mdp

November 22, 2020

Citation – Used multiple resources to understand and compile code for MDP algorithm.

[1]:
```
import gym
import time
import random
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from gym.envs.toy_text.frozen_lake import generate_random_map
from hiive.mdptoolbox.mdp import PolicyIteration, ValueIteration, QLearning,␣
↪PolicyIterationModified
from hiive.mdptoolbox.example import forest
from timeit import default_timer as timer
from datetime import timedelta

import warnings
warnings.filterwarnings('ignore')
```

1 Functions

[2]:
```
def frozen_lake_environment(size=4,p=0.80):
    __size = size
    environment  = 'FrozenLake-v0'
    desc_map = generate_random_map(size=__size,p=p)
    env = gym.make(environment,desc=desc_map,is_slippery=True)
    env.reset()

#     env = gym.make(environment)
    states = env.observation_space.n
    actions = env.action_space.n
    env = env.unwrapped
    desc = env.unwrapped.desc
    return env, desc, states, actions, __size
```

[3]:
```
def frozen_lake_get_PR(env,states,actions):
    prob_transitions = np.zeros((actions, states, states))
```

```
    rewards = np.zeros((states,actions))
    for state in env.P:
        for action in env.P[state]:
            for opt in env.P[state][action]:
                prob_transitions[action][state][opt[1]] += opt[0]
                rewards[state][action] += opt[2]
    return prob_transitions, rewards
```

[4]:
```
def forest_get_PR(input_state):
    prob_transitions, rewards = forest(S=input_state)
    return prob_transitions, rewards
```

[5]:
```
def␣
↪gamma_analysis(gamma_prob_transitions,gamma_rewards,gamma_size,gamma_gammas=np.
↪arange(0.1, 0.99, 0.01)):
    VI_ary, PI_ary = [],[]
#     gammas = np.arange(0.1, 0.99, 0.04)
    gammas = gamma_gammas
    for gamma in gammas:
        vi = ValueIteration(gamma_prob_transitions,gamma_rewards, gamma)
        vi.run()
        vi_time = vi.time
        vi_max_value = np.amax(vi.V)
        vi_mean_value = np.mean(vi.V)
        vi_value = vi.V
        vi_iters = vi.iter
        vi_policy = vi.policy
        VI_ary.
↪append(['VI',gamma_size,gamma,vi_time,vi_max_value,vi_mean_value,vi_iters,vi_policy,vi_valu

        pi = PolicyIteration(gamma_prob_transitions,gamma_rewards, gamma)
        pi.run()
        pi_time = pi.time
        pi_max_value = np.amax(pi.V)
        pi_mean_value = np.mean(pi.V)
        pi_value = pi.V
        pi_iters = pi.iter
        pi_policy = pi.policy
        PI_ary.
↪append(['PI',gamma_size,gamma,pi_time,pi_max_value,pi_mean_value,pi_iters,pi_policy,pi_valu

    df_vi = pd.
↪DataFrame(VI_ary,columns=['iter_type','size','gamma','time','max_value','mean_value','itera
    df_pi = pd.
↪DataFrame(PI_ary,columns=['iter_type','size','gamma','time','max_value','mean_value','itera
    return df_vi, df_pi
```

[6]:
```
def plot_gamma_analysis(gamma_df_vi, gamma_df_pi):
    fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(20,4))
    sns.lineplot(gamma_df_vi['gamma'], gamma_df_vi['iterations'], linestyle='-.
 ↪', label = "Value Iteration", ax=axs[0], alpha=0.5, color='red')
    sns.lineplot(gamma_df_pi['gamma'], gamma_df_pi['iterations'],␣
 ↪linestyle='--', label = "Policy Iteration", ax=axs[0], alpha=0.5,␣
 ↪color='blue')
    axs[0].set_title('Iterations vs Gamma')

    sns.lineplot(gamma_df_vi['gamma'], gamma_df_vi['time'], linestyle='-.',␣
 ↪label = "Value Iteration", ax=axs[1], alpha=0.5, color='red')
    sns.lineplot(gamma_df_pi['gamma'], gamma_df_pi['time'], linestyle='--',␣
 ↪label = "Policy Iteration", ax=axs[1], alpha=0.5, color='blue')
    axs[1].set_title('Time vs Gamma')

    sns.lineplot(gamma_df_vi['gamma'], gamma_df_vi['max_value'], linestyle='-.
 ↪', label = "Value Iteration", ax=axs[2], alpha=0.5, color='red')
    sns.lineplot(gamma_df_pi['gamma'], gamma_df_pi['max_value'],␣
 ↪linestyle='--', label = "Policy Iteration", ax=axs[2], alpha=0.5,␣
 ↪color='blue')
    axs[2].set_title('Max Value vs Gamma')

    sns.lineplot(gamma_df_vi['gamma'], gamma_df_vi['mean_value'], linestyle='-.
 ↪', label = "Value Iteration", ax=axs[3], alpha=0.5, color='red')
    sns.lineplot(gamma_df_pi['gamma'], gamma_df_pi['mean_value'],␣
 ↪linestyle='--', label = "Policy Iteration", ax=axs[3], alpha=0.5,␣
 ↪color='blue')
    axs[3].set_title('Mean Value vs Gamma')


    for ax in axs.flat:
        ax.legend(loc='best')
        ax.minorticks_on()
        ax.grid(b=True, which='major', color='k', linestyle='-', alpha=0.1)
        ax.grid(b=True, which='minor', color='r', linestyle='-', alpha=0.05)
    fig.tight_layout()
```

[7]:
```
def␣
 ↪epsilon_analysis(epsilon_prob_transitions,epsilon_rewards,epsilon_size,epsilon_epsilons=np.
 ↪arange(0.001,0.05, 0.001),epsilon_gamma=0.99):
    VI_ary, PI_ary = [],[]
    gamma = epsilon_gamma
#     epsilons = np.arange(0.001,0.05, 0.005)
    epsilons = epsilon_epsilons

    for epsilon in epsilons:
```

```
        vi = ValueIteration(epsilon_prob_transitions,epsilon_rewards, gamma,␣
↪epsilon=epsilon)
        vi.run()
        vi_time = vi.time
        vi_max_value = np.amax(vi.V)
        vi_mean_value = np.mean(vi.V)
        vi_value = vi.V
        vi_iters = vi.iter
        vi_policy = vi.policy
        VI_ary.
↪append(['VI',epsilon_size,epsilon,vi_time,vi_max_value,vi_mean_value,vi_iters,vi_policy,vi_

        pi = PolicyIterationModified(epsilon_prob_transitions,epsilon_rewards,␣
↪gamma, epsilon=epsilon)
        pi.run()
        pi_time = pi.time
        pi_max_value = np.amax(pi.V)
        pi_mean_value = np.mean(pi.V)
        pi_value = pi.V
        pi_iters = pi.iter
        pi_policy = pi.policy
        PI_ary.
↪append(['PI',epsilon_size,epsilon,pi_time,pi_max_value,pi_mean_value,pi_iters,pi_policy,pi_

    df_vi = pd.
↪DataFrame(VI_ary,columns=['iter_type','size','epsilon','time','max_value','mean_value','ite
    df_pi = pd.
↪DataFrame(PI_ary,columns=['iter_type','size','epsilon','time','max_value','mean_value','ite
    return df_vi, df_pi
```

[8]:
```
def plot_epsilon_analysis(epsilon_df_vi, epsilon_df_pi):
    fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(20,4))
    sns.lineplot(epsilon_df_vi['epsilon'], epsilon_df_vi['iterations'],␣
↪linestyle='-.', label = "Value Iteration", ax=axs[0], alpha=0.5, color='red')
    sns.lineplot(epsilon_df_pi['epsilon'], epsilon_df_pi['iterations'],␣
↪linestyle='--', label = "Policy Iteration", ax=axs[0], alpha=0.5,␣
↪color='blue')
    axs[0].set_title('Iterations vs Epsilon')

    sns.lineplot(epsilon_df_vi['epsilon'], epsilon_df_vi['time'], linestyle='-.
↪', label = "Value Iteration", ax=axs[1], alpha=0.5, color='red')
    sns.lineplot(epsilon_df_pi['epsilon'], epsilon_df_pi['time'],␣
↪linestyle='--', label = "Policy Iteration", ax=axs[1], alpha=0.5,␣
↪color='blue')
    axs[1].set_title('Time vs Epsilon')
```

```
    sns.lineplot(epsilon_df_vi['epsilon'], epsilon_df_vi['max_value'],␣
↪linestyle='-.', label = "Value Iteration", ax=axs[2], alpha=0.5, color='red')
    sns.lineplot(epsilon_df_pi['epsilon'], epsilon_df_pi['max_value'],␣
↪linestyle='--', label = "Policy Iteration", ax=axs[2], alpha=0.5,␣
↪color='blue')
    axs[2].set_title('Max Value vs Epsilon')

    sns.lineplot(epsilon_df_vi['epsilon'], epsilon_df_vi['mean_value'],␣
↪linestyle='-.', label = "Value Iteration", ax=axs[3], alpha=0.5, color='red')
    sns.lineplot(epsilon_df_pi['epsilon'], epsilon_df_pi['mean_value'],␣
↪linestyle='--', label = "Policy Iteration", ax=axs[3], alpha=0.5,␣
↪color='blue')
    axs[3].set_title('Mean Value vs Epsilon')


    for ax in axs.flat:
        ax.legend(loc='best')
        ax.minorticks_on()
        ax.grid(b=True, which='major', color='k', linestyle='-', alpha=0.1)
        ax.grid(b=True, which='minor', color='r', linestyle='-', alpha=0.05)
    fig.tight_layout()
```

```
[9]: def optimal_policy(optimal_df_vi,optimal_df_pi,optimal_size,epsilon,env_type):
    optimal_vi = optimal_df_vi[optimal_df_vi['epsilon'] == epsilon]
    optimal_pi = optimal_df_pi[optimal_df_pi['epsilon'] == epsilon]

    print(optimal_vi.shape)
    print(optimal_pi.shape)
    print(np.array(list(optimal_vi['policy'])).shape)
    print(np.array(list(optimal_pi['policy'])).shape)

    if env_type == 'frozen-lake':
        vi_policy_arr = np.array(list(optimal_vi['policy'])).
↪reshape(optimal_size,optimal_size).astype(str)
        vi_value_arr = np.round(np.array(list(optimal_vi['value'])).
↪reshape(optimal_size,optimal_size),2)


        vi_policy_arr[vi_policy_arr=='0'] = '←'
        vi_policy_arr[vi_policy_arr=='1'] = '↓'
        vi_policy_arr[vi_policy_arr=='2'] = '→'
        vi_policy_arr[vi_policy_arr=='3'] = '↑'

        vi_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(vi_policy_arr.
↪flatten(), vi_value_arr.flatten())])).reshape(optimal_size,optimal_size)
```

```
    elif env_type == 'forest':

        vi_policy_arr = np.array(list(optimal_vi['policy'])).
↪reshape(optimal_size).astype(str)
        vi_value_arr = np.round(np.array(list(optimal_vi['value'])).
↪reshape(optimal_size),2)

        vi_policy_arr[vi_policy_arr=='0'] = 'W'
        vi_policy_arr[vi_policy_arr=='1'] = 'C'

        vi_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(vi_policy_arr.
↪flatten(), vi_value_arr.flatten())])).reshape(optimal_size)


    if env_type == 'frozen-lake':
        pi_policy_arr = np.array(list(optimal_pi['policy'])).
↪reshape(optimal_size,optimal_size).astype(str)
        pi_value_arr = np.round(np.array(list(optimal_pi['value'])).
↪reshape(optimal_size,optimal_size),2)

        pi_policy_arr[pi_policy_arr=='0'] = '←'
        pi_policy_arr[pi_policy_arr=='1'] = '↓'
        pi_policy_arr[pi_policy_arr=='2'] = '→'
        pi_policy_arr[pi_policy_arr=='3'] = '↑'

        pi_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(pi_policy_arr.
↪flatten(), pi_value_arr.flatten())])).reshape(optimal_size,optimal_size)
    elif env_type == 'forest':

        pi_policy_arr = np.array(list(optimal_pi['policy'])).
↪reshape(optimal_size).astype(str)
        pi_value_arr = np.round(np.array(list(optimal_pi['value'])).
↪reshape(optimal_size),2)

        pi_policy_arr[pi_policy_arr=='0'] = 'W'
        pi_policy_arr[pi_policy_arr=='1'] = 'C'

        pi_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(pi_policy_arr.
↪flatten(), pi_value_arr.flatten())])).reshape(optimal_size)



    return vi_value_arr, vi_policy_viz, pi_value_arr, pi_policy_viz
```

[10]:
```
def plot_optimnal_policy(plot_vi_value_arr, plot_vi_policy_viz,␣
↪plot_pi_value_arr, plot_pi_policy_viz,plot_size,env_type):
```

```
    if env_type == 'frozen-lake':
        plt.figure(figsize=(plot_size,plot_size))
        plt.title("Value Iteration: Optimal Policy")
        sns.heatmap(plot_vi_value_arr, annot=plot_vi_policy_viz, fmt="")

        plt.figure(figsize=(plot_size,plot_size))
        plt.title("Policy Iteration: Optimal Policy")
        sns.heatmap(plot_pi_value_arr, annot=plot_pi_policy_viz, fmt="")
    elif env_type == 'forest':
        print("Value Iteration: Optimal Policy")
        line=""
        for val in plot_vi_policy_viz:
            line += (" {} |".format(val))
        print(line)

        print("Value Iteration: Optimal Policy")
        line=""
        for val in plot_pi_policy_viz:
            line += (" {} |".format(val))
        print(line)
```

[11]:
```
def size_analysis(size_env_type,size_sizes,size_gamma=0.99):

#     fl_size = 8
# fl_p = 0.85
# env, desc, states, actions, size = frozen_lake_environment(fl_size,fl_p)

# fl_prob_transitions, fl_rewards = frozen_lake_get_PR(env,states,actions)

# fm_size = 1000
# fm_prob_transitions, fm_rewards = forest_get_PR(fm_size)

    sizes = size_sizes
    gamma = size_gamma
    VI_ary, PI_ary = [],[]

    for size in sizes:
        if size_env_type == 'frozen-lake':
            fl_p = 0.85
            env, desc, states, actions, _ = frozen_lake_environment(size,fl_p)
            size_prob_transitions, size_rewards =␣
↪frozen_lake_get_PR(env,states,actions)
        elif size_env_type == 'forest':
            size_prob_transitions, size_rewards = forest_get_PR(size)

        vi = ValueIteration(size_prob_transitions, size_rewards, gamma)
```

```
        vi.run()
        vi_time = vi.time
        vi_max_value = np.amax(vi.V)
        vi_mean_value = np.mean(vi.V)
        vi_value = vi.V
        vi_iters = vi.iter
        vi_policy = vi.policy
        VI_ary.
↪append(['VI',size,vi_time,vi_max_value,vi_mean_value,vi_iters,vi_policy,vi_value])

        pi = PolicyIterationModified(size_prob_transitions, size_rewards, gamma)
        pi.run()
        pi_time = pi.time
        pi_max_value = np.amax(pi.V)
        pi_mean_value = np.mean(pi.V)
        pi_value = pi.V
        pi_iters = pi.iter
        pi_policy = pi.policy
        PI_ary.
↪append(['PI',size,pi_time,pi_max_value,pi_mean_value,pi_iters,pi_policy,pi_value])


    df_vi = pd.
↪DataFrame(VI_ary,columns=['iter_type','size','time','max_value','mean_value','iterations','
    df_pi = pd.
↪DataFrame(PI_ary,columns=['iter_type','size','time','max_value','mean_value','iterations','
    return df_vi, df_pi
```

[12]:

```
def plot_size_analysis(size_df_vi, size_df_pi):
    fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(20,4))
    sns.lineplot(size_df_vi['size'], size_df_vi['iterations'], linestyle='-.',␣
↪label = "Value Iteration", ax=axs[0], alpha=0.5, color='red')
    sns.lineplot(size_df_pi['size'], size_df_pi['iterations'], linestyle='--',␣
↪label = "Policy Iteration", ax=axs[0], alpha=0.5, color='blue')
    axs[0].set_title('Iterations vs size')

    sns.lineplot(size_df_vi['size'], size_df_vi['time'], linestyle='-.', label␣
↪= "Value Iteration", ax=axs[1], alpha=0.5, color='red')
    sns.lineplot(size_df_pi['size'], size_df_pi['time'], linestyle='--', label␣
↪= "Policy Iteration", ax=axs[1], alpha=0.5, color='blue')
    axs[1].set_title('Time vs size')

    sns.lineplot(size_df_vi['size'], size_df_vi['max_value'], linestyle='-.',␣
↪label = "Value Iteration", ax=axs[2], alpha=0.5, color='red')
    sns.lineplot(size_df_pi['size'], size_df_pi['max_value'], linestyle='--',␣
↪label = "Policy Iteration", ax=axs[2], alpha=0.5, color='blue')
```

```
    axs[2].set_title('Max Value vs size')

    sns.lineplot(size_df_vi['size'], size_df_vi['mean_value'], linestyle='-.',␣
 ↪label = "Value Iteration", ax=axs[3], alpha=0.5, color='red')
    sns.lineplot(size_df_pi['size'], size_df_pi['mean_value'], linestyle='--',␣
 ↪label = "Policy Iteration", ax=axs[3], alpha=0.5, color='blue')
    axs[3].set_title('Mean Value vs size')


    for ax in axs.flat:
        ax.legend(loc='best')
        ax.minorticks_on()
        ax.grid(b=True, which='major', color='k', linestyle='-', alpha=0.1)
        ax.grid(b=True, which='minor', color='r', linestyle='-', alpha=0.05)
    fig.tight_layout()
```

[13]:
```
def␣
 ↪ql_gamma_analysis(ql_gamma_prob_transitions,ql_gamma_rewards,ql_gamma_size,ql_gamma_gammas=
 ↪arange(0.1, 0.99, 0.01),ql_gamma_n_iter=100000):

    gammas = ql_gamma_gammas
    Q_ary = []

    for gamma in gammas:
        ql = QLearning(ql_gamma_prob_transitions,ql_gamma_rewards,␣
 ↪gamma,n_iter=ql_gamma_n_iter)
        ql.run()
        ql_time = ql.time
        ql_max_value = np.amax(ql.V)
        ql_mean_value = np.mean(ql.V)
        ql_value = ql.V
        ql_q_matrix = ql.Q
        ql_policy = ql.policy
#         ql_mean_discrepancy = ql.mean_discrepancy
#         Q_ary.
 ↪append(['QL',gamma_size,gamma,ql_time,ql_max_value,ql_mean_value,ql_q_matrix,ql_policy,ql_v
        Q_ary.
 ↪append(['QL',ql_gamma_size,gamma,ql_time,ql_max_value,ql_mean_value,ql_q_matrix,ql_policy,q

#     df_ql = pd.
 ↪DataFrame(VI_ary,columns=['iter_type','size','gamma','time','max_value','mean_value','q_mat
    df_gamma_ql = pd.
 ↪DataFrame(Q_ary,columns=['iter_type','size','gamma','time','max_value','mean_value','q_matr
    return df_gamma_ql
```

[14]:
```
def plot_ql_gamma_analysis(plot_gamma_df_ql):

    fig, axs = plt.subplots(nrows=1, ncols=3, figsize=(20,4))


#     sns.lineplot(plot_gamma_df_ql['gamma'], plot_gamma_df_ql['iterations'],␣
↪linestyle='-.', label = "Value Iteration", ax=axs[0], alpha=0.5, color='red')
#     axs[0].set_title('Iterations vs Gamma')

    sns.lineplot(plot_gamma_df_ql['gamma'], plot_gamma_df_ql['time'],␣
↪linestyle='-.', label = "Q Learning", ax=axs[0], alpha=0.5, color='red')
    axs[0].set_title('Time vs Gamma')

    sns.lineplot(plot_gamma_df_ql['gamma'], plot_gamma_df_ql['max_value'],␣
↪linestyle='-.', label = "Q Learning", ax=axs[1], alpha=0.5, color='red')
    axs[1].set_title('Max Value vs Gamma')

    sns.lineplot(plot_gamma_df_ql['gamma'], plot_gamma_df_ql['mean_value'],␣
↪linestyle='-.', label = "Q Learning", ax=axs[2], alpha=0.5, color='red')
    axs[2].set_title('Mean Value vs Gamma')


    for ax in axs.flat:
        ax.legend(loc='best')
        ax.minorticks_on()
        ax.grid(b=True, which='major', color='k', linestyle='-', alpha=0.1)
        ax.grid(b=True, which='minor', color='r', linestyle='-', alpha=0.05)
    fig.tight_layout()
```

[15]:
```
def␣
↪ql_n_iter_analysis(ql_n_iter_prob_transitions,ql_n_iter_rewards,ql_n_iter_size,ql_n_iter_ga
↪99,ql_n_iter_epsilon=0.9,ql_n_iter_n_iters=[10000,50000,100000]):

    n_iters = ql_n_iter_n_iters
    Q_ary = []
    run_stats = []

    for n_iter in n_iters:
#         ql = QLearning(ql_n_iter_prob_transitions,ql_n_iter_rewards,␣
↪ql_n_iter_gamma,epsilon=ql_n_iter_epsilon,n_iter=n_iter)
        ql = QLearning(ql_n_iter_prob_transitions,ql_n_iter_rewards,␣
↪ql_n_iter_gamma,epsilon=ql_n_iter_epsilon)
        ql.max_iter = n_iter
        ql_run_stat = ql.run()
        run_stats.extend(ql_run_stat)
#         print(run_stats)
```

```
        ql_time = ql.time
        ql_max_value = np.amax(ql.V)
        ql_mean_value = np.mean(ql.V)
        ql_value = ql.V
        ql_q_matrix = ql.Q
        ql_policy = ql.policy
#         ql_mean_discrepancy = ql.mean_discrepancy
#         Q_ary.
 ↪append(['QL',gamma_size,gamma,ql_time,ql_max_value,ql_mean_value,ql_q_matrix,ql_policy,ql_v
        Q_ary.
 ↪append(['QL',ql_n_iter_size,n_iter,ql_time,ql_max_value,ql_mean_value,ql_q_matrix,ql_policy

#     df_ql = pd.
 ↪DataFrame(VI_ary,columns=['iter_type','size','gamma','time','max_value','mean_value','q_mat
    df_n_iter_ql = pd.
 ↪DataFrame(Q_ary,columns=['iter_type','size','iterations','time','max_value','mean_value','q
    df_n_iter_run_stat = pd.DataFrame.from_dict(ql_run_stat,orient='columns')
    return df_n_iter_ql, df_n_iter_run_stat
```

[16]:
```
def plot_ql_n_iter_analysis(plot_n_iter_df_ql,plot_df_n_iter_run_stat):

    fig, axs = plt.subplots(nrows=1, ncols=3, figsize=(20,5))


#     sns.lineplot(plot_n_iter_df_ql['gamma'], plot_n_iter_df_ql['iterations'],␣
 ↪linestyle='-.', label = "Value Iteration", ax=axs[0], alpha=0.5, color='red')
#     axs[0].set_title('Iterations vs Gamma')

    sns.lineplot(plot_n_iter_df_ql['iterations'], plot_n_iter_df_ql['time'],␣
 ↪linestyle='-.', label = "Q Learning", ax=axs[0], alpha=0.5, color='red')
    axs[0].set_title('Time vs Iterations')

    sns.lineplot(plot_n_iter_df_ql['iterations'],␣
 ↪plot_n_iter_df_ql['max_value'], linestyle='-.', label = "Q Learning",␣
 ↪ax=axs[1], alpha=0.5, color='red')
    axs[1].set_title('Max Value vs Iterations')

    sns.lineplot(plot_n_iter_df_ql['iterations'],␣
 ↪plot_n_iter_df_ql['mean_value'], linestyle='-.', label = "Q Learning",␣
 ↪ax=axs[2], alpha=0.5, color='red')
    axs[2].set_title('Mean Value vs Iterations')

#     sns.lineplot(plot_df_n_iter_run_stat['Iteration'],␣
 ↪plot_df_n_iter_run_stat['Mean V'], linestyle='-.', label = "Q Learning",␣
 ↪ax=axs[3], alpha=0.5, color='red')
#     axs[3].set_title('Mean Value vs Iterations')
```

```python
for ax in axs.flat:
    ax.legend(loc='best')
    ax.minorticks_on()
    ax.grid(b=True, which='major', color='k', linestyle='-', alpha=0.1)
    ax.grid(b=True, which='minor', color='r', linestyle='-', alpha=0.05)
fig.tight_layout()
```

2 Set Environments

2.1 Frozen Lake

[21]:
```python
fl_size = 8
fl_p = 0.95
fl_env, fl_desc, fl_states, fl_actions, fl_size =␣
↪frozen_lake_environment(fl_size,fl_p)
```

[22]:
```python
fl_prob_transitions, fl_rewards =␣
↪frozen_lake_get_PR(fl_env,fl_states,fl_actions)
```

[ ]:
```python

```

[ ]:
```python

```

2.2 Forest Management

[23]:
```python
fm_size = 1000
fm_prob_transitions, fm_rewards = forest_get_PR(fm_size)
```

[24]:
```python
fm_states = fm_size
fm_actions = 2
```

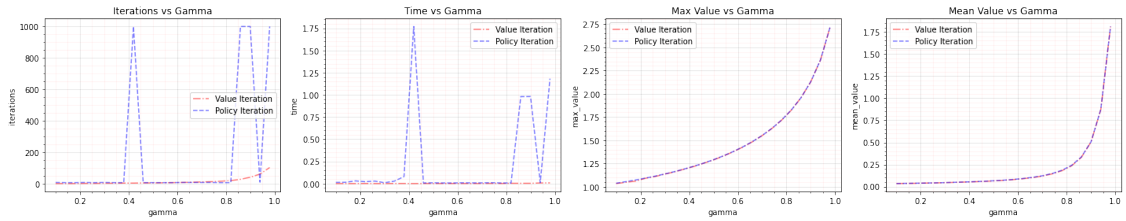
3 Value Iteration and Policy Iteration

3.1 Gamma Analysis

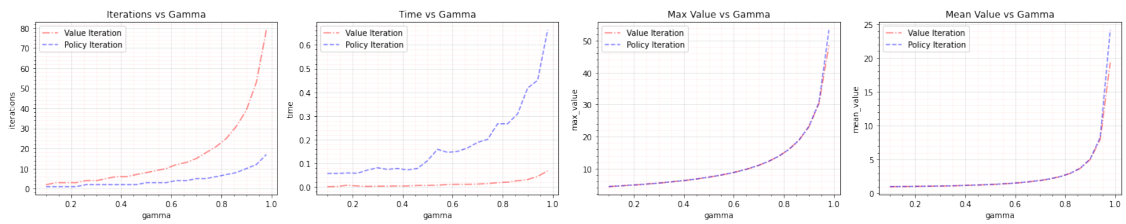
[25]:
```python
fl_gammas = np.arange(0.1, 0.99, 0.04)
fl_df_gamma_vi, fl_df_gamma_pi = gamma_analysis(fl_prob_transitions,␣
↪fl_rewards,fl_size,fl_gammas)
```

[26]:
```python
fm_gammas = np.arange(0.1, 0.99, 0.04)
fm_df_gamma_vi, fm_df_gamma_pi= gamma_analysis(fm_prob_transitions,␣
↪fm_rewards,fm_size,fm_gammas)
```

[27]:
```python
plot_gamma_analysis(fl_df_gamma_vi, fl_df_gamma_pi)
```



[28]:
```python
plot_gamma_analysis(fm_df_gamma_vi, fm_df_gamma_pi)
```

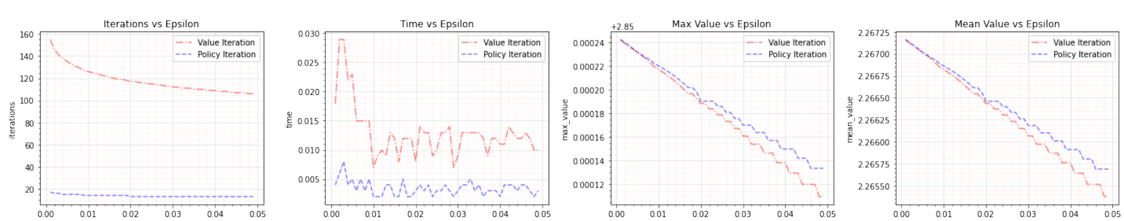


3.2 Epsilon Analysis

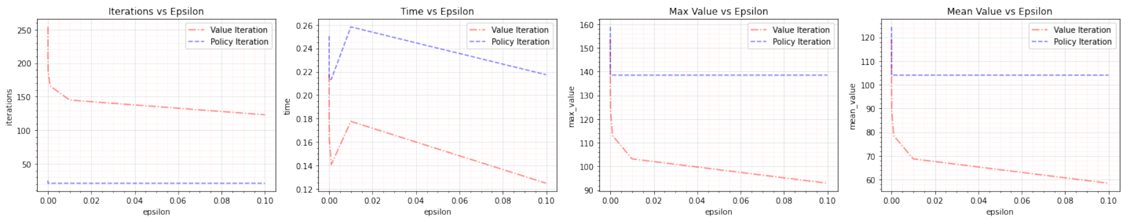
[29]:
```python
fl_epsilons = np.arange(0.001,0.05, 0.001)
fl_gamma = 0.99
fl_df_epsilon_vi, fl_df_epsilon_pi = epsilon_analysis(fl_prob_transitions,␣
↪fl_rewards,fl_size,fl_epsilons,fl_gamma)
```

[30]:
```python
# fm_epsilons = np.arange(0.001,0.05, 0.001)
# fm_epsilons = np.array([0.1,0.01,0.001, 0.0001, 0.00001, 0.000001, 0.
↪0000001,0.00000001,0.0000000001])
fm_epsilons = np.array([0.1,0.01,0.001, 0.0001, 0.00001, 0.000001, 0.0000001])
fm_gamma = 0.9999
fm_df_epsilon_vi, fm_df_epsilon_pi = epsilon_analysis(fm_prob_transitions,␣
↪fm_rewards,fm_size,fm_epsilons,fm_gamma)
```

[31]:
```python
plot_epsilon_analysis(fl_df_epsilon_vi, fl_df_epsilon_pi)
```



[32]:
```
plot_epsilon_analysis(fm_df_epsilon_vi, fm_df_epsilon_pi)
```



3.3 Optimal Policy

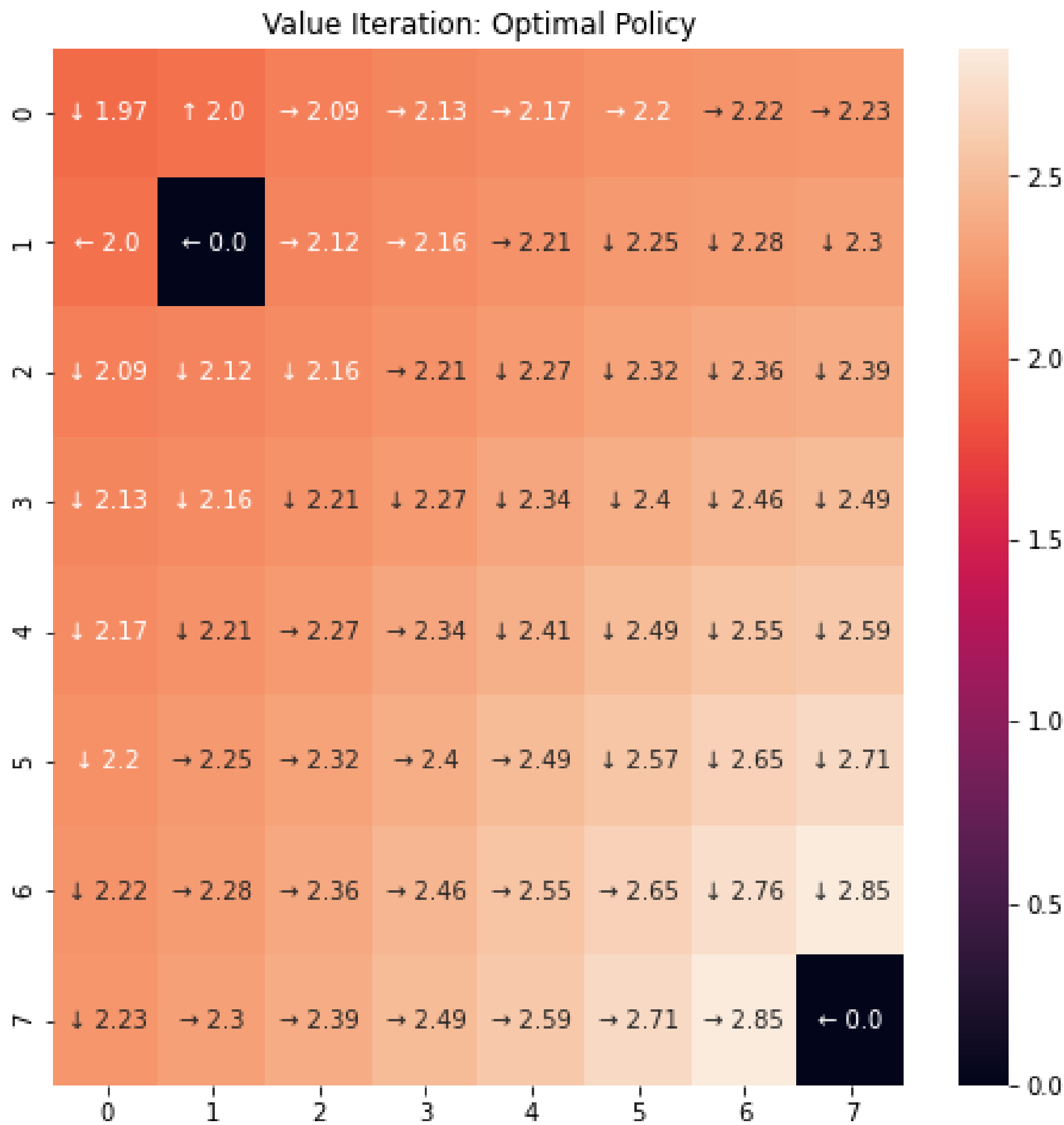
[33]:
```
fl_vi_value_arr, fl_vi_policy_viz, fl_pi_value_arr, fl_pi_policy_viz =␣
↪optimal_policy(fl_df_epsilon_vi,fl_df_epsilon_pi,fl_size,0.011,'frozen-lake')
```

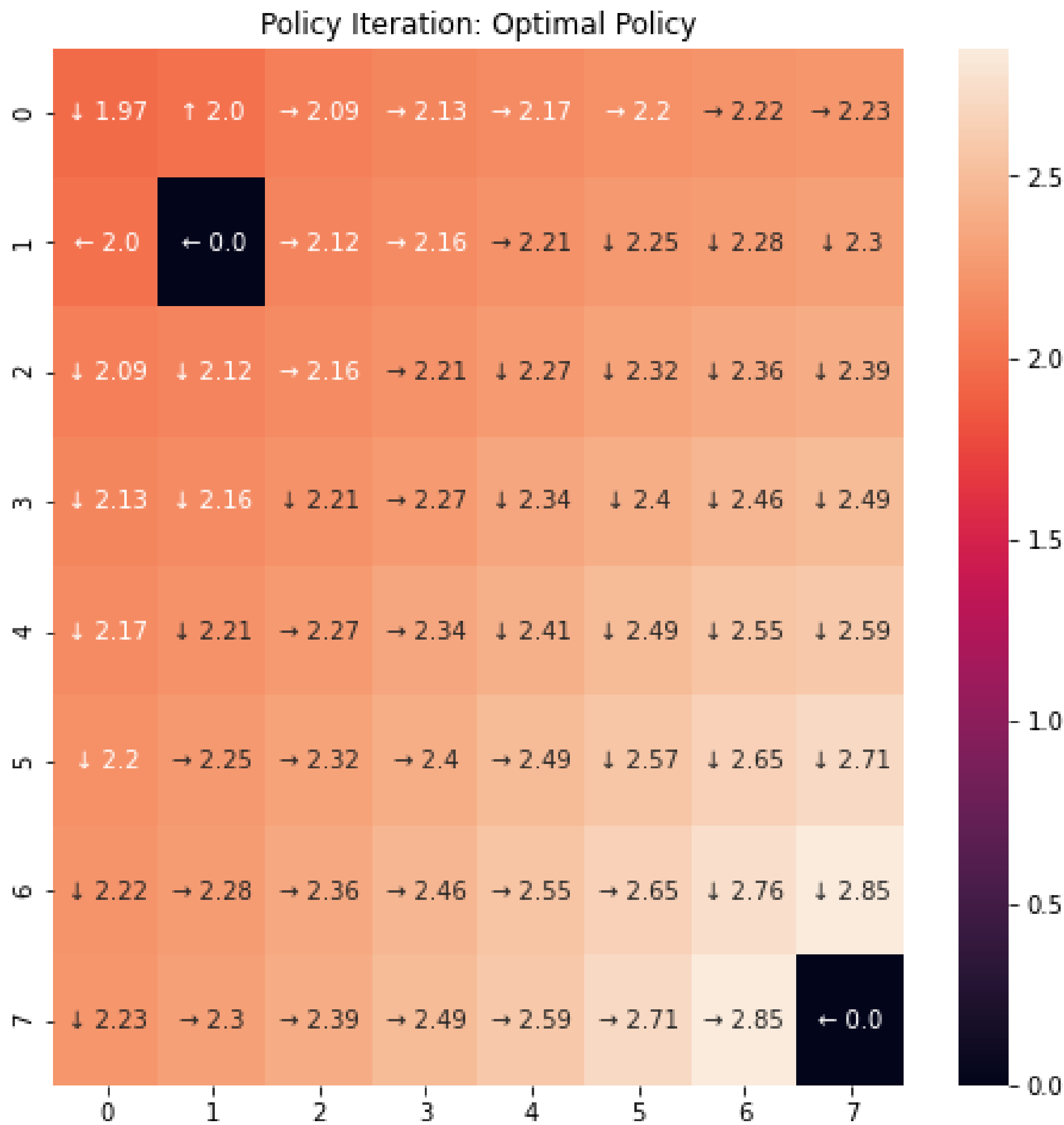
```
(1, 9)
(1, 9)
(1, 64)
(1, 64)
```

[34]:
```
fm_vi_value_arr, fm_vi_policy_viz, fm_pi_value_arr, fm_pi_policy_viz =␣
↪optimal_policy(fm_df_epsilon_vi,fm_df_epsilon_pi,fm_size,0.0001,'forest')
```

```
(1, 9)
(1, 9)
(1, 1000)
(1, 1000)
```

[35]:
```
plot_optimnal_policy(fl_vi_value_arr, fl_vi_policy_viz, fl_pi_value_arr,␣
↪fl_pi_policy_viz,fl_size,'frozen-lake')
```


Value Iteration: Optimal Policy
↓ 1.97
↑ 2.0
→ 2.09
→ 2.13
→ 2.17
→ 2.2
→ 2.22
→ 2.23
← 2.0
← 0.0
→ 2.12
→ 2.16
→ 2.21
↓ 2.25
↓ 2.28
↓ 2.3
↓ 2.09
↓ 2.12
↓ 2.16
→ 2.21
↓ 2.27
↓ 2.32
↓ 2.36
↓ 2.39
↓ 2.13
↓ 2.16
↓ 2.21
↓ 2.27
↓ 2.34
↓ 2.4
↓ 2.46
↓ 2.49
↓ 2.17
↓ 2.21
→ 2.27
→ 2.34
↓ 2.41
↓ 2.49
↓ 2.55
↓ 2.59
↓ 2.2
→ 2.25
→ 2.32
→ 2.4
→ 2.49
↓ 2.57
↓ 2.65
↓ 2.71
↓ 2.22
→ 2.28
→ 2.36
→ 2.46
→ 2.55
→ 2.65
↓ 2.76
↓ 2.85
↓ 2.23
→ 2.3
→ 2.39
→ 2.49
→ 2.59
→ 2.71
→ 2.85
← 0.0
0
1
2
3
4
5
6
7
2.5
2.0
1.5
1.0
0.5
0.0



[36]:

```
plot_optimnal_policy(fm_vi_value_arr, fm_vi_policy_viz, fm_pi_value_arr,
↪fm_pi_policy_viz,fm_size,'forest')
```

```
Value Iteration: Optimal Policy
 W 87.98 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
```

C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5
C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5	C 88.5

```
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
```

| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5
| C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | C 88.5 | W
88.63 | W 89.23 | W 89.9 | W 90.64 | W 91.46 | W 92.37 | W 93.39 | W 94.52 | W
95.77 | W 97.16 | W 98.71 | W 100.43 | W 102.34 | W 104.47 | W 106.83 | W 109.45
| W 112.37 | W 115.61 | W 119.21 | W 123.21 |
Value Iteration: Optimal Policy
 W 103.29 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C 103.82 | C
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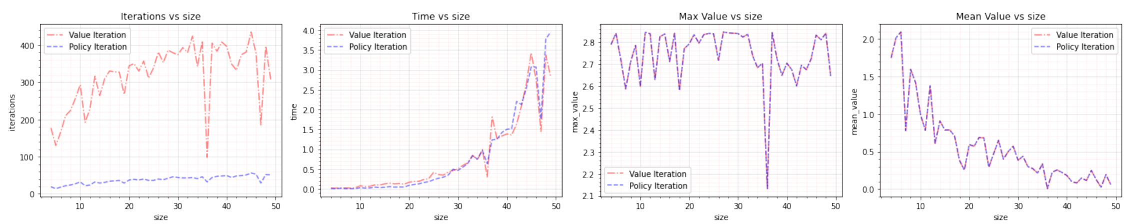
103.95 | W 104.55 | W 105.21 | W 105.95 | W 106.78 | W 107.69 | W 108.7 | W
109.83 | W 111.09 | W 112.48 | W 114.03 | W 115.75 | W 117.66 | W 119.79 | W
122.15 | W 124.77 | W 127.68 | W 130.92 | W 134.52 | W 138.52 |
```

3.4 Size Analysis

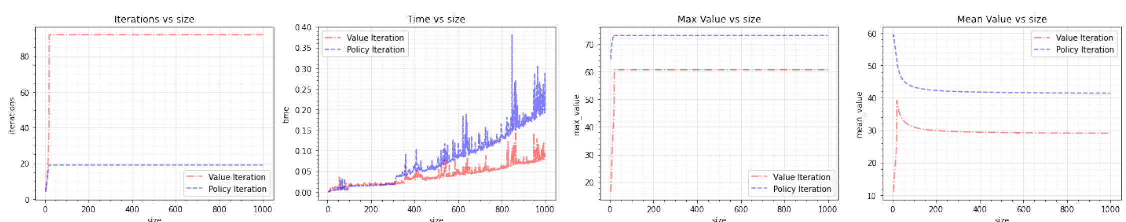
[37]:
```
fl_gridsizes = range(4, 50, 1)
fl_df_size_vi, fl_df_size_pi =␣
↪size_analysis('frozen-lake',fl_gridsizes,size_gamma=0.99)
```

[38]:
```
fm_sizes = range(4, 1000, 1)
fm_df_size_vi, fm_df_size_pi = size_analysis('forest',fm_sizes,size_gamma=0.99)
```

[39]:
```
plot_size_analysis(fl_df_size_vi, fl_df_size_pi)
```



[40]:
```
plot_size_analysis(fm_df_size_vi, fm_df_size_pi)
```



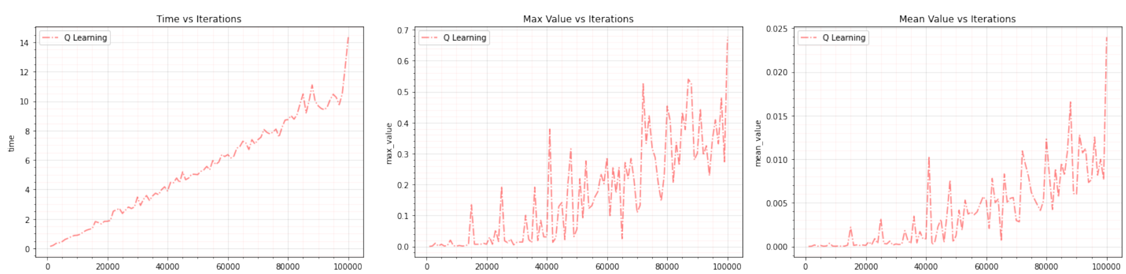
4 Q Learning

4.1 N-Iter Analysis

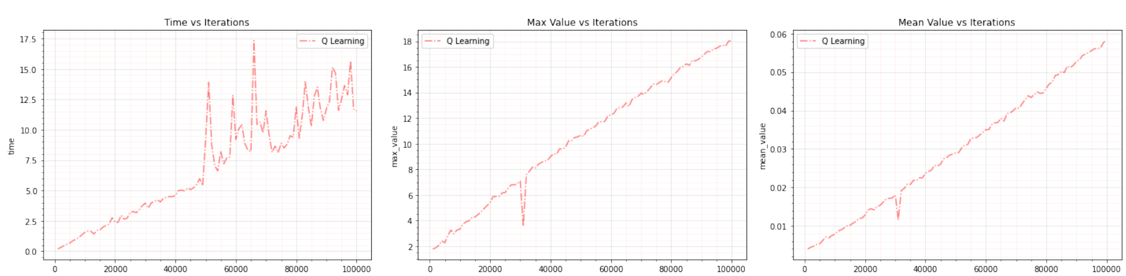
[45]:
```
fl_ql_gamma = 0.99
fl_ql_epsilon = 0.9
# fl_ql_n_iters = [10000, 50000, 100000, 500000, 1000000]
fl_ql_n_iters = np.arange(1000,100100,1000)
# fl_ql_n_iters = np.arange(100,201,100)
fl_df_n_iter_ql, fl_df_n_iter_run_stat = ql_n_iter_analysis(fl_prob_transitions, fl_rewards,fl_size,fl_ql_gamma,fl_ql_epsilon,fl_ql_n_iters)
```

[46]:
```
fm_ql_gamma = 0.99
fm_ql_epsilon = 0.9
# fm_ql_n_iters = [10000,50000,100000,500000,1000000]
fm_ql_n_iters = np.arange(1000,100100,1000)
fm_df_n_iter_ql, fm_df_n_iter_run_stat = ql_n_iter_analysis(fm_prob_transitions, fm_rewards,fm_size,fm_ql_gamma,fm_ql_epsilon,fm_ql_n_iters)
```

[47]:
```
plot_ql_n_iter_analysis(fl_df_n_iter_ql,fl_df_n_iter_run_stat)
```



[48]:
```
plot_ql_n_iter_analysis(fm_df_n_iter_ql,fm_df_n_iter_run_stat)
```



[ ]:
```

```

[69]:
```
fl_df_n_iter_ql[fl_df_n_iter_ql['iterations'] == 10000]
```

[69]:
```
  iter_type  size  iterations      time  max_value  mean_value  \
9        QL     8       10000  0.889622      0.001    0.000016

                                            q_matrix  \
9  [[0.0, 0.0, 0.0, 0.0], [0.0, 0.0, 0.0, 0.0], […

                                              policy  \
9  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, …

                                               value
9  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, …
```

[70]:
```
def optimal_policy_ql(optimal_df_ql,optimal_size,iterations,env_type):
    optimal_ql = optimal_df_ql[optimal_df_ql['iterations'] == iterations]

    print(optimal_ql.shape)
    print(np.array(list(optimal_ql['policy'])).shape)

    if env_type == 'frozen-lake':
        ql_policy_arr = np.array(list(optimal_ql['policy'])).
↪reshape(optimal_size,optimal_size).astype(str)
        ql_value_arr = np.round(np.array(list(optimal_ql['value'])).
↪reshape(optimal_size,optimal_size),2)


        ql_policy_arr[ql_policy_arr=='0'] = '←'
        ql_policy_arr[ql_policy_arr=='1'] = '↓'
        ql_policy_arr[ql_policy_arr=='2'] = '→'
        ql_policy_arr[ql_policy_arr=='3'] = '↑'

        ql_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(ql_policy_arr.
↪flatten(), ql_value_arr.flatten())])).reshape(optimal_size,optimal_size)

    elif env_type == 'forest':

        ql_policy_arr = np.array(list(optimal_ql['policy'])).
↪reshape(optimal_size).astype(str)
        ql_value_arr = np.round(np.array(list(optimal_ql['value'])).
↪reshape(optimal_size),2)

        ql_policy_arr[ql_policy_arr=='0'] = 'W'
        ql_policy_arr[ql_policy_arr=='1'] = 'C'

        ql_policy_viz = (np.asarray([a+" "+str(v) for a, v in zip(ql_policy_arr.
↪flatten(), ql_value_arr.flatten())])).reshape(optimal_size)
```

```python
    return ql_value_arr, ql_policy_viz
```

[84]:
```python
def plot_optimnal_policy_ql(plot_ql_value_arr,␣
↪plot_ql_policy_viz,plot_size,env_type):

    if env_type == 'frozen-lake':
        plt.figure(figsize=(plot_size,plot_size))
        plt.title("Q Learning: Optimal Policy")
        sns.heatmap(plot_ql_value_arr, annot=plot_ql_policy_viz, fmt="")

    elif env_type == 'forest':
        print("Q Learining: Optimal Policy")
        line=""
        for val in plot_ql_policy_viz:
            line += (" {} |".format(val))
        print(line)
```

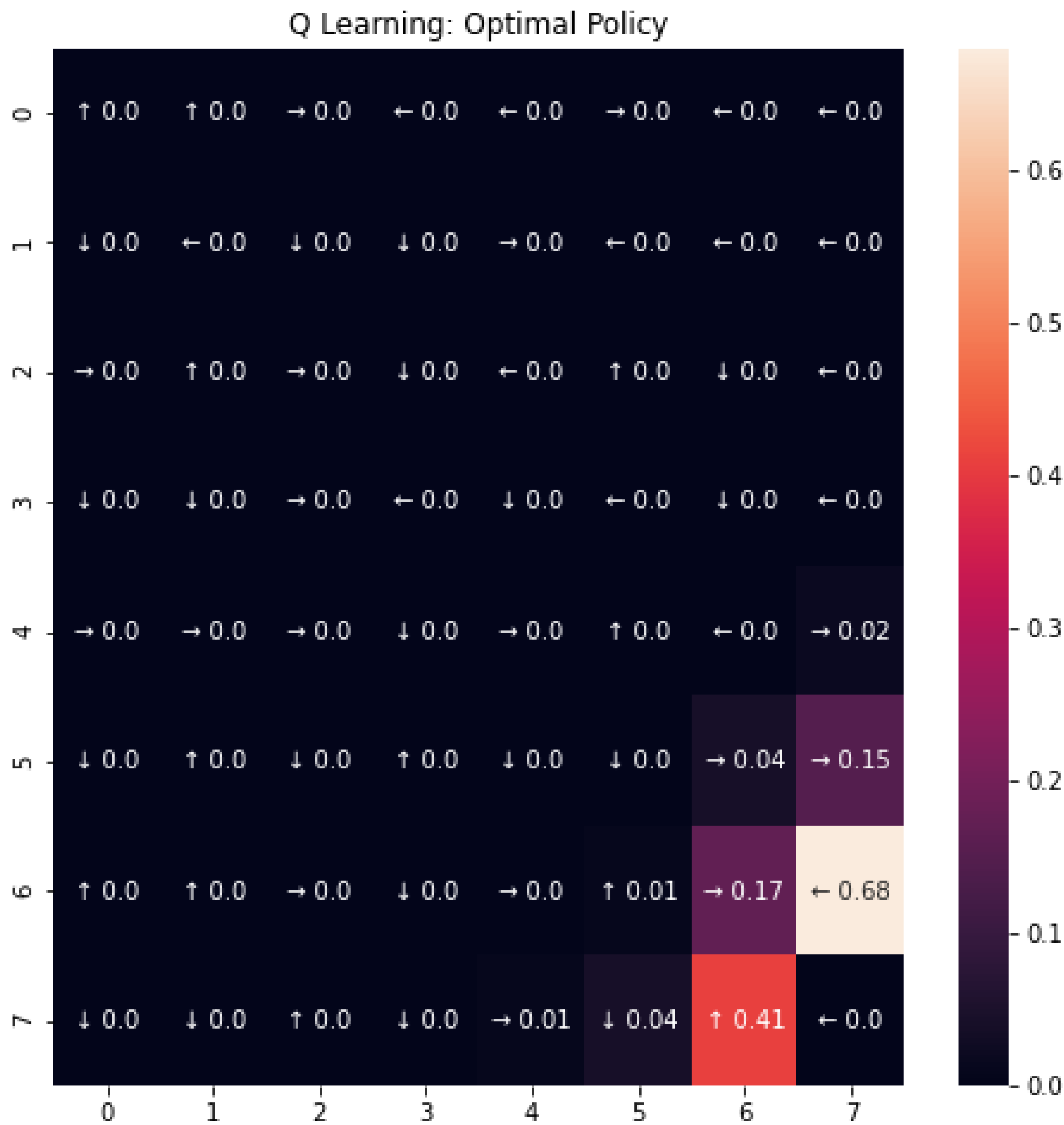
[85]:
```python
fl_ql_value_arr, fl_ql_policy_viz =␣
↪optimal_policy_ql(fl_df_n_iter_ql,fl_size,100000,'frozen-lake')
```

```
(1, 9)
(1, 64)
```

[86]:
```python
fm_ql_value_arr, fm_ql_policy_viz =␣
↪optimal_policy_ql(fm_df_n_iter_ql,fm_size,100000,'forest')
```

```
(1, 9)
(1, 1000)
```

[87]:
```python
plot_optimnal_policy_ql(fl_ql_value_arr, fl_ql_policy_viz,fl_size,'frozen-lake')
```



[88]:
```python
plot_optimnal_policy_ql(fm_ql_value_arr, fm_ql_policy_viz,fm_size,'forest')
```

```
Q Learining: Optimal Policy
 W 17.55 | C 18.06 | C 11.17 | C 1.11 | W 0.0 | W 0.0 | C 0.06 | W 0.0 | W 0.01
| W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.01 | C 0.08 | W 0.0 | W 0.01 | W 0.0 | W
0.01 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.01 | W
0.01 | W 0.03 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.01 | W 0.01 | W 0.01 | W
0.0 | W 0.02 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.01 | W 0.0
| C 0.01 | W 0.0 | W 0.0 | C 0.07 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W
0.0 | W 0.0 | W 0.0 | C 0.01 | C 0.05 | C 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 |
W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 |
```

W 0.0 | W 0.0 | W 0.0 | C 0.09 | C 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0
| W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | W
0.0 | C 0.03 | W 0.0 | W 0.0 | C 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.05 |
W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.01 | C 0.05 | C 0.03 | W 0.0 | W
0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.02 | W 0.01 | C 0.02
| W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.05 | W 0.0 | W 0.0 | C 0.03 | W
0.0 | W 0.01 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | C 0.07 | W 0.0 | C 0.02 | W 0.0
| W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.03 | W 0.01 | W 0.01 | W 0.0 | W 0.0 | W
0.0 | W 0.01 | W 0.0 | C 0.09 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 |
W 0.0 | C 0.03 | W 0.01 | W 0.0 | W 0.01 | C 0.03 | C 0.03 | W 0.0 | C 0.1 | W
0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.02 | W 0.0 |
W 0.0 | W 0.01 | W 0.0 | W 0.01 | C 0.03 | W 0.0 | W 0.0 | C 0.02 | W 0.0 | W
0.0 | C 0.06 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | C 0.05 | C 0.02
| C 0.03 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | C
0.02 | W 0.0 | W 0.02 | W 0.0 | W 0.02 | W 0.0 | W 0.01 | W 0.0 | C 0.09 | W 0.0
| W 0.0 | W 0.0 | W 0.02 | C 0.03 | W 0.01 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | W
0.0 | W 0.0 | C 0.04 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 |
W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.07 | W 0.0 | W 0.01 | W 0.0 | W 0.0
| W 0.01 | W 0.02 | W 0.0 | C 0.02 | W 0.0 | C 0.01 | W 0.0 | W 0.0 | W 0.0 | W
0.0 | C 0.01 | W 0.01 | W 0.01 | W 0.0 | C 0.07 | W 0.01 | W 0.0 | W 0.04 | W
0.01 | W 0.01 | W 0.01 | W 0.0 | C 0.02 | W 0.0 | C 0.01 | W 0.0 | W 0.0 | W
0.02 | C 0.02 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.02
| W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W
0.0 | W 0.01 | W 0.0 | W 0.03 | W 0.0 | W 0.01 | W 0.02 | W 0.0 | W 0.0 | C 0.04
| W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.03 | W 0.03 | W 0.02 | W 0.0 | C
0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.02
| W 0.0 | W 0.0 | W 0.0 | C 0.04 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W
0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.01 | W 0.01 | W 0.0 | W 0.01 | C 0.03 | W 0.0
| W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W
0.02 | W 0.0 | C 0.04 | W 0.01 | W 0.0 | W 0.01 | C 0.01 | C 0.05 | W 0.0 | W
0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | C 0.04 | W 0.0 | W 0.0 | W 0.0 |
W 0.0 | C 0.08 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.01 | W 0.0 | C
0.03 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | C 0.06
| C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.01 | W 0.0 | W 0.0 | W 0.02 | W
0.0 | C 0.03 | W 0.0 | W 0.0 | C 0.03 | W 0.01 | C 0.01 | W 0.01 | W 0.0 | W
0.02 | C 0.01 | W 0.01 | W 0.0 | W 0.02 | W 0.02 | W 0.0 | W 0.01 | W 0.02 | W
0.0 | C 0.04 | C 0.03 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | W 0.0 | W 0.0 |
C 0.04 | W 0.0 | W 0.0 | W 0.0 | C 0.01 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | C
0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.02 | W 0.01 | W 0.01 | W 0.02
| W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | C
0.03 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | C 0.07 | C 0.01 | W 0.0 | W 0.0
| W 0.0 | W 0.0 | C 0.05 | W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W
0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | C 0.09 | W 0.0 | C 0.01 |
W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 |
W 0.01 | C 0.02 | W 0.02 | W 0.0 | W 0.0 | W 0.02 | C 0.02 | C 0.03 | W 0.0 | C
0.04 | W 0.01 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | C 0.05 | W
0.01 | W 0.02 | C 0.04 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0
| C 0.09 | W 0.0 | C 0.03 | W 0.01 | W 0.0 | W 0.01 | W 0.01 | C 0.04 | W 0.0 |

W 0.0 | C 0.07 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | W 0.01 | W 0.0 | W 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.03 | W 0.02 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.05 | W 0.0 | W 0.0 | C 0.05 | C 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.03 | C 0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | C 0.05 | C 0.03 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.01 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.06 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.02 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.01 | W 0.02 | W 0.0 | W 0.01 | W 0.0 | C 0.05 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | C 0.02 | W 0.0 | C 0.02 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | C 0.04 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.01 | C 0.07 | W 0.0 | C 0.02 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | C 0.02 | W 0.01 | C 0.02 | W 0.01 | W 0.0 | W 0.02 | W 0.03 | W 0.0 | W 0.0 | C 0.06 | W 0.0 | W 0.01 | W 0.03 | W 0.0 | C 0.04 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | C 0.08 | W 0.0 | C 0.03 | W 0.02 | C 0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | C 0.08 | W 0.0 | W 0.0 | W 0.0 | C 0.08 | W 0.0 | W 0.0 | W 0.0 | C 0.03 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | C 0.07 | W 0.0 | W 0.0 | W 0.0 | C 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.03 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.03 | W 0.0 | W 0.0 | W 0.0 | C 0.05 | W 0.02 | W 0.01 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.09 | W 0.01 | W 0.01 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.03 | W 0.01 | W 0.01 | W 0.0 | W 0.0 | C 0.02 | W 0.01 | W 0.0 | C 0.08 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | C 0.02 | C 0.05 | W 0.02 | W 0.0 | W 0.01 | W 0.01 | C 0.03 | W 0.0 | W 0.0 | C 0.02 | C 0.03 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | C 0.08 | C 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.02 | W 0.01 | W 0.02 | C 0.04 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | W 0.01 | W 0.01 | W 0.0 | W 0.04 | W 0.0 | W 0.0 | W 0.02 | W 0.02 | W 0.0 | W 0.0 | W 0.03 | W 0.0 | W 0.02 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | C 0.07 | W 0.0 | C 0.01 | C 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.03 | C 0.07 | W 0.0 | W 0.01 | W 0.01 | W 0.0 | W 0.0 | C 0.02 | C 0.06 | W 0.0 | W 0.02 | W 0.0 | C 0.02 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.01 | C 0.05 | W 0.01 | W 0.0 | W 0.0 | W 0.01 | W 0.02 | W 0.0 | C 0.04 | W 0.0 | C 0.12 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.01 | W 0.0 | W 0.01 | W 0.0 | W 0.0 | W 0.0 | C 0.04 | W 0.0 | C 0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.03 | W 0.0 | W 0.02 | W 0.02 | C 0.04 | W 0.0 | W 0.0 | W 0.01 | C 0.05 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | C 0.07 | C 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | C 0.11 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.0 | W 0.02 | W 0.0 | W 0.0 | W 0.0 | C 0.08 | W 0.0 |

```
W 0.01 | W 0.0 | W 0.0 | W 0.0 | W 0.03 | W 0.0 | W 0.0 | C 0.05 | W 0.0 | W 0.0
| W 0.0 | W 0.0 | W 0.03 | W 0.0 | W 0.0 | W 0.16 |
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