

ANSWERS, HINTS & SOLUTIONS

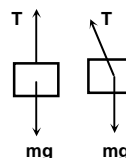
**FULL TEST –I
(Paper-1)**

Q. No.	PHYSICS	CHEMISTRY	MATHEMATICS
1.	B	B	B
2.	B	C	A
3.	A	D	A
4.	B	D	A
5.	A	A	C
6.	A	D	B
7.	C	B	A
8.	D	B	C
9.	B	B	A
10.	A	A	A
11.	D	B	C
12.	B	B	D
13.	B	A	C
14.	A	B	A
15.	B	C	C
16.	C	A	C
17.	A	B	A
18.	C	A	B
19.	C	C	C
1.	(A) \rightarrow (r, s), (B) \rightarrow (r, s), (C) \rightarrow (q, s), (D) \rightarrow (p, s)	A \rightarrow (q, r) B \rightarrow (q, r) C \rightarrow (p, s) D \rightarrow (p, s)	(A) \rightarrow (s), (B) \rightarrow (p), (C) \rightarrow (q), (D) \rightarrow (r)
2.	(A) \rightarrow (p, s), (B) \rightarrow (p, s), (C) \rightarrow (q, s), (D) \rightarrow (p, s)	A \rightarrow (r, s) B \rightarrow (r, s) C \rightarrow (p, r) D \rightarrow (p, q)	(A) \rightarrow (q), (B) \rightarrow (s), (C) \rightarrow (p), (D) \rightarrow (r)
3.	(A) \rightarrow (p, q, r, s), (B) \rightarrow (q, s) (C) \rightarrow (q, s), (D) \rightarrow (p, q, r, s)	A \rightarrow (q) B \rightarrow (r) C \rightarrow (s) D \rightarrow (p)	(A) \rightarrow (q), (B) \rightarrow (s), (C) \rightarrow (p), (D) \rightarrow (r)

Physics

PART – I

1. Now downward force on the right block is more.



$$2. \quad I_C = I_0 + M(OC)^2 = I_0 + M(OB^2 + BC^2) = I_B + M(BC)^2$$

3. Potential difference across AC is zero as

$$I_{AC} = 0$$

$$5 - 2I = 0$$

$$I = 2.5 \text{ A}$$

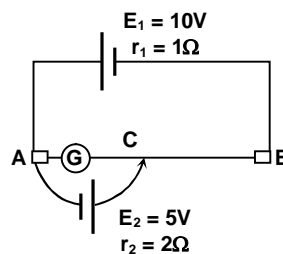
Let the resistance of part BC be r
Applying KVL

$$10 + 5 - 2I - Ir - I = 0$$

$$2.5r = 7.5 \Rightarrow r = 3 \Omega$$

As resistance of part AB = 9Ω

\therefore Length AC = 66.7 cm



5. Initial energy of electron = 2eV
Energy after formation of hydrogen atom in the ground state = -13.6 eV.

$$\text{Energy released} = 2 - (-13.6) = 15.6 \text{ eV}$$

$$\lambda = \frac{hc}{15.6} = 793 \text{ Å}$$

6. Threshold wavelength = 5000 Å

$$\text{Work function} = \frac{hc}{\lambda} = 2.48 \text{ eV}$$

$$\text{K.E.} = eV = 3 \text{ eV}$$

$$\lambda = \frac{hc}{5.48} = 2258 \text{ Å}$$

$$7. \quad P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{300^2}{100} = 900 \Omega$$

$$\therefore \text{current } i = \frac{V}{R} = \frac{300}{900} = \frac{1}{3} \text{ A}$$

Let L be the required inductance, then

$$\frac{500}{z} = \frac{1}{3} \text{ or } \frac{500}{\sqrt{(900)^2 + X_L^2}} = \frac{1}{3}$$

$$X_L = 1200 \Omega$$

$$L = \frac{1200}{\omega} = \frac{1200}{2\pi f} = \frac{1200}{2\pi \times \frac{150}{\pi}} = 4 \text{ H}$$

8. In steady state potential difference across each capacitor = E

$$9. \quad H = \int_0^4 \frac{E^2}{R} dt = \int_0^4 \frac{(6t)^2}{12} dt = 64 \text{ J}$$

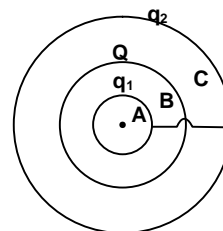
$$14. \quad q_1 + q_2 = 0$$

$$V_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$$

$$V_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$$

$$V_A = V_C$$

$$\Rightarrow q_1 = -Q/3 \text{ and } q_2 = Q/3$$



$$15. \quad V_A = k \left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{Q}{16\pi\epsilon_0 R}$$

$$16. \quad V_B = k \left[\frac{-Q}{6R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{5Q}{48\pi\epsilon_0 R}$$

- 17-19. When current is maximum $\frac{di}{dt} = 0$

\therefore emf across L = 0 and potential difference across the capacitor will be same.

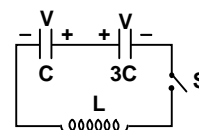
From conservation of charge

$$3CV + CV = 6CV_0 - CV_0$$

$$\Rightarrow V = \frac{5CV_0}{4}$$

Loss in energy of capacitor = energy stored in inductor

$$\Rightarrow I_{\max} = \frac{3V_0}{2} \sqrt{\frac{3C}{L}}$$



Chemistry

PART – II

- $$\begin{array}{l} \text{Ag} + \text{I}^- \longrightarrow \text{AgI} + \text{e}^- \\ \text{Ag}^+ + \text{e}^- \longrightarrow \text{Ag} \\ \hline \text{Ag}^+ + \text{I}^- \longrightarrow \text{AgI} \end{array}$$

$$\begin{array}{l} E^\circ = 0.152 \text{ V} \\ E^\circ = -0.800 \text{ V} \\ \hline E_{\text{cell}} = 0.952 \text{ V} \end{array}$$

At eqm, $E^\circ = \frac{2.303RT}{[Ag^+][I^-]} = \frac{1}{K_{\text{sp}}}$

$$\therefore 0.952 = \frac{-2.303RT}{F} \log K_{\text{sp}} = -0.059 \log K_{\text{sp}}$$

$$\therefore \log K_{\text{sp}} = -16.13$$
- $$A^2 = \alpha t + \beta$$

on differentiating the equation we get:

$$-2AdA = \alpha dt \text{ or } \frac{-dA}{dt} = \frac{\alpha}{2A} = \frac{\alpha}{2}[A]^{-1}$$

Hence order is -1.
- $$\frac{r_{A^+}}{r_{B^-}} = \frac{1}{2} = 0.5$$

As it lies in the range 0.414 to 0.732. AB has octahedral structure like that of NaCl.

$$\therefore a = 2(r_{A^+} + r_{B^-}) = 2(1 + 2) = 6 \text{ pm}$$

$$\text{Volume} = a^3 = (6 \text{ pm})^3 = 216 \text{ pm}^3$$
- $$\text{BaF}_2(\text{s}) \rightleftharpoons \text{Ba}^{2+}(\text{aq}) + 2\text{F}^-(\text{aq})$$

$$K_{\text{sp}} = [\text{Ba}^{2+}][\text{F}^-]^2 \quad \therefore \text{F}^- = \sqrt{\frac{K_{\text{sp}}}{[\text{Ba}^{2+}]}}$$

Again $K_{\text{sp}} = 2 \times [\text{Ba}^{2+}][\text{F}^-]^2$

$$\therefore [\text{F}^-] = \sqrt{\frac{K_{\text{sp}}}{2[\text{Ba}^{2+}]}}$$
- $$\text{Co}^{3+} \quad 1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$$

3d ⁶					
↑↓	↑	↑	↑	↑	↑

4s

..

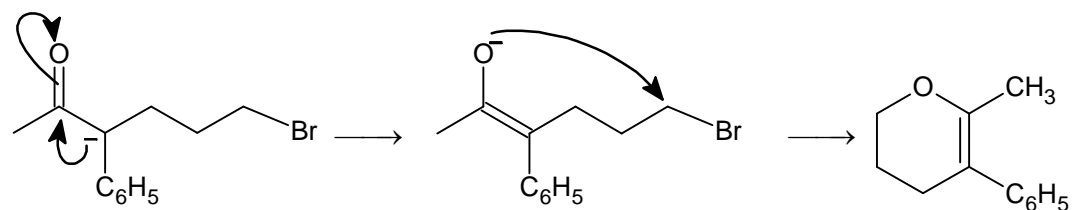
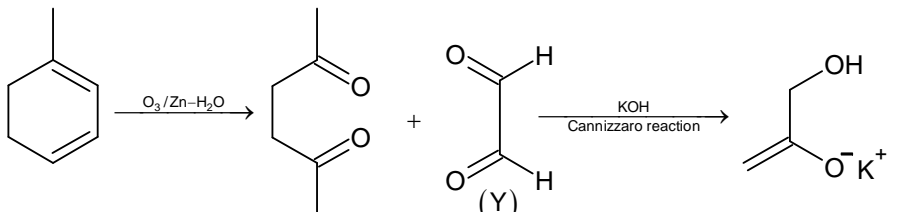
4p

..
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4d

..	..
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sp³ hybridisation
- Bond order of N₂, N₂⁺, NO⁺, NO, CN⁻ and CN are 3, 2.5, 3, 2.5, 3, 2.5 respectively. Higher is bond order smaller is bond length. Bond order of CO and CO⁺ are 3 and 3.5.
- $$\text{CH}_4 \xrightarrow{\text{Cl}_2/h\nu} \text{CH}_3\text{Cl} \xrightarrow{\text{MgCl}} \text{CH}_3\text{MgCl} \xrightarrow{\text{Br}_2/h\nu} \text{CH}_3\text{CH}_2\text{Br} \xrightarrow{\text{KCN}} \text{CH}_3\text{CH}_2\text{C}\equiv\text{N} \xrightarrow{\text{H}_3\text{O}^+} \text{CH}_3\text{CH}_2\text{COOH}$$
- $$\text{CH}_3\text{C}(=\text{O})\text{CH}(\text{C}_6\text{H}_5)\text{CH}_2\text{CH}_2\text{CH}_2\text{Br} \xrightarrow{\text{CH}_3\text{CH}_2\text{O}^-}$$

9. 
9. 
14.
$$\Delta H^\circ = H\left[\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}(\text{s})\right] + H\left[\frac{3}{2}\text{H}_2\text{O}(\text{g})\right]$$

$$-H\left[\text{CaSO}_4 \cdot 2\text{H}_2\text{O}(\text{s})\right] = 833 \text{ kJ mol}^{-1}$$

$$= +484 \text{ kJ for 1 kg}$$
15.
$$\Delta H^\circ = \Delta H^\circ - \Delta S^\circ$$

$$= 17920 \text{ J mol}^{-1}$$

$$\Delta G^\circ = -2.303RT \log K_p$$

$$\log K_p = \frac{\Delta G^\circ}{2.303RT} = 7.22 \times 10^{-4} (p_{\text{H}_2\text{O}})^{\frac{3}{2}}$$

$$p_{\text{H}_2\text{O}} = 8.1 \times 10^{-3} \text{ atm}$$
16.
$$p_{\text{H}_2\text{O}} = 1, K_p = 1$$

$$\Delta G^\circ = 0 \text{ at eqm.}$$

$$\Delta H^\circ = T\Delta S^\circ \Rightarrow T = \frac{\Delta H^\circ}{\Delta S^\circ} = 380 \text{ K}$$

$$= 107^\circ\text{C}$$

Mathematics

PART - III

1. $x^2 + y^2 - 2x - 2y - 1 = 0$
 $x^2 + y^2 - 1 = 0$
 Common chord is $-2x - 2y = 0 \Rightarrow 2x + 2y = 0$
 $y = -x$
 Points of intersection of circles are $\left(\pm \frac{1}{\sqrt{2}}, \mp \frac{1}{\sqrt{2}} \right)$

2. $P(E) = \frac{\sum_{k=1}^4 k}{{}^8C_2} = \frac{10 \times 2}{8 \times 7} = \frac{5}{14}$

3. The line can be written as $y = mx$ and curve as $x^2 + y^2 = 4$
 Let, $C(h, k)$ be a point on the circles and $A(\sqrt{3}, 1)$ be

given point then, $\frac{h + 2\sqrt{3}}{3} = \alpha$

$\Rightarrow h = 3\alpha - 2\sqrt{3}$

$\frac{k + 2}{3} = m\alpha$

$\Rightarrow k = 3m\alpha - 2$

Now, this point (h, k) lies on the circle

$\Rightarrow (3\alpha - 2\sqrt{3})^2 + (3m\alpha - 2)^2 = 4$

$9\alpha^2 + 12 - 12\sqrt{3}\alpha + 9m^2\alpha^2 + 4 - 12m\alpha = 4$

$\Rightarrow 9(1+m^2)\alpha^2 - 12\alpha(\sqrt{3}+m) + 12 = 0$

$3(1+m^2)\alpha^2 - 4\alpha(\sqrt{3}+m) + 4 = 0$

$16(\sqrt{3}+m)^2 - 4 \times 3(1+m^2)(4) > 0$

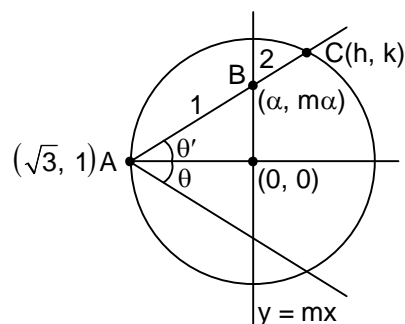
$(\sqrt{3}+m)^2 - 3(1+m^2) > 0$

$3 + m^2 + 2\sqrt{3}m - 3 - 3m^2 > 0$

$2\sqrt{3}m - 2m^2 > 0$

$2m^2 - 2\sqrt{3}m < 0$

$m \in (0, \sqrt{3})$



4. Suppose m is an integer root of $x^4 - ax^3 - bx^2 - cx - d = 0$ as $d \neq 0$
 $\Rightarrow m \neq 0$

(I) $m > 0$

$m^4 - am^3 - bm^2 - cm = d$

$\Rightarrow d = m(m^3 - am^2 - bm - c)$

$\Rightarrow d \geq m$

Also, $m^4 - am^3 = bm^2 + cm + d$

$m^3(m - a) = bm^2 + cm + d$

$\Rightarrow m > a \Rightarrow \text{contradiction}$

(II) $m < 0$

$m = -n$

$\Rightarrow n^4 + an^3 - bn^2 + cn - d = 0$

$$n^4 + n^2(an - b) + (cn - d) > 0$$

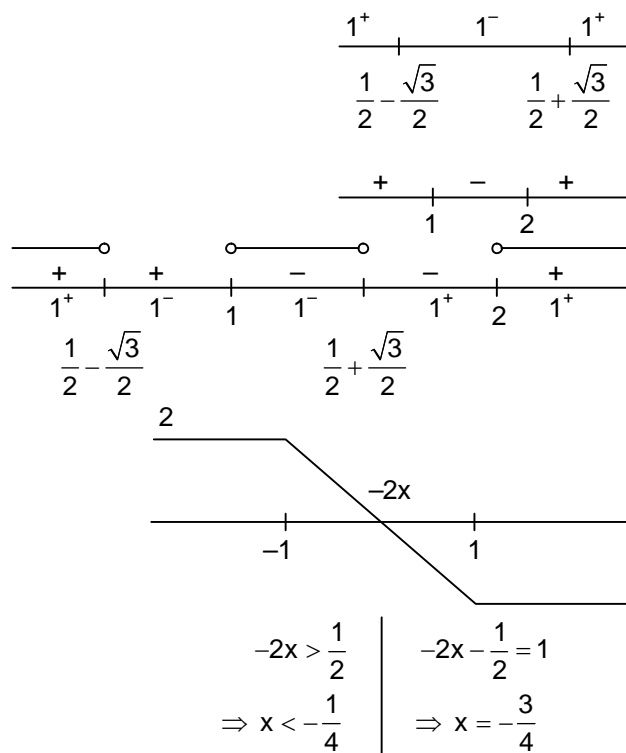
So, contradiction

Hence, equation has no integral solution

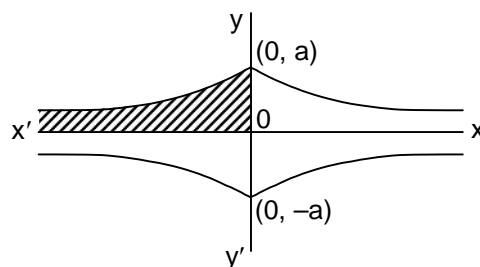
5. (I) $x^2 - x + \frac{1}{2}$
 $\left(x - \frac{1}{2}\right)^2 + \frac{1}{4} = 1$
 $\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

(II) $x^2 - 3x + 2$

(III) $|x - 1| - |x + 1| - \frac{1}{2}$
 \therefore set x can be $\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{3}{4}, \infty\right)$



6. Required area, $A = 4 \int_0^{\pi/2} y \cdot \frac{dx}{dt} dt$
 $= 4 \int_0^{\pi/2} (a \sin t) \left(\frac{a}{\sin t} \cos^2 t \right) dt$
 $= 4a^2 \int_0^{\pi/2} \cos^2 t dt = \pi a^2$



7. Distance from origin, $D = \sqrt{x^2 + y^2 + z^2}$ where $P(x, y, z)$ is any point on the curve

$$D^2 = x^2 + y^2 + z^2 = x^2 + y^2 + \frac{2}{xy} \geq 2xy + \frac{2}{xy} \geq 4$$

$$\Rightarrow D = 2 \text{ and occurs at point(s) } (1, 1, \sqrt{2}), (-1, -1, \sqrt{2}), (1, 1, -\sqrt{2}), (-1, -1, -\sqrt{2})$$

8. Differentiating w.r.t. 'r', $2r dr = 2a \cos 2\theta d\theta$, $a = \frac{r^2}{\sin 2\theta}$

$$\Rightarrow r dr = \frac{r^2}{\sin 2\theta} \cos 2\theta d\theta$$

$$\Rightarrow r \cdot \frac{d\theta}{dr} = \tan 2\theta$$

So, differential equation of orthogonal trajectory, $r \cdot \frac{d\theta}{dr} = -\cot 2\theta$

$$\text{Solving } -\tan 2\theta d\theta = \frac{dr}{r}$$

$$\Rightarrow \ln(\cos 2\theta) = 2 \ln r + k \Rightarrow r^2 = c \cos 2\theta$$

9. Let $f(x) = x^3 + bx^2 + cx + 1$. $f(0) = 1 > 0$, $f(-1) = b - c < 0$

So, $\alpha \in (-1, 0)$. So, $2 \tan^{-1}(\operatorname{cosec} \alpha) + \tan^{-1}(2 \sin \alpha \sec^2 \alpha)$

$$= 2 \tan^{-1}\left(\frac{1}{\sin \alpha}\right) + \tan^{-1}\left(\frac{2 \sin \alpha}{1 - \sin^2 \alpha}\right) = 2 \left[\tan^{-1}\left(\frac{1}{\sin \alpha}\right) + \tan^{-1}(\sin \alpha) \right]$$

$$= 2 \left(-\frac{\pi}{2} \right) = -\pi \quad (\text{as } \sin \alpha < 0)$$

10. $\frac{1}{1+z^2}$ is continuous every where except where $1+z^2=0$

$\Rightarrow z = \pm i$ so when $|z| < 1$ then above points are excluded so $f(z)$ is continuous

11. Continuity : for any point z , $\lim_{z \rightarrow z_0} f(z) = \lim_{z \rightarrow z_0} x = x_0 = f(z_0)$. Non differentiable : for any point z ,

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{\Delta x}{\Delta x + i \Delta y} \quad \text{now } \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{\Delta x}{\Delta x + i \Delta y} = 0 \quad \text{and} \quad \lim_{\substack{\Delta y \rightarrow 0 \\ \Delta x \rightarrow 0}} \frac{\Delta x}{\Delta x + i \Delta y} = 1$$

12. $y_1 = \cos(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$

$$\sqrt{1-x^2} y_1 = m \cos(m \sin^{-1} x)$$

$$\Rightarrow (1-x^2) y_2 - x y_1 + m^2 y = 0$$

$$\left[(1-x^2) y_{n+2} + n(-2x) y_{n+1} + \frac{n(n-1)}{2} (-2) y_n \right] - [x y_{n+1} + n \cdot 1 \cdot y_n] + m^2 y_n = 0$$

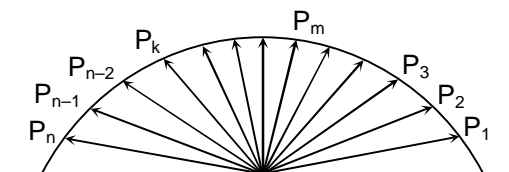
$$\text{Simplifying we get, } (1-x^2) y_{n+2} - (2n+1) y_{n+1} - (n^2 - m^2) y_n = 0$$

13. $\frac{dy}{dt} = \frac{3at(2-t^3)}{(1+t^3)^2}$

$$\frac{dx}{dt} = 6a \frac{\left(\frac{1}{2} - t^3 \right)}{(1+t^3)^2}$$

$$\text{Now } \frac{dy}{dx} \rightarrow \infty \text{ at } t \rightarrow (2)^{1/3}$$

14.-16. Here, $\vec{P}_v = \vec{P}_2 + \vec{P}_K$ so \vec{P}_v will make acute angle with all the vectors from $\vec{P}_2, \vec{P}_3, \dots, \vec{P}_K$



$$\text{So, } \overrightarrow{OP_v} \cdot (\overrightarrow{OP_2} + \overrightarrow{OP_3} + \dots + \overrightarrow{OP_{k-1}}) > 0$$

Again, if n is odd $\Rightarrow n = 2k - 1$

$$\overrightarrow{OP_1} + \overrightarrow{OP_{2k-1}} = \overrightarrow{OP_k}$$

Now $\overrightarrow{OP_k}$ will make acute angle with all the vectors from $\overrightarrow{OP_1}, \overrightarrow{OP_2}, \dots, \overrightarrow{OP_{2k-1}}$

$$\overrightarrow{OP_k} \cdot (\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_k} + \overrightarrow{OP_{k+1}} + \dots + \overrightarrow{OP_{2k-1}}) \geq \overrightarrow{OP_k} \cdot \overrightarrow{OP_k} = 1$$

$$\text{Now, } \overrightarrow{OP_k} \cdot (\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_{2k-1}}) \leq |\overrightarrow{OP_k}| |\overrightarrow{OP_1} + \dots + \overrightarrow{OP_{2k-1}}| = |\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_n}|$$

Again, \vec{V} makes acute angle with all the vectors $\overrightarrow{OP_1}, \overrightarrow{OP_2}, \dots, \overrightarrow{OP_7}$

$$\text{So, } |\vec{V} + \vec{S}| = |\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_7}| \geq 1 \Rightarrow |\vec{V}| + |\vec{S}| \geq 1$$

17. Eigen values are roots of the equation $A - \lambda X = 0 \Rightarrow (A - \lambda I)X = 0$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 4)(\lambda - 6) = 0$$

$$\therefore \lambda = 4, 6$$

18. When $\lambda = 6$

$$\begin{bmatrix} 8-6 & -4 \\ 2 & 2-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2x_1 - 4x_2 = 0 \Rightarrow x_1 = 2x_2$$

$$\therefore X = C \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

19. $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \Rightarrow \lambda = 1, 2, 3,$$

For $\lambda = 3$

$$X = C \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \text{ which is orthogonal}$$

SECTION - B

1. (A) Assume sphere as, $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

Now P, Q, R are $(-2u, 0, 0), (0, -2v, 0), (0, 0, -2w)$

$$\text{So, equation of plane is } \frac{x}{-2u} + \frac{y}{-2v} + \frac{z}{-2w} = 1$$

$$\text{Since it passes through } (1, 2, 3) \quad \frac{1}{-2u} + \frac{2}{-2v} + \frac{3}{-2w} = 1$$

If centre is $(x, y, z) \equiv (-u, -v, -w)$

$$\therefore \text{locus is } \frac{1}{x} + \frac{2}{y} + \frac{3}{z} = 2$$

- (B) Assume equation of sphere as, $x^2 + y^2 + 2ux + 2vy + 2wz = 0$

It passes through $(\alpha, 0, 0), (0, \beta, 0), (0, 0, \gamma)$

$$\Rightarrow u = -\frac{\alpha}{2}, v = -\frac{\beta}{2}, w = -\frac{\gamma}{2}$$

$$\text{Radius} = 1 \Rightarrow \sqrt{\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 1$$

$\alpha^2 + \beta^2 + \gamma^2 = 4$ if (x, y, z) are coordinates of centroid

$$\Rightarrow x = \frac{\alpha}{3}, y = \frac{\beta}{3}, z = \frac{\gamma}{3} \therefore x^2 + y^2 + z^2 = \frac{4}{9}$$

(C) $P \equiv (\alpha, 0, 0), Q \equiv (0, \beta, 0), \angle OPQ = 15^\circ$

$$\Rightarrow \tan 15^\circ = \frac{\beta}{\alpha} \quad \dots (1)$$

Now if sphere is made with PQ as diameter $(x - \alpha)x + (y - \beta)y + z^2 = 0$

$$\Rightarrow x^2 + y^2 + z^2 = ax + by \quad \dots (2)$$

A plane through PQ parallel to z-axis is

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1 \quad \dots (3)$$

Using (1), (2), (3) we get $x^2 + y^2 + z^2 = (\alpha x + \beta y)\left(\frac{x}{\alpha} + \frac{y}{\beta}\right)$

$$\Rightarrow z^2 = xy(\tan 15^\circ + \cot 15^\circ) \Rightarrow 2xy - \frac{z^2}{2} = 0$$

(D) Using family of planes any plane through line of intersection is $x + y - 6 + \lambda(x - 2z - 3) = 0$

$$\text{Now, its distance from centre of sphere is radius } \frac{|-6 - 3\lambda|}{\sqrt{(1 + \lambda)^2 + 1 + 4\lambda^2}} = 3, \lambda = 1, -\frac{1}{2}$$

\therefore Equation is $x + 2y + 2z = 9; 2x + y - 2z = 9$

2. (A) Let $p = \cos x \sin y \cos z$. As, $\frac{\pi}{2} \geq y \geq z, \sin(y - z) \geq 0$

$$p = \frac{1}{2} \cos z [\sin(x + y) - \sin(x - y)] \leq \frac{1}{2} \cos^2 z$$

As, $\sin(x - y) \geq 0$ and $\sin(x + y) = \cos z$

$$p \leq \frac{1}{4}(1 + \cos 2z) \leq \frac{1}{4}\left(1 + \cos \frac{\pi}{6}\right) = \frac{2 + \sqrt{3}}{8}$$

(B) The equation can be written as $3u^2 + 8u + 3 = 0 \Rightarrow u_1 = \frac{-8 + 2\sqrt{7}}{6} \Rightarrow u_2 = \frac{-8 - 2\sqrt{7}}{6}$

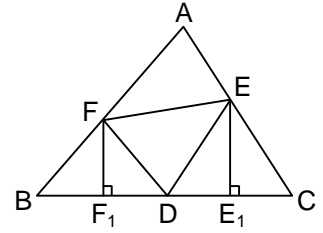
Clearly, u_1, u_2 are negative so $\frac{\pi}{2} < x_1, x_2 < \pi$

$$\Rightarrow \pi < x_1 + x_2 < 2\pi \text{ as } \cot x \tan x = 1 = \cot x \cot\left(\frac{\pi}{2} - x\right) = \cot x \cdot \cot\left(\frac{3\pi}{2} - x\right)$$

$$\therefore x_1 + x_2 = \frac{3\pi}{2} \text{ and another pair, } x'_1 + x'_2 = \frac{7\pi}{2} [\pi < x'_1, x'_2 < 2\pi]$$

$$\therefore x_1 + x'_1 + x_2 + x'_2 = 5\pi$$

(C) $|EF| \geq |E_1F_1| = a - [|BF| \cos B + |CF| \cos C]$
 $|DE| \geq c - [|AE| \cos A + |BD| \cos B]$
 $|FD| \geq b - [|CD| \cos C + |AF| \cos A]$
 $|DC| + |CE| = |EA| + |AF| = |FB| + |BD| = \frac{1}{3}(a+b+c)$
 $\therefore |DE| + |EF| + |FD|$
 $\geq a+b+c - \frac{1}{3}((a+b+c)(\cos A + \cos B + \cos C)) \geq \frac{1}{2}(a+b+c)$



So, minimum value is $\frac{1}{2}$

(D) $\sin^2 x_1 + \sin^2 x_2 + \dots + \sin^2 x_{10} = 1$

$$\cos x_i = \sqrt{\sum_{j=1}^{10} \sin^2 x_j}$$

For each $1 \leq i \leq 10$ we have $\cos x_i = \sqrt{\sum_{j \neq i} \sin^2 x_j} \geq \frac{\sum_{j \neq i} \sin x_j}{3}$

$$\Rightarrow \sum_{i=1}^{10} \cos x_i \geq \sum_{i=1}^{10} \sum_{j \neq i} \frac{\sin x_j}{3} = \sum_{i=1}^{10} 9 \cdot \frac{\sin x_i}{3}$$

$$\Rightarrow \frac{\cos x_1 + \cos x_2 + \dots + \cos x_{10}}{\sin x_1 + \sin x_2 + \dots + \sin x_{10}} \geq 3$$

3. (A) Property of ellipse

(B) Normal chord is made between points $A(2, 2\sqrt{2})$ and $B(8, -4\sqrt{2})$ so its length is $6\sqrt{3}$

(C) $Q \equiv (4, 2)$, $S : (x-4)^2 + (y-2)^2 = 16$ then required circle is $(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

Here, $\lambda = 9$ and radius = $\frac{\sqrt{162}}{2}$

(D) $x^2 = \frac{3y^2}{2} + c$

$$e = \sqrt{\frac{5}{3}} \quad (c > 0) \text{ and } e = \sqrt{\frac{5}{2}} \quad (c < 0)$$