The Complex plane frame of reference is an Even function.

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$$\ln(i) = \frac{\pi}{2} * i$$
 and $\frac{1}{i} = -i$ and $-\frac{1}{i} = i \rightarrow (A)$

X	$\ln\left(i * X - \frac{X}{i}\right)$	$= \ln(2 * i * X)$	$= \ln(2 * X) + \ln(i)$	$= \ln(2) + \ln(i * X)$
0	1	1	1	-∞
1	$\ln\left(i-\frac{1}{i}\right)$	$= \ln(2*i)$	$= \ln(2) + \ln(i)$	$= \ln(2) + \ln(i)$
2	$\ln\left(2*i-\frac{2}{i}\right)$	$= \ln(4*i)$	$= \ln(4) + \ln(i)$	$= \ln(2) + \ln(i * 2)$
3	$\ln\left(3*\ i-\frac{3}{i}\right)$	$= \ln(6*i)$	$= \ln(6) + \ln(i)$	$= \ln(2) + \ln(i*3)$
4	$\ln\left(4*i-\frac{4}{i}\right)$	$= \ln(8*i)$	$= \ln(8) + \ln(i)$	$= \ln(2) + \ln(i * 4)$
5	$\ln\left(5*\ i-\frac{5}{i}\right)$	$= \ln(10*i)$	$= \ln(10) + \ln(i)$	$= \ln(2) + \ln(i * 5)$
6	$\ln\left(6*i-\frac{6}{i}\right)$	$= \ln(12*i)$	$= \ln(10) + \ln(i)$	$= \ln(2) + \ln(i * 5)$
•••				
•••				

AT X = -X

-X	$\ln\left(i*-1*X-\frac{-1*X}{i}\right) = \ln\left(\frac{X}{i}+\frac{X}{i}\right)$	$= \ln\left(\frac{2*X}{i}\right)$	$= \ln(2 * X) - \ln(i)$	$= \ln(2) + \ln(X) - \ln(i)$
-1	$\ln\left(\frac{1}{i} + \frac{1}{i}\right)$	$= \ln\left(\frac{2}{i}\right)$	$= \ln(2) - \ln(i)$	$= \ln(2) + \ln(1) - \ln(i)$
-2	$\ln\left(\frac{2}{i} + \frac{2}{i}\right)$	$= \ln\left(\frac{4}{i}\right)$	$= \ln(4) - \ln(i)$	$= \ln(2) + \ln(2) - \ln(i)$
-3	$\ln\left(\frac{3}{i} + \frac{3}{i}\right)$	$= \ln\left(\frac{6}{i}\right)$	$= \ln(6) - \ln(i)$	$= \ln(2) + \ln(3) - \ln(i)$
-4	$\ln\left(\frac{4}{i} + \frac{4}{i}\right)$	$= \ln\left(\frac{8}{i}\right)$	$= \ln(8) - \ln(i)$	$= \ln(2) + \ln(4) - \ln(i)$
-5	$\ln\left(\frac{5}{i} + \frac{5}{i}\right)$	$= \ln\left(\frac{10}{i}\right)$	$= \ln(10) - \ln(i)$	$= \ln(2) + \ln(5) - \ln(i)$
-6	$\ln\left(\frac{6}{i} + \frac{6}{i}\right)$	$= \ln\left(\frac{12}{i}\right)$	$= \ln(12) - \ln(i)$	$= \ln(2) + \ln(6) - \ln(i)$
-7	$\ln\left(\frac{7}{i} + \frac{7}{i}\right)$	$= \ln\left(\frac{14}{i}\right)$	$= \ln(14) - \ln(i)$	$= \ln(2) + \ln(7) - \ln(i)$

Therefore.
$$\ln(2 * X) = \ln\left(i * -1 * X - \frac{-1 * X}{i}\right) + \ln(i) \implies (1)$$

$$\ln(2 * X) = \ln\left(i * X - \frac{X}{i}\right) - \ln(i) \implies (2)$$

$$\ln(2 * X) = \ln\left(i * -1 * X - \frac{-1 * X}{i}\right) + \frac{\pi}{2} * i \implies (1)$$

$$\ln(2 * X) = \ln\left(i * X - \frac{X}{i}\right) - \frac{\pi}{2} * i \implies (2)$$

Taking the exponent with base = [e]

Х	$X = \frac{1}{2} * \left(\frac{X}{i} - i * X\right) * e^{\frac{\pi}{2}*i}$	$\frac{1}{2} * \left(i * X - \frac{X}{i}\right) * e^{-\frac{\pi}{2}*i}$
-4	$-4 = \frac{1}{2} * \left(-\frac{4}{i} + 4 * i \right) * e^{-\frac{\pi}{2} * i}$	$-4 = \frac{1}{2} * \left(-4 * i + \frac{4}{i}\right) * e^{-\frac{\pi}{2} * i}$
-3	$-3 = \frac{1}{2} * \left(-\frac{3}{i} + 3 * i\right) * e^{-\frac{\pi}{2} * i}$	$-3 = \frac{1}{2} * \left(-3 * i + \frac{3}{i}\right) * e^{-\frac{\pi}{2} * i}$
-2	$-2 = \frac{1}{2} * \left(\frac{-2}{i} + 2 * i\right) * e^{\frac{\pi}{2} * i}$	$-2 = \frac{1}{2} * \left(-2 * i + \frac{2}{i}\right) * e^{-\frac{\pi}{2} * i}$
-1	$-1 = \frac{1}{2} * \left(\frac{-1}{i} + i\right) * e^{\frac{\pi}{2} * i}$	$-1 = \frac{1}{2} * \left(-i + \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i}$
0	0	0
1	$1 = \frac{1}{2} * \left(\frac{1}{i} - i\right) * e^{\frac{\pi}{2} * i}$	$1 = \frac{1}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i}$
2	$2 = \frac{1}{2} * \left(\frac{2}{i} - 2 * i\right) * e^{\frac{\pi}{2} * i}$	$2 = \frac{1}{2} * \left(2 * i - \frac{2}{i}\right) * e^{-\frac{\pi}{2} * i}$
3	$3 = \frac{1}{2} * \left(\frac{3}{i} - 3 * i\right) * e^{\frac{\pi}{2} * i}$	$3 = \frac{1}{2} * \left(3 * i - \frac{3}{i} \right) * e^{-\frac{\pi}{2} * i}$
4	$4 = \frac{1}{2} * \left(\frac{4}{i} - 4 * i\right) * e^{\frac{\pi}{2} * i}$	$4 = \frac{1}{2} * \left(4 * i - \frac{4}{i} \right) * e^{-\frac{\pi}{2} * i}$

5	$5 = \frac{1}{2} * \left(\frac{5}{i} - 5 * i\right) * e^{\frac{\pi}{2} * i}$	$5 = \frac{1}{2} * \left(5 * i - \frac{5}{i}\right) * e^{-\frac{\pi}{2} * i}$
6	$6 = \frac{1}{2} * \left(\frac{6}{i} - 6 * i\right) * e^{\frac{\pi}{2} * i}$	$6 = \frac{1}{2} * \left(6 * i - \frac{6}{i} \right) * e^{-\frac{\pi}{2} * i}$

$$X = \frac{1}{2} * \left(\frac{X}{i} - i * X\right) * e^{\frac{\pi}{2} * i} = \frac{1}{2} * e^{\frac{\pi}{2} * i + \ln\left(\frac{X}{i} - i * X\right)} \rightarrow (B)$$

$$X = \frac{1}{2} * \left(i * X - \frac{X}{i}\right) * e^{-\frac{\pi}{2} * i} = \frac{1}{2} * e^{\ln\left(i * X - \frac{X}{i}\right) - \frac{\pi}{2} * i}$$

Therefore

$$f(X) = X = \frac{1}{2} * \left(\frac{X}{i} - i * X\right) * e^{\frac{\pi}{2} * i} = \frac{1}{2} * e^{\frac{\pi}{2} * i + \ln\left(\frac{X}{i} - i * X\right)} \rightarrow (B)$$

$$f(X) = X = \frac{1}{2} * \left(i * X - \frac{X}{i}\right) * e^{-\frac{\pi}{2} * i} = \frac{1}{2} * e^{\ln\left(i * X - \frac{X}{i}\right) - \frac{\pi}{2} * i}$$

And if

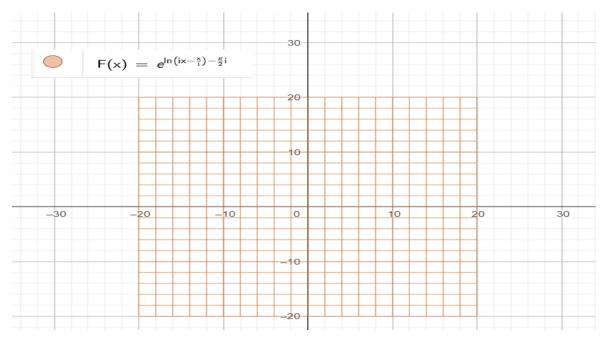
f(X) = X this means for each X we will have Zero value for Y

$$f(2*X) = 2*X = \left(\frac{X}{i} - i*X\right) * e^{\frac{\pi}{2}*i} = e^{\frac{\pi}{2}*i + \ln\left(\frac{X}{i} - i*X\right)} \rightarrow (D)$$

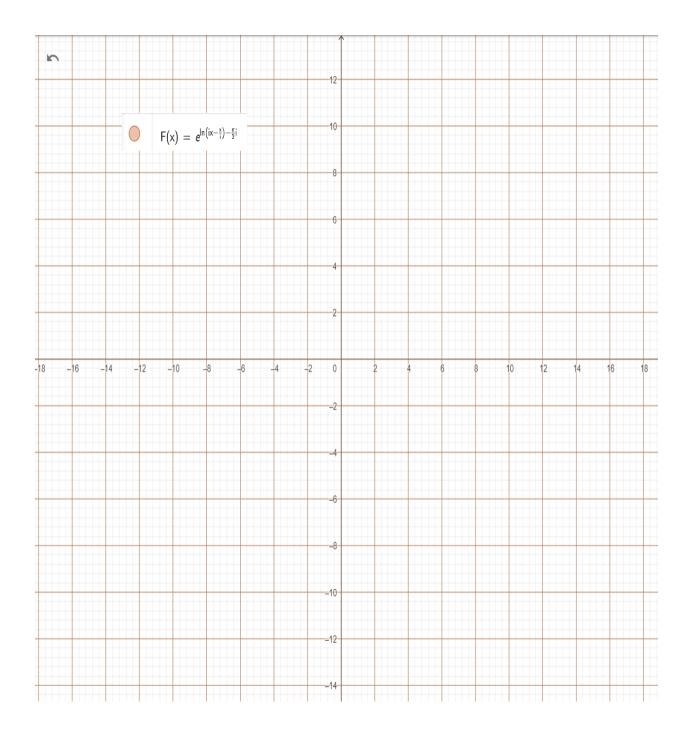
$$f(2*X) = 2*X = \left(i*X - \frac{X}{i}\right)*e^{-\frac{\pi}{2}*i} = e^{\ln\left(i*X - \frac{X}{i}\right) - \frac{\pi}{2}*i}$$

This is function is an even function and all its Zeros are Even numbers for any natural number X.

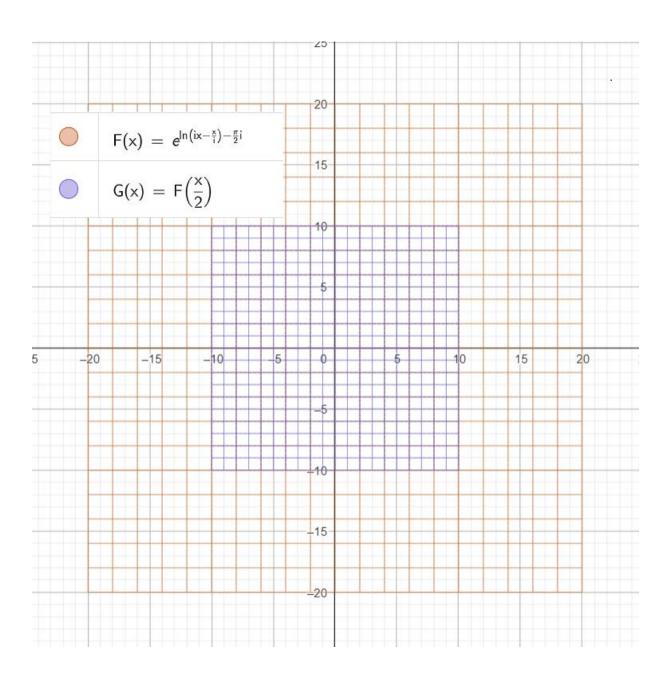
And it is 2 * complex plane frame of reference.

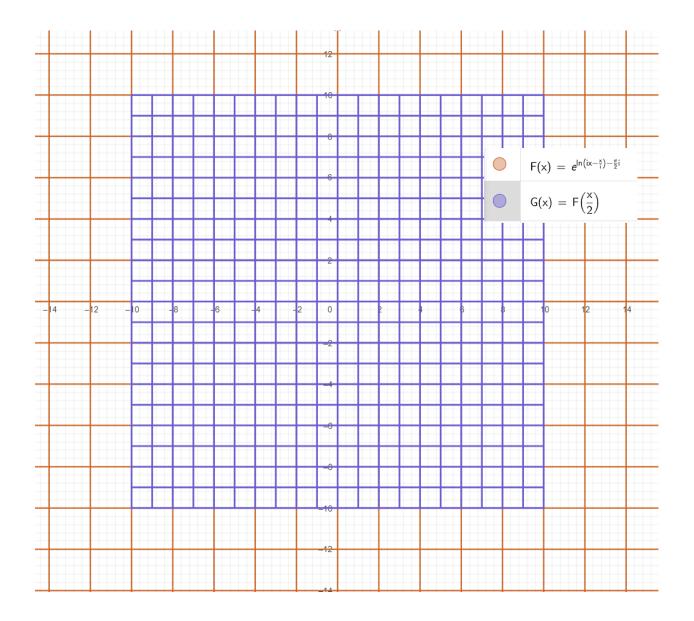


Even Function all its Zero are even numbers only.



In F(X) let X = X/2 we get the exact complex plane frame of reference, and we get Odd Zeros from an Even function



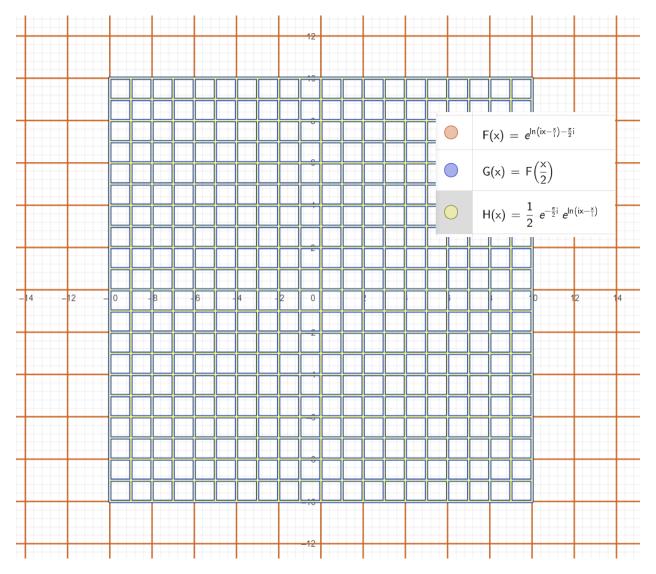


$$f\left(2*\frac{X}{2}\right) = 2*\frac{X}{2} = e^{\frac{\pi}{2}*i + \ln\left(\frac{X}{2*i} - i*\frac{X}{2}\right)} = \frac{1}{2}*e^{\frac{\pi}{2}*i + \ln\left(\frac{X}{i} - i*X\right)} \rightarrow (B)$$

$$f(X) = X = e^{\frac{\pi}{2}*i + \ln\left(\frac{X}{2*i} - i*\frac{X}{2}\right)} = \frac{1}{2}*e^{\frac{\pi}{2}*i + \ln\left(\frac{X}{i} - i*X\right)} \rightarrow (B)$$

$$f(X) = X = \frac{1}{2}*e^{\ln\left(i*X - \frac{X}{i}\right) - \frac{\pi}{2}*i} \rightarrow (C)$$

And this what the analytical continuity did by multiplying by $\frac{1}{2}$; that converted the complex plane even function into and odd function (i * X) and showed all odd numbers zeros as well!



As these even function F(X) is the frame of reference for the complex plane which = (i * X)

Then in Euler's Identity we can write it with F(X) instead of use (i * X)

Therefore!

Taking exponent with base [e] for both sides

Therefore, we can re write Euler's Identity as

$$e^{i*X} = e^{\frac{1}{2}*} e^{\ln\left(i*X - \frac{X}{i}\right) - \frac{\pi}{2}*i}$$

$$e^{i*X} = e^{e^{\ln(i*\frac{X}{2} - \frac{X}{2*i}) - \frac{\pi}{2}*i}}$$

In this Equation G

$$e^{i*X} = e^{e^{\ln(i*\frac{X}{2} - \frac{X}{2*i}) - \frac{\pi}{2}*i}}$$

Let X = 2 * X we did not change any thing in the numbers nature the even number will remain even, and the odd numbers will remain odd.

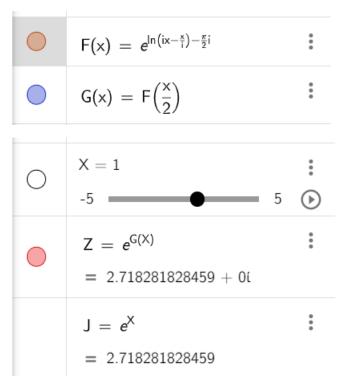
$$e^{i*2*X} = e^{e^{\ln\left(i*X - \frac{X}{i}\right) - \frac{\pi}{2}*i}}$$

If we consider eft hand side one function of X called E(X) and the right-hand side another function called G(X)

Then we can say that the relation between these two functions is

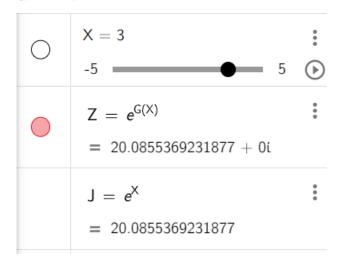
$$E(2 * X) = G(X)$$
 and $E(X) = G\left(\frac{X}{2}\right)$

And this is the calculation for some values of the functional relation between these equation and Euler's Identity



$$F(x) = e^{\ln(ix - \frac{x}{i}) - \frac{\pi}{2}i}$$

$$G(x) = F(\frac{x}{2})$$



$$X = \ln\left(\frac{1}{2}\right)$$

$$= -0.6931471805599$$

$$Z = e^{G(X)}$$

$$= 0.5 - 0i$$

$$J = e^{X}$$

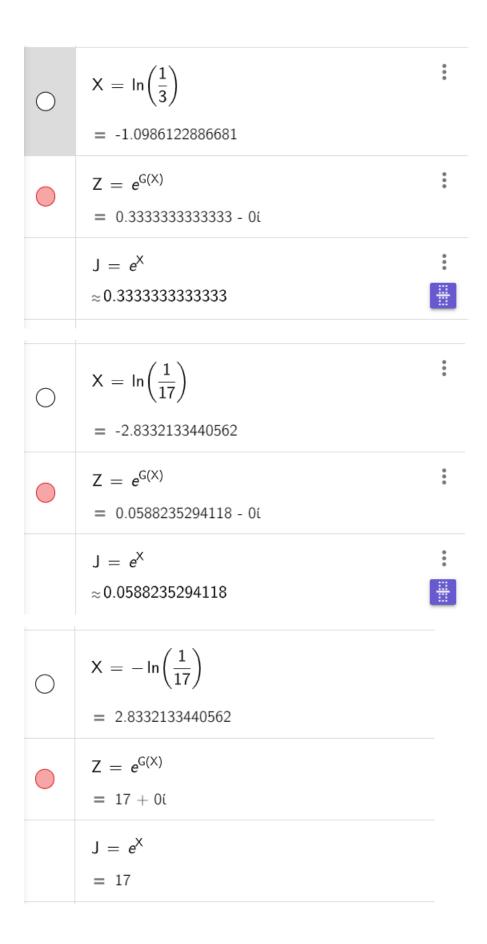
$$\approx 0.5$$

$$X = -\ln\left(\frac{1}{2}\right)$$
= 0.6931471805599
$$Z = e^{G(X)}$$
= 2 + 0i
$$J = e^{X}$$
= 2

$$X = -\ln\left(\frac{1}{3}\right)$$
= 1.0986122886681

$$Z = e^{G(X)}$$
= 3 + 0i

$$J = e^{X}$$
= 3



Therefore, this new functional representation for Euler's Identity will reach Exact natural number at each value of

at
$$X = \ln\left(\frac{1}{N}\right)$$
 then $e^{i*X} = e^{e^{\ln\left(i*\frac{X}{2} - \frac{X}{2*i}\right) - \frac{\pi}{2}*i}} = \frac{1}{N}$ \rightarrow (H)

at
$$X = -\ln\left(\frac{1}{N}\right)$$
 then $e^{i*X} = e^{e^{\ln\left(i*\frac{X}{2} - \frac{X}{2*i}\right) - \frac{\pi}{2}*i}} = N \rightarrow (H)$

$$X = -\ln\left(\frac{1}{\pi}\right)$$
= 1.1447298858494

$$Z = e^{G(X)}$$
= 3.1415926535898 + 0i

$$J = e^{X}$$
= 3.1415926535898

$$Z = e^{G(X)}$$
= 23.1406926327793 + 06

$$J = e^{X}$$
= 23.1406926327793

$$T = e^{iX}$$

$$= -1 + 0i$$

$$z_{38} = e^{\pi}$$
= 23.1406926327793 + 06

$$F(x) = e^{\ln(ix - \frac{x}{i}) - \frac{\pi}{2}i}$$

$$G(x) = F\left(\frac{x}{2}\right)$$

$$X = \frac{\pi}{i}$$
= -3.1415926535898i

$$Z = e^{G(X)}$$

$$= -1 - 0i$$

$$\int J = e^{X}$$

$$= (-1, 0)$$

$$T = e^{iX}$$
= 23.1406926327793 + 0i

$$\begin{array}{ccc}
z_{38} = e^{\frac{\pi}{1}} \\
= -1 - 0i
\end{array}$$

Therefore, this form of Euler's Identity works on Cycle.

=
$$X = \frac{\pi}{i}$$
 while Euler's Identity works on Cycle of $X = \pi$

Therefore, this form of Euler's Identity works on Cycle.

= $X = \frac{\pi}{i}$ while Euler's Identity works on Cycle of $X = \pi$

 $X = \frac{\pi}{2 i}$ = -1.5707963267949í $Z = e^{G(X)}$ = 0 - ί $J = e^{X}$ = (0, -1) $T = e^{iX}$ = 4.8104773809654 + 0i

 $\mathsf{z}_{38} = e^{\frac{\pi}{2\mathsf{i}}}$ = 0 - ί

Therefore, this form of Euler's Identity works on Cycle.

= $X = \frac{\pi}{i}$ while Euler's Identity works on Cycle of $X = \pi$

 $X = \frac{\pi}{2}$ = 1.5707963267949 + 0i $Z = e^{G(X)}$ = 4.8104773809654 + 0í

 $J = e^{X}$ = (4.8104773809654, 0)

 $T = e^{iX}$ = 0 + i

 $z_{38}=e^{\frac{\pi}{2}}$ = 4.8104773809654 + 0i

$$X = -\frac{\pi}{2 i}$$
= 1.5707963267949i

$$Z = e^{G(X)}$$
= 0 + i

$$J = e^{X}$$
= (0, 1)

$$T = e^{iX}$$
= 0.2078795763508 + 0i

$$z_{38} = e^{\frac{-\pi}{2i}}$$
= 0 + i

And this observation says that with our even functional new representation for Euler's function, we will reach Zeros only when X is complex number Cycle of PI $X=\frac{\pi}{i}$ or $X=-\pi*i$

$$e^{e^{\ln\left(i*X*\frac{\pi*i}{2}-X*\frac{\pi*i}{2*i}\right)-\frac{\pi}{2}*i}}=-1$$

$$e^{e^{\ln\left(i*X*\pi*i-X*\frac{\pi*i}{i}\right)-\frac{\pi}{2}*i}}=1$$

$$z_{39} = e^{\ln\left(i\frac{\pi}{2}i - \frac{\pi}{2}i\right) - \frac{\pi}{2}i}$$

$$= 0 + 3.1415926535898i$$

$$z_{40} = e^{\ln\left(i\frac{-\pi}{2}i - \frac{-\pi}{2}i\right) - \frac{\pi}{2}i}$$

$$= 0 - 3.1415926535898i$$

$$z_{41} = e^{\ln\left(i\pi i - \frac{\pi i}{i}\right) - \frac{\pi}{2}i}$$

$$= 0 + 6.2831853071796i$$

$$z_{42} = e^{\ln\left(-i\pi i - \frac{-\pi i}{i}\right) - \frac{\pi}{2}i}$$

$$= 0 - 6.2831853071796i$$

$$z_{41} = e^{\ln(i\pi i - \frac{\pi i}{i}) - \frac{\pi}{2}i}$$

$$= 0 + 6.2831853071796i$$

$$z_{42} = e^{\ln(-i\pi i - \frac{\pi i}{i}) - \frac{\pi}{2}i}$$

$$z_{42} = e^{m(-m - \frac{1}{4})^{-2}}$$

$$= 0 - 6.2831853071796i$$

$$z_{43} = e^{\left(e^{\ln\left(-i\pi i - \frac{\pi i}{i}\right) - \frac{\pi}{2}i}\right)}$$

$$= 1 + 0i$$

$$z_{44} = e^{\left(e^{\ln\left(\ln i - \frac{\pi i}{i}\right) - \frac{\pi}{2}i\right)}$$
= 1 - 0i

$$z_{46} = e^{\ln\left(i\frac{\pi}{2}i - \frac{\pi}{2}i\right) - \frac{\pi}{2}i}$$

$$= 0 + 3.1415926535898i$$

$$z_{47} = e^{\ln\left(i\frac{-\pi}{2}i - \frac{-\pi}{2}i\right) - \frac{\pi}{2}i}$$

$$= 0 - 3.1415926535898i$$

$$z_{48} = e^{\left(e^{\ln\left(-i\cdot 3\cdot \frac{\pi}{2}i - \frac{-3\pi i}{2i}\right) - \frac{\pi}{2}i}\right)}$$
$$= -1 + 0i$$

$$z_{49} = e^{\left(e^{\ln\left(-i.5 \cdot \frac{\pi}{2}i - \frac{-5\pi i}{2i}\right) - \frac{\pi}{2}i}\right)}$$

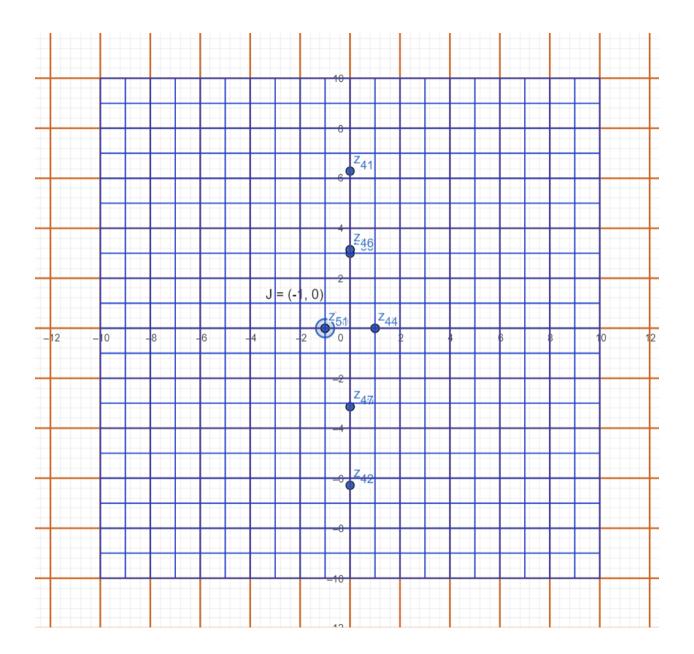
$$= -1 - 0i$$

$$z_{50} = e^{\left(e^{\ln\left(-i\cdot7\cdot\frac{\pi}{2}i-\frac{-7\pi i}{2i}\right)-\frac{\pi}{2}i}\right)}$$

$$= -1 - 0i$$

$$z_{51} = e^{\left(e^{\ln\left(-i\cdot 9\cdot \frac{\pi}{2}i - \frac{-9\pi i}{2i}\right) - \frac{\pi}{2}i}\right)}$$

$$= -1 - 0i$$



And these two forms sync together AT.

$$X = N * \left(-\ln\left(\frac{1}{i}\right) * \frac{1}{\pi} - \frac{1}{2}\right)$$

$$X = N * \left(\ln\left(\frac{1}{i}\right) * \frac{1}{\pi} + \frac{1}{2}\right)$$

$$X = N * \left(\ln(i) * \frac{1}{\pi} - \frac{1}{2}\right)$$

$$X = N * \left(-\ln(i) * \frac{1}{\pi} + \frac{1}{2}\right)$$

$$X = -\ln\left(\frac{1}{i}\right) \frac{1}{\pi} - \frac{1}{2}$$

$$= -0.5 + 0.5i$$

$$Z = e^{G(X)}$$

$$= 0.5322807302157 + 0.2907862882127i$$

$$J = e^{X}$$

$$= (0.5322807302157, 0.2907862882127)$$

$$T = e^{iX}$$

$$X = -\ln(i) \frac{1}{\pi} + \frac{1}{2}$$

$$= 0.5 - 0.5i$$

$$Z = e^{G(X)}$$

$$= 1.4468890365842 - 0.7904390832136i$$

$$J = e^{X}$$

$$= (1.4468890365842, -0.7904390832136)$$

= 1.4468890365842 + 0.7904390832136i

 $T = e^{iX}$

$$X = 3 \left(\ln \left(\frac{1}{i} \right) \frac{1}{\pi} + \frac{1}{2} \right)$$
$$= 1.5 - 1.5i$$

$$Z = e^{G(X)}$$
= 0.3170221435804 - 4.47046237918046

$$X = 3 \left(-\ln\left(\frac{1}{i}\right) \frac{1}{\pi} - \frac{1}{2} \right)$$
$$= -1.5 + 1.5i$$

$$Z = e^{G(X)}$$
= 0.0157836031366 + 0.2225712161082i

$$T = e^{iX}$$
= 0.0157836031366 - 0.22257121610826

Going back to Euler's Equation if we replace X with our functional form.

At $X = \pi$

$$e^{i\pi} = e^{i*\frac{\pi}{2}*\left(i-\frac{1}{i}\right)*e^{-\frac{\pi}{2}*i}} = Cos\left(\frac{\pi}{2}*\left(i-\frac{1}{i}\right)*e^{-\frac{\pi}{2}*i}\right) + i*Sin\left(\frac{\pi}{2}*\left(i-\frac{1}{i}\right)*e^{-\frac{\pi}{2}*i}\right) = -1 \implies (*)$$

At each X = multiplier of 60 degrees Cos term will Equal ± 0.5

$$X = \frac{N * \pi}{3}$$

$$e^{i * \frac{1}{2} * \left(i * \frac{N * \pi}{3} - \frac{N * \pi}{3 * i}\right) * e^{-\frac{\pi}{2} * i}} = Cos\left(\frac{N * \pi}{6} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i}\right) + i * Sin\left(\frac{N * \pi}{6} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i}\right) \rightarrow (*)$$

For any positive natural number N that is multiplier of 3 the Cos term will equal -1 and Sin imaginary Term = 0

For each even value Cos term = 1 and imaginary term = 0 (sin = 0)

S = 8 -5 30

 $z_{60} = e^{i\frac{\pi}{6}(i\cdot 3S - \frac{3S}{i})e^{-\frac{\pi}{2}i}}$ = 1 - 0i

 $z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 1 - 0*i*

 $z_{61} = \cos(\pi \cdot 3 \text{ S}) + i \sin(\pi \cdot 3 \text{ S})$ = 1 + 0i

 $z_{59} = \cos\left(\frac{\pi}{6} \left(i \cdot 3 S - \frac{3 S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ = 1 + 0i

 $z_{58} = \sin\left(\frac{\pi}{6} \left(i \cdot 3S - \frac{3S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ = 0 + 0i

For each odd number Cos term = -1 and Sin term = 0 (Sin= 0)

S = 23 -5 30

 $z_{60} = e^{i\frac{\pi}{6}(i\cdot 3S - \frac{3S}{i})e^{-\frac{\pi}{2}i}}$ = -1 + 0i

 $z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = -1 + 0i

 $z_{61} = \cos(\pi \cdot 3 S) + i \sin(\pi \cdot 3 S)$ = -1 + 0i

 $z_{59} = \cos\left(\frac{\pi}{6} \left(i \cdot 3 S - \frac{3 S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ = -1 - 0i

 $z_{58} = \sin\left(\frac{\pi}{6} \left(i \cdot 3 S - \frac{3 S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ = 0 - 0i

Odd numbers Cos term (real part of the complex number) will = 0.5 and if we divide each S by 3 Only if S is multiplier of 3 we get value = -1 (i.e. not primes) = -1 otherwise = 0.5 For each odd number S it reach pi at $\pi/3$ only when S = S/2 to get the for pi/6.

0	S = 17 -5 30
	$z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})}e^{-\frac{\pi}{2}i}$ = -1 - 0i
	$z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ $= 0.5 + 0i$
	$z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ $= -0.8660254037844 + 0i$
	$z_{61} = \cos(\pi S) + i \sin(\pi S)$ = -1 + 0i
0	$z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{\frac{-\pi}{2}i}}$ = 0.5 - 0.8660254037844i

Even numbers Cos term (real part of the complex number) will = 0.5 and if we divide each S by 3 Only if S is multiplier of 3 we get value = -1 (i.e. not primes) = -1 otherwise = 0.5 And because it is even number S then our pi value will be $\frac{\pi}{3}=60^\circ$

0	S = 8 -5
	$z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 1 - 0i
	$z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ $= -0.5 - 0i$
	$z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ $= 0.8660254037844 - 0i$
0	$z_{61} = \cos(\pi S) + i \sin(\pi S)$ = 1 + 0i
	$z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ $= -0.5 + 0.8660254037844i$

For Cases of S = S/2; complex number imaginary part will be = 0.5 and real part of complex number = cos(60)

S = 0 + 0.5= 0.5

 $z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 0 + 1i

 $z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ = 0.8660254037844 + 0i

 $z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ = 0.5 + 0i

 $z_{61} = \cos(\pi S) + i \sin(\pi S)$ = i

 $z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{\frac{-\pi}{2}i}}$ = 0.8660254037844 + 0.5i

For case S = 5/2 = 2.5

S = 2 + 0.5= 2.5

 $z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 0 + 1i

 $z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ = -0.8660254037844 - 0i

 $z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ = 0.5 - 0i

 $z_{61} = \cos(\pi S) + i \sin(\pi S)$ = i

 $z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = -0.8660254037844 + 0.5i

In case S = 19/2

$$S = 9 + 0.5$$

= 9.5

$$z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$$
= 0 - 1i

$$z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$$

$$= -0.8660254037844 + 0i$$

$$z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$$
$$= -0.5 - 0i$$

$$z_{61} = \cos(\pi S) + i \sin(\pi S)$$

$$= -1i$$

$$z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{-2\pi i}}$$
$$= -0.8660254037844 - 0.5i$$

In Case S is multiplier of 3; Like S = {3,6,9,12,15,}

Imaginary part of the complex number = i and Real part number of the complex number = 0.

S = 1 + 0.5= 1.5

 $z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 0 - 1i

 $z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$ = 0 - 0i

 $z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$ = 1 + 0i

 $z_{61} = \cos(\pi S) + i \sin(\pi S)$ = -1i

 $z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$ = 0 + 1i

$$z_{62} = e^{i\frac{\pi}{2}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$$
= 0 - i

$$z_{59} = \cos\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{-\frac{\pi}{2}i}\right)$$
$$= 0.8660254037844 + 0i$$

$$z_{58} = \sin\left(\frac{\pi}{6} \left(i S - \frac{S}{i}\right) e^{\frac{-\pi}{2}i}\right)$$
$$= -0.5 + 0i$$

$$z_{61} = \cos(\pi S) + i \sin(\pi S)$$

$$= -1i$$

$$z_{60} = e^{i\frac{\pi}{6}(iS - \frac{S}{i})e^{-\frac{\pi}{2}i}}$$
$$= 0.8660254037844 - 0.5i$$

One degree can be represented by this formula.

$$\frac{1}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} = 1$$

$$\left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} = 2$$

$$\frac{1}{3} \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} = \frac{1}{6}$$

$$\frac{\pi}{3} \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} = \frac{\pi}{6}$$

$$\frac{1}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} + \frac{1}{2} = \frac{3}{2}$$

$$\frac{2}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} + \frac{1}{2} = \frac{5}{2}$$

$$\frac{3}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} + \frac{1}{2} = \frac{7}{2}$$

$$\frac{4}{2} * \left(i - \frac{1}{i}\right) * e^{-\frac{\pi}{2} * i} + \frac{1}{2} = \frac{9}{2}$$

DEG1 =
$$\cos\left(1 \cdot \frac{\pi}{180^{\circ}}\right)$$

= 0.5403023058681



$$\mathsf{z}_{63} = \, \mathsf{cos}\!\left(\frac{1}{2} \, \left(-\mathsf{i} + \frac{1}{\mathsf{i}}\right) \, \mathsf{e}^{\frac{\pi}{2}\mathsf{i}}\right)$$

$$z_{57} = \cos\left(\left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= -0.4161468365471 + 0i$$

$$DEG2 = \cos\left(2 \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= -0.4161468365471$$

DEG3 =
$$\cos\left(3 \cdot \frac{\pi}{180^{\circ}}\right)$$

= -0.9899924966004
 $z_{65} = \cos\left(\frac{3}{2}\left(-i + \frac{1}{i}\right)e^{\frac{\pi}{2}i}\right)$

$$z_{65} = \cos\left(\frac{3}{2}\left(-i + \frac{1}{i}\right)e^{\frac{\pi}{2}i}\right)$$
$$= -0.9899924966004 + 0i$$

DEG103 =
$$\cos\left(\frac{1}{3} \cdot \frac{\pi}{180^{\circ}}\right)$$

= 0.9449569463147
$$z_{66} = \cos\left(\frac{1}{6} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$z_{67} = \cos\left(\frac{\pi}{6} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= 0.5 + 0i$$

$$DEG1O3PI = \cos\left(\frac{\pi}{3} \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= 0.5$$

= 0.9449569463147 + 0i

$$z_{68} = \sin\left(\frac{\pi}{6} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= 0.8660254037844 - 0i$$

$$DEG60 = \sin(60^{\circ})$$

$$= 0.8660254037844$$

$$\mathsf{DEG3O2} \,=\, \cos\!\left(1.5 \cdot \frac{\pi}{180^\circ}\right)$$

= 0.0707372016677

$$z_{69} = \cos\left(\frac{1}{2}\left(-i + \frac{1}{i}\right)e^{\frac{\pi}{2}i} + \frac{1}{2}\right)$$

= 0.0707372016677 + 0i

$$X3O2 = \cos\left(\frac{3}{2}\right)$$

= 0.0707372016677

$$z_{69} = \cos\left(\frac{2}{2}\left(-i + \frac{1}{i}\right)e^{\frac{\pi}{2}i} + \frac{1}{2}\right)$$

= -0.8011436155469 + 0i

$$X3O2 \,=\, cos \bigg(\frac{5}{2}\bigg)$$

= -0.8011436155469

$$z_{69} \, = \, \text{cos} \bigg(\frac{3}{2} \, \left(- \text{i} + \frac{1}{\text{i}} \right) \, e^{\frac{\pi}{2} \text{i}} + \frac{1}{2} \bigg)$$

= -0.9364566872908 - 0í

$$X3O2 = \cos\left(\frac{7}{2}\right)$$

= -0.9364566872908

$$z_{69} = \cos\left(\frac{4}{2}\left(-i + \frac{1}{i}\right)e^{\frac{\pi}{2}i} + \frac{1}{2}\right)$$

$$= -0.2107957994308 - 0i$$

$$X3O2 = \cos\left(\frac{9}{2} \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= -0.2107957994308$$

And these cases if we move by cycle of pi

$$z_{69} = \cos\left(\left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + 1\right)$$

$$= -0.9899924966004 + 0i$$

$$X302 = \cos\left(\frac{6}{2} \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= -0.9899924966004$$

$$e = \cos\left(\frac{6}{2}\right)$$

$$= -0.9899924966004$$

$$z_{69} = \cos\left(\left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} - 1\right)$$

$$= 0.5403023058681 + 0i$$

$$X3O2 = \cos\left(\left(\frac{6}{2} - 2\right) \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= 0.5403023058681$$

$$e = \cos\left(\frac{6}{2} - 2\right)$$

$$= 0.5403023058681$$

$$z_{69} = \cos\left(\left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} - \frac{1}{2}\right)$$

$$= 0.0707372016677 + 0i$$

$$X3O2 = \cos\left(\left(\frac{6}{2} - \frac{3}{2}\right) \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= 0.0707372016677$$

$$e = \cos\left(\frac{6}{2} - \frac{3}{2}\right)$$

$$= 0.0707372016677$$

$$z_{69} = \cos\left(\left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= -0.4161468365471 + 0i$$

$$X302 = \cos\left(\left(\frac{6}{2} - 1\right) \cdot \frac{\pi}{180^{\circ}}\right)$$

$$= -0.4161468365471$$

$$e = \cos\left(\frac{6}{2} - 1\right)$$

$$= -0.4161468365471$$

$$z_{69} = \cos\left(\frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= -0.5 + 0i$$

$$X3O2 = \cos\left(\frac{\pi}{3} \left(\frac{6}{2} - 1\right) \frac{\pi}{180^{\circ}}\right)$$

$$= -0.5$$

$$e = \cos\left(\frac{\pi}{3} \left(\frac{6}{2} - 1\right)\right)$$

$$= -0.5$$

$$z_{69} = \cos\left(\frac{\pi}{6} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i}\right)$$

$$= 0.5 + 0i$$

$$X3O2 = \cos\left(\frac{\pi}{6} \left(\frac{6}{2} - 1\right) \frac{\pi}{180^{\circ}}\right)$$

$$= 0.5$$

$$e = \cos\left(\frac{\pi}{6} \left(\frac{6}{2} - 1\right)\right)$$

$$= 0.5$$

$$z_{69} = \cos\left(\frac{\pi}{2} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \frac{1}{2}\right)$$

$$= -0.8775825618904 - 0i$$

$$X302 = \cos\left(\pi \left(\frac{6}{2} - \frac{1}{2\pi}\right) \frac{\pi}{180^{\circ}}\right)$$

$$= -0.8775825618904$$

$$e = \cos\left(\pi \left(\frac{6}{2} - \frac{1}{2\pi}\right)\right)$$

$$= -0.8775825618904$$

$$z_{69} = \cos\left(3 \cdot \frac{\pi}{2} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \frac{1}{2}\right)$$

$$= -0.8775825618904 - 0i$$

$$X3O2 = \cos\left(3 \pi \left(\frac{6}{2} - \frac{1}{6 \pi}\right) \frac{\pi}{180^{\circ}}\right)$$

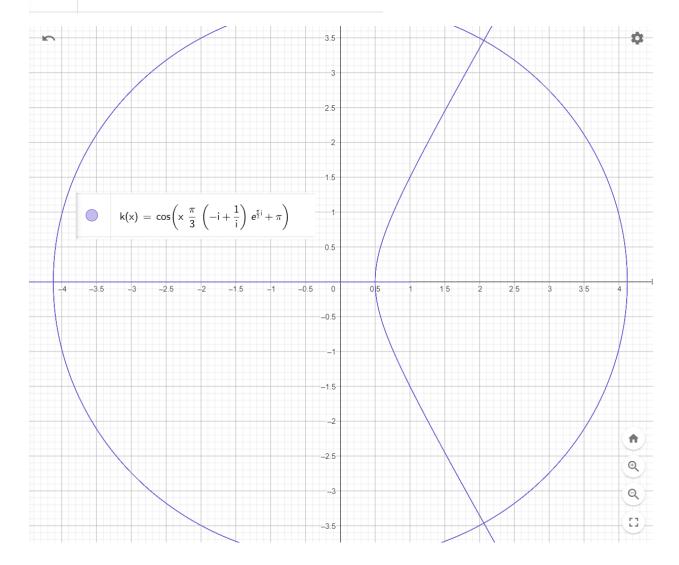
$$= -0.8775825618904$$

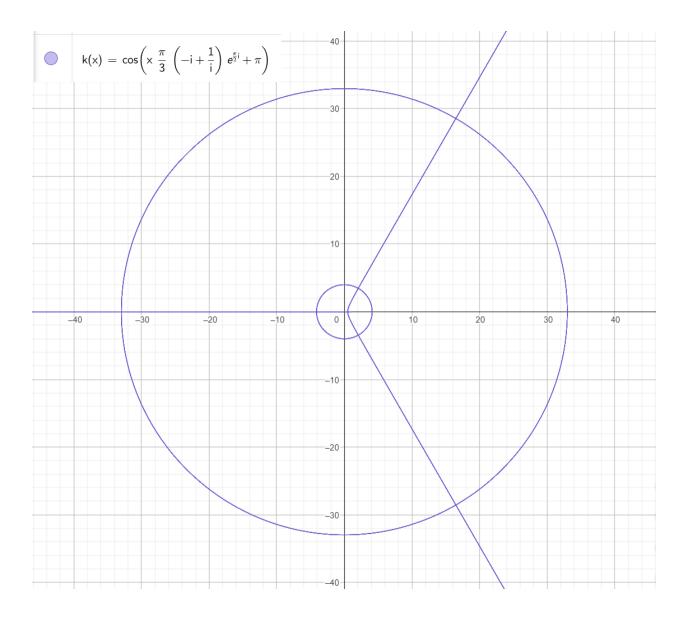
$$e = \cos\left(3 \pi \left(\frac{6}{2} - \frac{1}{2 \cdot 3 \pi}\right)\right)$$

$$= -0.8775825618904$$

$$z_{69} = \cos\left(3 \cdot \frac{\pi}{2} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \frac{3\pi}{2}\right)$$
$$= 0 + 0i$$

$$z_{69} = \cos\left(5 \cdot \frac{\pi}{2} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \frac{5\pi}{2}\right)$$
$$= 0 - 0i$$





$$z_{71} = \cos\left(1 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= 0.5 - 0i$$

$$z_{72} = \cos\left(2 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= 0.5 + 0i$$

$$z_{73} = \cos\left(4 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= 0.5 - 0i$$

$$z_{74} = \cos\left(5 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= 0.5 + 0i$$

$$z_{75} = \cos\left(6 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= -1 + 0i$$

$$z_{76} = \cos\left(7 \cdot \frac{\pi}{3} \left(-i + \frac{1}{i}\right) e^{\frac{\pi}{2}i} + \pi\right)$$

$$= 0.5 - 0i$$

