① Joint Probability Density:

$$f_{x,y} = \begin{cases} e^{-x-y}, & 0<x<\infty,\ 0<y<\infty \\ 0, & \text{elsewhere} \end{cases}$$

$$P(x<y) = \int_0^\infty \int_0^y (e^{-x-y})\, dx\, dy$$

$$= \int_0^\infty \int_0^y \frac{1}{e^x \cdot e^y} \cdot dx \cdot dy$$

$$= \int_0^\infty -[e^{-x} \cdot e^{-y}]_0^y\, dy$$

$$= \int_0^\infty -[e^{-y} e^{-y} - 1 \times e^{-y}]_0^y\, dy$$

$$= \int_0^\infty (e^{-2y} + e^{-y})\, dy$$

$$= \left(\frac{e^{-2y}}{2} - e^{-y}\right]_0^\infty$$

$$= 0 - \left[\frac{1}{2} - 1\right]$$

$$= \underline{\underline{1/2}}$$