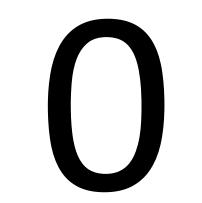
### **Lecture Plan**

- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Signed Integers
- Overflow
- Casting and Combining Types

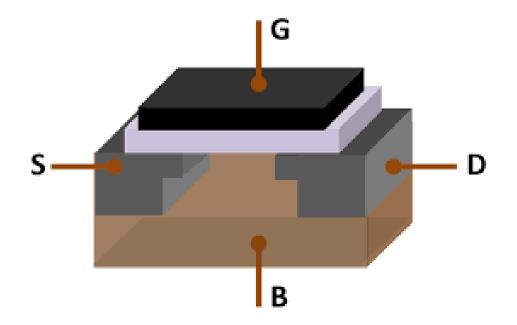
## **Lecture Plan**

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### **Bits**

• Computers are built around the idea of two states: "on" and "off". Transistors represent this in hardware, and bits represent this in software!



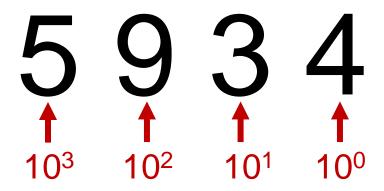
#### One Bit At A Time

- We can combine bits, like with base-10 numbers, to represent more data.
   bits = 1 byte.
- Computer memory is just a large array of bytes! It is byte-addressable; you can't address (store location of) a bit; only a byte.
- Computers still fundamentally operate on bits; we have just gotten more creative about how to represent different data as bits!
  - Images
  - Audio
  - Video
  - Text
  - And more...

5934

Digits 0-9 (0 to base-1)

$$= 5*1000 + 9*100 + 3*10 + 4*1$$



5 9 3 4 3 2 1 0

10<sup>X</sup>:

1 0 1 1 2<sup>x</sup>: 3 2 1 0

Digits 0-1 (0 to base-1)

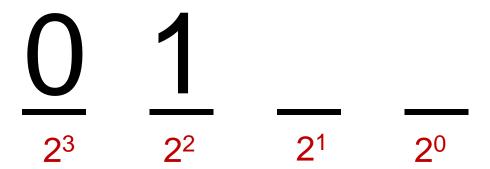




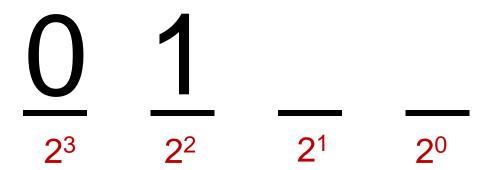
$$= 1*8 + 0*4 + 1*2 + 1*1 = 11_{10}$$

- Strategy:
  - What is the largest power of 2 ≤ 6?

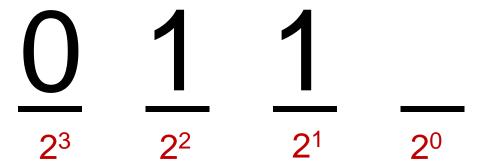
- Strategy:
  - What is the largest power of  $2 \le 6$ ?  $2^2=4$



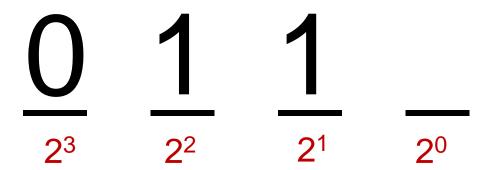
- Strategy:
  - What is the largest power of  $2 \le 6$ ?  $2^2=4$
  - Now, what is the largest power of  $2 \le 6 2^2$ ?



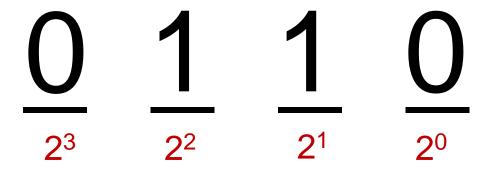
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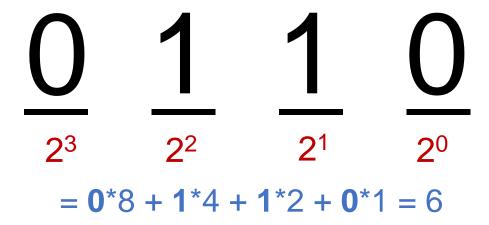
- Strategy:
  - What is the largest power of  $2 \le 6$ ?  $2^2=4$
  - Now, what is the largest power of  $2 \le 6 2^2$ ?  $2^1 = 2$
  - $6-2^2-2^1=0!$



- Strategy:
  - What is the largest power of  $2 \le 6$ ?  $2^2=4$
  - Now, what is the largest power of  $2 \le 6 2^2$ ?  $2^1 = 2$
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  - Now, what is the largest power of  $2 \le 6 2^2$ ?  $2^1=2$
  - $6-2^2-2^1=0!$



#### **Practice: Base 2 to Base 10**

What is the base-2 value 1010 in base-10?

- a) 20
- b) 101
- c) 10
- d) 5
- e) Other

### Practice: Base 10 to Base 2

What is the base-10 value 14 in base 2?

- a) 1111
- b) 1110
- c) 1010
- d) Other

• What is the minimum and maximum base-10 value a single byte (8 bits) can store?

• What is the minimum and maximum base-10 value a single byte (8 bits) can store? **minimum = 0 maximum = ?** 

What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = ?

2<sup>x</sup>:

What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = ?

• Strategy 1:  $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$ 

2<sup>x</sup>:

What is the minimum and maximum base-10 value a single byte (8 bits) can store?
 minimum = 0
 maximum = 255

- Strategy 1:  $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$
- Strategy 2:  $2^8 1 = 255$

2<sup>x</sup>:

## Multiplying by Base

$$1450 \times 10 = 1450$$
0
 $1100_2 \times 2 = 1100$ 0

Key Idea: inserting 0 at the end multiplies by the base!

## **Dividing by Base**

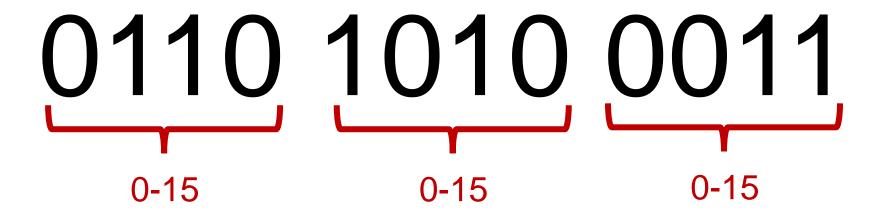
$$1450 / 10 = 145$$
 $1100_2 / 2 = 110$ 

Key Idea: removing 0 at the end divides by the base!

## **Lecture Plan**

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- When working with bits, oftentimes we have large numbers with 32 or 64 bits.
- Instead, we'll represent bits in base-16 instead; this is called hexadecimal.



- When working with bits, oftentimes we have large numbers with 32 or 64 bits.
- Instead, we'll represent bits in base-16 instead; this is called hexadecimal.



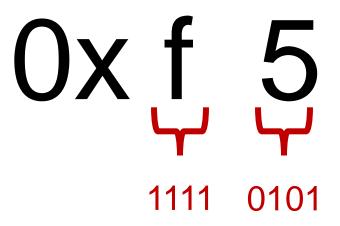
Each is a base-16 digit!

• Hexadecimal is *base-16*, so we need digits for 1-15. How do we do this?

0 1 2 3 4 5 6 7 8 9 a b c d e f

Hex digit	0	1	2	3	4	5	6	7
Decimal value	0	1	2	3	4	5	6	7
Binary value	0000	0001	0010	0011	0100	0101	0110	0111
Hex digit	8	9	Α	В	С	D	E	F
Decimal value	8	9	10	11	12	13	14	15
Binary value	1000	1001	1010	1011	1100	1101	1110	1111

- We distinguish hexadecimal numbers by prefixing them with 0x, and binary numbers with 0b.
- E.g. **0xf5** is **0b11110101**



## **Practice: Hexadecimal to Binary**

What is **0x173A** in binary?

Hexadecimal	1	7	3	A
Binary	0001	0111	0011	1010

### **Practice: Hexadecimal to Binary**

What is **0b1111001010** in hexadecimal? (Hint: start from the right)

Binary	11	1100	1010
Hexadecimal	3	C	A

### **Lecture Plan**

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### **Number Representations**

- Unsigned Integers: positive and 0 integers. (e.g. 0, 1, 2, ... 99999...
- **Signed Integers:** negative, positive and 0 integers. (e.g. ...-2, -1, 0, 1,... 9999...)

• Floating Point Numbers: real numbers. (e,g. 0.1, -12.2, 1.5x10<sup>12</sup>)

### **Number Representations**

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- Floating Point Numbers: real numbers. (e,g. 0.1, -12.2, 1.5x10<sup>12</sup>)
  - **→** Look up IEEE floating point if you're interested!

# **Number Representations**

C Declaration	Size (Bytes)
int	4
double	8
float	4
char	1
char *	8
short	2
long	8

# In The Days Of Yore...

C Declaration	Size (Bytes)
int	4
double	8
float	4
char	1
char *	4
short	2
long	4

### **Transitioning To Larger Datatypes**



- Early 2000s: most computers were 32-bit. This means that pointers were 4 bytes (32 bits).
- 32-bit pointers store a memory address from 0 to 2<sup>32</sup>-1, equaling **2<sup>32</sup> bytes of addressable memory**. This equals **4 Gigabytes**, meaning that 32-bit computers could have at most **4GB** of memory (RAM)!
- Because of this, computers transitioned to **64-bit**. This means that datatypes were enlarged; pointers in programs were now **64 bits**.
- 64-bit pointers store a memory address from 0 to 2<sup>64</sup>-1, equaling **2<sup>64</sup> bytes of addressable memory.** This equals **16 Exabytes**, meaning that 64-bit computers could have at most **1024\*1024\*1024 GB** of memory (RAM)!

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### **Unsigned Integers**

- An **unsigned** integer is 0 or a positive integer (no negatives).
- We have already discussed converting between decimal and binary, which is a nice 1:1 relationship. Examples:

```
0b0001 = 1
```

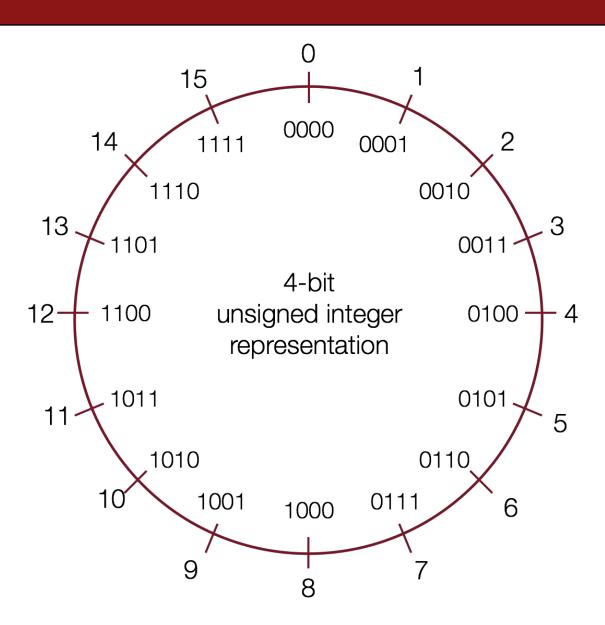
0b0101 = 5

0b1011 = 11

0b1111 = 15

• The range of an unsigned number is  $0 \rightarrow 2^w$ - 1, where w is the number of bits. E.g. a 32-bit integer can represent 0 to  $2^{32} - 1$  (4,294,967,295).

### **Unsigned Integers**



### Let's Take A Break

### To ponder during the break:

A **signed** integer is a negative, 0, or positive integer. How can we represent both negative and positive numbers in binary?

### **Lecture Plan**

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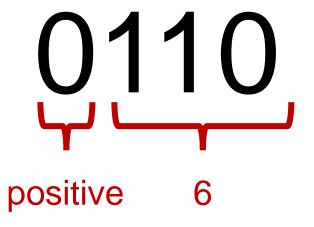
### **Signed Integers**

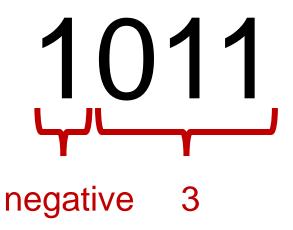
- A **signed** integer is a negative integer, 0, or a positive integer.
- *Problem:* How can we represent negative *and* positive numbers in binary?

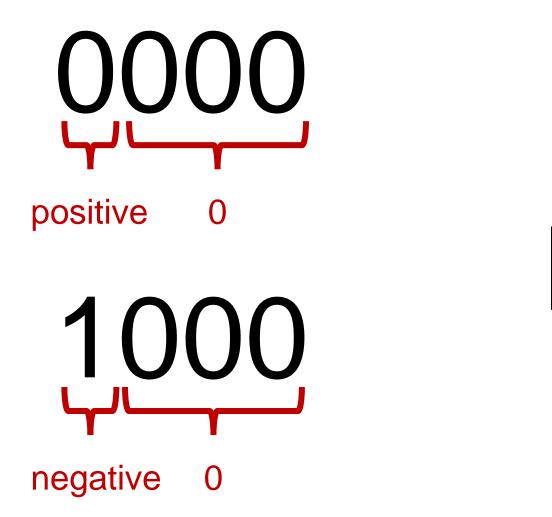
### **Signed Integers**

- A **signed** integer is a negative integer, 0, or a positive integer.
- *Problem:* How can we represent negative *and* positive numbers in binary?

Idea: let's reserve the *most* significant bit to store the sign.







```
1\ 000 = -0 0\ 000 = 0

1\ 001 = -1 0\ 001 = 1

1\ 010 = -2 0\ 010 = 2

1\ 011 = -3 0\ 011 = 3

1\ 100 = -4 0\ 100 = 4

1\ 101 = -5 0\ 101 = 5

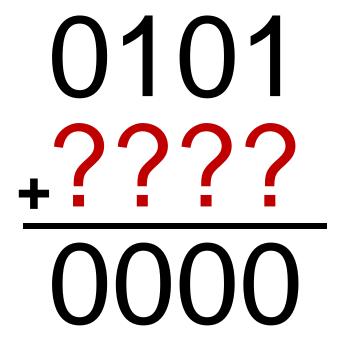
1\ 110 = -6 0\ 110 = 6

1\ 111 = -7 0\ 111 = 7
```

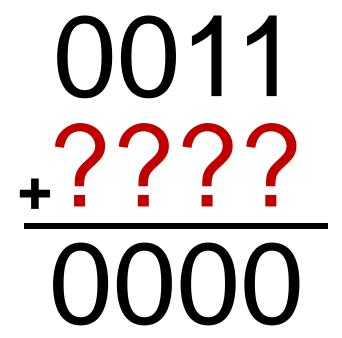
We've only represented 15 of our 16 available numbers!

- **Pro:** easy to represent, and easy to convert to/from decimal.
- Con: +-0 is not intuitive
- Con: we lose a bit that could be used to store more numbers
- **Con:** arithmetic is tricky: we need to find the sign, then maybe subtract (borrow and carry, etc.), then maybe change the sign. This complicates the hardware support for something as fundamental as addition.

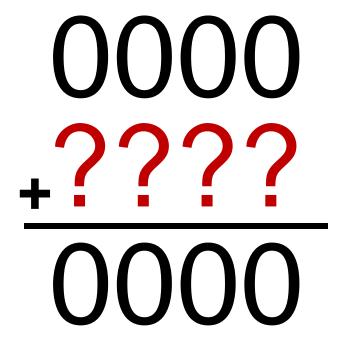
Can we do better?

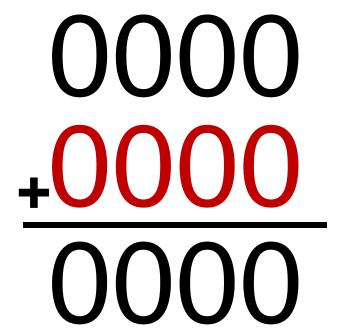


$$0101 \\
+101 \\
\hline
0000$$



$$0011 \\ +1101 \\ \hline 0000$$

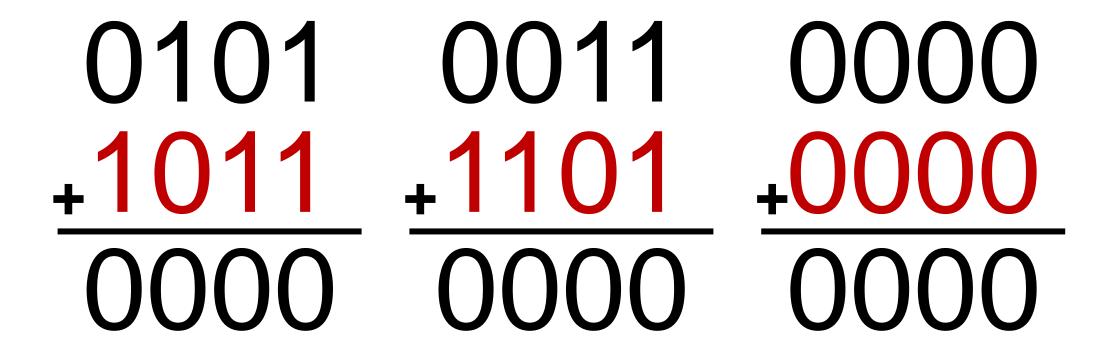




Decimal	Positive	Negative
0	0000	0000
1	0001	1111
2	0010	1110
3	0011	1101
4	0100	1100
5	0101	1011
6	0110	1010
7	0111	1001

Decimal	Positive	Negative
8	1000	1000
9	1001 (same as -7!)	NA
10	1010 (same as -6!)	NA
11	1011 (same as -5!)	NA
12	1100 (same as -4!)	NA
13	1101 (same as -3!)	NA
14	1110 (same as -2!)	NA
15	1111 (same as -1!)	NA

### There Seems Like a Pattern Here...



The negative number is the positive number inverted, plus one!

### There Seems Like a Pattern Here...

A binary number plus its inverse is all 1s.

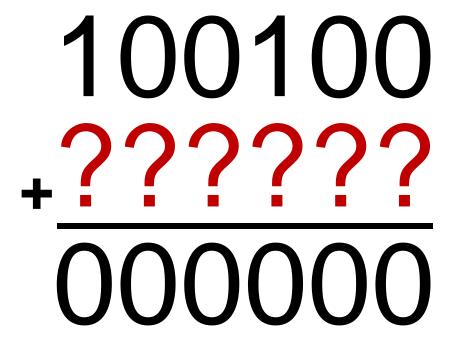
Add 1 to this to carry over all 1s and get 0!

0101 +1010 1111

1111 +001 0000

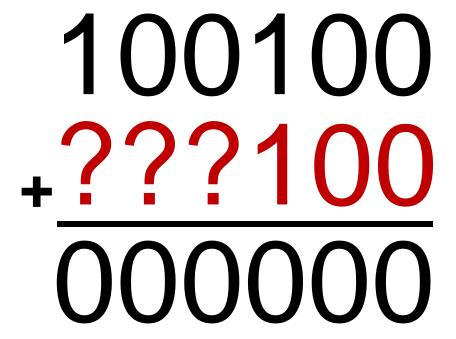
#### **Another Trick**

• To find the negative equivalent of a number, work right-to-left and write down all digits through when you reach a 1. Then, invert the rest of the digits.



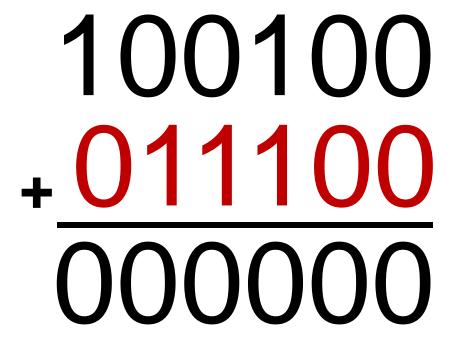
#### **Another Trick**

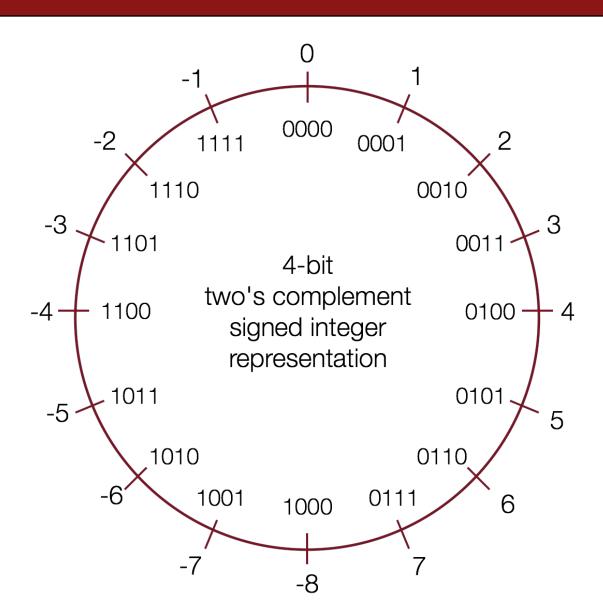
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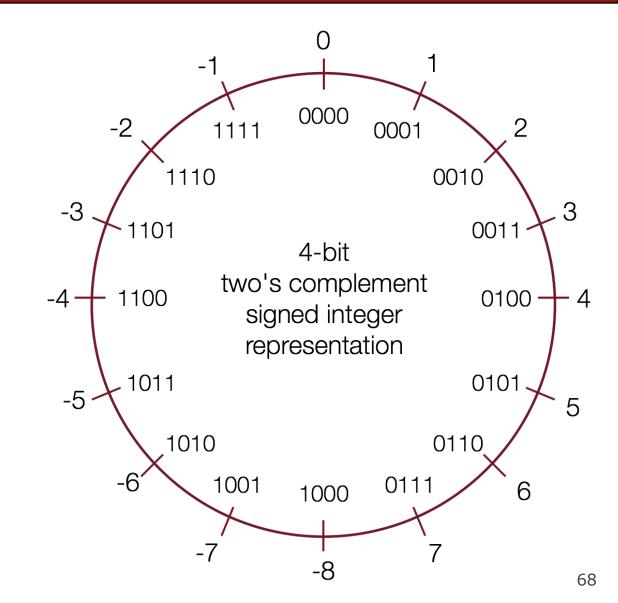
#### **Another Trick**

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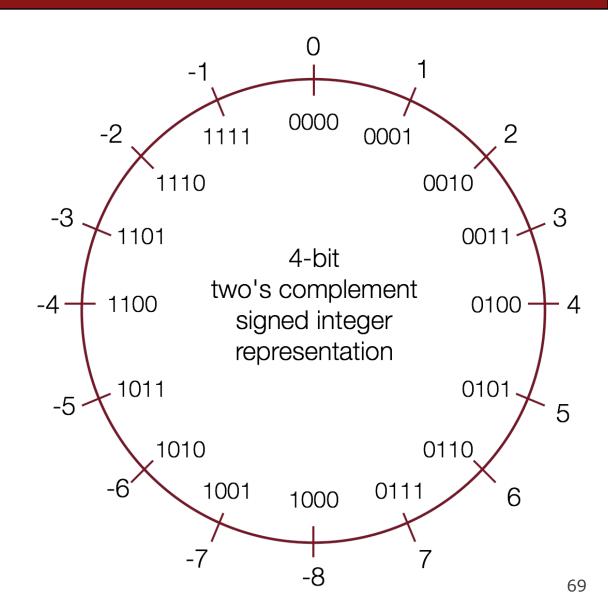




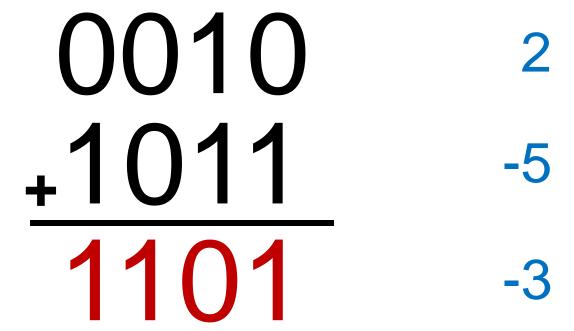
- In two's complement, we represent a positive number as itself, and its negative equivalent as the two's complement of itself.
- The **two's complement** of a number is the binary digits inverted, plus 1.
- This works to convert from positive to negative, and back from negative to positive!



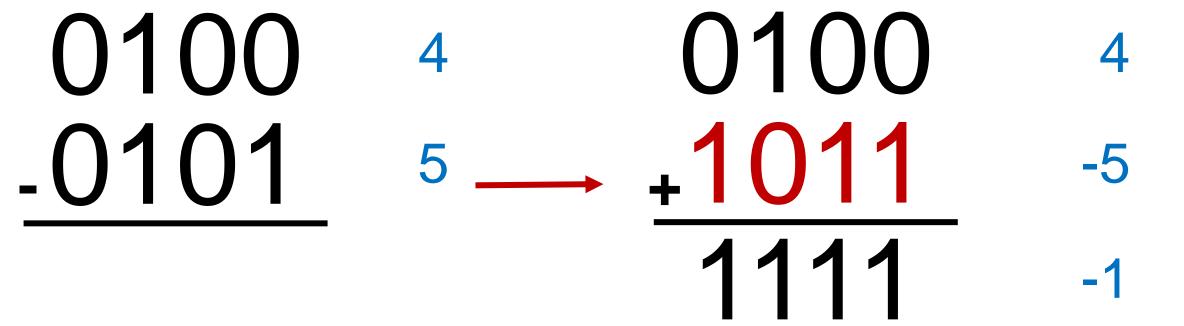
- Con: more difficult to represent, and difficult to convert to/from decimal and between positive and negative.
- **Pro:** only 1 representation for 0!
- **Pro:** all bits are used to represent as many numbers as possible
- **Pro:** the most significant bit still indicates the sign of a number.
- **Pro:** addition works for any combination of positive and negative!



• Adding two numbers is just...adding! There is no special case needed for negatives. E.g. what is 2 + -5?



• Subtracting two numbers is just performing the two's complement on one of them and then adding. E.g. 4-5=-1.



### **Practice: Two's Complement**

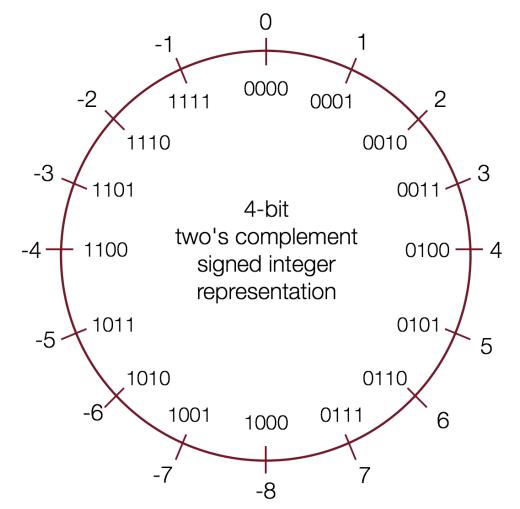
What are the negative or positive equivalents of the numbers below?

- a) -4 (1100)
- b) 7 (0111)
- c) 3 (0011)
- d) -8 (1000)

# **Practice: Two's Complement**

What are the negative or positive equivalents of the numbers below?

- a) -4 (1100)
- b) 7 (0111)
- c) 3 (0011)



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#### **Overflow**

• If you exceed the **maximum** value of your bit representation, you wrap around or overflow back to the **smallest** bit representation.

$$0b1111 + 0b1 = 0b0000$$

• If you go below the **minimum** value of your bit representation, you wrap around or overflow back to the **largest** bit representation.

$$0b0000 - 0b1 = 0b1111$$

# Min and Max Integer Values

Туре	Size (Bytes)	Minimum	Maximum
char	1	-128	127
unsigned char	1	0	255
short	2	-32768	32767
unsigned short	2	0	65535
int	4	-2147483648	2147483647
unsigned int	4	0	4294967295
long	8	-9223372036854775808	9223372036854775807
unsigned long	8	0	18446744073709551615

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# printf and Integers

- There are 3 placeholders for 32-bit integers that we can use:
  - %d: signed 32-bit int
  - %u: unsigned 32-bit int
  - %x: hex 32-bit int
- The placeholder—not the expression filling in the placeholder—dictates what gets printed!

# Casting

What happens at the byte level when we cast between variable types? The
bytes remain the same! This means they may be interpreted differently
depending on the type.

```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);

This prints out: "v = -12345, uv = 4294954951". Why?
```

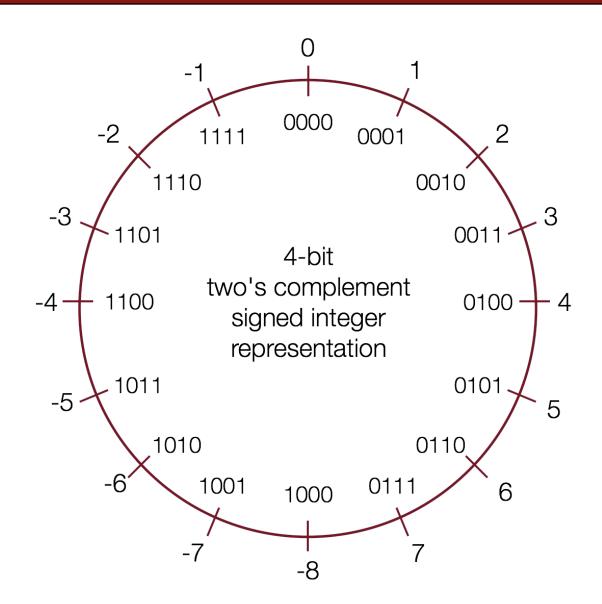
# Casting

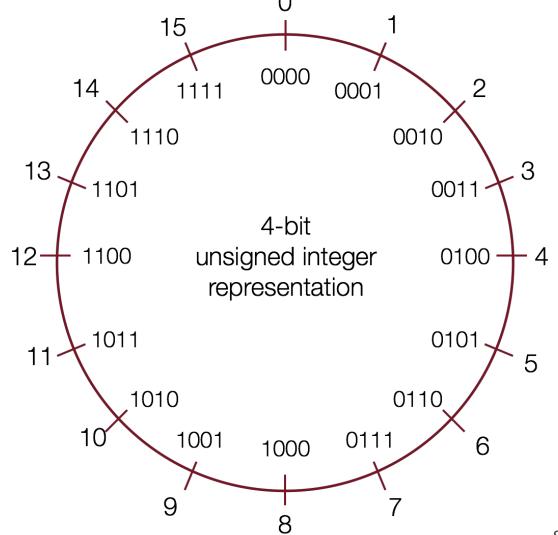
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```

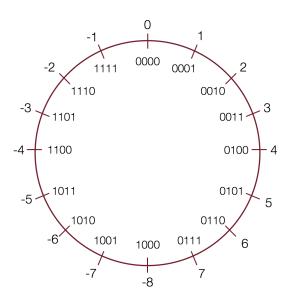
If we treat this binary representation as a positive number, it's huge!

# Casting

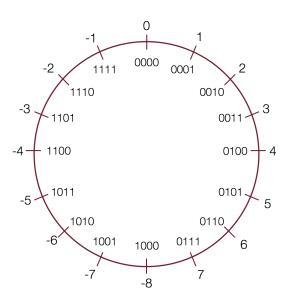




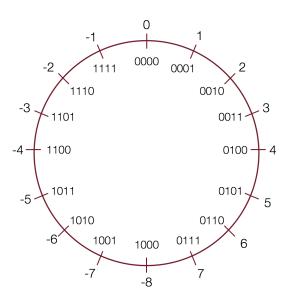
Expression	Туре	Evaluation	Correct?
0 == 0U			
-1 < 0			
-1 < 0U			
2147483647 > -2147483647 - 1			
2147483647U > -2147483647 -			
1			
2147483647 >			
(int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



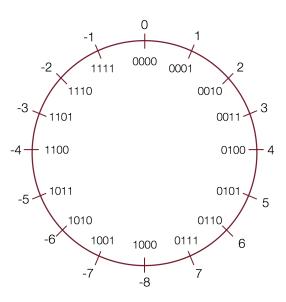
Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0			
-1 < 0U			
2147483647 > -2147483647 - 1			
2147483647U > -2147483647 -			
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



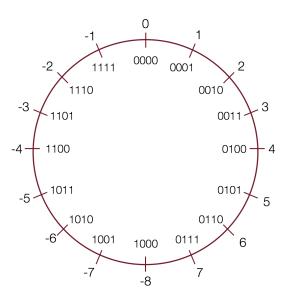
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-1 < 0	Signed	1	yes
-1 < 0U			
2147483647 > -2147483647 - 1			
2147483647U > -2147483647 -			
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



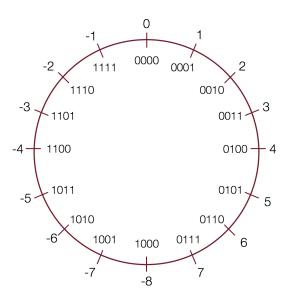
Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
2147483647 > -2147483647 - 1			
2147483647U > -2147483647 -			
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



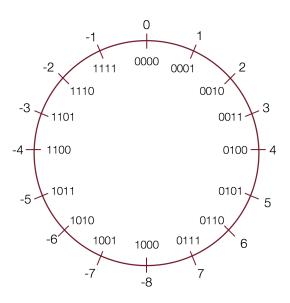
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-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
2147483647 > -2147483647 - 1	Signed	1	yes
2147483647U > -2147483647 -			
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



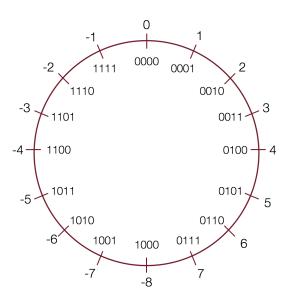
Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
2147483647 > -2147483647 - 1	Signed	1	yes
2147483647U > -2147483647 -	Unsigned	0	No!
2147483647 > (int)2147483648U			
-1 > -2			
(unsigned)-1 > -2			



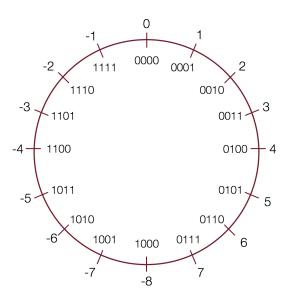
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-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
2147483647 > -2147483647 - 1	Signed	1	yes
2147483647U > -2147483647 -	Unsigned	0	No!
2147483647 > (int)2147483648U	Signed	1	No!
-1 > -2			
(unsigned)-1 > -2			



Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0	Signed	1	yes
-1 < 0U	Unsigned	0	No!
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-1 > -2	Signed	1	yes
(unsigned)-1 > -2	Unsigned	1	yes



# **Expanding Bit Representations**

- Sometimes, we want to convert between two integers of different sizes (e.g. short to int, or int to long).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a **smaller** data type to a **bigger** data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For signed values, we can repeat the sign of the value for new digits ("sign extension"
- Note: when doing <, >, <=, >= comparison between different size types, it will promote to the larger type.

# **Expanding Bit Representation**

# **Expanding Bit Representation**

```
short s = 4;
// short is a 16-bit format, so
                               s = 0000 0000 0000 0100b
int i = s;
— or —
short s = -4;
// short is a 16-bit format, so
                               s = 1111 1111 1111 1100b
int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 11100b
```

# **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

0000 0000 0000 0000 1100 1111 1100 0111

When we cast x to a short, it only has 16-bits, and C truncates the number:

1100 1111 1100 0111

This is -12345! And when we cast sx back an int, we sign-extend the number.

**1111 1111 1111 1111 1100 1111 1100 0111** // still -12345

# **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

```
1111 1111 1111 1111 1111 1111 1111 1101
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1111 1111 1111 1101
```

This is -3! If the number does fit, it will convert fine. y looks like this:

# **Truncating Bit Representation**

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

0000 0000 0000 0001 1111 0100 0000 0000

When we cast x to a short, it only has 16-bits, and C truncates the number:

1111 0100 0000 0000

This is 62464! Unsigned numbers can lose info too. Here is what y looks like:

**0000 0000 0000 1111 0100 0000 0000** // still 62464

# The size of Operator

```
long sizeof(type);

// Example
long int_size_bytes = sizeof(int); // 4
long short_size_bytes = sizeof(short); // 2
long char_size_bytes = sizeof(char); // 1
```

sizeof takes a variable type as a parameter and returns the size of that type, in bytes.

### Recap

- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Signed Integers
- Overflow
- Casting and Combining Types

Next time: How can we manipulate individual bits and bytes?

### Recap

- Bits and Bytes
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Next time: How can we manipulate individual bits and bytes?

# **Practice: Two's Complement**

What are the negative or positive equivalents of the 8-bit numbers below?

- a) -4 (0b11111100)
- b) 24 (0b11000)
- c) 36 (0b100100)
- d) -17 (0b11101111)

# **Practice: Two's Complement**

What are the negative or positive equivalents of the 8-bit numbers below?

- a) -4 (0b11111100) -> **0b100**
- b) 24 (0b11000) -> **0b11101000**
- c) 36 (0b100100) -> **0b11011100**
- d) -17 (0b11101111) -> **0b10001**

```
short x = 130; // 0b1000 0010
char cx = x;
```

```
short x = -132 // 0b1111 1111 0111 1100 char cx = x;
```

```
short x = 25; // 0b1 1001
char cx = x;
```

```
short x = 130; // 0b1000 0010
char cx = x; // -126
```

```
short x = -132 // 0b1111 1111 0111 1100
char cx = x; // 124
```

```
short x = 25; // 0b1 1001
char cx = x; // 25
```

```
short x = 390; // 0b1 1000 0110 char cx = x;
```

```
short x = -15; // 0b1111 1111 1111 0001
char cx = x;
```

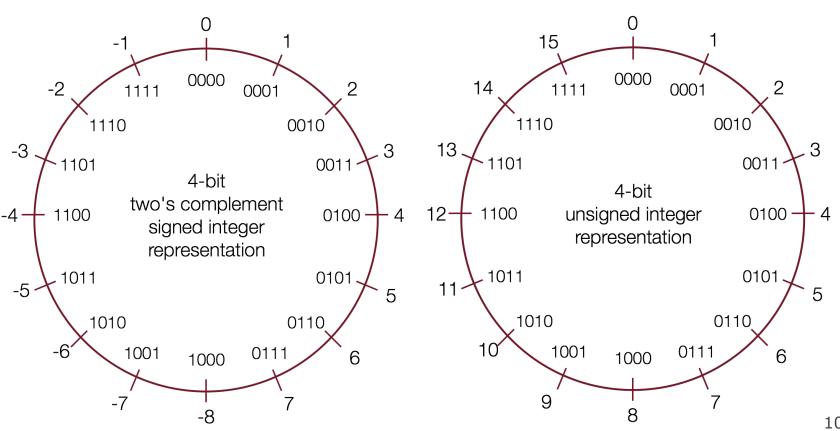
```
short x = 390; // 0b1 1000 0110 char cx = x; // -122
```

```
short x = -15; // 0b1111 1111 1111 0001 char cx = x; // -15
```

# **Practice: Comparisons**

What are the results of the char comparisons performed below?

- 1. -7 < 4
- 2. -7 < 4U
- (char)1000 > 4
- (char)-500 > 4



### **Practice: Comparisons**

What are the results of the char comparisons performed below?

- 1. -7 < 4 true
- 2. -7 < 4U -false
- 3. (char)1000 > 4 false
- 4. (char)-500 > 4 **true**

