Pacing Equilibrium in First-Price Auction Markets

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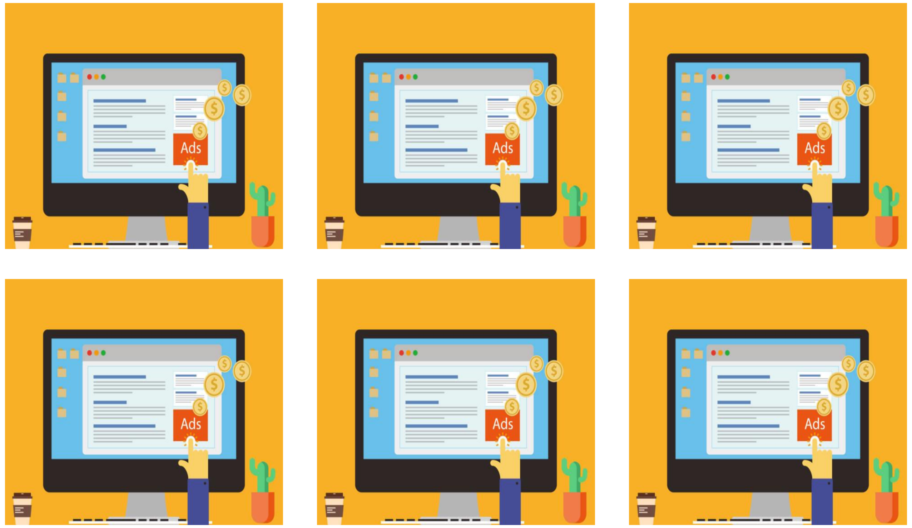
Presented by: Abhigyan, Issac, Shaily

Selling Single item

- First Price Auctions
  - Winner pays highest bid
  - Bidders have incentive to cheat
- Second Price Auctions
  - Winner pays second highest bid
  - Truthful bidding is dominant strategy
- E.g. – Internet Ads – selling a single slot

Selling single item in larger system

- Second price auction, truthful bids is not the best strategy
- Companies have budgets, so winning all slots with true valuations, is not that profitable
- Plus running single item auctions for all items is not incentive compatible



Single item auction in larger system (Ad Markets)

Bidder Selection

- Choose a subset of bidders whose budgets have not been exhausted for each auction
- For any item, bidders pay their original bids

Bid Modification

- Individual bids are scaled using pacing multipliers
- Winner pays scaled bid instead of original bid

Allocate impression such that it optimizes the use of bidder's overall budgets

Model

- N bidders {1,2,…,n}
- M divisible goods {1,2,…,m}
- Bidder i's valuation for good j $v_{ij} >= 0$
- Bidder i's budget $B_i > 0$

Goal

- Find pacing multipliers α_i in [0,1] such that it smooths out the spending of each bidders, and fractional allocation x_{ij}
- So bidder i bids $\alpha_i v_{ij}$ for good j and will pay that value if it wins
- In case of ties, auctioneer does fractional allocation

Assumptions

- Valuations, budgets are known to auctioneer
- Pacing multipliers will remain constant for a bidder

Budget feasible Pacing Multipliers BFPM (α,x)

Pacing multipliers α and allocation x, should satisfy:

- Prices (Unit price) $\mathrm{p_{ij}} = \max\limits_{\mathrm{i \in N}} \alpha_i . v\_ij$
- Goods go to highest bidders $\mathrm{x_{ij}} > 0 \Rightarrow \alpha_i . v_{ij} = \max\limits_{\mathrm{i \in N}} \alpha_i . v_{ij}$
- Budget feasible $\sum_{j \in M} x_{ij} . p_j \leq B_i \; \forall \, i \in \mathrm{N}$
- Demanded goods sold completely $\mathrm{p_j} > 0 \Rightarrow \sum_{i \in N} x_{ij} = 1$
- No overselling $\sum_{i \in N} x_{ij} \leq 1$

FPPE First Price Pacing Equilibrium is BFPM (α,x) when there is no unnecessary pacing, i.e.

If $\sum_{j \in M} x_{ij} . p_j \leq B_i \Rightarrow \alpha_i = 1$

SOLUTION

	Item1	Item2	Budget
Bidder1	10	0	10
Bidder2	5	8	6

BFPM

Alpha = (0.5,0.5)

P1 = max(5,2.5) = 5
P2 = max(0,4) = 4
X11 = 1, x22 = 1
Budget feasible :
bidder1 => 5*1 + 4*0 <= 10
bidder2 => 5*0 + 4*1 <= 6

Alpha = (1,0.75)

P1 = max(10,3.75) = 10
P2 = max(0,6) = 6
X11 = 1, x22 = 1
Budget feasible :
bidder1 => 10*1 + 6*0 <= 10
bidder2 => 10*0 + 6*1 <= 6

BFPM and FPPE

BFPM

Alpha = (0.7,0.75)

P1 = max(7,3.75) = 7
P2 = max(0,6) = 6
X11 = 1, x22 = 1
Budget feasible :
bidder1 => 7*1 + 6*0 <= 10
bidder2 => 7*0 + 6*1 <= 6

Properties of FPPE

Existence and Uniqueness

Pacing multipliers are exists and are unique, however allocation may not be

Equal Rate Competitive Equilibrium

Competitive equilibrium where buyers have constant bang-per-buck,

$$\beta_i = \frac{v_{ij}}{p_i} \ if \ x_{ij} > 0$$

Shill proofness

Sellers cannot benefit from adding fake bids in auction

Core

No group of bidders has incentive to form coalition with seller

Not Credible Mechanism

Seller can benefit by lying what other agents have done

Existence and Uniqueness

Lemma 1 : There exists a Pareto dominant BFPM (α,x)

Lemma 2 : The Pareto dominant BFPM has no unnecessarily paced bidders, so it forms FPPE

Lemma 3 : If a BFPM (1) dominates another BFPM (2), then BFPM(2) must have unnecessarily paced bidder.

Lemma 4 : If a BFPM 1 dominates another BFPM 2, revenue of BFPM1 is atleast revenue of BFPM 2

Monotonicity

	Add Bidder	Add Good	Increase Budget	Increase Valuation
Revenue	Increases	Increases	Increases	Increase/Decrease
Social Welfare	Increase/Decrease	Increases	Increase/Decrease	Increase/Decrease

	Item1	Item2	Budget
Bidder1	10	2	10
Bidder2	5	8	6

α= (1,0.75)
p1 = max(10,3.75) = 5
P2 = max(5,6) = 6
X11 = 1 , x22 = 1
Revenue = 10+6 = 16

Adding Bidder

	Item1	Item2	Budget
Bidder1	10	2	10
Bidder2	5	8	6
Bidder3	7	10	10

α= (1,1,1)
p1 = max(10,5,7) = 10
P2 = max(2,8,10) = 10
X11 = 1 , x32 = 1
Revenue = 10+10 = 20

	Item1	Item2	Budget
Bidder1	10	2	13
Bidder2	5	8	6

α= (1,0.75)
p1 = max(10,3.75) = 5
p2 = max(2,6) = 6
x11 = 1 , x22 = 1
Revenue = 10+6 = 16

Adding Good

	Item1	Item2	Item3	Budget
Bidder1	10	2	3	13
Bidder2	5	8	1	6

α= (1,0.75)
p1 = max(10,3.75) = 5
p2 = max(2,6) = 6
p3 = max(3,0.75) = 3
x11 = 1 , x22 = 1, x13 = 1
Revenue = 13+6 = 19

	Item1	Item2	Item3	Budget
Bidder1	10	2	6	10
Bidder2	5	8	1	6

α= (0.625,0.75)
p1 = max(6.25,3.75) = 6.25
p2 = max(1.25,6) = 6
P3 = max(3.75,0.75) = 3.75
x11 = 1 , x22 = 1 , x13 = 1
Revenue = 16

Increase Budget

	Item1	Item2	Item3	Budget
Bidder1	10	2	6	16
Bidder2	5	8	1	6

α= (1,0.75)
p1 = max(10,3.75) = 5
p2 = max(2,6) = 6
p3 = max(6,0.75) = 6
x11 = 1 , x22 = 1, x13 = 1
Revenue = 16+6 = 22

	Item1	Item2	Item3	Budget
Bidder1	10	2	2	12
Bidder2	5	8	1	6

Inc Valuation →

	Item1	Item2	Item3	Budget
Bidder1	10	2	2	12
Bidder2	5	8	7	6

α= (1,0.75)
p1 = max(10,3.75) = 5
p2 = max(2,6) = 6
x11 = 1 , x22 = 1, x13 = 1
Revenue = 12+6 = 18

α= (1,0.4)
p1 = max(10,2) = 10
p2 = max(2,3.2) = 3.2
p3 = max(2,2.8) = 2.8
x11 = 1 , x22 = 1, x23 = 1
Revenue = 10+6 = 16

	Item1	Item2	Item3	Budget
Bidder1	10	2	2	12
Bidder2	5	8	7	6

Inc Valuation →

	Item1	Item2	Item3	Budget
Bidder1	10	2	10	12
Bidder2	5	8	7	6

α= (1,0.4)
p1 = max(10,2) = 10
p2 = max(2,3.2) = 3.2
p3 = max(2,2.8) = 2.8
x11 = 1 , x22 = 1, x23 = 1
Revenue = 10+6 = 16

α= (0.6,0.75)
p1 = max(6,3.75) = 6
p2 = max(1.2,6) = 6
p3 = max(6,5.25) = 6
x11 = 1 , x22 = 1, x13 = 1
Revenue = 12+6 = 18

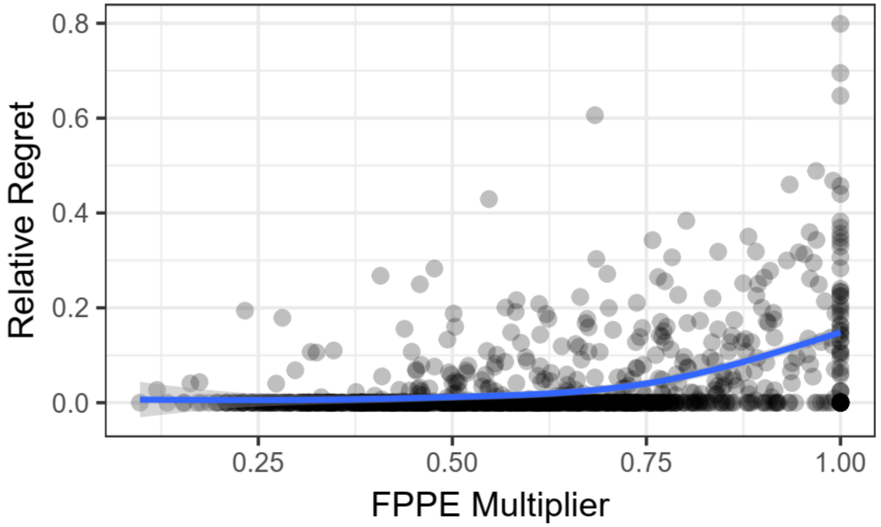
Computation

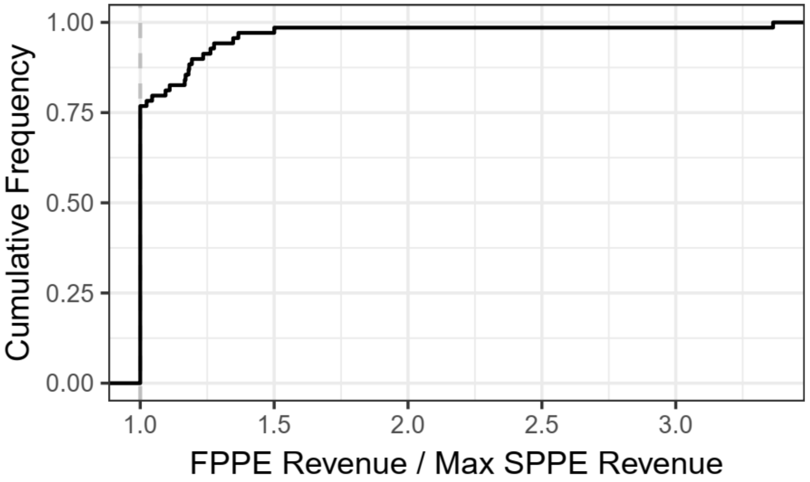
- Eisenberg-Gale convex program for Fisher markets with quasi-linear utilities correspond exactly to FPPE in our setting

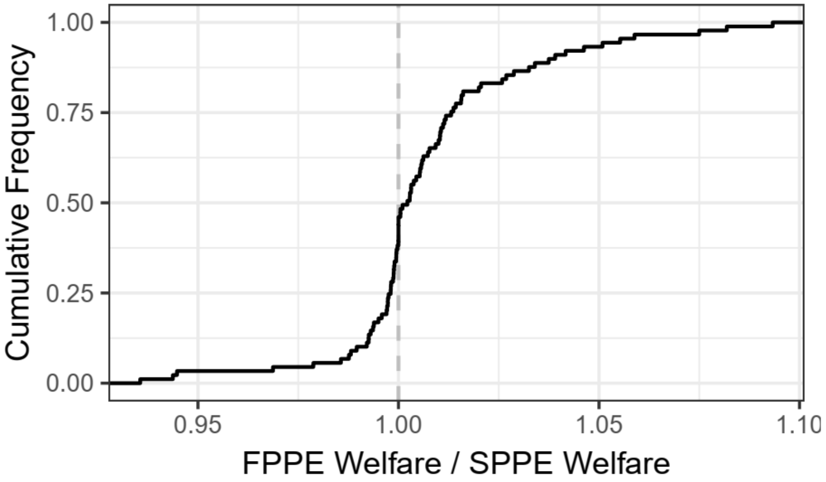
Theorem 3 : *An optimal solution to CP corresponds to a FPPE with pacing multiplier* $\alpha_i = \beta i$ *and allocation* x_{ij}*, and vice versa.*

Experiments

- Under FPPE, how high is bidder regret for reporting truthfully?
- How does FPPE compare to SPPE in terms of revenue and social welfare?

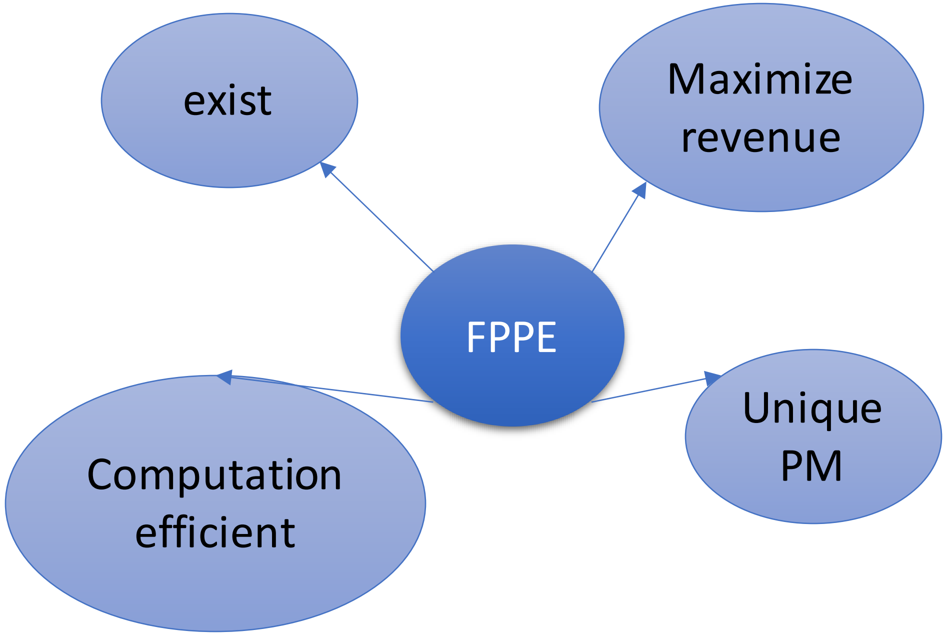


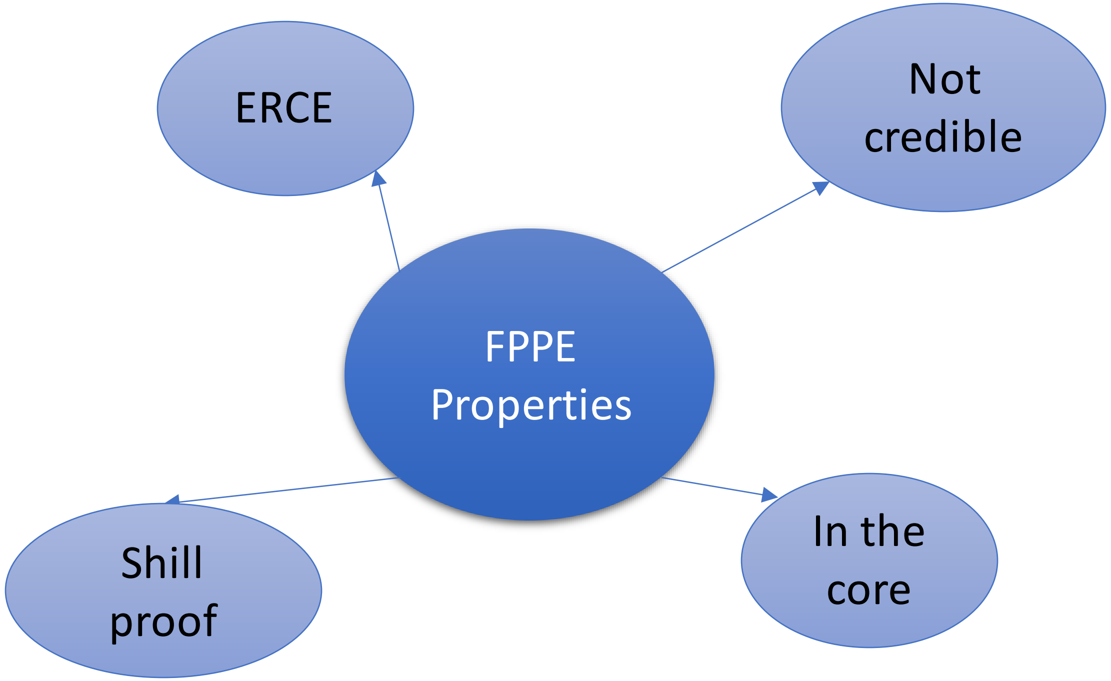




Data : 8 bidders, 14 goods, valuations U[0,1]
Algo : Eisenberg-Gale CP to find FPPE (in ms)

Conclusion





Empirical Results

- Revenue is better than SPPE
- Social Welfare is comparable to SPPE
- For budget constraints bidders, little incentive to misreport
- For non-budget constraints bidders, incentive to misreport vanishes as markets becomes thick