

Explanation of Riemann Hypothesis Conjecture About the Critical Strip and The None-trivial Zeros of Zeta Function

Shaimaa said soltan¹

¹ Computer Engineer, Toronto, Canada

Correspondence: Shaimaa Soltan, 3050 Constitution Blvd, Mississauga, ON., L4Y 3X1, Canada. Tel: 1-647-801-6063 E-mail: shaimaasultan@hotmail.com

Abstract

1.1 Hypotheses Explanation

- A) All Prime numbers are not even numbers.
- B) Every Prime number multiplied by 2 is an even number.
- C) All even numbers are a composite prime \forall 2 is one of its multipliers, (2 is a prime number).
- D) All Prime numbers can't be divided by 2.
- E) All Primes are odd numbers.
- F) Any odd number multiply a prime number is a prime number
- G) Any Prime number exists on a complex plane as a pure prime number or (composite prime).
- H) A composite prime is a prime shifted by an arithmetic operation like (+, -, *, /).

Knowing that 2 is a prime number, then all the even numbers are composite primes \forall 2 is one of its multipliers.

We can argue that all numbers are prime numbers with an arithmetic operation, (+, -, *, /) applied to the number will produce another (even, odd, prime) number.

For example, number 2 can be represented as 1+1, except (1, zero and ∞) cannot be represented by these operations and produce another number other than (1, zero and ∞). Therefore, Zeta Function has a residue = 1 when it reaches its pole.

Any arithmetic operation can be represented in a complex plan as shifting or rotation on the complex plan.

Conclusion (1) The odd number set contains all prime numbers except number 2. (Odd numbers are Prime numbers or composite primes)

Conclusion (2) The even number set contains all prime numbers, such that, an even number represented as composite prime and one of the factors is a prime number 2.

Conclusion (3) All the none-trivial zeros for Zeta function are a trivial zero but shifted by a magnitude of 1/2 on the complex plane, for any $S = a + ib$ when $a = 1/2$. on the Critical strip.

2. Method

1.1 Definitions and symbols

A) Let us call all the Set of odd numbers O and all the Set of even number E.

B) Function $E(S) = \frac{1}{4^S} + \frac{1}{6^S} + \frac{1}{8^S} + \frac{1}{10^S} + \frac{1}{12^S} + \frac{1}{14^S} + \frac{1}{16^S} + \frac{1}{18^S} + \frac{1}{20^S} + \dots \infty$

$$E(S) = \sum_{n=1}^{\infty} \frac{1}{(2n+2)^s}$$

C) Function $O(S) = \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{9^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{15^s} + \frac{1}{17^s} + \frac{1}{19^s} + \dots \infty$

$$O(S) = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^s}$$

D) Function $Z(S) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \frac{1}{8^s} + \frac{1}{9^s} + \frac{1}{10^s} + \frac{1}{11^s} + \dots \infty$

$$Z(S) = \sum_{n=1}^{\infty} \frac{1}{(n)^s}$$

Add Function E + Function O

$$O(S) + E(S) = \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \frac{1}{8^s} + \frac{1}{9^s} + \frac{1}{10^s} + \frac{1}{11^s} + \dots \infty$$

Rewrite Function Z as Function E + Function O

$$E(S) + O(S) = Z(S) - 1 - \frac{1}{2^s}$$

Factor out $\frac{1}{2^s}$ from Function E

$$E(S) = \frac{1}{2^s} * \left[\frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \frac{1}{8^s} + \frac{1}{9^s} + \frac{1}{10^s} + \dots \infty \right]$$

Then Function E is

$$E(S) = \frac{1}{2^s} [Z(S) - 1]$$

1.2) $S = \frac{1}{2}$, $S = a + ib \quad \forall \quad Re(S) = 0.5 \text{ and } S \neq 0 \text{ and } b = 0, S = 1/2$

From definition of function E, let $S = 1/2$.

$$E(0.5) = \frac{1}{\sqrt[2]{4}} + \frac{1}{\sqrt[2]{6}} + \frac{1}{\sqrt[2]{8}} + \frac{1}{\sqrt[2]{10}} + \frac{1}{\sqrt[2]{12}} + \frac{1}{\sqrt[2]{14}} + \frac{1}{\sqrt[2]{16}} + \frac{1}{\sqrt[2]{18}} + \frac{1}{\sqrt[2]{20}} + \dots \infty$$

Re arrange E(S) and factor out $\frac{1}{\sqrt[2]{2}}$

$$E(0.5) = \frac{1}{\sqrt[2]{2} * \sqrt[2]{2}} + \frac{1}{\sqrt[2]{2} * \sqrt[2]{3}} + \frac{1}{\sqrt[2]{2} * \sqrt[2]{4}} + \frac{1}{\sqrt[2]{2} * \sqrt[2]{5}} + \frac{1}{\sqrt[2]{2} * \sqrt[2]{6}} + \frac{1}{\sqrt[2]{2} * \sqrt[2]{7}} \\ + \frac{1}{\sqrt[2]{2} * \sqrt[2]{8}} + \dots \infty$$

$$E(0.5) = \frac{1}{\sqrt[2]{2} * \sqrt[2]{2}} + \frac{1}{\sqrt[2]{2}} \left[\frac{1}{\sqrt[2]{3}} + \frac{1}{\sqrt[2]{5}} + \frac{1}{\sqrt[2]{7}} + \frac{1}{\sqrt[2]{9}} + \frac{1}{\sqrt[2]{11}} + \dots \infty \right] \\ + \frac{1}{\sqrt[2]{2}} \left[\frac{1}{\sqrt[2]{4}} + \frac{1}{\sqrt[2]{6}} + \frac{1}{\sqrt[2]{8}} + \frac{1}{\sqrt[2]{10}} + \frac{1}{\sqrt[2]{12}} + \dots \infty \right]$$

From definition of function O, at $S = a + ib \quad \forall \quad Re(S) = 0.5 \text{ and } S \neq 0 \text{ and } b = 0, S=1/2$.

$$O(0.5) = \frac{1}{\sqrt[2]{3}} + \frac{1}{\sqrt[2]{5}} + \frac{1}{\sqrt[2]{7}} + \frac{1}{\sqrt[2]{9}} + \frac{1}{\sqrt[2]{11}} + \frac{1}{\sqrt[2]{13}} + \frac{1}{\sqrt[2]{15}} + \frac{1}{\sqrt[2]{17}} + \frac{1}{\sqrt[2]{19}} + \dots \infty$$

We can rewrite E function, at $S = a + ib \quad \forall \quad Re(S) = 0.5 \text{ and } S \neq 0 \text{ and } b = 0, S=1/2$.

$$E(0.5) = \frac{1}{2} + \frac{1}{\sqrt[2]{2}} E(0.5) + \frac{1}{\sqrt[2]{2}} O(0.5)$$

New E Function at $S=0.5$,

$$E(0.5) = \sqrt[2]{2} E(0.5) - O(0.5) - \frac{\sqrt[2]{2}}{2}$$

But Z function can be written as $E(S) + O(S) = Z(S) - 1 - \frac{1}{2^S}$

$$E(0.5) + O(0.5) = Z(0.5) - 1 - \frac{1}{\sqrt[2]{2}} = Z(0.5) - 1 - \frac{\sqrt[2]{2}}{2}$$

New Z Function as E Function + O Function at $S=0.5$,

$$E(0.5) + O(0.5) = Z(0.5) - 1 - \frac{\sqrt[2]{2}}{2}$$

Substitute E (0.5) in New Z Function by E (0.5) from and new E Function

$$\sqrt[2]{2} E(0.5) - O(0.5) - \frac{\sqrt[2]{2}}{2} + O(0.5) = Z(0.5) - 1 - \frac{\sqrt[2]{2}}{2}$$

$$\sqrt[2]{2} E(0.5) = Z(0.5) - 1 \quad \rightarrow \quad \forall S = a + ib \text{ and } a = 0.5 \text{ and } b = 0$$

Z Function (Zeta function) At $Re(0.5)$, represented only in term of E function and O function vanishes, (odd

numbers), including any prime number will vanish on the critical Strip at $\forall S = a + ib$ and $a = 0.5$ and $b = 0$.
(Their ration = 1).

1.3) At $S = \frac{1}{2} + ib$, $S = a + ib \quad \forall \quad \text{Re}(S) = 0.5$ and $S \neq 0$ and $b \neq 0$

$$E(S + 0.5) = \frac{1}{4^{s+0.5}} + \frac{1}{6^{s+0.5}} + \frac{1}{8^{s+0.5}} + \frac{1}{10^{s+0.5}} + \frac{1}{12^{s+0.5}} + \frac{1}{14^{s+0.5}} \\ + \frac{1}{16^{s+0.5}} + \frac{1}{18^{s+0.5}} + \dots \infty$$

$$E(S + 0.5) = \frac{1}{\sqrt[2]{4} * 4^s} + \frac{1}{\sqrt[2]{6} * 6^s} + \frac{1}{\sqrt[2]{8} * 8^s} + \frac{1}{\sqrt[2]{10} * 10^s} + \frac{1}{\sqrt[2]{12} * 12^s} \\ + \frac{1}{\sqrt[2]{14} * 14^s} + \dots \infty$$

$$E(S + 0.5) = \frac{1}{\sqrt[2]{2} * \sqrt[2]{2} * 2^s * 2^s} + \frac{1}{\sqrt[2]{6} * 2^s * 3^s} + \frac{1}{\sqrt[2]{8} * 2^s * 4^s} \\ + \frac{1}{\sqrt[2]{10} * 2^s * 5^s} + \frac{1}{\sqrt[2]{12} * 2^s * 6^s} + \frac{1}{\sqrt[2]{14} * 2^s * 7^s} + \dots \infty$$

Factor out $\frac{1}{\sqrt[2]{2} * 2^s}$ from E Function

$$E(S + 0.5) = \frac{1}{2^{s+0.5}} \left[\frac{1}{\sqrt[2]{2} * 2^s} + \frac{1}{\sqrt[2]{3} * 3^s} + \frac{1}{\sqrt[2]{4} * 4^s} + \frac{1}{\sqrt[2]{5} * 5^s} + \frac{1}{\sqrt[2]{6} * 6^s} \right. \\ \left. + \frac{1}{\sqrt[2]{7} * 7^s} + \dots \infty \right]$$

$$E(S + 0.5) = \frac{1}{2^{s+0.5}} \left[\frac{1}{2^{s+0.5}} + \frac{1}{3^{s+0.5}} + \frac{1}{4^{s+0.5}} + \frac{1}{5^{s+0.5}} + \frac{1}{6^{s+0.5}} + \frac{1}{7^{s+0.5}} \right. \\ \left. + \dots \infty \right]$$

Rearrange E(S)

$$\begin{aligned}
E(S + 0.5) &= \frac{1}{2^{2s+1}} \\
&\quad + \frac{1}{2^{s+0.5}} \left[\frac{1}{4^{s+0.5}} + \frac{1}{6^{s+0.5}} + \frac{1}{8^{s+0.5}} + \frac{1}{10^{s+0.5}} + \frac{1}{12^{s+0.5}} + \frac{1}{14^{s+0.5}} + \dots \infty \right] \\
&\quad + \frac{1}{2^{s+0.5}} \left[\frac{1}{3^{s+0.5}} + \frac{1}{5^{s+0.5}} + \frac{1}{7^{s+0.5}} + \frac{1}{9^{s+0.5}} + \frac{1}{11^{s+0.5}} + \frac{1}{13^{s+0.5}} + \dots \infty \right] \\
E(S + 0.5) &= \frac{1}{2^{2s+1}} + \frac{1}{2^{s+0.5}} E(S + 0.5) + \frac{1}{2^{s+0.5}} O(0.5)
\end{aligned}$$

multiply by $(2^{s+0.5})$

$$2^{s+0.5} * E(S + 0.5) = \frac{2^{s+0.5}}{2^{2s+1}} + E(S + 0.5) + O(0.5)$$

$$2^{s+0.5} * E(S + 0.5) = \frac{1}{2^{s+0.5}} + E(S + 0.5) + O(0.5)$$

$$E(S + 0.5) = 2^{s+0.5} * E(S + 0.5) - O(S + 0.5) - \frac{1}{2^{s+0.5}}$$

But Z function can be written as $E(S) + O(S) = Z(S) - 1 - \frac{1}{2^s}$

$$\begin{aligned}
2^{s+0.5} * E(S + 0.5) - O(S + 0.5) - \frac{1}{2^{s+0.5}} &+ O(S + 0.5) \\
&= Z(S + 0.5) - 1 - \frac{1}{2^{s+0.5}}
\end{aligned}$$

$$2^{s+0.5} * E(S + 0.5) = Z(S + 0.5) - 1$$

Factor out $1/2^s$ or $1/2^{s+0.5}$ from E Function

$$\begin{aligned}
E(S + 0.5) &= \frac{1}{2^s} \left[\frac{1}{2^{s+0.5}} + \frac{1}{3^{s+0.5}} + \frac{1}{4^{s+0.5}} + \frac{1}{5^{s+0.5}} + \frac{1}{6^{s+0.5}} + \frac{1}{7^{s+0.5}} + \frac{1}{8^{s+0.5}} \right. \\
&\quad \left. + \frac{1}{9^{s+0.5}} + \dots \infty \right]
\end{aligned}$$

Rearrange Equations (20) and (21)

$$E(S + 0.5) = \frac{1}{2^s} [Z(S + 0.5) - 1] \rightarrow (22) \text{ if we factored out } \frac{1}{2^s}$$

$$E(S + 0.5) = \frac{1}{2^{s+0.5}} [Z(S) - 1] \rightarrow (23) \text{ if we factored out } \frac{1}{2^{s+0.5}}$$

From Equation (20) and (23)

$$Z(S + 0.5) - 1 = [Z(S) - 1] \rightarrow (24)$$

Conclusion (I)

$$Z(S) = Z(S+0.5) \text{ for any } S=S+0.5$$

for any complex number S at $S=S+0.5$ in the whole complex plane if $\text{Re}(S) = 0.5$ Zeta function will reach its zero as well at the critical Strip at $S=0.5+ib$ for any value for b in the complex plane.

1.3 Proof that $O(S+0.5)$ is meromorphic at Critical strip $O(S+0.5) = O(S-0.5)$

$$O(S + 0.5) = \frac{1}{3^{s+0.5}} + \frac{1}{5^{s+0.5}} + \frac{1}{7^{s+0.5}} + \frac{1}{9^{s+0.5}} + \frac{1}{11^{s+0.5}} + \frac{1}{13^{s+0.5}} + \frac{1}{15^{s+0.5}} \\ + \frac{1}{17^{s+0.5}} + \dots \infty \rightarrow (a)$$

$$O(S + 0.5) = \frac{1}{\sqrt[2]{3} * \frac{3^{s-0.5}}{\sqrt[2]{3}}} + \frac{1}{\sqrt[2]{5} * \frac{5^{s-0.5}}{\sqrt[2]{5}}} + \frac{1}{\sqrt[2]{7} * \frac{7^{s-0.5}}{\sqrt[2]{7}}} + \frac{1}{\sqrt[2]{9} * \frac{9^{s-0.5}}{\sqrt[2]{9}}} \\ + \frac{1}{\sqrt[2]{11} * \frac{11^{s-0.5}}{\sqrt[2]{11}}} + \dots \infty$$

$$O(S + 0.5) = \frac{1}{3^{s-0.5}} + \frac{1}{5^{s-0.5}} + \frac{1}{7^{s-0.5}} + \frac{1}{9^{s-0.5}} + \frac{1}{11^{s-0.5}} + \frac{1}{13^{s-0.5}} + \frac{1}{15^{s-0.5}} \\ + \frac{1}{17^{s-0.5}} + \dots \infty \rightarrow (b)$$

Conclusion (II)

$$O(S+0.5) = O(S-0.5)$$

All odd numbers including any prime numbers will vanish on the critical Strip at $S = S + 0.5$.

3. Results

Based on this Conjecture all prime numbers will vanish on the critical strip at $\text{Re}(S) = 0.5$ for any complex number S.

$$O(S+0.5) = O(S-0.5)$$

$$Z(S) = Z(S+0.5) \text{ at } \text{Re}(S) = 0.5 \text{ and zeta function has zeros at even numbers } -2, -4, -6, \dots$$

Next step will be to study the trivial zeros patterns in relation with the odd numbers and then the Prime numbers.

To get a formula for the distribution of the prime numbers on the Critical strip.