Multi Facility location

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1 Introduction

Given a space of locations in R^d, the goal of locating a single facilities such that it reduces social cost is single facility location problem. In the special case, where agents have single peaked preferences, the problem reduces to choosing medians of the agents peak (Moulin's generalized median rule) and its a strategyproof mechanism. For the problem involving locating multiple facilities, the current literature doesn't have complete characterization of strategy-proof mechanisms. These are mechanisms without money

2 Paper

- Title : Deep Learning for Multi-Facility Location Mechanism Design
- Goal : Design strategy-proof, multi-facility mechanisms that minimize expected social cost. Designing nn for already existing characterization results, and further for learning mechanism where characterization results are not available.
- Setting of paper : N agents 1,2,...N , set of location Ω = [0,1], K facilities. Each agent has single peaked preference over Ω. For each agent i, utility will be the based on the facility that was placed closet to its preferred location. i.e. $u_i(o) = \max_{k \in K} u_i(o_k)$. We assume $u(x) = -|x - a|$ where $a = \tau(u)$ is the peak

  The paper purposes MoulinNet for Generalized Median Rules for Single Facility Problem and RegretNet-nm For General Mechanisms
- MoulinNet for Generalized Median Rules
  - Single Facility Generalized Median Rule - A unanimous mechanism $f : U \to \Omega$ is strategy-proof if and only if it is a generalized median rule, i.e. for each $S \subset 1, 2, ..., n$, there exists some $a_S \in \Omega$ s.t. for all $(u_1, ..., u_n) \in U$. The parameters a_S are monotone, i.e. $a_S \geq a_T, \forall S \subset T$

    Equation Example
  - Multi Facility Generalized Median Rule - $f = (f_1, ..., f_K)$ where each f_k is a single facility generalized median rule for parameters $a^k_S \in \Omega$, $S \subseteq N$ This class is also strategyproof.
  - We use neural network to determine each a_S, it maps an n-dimensional binary representation of a set S i.e. v(S) to a real value $a_S = h(v(S))$

    EQUATION HERE of neural network (1) IMAGE of neural network

    Time complexitiy of the network is given as $O(z \cdot n + nlogn)$, where z is the time taken for a single invocation of $h^{w,b}$

    DOUBTS : How are they trainning the network? which initial values of a_S are they taking How are they solving multi facility location problem using this nn, this nn is only giving a_S
- RegretNet-nm For General Mechanisms
  - We use a NN, fully connected, L hidden layers, takes inputs as agents peaks and gives locations of K facilities as output
  - Goal here is to minimize $\mathcal{L}(w)$ expected social cost subject to expected ex post regret being zero for all agents. (such that it is almost strategyproof)

    IMAGE OF NN *EQUATION HERE*
- Experimental Results

- Compared the results with the existing mechanism - best percentile rule (A percentile rule locates each facility at a fixed percentile of the reported peaks), best dictatorial rule (A dictatorial rule locates each facility at the peak of a fixed agent) and best constant rule (A constant rule locates each facility at a fixed point)
- Evaluated over Unweighted social cost, Weighted social cost and Non Independent Valuations For Unweighted social cost, both kinds of networks yield similar performance as the best percentile rule. For weighted social cost, RegretNet-nm and MoulinNet yield significantly smaller social cost than the baseline mechanisms. For Non Independent Valuations, RegretNet-nm outperform all the benchmarks