Natural Logarithm with Euler Identity and Tylor expansion

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Abstract

This paper presents Euler's identity for any power while showing the equality reason between subtracting one half from any power of X and subtracting one from any power of X. then we showed how this equality is correct for all neutral numbers and its powers when subtracting one half as well as when subtraction -1.

Keywords: Euler Identity, Tylor Expansion

1. Introduction

Tylor expansion

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots , \qquad -\infty < x < \infty$$

$in\ Tylor\ expansion\ if\ we\ replaced\ [X = i * X]\ and\ using\ this\ realtion\ of\ [\ i = \frac{-1}{i}\ ]$

$$e^{i*x} = 1 + \frac{i*x}{1!} + \frac{(i*x)^2}{2!} + \frac{(i*x)^3}{3!} + \cdots , \qquad -\infty < x < \infty$$

$$e^{i*x} = 1 + \frac{i*x}{1!} + \frac{(i*x)^2}{2!} + \frac{(i*x)^3}{3!} + \frac{(i*x)^4}{4!} + \cdots , \qquad -\infty < x < \infty$$

$$e^{i*x} = 1 + \frac{i*x}{1!} - \frac{(x)^2}{2!} - \frac{(x)^3}{i*3!} + \frac{(x)^4}{4!} + \frac{i*(x)^5}{5!} - \frac{(x)^6}{6!} - \frac{(x)^7}{i*7!} \ldots , \rightarrow (A)$$

$$e^{i*x} = 1 - \frac{x}{i*1!} - \frac{(x)^2}{2!} + \frac{i*(x)^3}{3!} + \frac{(x)^4}{4!} - \frac{(x)^5}{i*5!} - \frac{(x)^6}{6!} + \frac{i\,(x)^7}{7!} \ldots \rightarrow (B)$$

Adding EQ (A) and (B) all the odd terms will be an imaginary part.

$$2*e^{i*x} = 2 + \frac{2*i*x}{1!} - \frac{2*(x)^2}{2!} + \frac{2*i*(x)^3}{3!} + \frac{2*(x)^4}{4!} + \frac{2*i*(x)^5}{5!} - \frac{2*(x)^6}{6!} + \frac{2*i\,(x)^7}{7!} + \cdots$$

$$e^{i*x} = 1 + \frac{i*x}{1!} - \frac{(x)^2}{2!} + \frac{i*(x)^3}{3!} + \frac{(x)^4}{4!} + \frac{i*(x)^5}{5!} - \frac{(x)^6}{6!} + \frac{i\,(x)^7}{7!} + \cdots \rightarrow (C)$$

$$Sinh(x) = \frac{x}{1!} + \frac{(x)^3}{3!} + \frac{(x)^5}{5!} + \frac{(x)^7}{7!} + \cdots$$

$$i*Sinh(x) = \frac{i*x}{1!} + \frac{i*(x)^3}{3!} + \frac{i*(x)^5}{5!} + \frac{i*(x)^7}{7!} + \cdots$$

$$Sin(i\,x) = \frac{i*x}{1!} + \frac{i*(x)^3}{3!} + \frac{i*(x)^5}{5!} + \frac{i\,(x)^7}{7!} + \cdots$$

$$Sinh(x) = \frac{(e^{x} - e^{-x})}{2}$$

$$i*Sinh(x) = i*\frac{(e^{x} - e^{-x})}{2} = -\frac{(e^{x} - e^{-x})}{2*i}$$

$$e^{i*x} = i * Sinh(x) + Cos(x) = \sin(ix) + \cos(x) = \cos(x) + i\sin(x) \to (D)$$

$$\sin(ix) + \cos(x) = \frac{-(e^{x} - e^{-x})}{2\,i} + \frac{(e^{xi} + e^{-xi})}{2}$$

$$at\ X = X + \frac{1}{2}$$

$$e^{i*\left(X+\frac{1}{2}\right)} = \sin\left(i * X + \frac{i}{2}\right) + \cos\left(X + \frac{1}{2}\right) = \begin{cases} \frac{\frac{-i}{2} * (i + 180 * X)}{2} + \cos\left(X + \frac{1}{2}\right) \\ 1 \qquad at\ X = \frac{-1}{2} \end{cases}$$

From Tylor equation $at\ X = \pi$; Tylor expansion converge to -1

And from Euler identity at $X = \pi$; Tylor expansion converge to -1

$$e^{i*\pi} = -1$$

Similarly, $\sin(ix) + \cos(x)\ \ at\ X = \pi = 180\ it\ will\ converge\ to - 1$

$e^{i*X} = \sin(ix) + \cos(x)$ = i $* \sin(180) + \cos(180) = -1$

Therefore, when we deal with Euler identity in Left hand side of EQ (D) we use the radian π and when we deal with trigonometric function in the Right-hand side, we use degrees.

$e^{i*\left(Y+\frac{Y}{2}\right)} = \sin\left(ix + \frac{i\,X}{2}\right) + \cos\left(x + \frac{X}{2}\right)$ = i $* \sin\left(x + \frac{X}{2}\right) + \cos\left(x + \frac{X}{2}\right)$→(E)

$in\ Left\ hand\ side\ we\ use\ radian\ So\ if\ Y = \pi\ so\ half\ Y = \frac{\pi}{2}$

$$and\ if\ we\ move\ by\ half\ step\ e^{i*\left(Y+\frac{Y}{2}\right)} = e^{i*\left(\frac{3*\pi}{2}\right)} = -i$$

$$in\ right\ hand\ side\ at\ X = 180\ degrees\ and\ half\ X = \frac{180}{2} = 90$$

$$therefore; I\ \ \sin\left(180 + \frac{180}{2}\right) + \cos\left(180 + \frac{180}{2}\right) = -i$$

$$e^{i*\left(N*\pi + \frac{\pi}{2}\right)} = \mathrm{i} * \sin\left(N * 180° + \frac{180°}{2}\right) + \cos\left(N * 180° + \frac{180°}{2}\right) \to (F)$$

In the next section we will show how this is correct for any power of X.

Let us write X^N in this form.

$$X^N = \left(\left(\frac{2 * \ln(i)}{X^N} - 1\right) * X^N + (X^N - 1)\right) * \frac{1}{i}$$

Or

$$X^N = \left(\left(\frac{2 * \ln(i)}{X^N} - \frac{1}{2}\right) * X^N + (\frac{X^N}{2} - \frac{1}{2})\right) * \frac{1}{i}$$

The Key point is the natural logarithm of the complex imaginary unit number.

$$2 * \ln(i) = \pi * i \quad ➔(1)$$

$$e^{i * \pi} = -1 \quad ➔(2)$$

Therefore, From EQ (1) and (2)

$$e^{2\ln(i)} = -1 \; and \; e^{\ln(i)} = i \;\; and \; e^{\ln\left(i \pm \frac{1}{2}\right)} = \pm\frac{1}{2} + i$$

$$A = \left(\left(\frac{2 * \ln(i)}{1} - 1\right) * 1 + 0\right) * \frac{1}{i} = \pi + i \quad ➔(3)$$

$$e^{i * A+1} = -1 \quad ➔(4)$$

$$B = \left(\left(\frac{2 * \ln(i)}{X} - 1\right) * X + (X - 1)\right) * \frac{1}{i} = \pi + i \quad ➔(5)$$

$$e^{i * B+1} = -1 \quad ➔(6)$$

$$C = \left(\left(\frac{2 * \ln(i)}{1} - \frac{1}{2}\right) * 1 + 0\right) * \frac{1}{i} = \pi + \frac{i}{2} \quad ➔(7)$$

$$e^{i * C+\frac{1}{2}} = -1 \quad ➔(8)$$

$$D = \left(\left(\frac{2 * \ln(i)}{X} - \frac{1}{2}\right) * X + (\frac{X}{2} - \frac{1}{2})\right) * \frac{1}{i} = \pi + \frac{i}{2} \quad ➔(9)$$

$$e^{i * D+\frac{1}{2}} = -1 \quad ➔(10)$$

$$F = \left(2 * \ln(i) - \left(\frac{1}{2}\right)\right) * \frac{1}{i} = \pi + \frac{i}{2} \quad ➔(11)$$

$$e^{i * F+\frac{1}{2}} = -1 \quad ➔(12)$$

$$G = \left(\left(\frac{2 * \ln(i)}{X^N} - \frac{1}{2}\right) * X^N + (\frac{X^N}{2} - \frac{1}{2})\right) * \frac{1}{i} = \pi + \frac{i}{2} \quad ➔(13)$$

$$e^{i * G+\frac{1}{2}} = -1 \quad ➔(14)$$

$$H = \left(\left(\frac{2 * \ln(i)}{X^N} - 1\right) * X^N + (X^N - 1)\right) * \frac{1}{i} = \pi + i \quad ➔(15)$$

$$e^{i * H+1} = -1 \quad ➔(16)$$

$$e^{i*\left(\left(\frac{2*\ln(i)}{X^N}-\frac{1}{2}\right)*X^N+\left(\frac{X^N}{2}-\frac{1}{2}\right)\right)*\frac{1}{i}+\frac{1}{2}} = e^{i*\left(\left(\frac{2*\ln(i)}{X^N}-1\right)*X^N+\left(X^N-1\right)\right)*\frac{1}{i}+1} = -1 \quad \rightarrow (17)$$

From EQ (1)

$$e^{\left(\left(\frac{\pi*i}{X^N}-\frac{1}{2}\right)*X^N+\left(\frac{X^N}{2}-\frac{1}{2}\right)\right)+\frac{1}{2}} = e^{\left(\left(\frac{\pi*i}{X^N}-1\right)*X^N+\left(X^N-1\right)\right)+1} = -1 \quad \rightarrow (18)$$

$$e^{\left(i*\left(\frac{\pi}{X^N}+\frac{i}{2}\right)*X^N+\frac{X^N}{2}\right)} = -1 \quad \rightarrow (19)$$

$$let\ X = \frac{\pi}{X}$$

$$for\ any\ Natural\ number\ X\ ; \qquad e^{\left(\left(i*X-\frac{1}{2}\right)*\frac{\pi}{X}+\frac{\pi}{2*X}\right)} = -1 \quad \rightarrow (20)$$

And this is because the default exponent rules (multiplication and division) therefore for any X we go back to Euler unit identity.

$$e^{\frac{\pi}{2*X}} * e^{\left(\left(i*X-\frac{1}{2}\right)*\frac{\pi}{X}\right)} = -1$$

$$e^{\frac{\pi}{2*X}} * e^{\left(i*\pi-\frac{\pi}{2*X}\right)} = -1$$

$$e^{\frac{\pi}{2*X}} * \frac{e^{(i*\pi)}}{e^{\frac{\pi}{2*X}}} = -1$$

$$e^{(i*\pi)} = -1$$

Adding 1 or ½ give us same result and we get only one pole for all natural numbers.

Conclusion

In This paper we showed how Euler identity and Tylor series converse to -1 for all natural numbers when we add one half or when we are adding 1.

$$e^{i*\left(\left(\frac{2*\ln(i)}{X^N}-\frac{1}{2}\right)*X^N+\left(\frac{X^N}{2}-\frac{1}{2}\right)\right)*\frac{1}{i}+\frac{1}{2}} = -1$$

$$e^{i*\left(\left(\frac{2*\ln(i)}{X^N}-1\right)*X^N+\left(X^N-1\right)\right)*\frac{1}{i}+1} = -1$$

$$e^{\left(i*\left(\frac{\pi}{X^N}+\frac{i}{2}\right)*X^N+\frac{X^N}{2}\right)} = -1$$

$$For\ each\ X = \frac{\pi}{2}\ ; \qquad e^{i*\left(X+\frac{X}{2}\right)} = \sin\left(ix+\frac{i\,X}{2}\right)+\cos\left(x+\frac{X}{2}\right) = \mathrm{i}*\sin\left(x+\frac{X}{2}\right)+\cos\left(x+\frac{X}{2}\right)$$

$$e^{i*\left(N*\pi+\frac{\pi}{2}\right)} = \mathrm{i}*\sin\left(N*180°+\frac{180°}{2}\right)+\cos\left(N*180°+\frac{180°}{2}\right)$$

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