

Mathematics of Symmetric Key Cryptography:Symmetric Encryption:-

- Symmetric Ciphers Use Symmetric algorithms to encrypt and decrypt data.
- These ciphers are Used in Symmetric Key Cryptography..
- A Symmetric algorithm Uses the Same Key to encrypt data as it does to decrypt data.

Example:-

A Symmetric algorithm will use key  $K$  to encrypt some plaintext information like a password into a ciphertext.

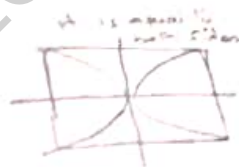
Then it uses  $K$  again to take the ciphertext and turn it back into the password.

- Symmetric Ciphers are opposite of asymmetric Ciphers, like those used in public-key cryptography.
- Ciphers Use asymmetric algorithm which use one key to encrypt data and a different key to decrypt Ciphers.
- These two keys are called public and private keys as in the case with RSA encryption.
- The public key is used to encrypt data and private key is used to decrypt data.

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# Mathematics of Symmetric Key Cryptography

## ① Chinese Remainder Theorem:-



General Form

$$\left. \begin{aligned} x &= a_1 \pmod{m_1} \\ x &= a_2 \pmod{m_2} \\ x &= a_n \pmod{m_n} \end{aligned} \right\} \begin{array}{l} \text{Constant Value } x \\ \text{Change Value for} \\ \text{each equation} \end{array}$$

Consistently  
change.

Let fold a paper  
under the fold of  
the paper coincide  
with the line fold  
of the paper.

Generalize defined  
with multiple congruence  
equations.

Step: 1:-

Find out Common modulus  $M$

$$M = m_1 \times m_2 \times m_3 \times \dots \times m_n$$

Step: 2:-

(i)  $M = m_1 m_2 m_3$

(ii)  $M_1 = \frac{M}{m_1}, M_2 = \frac{M}{m_2}, M_3 = \frac{M}{m_3}$

Find  $M_1 = \frac{M}{m_1}, M_2 = \frac{M}{m_2}, \dots, M_n = \frac{M}{m_n}$

Step: 3:-

(iii)  $M_1^{-1} \pmod{m_1}, M_2^{-1} \pmod{m_2}, \dots, M_n^{-1} \pmod{m_n}$

Step: 4:-

Find out Inverse  $M_1^{-1}, M_2^{-1}, \dots, M_n^{-1}$

$$x = (a_1 \times M_1 \times M_1^{-1}) + (a_2 \times M_2 \times M_2^{-1}) + \dots$$

(iv)  $x = (a_1 \times m_1 \times M_1^{-1}) + (a_2 \times m_2 \times M_2^{-1}) + (a_3 \times m_3 \times M_3^{-1}) \pmod{M}$

Ex:-

$$x \equiv 4 \pmod{11}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 6 \pmod{13}$$

$$\left\{ \begin{array}{l} x \equiv 2 \pmod{3} \\ x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \end{array} \right.$$

From the above simplify.

$$m_1 = 11$$

$$m_2 = 7$$

$$m_3 = 13$$

$$a_1 = 4$$

$$a_2 = 5$$

$$a_3 = 6$$

Step 1:

$$N = m_1 \times m_2 \times m_3$$

$$= 11 \times 7 \times 13$$

$$= 1001$$

Step 2:

$$M_1 = \frac{N}{m_1} = \frac{1001}{11} = 91$$

$$M_2 = \frac{N}{m_2} = \frac{1001}{7} = 143$$

$$M_3 = \frac{N}{m_3} = \frac{1001}{13} = 77$$

Step 3:

To find:  $M_1^{-1} = 91^{-1} \pmod{11} \Rightarrow \frac{1}{91} \pmod{11}$

$$(n \times 91) \pmod{11} = 1$$

Here  $n = 1, 2, 3, 4, \dots$

$$(4 \times 91) \pmod{11} = 1$$

$$M_1^{-1} = 4$$

To find:  $M_2^{-1} = 143^{-1} \pmod{7} = \frac{1}{143} \pmod{7}$

$$= (n \times 143) \pmod{7}$$

Here  $n = 1, 2, 3, 4, \dots$

$$= (5 \times 143) \pmod{7}$$

$$M_2^{-1} = 5$$

$$\begin{array}{r} 33 \\ 33 \overline{) 364} \\ \underline{33} \phantom{0} \\ 34 \\ 33 \phantom{0} \\ \underline{1} \phantom{0} \end{array}$$

$$364 \pmod{11} = 1$$

$$5 \times 143 = 715$$

$$\begin{array}{r} 7 \overline{) 715} \phantom{0} \\ \underline{714} \phantom{0} \\ 1 \phantom{0} \end{array}$$

5

To find:  $M_3^{-1} = 77 \pmod{13} = \frac{1}{77} \pmod{12}$

$$= (n \times 77) \pmod{13} = 1$$

Here  $n = 1, 2, 3, 4, \dots$

$$= (12 \times 77) \pmod{13} = 1$$

$$= 924 \pmod{13} = 1$$

$$\boxed{M_3^{-1} = 12}$$

$$\begin{array}{r} 12 \overline{) 924} \\ \underline{14} \\ 12 \\ \underline{1} \end{array}$$

$$\begin{array}{r} 12 \overline{) 924} \\ \underline{14} \\ 12 \\ \underline{1} \end{array}$$

Formula:

$$x = (a_1 \times m_1 \times M_1^{-1}) + (a_2 \times m_2 \times M_2^{-1}) + \dots + (a_3 \times m_3 \times M_3^{-1}) \pmod{M}$$

$$= (4 \times 91 \times 4) + (5 \times 143 \times 5) + (6 \times 77 \times 12) \pmod{1001}$$

$$= (1456 + 3575 + 5544) \pmod{1001}$$

$$= 10575 \pmod{1001}$$

$$\boxed{n = 565}$$

Here  $n$  value remains same but  $a_1, a_2, a_3$  & mod values changes.

$$565 \equiv a_1 \pmod{11}$$

$$a_1 = 4$$

$$565 \equiv a_2 \pmod{7}$$

$$a_2 = 5$$

$$565 \equiv a_3 \pmod{13}$$

$$a_3 = 6$$

Hence proved that  $n$  is constant.

$n$  is a Unique solution for all the equations.



b

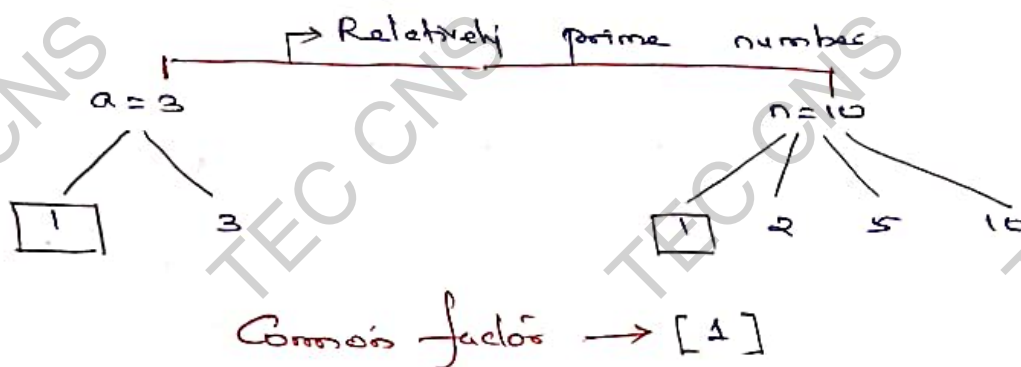
Euler's Theorem :- (Example of Symmetric Key).

For every positive integer 'a' & any number 'n' where it is said to be relatively prime.

Then

$$a^{\phi(n)} \equiv 1 \pmod{n}$$

where  $\gcd(a, n) = 1$



# Chinese Remainder Theorem:-

Q:-

$$\begin{aligned} x &= 2 \pmod{3} \\ x &= 3 \pmod{5} \\ x &= 2 \pmod{7} \end{aligned}$$

Steps:-

- i)  $M = m_1 m_2 m_3$
- ii)  $M_1 = \frac{M}{m_1}, M_2 = \frac{M}{m_2}, M_3 = \frac{M}{m_3}$
- iii)  $M_1^{-1} \pmod{m_1}, \dots$
- iv)  $x = (a_1 m_1 M_1^{-1} + a_2 m_2 M_2^{-1} + \dots)$   
 $x = (a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + \dots)$

Step 1:-

$$\begin{aligned} a_1 &= 2 & a_2 &= 3 & a_3 &= 2 \\ m_1 &= 3 & m_2 &= 5 & m_3 &= 7 \end{aligned}$$

Step 2:-

$$\begin{aligned} M &= m_1 \times m_2 \times m_3 \\ &= 3 \times 5 \times 7 \\ &= 105 \end{aligned}$$

Step 3:-

$$\begin{aligned} M_1 &= \frac{M}{m_1} = \frac{105}{3} = 35, & M_2 &= \frac{M}{m_2} = \frac{105}{5} = 21, & M_3 &= \frac{M}{m_3} = \frac{105}{7} = 15 \end{aligned}$$

Step 4:-

$$M_1^{-1} \pmod{m_1} = (35)^{-1} \pmod{3}$$

$$\begin{aligned} &= 35 \pmod{3} \\ &= 2 \end{aligned}$$

$$M_2^{-1} \pmod{m_2} = (21)^{-1} \pmod{5}$$

$$\begin{aligned} &= (21)^3 \pmod{5} \\ &= 9261 \pmod{5} \\ &= 1 \end{aligned}$$

$$M_3^{-1} \pmod{m_3} = (15)^{-1} \pmod{7}$$

$$\begin{aligned} &= 15^5 \pmod{7} \\ &= 759275 \pmod{7} \\ &= 1 \end{aligned}$$

$$\begin{array}{r} 35 \\ 2 \overline{) 35} \\ \underline{11} \times 3 \\ 33 \\ \underline{35-33} \\ 2 \end{array}$$

5-2=3  
3-2=1

$$\begin{array}{r} 759275 \\ 7 \overline{) 759275} \\ \underline{7} \\ 1004025 \\ \underline{1004025} \\ 0 \end{array}$$

$$x = (a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + a_3 M_3 M_3^{-1}) \bmod M$$

$$= (2 \times 35 \times 2 + 3 \times 25 \times 1 + 2 \times 15 \times 1) \bmod 105$$

$$= (140 + 75 + 30) \bmod 105$$

$$= 233 \bmod 105$$

$$= 23$$

$$\begin{array}{r} 2 \\ 105 \overline{) 233} \\ \underline{210} \\ 23 \end{array}$$

Hence the  $x$  value remain same  
but  $a_1, a_2, a_3$  mod value changes.

$23 \equiv a_1 \bmod 3$
$23 \equiv a_2 \bmod 5$
$23 \equiv a_3 \bmod 7$

$$a_1 = 2$$

$$a_2 = 3$$

$$a_3 = 2$$

Hence proved that  $x$  is constant.

$x$  is a unique solution for all the  
equations.

□

Explain the network security model with neat sketch?

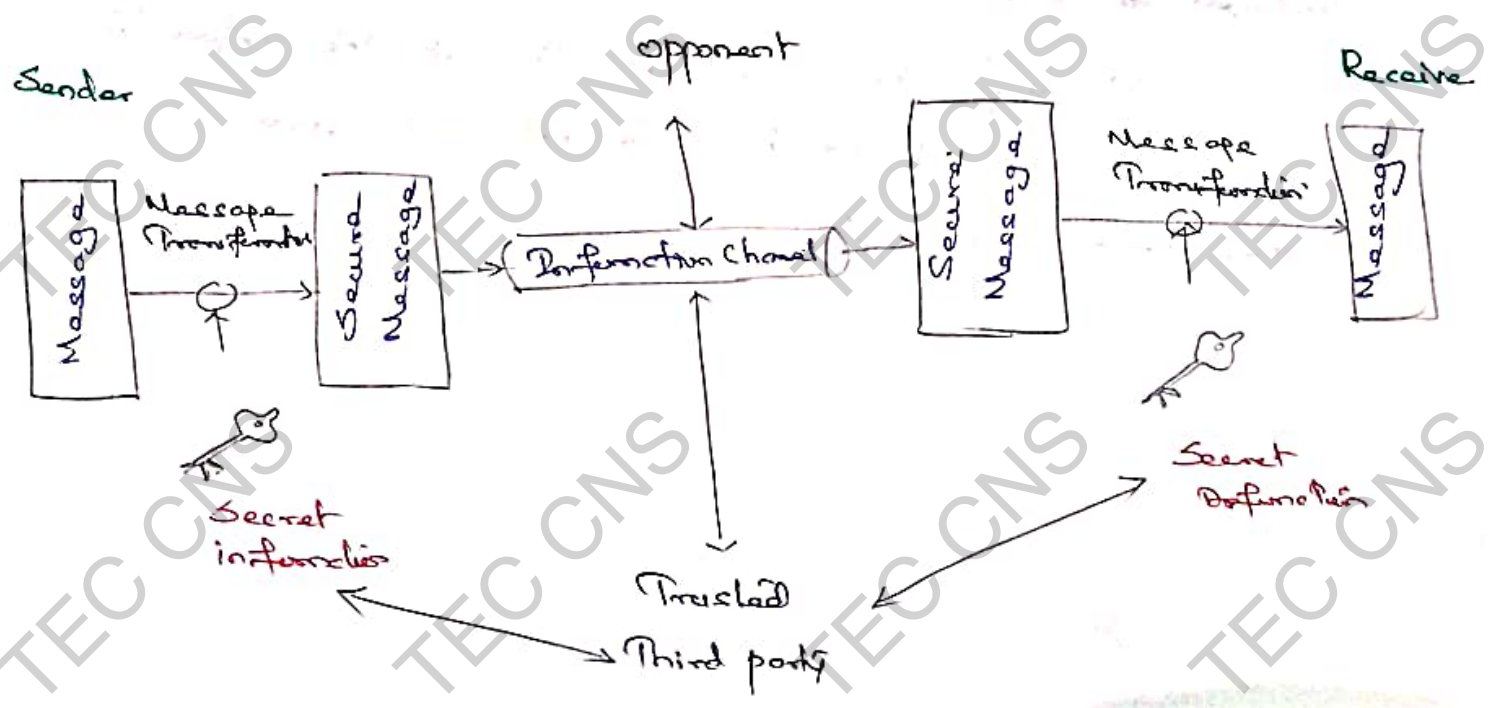
N/w Security Model with neat sketch:

N/w security Model exhibits how the security service has been designed over the net to prevent the opponent from causing a threat to the confidentiality or authenticity of the information that is being transmitted through the net.

1. Confidentiality of the information which has to be sent to the receiver, so that any opponent present at the information channel is unable to read the message.

This involves the encryption of the message.

It involves the addition of code along with the transmission of the information, which will be used to verify the identity of the authentic receiver.





2. Secret information. b/w sender and the receiver.  
of the which the opponent must not any else.

The encryption key which is used during the encryption of the message at the sender's end and also during the decryption of message at receiver's end.

3. Trusted third party: which should let the responsibility of distributing the secret information (key) to both the communicating parties and also prevent it from any opponent.

→ The other security model prevent the two communicating parties Sender and receiver who mutually agrees to exchange information.

→ But sender cannot send the message as the information channel is the readable form as it will have a threat being attacked by the opponent.

→ Sending the message through the information channel, it should be transformed into an unreadable format.

## DES (Data Encryption Standard)

- The Data Encryption Standard (DES) is a Symmetric-Key block cipher.
- DES is published by the National Institute of Standards and Technology (NIST) in the year 1977.
- It is based on the Feistel Structure in which the plaintext is separated into two halves.
- It takes input as 64-bit plaintext and 56-bit key to produce 64-bit ciphertext.
- Before processing, the entire plaintext is separated into two pieces of 32 bits each.
- Same operation is done on each portion.
- Each piece goes through 16 rounds of permutation and then used to obtain the 64-bit ciphertext.

### DES Algorithm Steps:-

- 1) In the first step, the 64-bit plaintext block is handed over to an initial Permutation (IP) function.
- 2) The initial permutation is performed on Plaintext.
- 3) Next, the initial Permutation (IP) produces two halves of the permutation block.

Say: Left Plain Text (LPT 32 bit)

Right Plain Text (RPT 32 bit)



- Test -



Initially, 64 bits, 8 parity bits are to be removed from every 8th position.

64 = (8x8) i.e. 56

Then apply Left Circular shift after dividing 56 bits into 2 parts.

C0 and D0 each having 28 bits.

Initial permutation

Round 1

Round 2

Round 16

Final permutation

Ciphertext

Initial key

PC1

C0 28 bit

D0 28 bit

L0

R0

C1

D1

L2

R2

C2

D2

L4

R4

C16

D16

PC2

PC2

PC2

Parity bit

odd parity

even parity

1 1 0 1 0 1 1 0

0 1 1 0 1 0 1 1

Left Circular shift.

Move the bits based on Round number.

→ For Round 1, 2, 9, 16 — 1 bit shift  
→ other Rounds — 2 bit shift

In PC2,

C1 + C2 are combined to form 56 bits again.

Permutation choice 2 is applied.

56 bit are rearranged, permuted and

48 bits are selected

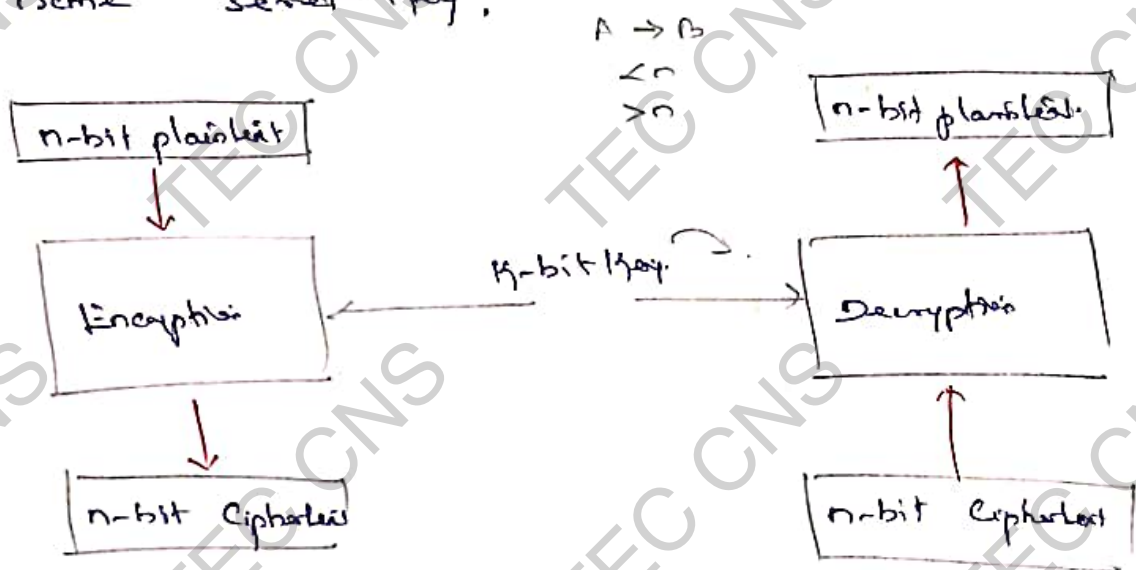
Key for Round 1



## Symmetric-Key Modern Block Cipher:

A Symmetric-Key modern block cipher encrypts an  $n$ -bit block of plaintext or decrypts an  $n$ -bit block of ciphertext.

- \* The encryption or decryption algorithm uses a  $k$ -bit key.
- \* The decryption algorithm must be the inverse of the encryption algorithm, and both operations must use the same secret key.



- \* If the message has fewer than  $n$  bits, padding must be added to make it an  $n$ -bit block.
- \* The message more than  $n$  bits it should be divided into  $n$ -blocks and the appropriate padding must be added to the last block if necessary.
- \* The common values for  $n$  are 64, 128, 256 or 512 bits.

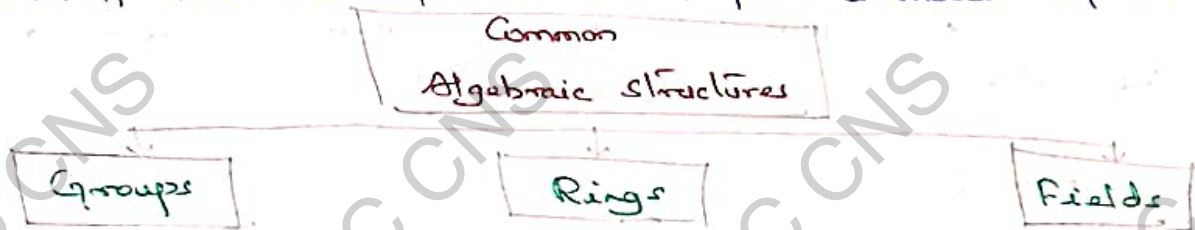
$$|M| + \text{pad} \equiv 0 \pmod{\text{block size.}}$$

## Algebraic structures :-

Cryptography requires set of integers and specific operations that are defined for those sets.

The combination of the set and the operations that are applied to the elements of the set is called an algebraic structure.

Groups, rings, and fields are the fundamental elements of a branch of mathematics known as abstract algebra or modern algebra.



### Groups:-

A group  $G$ , denoted by  $\{G, *\}$  is a set of elements with a binary operation denoted by  $*$  that associates to each ordered pair  $(a, b)$  of elements in  $G$  an element  $(a \cdot b)$  in  $G$ , such that the following axioms are satisfied.

i) Closure :- if  $a$  and  $b \in G$  then

$$a \cdot b \in G$$

ii) Associative :-

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \quad \forall a, b, c \in G$$

iii) Identity element :-

There is an element  $e$  in  $G$ , such that

$$a \cdot e = e \cdot a = a \quad \forall a \in G$$

$$a + 0 = 0 + a = a \quad \forall a \in G$$

iv) Inverse element :-

$$a \cdot a' = a' \cdot a = e$$

For each  $a$  in  $G$ , there is an element

$a'$  in  $G$ , such that

$$a \cdot a' = a' \cdot a = e$$

v) Commutative :-  $a \cdot b = b \cdot a$  for all  $a, b$  in  $G$ .

If a group has a finite no. of elements it is referred to as a finite group and the order of the group is equal to the number of elements in the group. Otherwise the group is an infinite group.

A group is said to be abelian, if it satisfies the following additional condition

v) Commutative :-

$$a \cdot b = b \cdot a \quad \forall a, b \text{ in } G$$

### Cyclic Groups :-

A Group  $G$  is Cyclic if every element of  $G$  is a power  $a^k$  ( $k$  is an integer) of a fixed element  $a \in G$ .

The element  $a$  is said to generate the group  $G$  or to be a generator of  $G$ .

A Cyclic Group is always abelian and may be finite or infinite.

### Cyclic Subgroup :-

If a Subgroup of a group can be generated using the power of an element, the subgroup is called the Cyclic Subgroup.

$$a^n \rightarrow a \cdot a \cdot a \cdot \dots \cdot a \quad (n \text{ times})$$

### Example : Cyclic Groups :-

✓ The group  $G = (\mathbb{Z}_6, +)$  is a Cyclic group with two generators  $g=1$  and  $g=5$

✓ The group  $G = (\mathbb{Z}_6, *)$  is a Cyclic group with two generators  $g=3$  and  $g=7$

### Lagrange's Theorem :-

Assume that  $G$  is a group, and  $H$  is a subgroup of  $G$ .

If the order of  $G$  and  $H$  are  $|G|$  and  $|H|$ , resp/- then based on this theorem

$$\frac{o(G)}{o(H)}$$



### Order of an Element:-

The order of an element is the order of the cyclic group it generates.

Ex:-

In the group  $G = \langle \mathbb{Z}_6, + \rangle$  the order of the elements are

$$\text{ord}(0) = 1$$

$$\text{ord}(1) = 6$$

$$\text{ord}(2) = 3$$

$$\text{ord}(3) = 2$$

$$\text{ord}(4) = 3$$

$$\text{ord}(5) = 6$$

### Rings:-

A ring  $R$ , sometimes denoted by  $(R, +, \times)$  is a set of elements with two binary operations called addition and multiplication, such that for all  $a, b, c$  in  $R$  the following axioms are satisfied

$\leftarrow$  (i) (ii) (iii) (iv) (v)

$R$  is an abelian group with respect to addition

#### (vi) closure under multiplication:

$$a, b \in R$$

then

$$ab \in R$$

#### (vii) Associativity of Multiplication:-

$$a(bc) = (ab)c \quad \forall a, b, c \in R$$

#### (viii) Distributive laws:

$$(a+b)c = ac + bc \quad \forall a, b, c \in R$$

$$a(b+c) = ab + ac \quad \forall a, b, c \in R$$



A ring is said to be commutative if it satisfies following additional condition:

i) Commutativity of Multiplication:

$$ab = ba \quad \forall a, b \in R$$

ii) Multiplicative identity:

$$1 \in R \text{ such that } a1 = 1a = a \quad \forall a \in R$$

iii) No Zero divisors:-

$$\begin{aligned} a, b \in R \text{ and } ab &= 0 \\ \text{Then} \\ a &= 0 \text{ or } b = 0 \end{aligned}$$

Fields:

A field  $F$ , sometimes denoted by  $(F, +, \times)$  is a set of elements with two binary operations called addition and multiplication such that for all  $a, b, c \in F$  the following axioms are obeyed

i) - ii)

iii) Multiplicative inverse:-

$$\begin{aligned} a \in F, \text{ except } 0 \\ a^{-1} \in F \text{ then} \\ aa^{-1} = a^{-1}a = 1 \end{aligned}$$

Permutation Groups:-

A permutation of a set  $A$  is a function from  $A$  to  $A$  that is both 1-1 and onto.

Q6: Find the 8-bit word related to the polynomial  $x^6 + x^3 + x$

$x^7$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$	$x^1$	$x^0$
0	1	0	0	1	0	1	0

This is related to the 8-bit word 01001010

Note: Polynomial representing n-bit words use two fields.

$GF(2)$  and  $GF(2^n)$

Qn: Figure show how we can represent the 8-bit word

(10011001) using a polynomial.

n-bit word 1 0 0 1 1 0 0 1

Polynomial

$1x^7 + 0x^6 + 0x^5 + 1x^4 + 1x^3 + 0x^2 + 0x^1 + 1x^0$

First Simplification

$$1x^7 + 1x^4 + 1x^3 + 1x^0$$

$$x^0 = 1$$

Second Simplification

$$x^7 + x^4 + x^3 + 1$$

Points:

\* A prime polynomial cannot be factored into a polynomial with degree of less than n. Such polynomials are referred to as irreducible polynomials.

Q:

$(x+1)$  - degree 1

$(x^2+x+1)$  - degree 2

$(x^3+x^2+1)$  - degree 3

$(x^4+x^3+x^2+x+1)$  - degree 4

} Irreducible polynomials.

Galois Field (GF)

Addition operation on polynomials are the same operation

Qn:  $(x^2 + x^2) \bmod 2$   $2^0$   $1 \bmod 2$

$= 2x^2$  (Coefficients belong to GF(2))

$= 2x^2 \bmod 2$

$= [(2 \bmod 2) \times (x^2 \bmod 2)] \bmod 2$

$= [0 \times (x^2 \bmod 2)] \bmod 2$

Subtraction:

Qn:  $(x^2 - x^2) \bmod 2$

$= (x^2 - x^2) \bmod 2$

$= 0 \bmod 2$

$= 0$

$\therefore (x^2 + x^2) = (x^2 - x^2) = 0$

Remember this

2  $\begin{array}{r} 2 \\ 5 \\ \hline 0 \end{array}$  remainder

XOR operation

x	y	z
0	0	0
0	1	1
1	0	1
1	1	0

Example:- Solve  $(x^5 + x^2 + x) \otimes (x^3 + x^2 + 1)$  in  $GF(2^8)$

Let us solve the above.

$\otimes$  - Symbol is mean polynomial addition.

For the given  $GF(2^8)$

$x^5 + x^2 + x = 0x^7 + 0x^6 + 1x^5 + 0x^4 + 0x^3 + 1x^2 + 1x^1 + 0x^0$

$x^3 + x^2 + 1 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 1x^3 + 1x^2 + 0x^1 + 1x^0$

Mean polynomial addition  $\otimes$

---

$0x^7 + 0x^6 + 1x^5 + 0x^4 + 1x^3 + 0x^2 + 1x^1 + 1x^0$

---

Thus given as,

$x^5 + x^3 + x + 1$

Note:-

- 0 XOR 0 = 0 (Same)
- 0 XOR 1 = 1 (Different)
- 1 XOR 0 = 1 (Different)
- 1 XOR 1 = 0 (Same)

## Encryption:

$$\begin{array}{r} 00110101 \text{ Plain Text} \\ 11100011 \text{ Secret Key} \\ \hline 11010110 \text{ Cipher Text} \end{array}$$

## Decryption:

$$\begin{array}{r} 11010110 \text{ Cipher Text} \\ 11100011 \text{ Secret Key} \\ \hline 00110101 \text{ Plain Text} \end{array}$$

XOR

The addition in  $GF(2)$  means the exclusive-or (XOR) operation. So we can exclusive-or the two words, bit by bit, to get the result.

In the example:

$$\begin{array}{r} x^5 + x^2 + x \text{ is } 00100110 \\ x^3 + x^2 + 1 \text{ is } 00001101 \\ \hline \text{Result is: } 00101011 \end{array}$$

The polynomial notation:  $x^5 + x^2 + x + 1$

Qn: Let us define a  $GF(2^2)$  field in which the set has four 2-bit words.

$$\{00, 01, 10, 11\}$$

We can redefine addition and multiplication for this field in such a way that all properties of these operations are satisfied.



$$GF(2^2) = \langle \{00, 01, 10, 11\}, \oplus, \otimes \rangle$$

Addition:

$\oplus$	00	01	10	11
00	00	01	10	11
01	01	00	11	10
10	10	11	00	01
11	11	10	01	00

$$\begin{array}{r} 10 \\ \oplus 00 \\ \hline 10 \end{array} \quad \begin{array}{r} 10 \\ \oplus 01 \\ \hline 11 \end{array} \quad \begin{array}{r} 10 \\ \oplus 10 \\ \hline 00 \end{array} \quad \begin{array}{r} 10 \\ \oplus 11 \\ \hline 01 \end{array}$$

Multiplication:

$\otimes$	00	01	10	11
00	00	00	00	00
01	00	01	10	11
10	00	10	11	01

The set is  $\{00, 01, 10, 11\}$   
Corresponding  
polynomial  
are:-

$$\begin{array}{l} 0x^1 + 1x^0 \\ 0x^1 + 0x^0 \\ 1x^1 + 0x^0 \\ 1x^1 + 1x^0 \end{array} \Rightarrow \begin{array}{l} 0 \\ 0 \\ x \\ x+1 \end{array}$$

Qn. Generate 16 elements of the field  $GF(2^4)$  using the irreducible polynomial  $f(x) = x^4 + x + 1$ .

The elements  $0, g^0, g^1, g^2$  and  $g^3$  can be easily generated, because they are the first repn of  $0, 1, x^1$  &  $x^3$

$$\begin{aligned} 0 &= 0000 \\ g^0 &= 0001 \\ g^1 &= 0010 \\ g^2 &= 0100 \\ g^3 &= 1000 \end{aligned}$$

$$g^3 \quad g^2 \quad g^1 \quad g^0$$

## Block Cipher modes of operation (DES).

1. Electronic Codebook (ECB)
2. Cipher Block chaining (CBC)
3. Cipher Feedback (CFB)
4. Output Feedback (OFB)

### Block Cipher

- \* We need a mode of operation
- \* Fixed-length block.
- \*  $b$  bits input and  $b$  bits output  $\langle n \rangle$
- \* If the amount of PT is the encrypted in  $> b$  bits
- \* Breaking the plaintext into  $b$  bits in each block.
- \* 5 modes of operation defined by NIST.
- \* Different applications - Different modes of operation.

### Modes of operation.

1- 4 + 1 Counter Mode.

#### ECB:-

Deterministic  
plaintext block  $P_1, P_2, \dots, P_m$  are  
encrypted twice under the  
same key.  
Partially - Initial and error

CBC:- Local n-bit data.  
Chain format.

XOR

Initialization Vector.

CFB:- Ciphertext into stream operation

OFB:- None

Block cipher  
- A single message  
- Encrypted  
- Multiple blocks  
- Can be decrypted

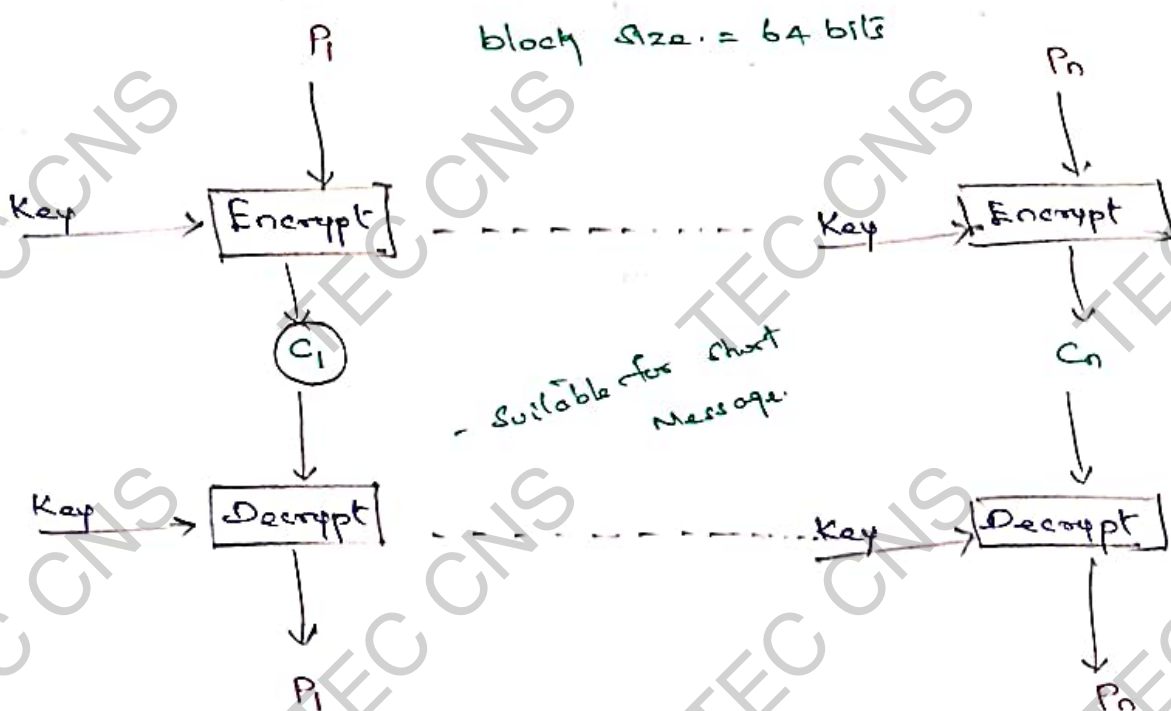
Security  
issue  
arise  
multiple  
block  
as  
encrypted

## Block Cipher Modes of operation:

There are 5 modes of operation:

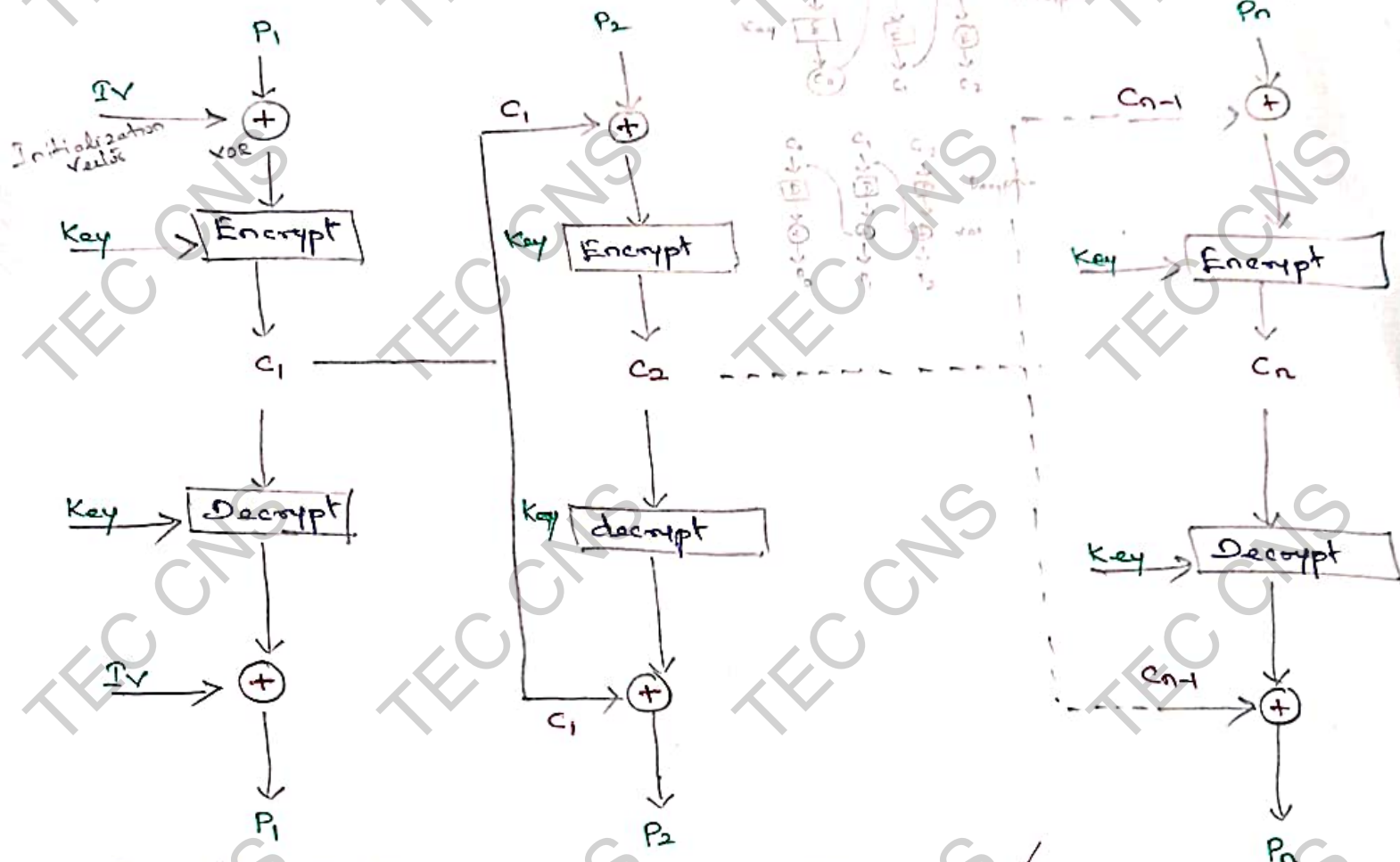
1. Electronic Code Book (ECB)
2. Cipher Block Chaining (CBC)
3. Cipher Feedback Mode (CFB)
4. Output Feedback Mode (OFB)
5. Counter Mode (CTR)

### (1) Electronic Code Book :: (ECB)



\* Each block of 64-bit plaintext is encoded independently using the same key

## (ii) Cipher block chaining (CBC)

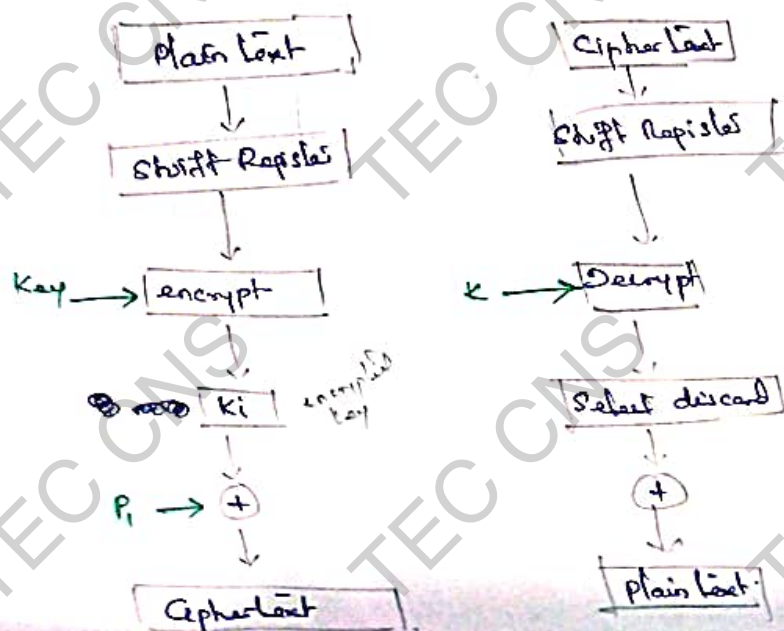


\* The i/p to the encryption algorithm is the XOR of the next 64-bits of plaintext and preceding 64-bits of ciphertext

## (iii) Cipher Feedback Mode (CFB)

I/p is processed  $j$  bits at a time. Preceding ciphertext is used as i/p to the encryption algorithm to produce pseudorandom o/p. which is XOR with plaintext to produce next unit of ciphertext.

Encryption performed is not parallel.





### iii. Output Feedback Mode (OFB)

- It is similar to CFB, except that it is the encryption algorithm is the preceding DES output.
- Key stream is generated independently of the plaintext.

Parallel encryption and decryption is advantage.

Turns a block cipher into a synchronous stream cipher.

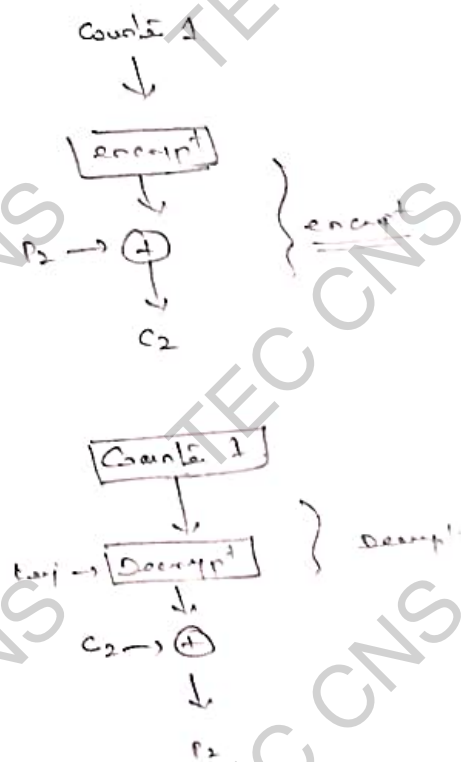
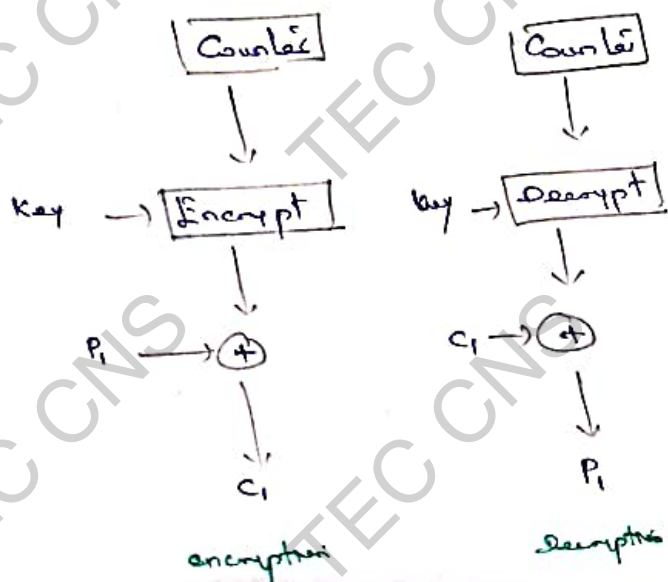
### Stream Cipher:

- plaintext: binary string
- Key stream: a pseudorandom bit string.
- Ciphertext: bit-wise XOR (addition mod 2) of plaintext and key stream.
- Decryption: bit-wise XOR of ciphertext and key stream.

Ex: - P: 10001101010111101101 C: 110001111000110110110  
K: 0100101011010011001101 where  $E = P \oplus K$  (encryption),  
 $P = E \oplus K$  (decryption).

### iv. Counter mode:

- Turns a block cipher into a stream cipher by encrypting successive values of a counter.
- Allows for parallel encryption and decryption.
- No need for padding.



## Ingredients Symmetric Cipher Model

There are 5 ingredients

- a) Plain Text : Original message to be communicated between sender and receiver
- b) Cipher text : encoded format of the original message that cannot be understood by humans.
- c) Secret Key :-  
(K) It is a value/string/text file used by the encryption and decryption algorithm to encode + decode.
- d) Encryption (or Enciphering):

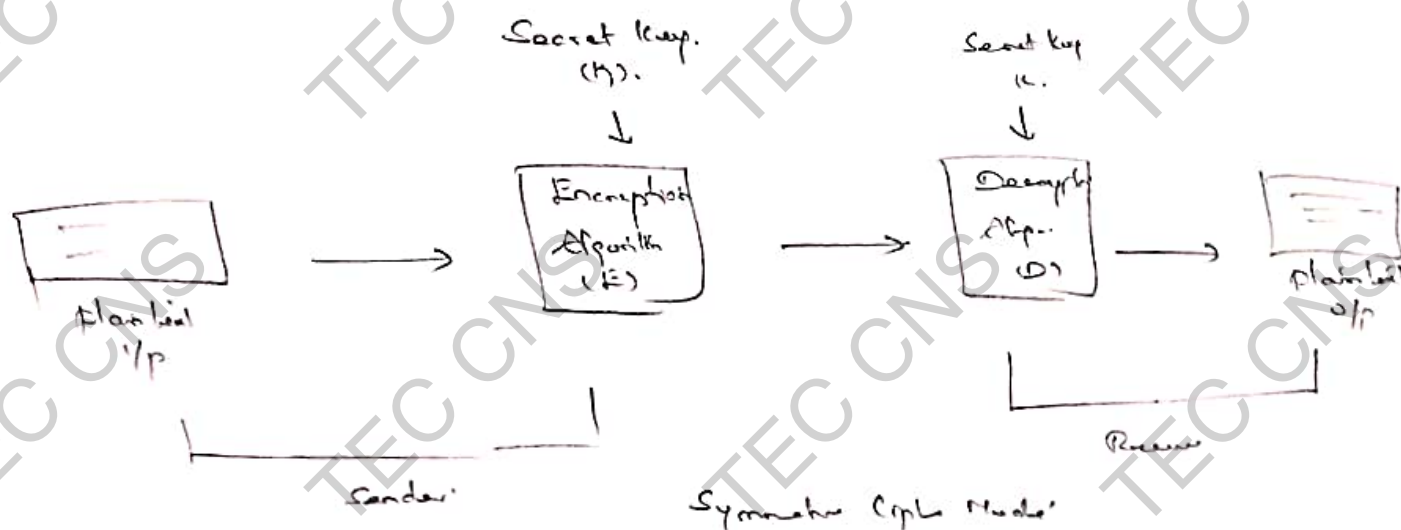
The Conversion of Plain text to Cipher text

$$E(K, X) = Y$$

- e) Decryption (or Deciphering):

The Conversion of Cipher text to plain text, i.e. reverse of encryption

$$D(Y, K) = X$$



Comparison b/w public key and private key algorithms:

Symmetric Key  
Cryptography.  
(private key).

1. Same key is used for encryption and decryption.
2. Very fast.
3. Key exchange is a big problem.
4. Also called.  
Secret key encryption.
5. The key must be kept secret.
6. Cannot be used for digital signature.

Asymmetric Key  
Cryptography  
(private key).

1. One key for encryption and another key for decryption.
2. Slower.
3. Key exchange is not a problem.
4. Also called.  
public key encryption.
5. One of the two keys must be kept secret.
6. Can be used for digital signature.

# AES

## Advanced Encryption Standard :-

### Introduction :-

- \* AES is Symmetric Key Cryptographic algorithm published by NIST.
- \* The algorithm was proposed by Rijndael. It is also known as Rijndael encryption algorithm.
- \* AES is replacement of DES.
- \* AES works on block cipher technique. Size of plain text and cipher text must be same.
- \* An i/p key is also required to the AES algorithm. Same size of plain text.
- \* In AES, the data length (Plain text size) of.
  - 128 bits - 10 rounds
  - 192 bits - 12 rounds
  - &
  - 256 bits - 14 roundsand supporting three different
- \* Key lengths
  - 128 bits
  - 192 bits
  - and
  - 256 bits
- \* AES consist of Multiple rounds of processing different key bit like.



10 rounds for processing

128-bit key.

12 rounds

192-bit key.

and,

14 rounds.

256-bit key

\* Encrypts data in blocks of 128 bits each.

\* It takes 128 bits as input and outputs 128 bits of encrypted cipher text as output.

\* It is performed using a series of linked operations which involves replacing and shuffling of the input data.

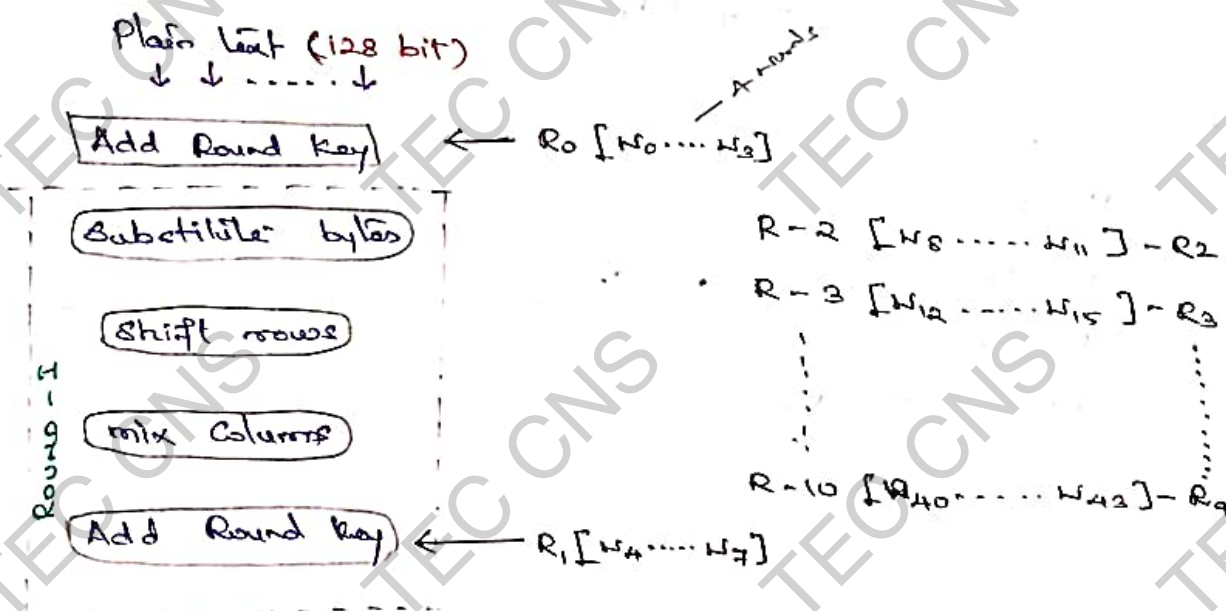
(Qn.) Difference between DES and AES algorithm:

DES	AES
1. Used to encrypt plain text of 64-bit	1. Used to encrypt plain text of 128-bit
2. The key is of 56-bit size	2. The key is of different sizes such as 128-bits, 192-bits and so on.
3. Less secure than AES	3. More secure than DES
4. It can be broken by brute force attacks	4. No data, AES has not been attacked
5. It is based on FEISTEL NETWORK	5. It is based on permutation and substitution network.

# ADVANCED ENCRYPTION STANDARD (AES)

## Overview:-

Block Size	-	128 bit Plain Text	Each = 1 byte
No: of Rounds	-	10 Rounds	
Key Size	-	128 bit (4 words / 16 Bytes)	
No: of Subkeys	-	44 Subkeys (44 words)	Subkey = 32 bit (1 word)
Each Subkey Size	-	32 bit / 1 word / 4 Bytes	
Each Round	-	4 Subkeys (128 bit / 4 words / 16 Bytes)	
Pre Round Calculation	-	4 Subkeys (128 bit / 4 words / 16 Bytes)	
Cipher Text	-	128 bit (4 words / 16 bytes)	
1 Subkey	-	32 bit	
4 bytes	-	32 bit [∵ 1 byte = 8 bit]	



1. Symmetric Cipher
2. Block Cipher
3. Plaintext - 128 bits / 16 bytes

Key 128/192/256

1 byte = 8 bits

$$\frac{16 \times 8}{128}$$

16 steps only  
not a table  
Need in relation to

Input:- array 4x4

$In_0$	$In_4$	$In_8$	$In_{12}$
$In_1$	$In_5$	$In_9$	$In_{13}$
$In_2$	$In_6$	$In_{10}$	$In_{14}$
$In_3$	$In_7$	$In_{11}$	$In_{15}$

each cell = 8 bits / 2 bytes  
Total = 16 cells

16 bytes  
=  $16 \times 8$   
= 128 bit  
= 4 words (32 each)

PT is represented in the i/p array

State:- array

(or) Intermediate results

word number

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

bits

Used in State & Intermediate States within the Rounds

Hex 00 11 22 33 44  
or  
44 33 22 11

16 bytes → 4 words  
Each word

Output:- array

$Out_0$	$Out_4$	$Out_8$	$Out_{12}$
$Out_1$	$Out_5$	$Out_9$	$Out_{13}$
$Out_2$	$Out_6$	$Out_{10}$	$Out_{14}$
$Out_3$	$Out_7$	$Out_{11}$	$Out_{15}$

Key:- array

$K_0$	$K_4$	$K_8$	$K_{12}$
$K_1$	$K_5$	$K_9$	$K_{13}$
$K_2$	$K_6$	$K_{10}$	$K_{14}$
$K_3$	$K_7$	$K_{11}$	$K_{15}$



$K_0$	$K_1$	$K_2$	...	$K_{15}$
-------	-------	-------	-----	----------

1 column (4 words) (64 bit)  
128 bit (4 words)

Actually, 4 words  
They are expanded into 44 words.  
each Round = 4 words  
= 4 words x 10 Rounds  
= 40 + 4 (for Add Round key)  
= 44 words

Initial Step

1. Preparation
2. Substitution
3. Shift Rows
4. New Column - Add Round key
5. Add key

# Substitution bytes:-

S-box

8 bits (each cell size).

Ex:-

0000 0001  
First 4 bits Last 4 bits

First Four bit  $\rightarrow$  Row number  $\rightarrow 0-15$

Next four bit  $\rightarrow$  Column number  $\rightarrow 0-15$

Size of Substitution array is  $16 \times 16$  [256 bytes].

256 Values

0000  $\rightarrow$  row = 0

0001  $\rightarrow$  Column = 1

$\Rightarrow 7C$  [From Table]

$\Rightarrow 0111 1100$

2 17  
 2 12  
 2 10  
 2 10  
 1 1 1100<sub>2</sub>

[ $7C = 12$ ]

## Shift Rows:-

Row 0  $\rightarrow$  0 bit shifted

Row 1  $\rightarrow$  1 bit shifted

Row 2  $\rightarrow$  2 bits shifted

Row 3  $\rightarrow$  3 bits shifted

[Circular right shift]

0 1 2 3  
 4 5 6 7

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,1}$	$S_{1,2}$	$S_{1,3}$	$S_{1,0}$
$S_{2,2}$	$S_{2,3}$	$S_{2,0}$	$S_{2,1}$
$S_{3,3}$	$S_{3,0}$	$S_{3,1}$	$S_{3,2}$



## Mix Columns:-

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

Inbuild  
Matrix  
Table:

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Column order.

$S'_{0,0}$	$S'_{0,1}$	$S'_{0,2}$	$S'_{0,3}$
$S'_{1,0}$	$S'_{1,1}$	$S'_{1,2}$	$S'_{1,3}$
$S'_{2,0}$	$S'_{2,1}$	$S'_{2,2}$	$S'_{2,3}$
$S'_{3,0}$	$S'_{3,1}$	$S'_{3,2}$	$S'_{3,3}$

## Add round key:

O/p from mix Column @ Key (4 words).

$$\begin{aligned} &R[0,3] \\ &R-1 \quad R[4,7] \\ &\vdots \\ &R-9 \quad R[40,43]. \end{aligned}$$

Total  
44 words.

Cipher text generated.