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Q-2. Microscopic Image $M \times N$

Each cell $\rightarrow$ BG $\Rightarrow \mathcal{N}(\mu_1, \sigma_1^2)$
$\hookrightarrow$ FG $\rightarrow \mathcal{N}(\mu_2, \sigma_2^2)$

$\mu_1 < \mu_2$

$x_{ij} < \theta$, $\rightarrow$ BG
else $\rightarrow$ FG

a) $P(BG) = P(FG)$, $\sigma_1 = \sigma_2$, then show $\theta^* = \frac{\mu_1 + \mu_2}{2}$

optimal.

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad \boxed{g(x) = P(x|w_1)\,P(w_1)}$$

$$\boxed{\log: \quad -\frac{1}{2}\log(2\pi) - \log(\sigma) - \frac{(x-\mu)^2}{2\sigma^2}}$$

$$g(x) \quad -\frac{(x^2 - 2\mu x + \mu^2)}{2\sigma^2} - \frac{1}{2}\log(2\pi) - \log(\sigma) + \log(P)$$

Decision Boundary: $g_1(x) = g_2(x)$

$$-\frac{(x^2 - 2\mu_1 x + \mu_1^2)}{2\sigma_1^2} - \log(\sigma_1) + \frac{(x^2 - 2\mu_2 x + \mu_2^2)}{2\sigma_2^2} + \log(\sigma_2)$$
$$+ \log(P(BG)) - \log(P(FG))$$

$$\Rightarrow \frac{x^2}{2}\left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}\right) + \frac{x}{2}\left(\frac{2\mu_1}{\sigma_1^2} - \frac{2\mu_2}{\sigma_2^2}\right) + \frac{\mu_2^2}{2\sigma_2^2} - \frac{\mu_1^2}{2\sigma_1^2} + \log\left(\frac{\sigma_2}{\sigma_1}\right) + \log\left(\frac{P(BG)}{P(FG)}\right)$$

☆ $\sigma_1 = \sigma_2 = \sigma$ and $P(BG) = P(FG)$

x^2 terms vanishes.

Decision boundary : $\frac{x}{2\sigma^2}(2\mu_1 - 2\mu_2) + \frac{(\mu_2^2 - \mu_1^2)}{2\sigma^2} = 0$

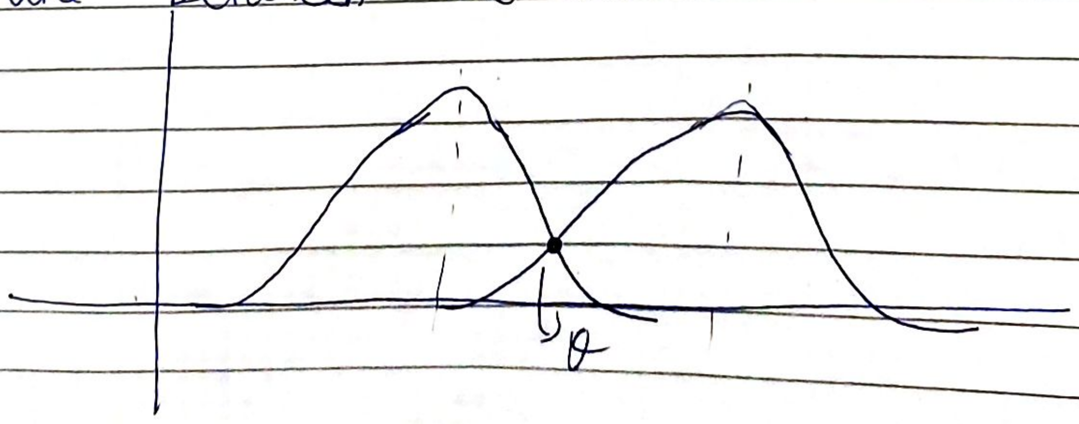
$$\frac{(\mu_1 - \mu_2)}{\sigma^2}\left[x - \left(\frac{\mu_2 + \mu_1}{2}\right)\right] = 0$$

$$\underset{\theta}{\shortparallel}$$

now since $\mu_1 < \mu_2$ so $\mu_1 - \mu_2 \neq 0$

$$\boxed{\theta = \frac{\mu_2 + \mu_1}{2}}$$

This is optimal because decision boundary is perpendicular bisector of the line between the mean



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(C) $P(BG) = 4\,P(FG)$

$\mu_1 = 100$ $\sigma_1 = \sigma_2$

$\mu_2 = 200$

$$\Rightarrow \frac{2x(\mu_1 - \mu_2)}{2\sigma^2} + \frac{\mu_2^2 - \mu_1^2}{2\sigma^2} + \log\left(\frac{P(BG)}{P(FG)}\right) = 0$$

$$2x(-100) + (200-100)(300) + 2\sigma^2 \log(4) = 0$$

$$x - \left(\frac{30000 + 2\sigma^2 \log(4)}{200}\right) = 0$$

$$\theta = \frac{30000 + 2\sigma^2 \log_e(4)}{200}$$

(b)

$$g_i(x) = -\frac{1}{2}\log(2\pi) - \log(\sigma_i) - \frac{(x-\mu_i)^2}{2\sigma_i^2} + \log(P_i)$$

$$g_1(x) = g_2(x)$$

$$-\log(\sigma_1) - \frac{(x-\mu_1)^2}{2\sigma_1^2} + \log(P(BG))$$

$$= -\log(\sigma_2) - \frac{(x-\mu_2)^2}{2\sigma_2^2} + \log(P(FG))$$

$$\frac{\left(\frac{\mu_1+\mu_2}{2} - \mu_2\right)^2}{2\sigma_2^2} - \frac{\left(\frac{\mu_1+\mu_2}{2} - \mu_1\right)^2}{2\sigma_1^2} + \log\left(\frac{\sigma_2}{\sigma_1}\right) + \log\left(\frac{P(BG)}{P(FG)}\right) = 0$$

$$\frac{(\mu_1-\mu_2)^2}{8\sigma_2^2} - \frac{(\mu_2-\mu_1)^2}{8\sigma_1^2} + \log\left(\frac{\sigma_2}{\sigma_1}\right) + \log\left(\frac{P(BG)}{P(FG)}\right) = 0$$

$$\frac{(\mu_1-\mu_2)^2}{8}\left[\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}\right] + \log\left(\frac{\sigma_2}{\sigma_1} \times \frac{P(BG)}{P(FG)}\right) = 0$$

$$(\mu_1-\mu_2)^2 = 8\log\left(\frac{\sigma_1 \times P(FG)}{\sigma_2 \times P(BG)}\right) \times \frac{\sigma_2^2\sigma_1^2}{(\sigma_1^2-\sigma_2^2)}$$

$$\boxed{\mu_1 - \mu_2 = 2\sqrt{2}\,\sigma_1\sigma_2\sqrt{\log\left(\frac{\sigma_1 P(FG)}{\sigma_2 P(BG)}\right)} \times \frac{1}{\sqrt{\sigma_1^2-\sigma_2^2}}}$$