GROUP WORK PROJECT # 1
GROUP NUMBER: 4994

MScFE 652: PORTFOLIO MANAGEMENT

FULL LEGAL NAME	LOCATION (COUNTRY)	EMAIL ADDRESS	MARK X FOR ANY NON-CONTRIBUTING MEMBER
Shailza Virmani	India	virmanishailza@gmail.com	
Julia Nakashwa Amukwa	Namibia	amukwajulia@yahoo.com	
Lelisa Kebena Bijiga	Ethiopia	lelisak66@gmail.com	

Remember: Any group members who did **not contribute to the project should be given all zero (0) points for the collaboration grade on the GWP submission page.*

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
Team member 1	**Shailza Virmani**
Team member 2	**Lelisa Kebena Bijiga**
Team member 3	**Julia Nakashwa Amukwa**

Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.
N/A

Step 1: Definitions of the five factors in FF5

a. The FAMA and French five-factor model equations can be written as -

$$R_{it} - R_{ft} = a_{it} + \beta_1(R_{Mt} - R_{ft}) + \beta_2(SMB_t) + \beta_3(HML_t) + \beta_4(CMA_t) + \beta_5(RMW_t)$$

where,
R_{it} = return of the asset i at time t
R_{ft} = risk-free return at time t
R_{Mt} = return on the market portfolio
$R_{it} - R_{ft}$ = excess return on asset i
$R_{Mt} - R_{ft}$ = excess return on the market portfolio and market premium

In the context of the Fama-French 5-factor model (FF5), the five factors are as follows:

1. Market Factor (Mkt-RF): The Market Factor often denoted as Mkt-RF, represents the excess return of the overall market portfolio over the risk-free rate. The elements that make it up are as follows:
   (a) Excess Return: The market factor, which is the return of the whole market less the risk-free rate, measures the excess return of the market portfolio. Investors want this additional return in exchange for assuming the risk involved in making market investments.
   (b) Overall Market: Generally, a broad equity markets index, like the S&P 500 in the US or the FTSE 100 in the UK, is referred to as the "market". It is a diverse stock portfolio that is frequently used as a standard for evaluating the performance of the stock market as a whole.
   (c) Risk-Free Rate: The return on an investment that is thought to carry no danger of financial loss is known as the risk-free rate. The yield on short-term government bonds, such as US Treasury bills, is frequently used as a stand-in for it. Any returns that exceed the risk-free rate are regarded by investors as compensation for taking on risk. This rate serves as a baseline return for investors.

   A crucial element of asset pricing models such as the Fama-French and Capital Asset Pricing Models (CAPM) is the Market Factor. It represents the performance of the entire market in comparison to a risk-free investment and acts as a measure of systematic risk or market risk. When the Market Factor has positive values, it means the market has fared better than the risk-free rate; when it has negative values, it has performed worse. It is employed to appraise the risk-adjusted returns of individual securities or asset classes as well as the performance of investment portfolios.

In the equation,

$$\beta_1(R_{Mt} - R_{ft})$$

is the factor. Where, $(R_{Mt} - R_{ft})$= Market premium, β_1 = market factor, R_{Mt}= market portfolio return, and R_{ft}= risk-free return.

2. Small Minus Big (SMB): The Fama-French three- and five-factor models, which aim to explain the cross-section of stock returns, include the Small Minus Big (SMB) factor as a component. The performance gap between small-cap equities and large-cap stocks is the specific area of attention for SMB. Here's how SMB is explained:
   (a) Small-cap Stocks: Companies having comparatively small market capitalizations are referred to as small-cap stocks. The number of outstanding shares multiplied by the stock price yields the market capitalization. Smaller, younger businesses that may have greater development potential but also have a tendency to be riskier and more volatile are generally represented by small-cap stocks.
   (b) Large-cap Stocks: Conversely, large-cap stocks are shares of businesses with substantial market capitalizations. These businesses are usually well-known, reputable, and have a solid financial foundation. Large-cap stocks frequently reflect market leaders with firmly established positions and reliable sources of income.
   (c) Performance Differential: The historical performance difference between small-cap equities and large-cap companies is captured by the SMB factor. It calculates the small-cap stocks' excess returns over large-cap stocks for a specific time frame.
   (d) Risk Premium: SMB represents the extra yield that small-cap stock investments provide over large-cap stock investments. Because small-cap companies are typically thought to be less liquid and riskier than large-cap stocks, investors want a higher expected return to offset these risks.
   (e) Investment Strategy For investors and portfolio managers who focus on small-cap investing, the SMB factor is crucial. It helps investors evaluate the risk-return trade-off of investing in small-cap stocks and offers insights into how small-cap firms perform in comparison to large-cap stocks.

   In the equation,

$$\beta_2(SMB_t)$$

   is the SMB factor. where, β_2 = value coefficient & SMB_t = value premium(SMB).

3. High Minus Low (HML): Another element of the Fama-French three-factor and five-factor models, which are extensively employed in finance to explain stock returns, is the High Minus Low (HML) factor. HML specializes in analyzing the performance gap between growth and value equities. Here's how HML is explained:

(a) Value Stocks: Value stocks are ones that are thought to be cheap in relation to their underlying values. These stocks frequently have low price-to-book ratios, which indicates that their market value is low in relation to their asset worth. Even with strong underlying fundamentals, value stocks may be momentarily disregarded by the market, which would result in reduced stock prices.

(b) Growth Stocks: On the other side, growth stocks are the stocks of businesses that are anticipated to rise faster than the market as a whole. These businesses usually reinvest their profits back into growing their businesses, creating new goods, or breaking into untapped areas. Price-to-book ratios for growth stocks are frequently higher, which indicates investor confidence in the stocks' ability to grow in the future.

(c) Performance Differential: The historical performance difference between growth and value equities is captured by the HML factor. It calculates the value stocks' excess returns above growth stocks for a specific time frame.

(d) Value Premium: The higher return that investors seek when purchasing value companies as opposed to growth equities is reflected in HML. Because value companies are sometimes thought of as riskier or less appealing in the near term, investors tend to demand a larger expected return before deciding to hold these firms.

(e) Investment Strategy: For investors and portfolio managers who concentrate on value investing strategies, the HML factor is crucial. It helps investors weigh the risk-return trade-off of investing in value-oriented equities and offers insights into the performance of value stocks in comparison to growth stocks.

In the equation,

$$\beta_3(HML_t)$$

is the HML factor. Where, β_3 = value coefficient & HML_t = value premium(HML)

4. Conservative Minus Aggressive (CMA): One of the extra components added to the Fama-French five-factor model to help explain the cross-section of stock returns is the Conservative Minus Aggressive (CMA) factor. The performance gap between companies with aggressive accounting methods and those with conservative accounting practices is the main emphasis of CMA. Here's how CMA is explained:

   (a) Conservative Accounting Practices: Businesses that use conservative accounting principles are typically cautious when it comes to recording revenues and assets on their financial accounts. They might recognize revenue, value assets, and recognize expenses more cautiously than enterprises with aggressive accounting techniques, which could lead to lower reported earnings and assets.

   (b) Aggressive Accounting Practices: On the other hand, businesses that use aggressive accounting techniques also frequently use aggressive asset and revenue recognition.

In comparison to companies with conservative accounting standards, they might use more lax approaches to revenue recognition and asset valuation, which could lead to higher reported earnings and assets.

(c) Performance Differential: The historical performance difference between the stocks of companies with aggressive accounting techniques and those with conservative accounting standards is captured by the CMA factor. It calculates the excess returns that conservative companies have over aggressive companies over a specific time frame.

(d) Risk Premium: When comparing the equities of aggressive and conservative companies, the CMA shows the extra return that investors require. Because aggressive corporations are thought to carry greater accounting risk, investors demand a larger projected return in order to hold stocks of conservative firms, which are thought to have more stable and dependable financial statements.

(e) Investment Strategy: The CMA factor sheds light on how the stocks of companies with various accounting procedures perform differently from one another. Investors and portfolio managers can use it to evaluate the risk-return trade-off between stocks of aggressive and conservative companies and to integrate accounting-based factors into their investment strategy.

Mathematical expression:

$$CMA = \frac{1}{2}(HML + RMW)$$

Where, HML and RMW measures returns difference between high and low book to market stocks and the returns difference between strong and weak portfolio stocks.

In the equation,

$$\beta_4(CMA_t)$$

represents the CMA factor. Where, β_4 =coefficient of the conservative investment and CMA_t = average return of the conservative investment portfolios less the average return on the two aggressive investment portfolios.

5. Robust Minus Weak (RMW): The Fama-French five-factor model includes the Robust Minus Weak (RMW) factor as well. This model tries to explain stock returns more thoroughly by including more sources of systematic risk. The performance gap between companies with strong operating profitability and those with weak operating profitability is the special emphasis of RMW. Here's how RMW is explained:

(a) Robust Operating Profitability: Businesses that exhibit robust and steady profitability in their primary activities are said to have robust operating profitability. These businesses often have strong profit margins, effective cost structures, and steady earn-

ings growth over time. Their profitability can be attributed to their strong market position, competitive advantage, or efficient operating techniques.

(b) Weak Operating Profitability: On the other hand, businesses with low operating profitability find it difficult to extract long-term value from their main business. Due to a variety of causes like fierce rivalry, inefficient operations, or unfavorable market conditions, they can see decreased profit margins, increased costs, or inconsistent profits performance.

(c) Performance Differential: The historical performance difference between the stocks of companies with strong operating profitability and those with weak operating profitability is captured by the RMW factor. It calculates the extra returns that profitable companies have over unprofitable companies over a given time frame.

(d) Profitability Premium: When comparing equities of companies with strong operating profitability to those with weak operating profitability, investors seek a higher return, which is reflected in RMW. Because companies with good profitability are thought to have better fundamentals and reduced financial risk, investors demand a larger projected return before purchasing stocks in companies with weak profitability.

(e) Investment Strategy: The RMW factor sheds light on the variations in stock performance of companies with varying operating profitability levels. Investors and portfolio managers can use it to evaluate the risk-return trade-off between buying stocks of profitable companies versus those with weaker profitability, and to include profitability-based factors into their investment strategy.

Mathematical Expression:

$$RMW = \frac{1}{2}(WML - SMB)$$

Where, WML and SMB reflects the returns between past winnings and losing stocks and the returns difference between small-cap and big-cap stocks.
In the equation,

$$\beta_5(RMW_t)$$

represents the CMA factor. Where, β_5 = coefficient of the operating profitability and RMW_t = average return on the robust operating portfolios less the average return on the two weak operating profitability portfolios.

b. A non-technical paragraph (NO MORE!) as to why this factor helps to explain returns:

A more complex framework for understanding stock returns than standard models like the Capital Asset Pricing Model (CAPM) can be found in the five-factor model created by Eugene Fama

and Kenneth French. The model incorporates a wider range of systematic risk factors influencing stock performance by adding variables including Robust Minus Weak (RMW), Conservative Minus Aggressive (CMA), High Minus Low (HML), and Small Minus Big (SMB). These variables include corporate size, value vs growth spending, accounting procedures, and operating profitability, according to a study by Fama and French (2015). By taking into consideration the various sources of risk and return in the equity markets, this enlarged approach enables analysts and investors to evaluate individual stock performance more precisely and build portfolios that are customized to their risk tolerance and investing goals. Because it offers greater insights into the factors influencing stock returns under different market conditions, the Fama-French five-factor model has consequently gained popularity as a useful tool in empirical asset pricing research and portfolio management. Fama and French (2015); Doğan et al. (2023)

Step 2: Select 3 years and use daily data

We have imported the 3 years data, and structured and plotted the graph of the daily factors. Fig. 1 shows the daily factor returns of the five factors. We have also collected the 10-year treasury rate from 2022-01-01 to 2024-01-01. Fig. 2 explains the correlation matrix of changes in the factor returns.

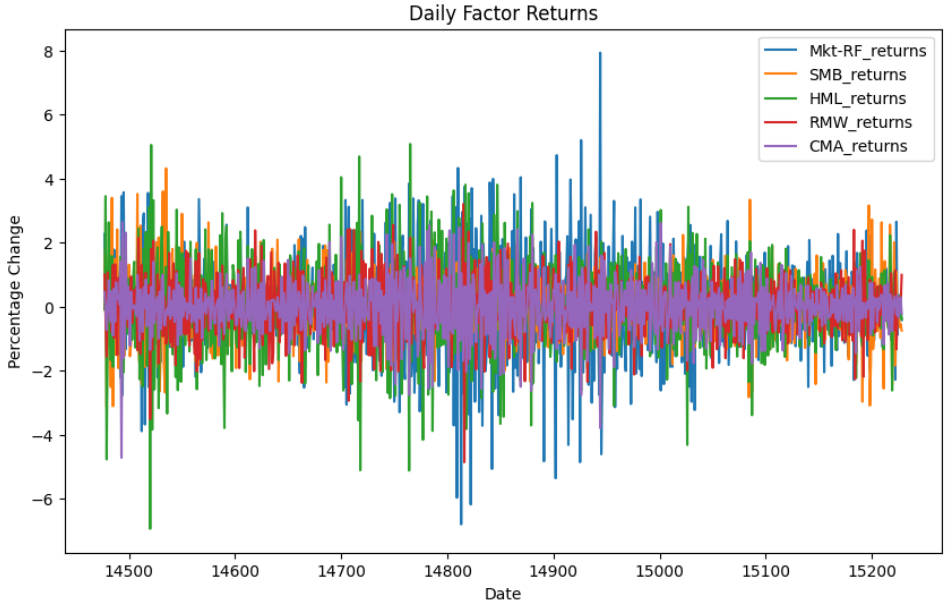

Fig. 1: Daily factor returns

Fig. 2: Correlation matrix of changes in factor returns

Step 3: The FF3 betas using LS and robust regressions

We split the data into train and test samples using the test size of 0.2, i.e. 80% of the data is used for training the regression model. Then we run both the LS and robust regression on the training data to get the betas of FF3. The detailed steps are found in the accompanying Jupyter Notebook.

Summary of the coefficients and model metrics.

OLS Regression Summary:

Coefficients:

Constant (Intercept): The coefficient is approximately $-1.284e-16$, indicating the expected change in Mkt-RF returns when all other predictors are zero.

CMA returns: The coefficient is $6.731e-16$, indicating the expected change in Mkt-RF returns for a one-unit increase in CMA returns.

SMB returns: The coefficient is $-2.507e-17$, indicating a very small and potentially negligible impact on Mkt-RF returns.

HML returns: The coefficient is $-3.171e-16$, indicating the expected change in Mkt-RF returns for a one-unit increase in HML returns.

Model Metrics:

R-squared and Adj. R-squared: The model has a perfect R-squared and Adj. R-squared of 1.000, implying that the model perfectly predicts the dependent variable.

F-statistic: 1.270e+32 - A very high F-statistic suggests a significant relationship between the predictors and the response variable.

AIC: -3.910e+04 BIC: -3.908e+04 Skew: -0.037 Kurtosis: 4.372

Robust Regression Summary:

Coefficients:

Coefficients are similar to OLS.

Discussion:

The perfect R-squared in both regressions indicates that the model perfectly fits the training data. However, such a high R-squared might indicate overfitting, especially when dealing with financial data.

Coefficients for some predictors are very close to zero, suggesting potential multicollinearity or small practical significance. Robust regression can be more appropriate in the presence of outliers, and it adjusts standard errors accordingly. Fig. 3 explains the distribution of the residuals in ols model and robust regression model.

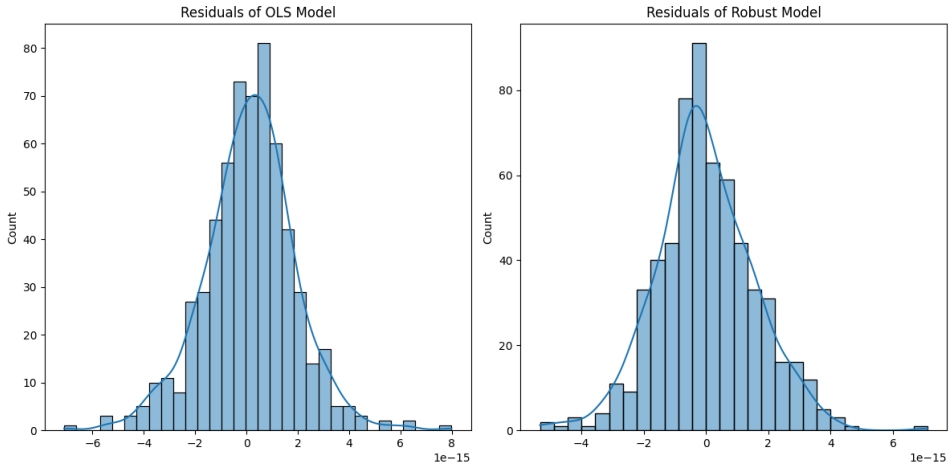

Fig. 3: Residuals of OLS and robust regression for FF3

Step 4: The FF5 betas using LS and robust regressions

We use the same regression methods to find the betas of FF5.

Summary of the coefficients and model metrics.

R-squared: 1.000 Adj. R-squared: 1.000 F-statistic: 3.029e+32 AIC: -3.976e+04 BIC: -3.973e+04 Kurtosis: 4.620

Coefficients:

Constant (Intercept): 1.784e-16

CMA returns: The coefficient is -2.594e-16

SMB returns: The coefficient is 9.758e-17

HML returns: The coefficient is 1.01e-16

RMW returns: The coefficient is 5.984e-16

The FF5 model metrics (F-Statistics, AIC and BIC, Kurtosis) are higher compared to FF3. This suggested that FF5 performed better. Fig. 4 explains the distribution of the residuals in ols model and robust regression model.

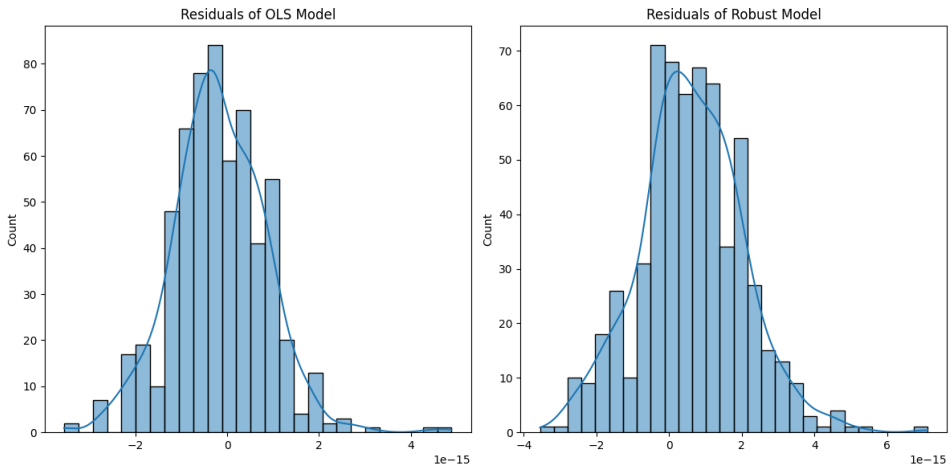

Fig. 4: Residuals of OLS and robust regression for FF5

Step 5: Comparison of covariance and correlation matrices using the daily factor data

Fig. 5 show that both matrices indicate a similar direction of relationships. Negative values are found with the same pair in both matrices, as well as positive values.

Covariance Matrix values are higher than Correlation Matrix values. This is because the covariance matrix shows the joint variability in the original units while the correlation matrix measures the strength and direction of linear relationships.

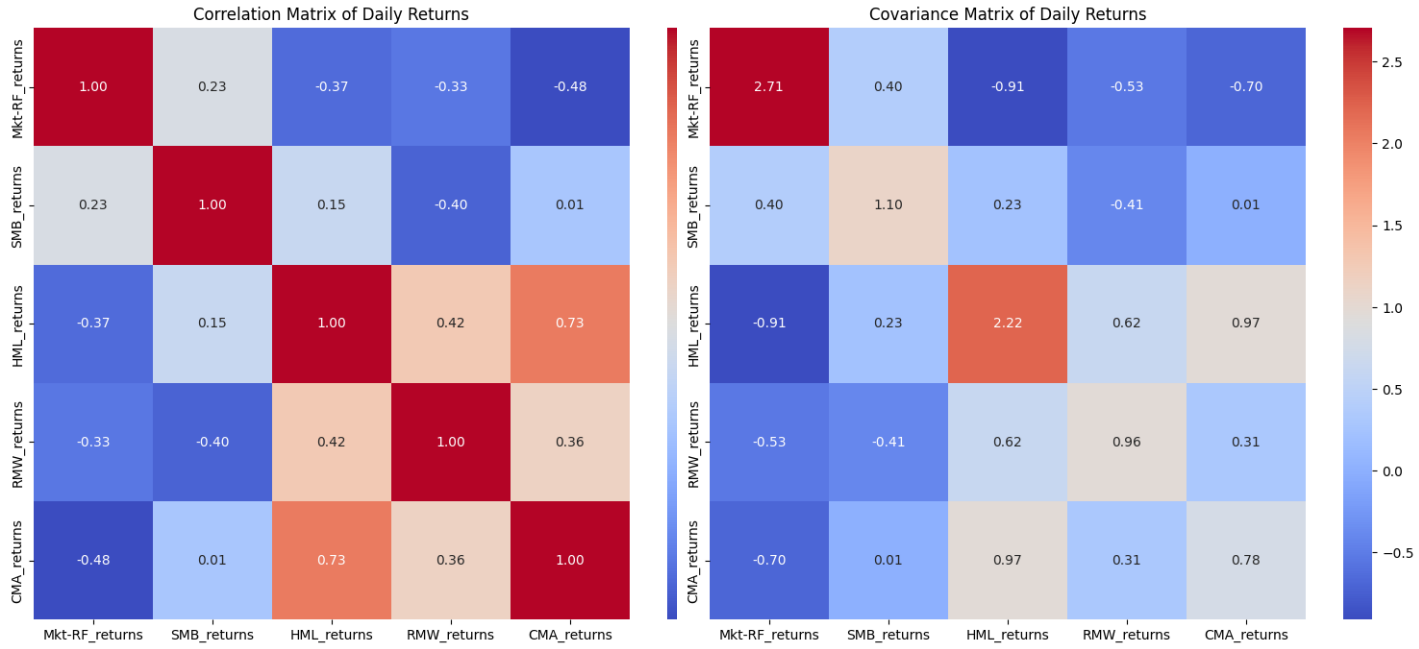

Fig. 5: Correlation matrix and covariance matrix of daily returns

Step 6: Influence of the extra 2 factors (CMA and RMW)

Table 1: Influence of the extra 2 factors (CMA and RMW)

	F-static	AIC	BIC	Skew	Kurtosis
FF3	1.270e+32	3.910e+04	3.908e+04	-0.037	4.372
FF5	3.029e+32	3.976e+04	3.973e+04	0.260	4.620

From Table 1 we see that the FF5 model metrics (F-Statistics, AIC and BIC, Kurtosis) are higher compared to FF3. This suggested that FF5 performed better.

Step 7: Markowitz portfolio optimization

In this step, we have used Markowitz portfolio optimization to find a set of optimal allocations. The optimal portfolio return is 0.001 and the optimal portfolio risk is 0.0152. The optimal weights are

$$\begin{bmatrix} 0.25114406 & -0.21671746 & 0.17199962 & 0.41341544 & 0.38015833 \end{bmatrix}$$

Depending on the factors in FF3 and FF5, the optimal protfolio return is 0.000859 and the optimal portfolio risk is 0.0148. The optimal weights are

$$\begin{bmatrix} 0.31848158 & -0.02792743 & 0.10065566 & 0.2507898 & 0.3580004 \end{bmatrix}$$

Step 8: Interpretation of the results

We have analyzed the five factors, namely the market risk factor (Rm-Rf), size factor (SMB), value factor (HML), profitability factor (RMW), and investment factor (CMA), using 3 years of data. We have also compared the influence of the FF3 and FF5 models on portfolio returns and risks. From the above results, we see that FF5 outperforms FF3. Rm-Rf factor: Exposure to overall market movements. SMB factor: captures the performance disparities between small-cap and large-cap companies, suggesting small-cap companies tend to yield higher returns.
HML factor: gives investors useful information for building diverse portfolios and making investing decisions, as well as aids in their understanding of the performance disparities between value and growth equities.
CMA factor: helps to provide a more thorough understanding of the performance of low-investment firms compared to high-investment firms.
RMW factor: improves our understanding of stock returns. By using profitability criteria, it assists investors in assessing the consistency and quality of businesses' earnings as well as in locating possible investment possibilities.

References

Doğan, M., A. Altinay, B. Demirel, and S. Bekteshi (2023, 04). The fama-french five-factor asset pricing model: A research on borsa istanbul. *Ikonomicheski Izsledvania 32*, 2023.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of financial economics 116*(1), 1–22.