Assignment: (Shailesh)

Inferential Statistics and Hypothesis Testing

Problem Statement

Comprehension

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Ans

1a) The above problem can be solved using Binomial Distribution as it satisfies the below conditions.

i) Fixed Number of Trials
ii) Each trial is independent of each other
iii) There are only 2 outcomes
iv) The probability of each outcome remains constant from trial to trial.

1b) Probability that the drug produces a satisfactory result = $P(S)$

Probability that the drug does not produce a satisfactory result = $P(NS)$

From Question..

$$P(S) = 4P(NS)$$

From Probability

$$P(S) + P(NS) = 1$$

$$4P(NS) + P(NS) = 1$$

$$P(NS) = 1/5 = 0.2$$

$$\therefore P(S) = 1 - 0.2 = 0.8$$

$$\boxed{\begin{aligned} P(S) &= 0.8 \\ P(NS) &= 0.2 \end{aligned}}$$

Probability that atmost 3 drugs are not able to do a satisfactory job is given below

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) \quad \text{--- (1)}$$

Binomial Distribution is given by.

$$P(X=r) = {}^{n}C_{r} (p)^{r} (1-p)^{n-r}$$

n = no. of trials

p = probability of success (failure in our problem)

r = no. of successes after 'n' trials

$\therefore P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$ ... ①

Solving individually of each probability

$P(X=0) = {}^{n}C_0 p^0 (1-p)^n = {}^{10}C_0 (0.2)^0 (0.8)^{10}$ ... ②

$P(X=1) = {}^{n}C_1 p^1 (1-p)^{n-1} = {}^{10}C_1 (0.2)^1 (0.8)^9$ ... ③

$P(X=2) = {}^{n}C_2 p^2 (1-p)^{n-2} = {}^{10}C_2 (0.2)^2 (0.8)^8$ ... ④

$P(X=3) = {}^{n}C_3 p^3 (1-p)^{n-3} = {}^{10}C_3 (0.2)^3 (0.8)^7$ ... ⑤

Solving for ① using ②, ③, ④ and ⑤

$P(X \leq 3) = 0.1073 + 0.2684 + 0.30186 + 0.2013$

$\boxed{P(X \leq 3) = 0.8780}$ or $\boxed{87.8\%} \simeq \boxed{88\%}$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.) Find the required range.

Ans.

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2a) The Sampling Distribution has some interesting properties and are collectively called "Central Limit Theorem" (CLT)

It states that, No matter how the original population is distributed, the sampling Distribution will follow these below properties.

i) Sampling Distribution's Mean ($\mu_{\bar{x}}$) = Population Mean (μ) (unknown)

ii) Sampling Distribution's Standard Deviation (Standard Error) = $\frac{\sigma}{\sqrt{n}}$ $\frac{\text{(Population Std. Dev)}}{\text{(Sample Size)}}$

iii) For $n > 30$, the sampling distribution becomes a Normal Distribution

∴ Using CLT, you can estimate population Mean from Sample Mean & Standard Deviation

Also there are other important concepts.

i) Probability associated with the claim is called "confidence Level"

ii) Maximum Error Made in Sample Mean is called "Margin of Error"

iii) Final interval of values is called "confidence Interval".

$$\therefore \text{Confidence Interval} = \left(\bar{x} - \frac{z^* s}{\sqrt{n}}, \bar{x} + \frac{z^* s}{\sqrt{n}} \right)$$

z^* - z score associated with y% confidence Level

s - Sample Std. Deviation

$\bar{x}$ - Sample Mean

n - Sample Size.

26). Sample Size $n = 100$

Sample Mean $\bar{x} = 207$ sec.

Sample Std. Deviation $S = 65$ sec

For 95% confidence level $z^* = 1.96$

$\therefore$ Confidence Interval $= \left(\bar{x} - \frac{z^* S}{\sqrt{n}}, \bar{x} + \frac{z^* S}{\sqrt{n}} \right)$

$= \left(207 - \frac{1.96 \times 65}{\sqrt{100}}, 207 + \frac{1.96 \times 65}{\sqrt{100}} \right)$

$= (207 - 12.74, 207 + 12.74)$

$= (194.26, 219.74)$

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$\therefore$ Confidence Level = 95%

$\therefore$ Margin of Error = 12.74 seconds

$\therefore$ Confidence Interval = (194.26, 219.74)

Hence, the population mean would lie between 194.26 seconds and 219.74 seconds.

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure(with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice (Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

3a). From the question it says almost 200 seconds

∴ Null Hypothesis (H_0) ≤ 200

Alternative Hypothesis (H_1) > 200

∴ One-Tailed Test Method is applicable.

Approach 1:

Critical Value Method

Mean (μ) = 200

Std. Deviation = (σ) = 65

Significance Level $\alpha = 0.05$

Cumulative Probability = $1 - 0.05 = 0.95$

Z_c (Z score for 0.95) = 1.645

Upper critical value has to be calculated since it follows Right Tailed Test.

$UCV = \mu + (Z * \sigma_{\bar{x}})$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 65/\sqrt{100} = 6.5$

$UCV = 200 + (1.645 * 6.5) = 210.692$

∴ UCV = 210.7

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Sample Mean lies in the Acceptance Range

$207 < 210.69$

"Hence, we fail to reject the Null Hypothesis"

Approach 2:

P-value Method

Calculate z-score for sample Mean

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

$\bar{x} = 207$, $\mu = 200$, $\sigma_{\bar{x}} = 65$

$\alpha = 5\% = 0.05$

$$\therefore z = \frac{207 - 200}{65} = 0.107$$

∴ Find Cumulative Probability 0.8577

$P = (1 - 0.8577)$

$\boxed{P = 0.1423}$

This signifies 1-tailed test

P-value is greater than α (significance level)

$0.1423 > 0.05$

$\therefore$ "Fail to Reject the Null Hypothesis"

Final Conclusion:

We have calculated the probabilities using critical value Method and p-value method.

Hence, we conclude to

"Fail to Reject the Null Hypothesis"

that time of effectiveness of a drug is atmost 200 seconds.

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3b) $\alpha = 0.05$ $\beta = 0.45$ (Scenario 1)

→ A QA engineer in an Automobile Manufacturing Company is assesing the power of a brake system.

→ False Negative in this scenario merely means that the part is powerful enough but the test fails to detect it.

→ This does not cause life harm

→ Type I Errors are worse in this situation because they indicate that the part is strong enough when it is not.

→ This when α should be minimal

0.05 or 0.01

(Scenario 2):

$\alpha = \beta = 0.15$

→ Shampoo Manufacturing Company claiming Hair Growth will be improved at a rate of 0.5 inches per month.

→ Type I and Type 2 error in this scenario will be minimal.

→ If $\alpha = 0.05$, it will affect the Product Marketing.

→ α and β can be kept at 0.15 each in this example.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Ans. 4

A/B testing allows individuals, teams, and companies to make careful changes to their user experiences while collecting data on the results. This allows them to construct hypotheses, and to learn better why certain elements of their experiences impact user behavior. In another way, they can be proven wrong—their opinion about the best experience for a given goal can be proven wrong through an A/B test.

It has extensive applications in the case of E-Commerce field.

A/B Testing Process

The following is an A/B testing framework you can use to start running tests:

- **Collect Data:** Identify High traffic generating areas of Website or app. Use attractive elements in the UI and enable better experience and collect the click or form data for analysis.

- **Identify Goals:** Your conversion goals are the metrics that you are using to determine whether or not the variation is more successful than the original version. Goals can be anything from clicking a button or link to product purchases and e-mail signups.

- **Generate Hypothesis:** Once you've identified a goal you can begin generating A/B testing ideas and hypotheses for why you think they will be better than the current version. Once you have a list of ideas, prioritize them in terms of expected impact and difficulty of implementation.

- **Create Variations:** Using your A/B testing software (like Optimizely), make the desired changes to an element of your website or mobile app experience. This might be changing the color of a button, swapping the order of elements on the page, hiding navigation elements, or something entirely custom. Many leading A/B testing tools have a visual editor that will make these changes easy. Make sure to QA your experiment to make sure it works as expected.

- **Run Experiment:** Kick off your experiment and wait for visitors to participate! At this point, visitors to your site or app will be randomly assigned to either the control or variation of your experience. Their interaction with each experience is measured, counted, and compared to determine how each performs.

- **Analyze Results:** Once your experiment is complete, it's time to analyze the results. Your A/B testing software will present the data from the experiment and show you the difference between how the two versions of your page performed, and whether there is a statistically significant difference.

Step 1:

Create 2 Taglines for the Online Ad Campaign for Sun Pharma.

Tagline 1: (Team 1)

No Pain, No Gain

Tagline 2: (Team 2)

Relieves pain in a Jiffy

Step 2:

With Tagline 1, release the Ad Campaign to certain users online (around 500 members) – **Control Group**

Monitor the Statistics and check the conversion rate i.e. the customers who watched the ad purchased the drug.

Release another version of the Ad with Tagline 2 and change the UX a little to distinguish it from the previous version. – **Variable Group**

Monitor the Statistics and check the conversion rate i.e. the customers who watched the ad purchased the drug.

Step 3:

Use effective Call to Action Buttons on the site to attract more customers.

Step 4:

Check the Frequency and Total Sample Size in each of Ad Campaigns (Control Group and variable group) and keep a track of it.

Step 5:

Using XLSTAT in Excel, enter the values of Frequency and Sample size for both the Campaigns and perform the 2 Sample Proportion Test.

Step 6:

Choose the Null and Alternate Hypothesis in the XLSTAT.

Null Hypothesis:

The difference between the proportions = 0.

Alternate Hypothesis:

The difference between the proportions < 0.

Step 7:

Calculate the P-Value,

If the p-value is > significance level, one cannot reject the null hypothesis.

If the p-value is < significance level, we fail to reject the null hypothesis.