

ALL INDIA TEST SERIES

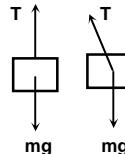
ANSWERS, HINTS & SOLUTIONS FULL TEST –I (Paper-1)

Q. No.	PHYSICS	CHEMISTRY	MATHEMATICS
1.	B	B	B
2.	B	C	A
3.	A	D	A
4.	B	D	A
5.	A	A	C
6.	A	D	B
7.	C	B	A
8.	D	B	C
9.	B	B	A
10.	A	A	A
11.	D	B	C
12.	B	B	D
13.	B	A	C
14.	A	B	A
15.	B	C	C
16.	C	A	C
17.	A	B	A
18.	C	A	B
19.	C	C	C
1.	(A) → (r, s), (B) → (r, s), (C) → (q, s), (D) → (p, s)	A → (q, r) B → (q, r) C → (p, s) D → (p, s)	(A) → (s), (B) → (p), (C) → (q), (D) → (r)
2.	(A) → (p, s), (B) → (p, s), (C) → (q, s), (D) → (p, s)	A → (r, s) B → (r, s) C → (p, r) D → (p, q)	(A) → (q), (B) → (s), (C) → (p), (D) → (r)
3.	(A) → (p, q, r, s), (B) → (q, s) (C) → (q, s), (D) → (p, q, r, s)	A → (q) B → (r) C → (s) D → (p)	(A) → (q), (B) → (s), (C) → (p), (D) → (r)

Physics

PART - I

1. Now downward force on the right block is more.



2. $I_C = I_0 + M(OC)^2 = I_0 + M(OB^2 + BC^2) = I_B + M(BC)^2$

3. Potential difference across AC is zero as

$$I_{AC} = 0$$

$$5 - 2I = 0$$

$$I = 2.5 \text{ A}$$

Let the resistance of part BC be r

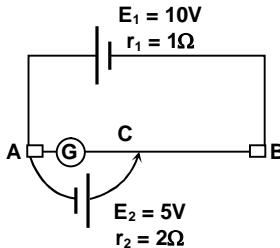
Applying KVL

$$10 + 5 - 2I - Ir - I = 0$$

$$2.5r = 7.5 \Rightarrow r = 3 \Omega$$

As resistance of part AB = 9Ω

$$\therefore \text{Length AC} = 66.7 \text{ cm}$$



5. Initial energy of electron = 2eV

Energy after formation of hydrogen atom in the ground state

$$= -13.6 \text{ eV}.$$

$$\text{Energy released} = 2 - (-13.6) = 15.6 \text{ eV}$$

$$\lambda = \frac{hc}{15.6} = 793 \text{ \AA}$$

6. Threshold wavelength = 5000\AA

$$\text{Work function} = \frac{hc}{\lambda} = 2.48 \text{ eV}$$

$$\text{K.E.} = \text{eV} = 3 \text{ eV}$$

$$\lambda = \frac{hC}{5.48} = 2258 \text{ \AA}$$

7. $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P^2} = \frac{300^2}{100} = 900 \Omega$

$$\therefore \text{current } i = \frac{V}{R} = \frac{300}{900} = \frac{1}{3} \text{ A}$$

Let L be the required inductance, then

$$\frac{500}{z} = \frac{1}{3} \text{ or } \frac{500}{\sqrt{(900)^2 + x_2^2}} = \frac{1}{3}$$

$$X_L = 1200\Omega$$

$$L = \frac{1200}{W} = \frac{1200}{2\pi f} = \frac{1200}{2\pi \times \frac{150}{\pi}} = 4\text{H}$$

8. In steady state potential difference across each capacitor = E

$$9. H = \int_0^4 \frac{E^2}{R} dt = \int_0^4 \frac{(6t)^2}{12} dt = 64 \text{ J}$$

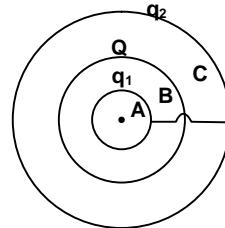
14. $q_1 + q_2 = 0$

$$v_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$$

$$v_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$$

$$v_A = v_C$$

$$\Rightarrow q_1 = -Q/3 \text{ and } q_2 = Q/3$$



$$15. v_A = k \left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{Q}{16\pi\epsilon_0 R}$$

$$16. v_B = k \left[\frac{-Q}{6R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{5Q}{48\pi\epsilon_0 R}$$

17-19. When current is maximum $\frac{di}{dt} = 0$

\therefore emf across L = 0 and potential difference across the capacitor will be same.

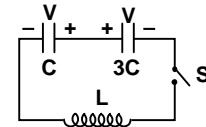
From conservation of charge

$$3CV + CV = 6CV_0 - CV_0$$

$$\Rightarrow V = \frac{5CV_0}{4}$$

Loss in energy of capacitor = energy stored in inductor

$$\Rightarrow I_{\max} = \frac{3V_0}{2} \sqrt{\frac{3C}{L}}$$



Chemistry**PART - II**

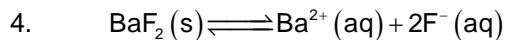
At eqm, $E^\circ = \frac{2.303RT}{[\text{Ag}^+][\text{I}^-]} = \frac{1}{K_{\text{sp}}}$
 $\therefore 0.952 = \frac{-2.303RT}{F} \log K_{\text{sp}} = -0.059 \log K_{\text{sp}}$
 $\therefore \log K_{\text{sp}} = -16.13$

2. $A^2 = \alpha t + \beta$
on differentiating the equation we get:

$$-2A dA = \alpha dt \text{ or } \frac{-dA}{dt} = \frac{\alpha}{2A} = \frac{\alpha}{2}[A]^{-1}$$

Hence order is -1.

3. $\frac{r_{\text{A}^+}}{r_{\text{B}^-}} = \frac{1}{2} = 0.5$. As it lies in the range 0.414 to 0.732. AB has octahedral structure like that of NaCl.
 $\therefore a = 2(r_{\text{A}^+} + r_{\text{B}^-}) = 2(1+2) = 6 \text{ pm}$
Volume = $a^3 = (6 \text{ pm})^3 = 216 \text{ pm}^3$

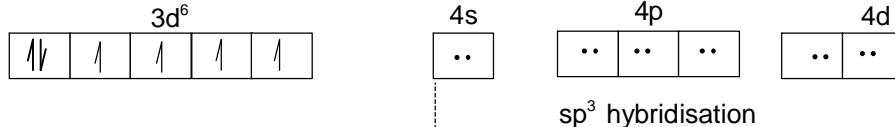


$$K_{\text{sp}} = [\text{Ba}^{2+}][\text{F}^-]^2 \quad \therefore \text{F}^- = \sqrt{\frac{K_{\text{sp}}}{[\text{Ba}^{2+}]}}$$

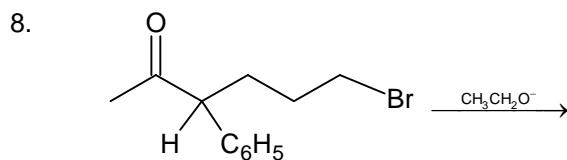
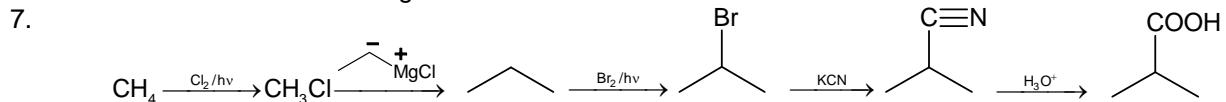
$$\text{Again } K_{\text{sp}} = 2 \times [\text{Ba}^{2+}][\text{F}^-]^2$$

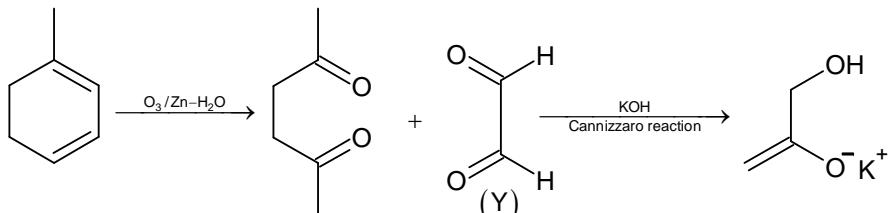
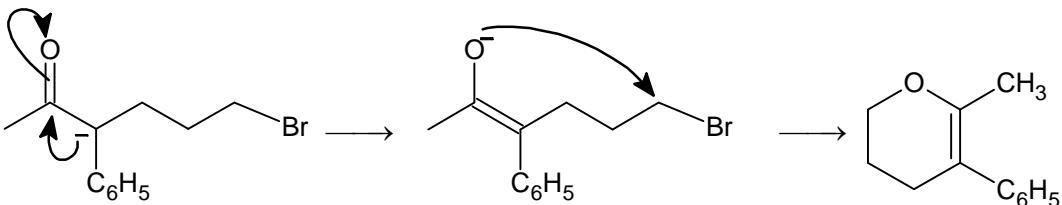
$$\therefore [\text{F}^-] = \sqrt{\frac{K_{\text{sp}}}{2[\text{Ba}^{2+}]}}$$

5. $\text{Co}^{3+} \quad 1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$



6. Bond order of N₂, N₂⁺, NO⁺, NO, CN⁻ and CN are 3, 2.5, 3, 2.5, 3, 2.5 respectively. Higher is bond order smaller is bond length. Bond order of CO and CO⁺ are 3 and 3.5.





$$14. \quad \Delta H^\circ = H\left[CaSO_4 \cdot \frac{1}{2}H_2O(s)\right] + H\left[\frac{3}{2}H_2O(g)\right]$$

$$-H[CaSO_4 \cdot 2H_2O(s)] = 833 \text{ kJ mol}^{-1}$$

$$= + 484 \text{ kJ for 1 kg}$$

$$15. \quad \Delta H^\circ = \Delta H^\circ - \Delta S^\circ$$

$$= 17920 \text{ J mol}^{-1}$$

$$\Delta G^\circ = -2.303RT \log K_p$$

$$\log K_p = \frac{\Delta G^\circ}{2.303RT} = 7.22 \times 10^{-4} (p_{H_2O})^{\frac{3}{2}}$$

$$p_{H_2O} = 8.1 \times 10^{-3} \text{ atm}$$

$$16. \quad p_{H_2O} = 1, K_p = 1$$

$$\Delta G^\circ = 0 \text{ at eqm.}$$

$$\Delta H^\circ = T \Delta S^\circ \Rightarrow T = \frac{\Delta H^\circ}{\Delta S^\circ} = 380 \text{ K}$$

$$= 107^\circ\text{C}$$

Mathematics

PART – III

1. $x^2 + y^2 - 2x - 2y - 1 = 0$
 $x^2 + y^2 - 1 = 0$

Common chord is $-2x - 2y = 0 \Rightarrow 2x + 2y = 0$

$$y = -x$$

Points of intersection of circles are $\left(\pm \frac{1}{\sqrt{2}}, \mp \frac{1}{\sqrt{2}}\right)$

2. $P(E) = \frac{\sum_{k=1}^4 k}{\frac{8}{2} C_2} = \frac{10 \times 2}{8 \times 7} = \frac{5}{14}$

3. The line can be written as $y = mx$ and curve as $x^2 + y^2 = 4$

Let, $C(h, k)$ be a point on the circles and $A(\sqrt{3}, 1)$ be

given point then, $\frac{h+2\sqrt{3}}{3} = \alpha$

$$\Rightarrow h = 3\alpha - 2\sqrt{3}$$

$$\frac{k+2}{3} = m\alpha$$

$$\Rightarrow k = 3m\alpha - 2$$

Now, this point (h, k) lies on the circle

$$\Rightarrow (3\alpha - 2\sqrt{3})^2 + (3m\alpha - 2)^2 = 4$$

$$9\alpha^2 + 12 - 12\sqrt{3}\alpha + 9m^2\alpha^2 + 4 - 12m\alpha = 4$$

$$\Rightarrow 9(1+m^2)\alpha^2 - 12\alpha(\sqrt{3}+m) + 12 = 0$$

$$3(1+m^2)\alpha^2 - 4\alpha(\sqrt{3}+m) + 4 = 0$$

$$16(\sqrt{3}+m)^2 - 4 \times 3(1+m^2)(4) > 0$$

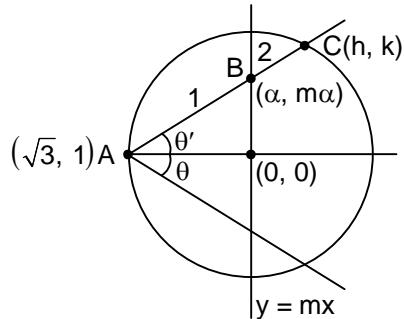
$$(\sqrt{3}+m)^2 - 3(1+m^2) > 0$$

$$3+m^2 + 2\sqrt{3}m - 3 - 3m^2 > 0$$

$$2\sqrt{3}m - 2m^2 > 0$$

$$2m^2 - 2\sqrt{3}m < 0$$

$$m \in (0, \sqrt{3})$$



4. Suppose m is an integer root of $x^4 - ax^3 - bx^2 - cx - d = 0$ as $d \neq 0$

$$\Rightarrow m \neq 0$$

(I) $m > 0$

$$m^4 - am^3 - bm^2 - cm = d$$

$$\Rightarrow d = m(m^3 - am^2 - bm - c)$$

$$\Rightarrow d \geq m$$

$$\text{Also, } m^4 - am^3 = bm^2 + cm + d$$

$$m^3(m-a) = bm^2 + cm + d$$

$$\Rightarrow m > a \Rightarrow \text{contradiction}$$

(II) $m < 0$

$$m = -n$$

$$\Rightarrow n^4 + an^3 - bn^2 + cn - d = 0$$

$$n^4 + n^2(an - b) + (cn - d) > 0$$

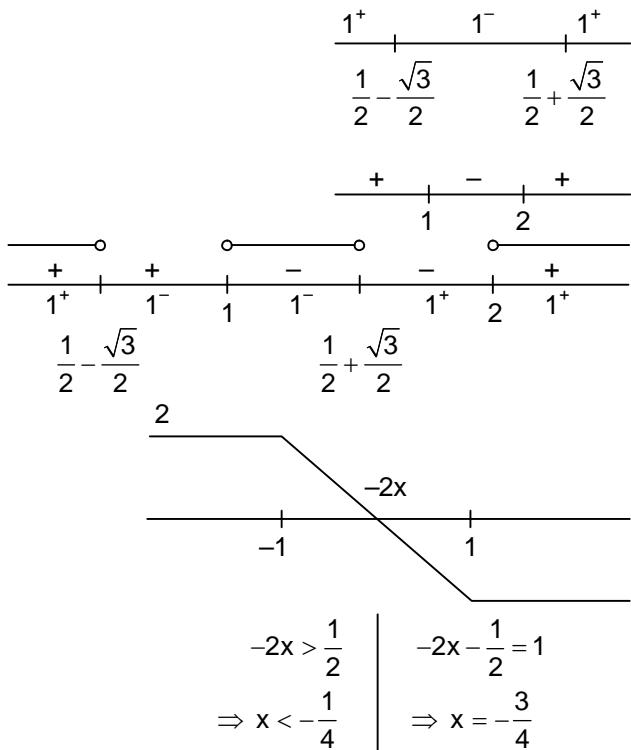
So, contradiction

Hence, equation has no integral solution

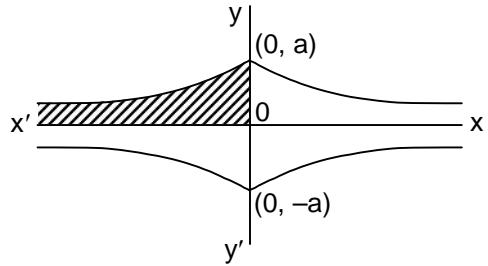
5. (I) $x^2 - x + \frac{1}{2}$
 $\left(x - \frac{1}{2}\right)^2 + \frac{1}{4} = 1$
 $\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

(II) $x^2 - 3x + 2$

(III) $|x - 1| - |x + 1| - \frac{1}{2}$
 \therefore set x can be $(-\infty, -\frac{1}{4}) \cup (-\frac{3}{4}, \infty)$



6. Required area, $A = 4 \int_0^{\pi/2} y \cdot \frac{dx}{dt} dt$
 $= 4 \int_0^{\pi/2} (a \sin t) \left(\frac{a}{\sin t} \cos^2 t \right) dt$
 $= 4a^2 \int_0^{\pi/2} \cos^2 t dt = \pi a^2$



7. Distance from origin, $D = \sqrt{x^2 + y^2 + z^2}$ where $P(x, y, z)$ is any point on the curve

$$D^2 = x^2 + y^2 + z^2 = x^2 + y^2 + \frac{2}{xy} \geq 2xy + \frac{2}{xy} \geq 4$$

$\Rightarrow D = 2$ and occurs at point(s) $(1, 1, \sqrt{2})$ and $(-1, -1, \sqrt{2})$, $(1, 1, -\sqrt{2})$, $(-1, -1, -\sqrt{2})$

8. Differentiating w.r.t. 'r', $2r dr = 2a \cos 2\theta d\theta$, $a = \frac{r^2}{\sin 2\theta}$
 $\Rightarrow r dr = \frac{r^2}{\sin 2\theta} \cos 2\theta d\theta$
 $\Rightarrow r \cdot \frac{d\theta}{dr} = \tan 2\theta$

So, differential equation of orthogonal trajectory, $r \cdot \frac{d\theta}{dr} = -\cot 2\theta$

$$\text{Solving } -\tan 2\theta d\theta = \frac{dr}{r}$$

$$\Rightarrow \ln(\cos 2\theta) = 2 \ln r + k \Rightarrow r^2 = c \cos 2\theta$$

9. Let $f(x) = x^3 + bx^2 + cx + 1$. $f(0) = 1 > 0$, $f(-1) = b - c < 0$

$$\text{So, } \alpha \in (-1, 0). \text{ So, } 2\tan^{-1}(\cosec \alpha) + \tan^{-1}(2\sin \alpha \sec^2 \alpha)$$

$$= 2\tan^{-1}\left(\frac{1}{\sin \alpha}\right) + \tan^{-1}\left(\frac{2\sin \alpha}{1 - \sin^2 \alpha}\right) = 2\left[\tan^{-1}\left(\frac{1}{\sin \alpha}\right) + \tan^{-1}(\sin \alpha)\right]$$

$$= 2\left(-\frac{\pi}{2}\right) = -\pi \quad (\text{as } \sin \alpha < 0)$$

10. $\frac{1}{1+z^2}$ is continuous every where except where $1+z^2 = 0$

$\Rightarrow z = \pm i$ so when $|z| < 1$ then above points are excluded so $f(z)$ is continuous

11. Continuity : for any point z , $\lim_{z \rightarrow z_0} f(z) = \lim_{z \rightarrow z_0} x = x_0 = f(z_0)$. Non differentiable : for any point z ,

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{\Delta x}{\Delta x + i\Delta y} \text{ now } \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{\Delta x}{\Delta x + i\Delta y} = 0 \text{ and } \lim_{\substack{\Delta y \rightarrow 0 \\ \Delta x \rightarrow 0}} \frac{\Delta x}{\Delta x + i\Delta y} = 1$$

12. $y_1 = \cos(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$

$$\sqrt{1-x^2} y_1 = m \cos(m \sin^{-1} x)$$

$$\Rightarrow (1-x^2) y_2 - xy_1 + m^2 y = 0$$

$$\left[(1-x^2) y_{n+2} + n(-2x) y_{n+1} + \frac{n(n-1)}{2} (-2) y_n \right] - [xy_{n+1} + n.1.y_n] + m^2 y_n = 0$$

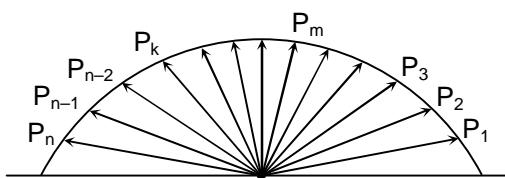
Simplifying we get, $(1-x^2) y_{n+2} - (2n+1) y_{n+1} - (n^2 - m^2) y_n = 0$

13. $\frac{dy}{dt} = \frac{3at(2-t^3)}{(1+t^3)^2}$

$$\frac{dx}{dt} = 6a \frac{\left(\frac{1}{2} - t^3\right)}{(1+t^3)^2}$$

Now $\frac{dy}{dx} \rightarrow \infty$ at $t \rightarrow (2)^{1/3}$

- 14.-16. Here, $\vec{P}_v = \vec{P}_2 + \vec{P}_K$ so \vec{P}_v will make acute angle with all the vectors from $\vec{P}_2, \vec{P}_3, \dots, \vec{P}_k$



$$\text{So, } \overrightarrow{OP_v} \cdot (\overrightarrow{OP_2} + \overrightarrow{OP_3} + \dots + \overrightarrow{OP_{k-1}}) > 0$$

Again, if n is odd $\Rightarrow n = 2k - 1$

$$\overrightarrow{OP_1} + \overrightarrow{OP_{2k-1}} = \overrightarrow{OP_k}$$

Now $\overrightarrow{OP_k}$ will make acute angle with all the vectors from $\overrightarrow{OP_1}, \overrightarrow{OP_2}, \dots, \overrightarrow{OP_{2k-1}}$

$$\overrightarrow{OP_k} \cdot (\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_k} + \overrightarrow{OP_{k+1}} + \dots + \overrightarrow{OP_{2k-1}}) \geq |\overrightarrow{OP_k}| |\overrightarrow{OP_1} + \dots + \overrightarrow{OP_{2k-1}}| = |\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_n}| = 1$$

$$\text{Now, } \overrightarrow{OP_k} \cdot (\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_{2k-1}}) \leq |\overrightarrow{OP_k}| |\overrightarrow{OP_1} + \dots + \overrightarrow{OP_{2k-1}}| = |\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_n}|$$

Again, \vec{V} makes acute angle with all the vectors $\overrightarrow{OP_1}, \overrightarrow{OP_2}, \dots, \overrightarrow{OP_7}$

$$\text{So, } |\vec{V} + \vec{S}| = |\overrightarrow{OP_1} + \overrightarrow{OP_2} + \dots + \overrightarrow{OP_7}| \geq 1 \Rightarrow |\vec{V}| + |\vec{S}| \geq 1$$

17. Eigen values are roots of the equation $A - \lambda X = 0 \Rightarrow (A - \lambda I)X = 0$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 4)(\lambda - 6) = 0$$

$$\therefore \lambda = 4, 6$$

18. When $\lambda = 6$

$$\begin{bmatrix} 8-6 & -4 \\ 2 & 2-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2x_1 - 4x_2 = 0 \Rightarrow x_1 = 2x_2$$

$$\therefore X = C \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

19. $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \Rightarrow \lambda = 1, 2, 3,$$

For $\lambda = 3$

$$X = C \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \text{ which is orthogonal}$$

SECTION – B

1. (A) Assume sphere as, $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

Now P, Q, R are $(-2u, 0, 0), (0, -2v, 0), (0, 0, -2w)$

$$\text{So, equation of plane is } \frac{x}{-2u} + \frac{y}{-2v} + \frac{z}{-2w} = 1$$

$$\text{Since it passes through } (1, 2, 3) \quad \frac{1}{-2u} + \frac{2}{-2v} + \frac{3}{-2w} = 1$$

If centre is $(x, y, z) \equiv (-u, -v, -w)$

$$\therefore \text{locus is } \frac{1}{x} + \frac{2}{y} + \frac{3}{z} = 2$$

- (B) Assume equation of sphere as, $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

It passes through $(\alpha, 0, 0), (0, \beta, 0), (0, 0, \gamma)$

$$\Rightarrow u = -\frac{\alpha}{2}, v = -\frac{\beta}{2}, \omega = -\frac{r}{2}$$

$$\text{Radius} = 1 \Rightarrow \sqrt{\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 1$$

$\alpha^2 + \beta^2 + \gamma^2 = 4$ if (x, y, z) are coordinates of centroid

$$\Rightarrow x = \frac{\alpha}{3}, y = \frac{\beta}{3}, z = \frac{\gamma}{3} \therefore x^2 + y^2 + z^2 = \frac{4}{9}$$

(C) $P \equiv (\alpha, 0, 0), Q \equiv (0, \beta, 0), \angle OPQ = 15^\circ$

$$\Rightarrow \tan 15^\circ = \frac{\beta}{\alpha} \quad \dots \dots (1)$$

Now if sphere is made with PQ as diameter $(x - \alpha)x + (y - \beta)y + z^2 = 0$

$$\Rightarrow x^2 + y^2 + z^2 = ax + by \quad \dots \dots (2)$$

A plane through PQ parallel to z-axis is

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1 \quad \dots \dots (3)$$

$$\text{Using (1), (2), (3) we get } x^2 + y^2 + z^2 = (\alpha x + \beta y) \left(\frac{x}{\alpha} + \frac{y}{\beta} \right)$$

$$\Rightarrow z^2 = xy(\tan 15^\circ + \cot 15^\circ) \Rightarrow 2xy - \frac{z^2}{2} = 0$$

(D) Using family of planes any plane through line of intersection is $x + y - 6 + \lambda(x - 2z - 3) = 0$

$$\text{Now, its distance from centre of sphere is radius } \frac{|-6 - 3\lambda|}{\sqrt{(1+\lambda)^2 + 1 + 4\lambda^2}} = 3, \lambda = 1, -\frac{1}{2}$$

$$\therefore \text{Equation is } x + 2y + 2z = 9; 2x + y - 2z = 9$$

2. **(A)** Let $p = \cos x \sin y \cos z$. As, $\frac{\pi}{2} \geq y \geq z, \sin(y - z) \geq 0$

$$p = \frac{1}{2} \cos z [\sin(x+y) - \sin(x-y)] \leq \frac{1}{2} \cos^2 z$$

As, $\sin(x-y) \geq 0$ and $\sin(x+y) = \cos z$

$$p \leq \frac{1}{4}(1 + \cos 2z) \leq \frac{1}{4} \left(1 + \cos \frac{\pi}{6}\right) = \frac{2 + \sqrt{3}}{8}$$

(B) The equation can be written as $3u^2 + 8u + 3 = 0 \Rightarrow u_1 = \frac{-8 + 2\sqrt{7}}{6} \Rightarrow u_2 = \frac{-8 - 2\sqrt{7}}{6}$

Clearly, u_1, u_2 are negative so $\frac{\pi}{2} < x_1, x_2 < \pi$

$$\Rightarrow \pi < x_1 + x_2 < 2\pi \text{ as } \cot x \tan x = 1 = \cot x \cdot \cot\left(\frac{\pi}{2} - x\right) = \cot x \cdot \cot\left(\frac{3\pi}{2} - x\right)$$

$$\therefore x_1 + x_2 = \frac{3\pi}{2} \text{ and another pair, } x'_1 + x'_2 = \frac{7\pi}{2} [\pi < x'_1, x'_2 < 2\pi]$$

$$\therefore x_1 + x'_1 + x_2 + x'_2 = 5\pi$$

(C) $|EF| \geq |E_1F_1| = a - [|BF| \cos B + |CF| \cos C]$

$|DE| \geq c - [|AE| \cos A + |BD| \cos B]$

$|FD| \geq b - [|CD| \cos C + |AF| \cos A]$

$$|DC| + |CE| = |EA| + |AF| = |FB| + |BD| = \frac{1}{3}(a + b + c)$$

$\therefore |DE| + |EF| + |FD|$

$$\geq a + b + c - \frac{1}{3}((a + b + c)(\cos A + \cos B + \cos C)) \geq \frac{1}{2}(a + b + c)$$

So, minimum value is $\frac{1}{2}$

(D) $\sin^2 x_1 + \sin^2 x_2 + \dots + \sin^2 x_{10} = 1$

$$\cos x_i = \sqrt{\sum_{j=1}^{10} \sin^2 x_j}$$

$$\text{For each } 1 \leq i \leq 10 \text{ we have } \cos x_i = \sqrt{\sum_{j \neq i} \sin^2 x_j} \geq \frac{\sum \sin x_j}{3}$$

$$\Rightarrow \sum_{i=1}^{10} \cos x_i \geq \sum_{i=1}^{10} \sum_{j \neq i} \frac{\sin x_j}{3} = \sum_{i=1}^{10} 9 \cdot \frac{\sin x_i}{3}$$

$$\Rightarrow \frac{\cos x_1 + \cos x_2 + \dots + \cos x_{10}}{\sin x_1 + \sin x_2 + \dots + \sin x_{10}} \geq 3$$

3. (A) Property of ellipse

(B) Normal chord is made between points $A(2, 2\sqrt{2})$ and $B(8, -4\sqrt{2})$ so its length is $6\sqrt{3}$

(C) $Q \equiv (4, 2)$, $S : (x - 4)^2 + (y - 2)^2 = 16$ then required circle is $(x - 1)^2 + (y - 1)^2 + \lambda(x - y) = 0$

$$\text{Here, } \lambda = 9 \text{ and radius} = \frac{\sqrt{162}}{2}$$

(D) $x^2 = \frac{3y^2}{2} + c$

$$e = \sqrt{\frac{5}{3}} \quad (c > 0) \text{ and } e = \sqrt{\frac{5}{2}} \quad (c < 0)$$

