Zeta Function SUM using Geometric series Between 0 and 1

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Zeta Function SUM using Geometric series Between 0 and 1

Abstract

We are going to calculate Zeta function Sum between X interval [0,1] in a complex plan, using a geometric series. And for simplicity, we are going to represent each term in zeta function as a separate geometric series of order 3. (These series can be expanded to higher order as well).
Keywords: Prime Numbers, Composite Prime Numbers, Prime Number Distribution, Zeta function

1. Introduction

We are going to represent each term in zeta function as a separate geometric series of order 3 for simplicity (but these series can be expanded to higher order.

$$a * R^3 + a * R^2 + a * \ R + R$$

Where R is a common ratio linear function, where its zero is a zeta function Term. In other words, Zeta function term is a 1/slop of a linear function (R = common ratio for a geometric series).

Zeta function Term	R (common ratio) in geometric series
$\frac{1}{2}$	2x-1
$\frac{1}{3}$	3x-1
$\frac{1}{4}$	4x-1
$\frac{1}{5}$	5x-1
$\frac{1}{6}$	6x-1
$\frac{1}{7}$	7x-1
…	…..

Therefore, write each term as a separate geometric series as

Zeta function Term	geometric series
$\frac{1}{2}$	$a * (2 * X - 1)^3 + a * (2 * X - 1)^2 + a * (2 * X - 1)^1 + (2 * X - 1)$
$\frac{1}{3}$	$a * (3 * X - 1)^3 + a * (3 * X - 1)^2 + a * (3 * X - 1)^1 + (3 * X - 1)$
$\frac{1}{4}$	$a * (4 * X - 1)^3 + a * (4 * X - 1)^2 + a * (4 * X - 1)^1 + (4 * X - 1)$
$\frac{1}{5}$	$a * (5 * X - 1)^3 + a * (5 * X - 1)^2 + a * (5 * X - 1)^1 + (5 * X - 1)$
$\frac{1}{6}$	$a * (6 * X - 1)^3 + a * (6 * X - 1)^2 + a * (6 * X - 1)^1 + (6 * X - 1)$
$\frac{1}{7}$	$a * (7 * X - 1)^3 + a * (7 * X - 1)^2 + a * (7 * X - 1)^1 + (7 * X - 1)$
…	…..

Properties for these geometric series

1- $at\ X = 0\ ; the\ geometric\ series\ SUM = -(\ a + 1)$

2- $at\ X = Zeta\ term\ ; the\ geometric\ series\ SUM = 0$

3- $[a] is\ a\ scaler\ value\ for\ the\ geometric\ series; where\ [a]\ is\ any\ Real\ number.$

4- At [a] = 0; these series converge to linear functions = R = Common ration for the geometric series.

5- At [a] = 0; and X =1; these series converge to linear functions = R = Common ration for the geometric series with SUM = (reciprocal of zeta term)-1.

6- $Between\ X\ interval\ [0,1]; zeta\ function\ SUM\ Converges = (a + 1)$

7- $each\ of\ these\ geometric\ series\ will\ have\ a\ Zero\ at\ its\ own\ Zeta\ function\ term$

$and\ these\ zeros\ will\ be\ in\ between\ intereval\ X = [0,1].$

A) If $at\ X = 0\ ; the\ geometric\ series\ SUM = -(\ a + 1)$; then For A = 1; all These geometric series will Y-intercept at Y = -2. At X =0.

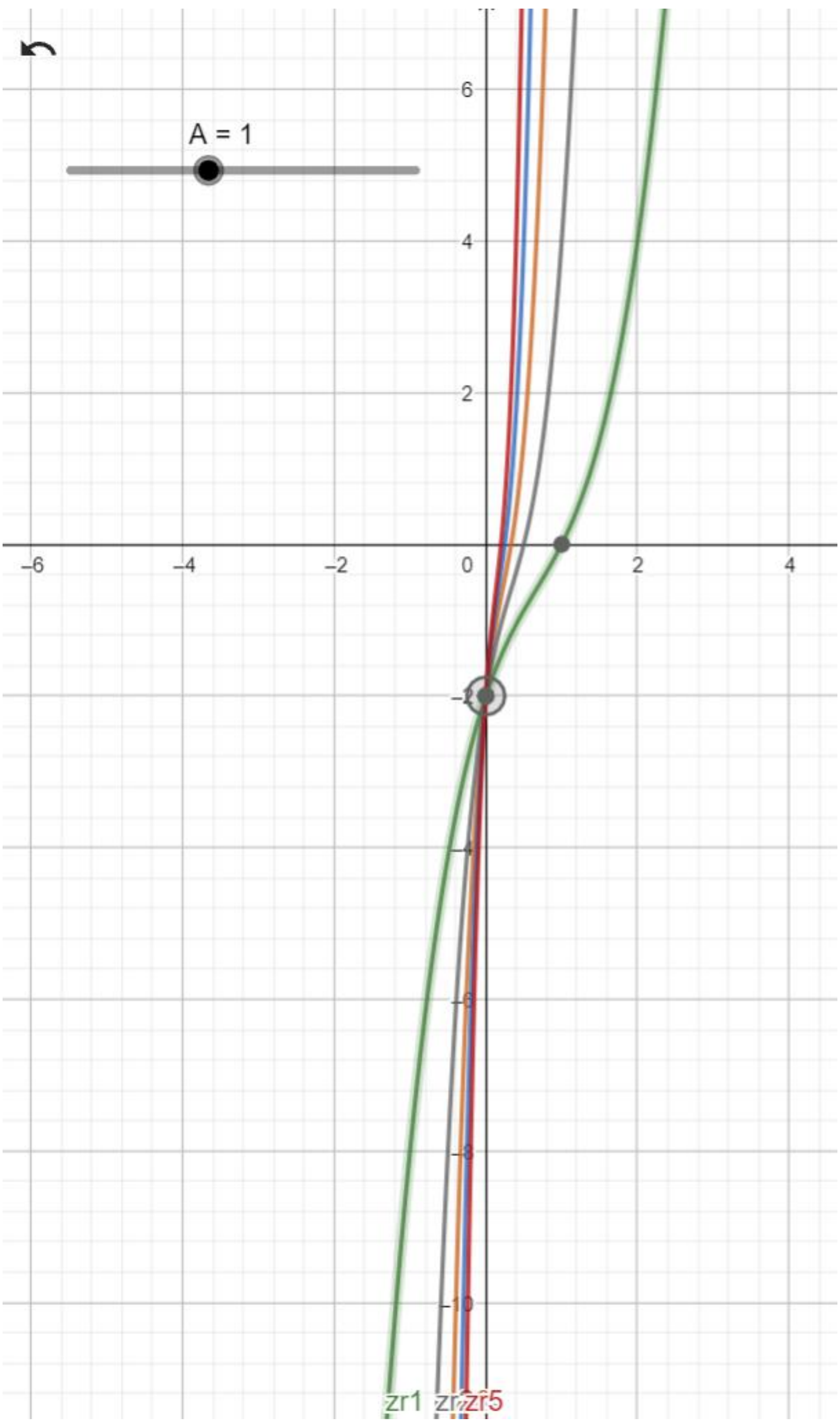
$$zr1(x) = a\ r1(x)\left((r1(x))^2 + r1(x) + 1\right) + r1(x)$$
$$\rightarrow 1\ (x-1)\left((x-1)^2 + x - 1 + 1\right) + x - 1$$

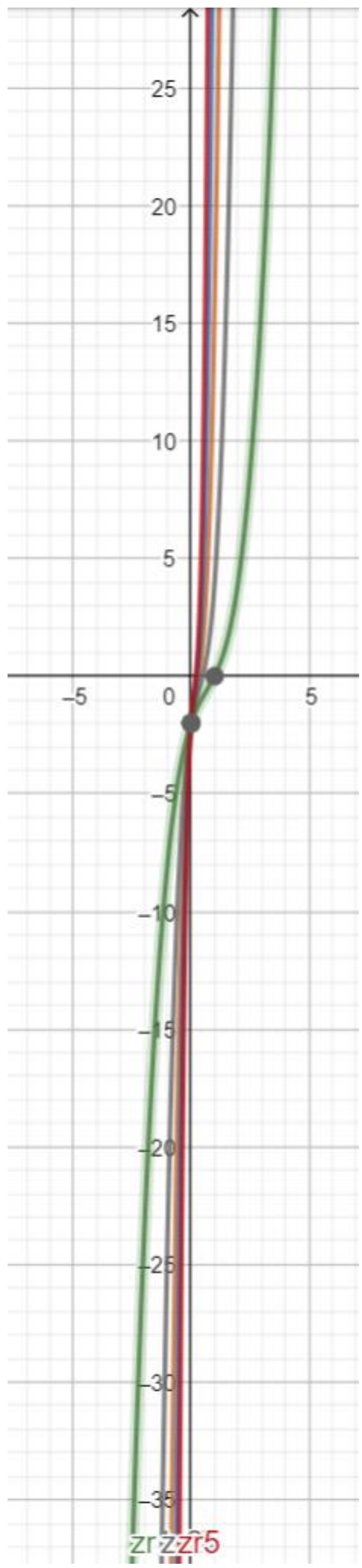
$$zr2(x) = a\ r2(x)\left((r2(x))^2 + r2(x) + 1\right) + r2(x)$$
$$\rightarrow 1\ (2\,x-1)\left((2\,x-1)^2 + 2\,x - 1 + 1\right) + 2\,x - 1$$

$$zr3(x) = a\ r3(x)\left((r3(x))^2 + r3(x) + 1\right) + r3(x)$$
$$\rightarrow 1\ (3\,x-1)\left((3\,x-1)^2 + 3\,x - 1 + 1\right) + 3\,x - 1$$

$$zr4(x) = a\ r4(x)\left((r4(x))^2 + r4(x) + 1\right) + r4(x)$$
$$\rightarrow 1\ (4\,x-1)\left((4\,x-1)^2 + 4\,x - 1 + 1\right) + 4\,x - 1$$

$$zr5(x) = a\ (r5(x))^3 + a\ (r5(x))^2 + a\ r5(x) + r5(x)$$
$$\rightarrow 1\ (5\,x-1)^3 + 1\ (5\,x-1)^2 + 1\ (5\,x-1) + 5\,x - 1$$


A = 1
zr1
zr5


zr5

B) $at\ X = Zeta\ term\,; the\ geometric\ series\ SUM = 0$; as the series is based on using the Zeta function term as common ratio for the series

$$zr5(x) = a\,(r5(x))^3 + a\,(r5(x))^2 + a\,r5(x) + r5(x)$$
$$\rightarrow 1\,(5x-1)^3 + 1\,(5x-1)^2 + 1\,(5x-1) + 5x - 1$$

Zr5(x) its Zeta function term = 1/5
Therefore Zr5(x) geometric SUM = 0 at X = 1/5 = 0.2
Same will be for all Zr2(X), zr3(X) , zr4(X), …

x	zr3(x)	zr5(x)	zr4(x)
-0.1	-3.107	-4.125	-3.584
-0.05	-2.498375	-2.890625	-2.688
0	-2	-2	-2
0.05	-1.591625	-1.359375	-1.472
0.1	-1.253	-0.875	-1.056
0.15	-0.963875	-0.453125	-0.704
0.2	-0.704	0	-0.368
0.25	-0.453125	0.578125	0
0.3	-0.191	1.375	0.448
0.35	0.102625	2.484375	1.024
0.4	0.448	4	1.776
0.45	0.865375	6.015625	2.752
0.5	1.375	8.625	4
0.55	1.997125	11.921875	5.568

C) $[a]$ *is a scaler value for the geometric series; where* $[a]$ *is any Real number.*

$[a]$ *can take any real value by changing A or the denominator.*

$A = 1$

-5 10

$a = \frac{A}{1}$

$\rightarrow 1$

One of the usages of this scaler parameter [a]; when [a = 0] this parameter reduces the dimension of our geometric series into a linear function

$a * R^3 + a * R^2 + a * R + R$

D) At [a] = 0; and X =1; these series converge to linear functions = R = Common ration for the geometric series with SUM = (reciprocal of zeta term)-1.

$$zr3(x) = a\, r3(x) \left((r3(x))^2 + r3(x) + 1\right) + r3(x)$$
$$\rightarrow 0\,(3x - 1)\left((3x - 1)^2 + 3x - 1 + 1\right) + 3x - 1$$

$$zr4(x) = a\, r4(x) \left((r4(x))^2 + r4(x) + 1\right) + r4(x)$$
$$\rightarrow 0\,(4x - 1)\left((4x - 1)^2 + 4x - 1 + 1\right) + 4x - 1$$

$$zr1(x) = a\, r1(x) \left((r1(x))^2 + r1(x) + 1\right) + r1(x)$$
$$\rightarrow 0\,(x - 1)\left((x - 1)^2 + x - 1 + 1\right) + x - 1$$

$$zr2(x) = a\, r2(x) \left((r2(x))^2 + r2(x) + 1\right) + r2(x)$$
$$\rightarrow 0\,(2x - 1)\left((2x - 1)^2 + 2x - 1 + 1\right) + 2x - 1$$

$$zr5(x) = a\,(r5(x))^3 + a\,(r5(x))^2 + a\, r5(x) + r5(x)$$
$$\rightarrow 0\,(5x - 1)^3 + 0\,(5x - 1)^2 + 0\,(5x - 1) + 5x - 1$$

Each one of these liner functions intersects with X axis at Zeta Term.

So Zr1(X) intersects with X-axis at X = 1

Zr2(X) intersects with X-axis at X = ½

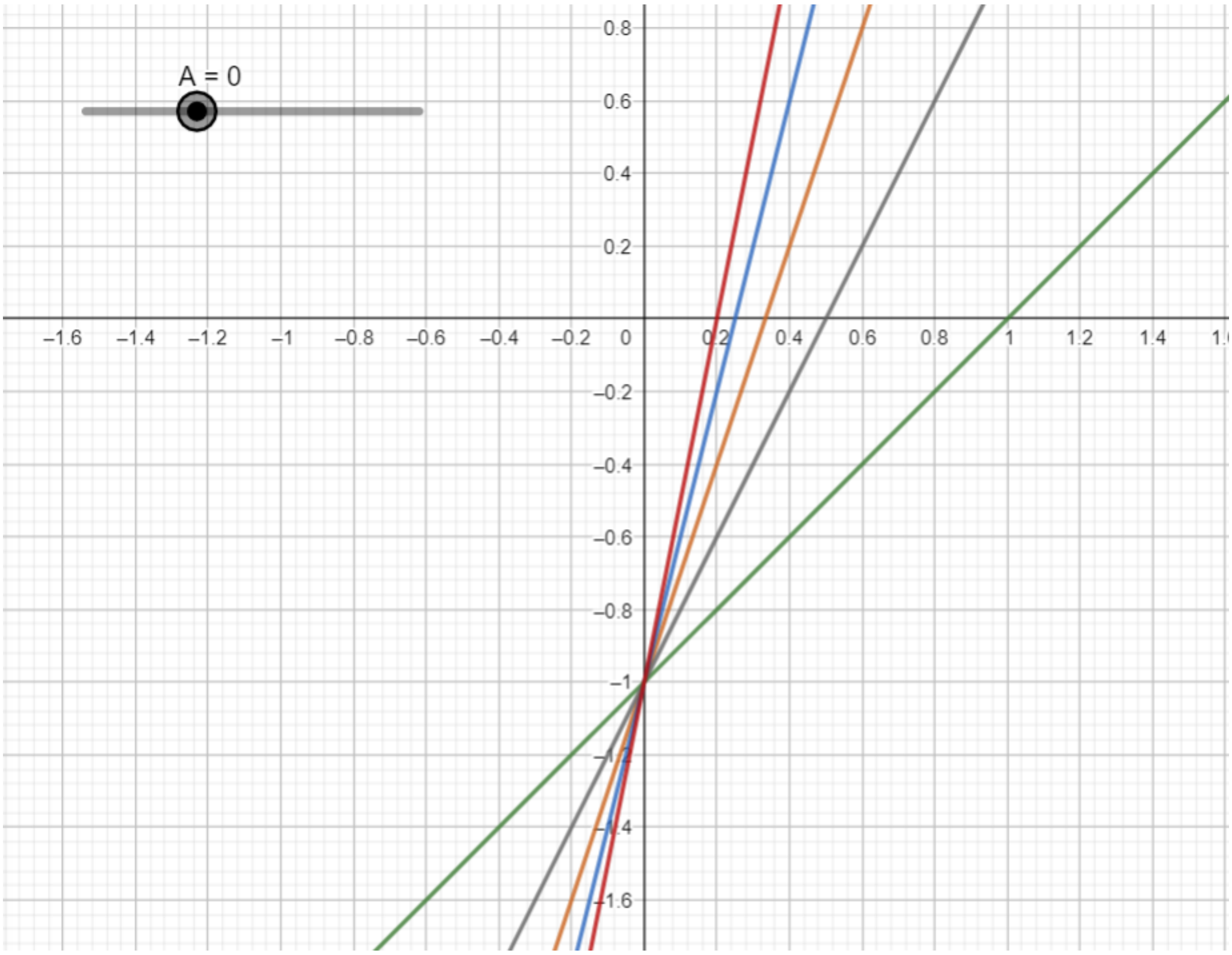
Zr3(X) intersects with X-axis at X = 1/3

Zr4(X) intersects with X-axis at X = ¼

Zr5(X) intersects with X-axis at X =1/5

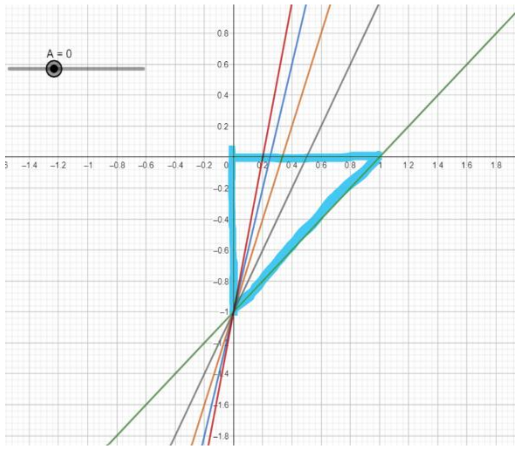
As we add new geometric series for each term in Zeta function, we get new linear function for this term intersects with X-axis at Zeta function Term. (Infinity number or linear functions in X interval between [0,1])

We are going to use the area of these triangles to get a SUM of these Zeta function terms between [0,1]. (Integral between 0 and 1) but without using integrals

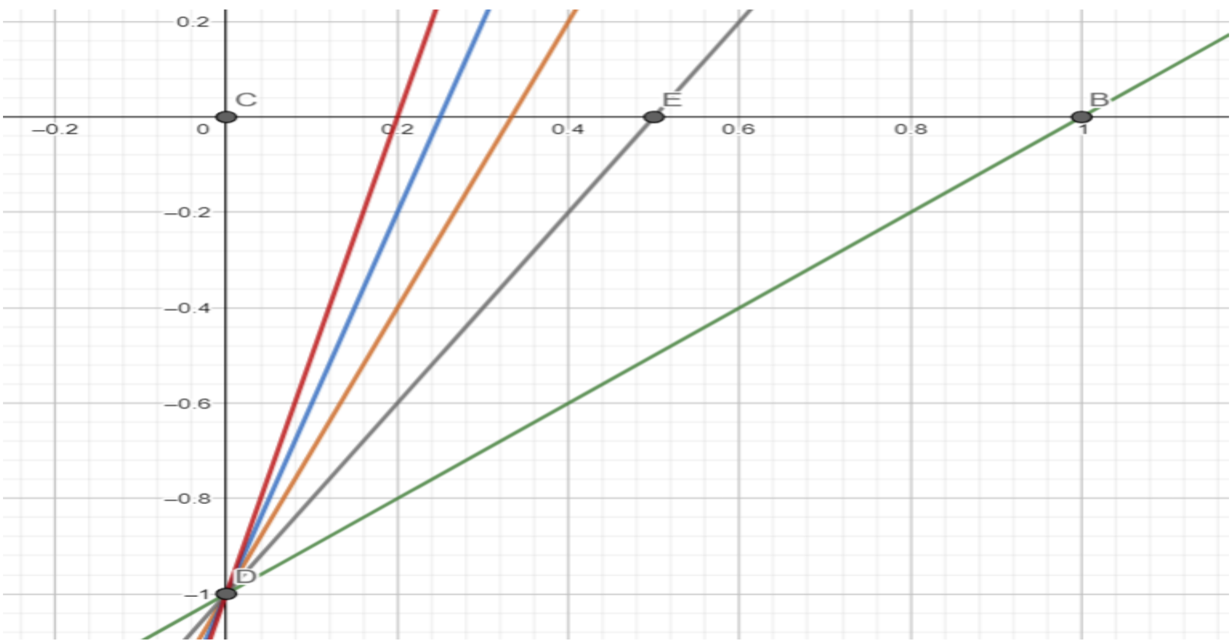


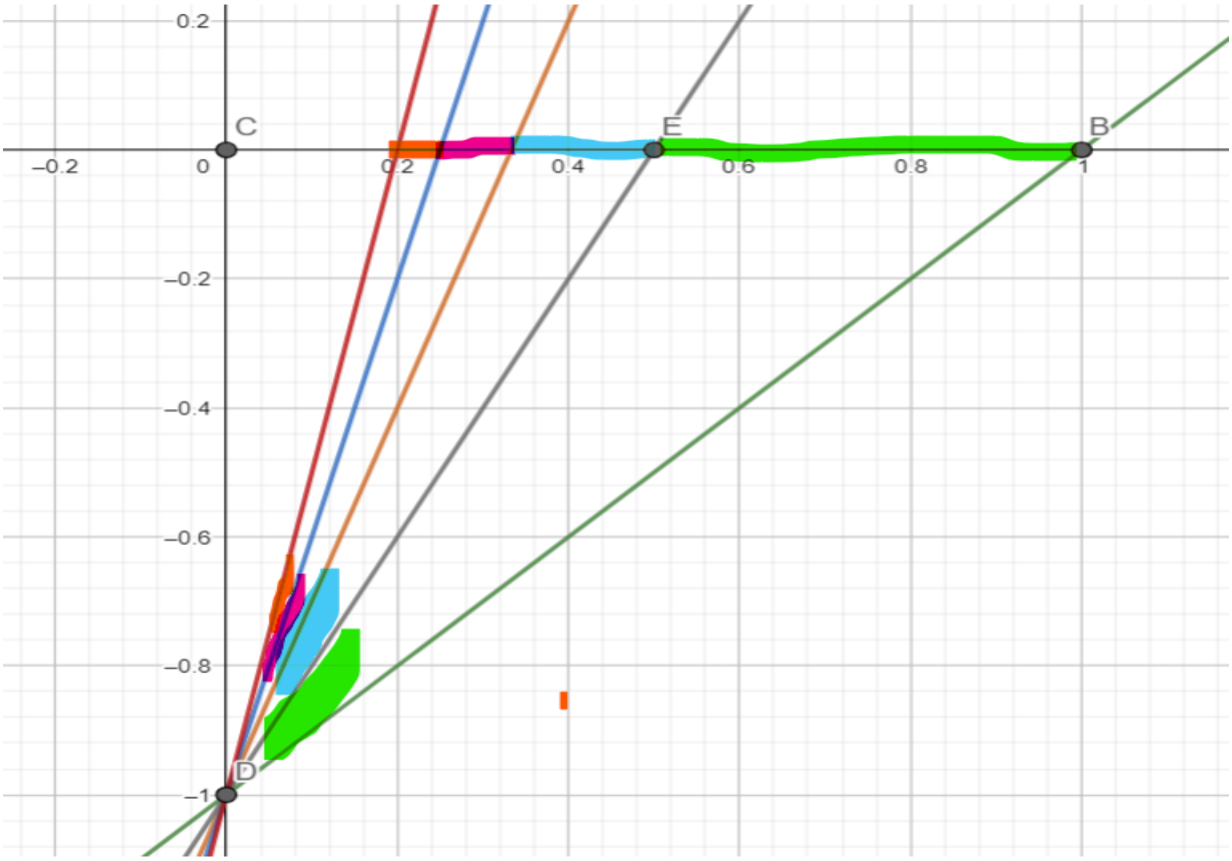
Using X-axis and Y-axis as our other sides of the triangle

Geometric Series	Area
Zr1(X)	0.5 * 1
Zr2(X)	0.5 * 0.5 * 1
Zr3(X)	0.5 * 1/3 * 1
Zr4(X)	0.5 * ¼ * 1
Zr5(X)	0.5 * 1/5 * 1
Zr6(X)	0.5 * 1/6 * 1
Zr7(X)	0.5 * 1/7 * 1
…	…



To get the SUM we need to remove the duplicates areas by subtracting each two terms from each other; Area (BDE) = Area (CBD) – Area (CED); by doing these we sum the Difference between areas (i.e., integrate all terms between [0,1]).





Areas without duplicates

Geometric Series	Area
Zr1(X)	0.5
Zr1(X) - Zr2(X)	0.5 - 0.5 * 0.5
Zr2(X) - Zr3(X)	0.5 * 0.5 - 0.5 * 1/3
Zr3(x) - Zr4(X)	0.5 * 1/3 - 0.5 * ¼
Zr4(X) - Zr5(X)	0.5 * ¼ - 0.5 * 1/5
Zr5(X) - Zr6(X)	0.5 * 1/5 -0.5 * 1/6 * 1
Zr6(X) - Zr7(X)	o.5 * 1/6 - 0.5 * 1/7 * 1
…	…

As Zr0(X) = X then Area difference will be (1-1/2)

$$(1-\frac{1}{2})+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{20}\right)+\left(\frac{1}{20}-\frac{1}{10}\right)+\left(\frac{1}{10}-\frac{1}{12}\right)+\left(\frac{1}{12}-\frac{1}{14}\right)+\cdots.$$

With Z0r(X) the sum converges to 1 and without the Zr0(X) the sum convers to 0.5.

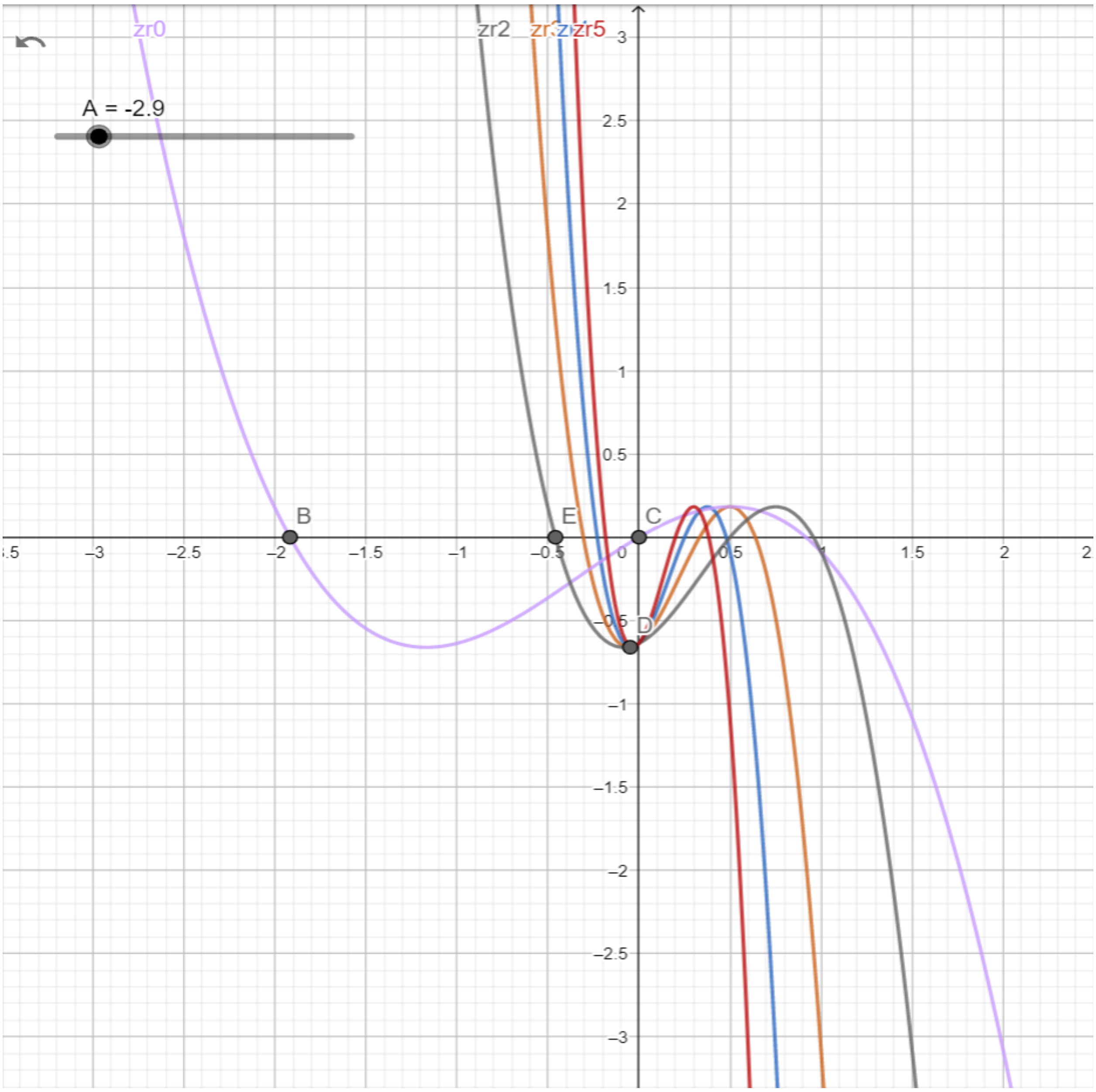
b	0.5/b	h	h * 0.5 * 1/b	Area Difference
1	0.5	1	0.5	
2	0.25	1	0.25	0.25
3	0.16666667	1	0.166666667	0.083333333
4	0.125	1	0.125	0.041666667
5	0.1	1	0.1	0.025
6	0.08333333	1	0.083333333	0.016666667
7	0.07142857	1	0.071428571	0.011904762
8	0.0625	1	0.0625	0.008928571
9	0.05555556	1	0.055555556	0.006944444
10	0.05	1	0.05	0.005555556
11	0.04545455	1	0.045454545	0.004545455
12	0.04166667	1	0.041666667	0.003787879
13	0.03846154	1	0.038461538	0.003205128
14	0.03571429	1	0.035714286	0.002747253
15	0.03333333	1	0.033333333	0.002380952
16	0.03125	1	0.03125	0.002083333
17	0.02941176	1	0.029411765	0.001838235
18	0.02777778	1	0.027777778	0.001633987
19	0.02631579	1	0.026315789	0.001461988
20	0.025	1	0.025	0.001315789
21	0.02380952	1	0.023809524	0.001190476
22	0.02272727	1	0.022727273	0.001082251
23	0.02173913	1	0.02173913	0.000988142
24	0.02083333	1	0.020833333	0.000905797
25	0.02	1	0.02	0.000833333
26	0.01923077	1	0.019230769	0.000769231
27	0.01851852	1	0.018518519	0.000712251
28	0.01785714	1	0.017857143	0.000661376
29	0.01724138	1	0.017241379	0.000615764
30	0.01666667	1	0.016666667	0.000574713
31	0.01612903	1	0.016129032	0.000537634
32	0.015625	1	0.015625	0.000504032
33	0.01515152	1	0.015151515	0.000473485

If r1(X) = X.

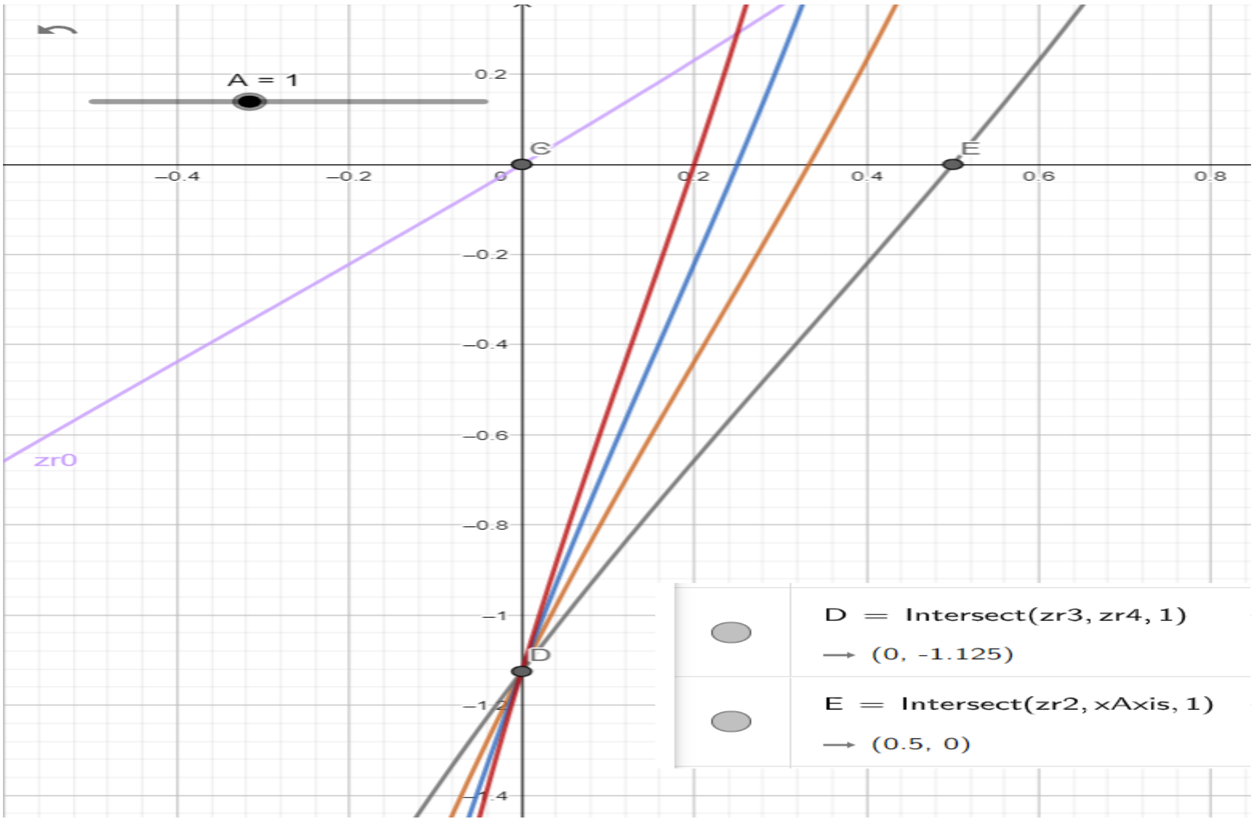
$$zr0(x) = a\, r1(x)\left(\left(r1(x)\right)^2 + r1(x) + 1\right) + r1(x)$$

$$\rightarrow\ 0\,x\left(x^2 + x + 1\right) + x$$

$$r1(x) = x$$



Scale by [a] = 1/8; SUM will be converged to 1 + a = 1+1/8 = 1.125.   (Y Intercept Point)



This converges to 0.5 + 1/16 in interval X = [0,1]
i.e., SUM in interval X = [0,1] Converges to 0.5 + 0.5 * a.

$$zr3(x) = a\, r3(x) \left((r3(x))^2 + r3(x) + 1\right) + r3(x)$$
$$\rightarrow \frac{1}{8} (3x - 1) \left((3x - 1)^2 + 3x - 1 + 1\right) + 3x - 1$$

$$zr4(x) = a\, r4(x) \left((r4(x))^2 + r4(x) + 1\right) + r4(x)$$
$$\rightarrow \frac{1}{8} (4x - 1) \left((4x - 1)^2 + 4x - 1 + 1\right) + 4x - 1$$

$$zr0(x) = a\, r1(x) \left((r1(x))^2 + r1(x) + 1\right) + r1(x)$$
$$\rightarrow \frac{1}{8} x (x^2 + x + 1) + x$$

$$zr2(x) = a\, r2(x) \left((r2(x))^2 + r2(x) + 1\right) + r2(x)$$
$$\rightarrow \frac{1}{8} (2x - 1) \left((2x - 1)^2 + 2x - 1 + 1\right) + 2x - 1$$

$$zr5(x) = a\, (r5(x))^3 + a\, (r5(x))^2 + a\, r5(x) + r5(x)$$
$$\rightarrow \frac{1}{8} (5x - 1)^3 + \frac{1}{8} (5x - 1)^2 + \frac{1}{8} (5x - 1) + 5x - 1$$

3. Results

Conclusion: - writing Zeta function as a Geometric series of $a * R^3 + a * R^2 + a * R + R$

Where R = (n * X - 1) and Zeta Term = 1/n.

1- $at\ X = 0\ ; the\ geometric\ series\ SUM = -(\ a + 1)$

2- $at\ X = Zeta\ term\ ; the\ geometric\ series\ SUM = 0$

3- $[a] is\ a\ scaler\ value\ for\ the\ geometric\ series; where\ [a]\ is\ any\ Real\ number.$

4- At [a] = 0; these series converge to linear functions = R = Common ration for the geometric series.

5- At [a] = 0; and X =1; these series converge to linear functions = R = Common ration for the geometric series with SUM = (reciprocal of zeta term)-1.

6- $Between\ X\ interval\ [0,1]; zeta\ function\ SUM\ Converges = (a + 1)$

7- $each\ of\ these\ geometric\ series\ will\ have\ a\ Zero\ at\ its\ own\ Zeta\ function\ term$

$and\ these\ zeros\ will\ be\ in\ between\ intereval\ X = [0,1].$

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