① Joint Probability Density:

$$f_{x,y} = \begin{cases} e^{-x-y} & ,\ 0 < x < \infty,\ 0 < y < \infty \\ 0 & ,\ \text{elsewhere} \end{cases}$$

$$P(x < y) = \int_0^\infty \int_0^Y (e^{-x-y})\ dx\,dy$$

$$= \int_0^\infty \int_0^Y \frac{1}{e^x \cdot e^Y}\ dx\,dy$$

$$= \int_0^\infty -[e^{-x} \cdot e^{-Y}]_0^Y\ dy$$

$$= \int_0^\infty -[e^{-Y} e^{-Y} - 1 \cdot e^{-Y}]_0^\infty\ dy$$

$$= \int_0^\infty \left(-e^{-2y} + e^{-y}\right) dy$$

$$= \left[\frac{e^{-2y}}{2} - e^{-y}\right]_0^\infty$$

$$= 0 - \left[\frac{1}{2} - 1\right]$$

$$= \frac{1}{2}$$