C-CM–102

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BOARD DIPLOMA EXAMINATION, (C-20)

MAY—2023

DCE - FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 Hours]   [*Total Marks* : 80

PART—A   3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. If $A = \left\{0, \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}, \dfrac{\pi}{2}\right\}$ and $f : A \to B$ is a function such that $f(x) = \cos x$, then find the range of f.

2. Resolve $\dfrac{x-1}{(x-2)(x+3)}$ into partial fractions.

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $3B - 2A$.

4. If $A + B = \dfrac{\pi}{4}$, then prove that $(1 + \tan A)(1 + \tan B) = 2$.

5. Prove that $\sin\theta \sin(60 - \theta)\sin(60 + \theta) = \dfrac{1}{4}\sin 3\theta$.

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6. Find the modulus of the complex number $\dfrac{7 + 24i}{3 - 4i}$.
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7. Find the equation of the straight line passing through $(3, -4)$ and parallel to the line $x + 7y + 1 = 0$.

8. Evaluate $\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^3 - 8}$

9. Find the derivative of $3\cos x + \log x + 21x + 5$.

10. Find the derivative of $e^{\sin^{-1} x}$.

PART—B

8×5=40

Instructions : (1) Answer **all** questions.

(2) Each question carries **eight** marks.

11. *(a)* Find the value of x, if the matrix $\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix}$ is singular.

(OR)

(b) Solve the following system of equations using Cramer's rule $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.

12. *(a)* Prove that $\dfrac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A + B),\ A \neq B$.

(OR)

(b) Prove that $\tan^{-1}\left(\dfrac{1}{7}\right) + \tan^{-1}\left(\dfrac{1}{13}\right) = \tan^{-1}\left(\dfrac{2}{9}\right)$.

13. *(a)* Solve $\cos\theta + \sin\theta = \sqrt{2}$.

*   **(OR)**

(b) In any $\triangle ABC$, if $\angle A = 60°$, then prove that $\dfrac{b}{c+a} + \dfrac{c}{a+b} = 1$.

14. *(a)* Find the equation of the circle passing through the points $(0, 0)$, $(6, 0)$ and $(8, 4)$.

(OR)

(b) Find the equation of the rectangular hyperbola whose focus is the point $(-1, -3)$, and directrix is the line $2x + y + 1 = 0$.

15. *(a)* If $y = x^{x^{x^{\cdots\infty}}}$, then show that $\dfrac{dy}{dx} = \dfrac{y^2}{x(1 - y \log x)}$.

(OR)

(b) If $u(x,y) = \sin^{-1}\left[\dfrac{x^2 + y^2}{x+y}\right]$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

PART—C   10×1=10

Instructions :   (1) Answer the following question.

                        (2) The question carries **ten** marks.

16. Find the lengths of tangent, normal, sub tangent and sub normal to the parabola $y^2 = 4x$ at $(1, 2)$.

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