Power Series Zeros using Exponential Formula at Half Factorial

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Abstract

In this document, we will introduce new formula to calculate a numerical value for half a factorial.

Then Will use this formula to Proof that exponential formula for the power series will has zeros at half factorial at the same values as X=1. This proofs Riemann hypotheses. Then we will show how using this half factorial function makes exponential formula of power series converges faster for all values of R.

Keywords: Prime Numbers, Composite Prime Numbers, Prime Number Distribution, Zeta function

1. Introduction

1.1 Introduce the Problem

Exponential formula for power series is an infinite series, so we do not know the final term in this series to calculate exact natural value for this summation and the formula uses fractions of factorials or decimal calculations, so getting actual natural values will be dependent two conditions finding the final term of infinite series (which will be even very big or very small) which needs and depends on the system used in the calculations. And both conditions relate to each other. very big or very small number needs specific machine and each time we find new terms (Primes) we need more advance machines, so this way of finding exact natural number solutions is kind of going in circles.

In this document we are going to introduce a graphical proof using new half factorial formula.

Half Fact (HF)

HF(N) = $HF(N) = \frac{N!}{2^{N-1}}$

N	N!	2^{N-1}	HF(N)= $\frac{N!}{2^{N-1}}$
1	1	1	1
2	2	2	1
3	6	4	1.5
4	24	8	3
5	120	16	7.5
6	720	32	22.5
7	5040	64	78.75
..	…	..	….

N	N-1	N!	2^N-1	HALFFACT	Factor 2 out of N!	2 * HALF FACT
1	0	1	1	1.00	2 * (0.5 * 1)	2.00
2	1	2	2	1.00	2 * (0.5 * 1)	2.00
3	2	6	4	1.50	2 * (0.5 * 1 * 1.5)	3.00
4	3	24	8	3.00	2* (0.5 * 1* 1.5 * 2)	6.00
5	4	120	16	7.50	2*(0.5 * 1 * 1.5 * 2* 2.5)	15.00
6	5	720	32	22.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3)	45.00
7	6	5040	64	78.75	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5)	157.50
8	7	40320	128	315.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4)	630.00
9	8	362880	256	1417.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5)	2835.00
10	9	3628800	512	7087.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5)	14175.00
11	10	39916800	1024	38981.25	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5)	77962.50
12	11	479001600	2048	233887.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6)	467775.00
13	12	6227020800	4096	1520268.75	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5)	3040537.50
14	13	87178291200	8192	10641881.25	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7)	21283762.50
15	14	1307674368000	16384	79814109.38	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5)	159628218.75
16	15	20922789888000	32768	638512875.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8)	1277025750.00
17	16	355687428096000	65536	5427359437.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5)	10854718875.00
18	17	6402373705728000	131072	48846234937.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9)	97692469875.00
19	18	121645100408832000	262144	464039231906.25	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5)	928078463812.50
20	19	2432902008176640000	524288	4640392319062.50	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10)	9280784638125.00
21	20	51090942171709400000	1048576	48724119350156.20	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10* 10.5)	97448238700312.50
22	21	1124000727777610000000	2097152	535965312851719.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10*10.5*11)	1071930625703440.00
23	22	25852016738885000000000	4194304	6163601097794770.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10*10.5*11*11.5)	12327202195589500.00
24	23	620448401733239000000000	8388608	73963213173537200.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10*10.5*11*11.5*12)	147926426347074000.00
25	24	15511210043331000000000000	16777216	924540164669215000.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10*10.5*11*11.5*12*12.5)	1849080329338430000.00
26	25	403291461126606000000000000	33554432	12019022140699800000.00	2*(0.5 * 4 * 4.5 * 2* 2.5 * 3 * 3.5 * 4*4.5*5*5.5*6 *6.5*7*7.5*8*8.5*9*9.5*10*10.5*11*11.5*12*12.5*13)	24038044281399600000.00

$$HF(N) = \frac{N!}{2^{N-1}}$$

Factoring out 2 in multiplication from factorial

2 * 4 * 6; not equal 2 * 2 * (2 * 3)

$$h(x) = 1 + \frac{x}{2\ast\ (0.5)} + \frac{x^2}{2\ast(0.5\ast 1)} + \frac{x^3}{2\ast\ (0.5\ast 1\ast 1.5)} + \frac{x^4}{2\ast(0.5\ast 1\ast 1.5\ast 2)} + \frac{x^5}{2\ast\ (0.5\ast 1\ast 1.5\ast 2\ast 2.5)} + \frac{x^6}{2\ast\ (0.5\ast 1\ast 1.5\ast 2\ast 2.5\ast 3)} + \frac{x^7}{2\ast\ (0.5\ast 1\ast 1.5\ast 2\ast 2.5\ast 3\ast 3.5)} + \cdots.$$

$$h(x) = 1 + \frac{x}{HF(1)} + \frac{x^2}{HF(2)} + \frac{x^3}{HF(3)} + \frac{x^4}{HF(4)} + \frac{x^5}{HF(5)} + \frac{x^6}{HF(6)} + \frac{x^7}{HF(7)} + \cdots.$$

$$h(x) = 1 + \frac{x}{\frac{1!}{2^{1-1}}} + \frac{x^2}{\frac{2!}{2^{2-1}}} + \frac{x^3}{\frac{3!}{2^{3-1}}} + \frac{x^4}{\frac{4!}{2^{4-1}}} + \frac{x^5}{\frac{5!}{2^{5-1}}} + \frac{x^6}{\frac{6!}{2^{6-1}}} + \frac{x^7}{\frac{7!}{2^{7-1}}} + \cdots.$$

$$h(x) = 1 + \frac{x}{1!} + \frac{2\ast x^2}{2!} + \frac{4\ast x^3}{3!} + \frac{8\ast x^4}{4!} + \frac{16\ast x^5}{5!} + \frac{32\ast x^6}{6!} + \frac{64\ast x^7}{7!} + \cdots.$$

$$2\ast h(x) = 2\ast(1 + \frac{x}{1!} + \frac{2\ast x^2}{2!} + \frac{4\ast x^3}{3!} + \frac{8\ast x^4}{4!} + \frac{16\ast x^5}{5!} + \frac{32\ast x^6}{6!} + \frac{64\ast x^7}{7!} + \cdots).$$

$$2\ast h(x) = 2 + \frac{2\ast x}{1!} + \frac{4\ast x^2}{2!} + \frac{8\ast x^3}{3!} + \frac{16\ast x^4}{4!} + \frac{32\ast x^5}{5!} + \frac{64\ast x^6}{6!} + \frac{128\ast x^7}{7!} + \cdots).$$

$$2\ast h(x) = 2 + \frac{(2\ast x)}{1!} + \frac{(2\ast x)^2}{2!} + \frac{(2\ast x)^3}{3!} + \frac{(2\ast x)^4}{4!} + \frac{(2\ast x)^5}{5!} + \frac{(2\ast x)^6}{6!} + \frac{(2\ast x)^7}{7!} + \cdots).$$

and

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots.$$

At X = 2 *X

$$e^{2x} = 1 + \frac{(2*x)}{1!} + \frac{(2*x)^2}{2!} + \frac{(2*x)^3}{3!} + \frac{(2*x)^4}{4!} + \frac{(2*x)^5}{5!} + \frac{(2*x)^6}{6!} + \frac{(2*x)^7}{7!} + \cdots.$$

$$2*h(x) = 1 + e^{2x}$$

$$h(x) = \frac{1}{2} + \frac{e^{2x}}{2}$$

$$e^{2x} = \sinh 2x + \cosh 2x$$

$$h(x) = \frac{1}{2} + \frac{\sinh 2x}{2} + \frac{\cosh 2x}{2}$$

$$2*h(x) - 1 = e^{2x}$$

Divide both sides by e^x

$$\frac{2*h(x) - 1}{e^x} = e^{-x}$$

At x = 1

$$\frac{1}{2} + \frac{e^2}{2} = h(1) = 1 + \frac{1}{1} + \frac{1}{1} + \frac{1}{3/2} + \frac{1}{3} + \frac{1}{15/2} + \frac{1}{45/2} + \frac{1}{157.5/2} + \cdots.$$

$$\frac{1}{2} + \frac{e^2}{2} = h(1) = 1 + \frac{1}{1} + \frac{1}{1} + \frac{1}{3/2} + \frac{1}{3} + \frac{1}{15/2} + \frac{1}{45/2} + \frac{2}{315/2} + \cdots.$$

$$\frac{1}{2} + \frac{e^2}{2} = h(1) = 1 + \frac{1}{1} + \frac{1}{1} + \frac{1}{3/2} + \frac{1}{3} + \frac{1}{15/2} + \frac{1}{45/2} + \frac{2}{315/2} + \cdots.$$

Euler at X =2

$$e^2 = 1 + \frac{2}{1!} + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!} + \frac{32}{5!} + \frac{64}{6!} + \frac{128}{7!} + \cdots.$$

$$e^2 = 1 + 2 + 2 + \frac{2}{3/2} + \frac{2}{3} + \frac{2}{15/2} + \frac{2}{45/2} + \frac{4}{315/2} + \cdots.$$

Divide by 2

$$\frac{e^2}{2} = \frac{1}{2} + 1 + 1 + \frac{1}{3/2} + \frac{1}{3} + \frac{1}{15/2} + \frac{1}{45/2} + \frac{2}{315/2} + \cdots.$$

$$\frac{e^2}{2} = \frac{1}{2} + 1 + 1 + \frac{1}{\frac{3}{2}} + \frac{1}{3} + \frac{1}{\frac{15}{2}} + \frac{1}{\frac{45}{2}} + \frac{2}{\frac{315}{2}} + \cdots. = h(1) - \frac{1}{2}$$

$$e^2 = 2 * h(1) - 1$$

For Zeta function

$$\sum_{s=0}^{n-1} \frac{(xr)^s}{s\,!} = \frac{\Gamma(n, xr)}{\Gamma(n)} e^{xr}$$

If we can show that h (0.5) equal to zero at X=0, then

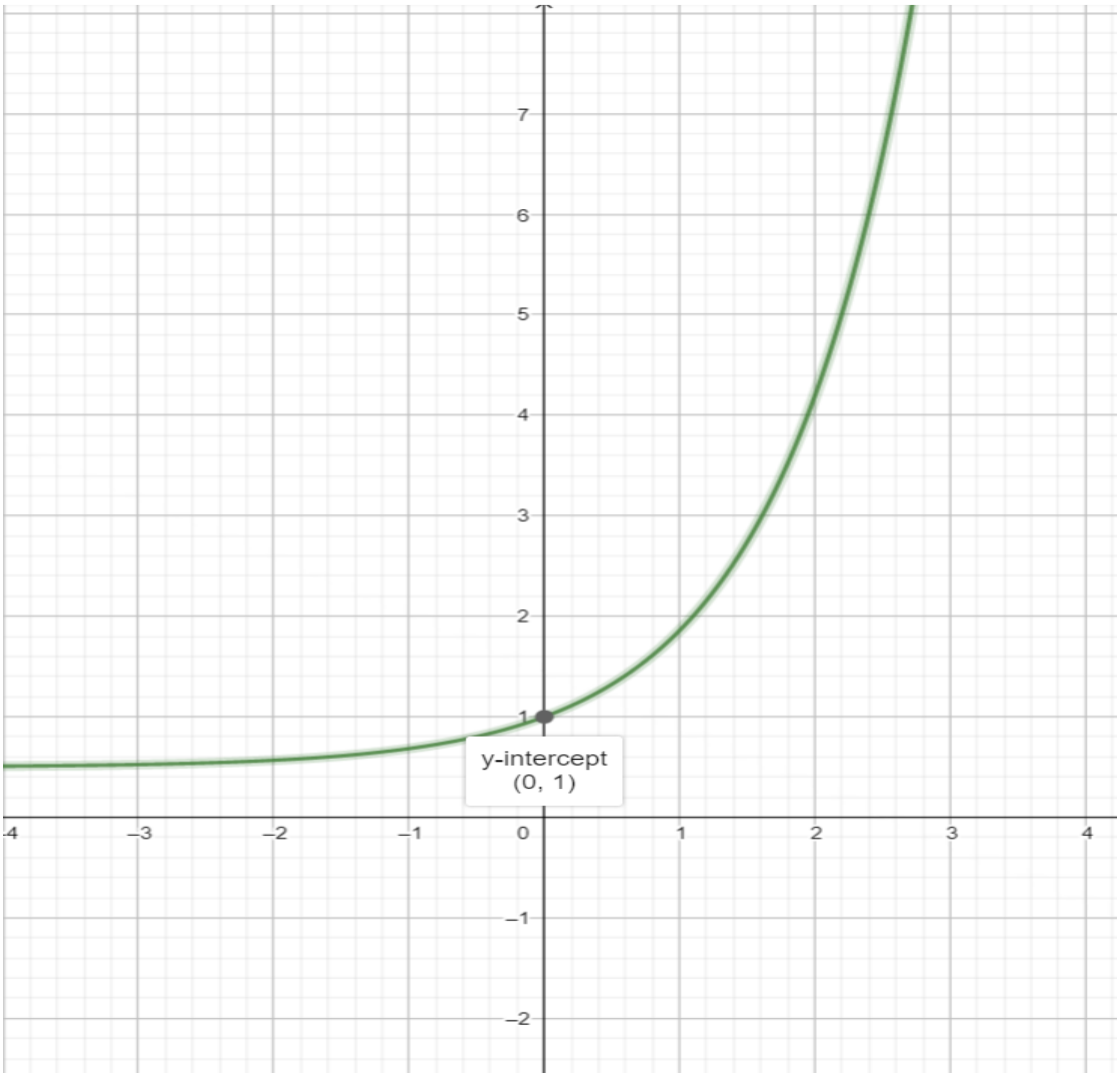
e^{xr} *will have zero at intersection with Y axis and then will have* $Z(S)$ *with has zero at stipe line.*

A) Mapping Factorial X into Half Factorial function h(x).

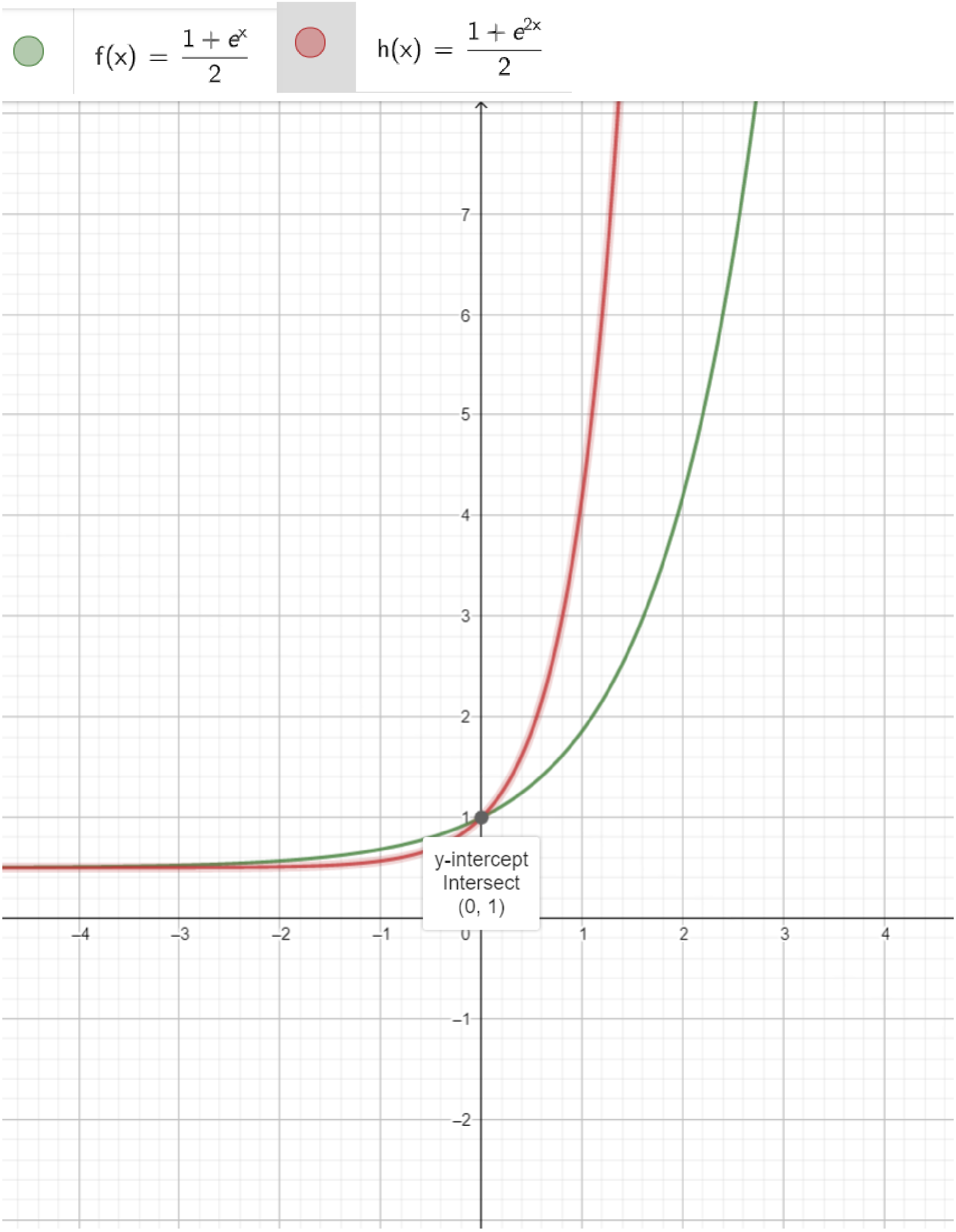
This mapping uses X = 2X; instead of X = X/2, but with half factorial not full factorial numbers.
For odd Numbers mapping: for X =3; Then X = 6 in Euler instead of X = 1.5.

In Figure 1., original half exponent formula.

$$f(x) = \frac{1+e^x}{2}$$



In Figure 2., h(x) power series exponential formula using half factorial.



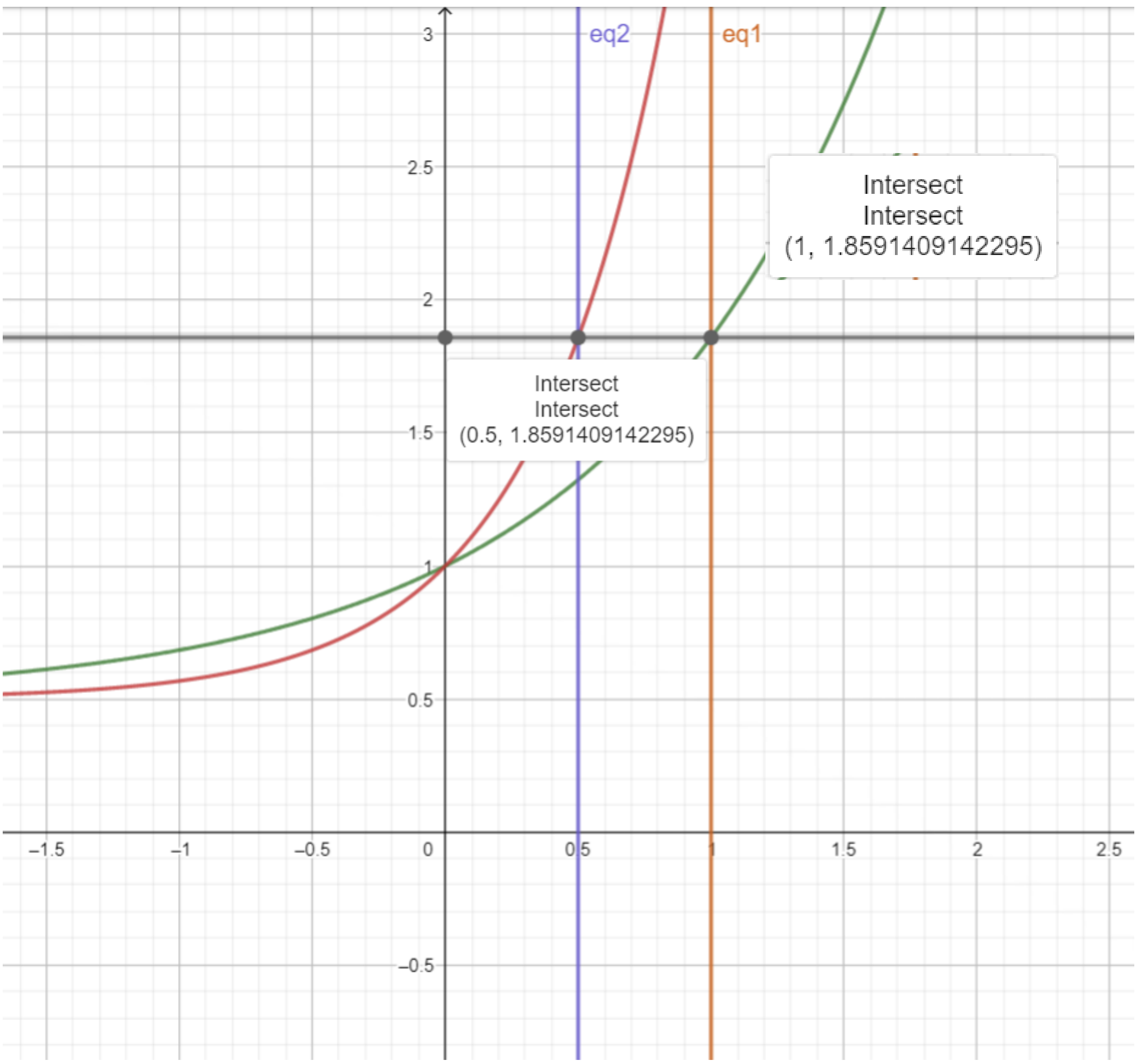
1- Using half factorial makes exponential function formula converges faster.

Example (1): f(x) at X = 1 equal h(x) at X = 0.5

$$at\ Y = \frac{1+e}{2}\ then\ f(1) = h(0.5)$$

In Figure 3., h(x) power series exponential formula using half factorial at X = 0.5 equal f(x) at X = 1, this graph shows how f(1) = h(0.5); where h(x) uses half factorial and not full factorial so it starts from 0.5.

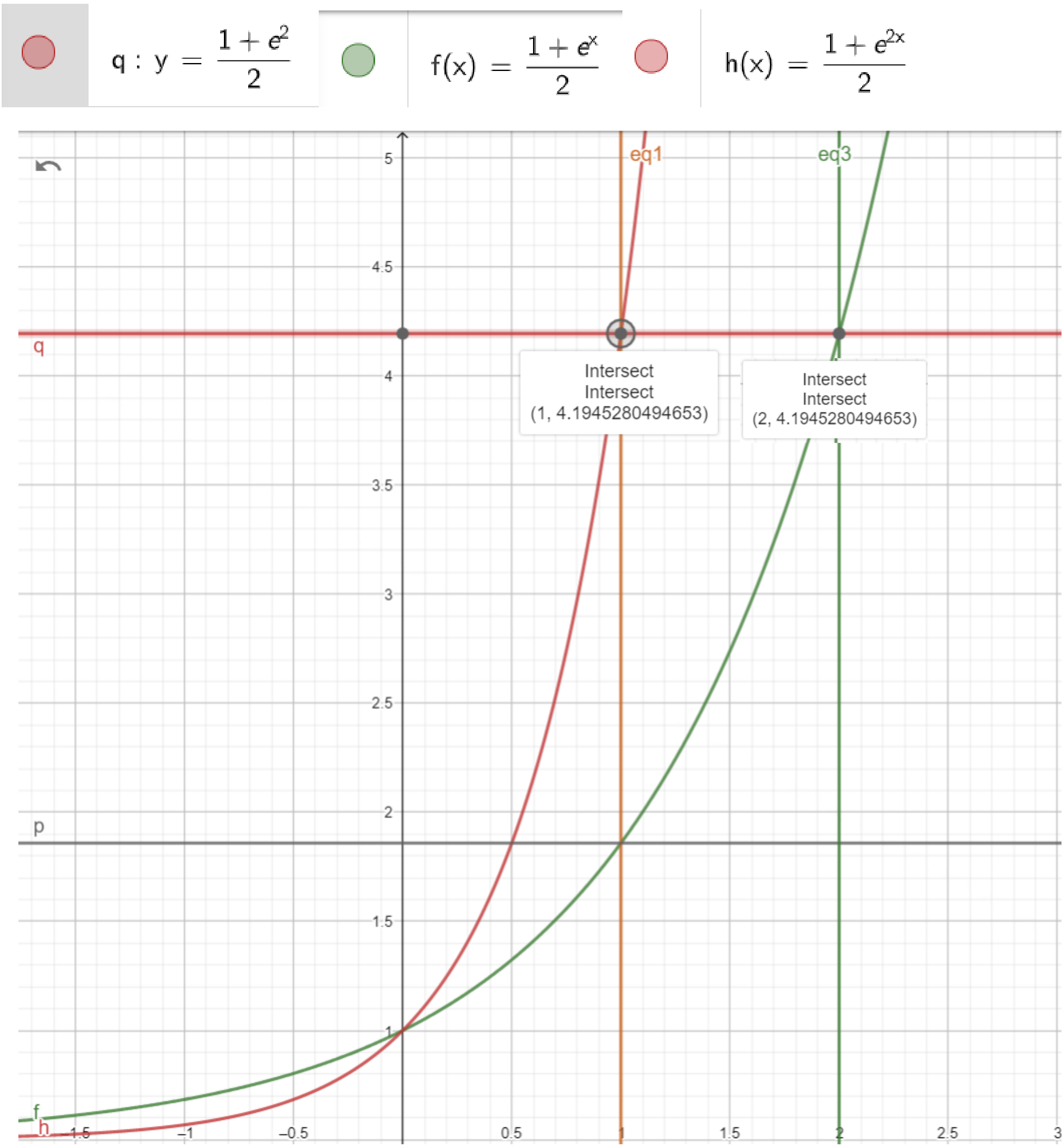
$$h(x) = \frac{1+e^{2x}}{2} \qquad f(x) = \frac{1+e^{x}}{2}$$



Example (2): f(x) at X =2 equal h(x) at X = 1

$$at\ Y = \frac{1+e^2}{2}\ then\ f(2) = h(1); So, h(x)\ converges\ faster$$

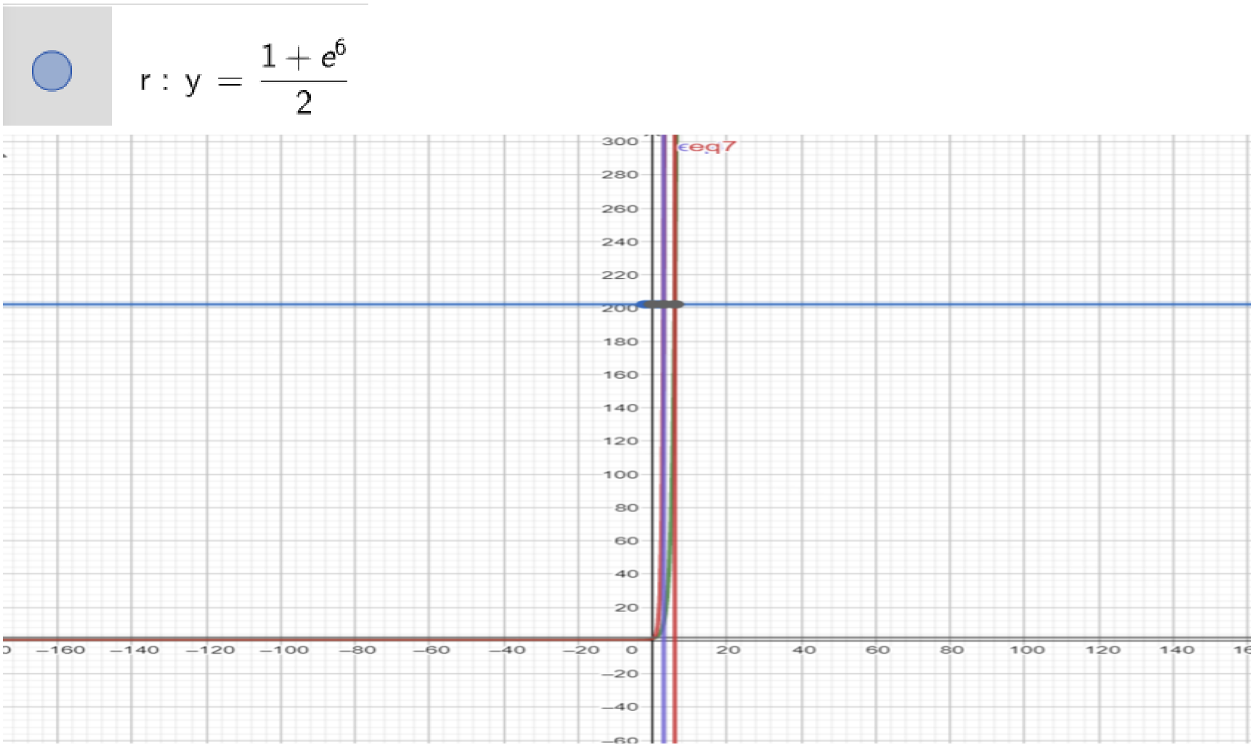
In Figure 4., h(x) power series exponential formula using half factorial at X = 1 and at X=2 for f(x) are equal.

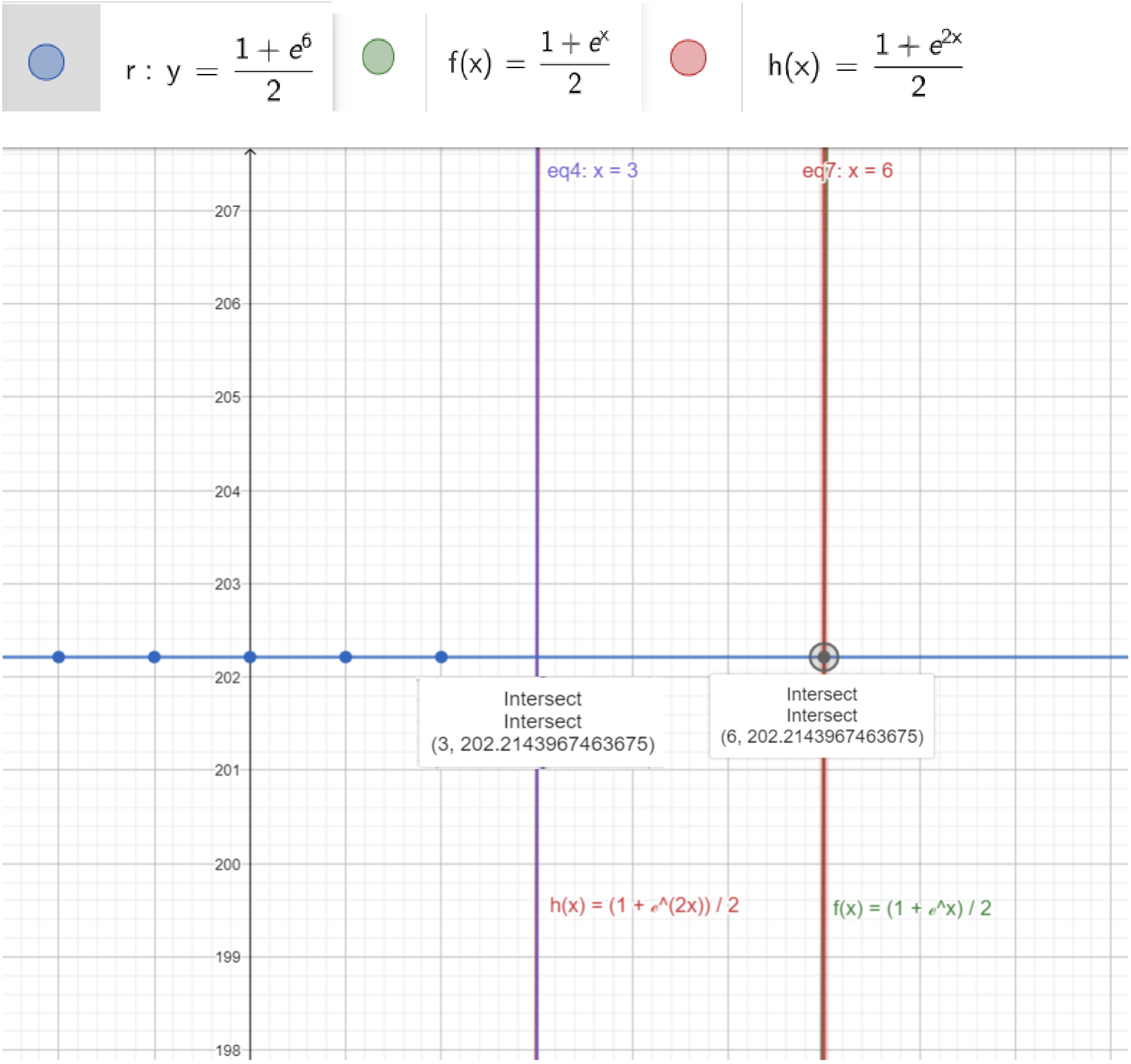


Example (3): f(x) at X =6 equal h(x) at X = 3

$$at\ Y = \frac{1+e^{6}}{2}\ then\ f(6) = h(3); h(x) Converges\ faster$$

In Figure 5., h(x) power series exponential formula using half factorial at X = 3 and at X=6 for f(x) are equal.



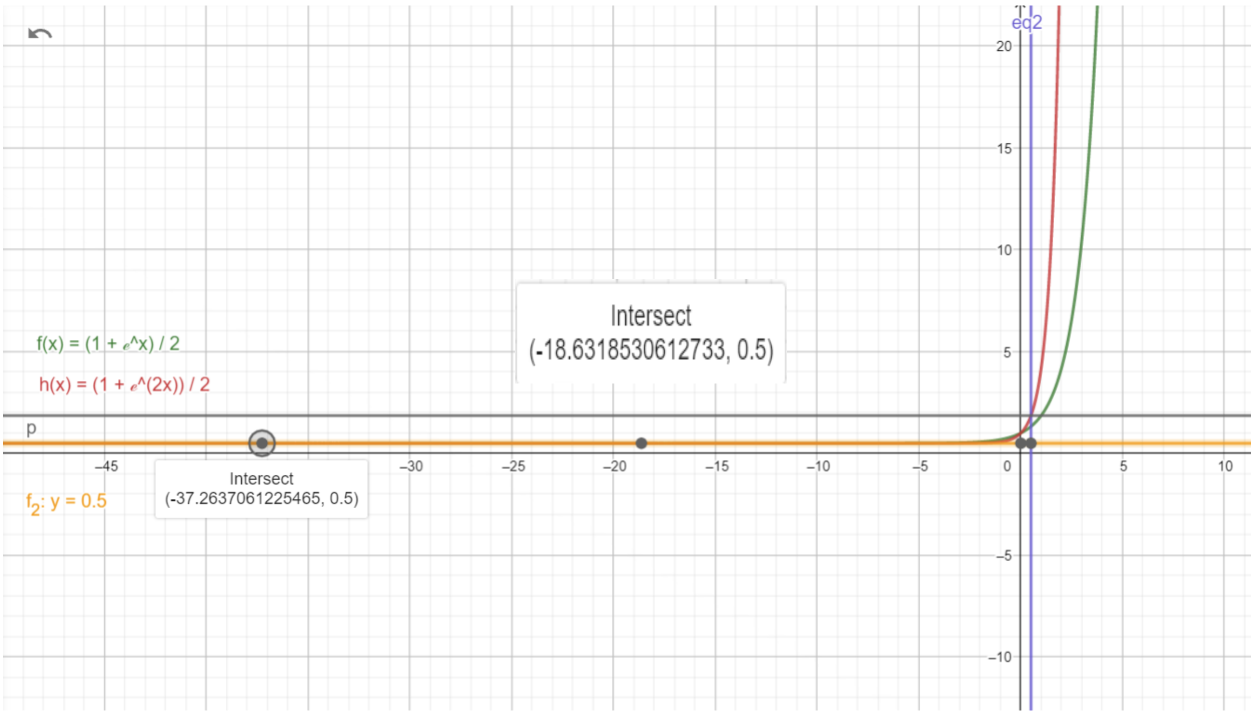

r : y = (1 + e^6) / 2
f(x) = (1 + e^x) / 2
h(x) = (1 + e^(2x)) / 2
eq4: x = 3
eq7: x = 6
207
206
205
204
203
202
201
200
199
198
Intersect
Intersect
(3, 202.2143967463675)
Intersect
Intersect
(6, 202.2143967463675)
h(x) = (1 + e^(2x)) / 2
f(x) = (1 + e^x) / 2

Example (4): H(x) Converges to 0.5 two times faster than f(x)

Y=0.5 intersects with H(x) at X and intersects with f(x) at 2X.

Y = 0.5 at X = 18 for H(X) but at X = 37 for f(X).

In Figure 6., h(x) = f(x) = 0.5; h(x) power series exponential formula using half factorial converges faster than f(x). h(x) converges at around X= 18 while f(x) converges at around x =37.



B) For each N = {0.5 ,1, 2, 3, 4, 5……}

$$There\ is\ function\ H1(X) = \frac{h(N)}{f(X)}\ that\ intersects\ once\ with\ Y\ axis\ at\ X = 0\ , Y`.$$

Examples (5): at N = 1 then h(N) will have zero at interstation with g1(X) (i.e., at Y =1) so at X = 2.

G1(X) = 1 is the ratio between f (1) and h (0.5)

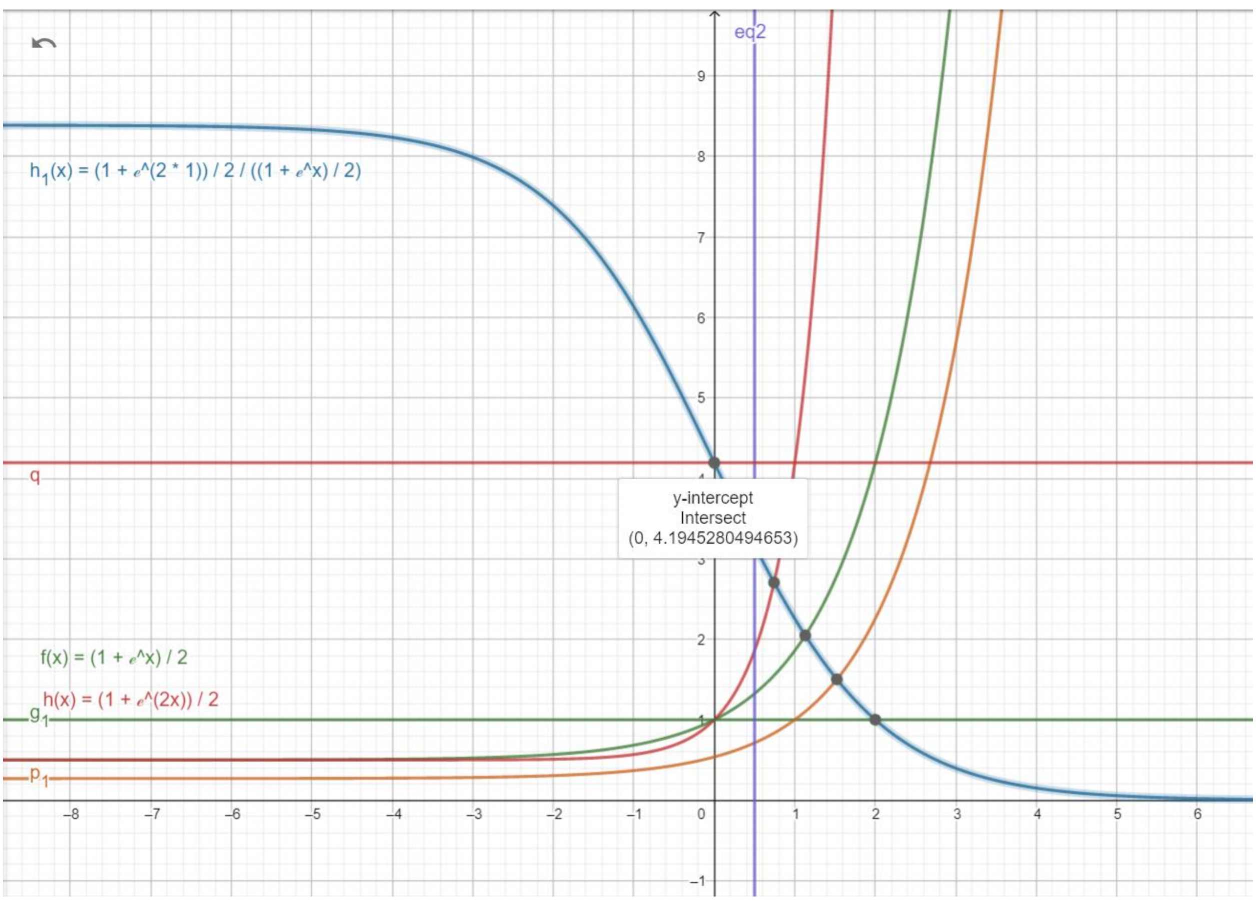
In Figure 7., graphical proof that f (1) = h (0.5) as the ration between both = 1; also showing how exponential formula intersects with Y-axis at X=0 means having imaginary solution only, and h (1) intersects with g1(x) at X=2 (i.e., zero at X=2).

$$q : y = \frac{1+e^2}{2}$$

$$h_1(x) = \frac{h(1)}{f(x)} \rightarrow \frac{\frac{1+e^{2\cdot 1}}{2}}{\frac{1+e^x}{2}}$$

$$g_1(x) = \frac{f(1)}{h(0.5)} \rightarrow \frac{\frac{1+e}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$

$$p_1(x) = \frac{f(x)}{h(0.5)} \rightarrow \frac{\frac{1+e^x}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$



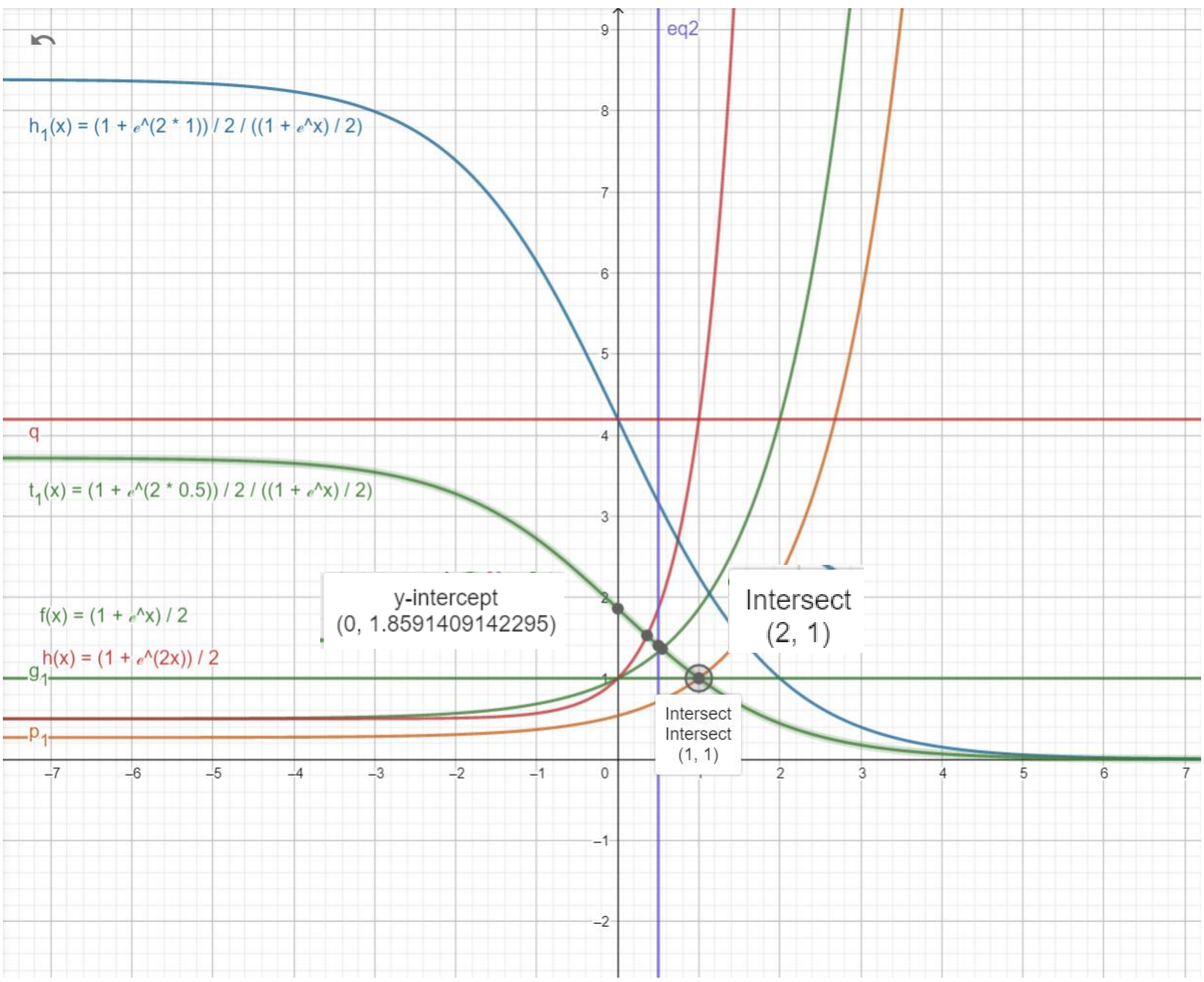
Example (7): Zero H (0.5)

$$There\ is\ function\ T1(X) = \frac{h(N)}{f(X)}\ that\ intersects\ once\ with\ Y\ axis\ at\ X = 0\ , Y^{`}\ .$$

In Figure 8., graphical proof showing how h (0.5) exponential formula intersects with Y-axis at X=0 means having imaginary solution only and intersects with g1(X) at X =1 (i.e., zero at X =1 for h (0.5))

$$g_1(x) = \frac{f(1)}{h(0.5)} \rightarrow \frac{\frac{1+e}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$

$$p_1(x) = \frac{f(x)}{h(0.5)} \rightarrow \frac{\frac{1+e^{x}}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$



Example (8): based on Euler formula Zeros starts at 1

$F(1) = H(0.5)\ as\ g1(x) = 1\ ;and\ e^x - 1 =$
$0\ ;then\ Q1(x)\ and\ R1(x)\ and\ S1(x)\ and\ P1(x)\ thier\ intersection\ point\ with\ G1(x)\ is\ the\ Zeros$

$have\ Zeros\ intersection\ with\ G1(X)\ at\ N\ for\ each\ function\ K(X) = \dfrac{f(X)}{h(N)}$

In Figure 9., graphical proof showing how h (0.5), h(1), h(2), h(3) ratios to f(x) exponential formula intersects with Y-axis at X=0 (means having imaginary solution only); and intersects with g1(X) at X ={1, 2 , 4 ,6} (i.e., zero at X ={1,2,4,6 } for h(0.5) ,h(1) ,h(2) ,h(3))

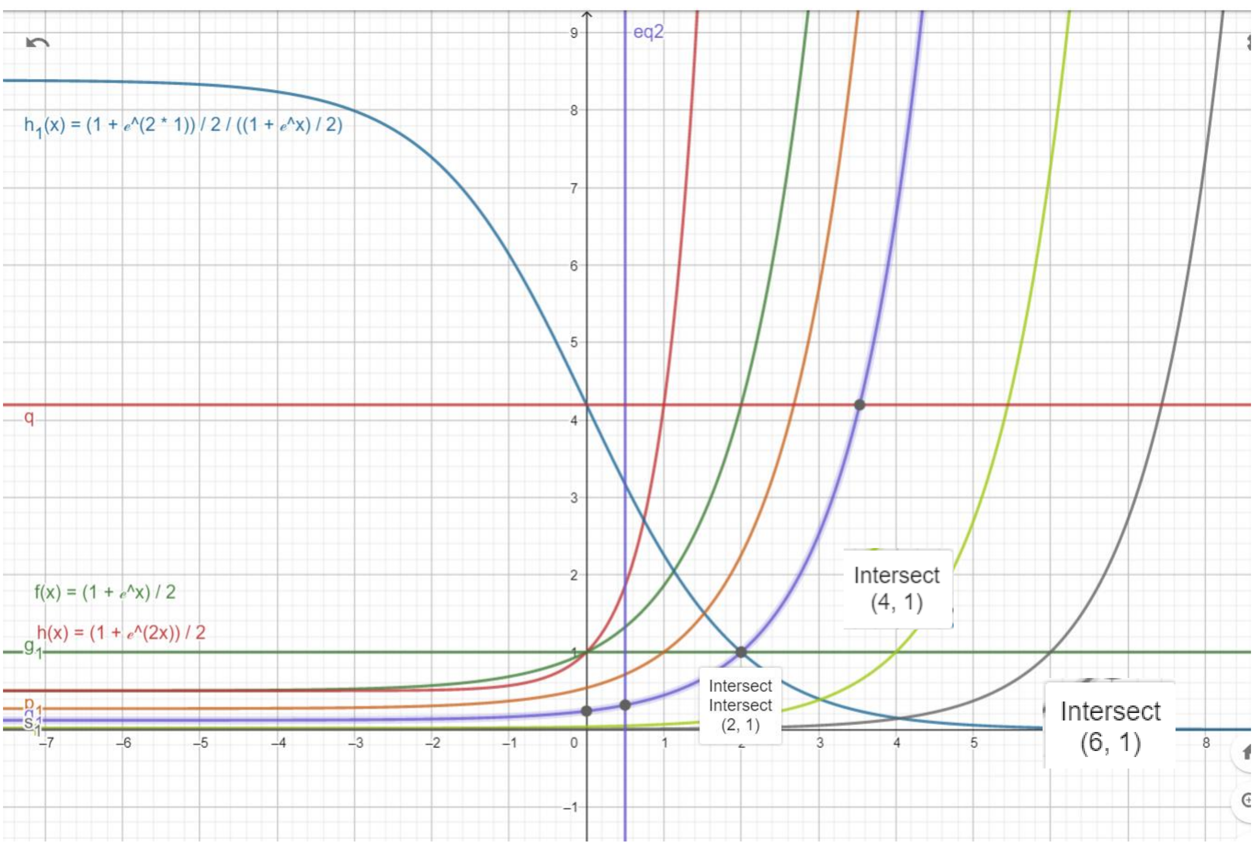
$$g_1(x) = \frac{f(1)}{h(0.5)} \rightarrow \frac{\frac{1+e}{2}}{\frac{1+e^{2 \cdot 0.5}}{2}}$$

$$q_1(x) = \frac{f(x)}{h(1)} \rightarrow \frac{\frac{1+e^x}{2}}{\frac{1+e^{2 \cdot 1}}{2}}$$

$$r_1(x) = \frac{f(x)}{h(2)} \rightarrow \frac{\frac{1+e^x}{2}}{\frac{1+e^{2 \cdot 2}}{2}}$$

$$s_1(x) = \frac{f(x)}{h(3)} \rightarrow \frac{\frac{1+e^x}{2}}{\frac{1+e^{2 \cdot 3}}{2}}$$

$$p_1(x) = \frac{f(x)}{h(0.5)} \rightarrow \frac{\frac{1+e^x}{2}}{\frac{1+e^{2 \cdot 0.5}}{2}}$$



In Figure 9., graphical proof showing how h (0.5), h(1), h(2), h(3) ratios to f(x) and its reciprocal ratio for exponential formula intersects with Y-axis at X=0 (means having imaginary solution only); and intersects with g1(X) at X ={1, 2 , 4 ,6} (i.e., zero at X ={1,2,4,6 } for h(0.5) ,h(1) ,h(2) ,h(3))

$$g_1(x) = \frac{f(1)}{h(0.5)} \rightarrow \frac{\frac{1+e}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$

$$p_1(x) = \frac{f(x)}{h(0.5)} \rightarrow \frac{\frac{1+e^{x}}{2}}{\frac{1+e^{2\cdot 0.5}}{2}}$$

$$p_2(x) = \frac{h(0.5)}{f(x)} \rightarrow \frac{\frac{1+e^{2\cdot 0.5}}{2}}{\frac{1+e^{x}}{2}}$$

$$q_1(x) = \frac{f(x)}{h(1)} \rightarrow \frac{\frac{1+e^{x}}{2}}{\frac{1+e^{2\cdot 1}}{2}}$$

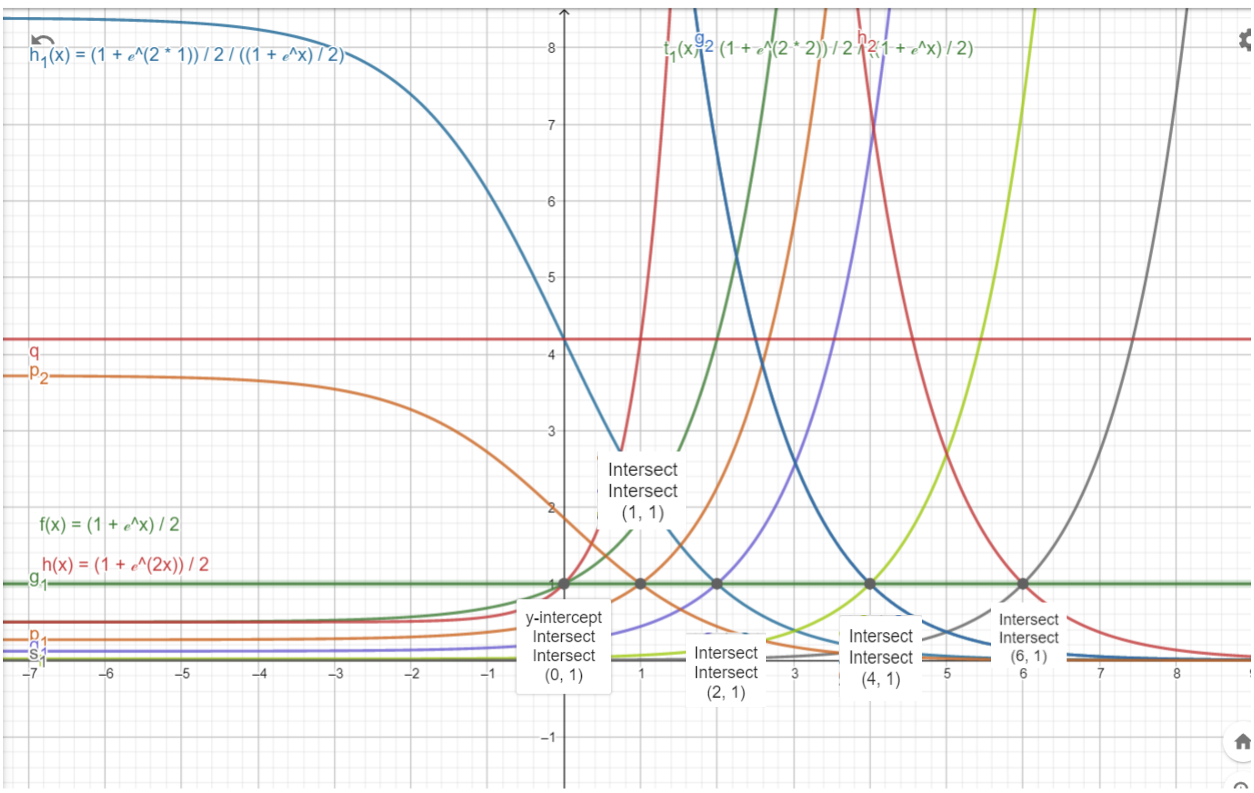
$$r_1(x) = \frac{f(x)}{h(2)} \rightarrow \frac{\frac{1+e^{x}}{2}}{\frac{1+e^{2\cdot 2}}{2}}$$

$$s_1(x) = \frac{f(x)}{h(3)} \rightarrow \frac{\frac{1+e^{x}}{2}}{\frac{1+e^{2\cdot 3}}{2}}$$

$$h_1(x) = \frac{h(1)}{f(x)} \rightarrow \frac{\frac{1+e^{2\cdot 1}}{2}}{\frac{1+e^{x}}{2}}$$

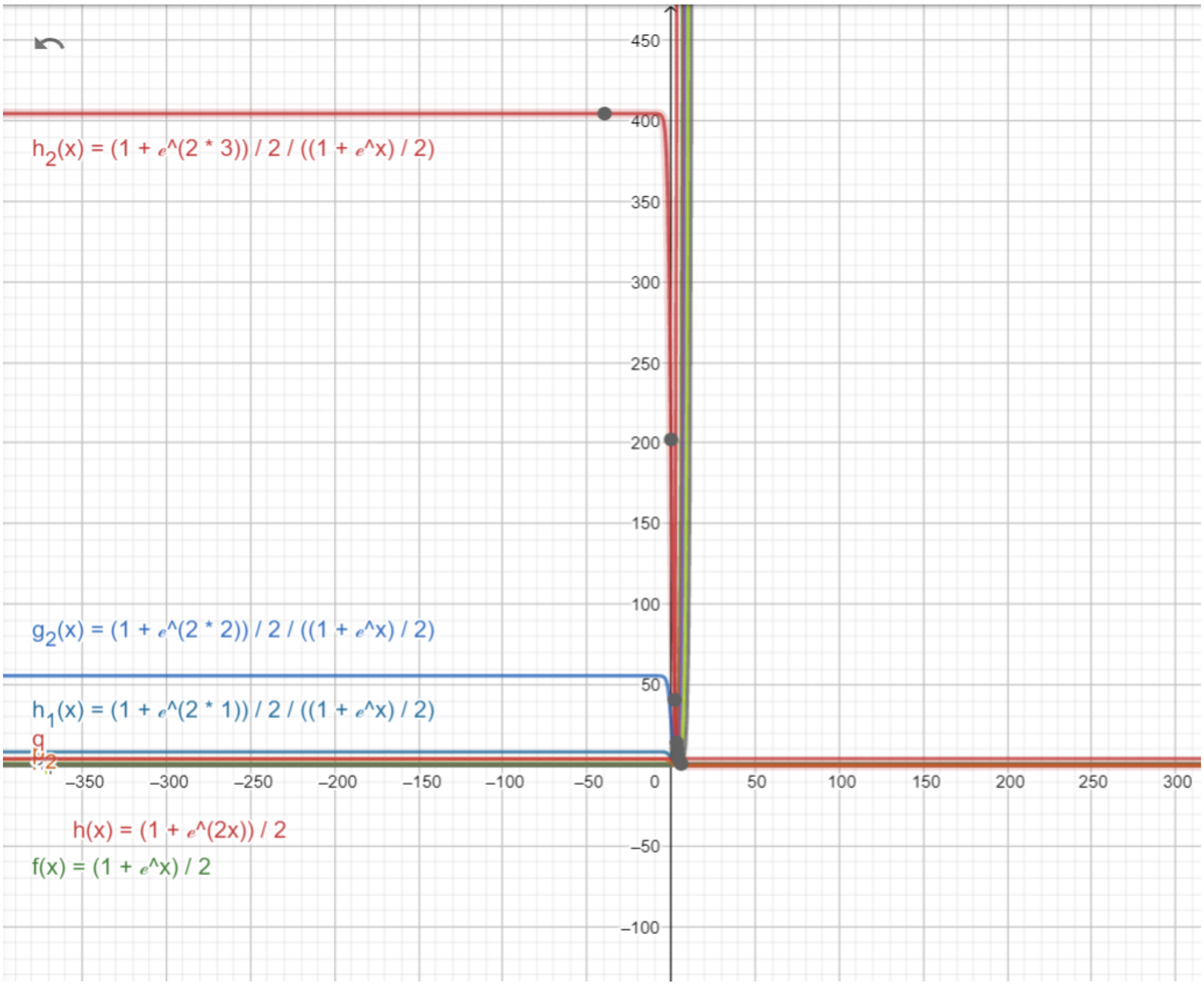
$$g_2(x) = \frac{h(2)}{f(x)} \rightarrow \frac{\frac{1+e^{2\cdot 2}}{2}}{\frac{1+e^{x}}{2}}$$

$$h_2(x) = \frac{h(3)}{f(x)} \rightarrow \frac{\frac{1+e^{2\cdot 3}}{2}}{\frac{1+e^{x}}{2}}$$



In Figure 10., graphical proof showing how h(x) using half factorial have imaginary solutions only at Y-intersection at X =0 and Y = 1 at X is even number. As shown at Figure 9.

As all exponential formula > 0 and as it shows in the graph it looks like step function for h(X) using half factorial. h (2) has Y-intersect around Y =27 and h (3) have Y-intersects around Y = 200.



3. Results

Conclusion: -

1- Half factorial formula makes power series exponential formula converges faster than using full factorial in exponential formula for power series.

2- h(X) Exponential formula using half factorial starts at 0.5 which is an exact equal to f(X) but at X =1 for f(X). [i.e., h (0.5) = f (1)]

3- Euler condition for unit Circle still valid by using the ratio between f(X) and h(X); as we showed that the ratio between [f (1) and h (0.5) = 1]. Which we represented it as g1(X) in our graphs.

4- g(X) is our Zero line as it is the unit Circle for this graph representation. Therefore, any function that intersects with g1(X) (i.e., Y =1) will be Zero. As we showed before.

5- As we show that h (0.5) has Zero at X =0 (i.e., when h (0.5) intersects with g1(x) at point (1,1). Therefore, replacing exponential formula in Zeta function with exponential formula that uses half factorial will have Zero at h (0.5). which gives us the none-trivial zeros at X = 0.5 for h(x)

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