Fraction Expansion using Step Pyramid Distribution for Prime Numbers and Rhind Mathematical Papyrus (RMP)

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Abstract

In this document, we will present an algorithm for fraction expansion by reconstructing the Rhind Papyrus Fraction table (RMP) using the natural number distribution explained in a Previous paper, Step Pyramid Distribution for Prime numbers[1].

Keywords: Papyrus, Rhind Mathematical Papyrus (RMP), Prime Numbers, Composite Prime Numbers, Prime Number Distribution

1. Introduction

1.1 Introduce the Problem

Rhind Papyrus Fraction Expansion Table was the first attempt known in history to use fraction expansion. But the paper was old and no general methodology to reproduce any fraction Expansion for all-natural Numbers.

In this paper, we will go through logic to reconstruct the full fraction table in Papyrus Fraction table using Step Pyramid Distribution for Prime numbers1. We will extend (RMP) table to include fraction expansion for [1/N] fractions as well.

Table 1. [2/n] table from the Rhind Mathematical Papyrus (RMP)

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330
2/57 = 1/38 + 1/114	2/59 = 1/36 + 1/236 + 1/531	2/61 = 1/40 + 1/244 + 1/488 + 1/610
2/63 = 1/42 + 1/126	2/65 = 1/39 + 1/195	2/67 = 1/40 + 1/335 + 1/536
2/69 = 1/46 + 1/138	2/71 = 1/40 + 1/568 + 1/710	2/73 = 1/60 + 1/219 + 1/292 + 1/365
2/75 = 1/50 + 1/150	2/77 = 1/44 + 1/308	2/79 = 1/60 + 1/237 + 1/316 + 1/790
2/81 = 1/54 + 1/162	2/83 = 1/60 + 1/332 + 1/415 + 1/498	2/85 = 1/51 + 1/255
2/87 = 1/58 + 1/174	2/89 = 1/60 + 1/356 + 1/534 + 1/890	2/91 = 1/70 + 1/130
2/93 = 1/62 + 1/186	2/95 = 1/60 + 1/380 + 1/570	2/97 = 1/56 + 1/679 + 1/776

Table 1., part of [2/n] Fraction Table in Rhind Mathematical Papyrus (RMP)

1.2 Fraction Expansion for numbers in Base Branch[1]

The Base branch will include all Natural numbers that are divisible by 3 and the result is an integer. The difference between numbers in each row is 3. As you see the similarity between the Step Pyramid distribution Base branch and the first column in the Fraction expansion table in (RMP).

Table 2. Base Branch

ID	Natural Number	Pyramid Layer (Number / 3)	Composite Prime
1	3	1	0
2	6	2	1
3	9	3	1
4	12	4	1
5	15	5	1
..	..	..	1
9999	9999	3333	1

Table 2., includes all-natural Numbers such that (N mod 3 = 0).

1- Add the first column from the Fraction expansion table (Table 1.) to the Base Branch table (Table 2.).
2- All green cells in Table2.1. generated from Pyramid distribution using Pyramid Layer Parameter.
3- Any fraction in the left hand side repeated between two Different Fractions means the numbers have a common factor between them.

Table 2.1. Fraction Expansion for numbers in Base Branch

ID	Natural Number	Pyramid Layer L = (Number / 3)	[1/N] Fraction Expansion (1/N) = 1/ [N * (L+1)] + 1/ [3 * (L + 1)]	[2/N] Fraction Expansion (2/N) = 1/ [2*L] + 1/(2N)
1	3	1	1/3 = 1/6 + 1/6	2/3 = 1/2 + 1/6
2	6	2	1/6 = 1/18 + 1/9	2/6 = 1/4 + 1/12
3	9	3	1/9 = 1/36 + 1/12	2/9 = 1/6 + 1/18
4	12	4	1/12 = 1/60 + 1/15	2/12 = 1/8 + 1/24
5	15	5	1/15 = 1/90 +1/18	2/15 = 1/10 + 1/30
6	18	6	1/18 = 1/126+1/21	2/18 = 1/12 + 1/36
7	21	7	1/21 = 1/168 + 1/24	2/21 = 1/14 + 1/42
8	24	8	1/24 = 1/216+1/27	2/24 = 1/16 + 1/48
9	27	9	1/27 = 1/270 + 1/30	2/27 = 1/18 + 1/54
10	30	10	1/30 = 1/330+1/33	2/30= 1/20 + 1/60
11	33	11	1/33=1/396 +1/36	2/33 = 1/22 + 1/66
12	36	12	1/36 = 1/468+1/39	2/36 = 1/24 + 1/72
13	39	13	1/39 = 1/546 + 1/42	2/39 = 1/26 + 1/78

Table 2.1, includes one column for [1/N] and [2/N] Fraction expansion

1.3 Fraction Expansion for numbers in Top Branch[1]

The Top branch is the Top branch in the Pyramid and will include all Natural Numbers that can be generated using Number 5.

Table 3. Top Branch

ID	Natural Number	Pyramid Layer (ID * 2) + 2	Top Branch Formula N = (5 * [ID + 1]) + ID	Base B= (Pyramid Layer * 3)	Composite Prime
0	5	2	(5 * 1) + 0	6	0
1	11	4	(5 * 2) +1	12	0
2	17	6	(5 * 3) + 2	18	0
3	23	8	(5 * 4) + 3	24	0
4	29	10	(5 * 5) + 4	30	0
5	35	12	(5 * 6) + 5	36	1
..	..	..	….		..

In Table 3. Each Natural Number in each row will be (+6) from the natural number in the previous row.

1- Add the first column from the Fraction expansion table (Table 1.) to the Base Branch table (Table 3.)
2- All green cells in Table2.1. generated from Pyramid distribution using Pyramid Layer Parameter.
3- We will use Table 2.1 to get the formula for [1/N] to substitute the base fraction on the [2/N] formula. For example, 1/3 in Table3.1 can be replaced by its fraction expansion from Table 2.1.
4- Substitute fraction from the base branch in Table 2.1 for (1/3, 1/6, 1/9, 1/12,) to get [2/N] fraction from [1/N] fraction formula.
5- In the [2/N] fraction Expansion column (from Rhind Papyrus (RMP)) some of its fraction's denominators are factors for the second fraction denominator in column [1/N].

Table 3.1. Fraction Expansion for numbers in Top Branch

ID	Natural Number	Pyramid Layer L = (ID * 2) + 2	[1/N] Fraction Expansion (1/N) = 1/ [B] + 1/ [N * B]	Base B= (Pyramid Layer * 3)	[2/N] Fraction Expansion
0	5	2	1/5 = 1/6 + 1/30	6	2/5 = 1/3 + 1/15
1	11	4	1/11 = 1/12 + 1/132	12	2/11 = 1/6 + 1/66
2	17	6	1/17 = 1/18 + 1/306	18	2/17 = 1/12 + 1/51 + 1/68
3	23	8	1/23 = 1/24 + 1/552	24	2/23 = 1/12 + 1/276
4	29	10	1/29 = 1/30 + 1/870	30	2/29 = 1/24 + 1/58 + 1/174 + 1/232
5	35	12	1/35 = 1/36 + 1/1260	36	2/35 = 1/30 + 1/42
6	41	14	1/41 = 1/42 + 1/1722	42	2/41 = 1/24 + 1/246 + 1/328

In Table 3.1. in the [1/N] Fraction column every N and First fraction denominator are the factors of the number in the second fraction denominator

For Example: - [1/41 = 1/42 + 1/1722] → 41 and 42 will be the factors of 1722. And in column [2/N]

Fraction Expansion [2/41 = 1/24 + 1/246 + 1/328] → 246 will be one of the factors of 1722.

1.4 Fraction Expansion for numbers in Right Branch[1]

This Branch will include all Natural Numbers that can be generated using Number 7.

Table 4. Right Branch

ID	Natural Number	Pyramid Layer (ID * 2) + 3	Top Branch Formula N = (7 * [ID + 1]) - ID	Base B= (Pyramid Layer * 3)	Composite Prime
0	7	3	(7 * 1) - 0	9	0
1	13	5	(7 * 2) - 1	15	0
2	19	7	(7 * 3) - 2	21	0
3	25	9	(7 * 4) - 3	27	1
4	31	11	(7 * 5) - 4	33	0
5	37	13	(7 * 6) - 5	39	0
..	..	..	….		..

In Table 4. Each Natural Number in each row will be (+6) from the natural number in the previous row.

1- Add the first column from the Fraction expansion table (Table 1.) to the Right Branch table (Table 4.)
2- All green cells in Table2.1. generated from Pyramid distribution using Pyramid Layer Parameter.
3- We will use Table 2.1 to get the formula for [1/N] to substitute the base fraction on the [2/N] formula. For example, 1/3 in Table4.1 can be replaced by its fraction expansion from Table 2.1.
4- Substitute fractions (1/3, 1/6, 1/9, 1/12,) from the base branch, Table 2.1., to get [2/N] fraction from [1/N] fraction formula.
5- In the [2/N] fraction Expansion column (from Rhind Papyrus (RMP)) some of its fraction's denominators are factors for the second fraction denominator in column [1/N].

Table 4.1. Fraction Expansion for numbers in Right Branch

ID	Natural Number	Pyramid Layer (ID * 2) + 3	[1/N] Fraction Expansion (1/N) = 1/ [B-1] + 1/ [N * B]	Base B= (Pyramid Layer * 3)	[2/N] Fraction Expansion
0	7	3	1/7 = 1/8 + 1/56	9	2/7 = 1/4 + 1/28
1	13	5	1/13 = 1/14 + 1/182	15	2/13 = 1/8 + 1/52 + 1/104
2	19	7	1/19 = 1/20 + 1/380	21	2/19 = 1/12 + 1/76 + 1/114
3	25	9	1/25 = 1/26 + 1/650	27	2/25 = 1/15 + 1/75
4	31	11	1/31 = 1/32 + 1/992	33	2/31 = 1/20 + 1/124 + 1/155
5	37	13	1/37 = 1/38 + 1/1406	39	2/37 = 1/24 + 1/111 + 1/296
6	43	15	1/43 = 1/44 + 1/1892	45	2/43 = 1/42 + 1/86 + 1/129 + 1/301

2. Results

These Fractions Expansion helps in number factorization and excluding composite Primes from natural numbers space.

References

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Rhind Mathematical Papyrus (RMP) Fraction expansion table
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