a) Prior probability :

$$P(\omega_1) = \frac{7}{14} = 0.5$$

$$P(\omega_2) = \frac{7}{14} = 0.5$$

b) mean of ω_1 : $\left[\frac{12}{7}, \frac{8}{7}\right]$ ω_2 : $\left[\frac{54}{7}, \frac{60}{7}\right]$

cov : $\frac{1}{42}\begin{bmatrix} 66 & 37 \\ 37 & 62 \end{bmatrix}$ cov : $\frac{1}{42}\begin{bmatrix} 24 & -27 \\ -27 & 152 \end{bmatrix}$

(c) w_1

$$\mu = \begin{bmatrix} \frac{12}{7} & \frac{8}{7} \end{bmatrix}^T \qquad X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & 0 \\ 3 & 2 \\ 3 & 3 \\ 2 & 2 \\ 2 & 0 \end{bmatrix}$$

$$X' = \frac{1}{7}\begin{bmatrix} -12 & -8 \\ -12 & -1 \\ 2 & -8 \\ 9 & 6 \\ 9 & 13 \\ 2 & 6 \\ 2 & -8 \end{bmatrix}$$

$$\Sigma = (X')(X') = \frac{1}{6\times 49}\begin{bmatrix} 462 & 259 \\ 259 & 434 \end{bmatrix}$$

$$= \frac{1}{6\times 7}\begin{bmatrix} 66 & 37 \\ 37 & 62 \end{bmatrix} \qquad \Sigma^{-1} = \frac{6}{389}\begin{bmatrix} 62 & -37 \\ -37 & 66 \end{bmatrix}$$

$$g(x) = x^T W_1 x + w_1^T x + w_{10}$$

$$W_1 = -\frac{1}{2}\Sigma^{-1} \qquad W_1 = -\frac{1}{2}\cdot\frac{6}{389}\begin{bmatrix} 62 & -37 \\ -37 & 66 \end{bmatrix}$$

$$w_1 = \Sigma_1^{-1}\mu_1 = \frac{1}{7}\times\frac{6}{389}\begin{bmatrix} 62 & -37 \\ -37 & 66 \end{bmatrix}\begin{bmatrix} 12 \\ 8 \end{bmatrix} = \frac{1}{7}\times\frac{6}{389}\begin{bmatrix} 448 \\ 84 \end{bmatrix}$$

$$= \frac{6}{389}\begin{bmatrix} 64 \\ 12 \end{bmatrix}$$

$$w_{10} = -1.1689$$

$$w_{i0} = -\frac{1}{2} u_i^T \underbrace{\Sigma_i^{-1} u_i}_{w_i} - \frac{1}{2}\ln|\Sigma_i| + \ln P(w_i)$$

$$= -\frac{1}{2} \times \frac{1}{7}\begin{bmatrix} 12 & 8 \end{bmatrix} \times \frac{6}{389}\begin{bmatrix} 64 \\ 12 \end{bmatrix} - \frac{1}{2}\ln|\Sigma_i| + \ln P(w_i)$$

$$= -\frac{432 \times 6}{7 \times 389} - \frac{1}{2}\ln\left(\frac{389}{7 \times 6}\right) + \ln P(w_i)$$

= [illegible]

$$g_1(x) = \begin{bmatrix} x & y \end{bmatrix} W_1 \begin{bmatrix} x \\ y \end{bmatrix} + w_1^T \begin{bmatrix} x \\ y \end{bmatrix} + w_{10}$$

$$= \begin{bmatrix} x & y \end{bmatrix} \times -\frac{1}{2} \times \frac{6}{389}\begin{bmatrix} 62 & -37 \\ -37 & 66 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \frac{6}{389}\begin{bmatrix} 64 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + w_{10}$$

$$= -\frac{1}{2} \times \frac{6}{389}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 62x - 37y \\ -37x + 66y \end{bmatrix} + \frac{6}{389}(64x + 12y) + w_{10}$$

$$= -\frac{1}{2} \times \frac{6}{389}\left(62x^2 - 37xy - 37xy + 66y^2\right) + \frac{6}{389}(64x + 12y) + w_{10}$$

$$= \frac{6}{389}\left(-31x^2 + 37xy - 33y^2 + 64x + 12y\right) + w_{10}$$

$$\mu = \frac{1}{7}\begin{bmatrix} 54 \\ 60 \end{bmatrix} \qquad X = \begin{bmatrix} 7 & 7 \\ 8 & 6 \\ 9 & 7 \\ 8 & 10 \\ 7 & 10 \\ 8 & 9 \\ 7 & 11 \end{bmatrix}$$

$$x' = \frac{1}{7}\begin{bmatrix} -\cancel{8}5 & -11 \\ 2 & -18 \\ 9 & -11 \\ 2 & 10 \\ -\cancel{8}5 & 10 \\ 2 & 3 \\ -\cancel{8}5 & 17 \end{bmatrix}$$

$$cov = \frac{1}{6 \times 49}\begin{bmatrix} 168 & -189 \\ -189 & 1064 \end{bmatrix} = \frac{1}{6 \times 7}\begin{bmatrix} 24 & -27 \\ -27 & 152 \end{bmatrix}$$

$$\Sigma_2^{-1} = \frac{6}{417}\begin{bmatrix} 152 & 27 \\ 27 & 24 \end{bmatrix}$$

$$W_2 = \frac{-1}{2}\Sigma^{-1} = \frac{-1}{2} \times \frac{6}{417}\begin{bmatrix} 152 & 27 \\ 27 & 24 \end{bmatrix}$$

$$w_2 = \Sigma_2^{-1}\mu_2 = \frac{6}{417}\begin{bmatrix} 152 & 27 \\ 27 & 24 \end{bmatrix} \times \frac{1}{7}\begin{bmatrix} 54 \\ 60 \end{bmatrix}$$

$$= \frac{6}{417}\begin{bmatrix} \cancel{8}1404 \\ 414 \end{bmatrix}$$

$w_{20} = -103.7009$

$$w_{i0} = -\frac{1}{2}\mu_i^T \underbrace{\Sigma^{-1}\mu_i}_{w_i} - \frac{1}{2}\ln|\Sigma_i| + \ln P(w_i)$$

$$= -\frac{1}{2} \times \frac{1}{7}\begin{bmatrix} 54 & 60 \end{bmatrix} \times \frac{1}{417}\begin{bmatrix} 1404 \\ 417 \end{bmatrix} - \frac{1}{2}\ln\left(\frac{417}{7}\right) + \ln P(w_2)$$

$$= \cancel{\frac{6}{417}\left(-\frac{50418}{7} - \frac{417}{6\times 2}\ln\left(\frac{417}{6\times 7}\right)\right)}$$

$$g_2(x) = \begin{bmatrix} x & y \end{bmatrix} \times -\frac{1}{2} \times \frac{1}{417}\begin{bmatrix} 152 & 27 \\ 27 & 24 \end{bmatrix}\begin{bmatrix} x & y \end{bmatrix} + \frac{1}{417}\begin{bmatrix} 1404 & 414 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + w_{20}$$

$$= -\frac{1}{2} \times \frac{1}{417}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 152x + 27y \\ 27x + 24y \end{bmatrix} + \frac{1}{417}\left(1404x + 414y\right) + w_{20}$$

$$= -\frac{1}{2} \times \frac{1}{417}\left(152x^2 + 27xy + 27xy + 24y^2\right) + \text{''} \quad \text{''}$$

$$= \frac{6}{417}\left(-76x^2 - 27xy - 12y^2 + 1404x + 414y\right) + w_{20}$$

Decision boundary:

$$g_1(x) = g_2(x)$$

$$\Rightarrow \frac{6}{389}\left(-31x^2 + 37xy - 33y^2 + 64x + 12y\right) + w_{10}$$

$$= \frac{6}{417}\left(-76x^2 - 27xy - 12y^2 + 1404x + 414y\right) + w_{20}$$

$$\Rightarrow \frac{6}{389}\left(-31x^2 + 37xy - 33y^2 + 64x + 12y \; \overbrace{\cancel{- \frac{1322}{7} - \frac{389}{2\times 6}\ln\left(\frac{389}{7\times 6}\right)}}^{w_{10}}\right)$$

$$= \frac{6}{417}\left(-76x^2 - 27xy - 12y^2 + 1404x + 414y \; \overbrace{\cancel{- \frac{50418}{7} - \frac{417}{2\times 6}\ln\left(\frac{417}{6\times 7}\right)}}^{+w_{20}}\right)$$

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$$417\left[-31x^2+37xy-33y^2+64x+12y-\frac{832}{7}-\frac{389}{2\times 6}\ln\left(\frac{389}{7\times 6}\right)+w_{10}\right]$$

$$=389\left[-76x^2-27xy-12y^2+1404x+414y-\frac{5048}{7}-\frac{417}{2\times 6}\ln\left(\frac{417}{7\times 6}\right)+w_{20}\right]$$

$$\Rightarrow (-28356+29564)x^2+(15429+10503)xy+$$

$$(-13761+4668)y^2+(26688-546156)x$$

$$+(5004-161046)y+\left(-\frac{180144}{7}-\frac{389\times 417}{2\times 6}\ln\left(\frac{389}{7\times 6}\right)+\frac{389\times 5048}{7}+\frac{389\times 417}{2\times 6}\ln\left(\frac{417}{7\times 6}\right)\right)$$

$$+(417w_{10}-389w_{20})$$

$$\Downarrow$$

$$\frac{389\times 417}{2\times 6}\left[\ln\left(\frac{417}{7\times 6}\right)-\ln\left(\frac{389}{7\times 6}\right)\right]$$

$$\frac{389\times 417}{2\times 6}\ln\left(\frac{417}{389}\right)+\frac{1943 2958}{7}$$

$$=0$$

$$\Rightarrow 1208x^2+25932xy-9093y^2-519468x$$

$$-156042y+(939.57+277606 5.45)$$

$$\Downarrow$$

$$2781003 \quad 277005$$

$$=0$$

Boundary eq^n :

$$16637x^2 + 25932xy - 9093y^2 - 549468x$$

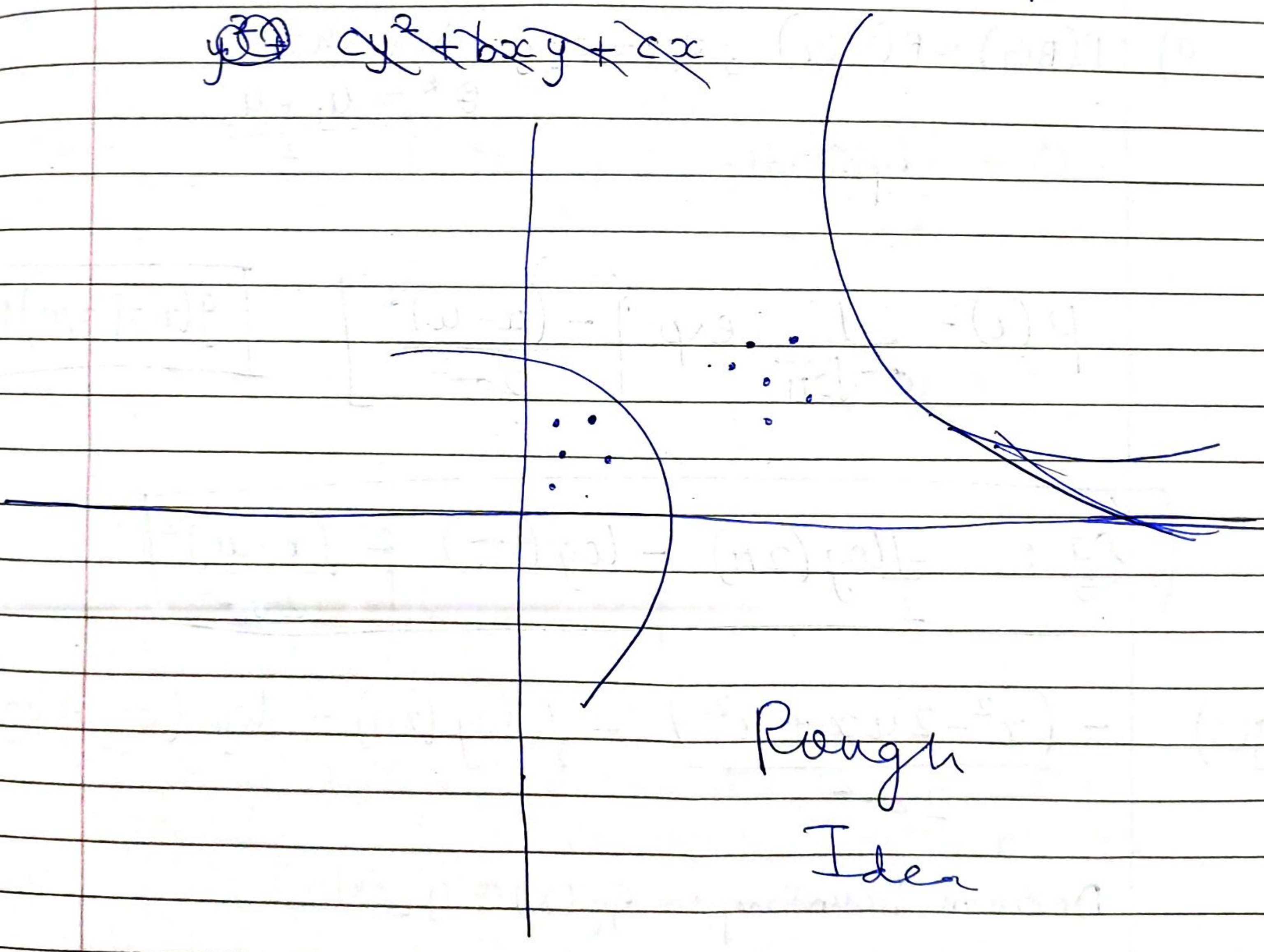
2844200

$$-156042y + \cancel{2281708} = 0$$

~~2844200~~

$$\cancel{ax^2 + bxy + cy^2 + dx + ey + f = 0}$$

$$\cancel{cy^2 + bxy + cx}$$



Rough Idea

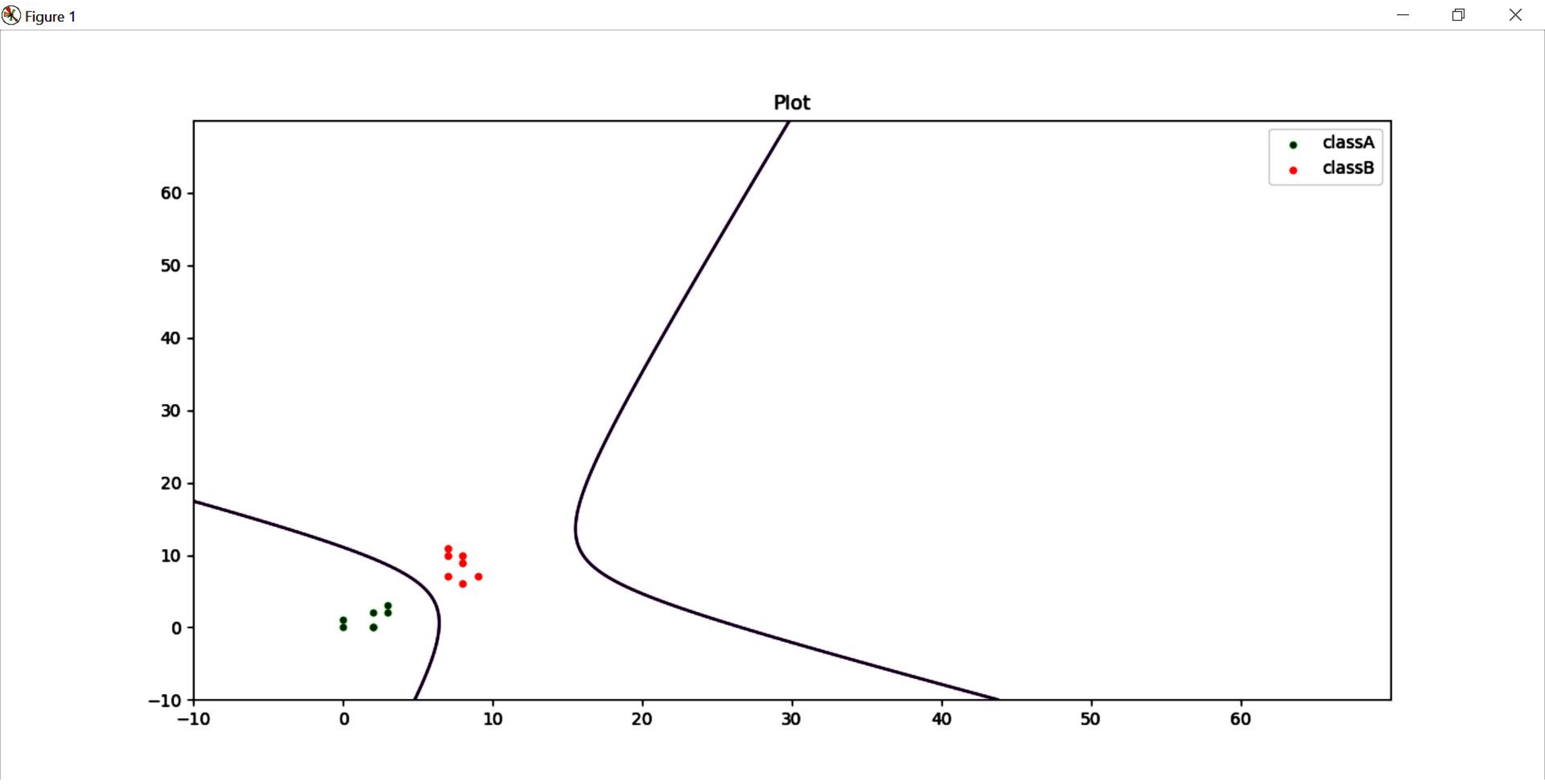


Figure 1
Plot
classA
classB
60
50
40
30
20
10
0
−10
−10
0
10
20
30
40
50
60

(c)

$$\cancel{g_1(x) = 2 g_2(x)}$$

$$P(x|\omega_1)\,P(\omega_1)\,C(\omega_1) = P(x|\omega_2)\,P(\omega_2)\,C(\omega_2)$$

$$C(\omega_1) = 2\,C(\omega_2)$$

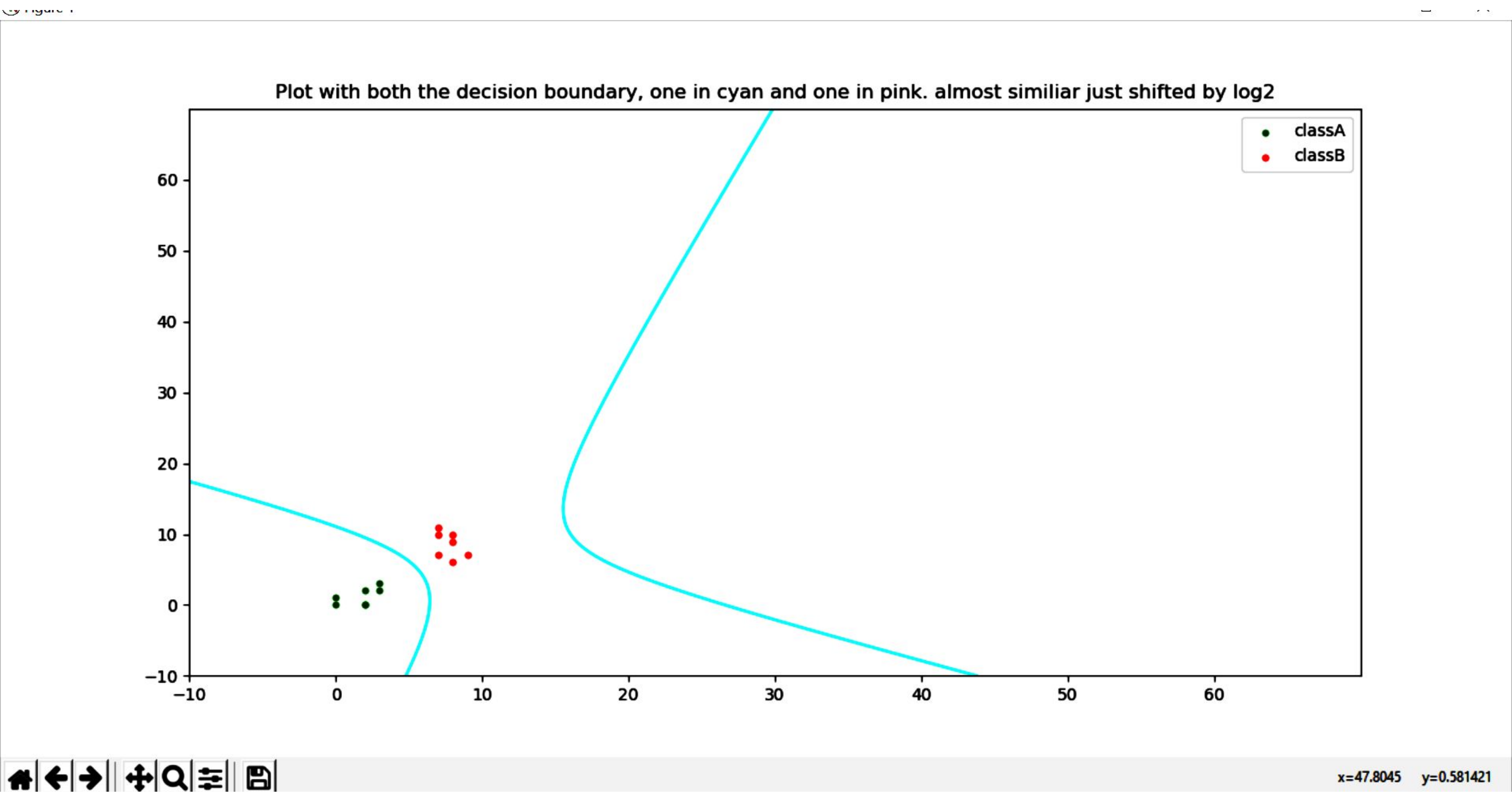
$$2P(x|\omega_1)\,P(\omega_1) = P(x|\omega_2)\,P(\omega_2)$$

$$\ln 2 + \underbrace{\ln(P(x|\omega_1)) + \ln P(\omega_1) = \ln(P(x|\omega_2)) + \ln(P(\omega_2))}_{\text{Earlier boundary condition.}}$$

The decision boundary still looks the same, it just has a different offset

$\Rightarrow$

$$16637x^2 + 25932xy - 9093y^2 - 549468x - 156042y + 284420 + \ln 2 = 0$$



Plot with both the decision boundary, one in cyan and one in pink. almost similiar just shifted by log2
classA
classB
60
50
40
30
20
10
0
−10
−10
0
10
20
30
40
50
60
x=47.8045 y=0.581421