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SECTION: D4

Date / /
Page No.

Assignment-1

1. Asymptotic notations are methods / languages using which we can define the running time of algorithm based on input size.

To represent the upper & lower bounds, we need some kind of syntax & this is represented in form of functional $f(n)$.

→ Logarithmic → $\log n$
→ Quadratic → n^2.
→ Exponential → a^n

Linear → n
polynomial → n^2

2. `for (i=1 to m) { i = i*2 }`
'i' is doubling everytime.

for k^{th} step → $2^k = n$ & for $(k+1)$ we are out of loop
Taking log both sides
$\log 2^k = \log n$
$k = \log_2 n$
Time complexity = $O(\log n)$

3. $T(n) = 3T(n-1)$ if $n>0$ otherwise 1
$T(n) = aT(n-b) + f(n)$ (Master theorem)
$a = 3, \quad b = 1$
$f(n) = 0 \quad , k \geq 0$
$T(n) = O(n^k a^{1/2})$
$T(n) = O(n\, a^n)$

$T(n) = O(3^n)$

Page No.
Date

④ $T(n) = 2T(n-1) - 1$

$T(n) = aT(n-b) + f(n)$

$a = 2$

$b = 1$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 2(2T(n-2) - 1) - 1$

$= 4T(n-2) - 3$

Similarly,

$T(n) = 8T(n-3) - 7$

$T(n) = 2^k T(n-k) - (2^k - 1)$

Let $n-1 = 1$

$T(n) = 2^k T(1) - 2^k + 1$

$= 2^k (T(1) - 1) + 1$

$\Rightarrow T(n) = O(2^k) = O(2^n)$ Ans.

⑤

```
int i = 1, s = 1;
while (s <= n) {
    i++; s = s + i;
    printf ("#");
}
```

We can see that 's' is increasing by 1.

$\therefore O(n)$ Ans.

⑥

```
void function (int n) {
    int i, count = 0;
    for (i = 1; i * i <= n; i++)
        count++;
}
```

$n = 5$

$i = 1 \quad 1 * 1 <= n$

$i = 2 \quad 2 * 2 <= n$

$i = 3 \quad \times$ out of loop.

Loop is working for $n/2$ time only.

$\therefore O(n/2) \Rightarrow O(n)$ Ans.

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Page No.
Date

⑦. void function (int n) {
int i, j, k, count = 0;
for (i = n/2 ; i <= n ; i++)
for (j = 1; j <= n ; j = j * 2)
for (k = 1 ; k <= n ; k = k * 2)
count ++;
}

i loop = O(n/2)
j loop = O(log n)
k loop = O (log n)

∴ Final = O(n log² n) Ans.

⑧. fun (int n) {
if (n == 1) return;
for (i = 1 to n){
for (j = 1 to n) {
print (n);
}
}
fun (n - 3);
}

Time Complexity T(n) = T(n²) - 3 Ans.

⑨. void func (int n) {
for (i = 1 to n) → O(n)
for (j = 1 ; j <= n; j = j + i) → O(log n)
printf ("n");

∴ O(n log n) Ans.

Page No.
Date

(10) $f(n) = n^k$ $\quad k >= 1$

$g(n) = a^n$ $\quad a > 1$

$f(n) = O(g(n))$

$n^k = O(a^n)$

Take log

$k \log n = n \log a$

$\frac{\log n}{\log a} = \frac{n}{k}$

$\log \frac{n}{a} = \frac{1}{k} n$

Let $\boxed{\frac{1}{k} = C}$

$\rightarrow O(Cn)$ $\therefore O(Cn)$ is time complexity.

(11) void fun (int n) { int j = 1, i = 0;
while (i < n) { i = i + j; j++; } }

→

i	j	loop runned
0	1	1
1	2	2
3	3	3

We can observe that loop is running for n/2+1 times.

$\therefore O(n/2 + 1)$.

$= O(n)$ Ans.

(12). int fibn(int n) {
line 1 if (n==0 || n==1) return n;
line 2 else return fib(n-1) + fib(n-2);

We know that line 1 takes ~~O(1)~~ O(1) time while line 2 takes T(n-1) + T(n-2)
∴ recursive eqn = T(n-1) + T(n-2) + O(1)

∴ T(n) = T(n-1) + T(n-2) - (1)
T(n-1) = T(n-2) + T(n-3) - (2)
T(n) = T(n-2) + T(n-2) + T(n-3) - (3)
T(n-2) = T(n-3) + T(n-4) - (4)

T(n) = 2T(n-3) + T(n-4)
∴ T(n) = 2T(n-k) + T(n-(k+1))

$O(n) = 2^n$ | Space Complexity = O(n) | Ans

(13) (i) (n log n)
→ void f(int n) {
for (i=0; i<n; i++)
for (j=0; j<n; j=j*2)
count++;
}

(ii) n^3
for (i=0; i<=n; i++)
for (j=0; j<n; j++)
for (k=0; k<n; k++)
{
printf ("GEU")
}

(iii) log (log n)

```
for (i = 0; i < log n; i = i * 2)
    printf ("Hi");
```

(14) $T(n) = T(n/4) + T(n/2) + cn^2$

Using Master's Theorem

We know that; $T(n/2) >= T(n/4)$

$\therefore T(n) <= 2T(n/2) + cn^2$

Apply Master's Theorem to RHS

$T(n) <= O(n^2)$

$T(n) = O(n^2)$

Also,

$T(n) >= cn^2$

$T(n) = O(n^2)$

$T(n) \Omega(n^2)$

Since

$T(n) = O(n^2)$

$T(n) = \Omega(n^2)$

$\therefore T(n) = O(n^2)$ Ans.

(15)

```
int fun(int n) {
    for (i = 1; i <= n; i++)
        for (j = 1; j < n; j = j + 1)   { O(1) }
```

for ~~the~~ i loop = $O(n)$

for j loop = $O(\log n)$

$\therefore O(n \log n)$ Ans.

(16)

```
for (i = 2; i <= n; i = pow(i, k))
{
    { if O(1) }
}
```

n = 7

k = 2

i	
→ 2	1
4	2
16	x

$2, 2^k, (2^k)^k, (2$

$2^{k \log_k \log(n)}$ $\quad 2^{\log(n)}$

$= O(\log(\log n))$ Ans.

(17) Quick Sort

When quick sort repeatedly divides the array into two parts of 99% and 1%, it is the worst case.

∴ Recurrence Relation

Let current problem = n

" (n-1) we have to call

Quicksort Worst Case

$$T(n) = T(n-1) + T(1) + Cn$$

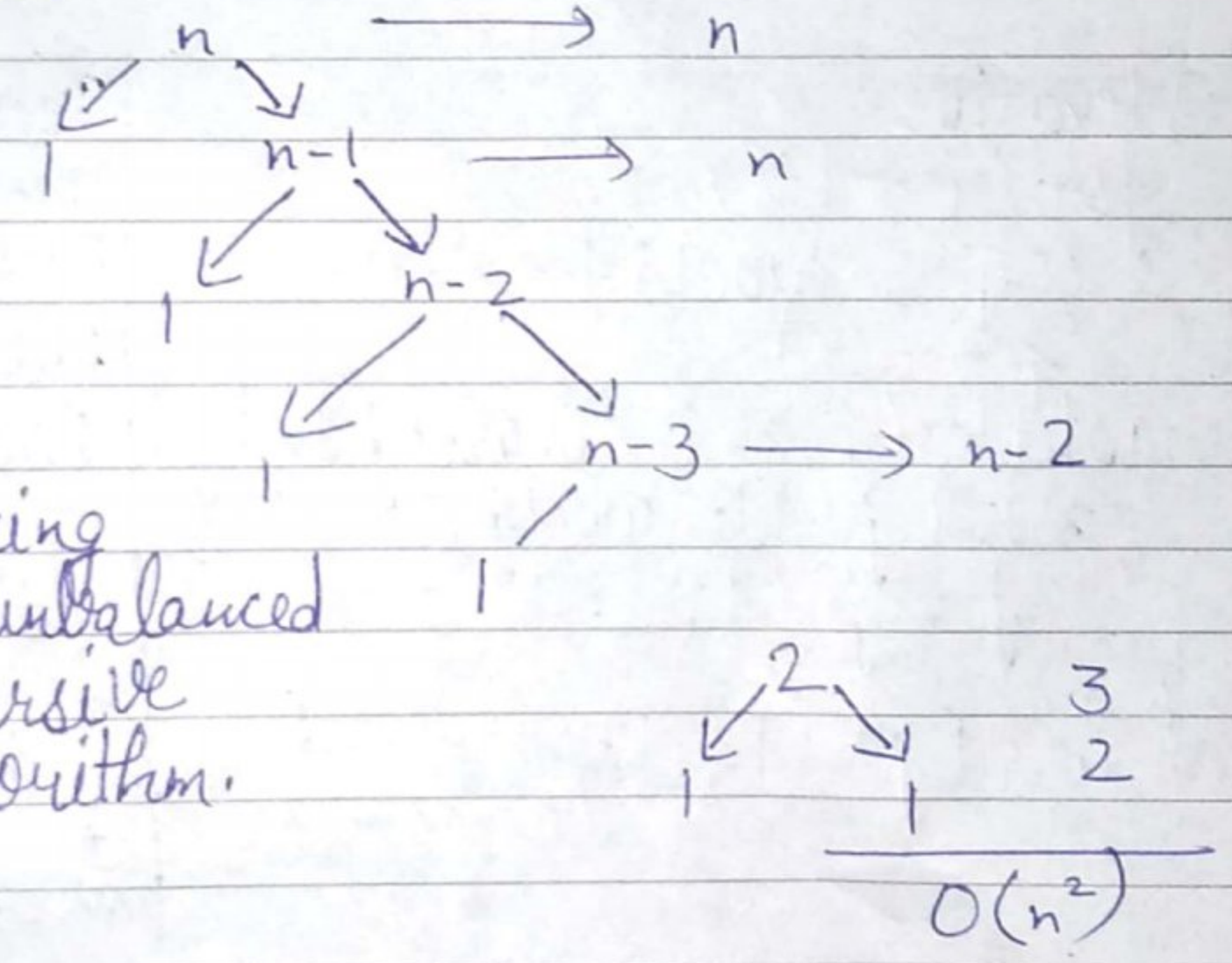
$$T(n) = T(n-1) + \Theta(n)$$

$$= \sum_{K=1}^{n} \Theta(K)$$

$$= \Theta\left(\sum_{1}^{n} K\right)$$

$= O(n^2)$ is the time complexity.

Recurssion Tree:



If the partioning is maximally unbalanced at every recursive step of algorithm.

(18). a) $100 < \log\log n < \log n < n \log n < \text{root}(n) < \log n! < n$
$< n^2 < 2^n < 2^{2n} < 4^n < n!$

(19). void linearSearch (int arr[], int n, int x)
{ if (arr[n/2] >= x) {
for (j = 0 to n/2 + 1)
if (arr[i] == x)
Cout << found;
break;
}
else { for (i = n/2 + 1 ; i < n ; i++)
{ if (arr[i] == x)
Cout << found;
break;
}
}
}

(20). Iterative
for (i = 1 to rel)
Key ← arr[i]
j = i - 1
while (j >= 0 && arr[j] > Key)
arr[j+1] = arr[j]
j = j - 1;
}
arr[j+1] ← Key;

Recursive
void insertion (arr, n)
{ if (n <= 1) return;
insertion (arr, n-1);
last ← arr[n-1];
j ← n-2;
while (j >= 0 && arr[j] > last)
{ arr[j+1] ← arr[j]
j--;
}
arr[j+1] = last;

It is known as online sort because it does not need to know anything about values, it will sort & the info is requested WHILE the algo is running
It guides new value at every iteration. Examples → Selection & Insertion

Page No.
Date

(21). Bubble Sort
Best = $O(n)$
Worst = $O(n^2)$
Avg = $O(n^2)$

Selection Sort
Best = $O(n^2)$
Worst = $O(n^2)$
Avg. = $O(n^2)$

Insertion Sort
Best = $O(n)$
Worst = $O(n^2)$
Avg = $O(n^2)$

Merge Sort
Best = $O(n \log n)$
Worst = $O(n \log n)$
Avg = $O(n \log n)$

Quick Sort
Best = $O(n \log n)$
Worst = $O(n^2)$
Avg = $O(n \log n)$

(22).

In Place	Stable	Online
Bubble	Insertion	Selection
Insertion	Merge	Insertion
Selection	Bubble	
Quick, Heap		* Offline
* Not Inplace	* Not Stable	Bubble, Quick,
Merge	Quick, Heap	Merge

(23). Iterative Binary Search

```
int binarySearch(int arr[], int l, int r, int x)
{   while (l <= r) {
    int mid = (l+r)/2;
    if (arr[mid] == x)
        return mid;
    if (arr[m] < x)
        l ← mid + 1;
    else
        r ← mid - 1;
}
return -1;
}
```

Page No.
Date

Recursive Binary Search

```
int binarySearch(int arr[], int l, int r, int x)
{       while (l <= r) {
            int mid = (l+r)/2;
            if (arr[mid] == x) return mid;
        else if (arr[mid] > x)
            return binarySearch(arr, l, mid-1, x);
        else
            return binarySearch(arr, mid+1, r, n);
    }
    return -1;
```

Time Complexity		Space Complexity	
Binary	Linear	Binary	Linear
Iterative → O(log n)	O(n)	O(1)	O(1)
Recursive → O(1)	O(n)	O(1) or O(log n)	O(1)
O(log n)			

(24). → T(n)

```
int bS(int arr, int l, int r, int n)
{       while (l <= r) {
            int mid = (l+r)/2;                    → T(1)
        if (arr[mid] == n) return mid;
        return bS(arr, mid+1, r, x);
        else
        return bs(arr, l, mid-1, x);              → T(n/2)
    }
    return -1;
}
```

$$T(n) = T(n/2) + T(n/2) + 1$$

$$\boxed{T(n) = T(n/2) + 1}$$ Ans.

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