New functional equation formula for Riemann's functional equation

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Abstract. **This paper introduces a new functional equation formula for Riemann's functional equation for the Zeta function. Using this new functional equation, we will show how Riemann's functional equation for the Zeta function evaluates to Zero when X = (X/2 + 1/2) for each X odd natural number.**

1 Introduction

Riemann's functional equation

$$\zeta(x) = 2^x * \pi^{x-1} * \sin\left(\pi * \frac{x}{2}\right) * \Gamma(1-x) * \zeta(1-x) \tag{1.1}$$

If we used this known equality

$$x = Cos\left(Cos^{-1}(x)\right) \tag{1.2}$$

$$Cos^{-1}\left(\text{Sin}\left(\frac{1}{X}\right)\right) = \frac{\pi}{2} - \frac{1}{X} \qquad (A)$$

$$Cos^{-1}\left(\text{Sin}\left(\frac{1}{X}\right)\right) = \frac{X * \pi - 2}{2 * X} \qquad (B)$$

$$Cos^{-1}(\text{Sin}(X)) - \frac{\pi}{2} + (-1)^X * X * 2 = \pm X \quad \pm A * \pi \qquad (C)$$

Therefore; from Equations (1.3), (A), (B), and (C) we can use this chain of trigonometric functions to rewrite the sinusoidal term in Riemann's functional equation as this chain of trigonometric functions has no effect on the sinusoidal term

$$Sin(x) = Cos(Cos^{-1}(\text{Sin}\,(x)\,)\,) \tag{1.3}$$

From Equation (1.1) and (1.3)

$$\zeta(x) = 2^x * \pi^{x-1} * Cos\left(Cos^{-1}\left(\sin\left(\pi * \frac{x}{2}\right)\right)\right) * \Gamma(1-x)\zeta(1-x) \tag{1.4}$$

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In Equation (1.4) replace each X=X+1/2

$$\zeta\left(x+\frac{1}{2}\right)=2^{x+\frac{1}{2}}*\ \pi^{x-\frac{1}{2}}*Cos\left(\ Cos^{-1}\left(\sin\left(\frac{\pi}{2}*\left(x+\frac{1}{2}\right)\right)\right)\right)*\ \Gamma\left(\frac{1}{2}-x\right)*\zeta\left(\frac{1}{2}-x\right) \qquad (1.5)$$

We can write the first two terms in Riemann's functional equation at X = X +1/2 as

$$f(x)=\left(2^{x+\frac{1}{2}}*\ \pi^{x-\frac{1}{2}}\right)=\ \sqrt{\frac{2}{\pi}}*\ \ (2^{x}*\ \pi^{x-1}) \qquad (1.6)$$

from Equations (1.5) and (1.6) we can write Equation (1.4) at X = X +1/2 as

$$\zeta\left(x+\frac{1}{2}\right)=2^{x}*\ \pi^{x-1}*\sqrt{\frac{2}{\pi}}*Cos\left(\ Cos^{-1}\left(\sin\left(\frac{\pi}{2}*\left(x+\frac{1}{2}\right)\right)\right)\right)*\ \Gamma\left(\frac{1}{2}-x\right)*\zeta\left(\frac{1}{2}-x\right) \qquad (1.7)$$

$$g(x)=Cos\left(\ Cos^{-1}\left(\sin\left(\frac{\pi}{2}*(2*\ x+1\ )\right)\right)\right)=\begin{cases}-1 & for\ each\ Odd\ X\\ 0 & for\ each\ X=X+\frac{1}{2}\\ 1 & for\ each\ even\ X\end{cases} \qquad (1.8)$$

$$h(x)=Cos\left(\ Cos^{-1}\left(\sin\left(\frac{\pi}{2}*\left(x+\frac{1}{2}\right)\right)\right)\right)=\begin{cases}1 & for\ each\ even\ X+\frac{1}{2}\\ 0 & for\ each\ odd\ X+\frac{1}{2}\\ \pm\sqrt{2} & for\ each\ X\end{cases} \qquad (1.9)$$

From equation (1.6) and (1.9)

$$for\ each\ odd\ X+\frac{1}{2}\quad we\ have\quad h(x)*\sqrt{\frac{2}{\pi}}\ =0 \qquad (1.10)$$

and this makes the sinusoidal term in Riemann's functional equation Equal Zero at each odd natural number plus or minus half

$$for\ each\ odd\ X+\frac{1}{2}\qquad we\ have\ h(x)*\sqrt{\frac{2}{\pi}}\ =0\quad therefore\ we\ have\quad \zeta\left(x+\frac{1}{2}\right)=0 \qquad (1.11)$$

$$\zeta\left(x+\frac{1}{2}\right)=2^{x}*\ \pi^{x-1}*\sqrt{\frac{2}{\pi}}*h(x)*\ \Gamma\left(\frac{1}{2}-x\right)*\zeta\left(\frac{1}{2}-x\right) \qquad (1.12)$$

$$\zeta\left(x-\frac{1}{2}\right) = 2^{x} * \pi^{x-1} * \sqrt{\frac{\pi}{2}} * Cos\left(Cos^{-1}\left(\sin\left(\frac{\pi}{2} * \left(x-\frac{1}{2}\right)\right)\right)\right) * \Gamma\left(\frac{3}{2}-x\right) * \zeta\left(\frac{3}{2}-x\right) \tag{1.13}$$

This new functional equation shows that Riemann's functional equation Equal Zero at each odd natural number plus or minus half because this representation for the sinusoidal term will evaluate to Zero

$f(x) = \cos\left(\cos^{-1}\left(\sin\left((2x+1)\,\pi\right)\right)\right)$

x	f(x)
-2.5	0
-2	0
-1.5	0
-1	0
-0.5	0
0	0
0.5	0
1	0
1.5	0
2	0
2.5	0
3	0
3.5	0
4	0

$f(x) = \cos\left(\cos^{-1}\left(\sin\left((2x+1)\cdot\frac{\pi}{2}\right)\right)\right)$

x	f(x)
-2.5	0
-2	1
-1.5	0
-1	-1
-0.5	0
0	1
0.5	0
1	-1
1.5	0
2	1
2.5	0
3	-1
3.5	0
4	1

Figure 1: This is the new representation for the sinusoidal term in Riemann's functional equation at X= 4X+2 and X=2x+1

$$f(x) = \cos\left(\cos^{-1}\left(\sin\left((3x+1)\cdot\frac{\pi}{2}\right)\right)\right)$$

x	f(x)
-2.5	0.707106781186...
-2	-1
-1.5	0.707106781186...
-1	0
-0.5	-0.70710678118...
0	1
0.5	-0.70710678118...
1	0
1.5	0.707106781186...
2	-1
2.5	0.707106781186...
3	0
3.5	-0.70710678118...
4	1

$$f(x) = \cos\left(\cos^{-1}\left(\sin\left(\left(x+\frac{1}{2}\right)\cdot\frac{\pi}{2}\right)\right)\right)$$

x	f(x)
-2.5	0
-2	-0.70710678118...
-1.5	-1
-1	-0.70710678118...
-0.5	0
0	0.707106781186...
0.5	1
1	0.707106781186...
1.5	0
2	-0.70710678118...
2.5	-1
3	-0.70710678118...
3.5	0
4	0.707106781186...

Figure 2: This is the new representation for the sinusoidal term in Riemann's functional equation at X= 3 X+1 and at X=X+1/2

$$f(x) = \cos\left(\cos^{-1}\left(\sin\left(\left(x+\frac{1}{2}\right)\cdot\frac{\pi}{2}\right)\right)\right)\sqrt{2}$$

x	f(x)
-2.5	0
-2	-1
-1.5	-1.41421356237...
-1	-1
-0.5	0
0	1
0.5	1.414213562373...
1	1
1.5	0
2	-1
2.5	-1.41421356237...
3	-1
3.5	0
4	1

$$f(x) = \cos\left(\cos^{-1}\left(\sin\left(\left(x+\frac{1}{2}\right)\cdot\frac{\pi}{2}\right)\right)\right)\sqrt{\frac{2}{\pi}}$$

x	f(x)
-2.5	0
-2	-0.56418958354...
-1.5	-0.79788456080...
-1	-0.56418958354...
-0.5	0
0	0.564189583547...
0.5	0.797884560802...
1	0.564189583547...
1.5	0
2	-0.56418958354...
2.5	-0.79788456080...
3	-0.56418958354...
3.5	0
4	0.564189583547...

Figure 3: This is the new representation for the sinusoidal term in Riemann's functional equation at X=X + 1/2 and includes the first and second term changes as well

2 Conclusion

In this paper, we introduced a new function formula for the Zeta function based on Riemann's functional equation and using this new representation for Riemann's functional equation which is based on this trigonometric equality

$$x = Cos\left(Cos^{-1}(x)\right) \tag{1.2}$$

using this trigonometric equality removes the floating point residuals that exist due to the changes between radial and degrees bases when using pi versus using 180 degrees. Using this new functional equation formula for the Zeta function we showed that the sinusoidal function term in Riemann's functional equation for the Zeta function evaluates to Zero at each odd natural number when we add 0.5 or subtract 0.5 from any odd natural number. as it is shown in Figure 3 the sinusoidal term in this new functional formula Zeta Function evaluates to Zero only at X = (X/2 + 1/2) for each X odd natural number.

$$h(x) = Cos\left(Cos^{-1}\left(\sin\left(\frac{\pi}{2} * \left(x + \frac{1}{2}\right)\right)\right)\right) = \begin{cases} 1 & for\ each\ even\ X + \frac{1}{2} \\ 0 & for\ each\ odd\ X + \frac{1}{2} \\ \pm\sqrt{2} & for\ each\ X \end{cases} \tag{1.9}$$

$$for\ each\ odd\ X + \frac{1}{2} \quad we\ have\ h(x) * \sqrt{\frac{2}{\pi}} = 0 \quad therefore\ we\ have\ \zeta\left(x + \frac{1}{2}\right) = 0 \tag{1.11}$$

$$\zeta\left(x + \frac{1}{2}\right) = 2^{x} * \pi^{x-1} * \sqrt{\frac{2}{\pi}} * h(x) * \Gamma\left(\frac{1}{2} - x\right) * \zeta\left(\frac{1}{2} - x\right) \tag{1.12}$$

$$\zeta\left(x + \frac{1}{2}\right) = 2^{x} * \pi^{x-1} * \sqrt{\frac{2}{\pi}} * Cos\left(Cos^{-1}\left(\sin\left(\frac{\pi}{2} * \left(x + \frac{1}{2}\right)\right)\right)\right) * \Gamma\left(\frac{1}{2} - x\right) * \zeta\left(\frac{1}{2} - x\right) \tag{1.7}$$

References

[1] Cassels, J.W.S., Frohlich, A., eds.: Algebraic Number Theory. London: Academic Press 1967.
[2] Gradshteyn, I.S., Ryzhik, I.M.: Table of Integrals, Series, and Products. New York: Academic Press 1980.
[3] Apostol, Tom M. (1976). Introduction to analytic number theory. Undergraduate Texts in Mathematics. Springer. ISBN 978-0-387-90163-3. Retrieved 2016-02-28.

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