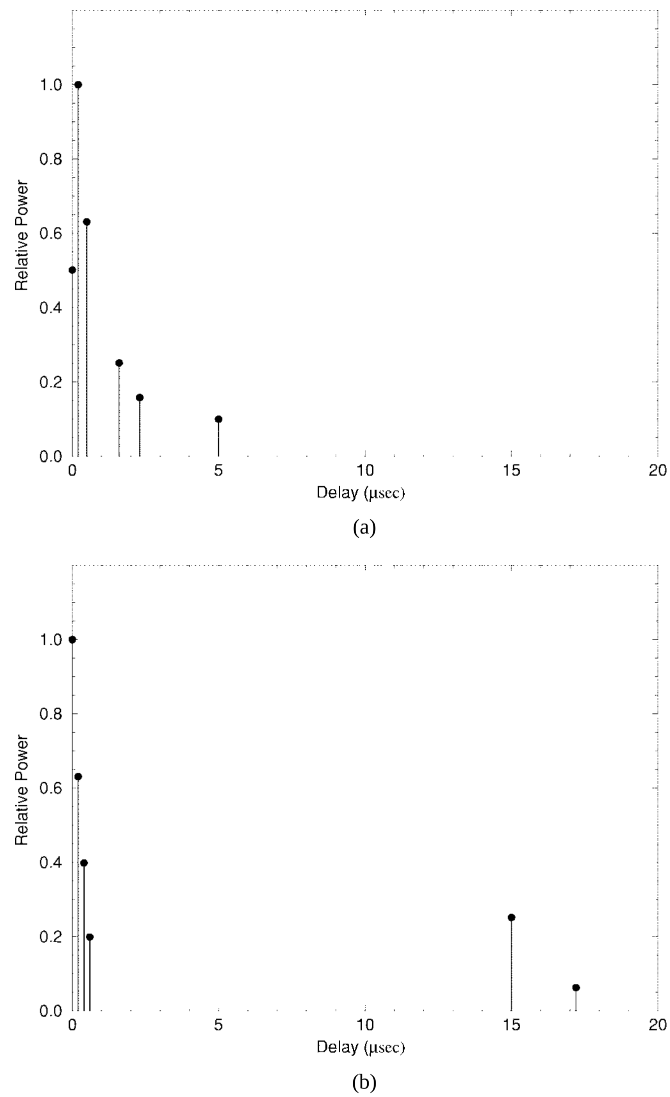

Fig. 2. (a) The TU delay profile ($\tau_d = 1.06\,\mu$s) and (b) the HT profile ($\tau_d = 5.04\,\mu$s).

III. Channel Parameter Estimation

As indicated in Section II, to decode a space-time code used for transmitter diversity in OFDM systems, the channel parameters must be provided. The difficulty of parameter estimation here is that, for each receiver antenna, every tone is associated with multiple channel parameters. Fortunately, channel parameters for the different tones of each channel are correlated. With this correlation, we are able to develop parameter estimation approaches.

A. Basic Approach

Following (6), the frequency response at the kth tone of the nth block corresponding to the ith transmitter antenna can be expressed as[1]

$$H_i[n, k] = \sum_{l=0}^{K_o-1} h_i[n, l]W_K^{kl}. \qquad (7)$$

Hence, to obtain $H_i[n, k]$, we only need to estimate $h_i[n, l]$.

From Section II, the received signal at each antenna can be expressed as

$$r[n, k] = \sum_{i=1}^{2} H_i[n, k]t_i[n, k] + w[n, k] \qquad (8)$$

for $k = 0, 1, \ldots, K-1$, and all n. If the transmitted signals $t_i[n, k]$'s, for $i = 1, 2$, are known through the use of a training block, the temporal estimation of $h_i[n, l]$ can be found by

[1] The index j for different receiver antennas is omitted from $H_{ij}[n, k]$, $r_j[n, k]$, and $w_j[n, k]$.

minimizing the following MSE cost function:

$$C\Big(\Big\{\tilde{h}_i[n,l];\ i=1,2\Big\}\Big) = \sum_{k=0}^{K-1}\left|r[n,k]-\sum_{i=1}^{2}\sum_{l=0}^{K_o-1}\tilde{h}_i[n,l]W_K^{kl}t_i[n,k]\right|^2. \quad (9)$$

Hence, $\tilde{h}_i[n,l]$ can be determined by

$$\frac{\partial C\Big(\Big\{\tilde{h}_i[n,l]\Big\}\Big)}{\partial \tilde{h}_i[n,l_o]} \triangleq \frac{1}{2}\left\{\frac{\partial C\Big(\Big\{\tilde{h}_i[n,l]\Big\}\Big)}{\partial \Re\Big(\tilde{h}_i[n,l_o]\Big)} - j\frac{\partial C\Big(\Big\{\tilde{h}_i[n,l]\Big\}\Big)}{\partial \Im\Big(\tilde{h}_i[n,l_o]\Big)}\right\} = 0 \quad (10)$$

where $\Re(*)$ and $\Im(*)$ denote the real and imaginary part of a complex number, respectively. Direct calculation yields that (10) is equivalent to

$$\sum_{k=0}^{K-1}\left(r[n,k]-\sum_{i=1}^{2}\sum_{l=0}^{K_o-1}\tilde{h}_i[n,l]W_K^{kl}t_i[n,k]\right) \cdot W_K^{-kl_o}t_j^*[n,k] = 0 \quad (11)$$

for $j=1,2$ and $l_o=0,1,\ldots,K_o-1$, where a^* denotes the complex conjugate of the complex number a.

Define

$$p_j[n,l] \triangleq \sum_{k=0}^{K-1} r[n,k]t_j^*[n,k]W_K^{-kl} \quad (12)$$

and

$$q_{ij}[n,l] \triangleq \sum_{k=0}^{K-1} t_i[n,k]t_j^*[n,k]W_K^{-kl}. \quad (13)$$

Then, (11) is equivalent to

$$\sum_{i=1}^{2}\sum_{l=0}^{K_o-1}\tilde{h}_i[n,l]q_{ij}[n,l_o-l] = p_j[n,l_o] \quad (14)$$

for $j=1,2$ and $l_0=1,2,\ldots,K_o-1$.

Equation (14) can be written in matrix form as shown at the bottom of the page in (15) for $i,j=1,2$. Hence, $\tilde{\mathbf{h}}[n]$ can be estimated by

$$\tilde{\mathbf{h}}[n] = \mathbf{Q}^{-1}[n]\mathbf{p}[n]. \quad (16)$$

With the temporal estimation of the channel parameters, a more accurate estimation can be obtained by exploiting the time correlation (i.e., the correlation between different blocks) of the channel parameters. Hence, using the results in [18], a robust channel estimator for OFDM systems with transmitter diversity can be constructed, as shown in Fig. 3. We have discussed a procedure for computing coefficients of the filter $\Phi(\omega)$ for robust estimator in [18], which is ideal f_d-band-limited, where f_d is the maximum Doppler frequency of the channel.

In the above derivation, we have assumed that the transmitted signals for each antenna are known. When the system is in data transmission mode, decoded data are used to generate the reference signals.

B. Simplified Approach

To get the temporal estimation of the channel parameters using the basic approach developed in Section III-A, $\mathbf{Q}^{-1}$ has to be calculated. For a system with a 160-μs symbol duration and 128 tones, K_o is 17 if the channel delay span is 20 μs. $\mathbf{Q}$ is a 34 × 34 matrix. Thus, calculating $\mathbf{Q}^{-1}$ requires intensive computations. Using the characteristics of the channel delay profiles, the estimation approach can be greatly simplified.

Most mobile wireless channels are characterized by discrete multipath arrivals, i.e., the magnitude of $h_i[n,l]$'s for most l's are zero or very small; hence, these channel taps can be ignored. Using this characteristic, we are able to reduce the computation complexity and improve the performance of the parameter estimation by identifying significant taps during training blocks.

$$\mathbf{Q}[n]\tilde{\mathbf{h}}[n] = \mathbf{p}[n]$$

where

$$\tilde{\mathbf{h}}[n] \triangleq \begin{pmatrix}\tilde{\mathbf{h}}_1[n]\\ \tilde{\mathbf{h}}_2[n]\end{pmatrix}, \quad \mathbf{p}[n] \triangleq \begin{pmatrix}\mathbf{p}_1[n]\\ \mathbf{p}_2[n]\end{pmatrix}, \quad \text{and } \mathbf{Q}[n] \triangleq \begin{pmatrix}\mathbf{Q}_{11}[n] & \mathbf{Q}_{21}[n]\\ \mathbf{Q}_{12}[n] & \mathbf{Q}_{22}[n]\end{pmatrix}$$

with

$$\tilde{\mathbf{h}}_i[n] \triangleq \Big(\tilde{h}_i[n,0], \tilde{h}_i[n,1], \cdots, \tilde{h}_i[n,K_o-1]\Big)^T, \qquad \mathbf{p}_i[n] \triangleq (p_i[n,0], p_i[n,1], \cdots, p_i[n,K_o-1])^T$$

and

$$\mathbf{Q}_{ij}[n] \triangleq \begin{pmatrix} q_{ij}[n,0] & q_{ij}[n,-1] & \cdots & q_{ij}[n,-K_o+1]\\ q_{ij}[n,1] & q_{ij}[n,0] & \cdots & q_{ij}[n,-K_o+2]\\ \vdots & \ddots & \ddots & \vdots\\ q_{ij}[n,K_o-1] & q_{ij}[n,K_o-2] & \cdots & q_{ij}[n,0]\end{pmatrix}. \quad (15)$$

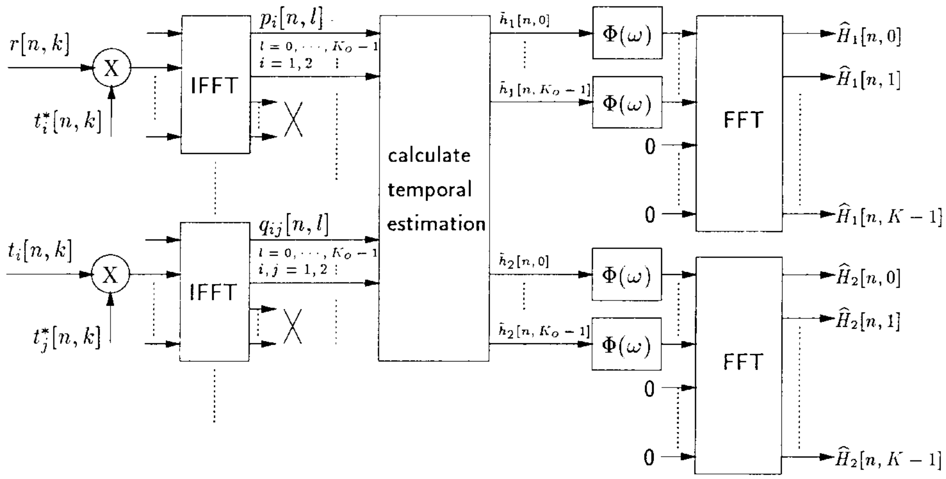

Fig. 3. Channel parameter estimator for transmitter diversity of OFDM systems.

For packet data transmission in wireless systems, the first OFDM block of a packet is the training block that is used for initial channel parameter estimation and for time and frequency synchronizations. Since the transmitted data for the training block is known, $\mathbf{Q}^{-1}$ in (16) can be calculated in advance and initial parameter estimation only requires multiplying $\mathbf{Q}^{-1}$ by $\mathbf{p}$.

Let $\tilde{h}_i[1, l]$'s for $i = 1, 2$ and $l = 0, 1, \ldots, K_o - 1$ be the estimated parameters from the training block. Then, we can identify the significant taps by finding the l's with large $\sum_{i=1,2} |h_i[1, l]|^2$. Assuming $h_i[1, l_m]$'s for $i = 1, 2$ and $m = 1, \ldots M$ $(0 \le 1_1 < l_2 < \ldots < l_M \le K_o - 1)$ to be the M significant taps, then we only need to estimate $h_i[1, l_m]$ corresponding to these taps and ignore the rest as shown in (16a) at the bottom of the page.

Similar to the derivation of Section III-A, the M significant channel taps for each channel corresponding to the same receiver antenna and different transmitter antennas can be estimated by

$$\overline{\mathbf{h}}[n] = \overline{\mathbf{Q}}^{-1}[n]\overline{\mathbf{p}}[n].$$

Here, $\overline{\mathbf{Q}}[n]$ is a $2M \times 2M$ matrix. As demonstrated by simulation, $M = 7$ is enough for all three delay profiles introduced in Section II-B.

As before, the time correlation of $h_i[n, l_m]$ can be used to further improve its estimation. Let $\hat{h}_i[n, l_m]$ be the time-averaged estimation; then the channel frequency response is determined by

$$H[n, k] = \sum_{m=1}^{M} \hat{h}[n, l_m] \exp\left(-j\frac{2\pi k l_m}{K}\right). \tag{17}$$

It should be indicated that the simplified approach is with zero error only if the nonsignificant taps are actually zeros. Consequently, the choice of M depends on both the computational complexity and the required performance of the simplified estimator. For systems with large SNR, the estimation error of $h[n, l_m]$ is small and the estimation error of $H[n, k]$ comes mainly from the leakage; hence, M should be larger to reduce the leakage. However, for systems with small SNR, the relative estimation error for those taps with small amplitude, is very large; hence, M should be smaller.

IV. Performance Analysis

We have developed a basic and simplified approach in Section III. In this section, we analyze the performance and derive the MSE bounds. We also briefly discuss channel identifiability and training signal design.

Let

$$\overline{\mathbf{h}}_i[n] \triangleq \left(\tilde{h}_i[n, l_1], \tilde{h}_i[n, l_2], \cdots, \tilde{h}_i[n, l_M]\right)^T, \qquad \overline{\mathbf{p}}_i[n] \triangleq (p_i[n, l_1], p_i[n, l_2], \cdots, p_i[n, l_M])^T$$

$$\overline{\mathbf{Q}}_{ij}[n] \triangleq \begin{pmatrix} q_{ij}[n, 0] & q_{ij}[n, l_1 - l_2] & \cdots & q_{ij}[n, l_1 - l_M] \\ q_{ij}[n, l_2 - l_1] & q_{ij}[n, 0] & \cdots & q_{ij}[n, l_2 - l_M] \\ \vdots & \ddots & \ddots & \vdots \\ q_{ij}[n, l_M - l_1] & q_{ij}[n, l_M - l_2] & \cdots & q_{ij}[n, 0] \end{pmatrix}$$

and

$$\overline{\mathbf{h}}[n] \triangleq \begin{pmatrix} \overline{\mathbf{h}}_1[n] \\ \overline{\mathbf{h}}_2[n] \end{pmatrix}, \qquad \overline{\mathbf{p}}[n] \triangleq \begin{pmatrix} \overline{\mathbf{p}}_1[n] \\ \overline{\mathbf{p}}_2[n] \end{pmatrix}, \qquad \overline{\mathbf{Q}}[n] \triangleq \begin{pmatrix} \overline{\mathbf{Q}}_{11}[n] & \overline{\mathbf{Q}}_{21}[n] \\ \overline{\mathbf{Q}}_{12}[n] & \overline{\mathbf{Q}}_{22}[n] \end{pmatrix}. \tag{16a}$$

A. MSE Bound of Estimator

From (8) and (12), we have

$$\begin{aligned} p_1[n, l_o] &= \sum_{k=0}^{K-1} r[n, k]t_1^*[n, k]W_K^{-kl_o} \\ &= \sum_{k=0}^{K-1} (H_1[n, k]t_1[n, k] + H_2[n, k]t_2[n, k] \\ &\quad + w[n, k])t_1^*[n, k]W_K^{-kl_o} \\ &= \sum_{k=0}^{K-1} H_1[n, k]|t_1[n, k]|^2 W_K^{-kl_o} \\ &\quad + \sum_{k=0}^{K-1} H_2[n, k]t_2[n, k]t_1^*[n, k]W_K^{-kl_o} \\ &\quad + \sum_{k=0}^{K-1} w[n, k]t_1^*[n, k]W_K^{-kl_o}. \end{aligned} \tag{18}$$

By means of the convolution theorem [23]

$$\sum_{k=0}^{K-1} H_1[n, k]|t_1[n, k]|^2 W_K^{-kl_o} = \sum_{l=0}^{K_o-1} q_{11}[n, l_o - l]h_1[n, l]$$

and

$$\begin{aligned} &\sum_{k=0}^{K-1} H_2[n, k]t_2[n, k]t_1^*[n, k]W_K^{-kl_o} \\ &\qquad = \sum_{l=0}^{K_o-1} q_{21}[n, l_o - l]h_2[n, l]. \end{aligned}$$

Let

$$W_i[n, l] \triangleq \sum_{k=0}^{K-1} w[n, k]t_i^*[n, k]W_K^{-kl} \tag{19}$$

for $i = 1, 2$. Then, for all $i, j = 1, 2$

$$\begin{aligned} &E\{W_i[n, l_1]W_j^*[n, l_2]\} \\ &\quad = E\left\{\sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1} w[n, k_1]w^*[n, k_2]t_i^*[n, k_1] \right. \\ &\qquad\qquad \left. \cdot\, t_j[n, k_2]W_K^{-k_1l_1+k_2l_2}\right\} \\ &\quad = \sum_{k=0}^{K-1} \sigma_n^2 t_i^*[n, k]t_j[n, k]W_K^{-k(l_1-l_2)} \\ &\quad = \sigma_n^2 q_{ji}[n, l_1 - l_2]. \end{aligned} \tag{20}$$

Hence

$$\mathbf{p}_1[n] = \mathbf{Q}_{11}[n]\mathbf{h}_1[n] + \mathbf{Q}_{21}[n]\mathbf{h}_2[n] + \mathbf{W}_1[n].$$

Similarly

$$\mathbf{p}_2[n] = \mathbf{Q}_{12}[n]\mathbf{h}_1[n] + \mathbf{Q}_{22}[n]\mathbf{h}_2[n] + \mathbf{W}_2[n]$$

where

$$\mathbf{W}_i[n] \triangleq (W_i[n, 0], \ldots, W_i[n, K_o - 1])^T$$

for $i = 1, 2$. Therefore,

$$\mathbf{p}[n] = \mathbf{Q}[n]\mathbf{h}[n] + \mathbf{W}[n] \tag{21}$$

where

$$\mathbf{W}[n] \triangleq \left(\mathbf{W}_1^T[n], \mathbf{W}_2^T[n]\right)^T$$

with

$$E\{\mathbf{W}[n]\mathbf{W}^H[n]\} = \sigma_n^2 \mathbf{Q}[n].$$

Hence, from (16)

$$\tilde{\mathbf{h}}[n] = \mathbf{h}[n] + \mathbf{Q}^{-1}[n]\mathbf{W}[n]. \tag{22}$$

From (22), the (temporal) estimation of $h_i[n, l]$ is the summation of the true value plus a term affected by channel noise.

From (22)

$$E\left\{\tilde{\mathbf{h}}[n]\right\} = \mathbf{h}[n] + \mathbf{Q}^{-1}[n]E\{\mathbf{W}[n]\} = \mathbf{h}[n] \tag{23}$$

which implies that (16) is an unbiased estimation of $h_i[n, l]$. The MSE of the temporal estimator is

$$\begin{aligned} \mathrm{MSE}[n] &\triangleq \frac{1}{2K_o} E\left\{\|\tilde{\mathbf{h}}[n] - \mathbf{h}[n]\|^2\right\} \\ &= \frac{1}{2K_o} E\left\{\left(\mathbf{Q}^{-1}[n]\mathbf{W}[n]\right)^H \mathbf{Q}^{-1}[n]\mathbf{W}[n]\right\} \\ &= \frac{1}{2K_o} \mathrm{Tr}\left\{E\left(\mathbf{Q}^{-1}[n]\mathbf{W}[n]\mathbf{W}^H[n]\mathbf{Q}^{-1}[n]\right)\right\} \\ &= \frac{\sigma_n^2}{2K_o} \mathrm{Tr}\left\{\mathbf{Q}^{-1}[n]\right\} \end{aligned} \tag{24}$$

where H denotes the Hermitian of a matrix or vector and Tr$(*)$ denotes the trace of a square matrix.

If $t_i[n, k]$'s, for $i = 1, 2$, all n and k, are constant modulus, then

$$|t_i[n, k]|^2 = 1$$

thus

$$\mathbf{Q}_{ii}[n] = K\mathbf{I}$$

where $\mathbf{I}$ is a $K_o \times K_o$ identity matrix. Since $\mathbf{Q}_{21}[n] = \mathbf{Q}_{12}^H[n]$, the result is shown in (24a) at the bottom of the page.

$$\mathbf{Q}^{-1}[n] = \frac{1}{K}\begin{pmatrix} \mathbf{I} + \frac{\mathbf{Q}_{21}[n]}{K}\left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1}\frac{\mathbf{Q}_{21}^H[n]}{K} & -\frac{\mathbf{Q}_{21}[n]}{K}\left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1} \\ -\left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1}\frac{\mathbf{Q}_{21}^H[n]}{K} & \left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1} \end{pmatrix} \tag{24a}$$

Hence, the average estimation error

$$\mathrm{MSE}[n] = \frac{\sigma_n^2}{2KK_o}\,\mathrm{Tr}\left\{\mathbf{I} + \frac{\mathbf{Q}_{21}[n]}{K}\left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1} \cdot \frac{\mathbf{Q}_{21}^H[n]}{K} + \left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1}\right\}$$
$$= \frac{\sigma_n^2}{2KK_o}\,\mathrm{Tr}\left\{\mathbf{I} + \left(\mathbf{I} - \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)^{-1} \cdot \left(\mathbf{I} + \frac{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]}{K^2}\right)\right\}. \quad (25)$$

Let $\lambda_k^2[n]$'s, for $k = 1, \ldots, K_o$, be the eigenvalues of $\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]$. Then, the MSE can also be expressed as

$$\mathrm{MSE}[n] = \frac{\sigma_n^2}{2KK_o}\left(K_o + \sum_{k=1}^{K_o} \frac{1 + (\lambda_k^2[n]/K^2)}{1 - (\lambda_k^2[n]/K^2)}\right). \quad (26)$$

From the above expression, the MSE of the temporal estimation depends on the transmitted sequence, and furthermore

$$\mathrm{MSE}[n] \geq \frac{\sigma_n^2}{K} \quad (27)$$

with equality if and only if $\lambda_l^2[n] = 0$ for $l = 1, \ldots, K_o$, or equivalently, $q_{ij}[n, l] = 0$ for $l = 0, \pm 1, \ldots, \pm(K_o - 1)$, or $\mathbf{Q}_{ij}[n] = \mathbf{0}$ for all $i \neq j$.

From [18], the lower bound of the MSE of the estimated $H_i[n, k]$ is

$$\mathrm{MSE}_o = \frac{K_o\sigma_n^2}{K}\left(1 - \frac{1}{[(\pi K/K_o\sigma_n^2\omega_d) + 1]^{\omega_d/\pi}}\right) \quad (28)$$

where ω_d is the Doppler frequency of the channel.

V. Remarks

1) *Identification Condition:* From (16), the necessary and sufficient condition for the channel parameters to be identifiable is that $\mathbf{Q}[n]$ is invertible. From (25), this condition is equivalent to that $\mathbf{I} - (\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]/K^2)$ being invertible. A sufficient identification condition is that

$$\mathrm{Tr}\{\mathbf{Q}_{21}^H[n]\mathbf{Q}_{21}[n]\} < K$$

or

$$\sum_{k=-(K_o-1)}^{K_o-1} \left(1 - \frac{|k|}{K_o}\right)|q_{21}[n, k]|^2 < \frac{K}{K_o}. \quad (29)$$

2) *Training Signal Design:* From (27), in order for the channel estimator to achieve its best performance, the training signals from the two transmitter antennas should satisfy

$$q_{21}[1, k] = 0 \quad (30)$$

for $k = 0, \pm 1, \ldots, \pm(K_o - 1)$, which can be easily constructed, for example, by letting $t_2[1, k] = (-1)^k t_1[1, k]$.

3) *Enhanced Parameter Estimation:* As indicated in [19], the second-pass channel estimation can also be used here to improve the performance of the channel estimator. In that case, the lower bound of the MSE is reduced to

$$\mathrm{MSE}_o = \frac{1}{(\pi K/\omega_d K_o \sigma_n^2) + 1}. \quad (31)$$

VI. Performance Evalutaion by Computer Simulation

Extensive computer simulations have been conducted to demonstrate the performance of transmitter diversity using space-time coding. Before presenting the simulation results, we first describe the parameters of the simulated OFDM systems.

A. Parameters of OFDM with Transmitter Diversity

In our simulation, channels with two-ray, TU, and HT delay profiles and a Doppler frequency of 40 and 200 Hz are used to represent different mobile environments. Two transmitter antennas and two receiver antennas are used for diversity. The links between different transmitter or receiver antennas are independent; however, they have the same global statistics.

The parameters of the simulated OFDM system are similar to those in [3], [18], and [19]. The entire channel bandwidth, 800 kHz, is divided into 128 subchannels. The four subchannels on each end are used as guard tones, and the rest (120 tones) are used to transmit data. To make the tones orthogonal to each other, the symbol duration is 160 μs. An additional 40 μs guard interval is used to provide protection from intersymbol interference due to channel multipath delay spread. This results in a total block length $T_f = 200$ μs and a subchannel symbol rate $r_b = 5$ kBd.

A 16-state space-time code with four PSK is used in the system. Each data block, containing 236 bits, is coded into two different blocks, each of which has exactly 120 symbols to form an OFDM block. Consequently, it is in fact a space-frequency code. The channel parameter estimation approaches developed in this paper are used to provide estimated parameters for decoding.

The described system can transmit data at a rate of 1.18 Mbits/s over an 800 kHz channel, i.e., the transmission efficiency is 1.475 bits/s/Hz.

B. Simulation Results

The performance of the systems is measured by WER, BER, and the estimator's MSE, each averaged over 10 000 OFDM blocks. Fig. 4 shows the WER, BER, and the estimator's MSE of the OFDM system with transmitter diversity for the TU and two-ray delay profile channels with 40-Hz Doppler frequency (f_d) and the same delay spread $\tau_d = 1.06$ μs. When ideal channel parameters of the previous OFDM block are used for decoding of the current OFDM block, the OFDM system for both delay profiles has almost the same performance, and the required SNR is 8 and 11 dB for 10 and 1% WER, respectively, and 6 and 8 dB for 1 and 0.1% BER, respectively. If a nine-tap significant-tap-catching (STC) estimator is used to estimate the channel parameters, then there is about a

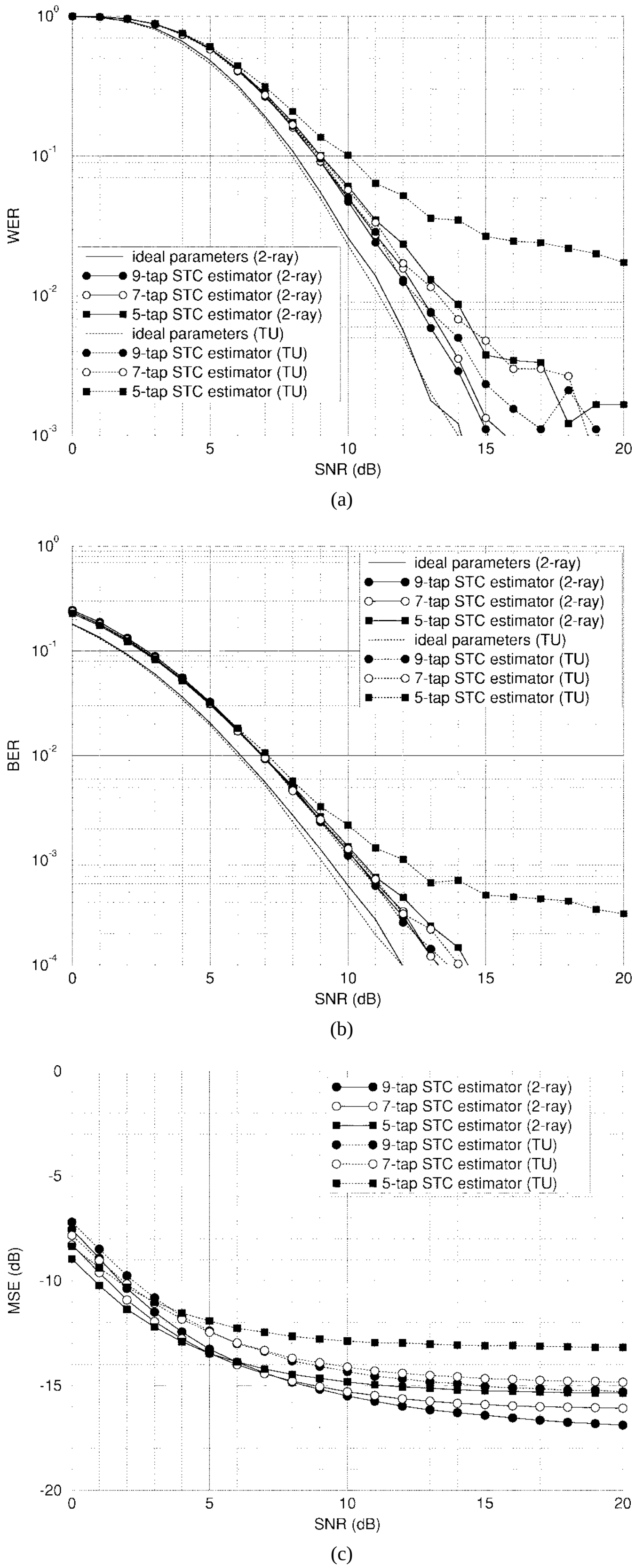

Fig. 4. (a) WER, (b) BER, and (c) MSE of OFDM systems with transmitter diversity for channels with the two-ray and the TU delay profiles, respectively, and $f_d = 40$ Hz when a different number of taps for the estimators is used.

1-dB degradation in the required SNR's. However, larger performance degradation can be seen for a seven or five-

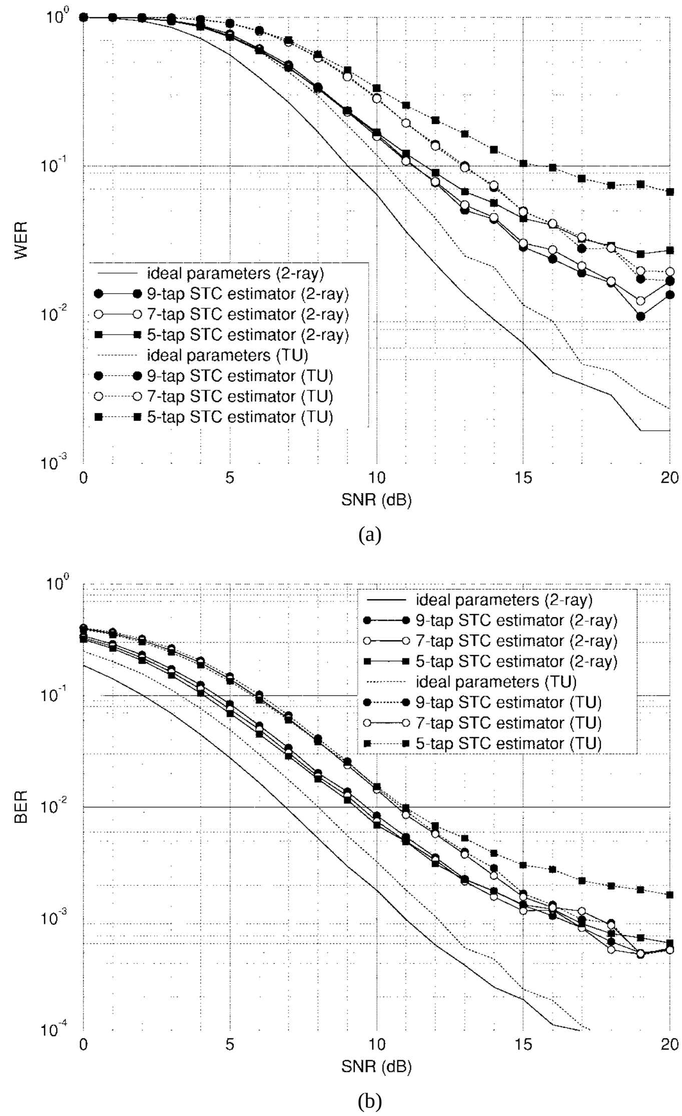

Fig. 5. (a) WER and (b) BER of OFDM systems with transmitter diversity for channels with the two-ray and the TU delay profiles, respectively, and $f_d = 200$ Hz when different number of taps for the estimators is used.

tap STC estimator. Since the estimator has more leakage for the TU delay profile, the system has a larger performance degradation than for the two-ray delay profile. Fig. 5 shows similar performance for $f_d = 200$ hz. Again, the receiver performance is generally better for the two-ray delay profile than for the TU delay profile.

Fig. 6 compares the performance between channels with the two-ray and the HT delay profiles with $f_d = 40$ Hz and $\tau_d = 5.04$ μs. Similar to the above results, for channels with $f_d = 40$ Hz, the system has the same performance when the ideal parameters of the previous OFDM block are used for decoding. However, when estimated parameters are used, the system has better performance for the two-ray delay profile than for the HT profile, since the estimator has lower MSE for the two-ray delay profile. When a seven or nine-tap STC estimator is used, the required SNR is 8 dB for a 10% WER, 6 dB for a 1% BER (for the two-ray delay profile), and about 8.6 dB for a 10% WER and 6.6 dB for a 1% BER, respectively.

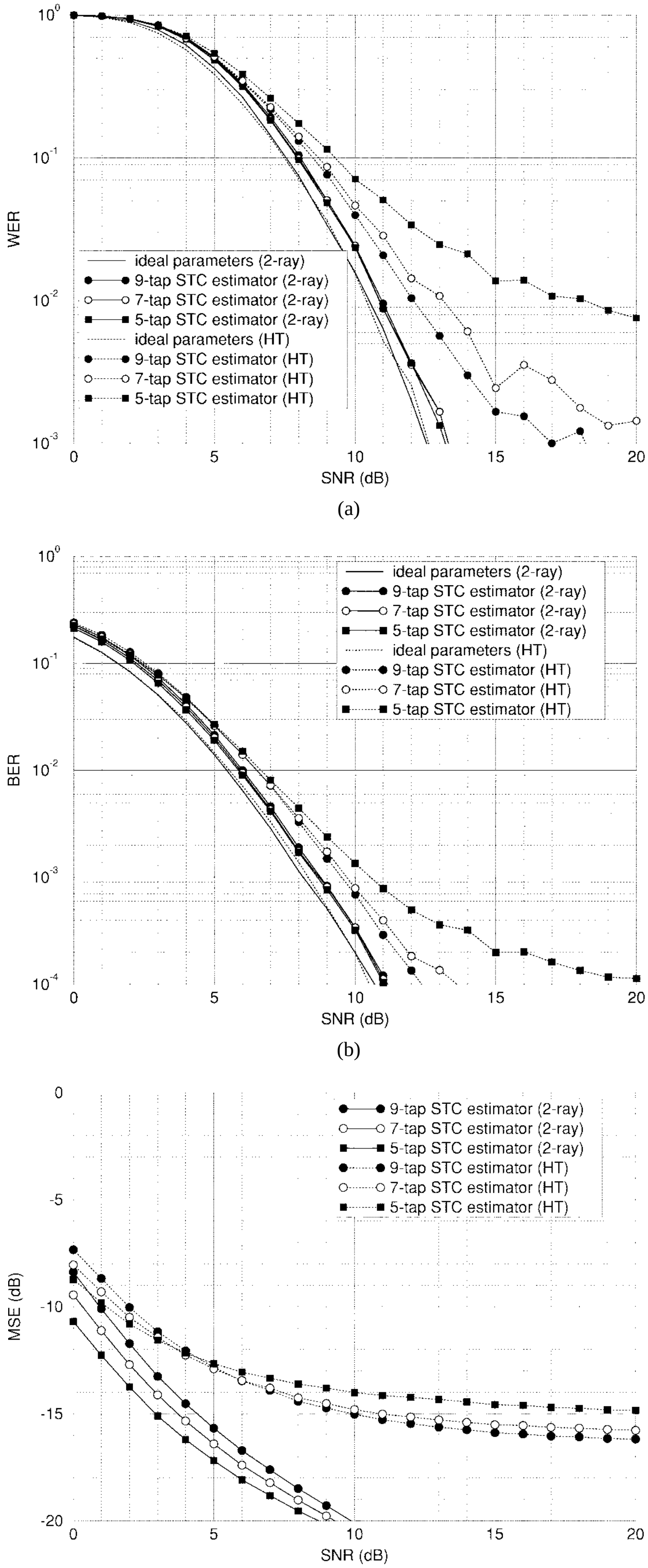

Fig. 6. (a) WER, (b) BER, and (c) MSE of OFDM systems with transmitter diversity for channels with the two-ray and the HT delay profiles, respectively, and $f_d = 40$ Hz when different number of taps for the estimators is used.

As indicated in Section IV-B, enhanced channel estimation can improve the OFDM system performance. Fig. 7 shows the performance improvement of the enhanced estimator for channels with $f_d = 40$ and 200 Hz, respectively. From the figure, for channels with a lower Doppler frequency ($f_d = 40$ Hz), the enhanced estimator does not improve the system performance; but for channels with a higher Doppler frequency ($f_d = 200$ Hz), the enhanced estimator improves the required SNR for a 10% WER by over 1 dB.

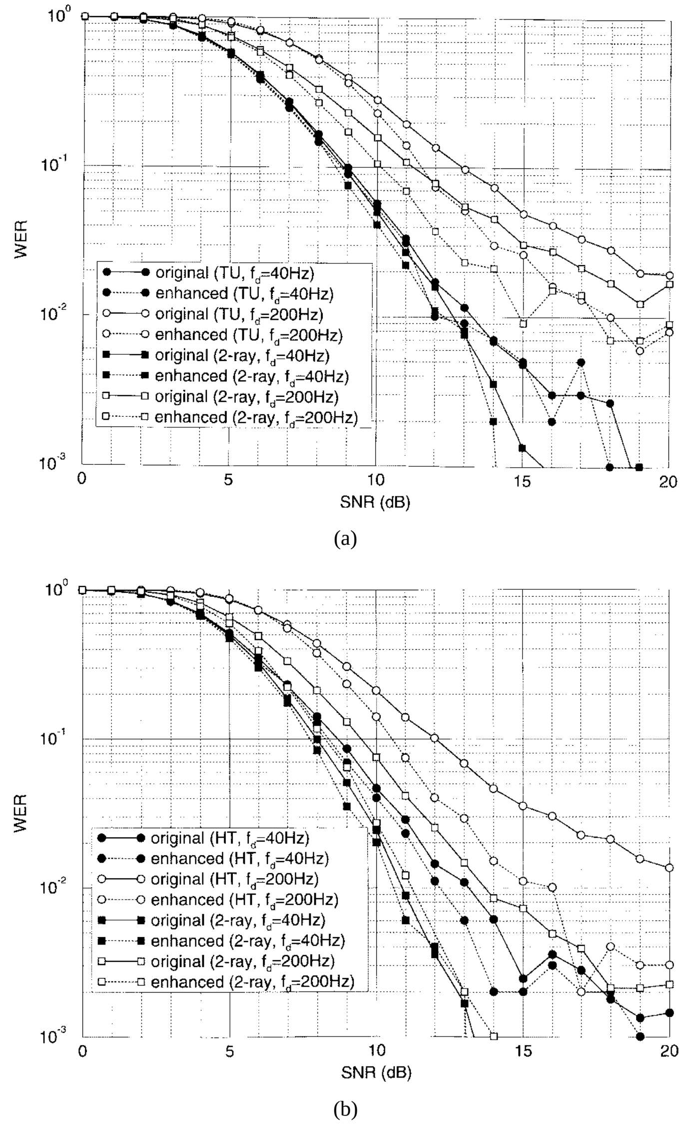

Fig. 7. WER's of original and enhanced estimators (seven-tap STC) for channels with (a) the two-ray and the TU delay profiles and (b) the two-ray and the HT delay profiles and with different Doppler frequencies.

VII. Discussions

In Section VI, we have shown the performance improvement of an OFDM system with transmitter diversity using a 16-state space-time code. A higher state, such as 64-state, space-time code can be used to further improve the system performance; however, the decoding complexity will increase.

It has been shown in [18] that an OFDM system with one transmitter antenna and two receiver antennas transmitting data at an efficiency of 0.75 bit/s/Hz requires about 6.5-dB SNR for a 10% WER. For the simulated system simulated in Section VI with two transmitter antennas, the data transmission rate is doubled and the required SNR is about 9 dB. Furthermore, considering the transmitted data rate and the length of codes (236 bits for the space-time code in this paper and 120 bits for the RS code in [18]), the required E_b/N_o for a 10% WER is about 1.5 dB less for the OFDM system studied in this paper, which is the gain of transmitter diversity using the space-time code.

However, the use of transmitter diversity using space-time codes may affect the system's interference suppression performance since each cochannel interference source would produce two interference signals for systems with two transmitter antennas. This effect is under investigation.

VIII. Conclusions

In this paper, we study transmitter diversity using space-time coding for OFDM systems. We develop channel parameter estimation approaches, which are crucial for the decoding of space-time codes, and we derive the MSE bounds for these estimation approaches. The overall receiver performance is evaluated by computer simulation. For an OFDM system with two transmitter antennas and two receiver antennas using space-time coding, permitting 1.475 bits/s/Hz, the required SNR is about 9 dB for 10% WER, and 7 dB for 1% BER, for channels with the two-ray, TU, and HT delay profiles and a Doppler frequency of 40 Hz. Therefore, OFDM systems with transmitter diversity using space-time coding can be used for highly efficient data transmission over mobile wireless channels.

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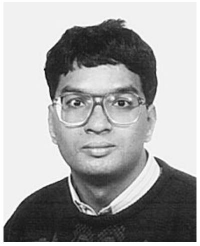

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