

## Derivation of Transfer Function Model based on Miniaturized Cryocooler Behavior

**Jiten H. Bhatt<sup>1</sup>, Shail J. Dave<sup>2</sup>, Manish M. Mehta<sup>3</sup>, Nitin Upadhyay<sup>3</sup>**

<sup>1</sup>Scientist/Engineer, SAC, ISRO, PhD Student, Nirma University, Ahmadabad, INDIA

<sup>2</sup>L.D. College of Engineering, Ahmadabad, INDIA

<sup>3</sup>Scientist/Engineer, SAC, ISRO, Ahmadabad, INDIA

---

### ABSTRACT

Infrared detectors are widely used in Space Borne Remote sensing satellites. These detectors are often operated at cryogenic temperatures to enhance their noise performance. In past cryogenic cooling of these detectors was achieved using passive cooling techniques. With advent of miniaturized long life active Stirling cycle cryocoolers, it has now become possible to use large area array detectors with enhanced spatial and spectral resolution. The control electronics for driving these Stirling cycle Cryocoolers is complex involves multi-disciplinary engineering. Often PID control is employed in these control electronics for cryocoolers. The transfer function of the cryocoolers is often not known and hence the tuning of the control loop is difficult and involves manual tuning and multiple iterations. It is however not advisable to have multiple tuning operations on flight cryocooler, since the life of the cryocooler is limited and every hour saved in ground testing adds to the mission life of the satellite. This paper describes the development of transfer function based model of the Stirling cycle based Cryocooler. The paper describes how to derive the transfer function of the cryocooler system based on the trial run with arbitrary PID coefficients. Here the cryocooler is modeled as a second order system, whose parameters are derived from the peak time and percentage overshoot of the system response. Subsequently the model is simulated and the system response is tuned using manual and/or software tuning techniques. After getting satisfactory performance in simulation, the tuned coefficients are used for actual testing. The comparison of the simulated results with the actual results is presented in this paper. Close matching between simulated and measurement results is obtained. The control electronics for Stirling cycle based cryocoolers can thus be tuned in very fast and efficient manner with minimum tuning iterations on hardware.

**Keywords:** Cryocoolers, PID, Transfer Function, Infrared Detectors

---

### INTRODUCTION

Miniaturized Stirling cycle cryocoolers are used for cooling of Infrared detectors. The control electronics for driving these cryocoolers is tasked with the tasks of providing the drive signal to cryocooler, controlling the cold tip temperature, providing health parameters through telemetry and ensuring the system safety against various anomalies such as over current, under voltage, over temperature etc. The control electronics must control the cold tip temperature of the cryocooler. The tuning of the controller is often a challenging task especially since the model of the cryocooler is not available. The paper describes how a simple transfer

function based model is sufficient to simulate the entire control loop and predict the system performance for PID parameters. The cryocooler system is assumed to be a second order system with large thermal inertia. In this method, initially arbitrary values of Proportional Gain (Kp) and Integral Gain (Ki) are selected. From the system response, the model of the cryocooler system using the Percentage Peak overshoot and the Peak time for the first overshoot is developed. The second order system model derived from these parameters is then used to simulate the system response against new values of Kp and Ki. Different PID tuning techniques can be employed and the system response

can be simulated in software tools such as MATLAB. Once the values of Kp and Ki are finalized based on the desired system response in simulation, the system can be retested with new values of Kp and Ki. The experimental results have shown close matching between the simulation and achieved results. Thus it is possible to have desired system response with minimum hardware iterations. This feature is particularly desired for limited life cryocoolers where every hour saved in hardware testing on ground will lead to additional life of the cryocooler and hence onboard payload of the satellite. The experimental results have shown that inspite of having a sampled system and a digital PID controller, our modeling was accurate and results were matching within +/- 0.5%.

### Block diagram of cryocooler control electronics

Figure 1 shows the simplified block diagram of Stirling cycle Cryocooler Control Electronics. The cold tip temperature is sensed by a temperature sensor, which is often a diode sensor. The sensor instrumentation circuit provides the biasing for the sensor and amplifies the sensor signal. Multiplexer and ADC are used to digitize temperature, voltage and current information. The heart of the circuit is Digital PID Controller, which provides the control signals to various blocks and provides the Pulse Width Modulated (PWM) drive signal to the Power Amplifier. In past analog PID controllers have been used for controlling the cold tip temperature. But with presently available state of art digital Field Programmable Gate Array (FPGA), it is feasible to accommodate large floating point multipliers required for digital PID Control. Hence digital PID control has been selected in order to have flexible on board programmability through tele-command. The cryocooler health is also measured remotely at the ground through Telemetry signals. Class D based topologies have been used for design of power amplifier which can provide high power (upto 80W) drive signals to the cryocooler through output LC filters. The linear drive Stirling cryocoolers need a AC signal drive, for which Direct Digital Synthesis (DDS) technique is used in Digital Controller. In this technique the sine wave coefficients stored in a ROM are used to generate a digital sine signal which is then transmitted to the power amplifier using PWM technique. The thermal

engineering for ensuring satisfactory thermal performance of the high power dissipating components of the power amplifier is also very challenging.

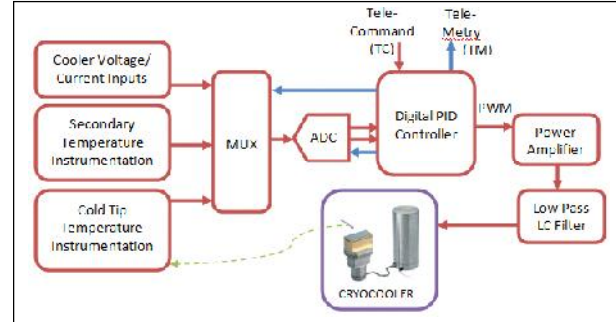


Fig. 1. Block Diagram of Cryocooler Control Electronics

### Modeling stirling cycle cryocooler

The complex thermal model of the cryocooler is generally not available and is not essential for the control loop design. The transfer function model itself is sufficient for the design of control loop, since the cryocooler system behavior can assumed to be second order system. In addition, since the cryocooler has high thermal inertia, often Proportional Integral (PI) type of control is sufficient for achieving satisfactory system response. The PID type of control is generally not necessary for the cryocooler system because the system itself is slow with limited bandwidth and it does not allow for very high speed disturbances in the system. Hence the PI control loop has been implemented for Stirling cycle cryocoolers.

The complete cryocooler system as shown in Fig. 1 can be modeled as second order system shown in Fig. 2. The open loop and closed loop transfer function of this system are given by the following equations.

$$\frac{C(s)}{R(s)} = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \quad (\text{closed loop}) \quad ..(1)$$

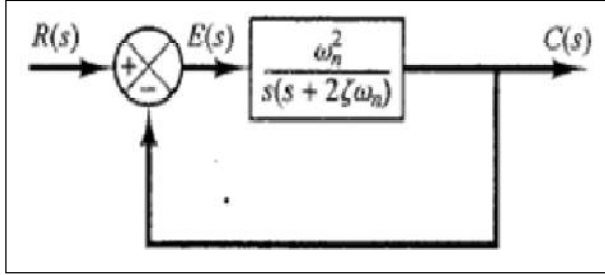
$$G(s) = \frac{C(s)}{E(s)} = \frac{w_n^2}{s(s + 2\zeta w_n)} \quad (\text{open loop}) \quad ..(2)$$

The Proportional Gain (Kp) and the Integral Gain (Ki) are so set in the first iteration of the control loop of the cryocooler system, so as to have an under damped system response. The damping factor  $\zeta$  and natural frequency of oscillation  $W_n$  of this second order system can be derived from the Percentage Peak

Overshoot ( $M_p$ ) and Peak Time ( $t_p$ ) using following equations.

$$\%M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 \quad ..(3)$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \quad ..(4)$$



**Fig. 2. Equivalent Block Diagram of Cryocooler System**

The open loop system transfer function obtained from equation (2) by this method can now be used for system modeling and PI parameter tuning. The overall open loop system transfer function can be considered to be composed of two components – First component is the Plant and Second component is PI Gain. Hence we can write as

$$G(s) = T.F_{total} = T.F_{plant} \times T.F_{PI}$$

$$\text{i.e. } T.F_{plant} = T.F_{total} / T.F_{PI} \quad ..(5)$$

Now, transfer function of PI control can be calculated as

$$\begin{aligned} T.F_{PI} &= K_p + \frac{K_i}{s} \\ &= \frac{K_p s + K_i}{s} \end{aligned}$$

So, transfer function of the plant can be calculated as

$$\begin{aligned} T.F_{plant} &= \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \times \frac{s}{K_p s + K_i} \\ &= \frac{\omega_n^2}{(s + 2\zeta\omega_n)(K_p s + K_i)} \quad (\text{open loop}) \quad ..(6) \end{aligned}$$

If the system uses PID instead of PI controller then, PID and plant transfer functions are

$$\begin{aligned} T.F_{PID} &= K_p + \frac{K_i}{s} + K_d s \\ &= \frac{K_d s^2 + K_p s + K_i}{s} \end{aligned}$$

$$\begin{aligned} T.F_{plant} &= \frac{\omega_n^2}{(s + 2\zeta\omega_n)(K_d s^2 + K_p s + K_i)} \quad (\text{open loop}) \quad ..(7) \end{aligned}$$

Here,  $K_p$ ,  $K_i$  and  $K_d$  are gains which were used in the first iteration for developing the model from corresponding response (with PI or PID closed loop control action).

Now, using this plant transfer function we can determine closed loop PI control response for unit step input through software tools such as MATLAB. Using this plant transfer function, step response with new gains of PI control for tuning purpose can be calculated as

$$\begin{aligned} G'(s) &= T.F_{system} \quad (\text{open loop}) \\ G'(s) &= \frac{\omega_n^2}{(s + 2\zeta\omega_n)(K_p s + K_i)} \\ &\times \frac{K_{p(new)} s + K_{i(new)}}{s} \quad (\text{open loop}) \quad ..(8) \end{aligned}$$

For PID control it will be

$$\begin{aligned} G'(s) &= \frac{\omega_n^2}{(s + 2\zeta\omega_n)(K_d s^2 + K_p s + K_i)} \\ &\times \frac{K_{d(new)} s^2 + K_{p(new)} s + K_{i(new)}}{s} \quad (\text{open loop}) \quad ..(9) \end{aligned}$$

This  $G'(s)$  should be used with unity negative feedback to yield the overall closed loop transfer function. Using this model the new system response for the new gains  $K_{p(new)}$ ,  $K_{i(new)}$  and  $K_{d(new)}$  can be obtained in software tools such as MATLAB. The response can be generated using step command by determining closed loop transfer function with open loop transfer function  $G'(s)$  and unity feedback. It will be

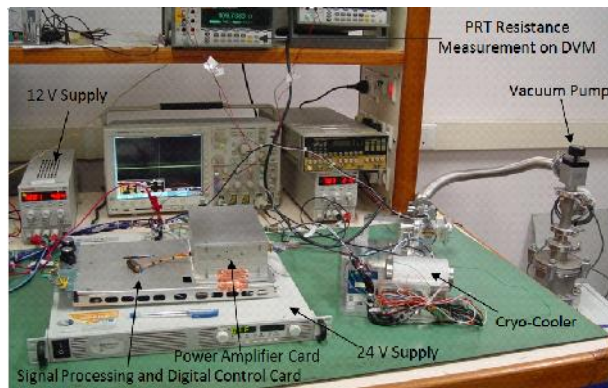
$$T.F_{overall} = \frac{G'(s)}{1 + G'(s)} \quad (\text{closed loop}) \quad ..(10)$$

Thus we now have a complete model of the system and we can simulate the system response for different values of new gains  $K_p(\text{new})$ ,  $K_i(\text{new})$  and  $K_d(\text{new})$ . In this manner the desired system response can be tuned in software, without the need for having multiple hardware iterations.

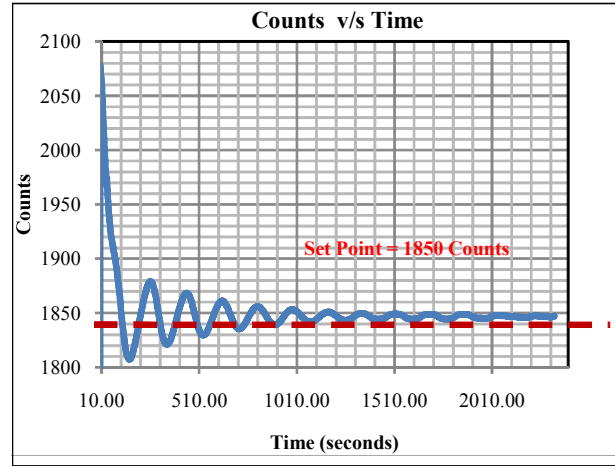
### Test Results

Figure 3 shows the complete cryocooler test setup and its control electronics. The control electronics consists of a Class D based Power amplifier, Instrumentation circuit based on 12 bit ADC and Digital PID controller based on Xilinx FPGA. The cold tip of cryocooler is cooled to cryogenic temperature (150K) using PI control loop. A vacuum pump is used to keep the cold finger in vacuum to avoid condensation of surrounding vapour on the cold tip. Two sensors are mounted on the cold tip. First sensor is diode temperature sensor, whose output is digitized and given to the digital PI controller. The second sensor is a PT100 sensor, whose resistance is measured separately on Digital Multi Meter (DMM) and is used for calibration of diode sensor. Two separate supplies are used – 12V supply is used for Digital PI controller and ADC instrumentation circuit. 24V supply is used for the Class D power amplifier.

The first iteration of the PI control loop was done with  $K_p=4$  and  $K_i=0.25$ . The decimal value was implemented in Digital PI controller using fixed point arithmetic (Multiplying by 1 and division by 4). The results of the first iteration are shown in Figure 4. From this graph the Percentage Overshoot and Peak time were computed



**Fig. 3. Cryocooler Test Set up and its Control Electronics**



**Fig. 4. First Iteration Closed Loop Test Results with  $K_p=4$  &  $K_i=0.25$**

and the transfer function of the system was derived based on procedure described above.

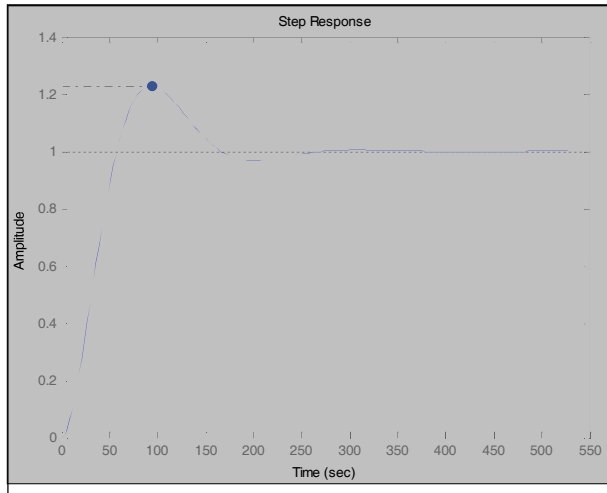
The plant transfer function derived from the first iteration is

$$T.F._{plant} = \frac{0.000788}{4s^2 + 0.3089s + 0.004467}$$

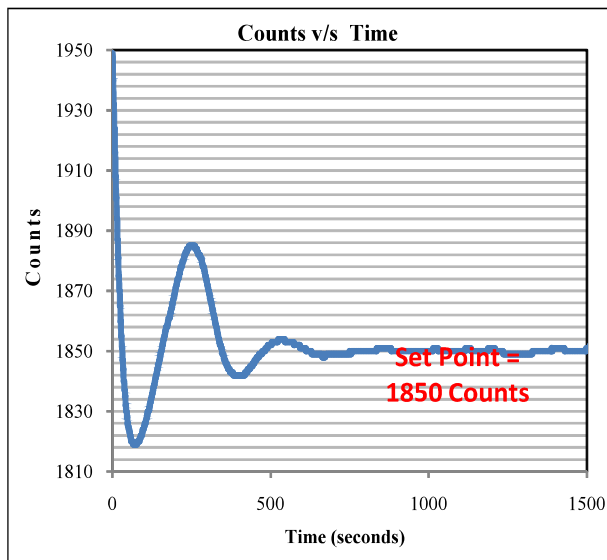
Now, using this plant transfer function, the closed loop PI control response for unit step input was computed in MATLAB. The overall transfer function is then calculated for the new gain and unit step response is simulated in MATLAB. Here known tuning techniques such as Ziegler–Nichols, Cohen–Coon or software tools based tuning methods can be employed to find the optimum values of  $K_p$  and  $K_i$ . Manual tuning methods can also be used for simulation. After couple of software simulation iterations, the new values of  $K_p=9$  and  $K_i=0.25$  were selected to have a desired system response. The overall closed-loop transfer function is

$$T.F._{overall} = \frac{0.007092s + 0.000197}{4s^3 + 0.3089s^2 + 0.01077s + 0.000197}$$

The step response for this MATLAB model is shown in the Figure 5.



**Fig. 5. Simulated Unit-Step Response with  $K_p = 9$  &  $K_i = 0.25$**



**Fig. 6. Measured Closed Loop Test Results with  $K_p=9$  and  $K_i=0.25$**

Figure 6 shows the system response obtained with  $K_p=9$  and  $K_i=0.25$ . As seen from the figure the response closely matches the simulation results. It may be noted that the step response appears inverted, because the circuit topology is such that the counts decrease with increasing temperature. But it may be seen that for both simulated and measured results the response settles

**Table 1: Comparison of Responses from Cooler and of MATLAB Model**

| Parameters               | Actual Response From Cooler | MATLAB Response of PI Model |
|--------------------------|-----------------------------|-----------------------------|
| Peak Time (seconds)      | 77                          | 91                          |
| Percentage Overshoot (%) | 1.67                        | 1.24                        |
| Settling Time (seconds)  | 674                         | 528                         |
| Total peak/valleys       | 5                           | 5                           |

down after second peak. The Table 1 gives comparison between simulated and measured results.

## CONCLUSION

The authors present their work on cryocoolers modeling using Transfer function based Plant modeling. The simulation and measured results are closely matching.

## ACKNOWLEDGEMENT

The authors would like to thank Shri Mukesh Shrivastav, Shri Zenish Patel and Shri Bhavesh Shrimali for their support in hardware realization. The authors would like to thank Shri Arup Roy Chowdhury, Shri Sanjeev Mehta, Shri Sandip Paul and Shri C. P. Sharma for their valuable guidance. The authors would like to acknowledge Deputy Director, Sensor Electronics Development Area and Director, Space Application Centre for providing an opportunity to work on this project.

## REFERENCES

- [1] Closed-loop Tuning of PID Controllers, Tor Steinar Schei, 1992 ACC/FA12
- [2] PID Tuning Via Classical Methods by James Bennett, Ajay Bhasin, Jamila Grant, <https://controls.engin.umich.edu/wiki/index.php/PIDTuningClassical>
- [3] Digital Control System Analysis and Design by Charles L. Phillips, H Troy Nagle, Jr.
- [4] Digital Control Systems, Design, Identification and Implementation by Ioan D. Landau and Gianluca Zito