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ACIT4321 Quantum Information Technology

Greenberg-Horne-Zeilinger Game

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Abstract

The Greenberg-Horne-Zeilinger (GHZ) game is a serious game for quantum computing that demonstrates the quantum mechanical property of entanglement. It is possible to explain entanglement with the GHZ game. Classical mathematical methods do not always succeed in this game. The quantum strategy is therefore to use qubits that are entangled with each other. As a result of the GHZ-state and GHZ-quantum circuit, you can always win the game using the three-qubit entangled state. Quantum and classical (mathematical) approaches to the three-player GHZ game are discussed in this project, and the quantum mechanical property of entanglement allows to always win the game while classical methods do not always.

I Introduction

The development of quantum computing (QC) and quantum computers is leading to dramatic changes in the way we think about computing, what problems one can solve, and to which extent we understand the environment of computing. QC is a new area of computer science concerned with the development of computer-based technologies based on quantum theory. The term "quantum theory" is widely used but poorly understood. So if you think quantum theory or quantum mechanics is challenging, you are by no means alone. Quantum theory explains this difficult behavior and gives us a more complete picture of our universe. We have realized that we can use this previously unexplained behavior to perform certain calculations that we previously thought were impossible. We

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call this quantum computing, which could help us understand unexplained behavior. QC is the improved version of quantum mechanical phenomena to perform the computation. Such computation can only be done physically or theoretically by a quantum computer.It is quantum computing that helps to understand the differences and additional potential of quantum computers. However, quantum entanglement, the core property of quantum computing that amplifies its power, is difficult to understand because it contradicts our everyday experience and intuition. The Greenberg-Horne-Zeilinger (GHZ) game is a serious game for quantum computing that demonstrates the quantum mechanical property of entanglement. The GHZ game can be used to explain the quantum mechanical property of entanglement.

II GHZ state

The GHZ states [1] is a certain type of multipartite entangled quantum state, which plays a crucial role in quantum information processing. The multipartite entangled states are entangled states of many qubits which are an immense resource for research in large-scale quantum computation [2], multipartite quantum communication [3], and quantum simulation simulation [4]. The GHZ states [1] are maximally entangled states, and generation of which has been paid much attention in recent quantum information processing. The GHZ state is an entangled quantum state for three qubits and its state is

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}. \tag{1}$$

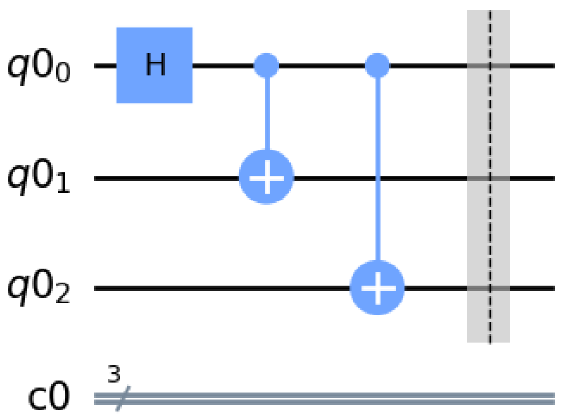

Figure 1. GHZ state circuit. This circuit creates a GHZ state and then measures all qubits in the standard basis. The measured results should be half 000 and half 111 [5].

The first gate we see in Figure 1 is the H-gate which brings the qubit into a superposition. The measurement output in this case has a 50% probability of being 0 and a 50% probability of being 1. The rest of the two other gates are the CNOT gates that operate on 2 qubits. The qubit with the dot is the control bit and the other one with the cross sign is the target bit.

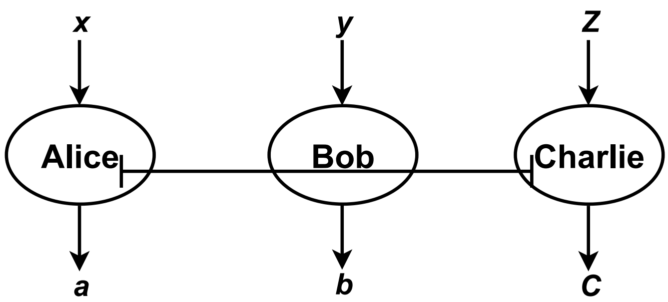

Figure 2. Three players Alice, Bob, and Charlie playing the GHZ game.

III The GHZ game

Let us consider the three players Alice, Bob, and Charlie as shown in figure 2, and each of them gets an input value x, y, and z with single bits $x \in \{0,1\}$, $y \in \{0,1\}$, and $z \in \{0,1\}$ respectively. Conveniently, suppose the referee of the game, gives each of the players a slip of paper with values 0 and 1. They have to produce an output a, b, and c with single bits and for convenient it is written as $x \in \{\pm 1\}$, $x \in \{\pm 1\}$, and $x \in \{\pm 1\}$ respectively. It means that the players have to shout out to the referee either the value $+1$ or -1. Also, the referee explains to the players that either two of them have received 1s on their slip of paper or none of them have received 1s. The winning conditions for Alice, Bob, and Charlie is shown in Table. 1 with what they have to shout out must be $\{+1,-1,-1,-1\}$ for the corresponding combinations $\{000,110,011,101\}$. The rules are as follows:

- Three players will be locked away in three separate rooms without the ability to communicate with each other once the game starts and their inputs are received.
- They win if $a \oplus b \oplus c = x \vee y \vee z$

The winning condition is

x	y	z	abc
0	0	0	+1
1	1	0	-1
0	1	1	-1
1	0	1	-1

Table 1. Truth table for the winning condition of Alice, Bob, and Charlie

Now, the task of Alice, Bob, and Charlie is to give the correct answer for any input given by the referee. At every turn, each of them has to answer with "+1" or with "-1". The question posed by the referee is the three bit-string as shown in the table for x, y, and z (000,110,011,101). The correct answer means the above condition of the table is to be fulfilled. The game proceeds as follows:

- Referee chooses one of the four possible ways taken from the set $\{000,011,101,110\}$

- The first bit of the chosen word is given to Alice, the second to Bob, and the third to Charlie.
- Alice, Bob, and Charlie win or lose depending on whether they gave the correct answer or not, according to the "truth table" shown above.

I will start the game with two approaches: the classical and the Quantum approach. Each of the approaches is further divided into two teams: Team ABC and Team Quantum respectively for classical strategy and quantum way of playing the games.

III.1 Classical Approach: Team Alice, Bob, and Charlie (ABC)

The simple python code for this game is made available at GitHub, `https://github.com/shailendrabhandari/GHZ-game.git` where both teams (Team ABC and Team Quantum) play the game and team quantum always wins while team ABC does not always. It is because classical strategies in this game are not going to do well. However, let's try to solve it mathematically to constantly win this game. Instead of 0 and 1 bit as input let's say each of the players get either X or Y from the referee. To write it all down symbolically, there are four possible combinations of X and Y, $\{XXX, XYY, YXY, YYX\}$, and the products of the output must be $\{-1, 1, 1, 1\}$ for these corresponding X and Y. The rest of the rules are already mentioned above and accordingly, we have the following variables:

$$X_A, X_B, X_C, Y_A, Y_B, Y_C$$

and each of these variables takes the value $+1$ or -1. To satisfy this requirements, the following condition must be true.

$$\begin{aligned} X_A X_B X_c &= -1 \\ X_A Y_B Y_C &= 1 \\ Y_A X_B Y_C &= 1 \\ Y_A Y_B X_C &= 1 \end{aligned} \tag{2}$$

When multiplying the last three lines of Equation. 2:

$$X_A (Y_A)^2 X_B (Y_B)^2 X_C (Y_C)^2 = 1 \tag{3}$$

Since $1^2 = (-1)^2 = 1$, Y^2 is always positive. So, there is a contradiction

$$X_A X_B X_c = 1.$$

This equation does not satisfy if Eqn.2 is true. There cannot exist ± 1 valued observables to satisfy these equations to always win the game. Thus, there is no way for Alice, Bob, and Charlie to always guarantee that they will win this game.

What happens if Alice, Bob, and Charlie decided to use probabilities in their game strategies? Let us suppose that $\langle X_A \rangle$ is the expectation value of Alice's outputs on getting either $+1$ or -1. Since the parties are not allowed to communicate with each other, so for each variable we can assign an independent expected value. Each of the variables is independent of each of the player's output, the expectation values will satisfy the same Eqn. 2:

$$\begin{aligned}\langle X_A\rangle\langle X_B\rangle\langle X_c\rangle &= -1\\ \langle X_A\rangle\langle Y_B\rangle\langle Y_C\rangle &= 1\\ \langle Y_A\rangle\langle X_B\rangle\langle Y_C\rangle &= 1\\ \langle Y_A\rangle\langle Y_B\rangle\langle X_C\rangle &= 1\end{aligned} \tag{4}$$

Again, there is a contradiction in this case too. The expectation values lie between -1 and 1 and the square of the expectation values in between these ranges is always positive. However, the product on the right-hand side of the Eqn. 4 is -1, which is a contradiction to win the game.

Solving the above two mathematical equations 24 there is no possible way for Alice, Bob, and Charlie to always win the game. If actually carried out this game with Alice, Bob and, Charlie, a random triple of questions can be generated and one can check whether the answers win or lose. If played this game once then they might win by luck. In fact, the winning probability is $\frac{3}{4}$. But, imagine this game is played several round in succession and they win win every single round? In general, if played n round in succession, the winning probability will be $(\frac{3}{4})^n$.

III.2 Quantum strategy: Team Quantum

Since the mathematical solutions of the classical approach are not always able to win this game, it is time to try the quantum strategy. The quantum strategy consists of using qubits that are entangled between the respective states of the three players. Suppose that Alice, Bob, and Charlie each share a part of the tripartite quantum state, as shown in equation (1). Now Alice, Bob and Charlie share this state and go back to their separate rooms. To satisfy the above winning condition, all three qubits must be entangled. Since the qubits do not communicate with each other, it is not against the rules of the game to win, because the entangled qubits are so correlated that they cannot be considered independent. And the GHZ state according to equation (1) and the entangled GHZ quantum circuit according to Fig. 1 create the dependencies between the qubits.

Since we are approaching quantum strategy, the maximum probability of winning should always be 1, i.e. $P_{win} = 1$. Let us see how this works. Each of the players has a qubit. This qubit can be visualized in a little sphere as in Fig. 3 with a vector pointing in one direction. And the particular vector describes the state the qubit is currently in. The vector in the sphere is between $|0\rangle$ (top of the sphere) and $|1\rangle$ (bottom of the sphere). If you measure this, the result is either 0 or 1 with a probability of 50% in each case.

The visualization of the Bloch sphere can be used only for a single qubit, but not for the whole (entangled) system of several qubits. However, the so-called Q-sphere can be used to visualize the measurement results of more than two states. The Q-sphere is shown in the figure. 4 (left). shows the measurement results of the GHZ state and it can only be $|000\rangle$ or $|111\rangle$ with equal probability. But the results of each player appear to be completely random and are highly correlated, not independent. The next step is to treat each of the four possible questions with their correct measurements. To do this, we need to measure each qubit of the GHZ state. We need to measure the X-value and the

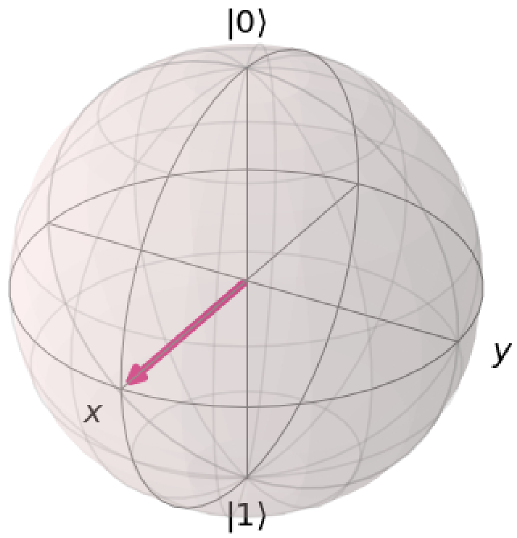

Figure 3. A Bloch sphere with a vector pointing in X- axis and this vector describes the state the qubit in X-axis.

Y-value separately to answer all four questions. The quantum information behind this is worked out in mathematical detail in section IV. Qiskit only allows the measurement in Z-basis. Therefore, the X and Y-basis can be created by our own using the combinations of gates. One can create the X-gate by sandwiching the Z-gate between the two H-gates. Starting in the Z-basis, the H-gate switches the qubit to the X-basis, the Z-gate performs a NOT in the X-basis, and the final H-gate returns the qubit to the Z-basis. Also, applying sdg-gate followed by the Hadamard gate gives the measurement of the Y-basis. The quantum circuit for measuring the Y-basis of a quantum state is shown in Figure. 5 (left). The possible question asked to the players will be the combination of X-basis and the both X and Y basis ($\{XXX, XYY, YYX, YXY\}$). For $\{XXX\}$, we have to apply Hadamard gate followed by measurement in qubit 0, 1, and, 2 of all three players in the GHZ-state. For this Qiskit[1] allow us to build and design quantum circuit with appropriate gates and their measurements. So, I used IBM quantum lab[2] to draw the desired quantum circuit by launching the IBM Quantum Lab. IBM quantum experience provides cloud based software to use their quantum computers anytime and, Qiskit which helps us in working with the quantum computers at the level of circuits, pulse, and algorithms. Similarly, this can be done by installing the Qiskit environment on Python Anaconda. The GHZ-state circuit generated with applying H-gate on each of the qubits followed by the measurements is shown in Figure. 5 (right). In this case the measurement results are distributed as $\{000, 110, 011, 101\}$ and is shown in the Qsphere Figure. 4 (right). For $\{XYY\}$, the quantum circuit can be generated by applying H-gate followed by measurement on qubit 0 (player 1), H-gate followed by sdg-gate and measurement on qubit 1 (player 2) and H-gate followed by sdg-gate and measurement on qubit 2 (player 3). The quantum circuit that can be used when the players are asked for the XYY-values, therefore, could look like this as shown in Figure. 6 (left). Similarly,

[1]qiskit.org
[2]https://quantum-computing.ibm.com/

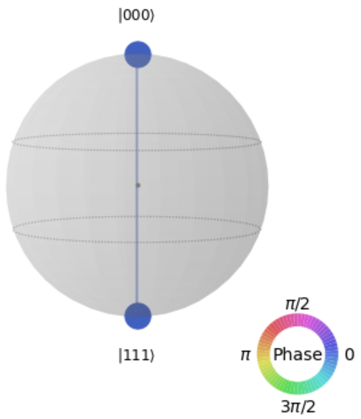

Figure 4. Qsphere showing the the measurement results of the GHZ-State and all the four measurement results respectively of the left and right of the figure

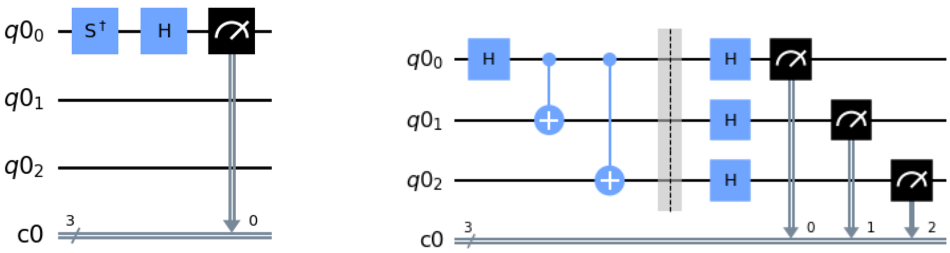

Figure 5. (Left) The quantum circuit for measuring the Y-basis of a quantum state. (Right) The GHZ-state circuit generated with applying H-gate on each of the qubits followed by the measurement.

the measurement results are distributed in this case is $\{010, 001, 100, 111\}$ and is shown in Figure. 6 (right). For the case with $YYXandYXY$, the case is similar to the previous case due to the symmetry of the state and and the strategies.

The above cases are randomly generated in a Python script and a loop is created with the appropriate gates, followed by the measurement for each of the cases and copied into the circuit. The created quantum circuit is run 10000 times on a simulator to get the measurement results for the mentioned condition of the truth table. It works and in each case the team ABC can now win no matter what question is asked. The results of this game can be better visualized in the Jupyter Notebook and I recommend playing with the Jupyter Notebook available at the GitHub link `https://github.com/shailendrabhandari/GHZ-game.git`. You can clone and run this notebook on IBM Quantum Lab by following the instructions in the README.md file and running the cell to view and play the game. This notebook was created according to the requirements of the project description and the material provided for this project. The original idea of this game is described in Ref. [6].

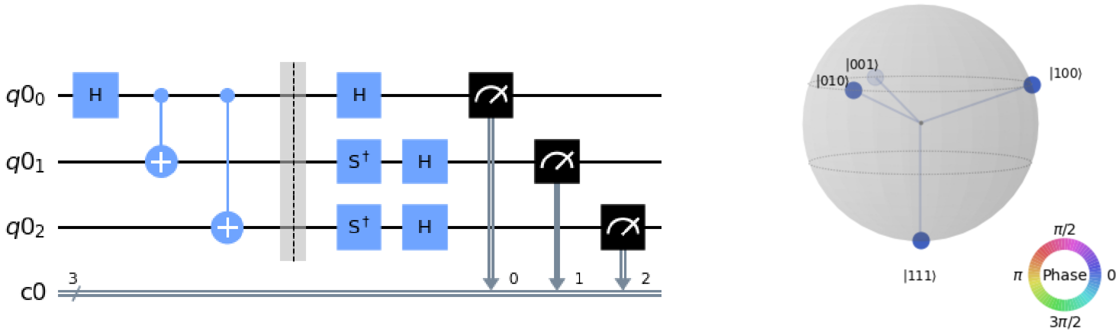

Figure 6. (Left) The quantum circuit that can be used when the players are asked for XYY-value. (Right) The measurement results distributed when the players are asked for XYY-value with the respective circuit.

IV The Quantum information behind the winning condition of the game

The three players, Alice, Bob, and Charlie, share their state and return to their separate rooms. The referee of the game gives each of them a slip of paper that says X or Y, and they measure the part of GHZ in the respective X or Y. In the GHZ state, for example, only the results $|000\rangle, |011\rangle, |101\rangle$ and $|110\rangle$ occur in a $\{XXX\}$ measurement. The interesting thing is that each qubit behaves completely randomly when only the first qubit is considered. However, if one considers the entire system, a dependence becomes apparent. A commonly known entangled state is the Bell state. The state can be $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$. If one measures the first qubit in state 1, one can already say with certainty that the result of the second measurement will also be 1, since a state with mixed results cannot occur. For example, in the case of the GHZ state, if the GHZ circuit is executed with the z-axis as the measurement base, the measurement result will be 50% in $|000\rangle$ and 50% in $|111\rangle$ and. In our experiment, the result is 49.8% probability for $|000\rangle$ state and 50.2% for $|111\rangle$ state as shown in Figure 7.

For example, when X is given to the players, they measure the part of their GHZ in the $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ basis and depending on this if their output is $|+\rangle$ then it is $+1$ and if $|-\rangle$ it is -1. The basis is the eigen states of the Pauli X operator and similar for the Y with the basis Y. When measuring on the $XXX-basis$, we apply a Hadamard-gate to each of the qubits. The Hadamard gate is a single-qubit operation that maps the basis state $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ and creates an equal superposition of the two basis states. The Hadamard gate can also be expressed as a 90° rotation around the Y-axis, followed by a 180° rotation around the X-axis. So, $H = XY^{1/2}$. A Hadamard-gate can be described with the following matrix:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

If we want to apply the H-gate three times (one time on each qubit) we can use the tensor product. For the tensor product $A \bigotimes B$ each block $a_{ij} * B$ is calculated. For the tensor product of three Hadamard gates this means:

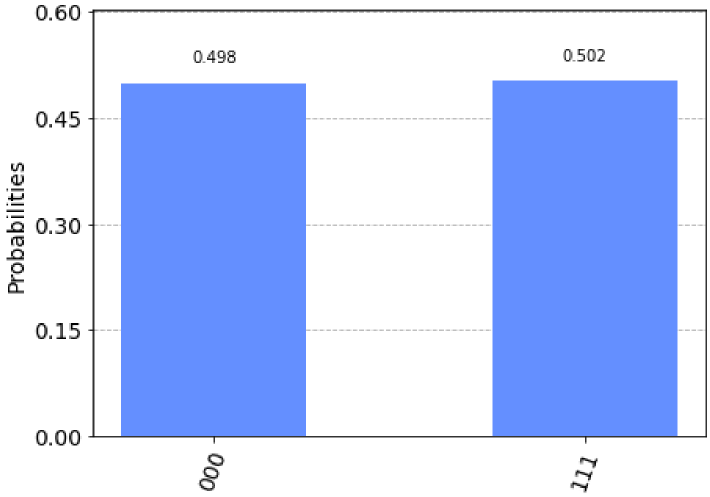

Figure 7. Visualization of the measurement results for GHZ-circuit, using z-axis as a measurement basis. The circuit for the GHZ-state was already shown in Figure 1.

$$H \bigotimes H \bigotimes H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \bigotimes \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \bigotimes \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \tag{5}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \bigotimes \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix} \tag{6}$$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1\\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1\\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1\\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1\\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1\end{pmatrix} \tag{7}$$

The states $|0\rangle$ and $|1\rangle$ can also be represented by the vectors $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ Therefore, the GHZ state can also be represented as a vector:

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}} = \frac{|0\rangle|0\rangle|0\rangle + |1\rangle|1\rangle|1\rangle}{\sqrt{2}}$$

$$= \frac{\begin{pmatrix}1\\0\end{pmatrix} \bigotimes \begin{pmatrix}1\\0\end{pmatrix} \bigotimes \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} \bigotimes \begin{pmatrix}0\\1\end{pmatrix} \bigotimes \begin{pmatrix}0\\1\end{pmatrix}}{\sqrt{2}}$$

$$= \frac{\begin{pmatrix}1\\0\end{pmatrix} \bigotimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} \bigotimes \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \bigotimes \begin{pmatrix}0\\1\end{pmatrix}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left(\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{pmatrix}$$

When applying three Hadamard-gates to the GHZ state:

$$\frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix} * \frac{1}{\sqrt{2}} \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} = \frac{1}{4} \begin{pmatrix}2\\0\\0\\2\\0\\2\\2\\0\end{pmatrix} \tag{8}$$

$$= \frac{1}{4}(2 * |000\rangle + 0 * |001\rangle + 0 * |010\rangle + 2 * |011\rangle + 0 * |100\rangle + 2 * |101\rangle + 2 * |110\rangle + 0 * |111\rangle) \tag{9}$$

$$= \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle) \tag{10}$$

As expected only the results with an even number of ones occur. When measuring a

Y-value we need an additional gate:

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

For the YYX-measurement we would apply the sdg-gate to qubit 0 and 1 followed by a Hadamard-gate on all three qubits. Therefore the following needs to be calculated:

$$HS^\dagger \bigotimes HS^\dagger \bigotimes H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \bigotimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \bigotimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{11}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \bigotimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \bigotimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{12}$$

When applying this measurement to the GHZ state:

$$HS^\dagger \bigotimes HS^\dagger \bigotimes H * \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \\ 2 \end{pmatrix} \tag{13}$$

$$= \frac{1}{4}(0 * |000\rangle + 2 * |001\rangle + 2 * |010\rangle + 0 * |011\rangle + 2 * |100\rangle + 0 * |101\rangle + 0 * |110\rangle + 2 * |111\rangle)$$

$$= \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$$

only the results with an even number of zeros.

V Conclusion

As can be seen above, a classical or mathematical strategy cannot correctly answer all four groups of questions. However, by using an entangled quantum state, namely the GHZ state, the three parties can always give a correct answer. The outcome distributions for each individual output bit is an unbiased bit. Therefore, the results of Alice's measurement contains absolutely no information about the rest of two players and similar for others. Therefore, any perfect strategy using entanglement must have the property that each output bit by itself is a random unbiased bit. Even, if considered a pair of bits, they are uncorrelated random bits and it is only tripartite correlations among all three output bits that contain information about the inputs. From this, we can conclude that quantum theory is not the same as a certain kind of classical theory. And for quantum computing, this means that quantum devices cannot be simulated by certain classical devices.

References

[1] Anton Zeilinger, Daniel M. Greenberger, and Michael A. Horne. *Bell's Theorem without Inequalities and Beyond*, pages 369–375. Springer US, Boston, MA, 1992.

[2] Robert Raussendorf and Hans J. Briegel. A one-way quantum computer. *Phys. Rev. Lett.*, 86:5188–5191, May 2001.

[3] Mark Hillery, Vladimír Bužek, and André Berthiaume. Quantum secret sharing. *Phys. Rev. A*, 59:1829–1834, Mar 1999.

[4] Seth Lloyd. Universal quantum simulators. *Science*, 273(5278):1073–1078, 1996.

[5] Ibm quantm composer. `https://quantum-computing.ibm.com/composer/docs/iqx/example-circuits/ghz`. Accessed: 2022-11-14.

[6] Hands-on interactive notebooks for the ghz game and quantum error mitigation:. `https://github.com/JanLahmann/Fun-with-Quantum`. Accessed: 2022-11-11.