2. Add $(432)_7 + (355)_7$

11 ← carry

$(432)_7$

$(355)_7$

1120

as $2 + 5 = (7)_{10} = (10)_7$

$3 + 5 + 1 = (9)_{10} = (12)_7$

$1 + 4 + 3 = (8)_{10} = (11)_7$

□□□□

Solved Examples

1. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the the changed number.

(a) 28 (b) 19

(c) 37 (d) 46

Ans: (a)

Going through options we get 82 – 28 = 54

2. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.

(a) 11, 4 (b) 12, 3

(c) 13, 2 (d) 10, 5

Ans: (b)

Going through options only 12 and 3 satisfices the conditon

$$AM = \frac{12+3}{2} = 7.5$$

$GM = \sqrt{12} \times 3 = 6\sqrt{3}$ which is 20% less than 7.5.

3. If A381 is divisible by 11, find the value of the smallest natural number A?

(a) 5 (b) 6

(c) 7 (d) 9

Ans. (c)

A 381 is divisible by 11 if and only if (A + 8) – (3 + 1) is divisible by 11.

So, A=7 Satisfies the condition

4. Find the LCM of 5/2, 8/9, 11/14.

(a) 280 (b) 360

(c) 420 (d) None of these

Ans: (d)

$$\text{LCM of fraction} = \frac{\text{LCM of numerators}}{\text{H. C. F of Denominators}}$$

Here, 5/2, 8/9, 11/14, so

$$LCM = \frac{\text{LCM of } (5, 8, 11)}{\text{HCF of } (2, 9, 14)} = \frac{440}{1} = 440$$

5. Find the number of divisors of 1420.

(a) 14 (b) 15

(c) 13 (d) 12

Ans: (d)

$1420 = 142 \times 10 = 71 \times 2 \times 2 \times 5 = 2^2 \times 5^1 \times 71^1$

No. of divisor = (2+1) (1+1) (1+1) = 12

6. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing?

(a) 34 (b) 46

(c) 26 (d) 44

Ans: (d)

It is given that gallons of

1st quality : 403

2nd quality : 465

3rd quality : 496

least number of bottles will be in size of HCF (403, 465 and 496)

$403 = 13 \times 31$

$465 = 15 \times 31$

$496 = 16 \times 31$

HCF = 31. So we required 13+15+16 = 44 bottles.

7. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?

(a) 9997 (b) 9793

(c) 9895 (d) 9487

Ans: (b)

LCM of 6, 9, 12, 17 = 612

greatest number of 4 digit divisible by 612 is 9792, to get remainder 1 number should be 9792+1

8. Which of the following is not a perfect square?

(a) 100858 (b) 3, 25, 137

(c) 945723 (d) All of these

Ans: (d)

Square of number never ends up with 2, 3, 7, 8

9. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
(a) $(x - 3)(x + 3)(4 - x^2)$
(b) $4(4 - x^2)(x + 3)$
(c) $(4 - x^2)(x - 3)$
(d) None of these
Ans: (d)

$$16 - x^2 = (4 - x)(4 + x)$$

$$(x^2 + x - 6) = (x + 3)(x - 2)$$

LCM will $(16 - x^2)(x^2 + x - 6)$

10. GCD of $x^2 - 4$ and $x^2 + x - 6$ is
(a) $x + 2$ (b) $x - 2$
(c) $x^2 - 2$ (d) $x^2 + 2$
Ans: (b)

$$x^2 - 4 = (x - 2)(x + 2)$$

$$(x^2 + x - 6) = (x + 3)(x - 2)$$

$$\text{GCD} = (x - 2)$$

11. Decompose the number 20 into two terms such that their product is the greatest.
(a) $x_1 = x_2 = 10$ (b) $x_1 = 5, x_2 = 15$
(c) $x_1 = 8, x_2 = 12$ (d) None of these
Ans: (a)
If $x + y$ = constant then xy will be maximum when $x = y$
here, $x_1 + x_2 = 20$
$x_1 = x_2 = 10$

12. For a number to be divisible by 88, it should be
(a) Divisible by 22 and 8
(b) Divisible by 11 and 8
(c) Divisible by 11 and thrice by 2
(d) Both (b) and (c)
Ans: (b)
A number to be divisible by 88 it should be divisible by 8 and 11 because 8 and 11 are co prime numbers whose multiplication gives 88.

13. Find the GCD of the polynomials $(x + 3)^2$ $(x - 2)(x + 1)^2$ and $(x + 1)^3(x + 3)(x + 4)$.
(a) $(x + 3)^3(x + 1)^2(x - 2)(x + 4)$
(b) $(x + 3)(x - 2)(x + 1)(x + 4)$
(c) $(x + 3)(x + 1)^2$
(d) None of these
Ans: (c)
GCD of $(x+3)(x-2)(x+1)^2$ and $(x+1)^3(x+3)(x+4)$ will be $(x+3)(x+1)^2$

14. Find the LCM of $(x + 3)(6x^2 + 5x - 4)$ and $(2x^2 + 7x + 3)(x + 3)$
(a) $(2x + 1)(x + 3)(3x + 4)$
(b) $(4x^2 - 1)(x + 3)^2(3x + 4)$
(c) $(4x^2 - 1)(x + 3)(3x + 4)$
(d) $(2x - 1)(x + 3)(3x + 4)$
Ans: (b)

$$(x+3)(6x^2 + 5x - 4) = (x+3)(2x-1)(3x+4)$$

$$(2x^2 + 7x + 3)(x+3) = (2x+1)(x+3)(x+3)$$

$$\text{LCM} = (2x+1)(2x-1)(x+3)^2(3x+4)$$

$$= (4x^2 - 1)(x+3)^2(3x+4)$$

15. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by
(a) 12 (b) 24
(c) 8 (d) All of these
Ans: (d)
Three consecutive number will be n(n + 1)(n + 2) if n is even number then (n + 2) will also be an even number and one of them will be divisible by 3. Hence number is always divisible by 12.

16. Find the pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13.
(a) 58 and 13 or 16 and 29
(b) 38 and 23 or 36 and 49
(c) 18 and 73 or 56 and 93
(d) 78 and 13 or 26 and 39
Ans: (d)
LCM = 78 and GCD = 13
Clearly 13, 78 and 26, 39 are the two numbers

17. Fill in the blank indicated by a star in the number 4* 56 so as to make it divisible by 33
(a) 3 (b) 4
(c) 5 (d) None of these
Ans. (a)
4*56 is divisible by 33 if and only if it is divisible by 3 and 11.

4* 56 will be divisible by 3 if * will be equal to 0, 3, 6, 9
4*56 is divisible by 11 if (4 + 5) – (* + 6) will be divisible by 11 so * should be 3.

18. Find the least number which being divided by 9, 12, 16 and 30 leaves in each case a remainder 3?

(a) 623 (b) 723
(c) 728 (d) None of these

Ans. (b)

LCM of 9, 12, 16 and 30 is 720 so required number is LCM + 3 = 723

19. Find the greatest number less than 10000 which is divisible by 48, 60 and 64

(a) 9600 (b) 8500
(c) 7600 (d) None of these

Ans. (a)

The required number will be the largest four digit number in form of n*(LCM) of 48, 60 and 64 LCM of 48, 60 and 64 is 960
So the largest four digit number will be 9600

20. Find the least multiple of 11 which when divided by 8, 9, 12, 14 leaves 4 as remainder in each case.

(a) 1012 (b) 1037
(c) 1090 (d) None of these

Ans. (a)

The number is divisible by 11 and can be written in form n(LCM) +4, LCM of 8, 9, 12, 14 is 504
So the number may be 508 & 1012 but 508 is not divisible by 11 so it is 1012

21. The LCM of two number is 12 times their HCF. The sum of HCF and LCM is 403. If one number is 93 find the other.

(a) 134 (b) 124
(c) 128 (d) None of these

Ans. (b)

It is given that LCM=12 times HCF
also LCM + HCF = 403
So, 13 HCF = 403, $\Rightarrow$ HCF = 31
LCM = 372 also we know that HCF

$$HCF \times LCM = Number(1) \times Number(2)$$

$$31 \times 372 = 93 \times N2 \quad \therefore \quad N2 = 124$$

22. I have to spend 1/10 of my income on house rent, 1/10 of remainder on conveyance 1/3 of further remainder on children's education after which I have Rs. 648 left over. What is my income?

(a) Rs. 1200 (b) Rs. 1400
(c) Rs. 1700 (d) None of these

Ans. (a)

One alternate method
Let I have x rupees

After spending $\frac{1}{10}$ of it on house rent I have $\frac{9x}{10}$.

Now out of $\frac{9x}{10}$ I spent $\frac{1}{10}$ of it i.e., $\frac{9}{100}x$ on conveyance so remainder will be

$$\frac{9}{10}x - \frac{9}{100}x = \frac{81x}{100}.$$

Further I spent $\frac{1}{3}$ of $\frac{81x}{100}$ i.e. $\frac{27x}{100}$ into childrens education now I have $\frac{54x}{100}$,

So, $\frac{54x}{100} = 648, \quad x = 1200$

23. A man had two sons. To the elder he gave $\frac{5}{11}$ of his property, to the younger $\frac{5}{11}$ of remainder, the rest to the widow. Find the Share of the sons if the widow gets Rs. 3600.

(a) Rs. 1200, 1000 (b) Rs. 6000, 2000
(c) Rs. 7500, 1000 (d) None of these

Ans. (d)

Younger son gets

$$3600 \times \left(\frac{1}{1-\frac{5}{11}}\right) \times \frac{5}{11} = \text{Rs. } 3000$$

Elder son gets

$$3000 \times \left(\frac{1}{1-\frac{5}{11}}\right) = \text{Rs. } 5500$$

□□□□

I Practice Exercise: I

1. $\sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}}=?$
   (a) $\sqrt{3\sqrt{5}}$ (b) 3
   (c) $3\sqrt{3}$ (d) $3+2\sqrt{5}$

2. x and y are integers and If $\frac{x^2}{y^3}$ is even integer then which of the following must be an even integer?
   (a) $x-y$ (b) $y+1$
   (c) $\frac{x^2}{y^4}$ (d) xy

3. What is the tens' digit of the sum of the first 50 terms of 1, 11, 111, 1111, 11111, 111111,......................?
   (a) 2 (b) 4
   (c) 5 (d) 8

4. If $81^y = \frac{1}{27^x}$, in terms of y, x=?
   (a) $\frac{3y}{4}$ (b) $-\frac{3y}{4}$
   (c) $\frac{4y}{3}$ (d) $-\frac{4y}{3}$

5. If $\frac{1}{n+1}<\frac{1}{31}+\frac{1}{32}+\frac{1}{33}<\frac{1}{n}$; then n ?
   (a) 9 (b) 10
   (c) 11 (d) 12

6. If one integer is greater than another integer by 3, and the difference of their cubes is 117, what could be their sum?
   (a) 11 (b) 7
   (c) 8 (d) 9

7. Which of these has total 24 positive factors?
   (a) $21^5 \times 2^3$ (b) $2^7 \times 12^3$
   (c) $2^6 \times 3^4$ (d) 63×55

8. What is the remainder of $\frac{3^{7^{11}}}{5}$?
   (a) 0 (b) 1
   (c) 2 (d) 3

9. Two numbers, x and y are such that when divided by 6, they leave remainder 4 and 5 respectively. Find the remainder when $x^3 + y^3$ is divided by 6?
   (a) 2 (b) 3
   (c) 4 (d) 5

10. What is the remainder when N = $(1! + 2! + 3!+...1000!)^{40}$ is divided by 10?
   (a) 1 (b) 3
   (c) 7 (d) 8

11. Set A is formed by selecting some of the numbers from the first 100 natural numbers such that the HCF of any two numbers in the set A is 5, what is the maximum number elements that set A can have?
   (a) 7 (b) 8
   (c) 9 (d) 10

12. Let x and y be positive integers such that x is prime and y is composite. Then,
   (a) $y-x$ cannot be an even integer
   (b) $\frac{x+y}{x}$ cannot be an even integer
   (c) $(x+y)$ cannot be even.
   (d) None of the above statements are true

13. Let N = 1421 × 1423 × 1425. What is the remainder when N is divided by 12?
   (a) 0 (b) 9
   (c) 3 (d) 6

14. When a four digit number is divided by 85 it leaves a remainder of 39. If the same number is divided by 17 the remainder would be ?
   (a) 2 (b) 5
   (c) 7 (d) 9

15. Let S be the set of integers such that
   1. $100 \le x \le 200$
   2. x is odd
   3. x is divisible by 3 but not by 7

   How many elements does S contain?
   (a) 16 (b) 12
   (c) 11 (d) 13

16. Integers 34041 and 32506 when divided by a three-digit integer n leave the same remainder. What is n?
   (a) 289 (b) 367
   (c) 453 (d) 307

17. Let T be the set of integers {3, 11, 19, 27,......, 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is ?

(a) 32 (b) 28

(c) 29 (d) 30

18. A box contains 100 tickets, numbered from 1 to 100. A person picks out three tickets from the box, such that the product of the numbers on two of the tickets yields the number on the third ticket. Which of the following tickets can never be picked as third ticket?

(a) 10 (b) 12

(c) 25 (d) 26

19. N is a natural number , then how many values of N are possible such that $\frac{6N^3+3N^2+N+24}{N}$ is also a Natural Number?

(a) 6 (b) 7

(c) 8 (d) 9

20. What is the unit digit of $39^{53} \times 27^{23} \times 36^{12}$?

(a) 2 (b) 4

(c) 6 (d) 8

21. How many number of zeros are there if we multiply all the prime numbers between 0 and 200.

(a) 1 (b) 2

(c) 3 (d) 4

22. A man wrote all the natural numbers starting from 1 in a series. What will be the 50^{th} digit of the number?

(a) 1 (b) 2

(c) 3 (d) 4

23. N = n(n + 1) (n + 2) (n + 3) (n + 4); where n is a natural number. Which of the following statement/s is/are true?

1. Unit digit of N is 0.
2. N is perfectly divisible by 24.
3. N is perfect square.
4. N is odd.

(a) 3 only (b) 3 and 4 only

(c) 1 only (d) 1 and 2 only

24. How many factors of $N = 12^{12} \times 14^{14} \times 15^{15}$ are multiple of $K = 12^{10} \times 14^{10} \times 15^{10}$

(a) $2 \times 4 \times 5$ (b) $3 \times 5 \times 6$

(c) $8 \times 7 \times 4 \times 5$ (d) $9 \times 8 \times 6 \times 5$

25. In a certain base

137 + 254 = 402 then

What is the sum of 342 + 562 in that base

(a) 904 (b) 1014

(c) 1104 (d) 1024

26. What is the remainder when $7^{7^{7^{\cdots\infty}}}$ is divided by 5?

(a) 2 (b) 3

(c) 1 (d) 2

□□□□

Solutions

1. (c)

Method (i) $\sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}}$ using rationalization

$$= \sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}\times\left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right)}$$

$$= \sqrt{3\sqrt{80}+\frac{(3\times9-3\times4\sqrt{5})}{9^2-(4\sqrt{5})^2}}$$

$$= \sqrt{3\sqrt{80}+\frac{27-12\sqrt{5}}{81-80}}$$

$$= \sqrt{3\sqrt{16\times5}+27-12\sqrt{5}}$$

$$= \sqrt{3\times4\times\sqrt{5}+27-12\sqrt{5}}$$

$$= \sqrt{12\sqrt{5}+27-12\sqrt{5}}$$

$$= \sqrt{27} = 3\sqrt{3}$$

Alternative Method

$$\sqrt{\left(3\sqrt{80}+\frac{3}{9+4\sqrt{5}}\right)}$$

$$3\sqrt{80} \cong 3\sqrt{81} \cong 27$$

and $\frac{3}{9+4\sqrt{5}} < 1$

Thus, $\sqrt{3\sqrt{80}+\frac{3}{9+4\sqrt{5}}} \cong \sqrt{3\sqrt{81}}$

$$\cong \sqrt{3\times9} = 3\sqrt{3}$$

2. (d)

if $\dfrac{x^2}{y^3}$ = even

$x^2 = y^3$ even

$\Rightarrow$ $x^2 \Rightarrow$ even

and x is integer

$\Rightarrow$ x = even

so only xy must be even.

3. (b)

$$\begin{array}{r} 1 \\ 1\,1 \\ 1\,1\,1 \\ \vdots\,\vdots\,\vdots \\ \text{50 terms.............}1\,1\,1 \\ \hline 40 \end{array}$$

unit digit (1 + 1...... 50 times)= 0

and carry = 5

tens digit (1 + 1 + ... 49 times)+ carry 5 = 4

4. (d)

$$81^y = \frac{1}{27^x}$$

$\Rightarrow$ $(3)^{4y} = (3^{-3})^x$

$\Rightarrow$ $(3)^{4y} = (3)^{-3x}$

as $a^m = a^n$

and $a \neq -1, 0, +1$

$\Rightarrow$ $m = n$

so $4y = -3x$

$$x = -\frac{4}{3}y$$

5. (b)

$\frac{1}{30} > \frac{1}{33}; \frac{1}{30} > \frac{1}{31}; \frac{1}{30} > \frac{1}{31};$

$\Rightarrow \frac{1}{30} + \frac{1}{30} + \frac{1}{30} > \frac{1}{31} + \frac{1}{32} + \frac{1}{33}$

$\Rightarrow \frac{3}{30} > \frac{1}{31} + \frac{1}{32} + \frac{1}{33}$

$\Rightarrow \frac{1}{10} > \frac{1}{31} + \frac{1}{32} + \frac{1}{33}$

$\Rightarrow \frac{1}{n} = \frac{1}{10} \Rightarrow n = 10$

and

OR $\frac{1}{31} > \frac{1}{33}; \frac{1}{32} > \frac{1}{33}; \frac{1}{33} = \frac{1}{33}$

$\Rightarrow \frac{1}{31} + \frac{1}{32} + \frac{1}{33} > \frac{3}{33}$

$\Rightarrow \frac{1}{31} + \frac{1}{32} + \frac{1}{33} > \frac{1}{11} = \frac{1}{n+1}$

n = 10

6. (b)

Use plugging in

as $(x + 3)^3 - x^3 = 117$

$x + 3 \geq 5$ as $(x + 3)^3 \geq 117$

put $x = 2$

$\Rightarrow$ $5^3 - 2^2 = 125 - 8 = 117$

$\Rightarrow$ $x = 2, \quad x + 3 = 5$

So, sum of both numbers = 7

Alternative Method

$$(x + 3)^3 - x^3 = 117$$

$x^3 + (3)^3 + 3(x)(3)(x + 3) - x^3 = 117$

$x^3 + 27 + 27x + 9x^2 - x^3 = 117$

$\Rightarrow 9x^2 + 27x - 90 = 0$

$\Rightarrow$ $x^2 + 3x - 10 = 0$

$\Rightarrow$ $(x + 5)(x - 2) = 0$

$x = 2, -5$

$\Rightarrow$ so either 2, 5, or –5, –2

Thus sum = 7, OR = –7

7. (d)

Put in prime factorization theorem.

(a) $21^5 \times 2^3 = 3^5 \times 7^5 \times 2^3$

Total factors $= 6 \times 6 \times 4 \neq 24$

(b) $2^7 \times (2^2 \times 3)^3 = 2^{13} \times 3^3$

Total factors $= 14 \times 4 \neq 24$

(c) $2^6 \times 3^4$ = factors $7 \times 5 \neq 24$

(d) $63 \times 55 = 3^2 \times 7^1 \times {}^15 \times 11^1$

Factors $= 3 \times 2 \times 2 \times 2 = 24$

8. (c)

For remainder we have to calculate the unit digit of $3^{7^{11}}$

$\Rightarrow$ Now, Rem$\dfrac{7^{11}}{4} = (-1)^{11}$

$= -1 \Rightarrow -1 + 4 = 3$

Thus, 3^{4K+3} gives unit

Digit $\Rightarrow$ $3 \times 3 \times 3 = 7$

so, Rem$\dfrac{3^{7^{11}}}{5} = \dfrac{(.......7)}{5}$

Thus remainder is 2. As for checking divisibilty by 5 is checked by dividing last digit of number.

$N = 12^3 \times 13^2 \times 14$

$= (2^2 \times 3)^3 \times 13^2 \times (7 \times 2)$

$= 2^7 \times 3^3 \times 7^1 \times 13^2$

Number of factors

(7 + 1) (3 + 1) (1 + 1) (2 + 1) = 192

9. (b)

$$\text{Rem}\frac{[x]}{6} = 4, \quad \text{Rem}\frac{[y]}{6} = 5$$

$$\text{Rem} = \frac{[x^3+y^3]}{6} = \frac{[4^3+5^3]}{6}$$

$$= \frac{[6+125]}{6}$$

$$\Rightarrow \text{Rem}\left[\frac{4+5}{6}\right] = \text{Rem}\left(\frac{9}{6}\right) = 3$$

10. (a)

N = (1! + 2! + 3! + 4! +....1000!)

Now we have to check only

1! + 2! + 3! + 4! as after that will factorial has unit digit as 0

5! = 120

6! = 720 and so on

Thus unit digit of (1! + 2! + 3! + 4! + 0) for all other

$\Rightarrow$ $(1 + 2 + 6 + 4 + 0)^{40}$

Unit digits of all factorial

$\Rightarrow$ $(3)^{40} = (3)^{4k}$

$\Rightarrow 3 \times 3 \times 3 \times 3 \Rightarrow 1$ unit digit. Hence when $(3)^{40} \div 10$, Remainder will be 1.

11. (c)

As there should be only 5 which should be common between any two number

so number may be $(5x_1, 5x_2, 5x_3.....5x_n)$

and HFC $(xi, xj) = 1$

Thus, number are

$(5 \times 1, 5 \times 2, 5 \times 3, 5 \times 5......)$

because $(xi, xj) = 1$

Then only prime number will work.

so $5 \times 19 = 95$ is biggest number.

so $5 \times 1, 5 \times 2, 5 \times 3, 5 \times 5, 5 \times 7, 5 \times 11, 5 \times 13, 5 \times 17, 5 \times 19$, total 9 number.

12. (d)

Plugging in

(a) $y = 10, x = 2$

$= 10 - 2 = 8$ X

(b) $\frac{x+y}{x} = 1 + \frac{y}{x}$

$y = 9, x = 3$

$1 + \frac{9}{3} = 1 + 3 = 4$ X

(c) $(x + y) = 10 + 2 = 12$ X

(d) None of above is true.

13. (c)

$$\text{Rem}\left[\frac{1421 \times 1423 \times 1425}{12}\right]$$

$$\Rightarrow \text{Rem}\frac{[5 \times 7 \times 9]}{12} = \text{Rem}\frac{[35 \times 9]}{17}$$

$$\text{Rem}\frac{[11 \times 9]}{12} = \text{Rem}\frac{[99]}{12} = 3$$

14. (b)

$n = 85K + 39$

Now $\frac{n}{17} = \frac{85K+39}{17} = 5K + \frac{34+5}{17}$

$= 5K + 2 + \frac{5}{17}$

$\Rightarrow$ Remainder is 5.

15. (d)

Number are 105, 111......195

$\Rightarrow$ Total $\frac{195-105}{6} + 1 = 16$

as number is odd and not multiple of 7.

Thus in total 16 number there are 3 numbers

$21 \times 5 = 125$

$21 \times 7 = 147$

$21 \times 9 = 189$

which are multiple of 7.

Thus $16 - 3 = 13$ numbers

16. (d)

Suppose division 'd' and remainder is 'r'.

$\Rightarrow$ $34041 = q_1 d + r$

and $32506 = q_2 d + r$

$\Rightarrow$ $15365 = (q_1 - q_2)d$

$\Rightarrow$ d should be factor 1535

So, only 307 is factor of 1535 in given option.

17. (c)

{3, 11,...... 459, 467}

This is an Arithmetic Progression

$467 = 3 + (n - 1)8$

Total $n = 59$

Now, (3, 467), (11, 459), (19, 451)........

Total $\frac{59}{2} = 29$ pair

All will make sum 470.

So we can take only 1 elements from each pair.

Thus total elements in T can be

{one of (3, 467), one of (11, 459)....}

Total 29.

18. (c)

(a) 100 = 5 × 2 possible

(b) 12 = 6 × 2 possible

(c) 25 = 5 × 5 not possible as only one 5 Number ticket

(d) 30 = 15 × 2 possible

19. (c)

Suppose

$$\frac{6N^3 + 3N^2 + N + 24}{N} = M = \text{Natural Number}$$

So, $M = 6N^2 + 3N + 1 + \frac{24}{N}$

Thus to be M as an natural number $\frac{24}{N}$ should be Natural number thus N should be factor of 24.

so $24 = 2^3 \times 3^1$

Factors = 4 × 2 = 8

8 values can be taken by N.

20. (a)

Unit digit of

$39^{53} \times 27^{23} \times 36^{12}$

Unit digit of $(9^{53} \times 7^{23} \times 6^{12})$

$= (9^{4K+1} \times 7^{4K+3} \times 6^{4K})$

$\Rightarrow (9 \times 3 \times 6) = 2$ = units digit

21. (a)

Number of zero's can be calculated by getting power of 10 in number or minimum among powers of 2 or 5 possible in n.

Thus 2 × 3 × 5 × 97

so only 1 → 5's and 1 → 2's

Thus only one zero as both 2 and five will occur only once.

22. (c)

123.....910........

One digit numbers (from 1 to 9) = 9

Now 2 digit number till 50 are 50 – 9 = 41 thus total digits used to make pair.

<u>10</u> <u>11</u> <u>12</u>.........29.

So total 20 × 2 = 40 digits would be required to write numbers from 10 to 29.

Total digits consumed so far till we write 29 will be 9 + 40 = 49.

50^{th} digit which we will write is 3 of 30.

Hence 50^{th} digit will be 3.

49 digits are used. So far till 29.

Now, 50^{th} digit would be first digit of 30 thus 3.

23. (d)

N = n(n+ 1) (n + 2) (n + 3) (n + 4) is product of 5 consecutive number.

Thus divisible by 5! = 120

So, (i) unit digit is 0 TRUE.

(ii) Perfectly divisible by 24 TRUE.

(iii) N is perfect square not TRUE.

(iv) N is odd X not TRUE.

Thus (d) (i) and (ii) only.

24. (d)

$N = 12^{12} \times 14^{14} \times 15^{15}$

$= (2^2 \times 3)^{12} \times (7 \times 2)^{14} \times (5 \times 3)^{15}$

$= 2^{24} \times 3^{12} \times 7^{14} \times 2^{14} \times 5^{15} \times 3^{15}$

$= 2^{38} \times 3^{27} \times 5^{15} \times 7^{14}$

$K = (12)^{10} \times (14)^{10} \times (15)^{10}$

$= 2^{20} \times 3^{10} \times 7^{10} \times 2^{10} \times 3^{10} \times 5^{10}$

$= 2^{30} \times 3^{20} \times 5^{10} \times 7^{10}$

Thus factors of N which are multiple of k are

$2^{30} \times 3^{20} \times 5^{10} \times 7^{10} (2^8 \times 3^7 \times 5^5 \times 7^4)$

will be (8 + 1) × (7 + 1) × (5 + 1) × (4 + 1)

9 × 8 × 6 × 5

25. (b)

137_9

$\underline{254_9}$

$\underline{402_9}$

$(7 + 4) = (11)_{10} = (2)$

unit digit is 2 thus 9 is possible base

Thus we have to add using base 9

so 342_9

$\underline{562_9}$

$\underline{1014_9}$

26. (b)

For remainder by 5 we have to calculate the unit digit only.

For unit digit of $7^{7^{7^{\cdots\infty}}}$ get $(7)^{4k+R}$ from

Now Rem $\left(\frac{7^{7^{\cdots\infty}}}{4}\right) = (-1)^{7^{7^{\cdots\infty}}} = (-1)^{odd}$

= (–1) Thus (3) Remainder

$= 7^{7^{\cdots\infty}} = 7^{4k+3}$

= unit digit is 7 × 7 × 7 = 3

So Rem$\left[\frac{3}{5}\right] = 3$ so (b)