On Collatz 1-Cycle

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Abstract

This paper introduces the flipping domain concept between even and odd domains. We will show how the flipping domain f(X) = 3 X +1 have a flipping point that happened at a specific frequency = 6 and starting from point Y = f(X) =-2. Which will alternate us between the even and the odd domains.

We also will show how both the flipping domain f(X) = 3 X +1 and the pure odd domain f(X) = 2 X +1 have one root at (X, Y) = (0,1). Which means they both evaluate to 1.

And how the flipping domain f(X) = 3 X +1 which is an odd domain, evaluates to value = 4 at X = 1 which alternates us between both even and odd domains, and this explains the Collatz Conjecture 1-cycle (4 ,2 ,1).

Keywords: Collatz conjecture, Pure even domain, Pure Odd domain, Collatz 1-Cycle

1. Introduction

1- Even domain and odd domain can be represented by two linear equations.
   f(x) = X and f(X) = X +1.
2- Odd domain linear function is parallel to the even domain linear function, because odd domain = even domain +1.
3- We alternate between even and odd domains based on the used value for X.
4- There are pure odd domains and pure even domains once we enter these domains we can not flip to the other domain even if X value changes.
5- We hit a flipping point (an alternating point) at each increase by 6 in the domain evaluation value.

Domain: - Is a linear function of X.

Even Domain: - Is a linear function f(X), which for each even and odd value of X evaluate to even value.

If X is an even natural number and f(X) = 2 X therefore f(X) is an Even domain.

If X is an odd natural number and f(X) = 3 X +1 therefore f(X) is an Even domain.

Odd Domain: - Is a linear function f(X), which for each even and odd value of X evaluate to odd value.

If X is an odd natural number and f(X) = 2 X +1 therefore f(X) value is an odd domain.

If X is an even natural number and f(X) = X +1 therefore f(X) value is an odd domain.

Pure Even Domain: - Is a domain where for any value of X even or odd the domain evaluates to only even value.

F(X) = 2 X is a pure Even Domain.

Pure Odd Domain: - is a domain where for any value of X even or odd the domain evaluates to only odd value.

F(X) = 2 X + 1 is a pure odd Domain.

In order to flip between these pure domains is to use one of the normal even and odd domains.

Every increase in the value of f(X) by 6 we flip between odd domain and even domain, starting from value (-1, -2) at Y = f(X) = -2.

2. Pure even domains and pure odd domains are parallel.

Even natural numbers and odd natural numbers never intersect, they are parallel to each other.

Then we need to find the best line that intersects these two domains to understand Collatz Conjecture which that says that these two domains flip to each other and both collapse to one [1]. Therefor we need to show that these two domains intersect and the reason for this intersection.

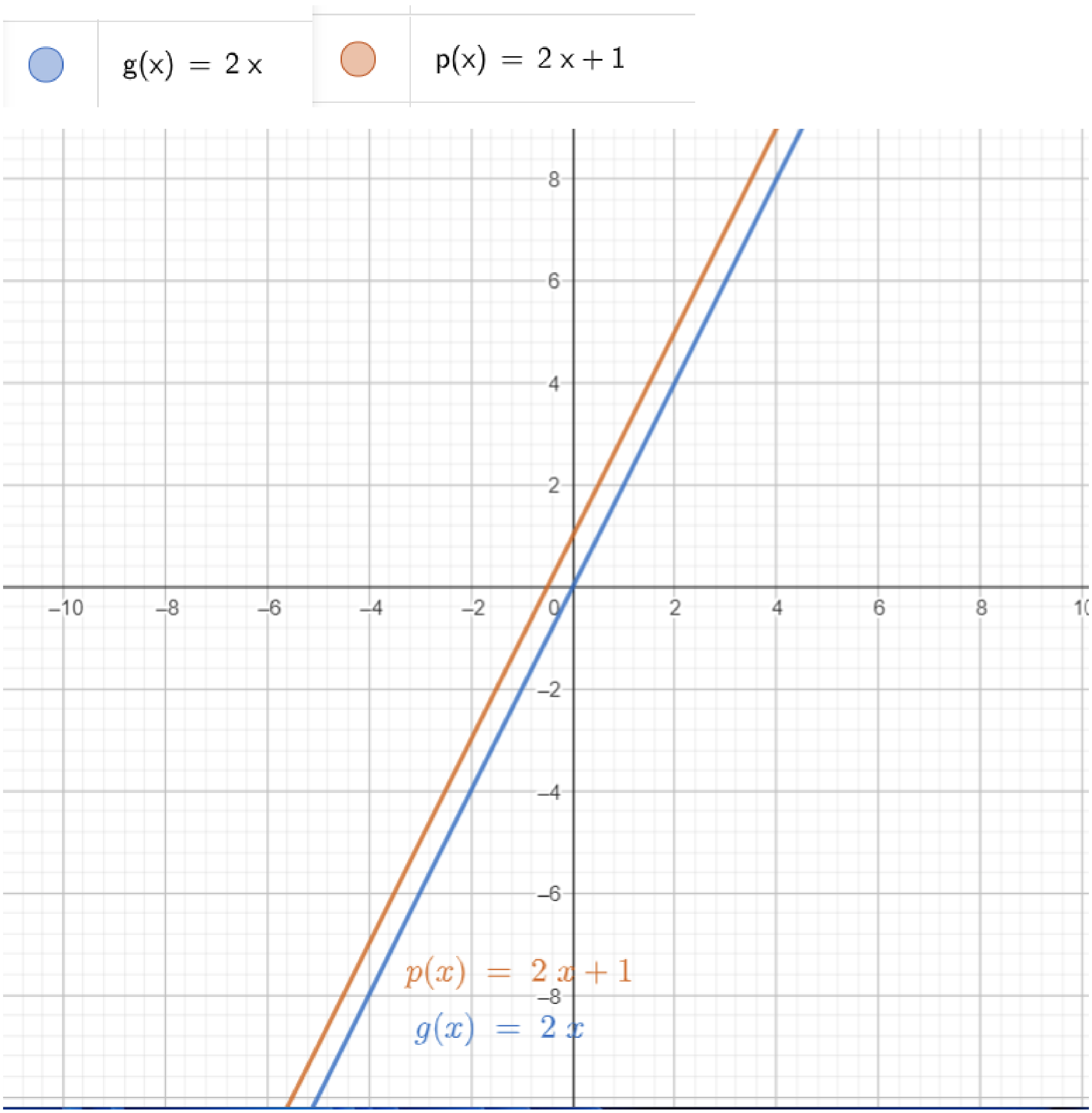

Figure 1. Two domains, pure even domain and pure odd domain parallel to each other.

3. Starting from Pure even domain f(X) = 2X

If we started from a pure even domain f(X) = 2X; then by dividing by 2. We get a new domain f(X) = X.

Therefor

2 * X becomes X and f(X) = X.

Both f(X) = 2*X and f(X) = X have one root at X = 0 and both intersects at (X, Y) = (0,0)

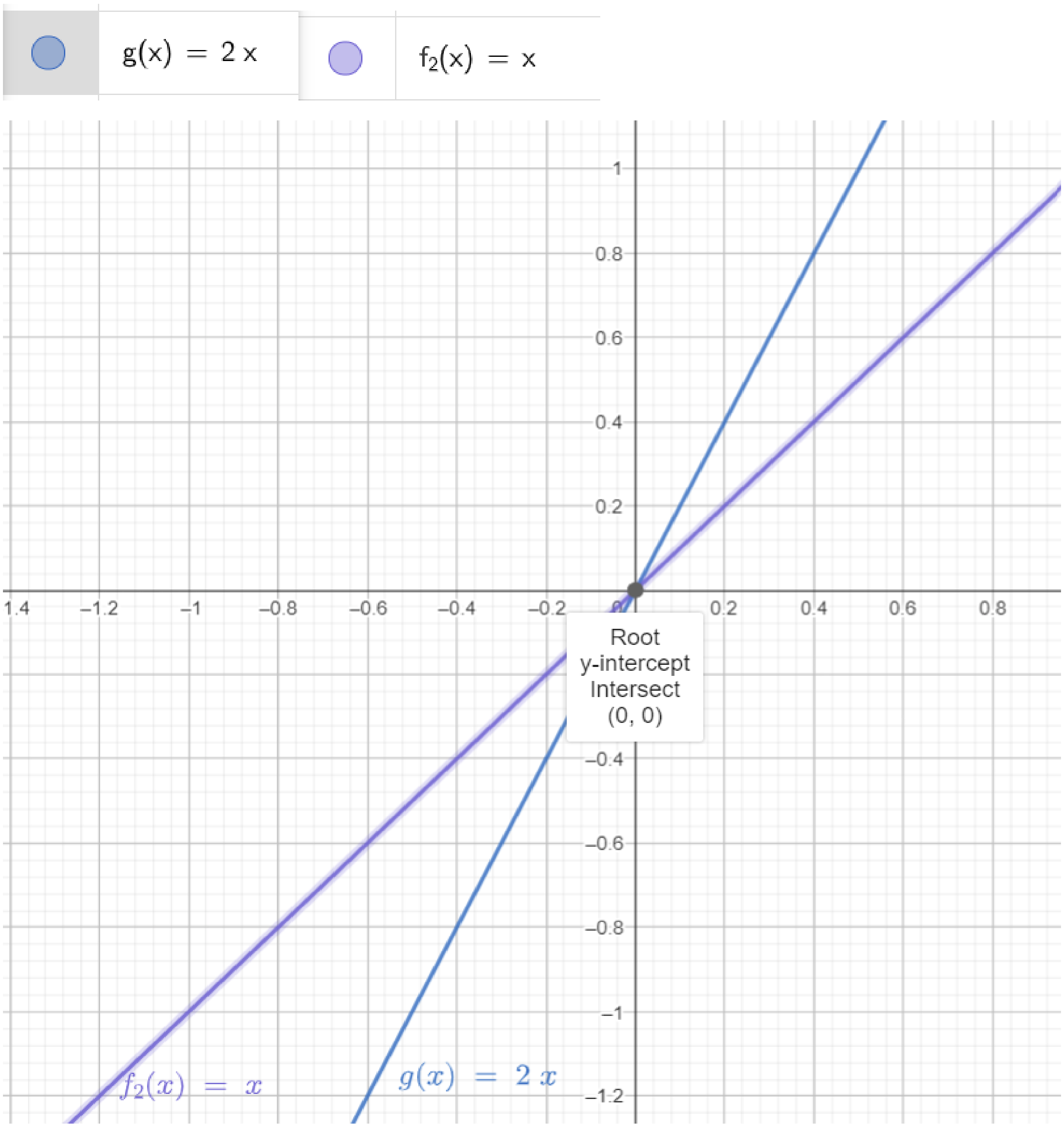

Figure 2. Apply Collatz first condition on a pure even domain f(X) = 2X; which divide by 2 if even.

The resulting domain is f(X) =X after we divide (2X) by 2, will intersects with the start pure odd domain f(X) = (2 X +1) at (-1, -1)

We need to find the sweet point on this line {f(X) = X}, that we can draw another line that will keep the properties of both domains which will give us the Collatz conjecture flipping feature between the two domains. (Pure odd domain and pure even domain)

The issue with this new domain f(X) is that it will give us infinite flipping feature. Therefore, we need to find another domain that will give us a Collatz Cycle that will hold us inside the loop (4, 2 ,1) instead of flipping between the two domains forever.

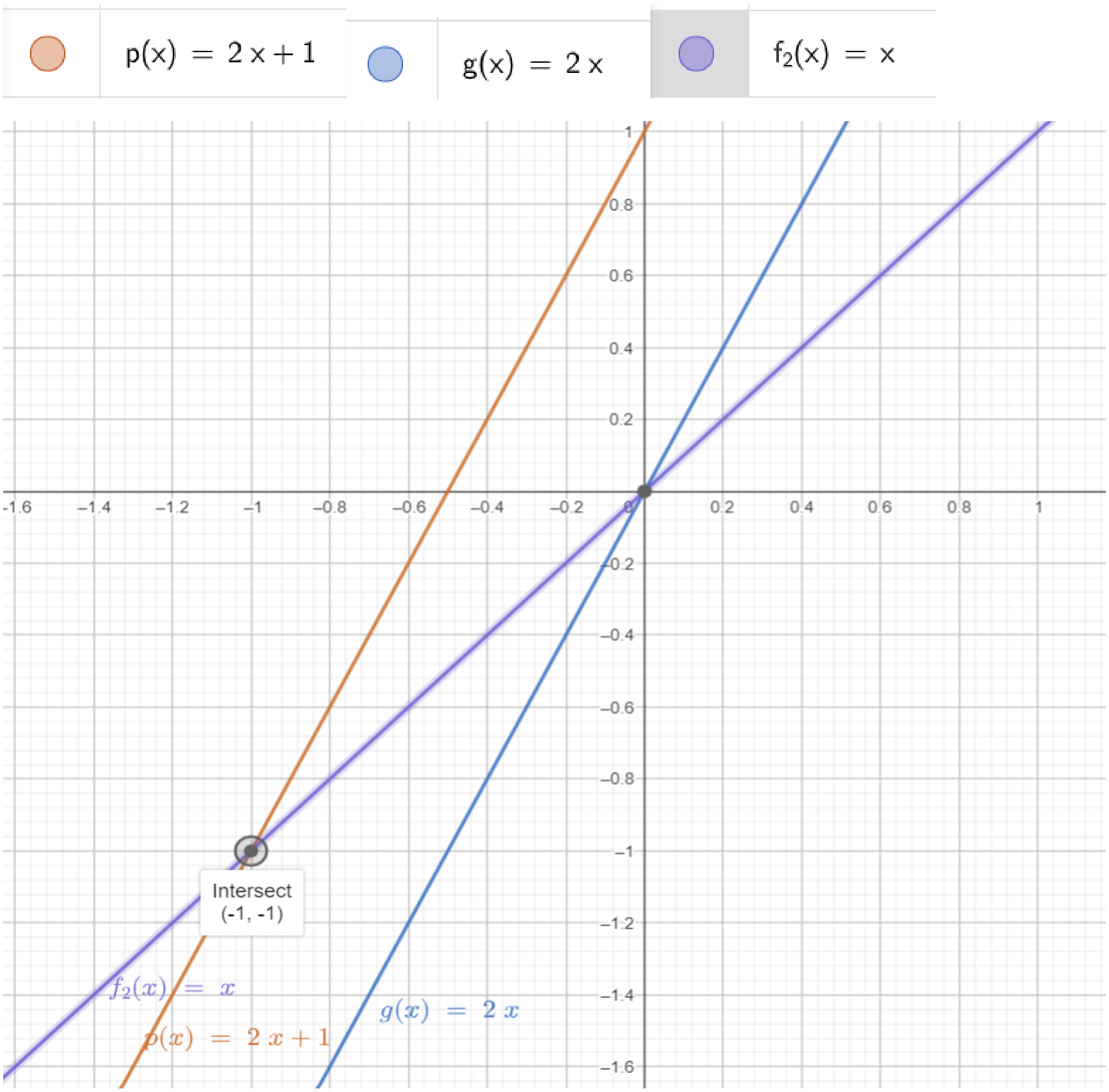

Figure 3. where the new resulting domain from first Collatz condition will intersect with a pure odd domain.

4. Finding the Flipping odd domain that satisfies Collatz Cycle (4,2,1)

To get the domain that will allow us to flip from one domain to the other (flip between odd and even domains) we need two points to get the linear function that will intersect these two domains.

To find the first sweet point it needs to be in the middle exactly on the line between the intersection line between the two domains f(X) = 2X and f(X) = 2X +1. As we added one to (2X). Adding one to a pure even domain will give use an odd domain to keep the Collatz definition.

Therefor, our first point will be at Z1 at (-0.5, -0.5) {i here is any number}

$$z_1 = -\frac{1}{2} - \frac{i}{2}$$

$$= -0.5 - 0.5i$$

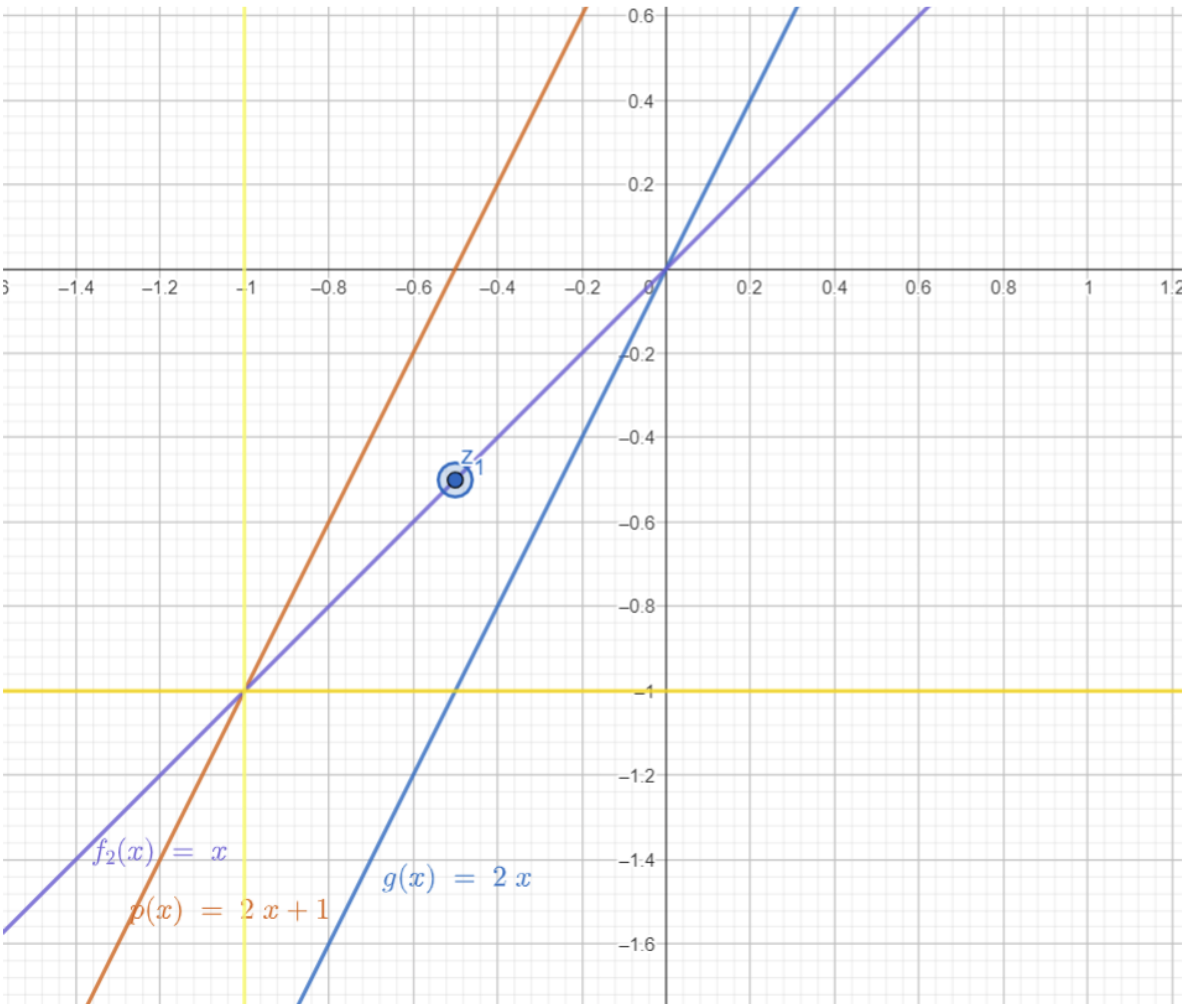

Figure 4. First point (-0.5, -0.5) for our required flipping domain.

To find the second point

Similarly, if we start with even number X and not (2 X)

Then its odd domain will be represented by line (X+1) and will be parallel to [X]

Therefor, our first point is (-0.5, -0.5) and second point is (0,1)

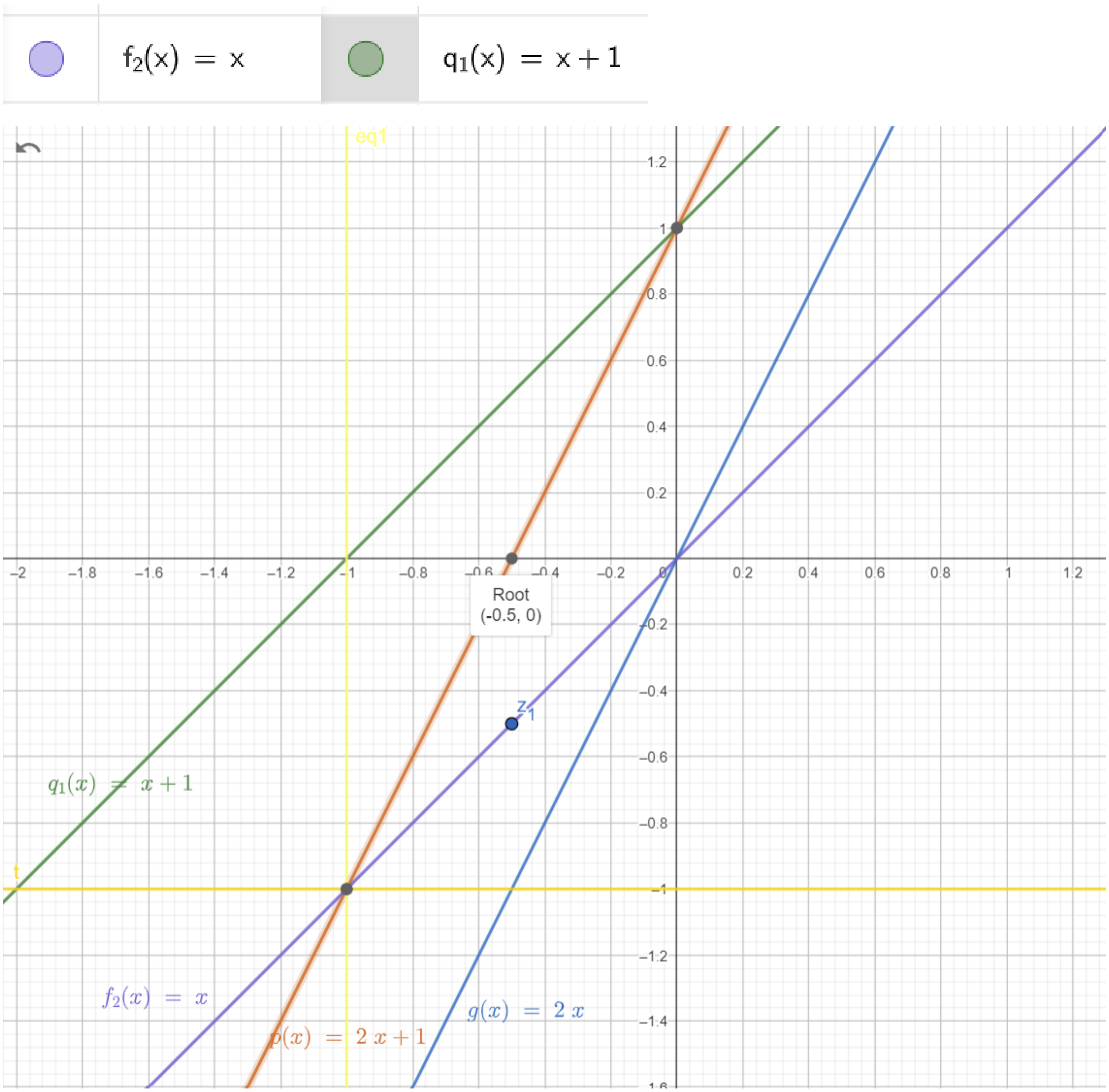

Figure 5. second point (0 ,1) for our required flipping domain.

Now we have two of our flipping domain points (-0.5, -0.5) and (0,1) if we draw line between these two points, we get our flipping domain.

F(X) = (3X +1)

As (3) is an odd number then (3 * even number = even number) and (3 * odd number = odd number)

If X is odd, then (3X+1) and (X+1) is even.

If X is even, then (3X+1) and (X+1) is Odd.

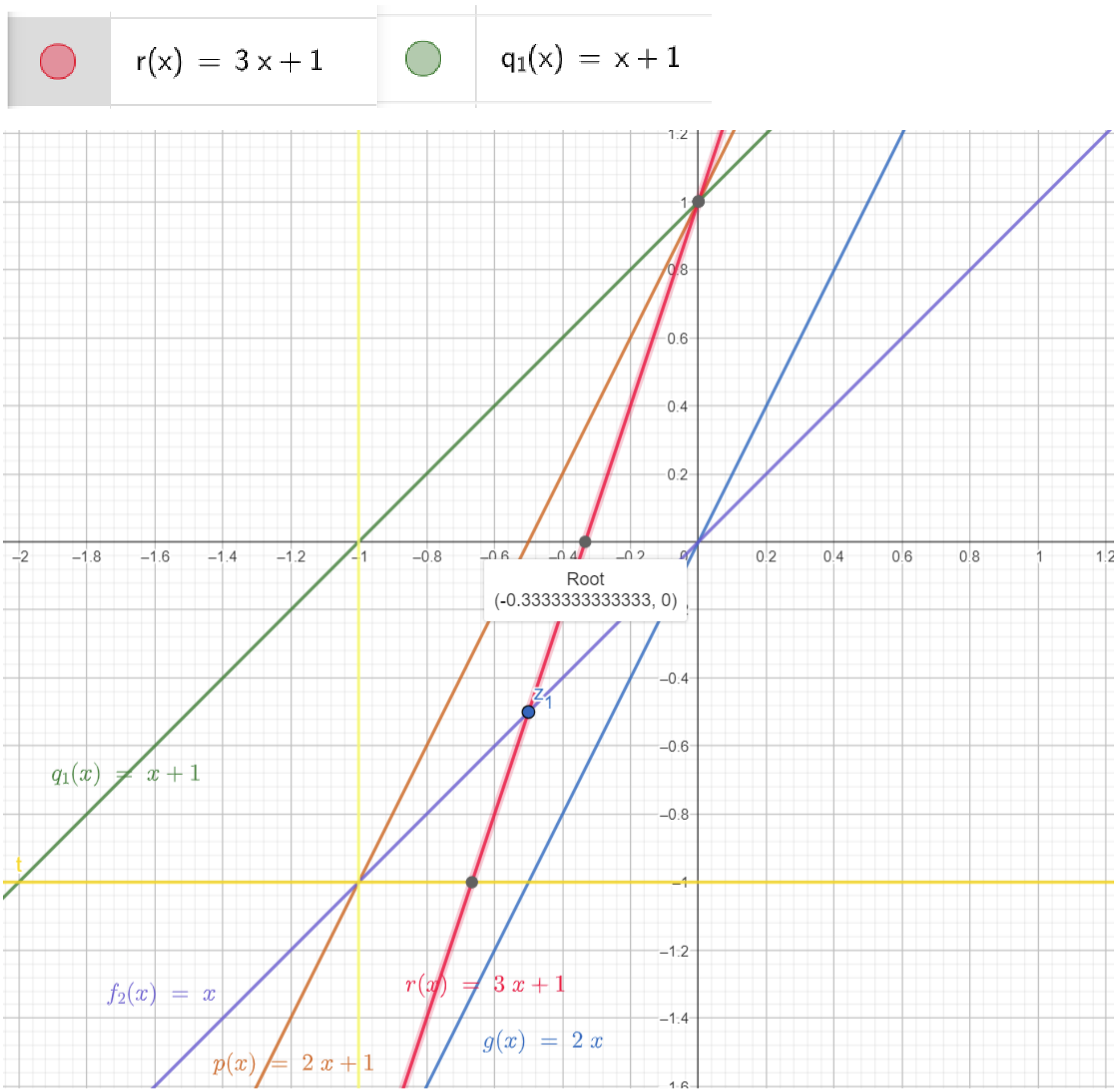

Figure 6. our flipping domain f(X) = 3 X +1 is line between two points (-0.5, -0.5) and (0,1).

To check this is the correct slope [1/3] for our flipping domain, we are going to check the inverse domain [X/2] these will intersect with the pure odd domain f(X) = [2X+1] at [-1/6, -1/3]

and as we are in the inverse domain (X/2) then Y value (-1/3) will be our slope.

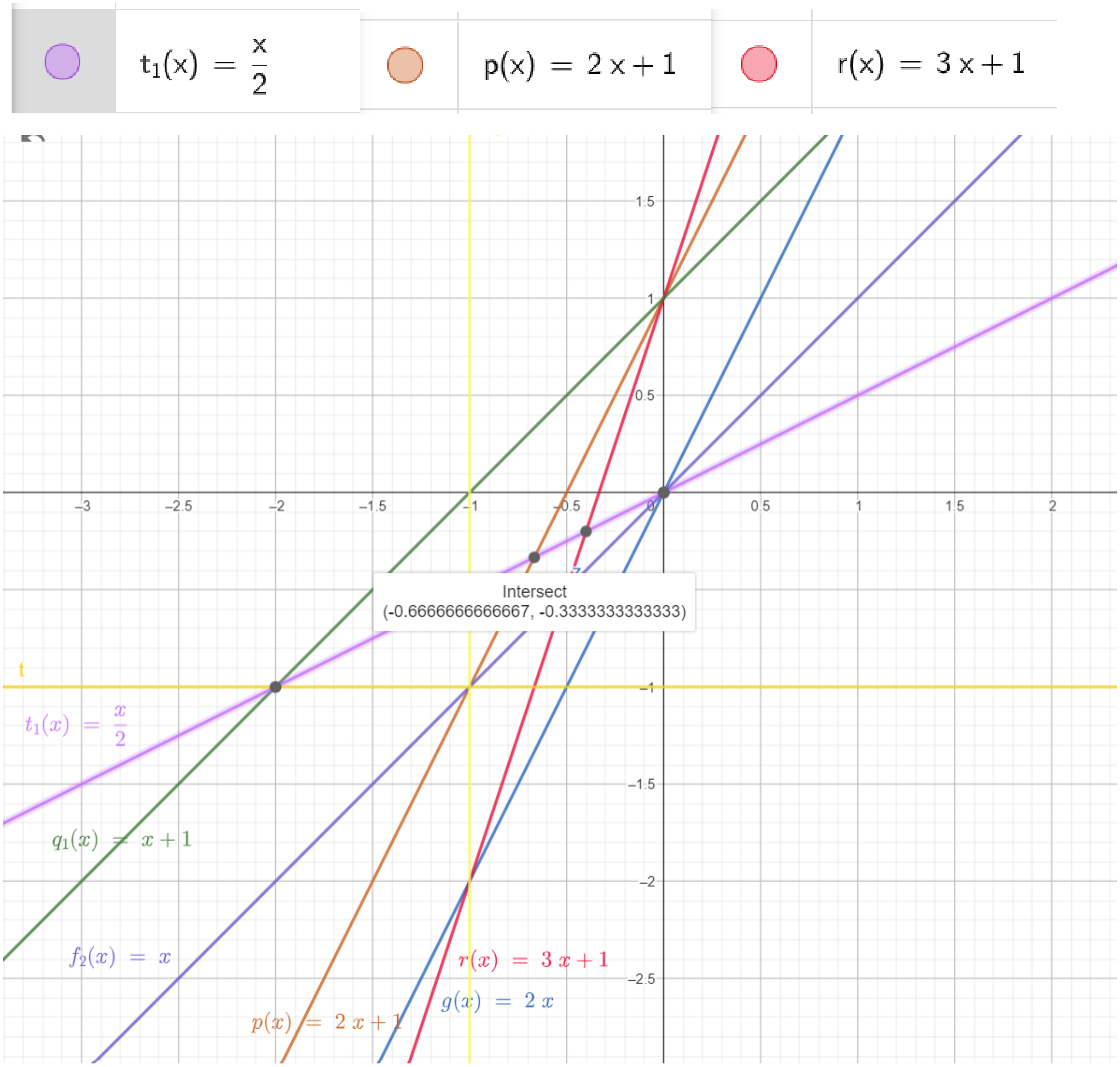

Figure 7. checking our flipping domain slope (-1/3] with the inverse domain (X/2).

Similarly, as the Collatz conjecture says both even and odd numbers collapse into one.

(3/2 X + 1/2) have one root at X = -1/3 and Y =0

Therefor once X = 1 then Y = 2 which means reach even number so we must divide by 2 = 2/2 = 1 then get odd number =1; Therefore, we go back to X= 1 and then Y = 2 again and we remain in this loop

The same will be for (3X+1) have one root at X = -1/3 and Y =0

Therefor once X = 1 then Y = 4 which means reach even number so we must divide by 2 = 4/2 = 2 then we reach even again so we must divide by 2 = 2/2 we get odd number =1; Therefore, we go back to X= 1 and then Y = 2 again and we remain in this loop

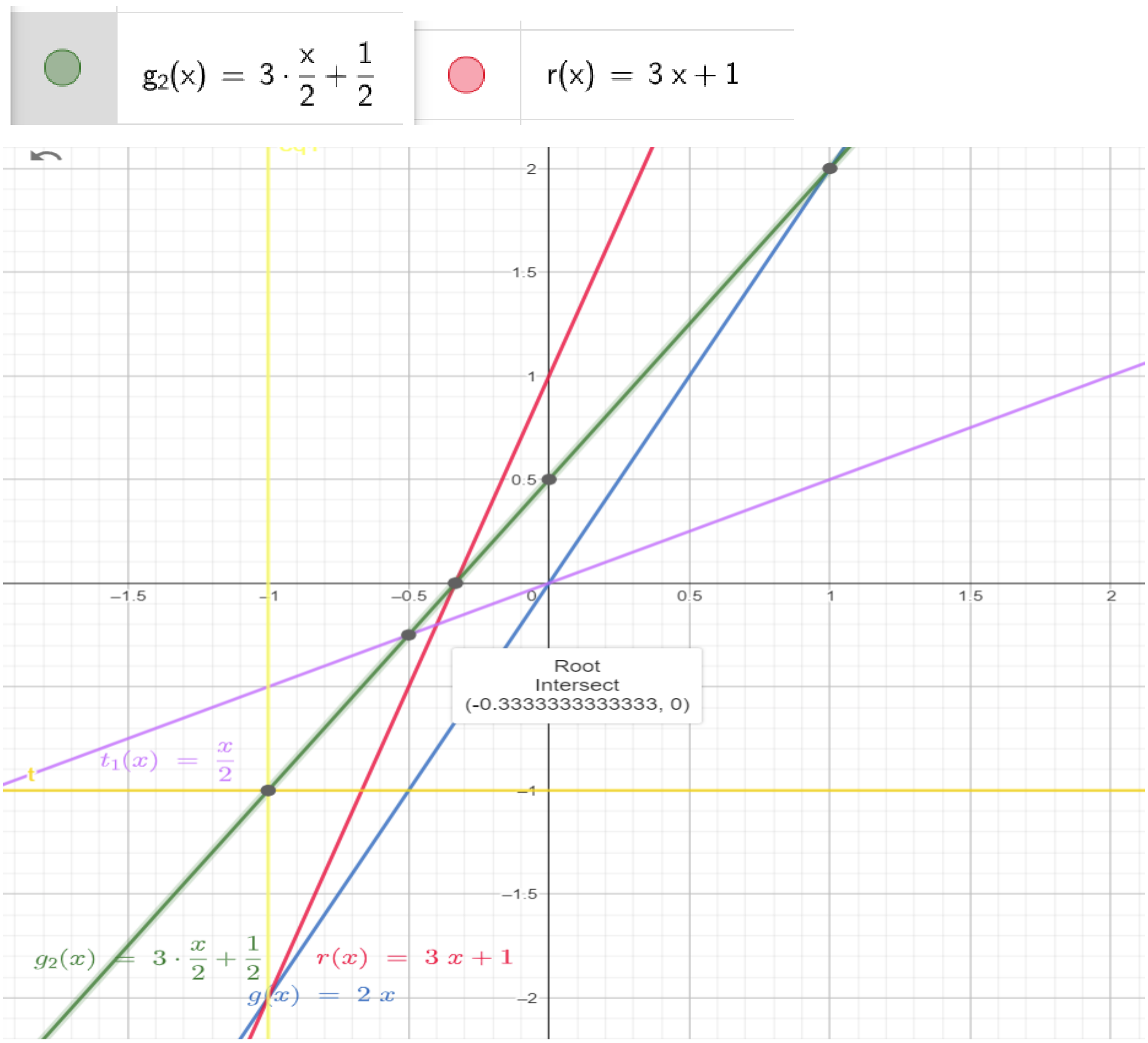

Figure 8. both domains f(X) = 3X+1 and its inverse f(X) = 3 X /2 +1.2 for applying the first Collatz condition divide by 2, both have root at [-1/3].

There is an intersection point between the two domains (odd domain and even domain) (X) and (3X+1)

If X is an odd number, then the odd domain is [X] and even domain is [3X+1]

If X is an even number, then the even domain is [X] and the odd domain is [3X=1]

And as both stratifies Collatz conjecture at (-0.5, -0.5)

As both domains reach -0.5 divide by 2.

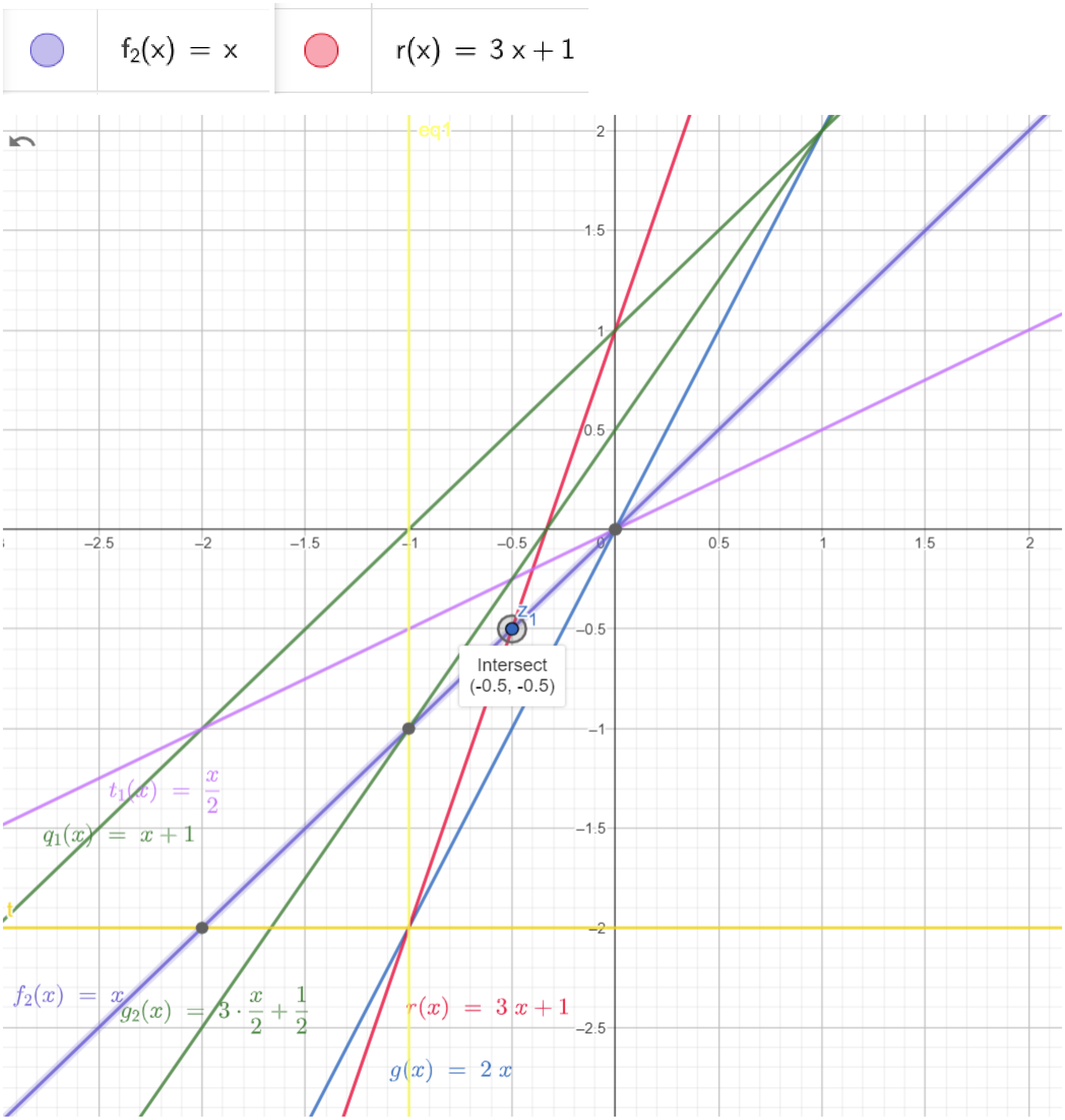

Figure 9. confirming the slope of the flipping domain with slope = [- 1/3] and running by point [-0.5, -0.5]

Both domains (3 X+1) and (2 X +1) intersect at (0,1) therefore both domains have the same root and evaluate to Y =1.

If X is an odd number, then (2 X+1) is the odd domain and (3 X +1) is the even domain.

If X is an even number, then (2 X +1) is the odd domain and (3X +1) is an odd domain.

Which means that (2 X +1) is an odd domain and once we enter it, we cannot go back to the even domain.

But (3 X +1) can work as an even or odd domain based on the value of X. and we could move between both domains if we used (3 X +1) instead of using (2 X +1) as an odd domain.

(2 X +1) is a pure odd domain similarly (2X) is a pure even domain.

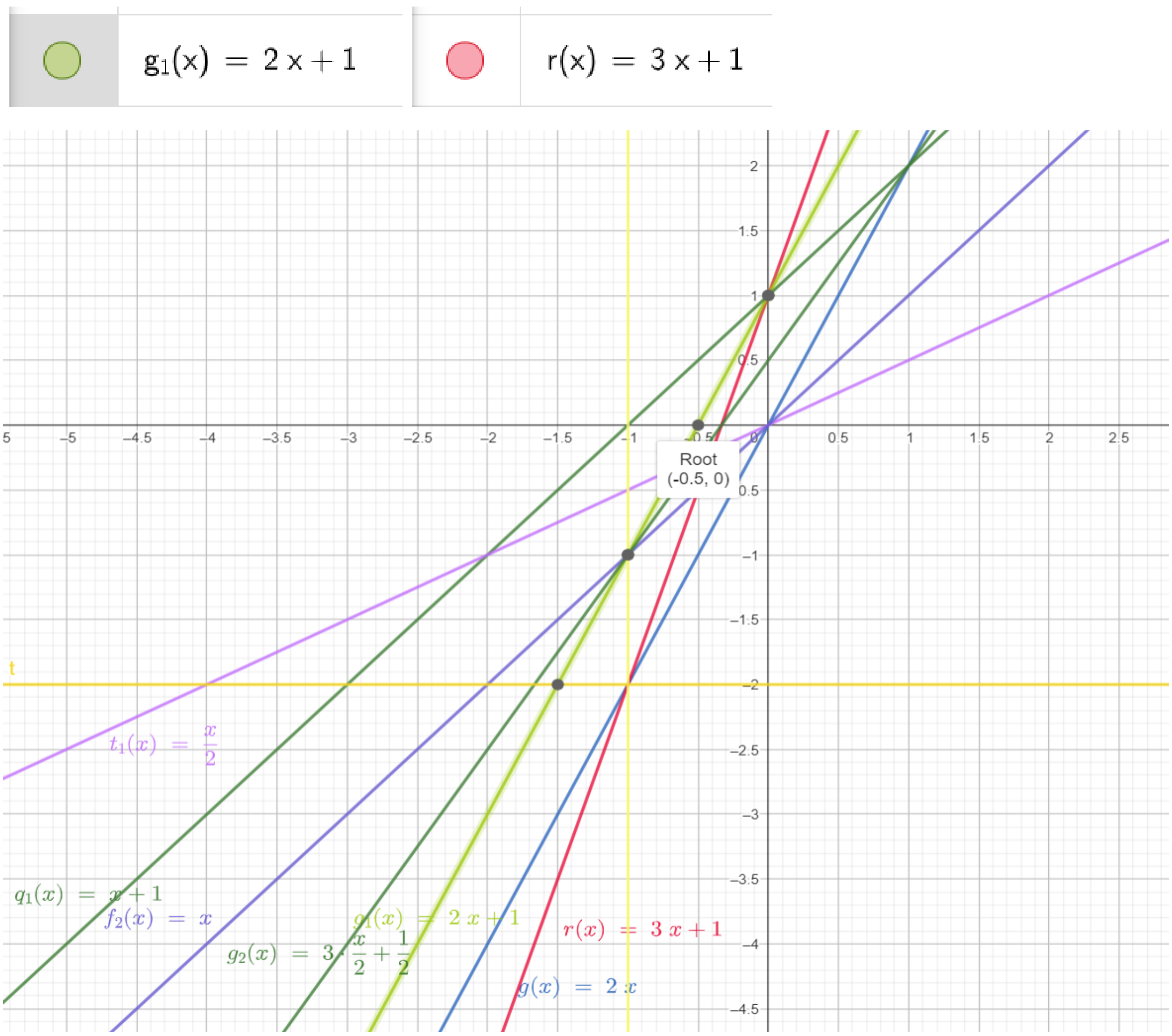

Figure 10. intersection point between Pure odd domain f(X) = 2 X +1 and odd domain f(X) = 3 X+1

5. The Flipping odd domain f(X) = 3 X +1 that satisfies Collatz Cycle (4,2,1)

What is unique about this odd flipping odd domain f(X) = (3 X +1) is that is the way to move between the two pure domains (the pure odd domain f(X) = 2X +1 and the pure even domain f(X) = 2 X)

Because it moves between the two pure domains by just shifting the origin from (0,0) to (-0.5, -0.5)

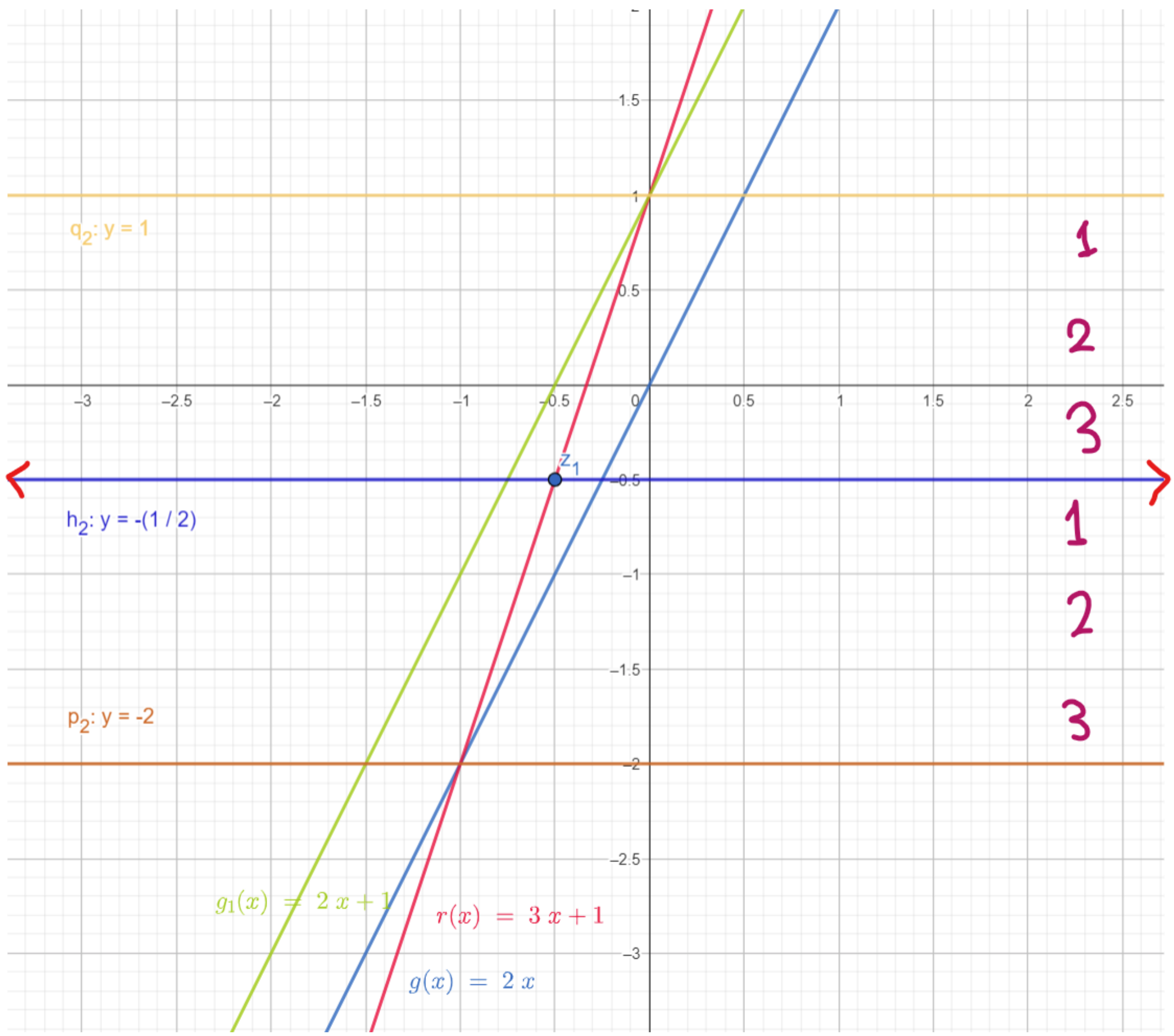

Figure 11. our odd flipping domain f(X) = 3 X + 1 is a transformation of moving (0,0) to (-0.5 , -0.5)

As in both domains if X =1 the value in the red line (odd domain f(X) = (3X+1) = 4) then we move to the even domain as you see it exactly intersect with the same line at X =2 on the even domain (2X).

Therefore, once X reaches 1 it will be kept moving between even and odd domain in an infinite cycle. (4,2,1). What is special about this cycle is that the Y value on both domain lines is the same and is a neutral number and this horizontal equality between the two domains exists along both lines and repeats every 6 natural numbers starting from 4.

Therefore, the next points that have the same property Y value is the same in both domains is at Y =4 + 6 = 10

And this is because the intersection point between the two domains starts at (-2) and not (0)

Therefore, the first point to flip between even and odd domain will be at Y = -2 +6 = 4.

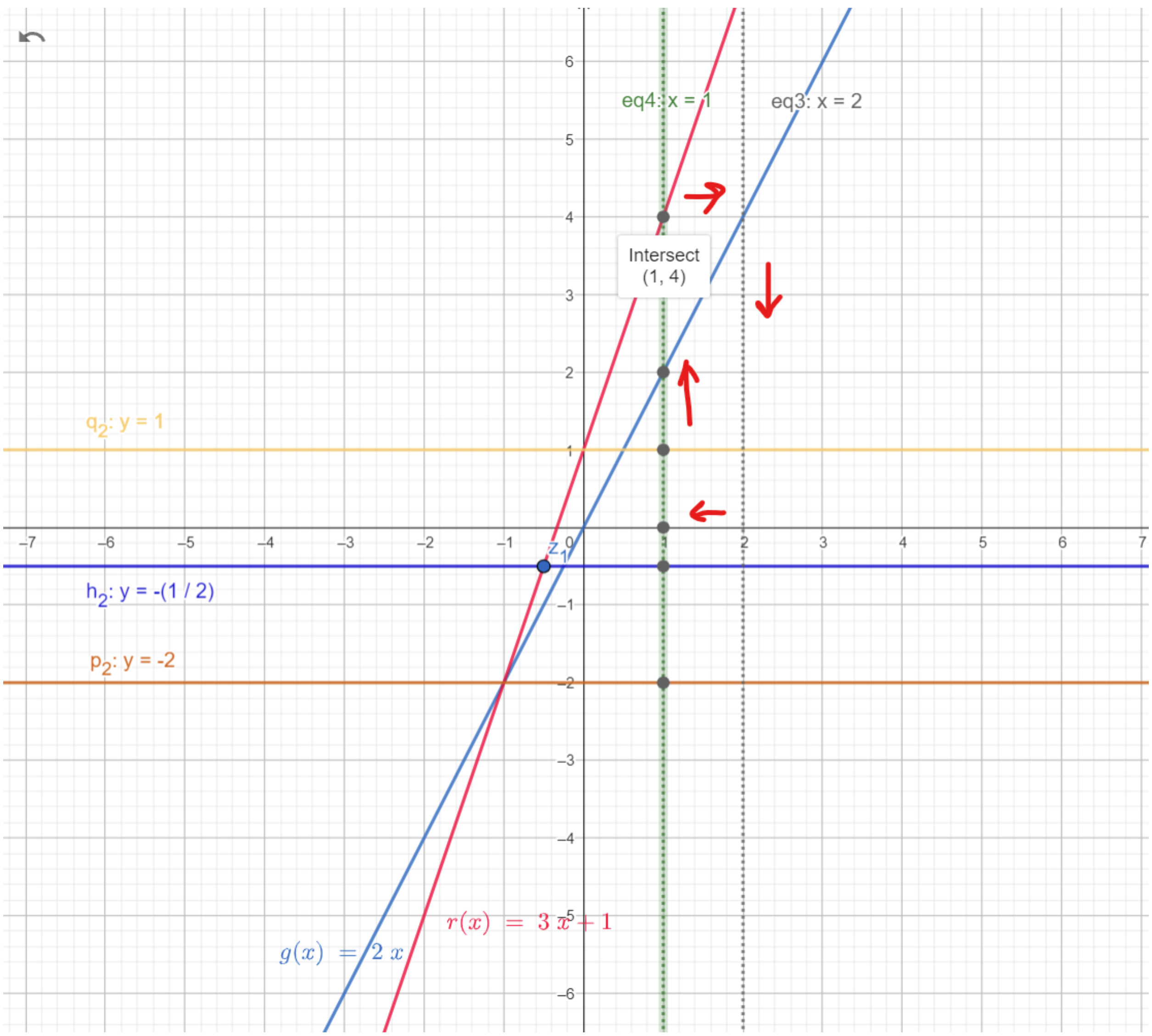

Figure 12. Collatz Cycle using a flipping domain f(X) = 3 X +1 instead of f(X) = X halt the flipping point inside this cycle (4 ,2 1) instead of going to flip for ever if we used f(X) = X.

As we show the first flipping pint between the two domains is at Y =4 and points (1,4) and (2,4)

The second flipping point to flip between the two domains is at Y =10 and points (3,10) and (5,10)

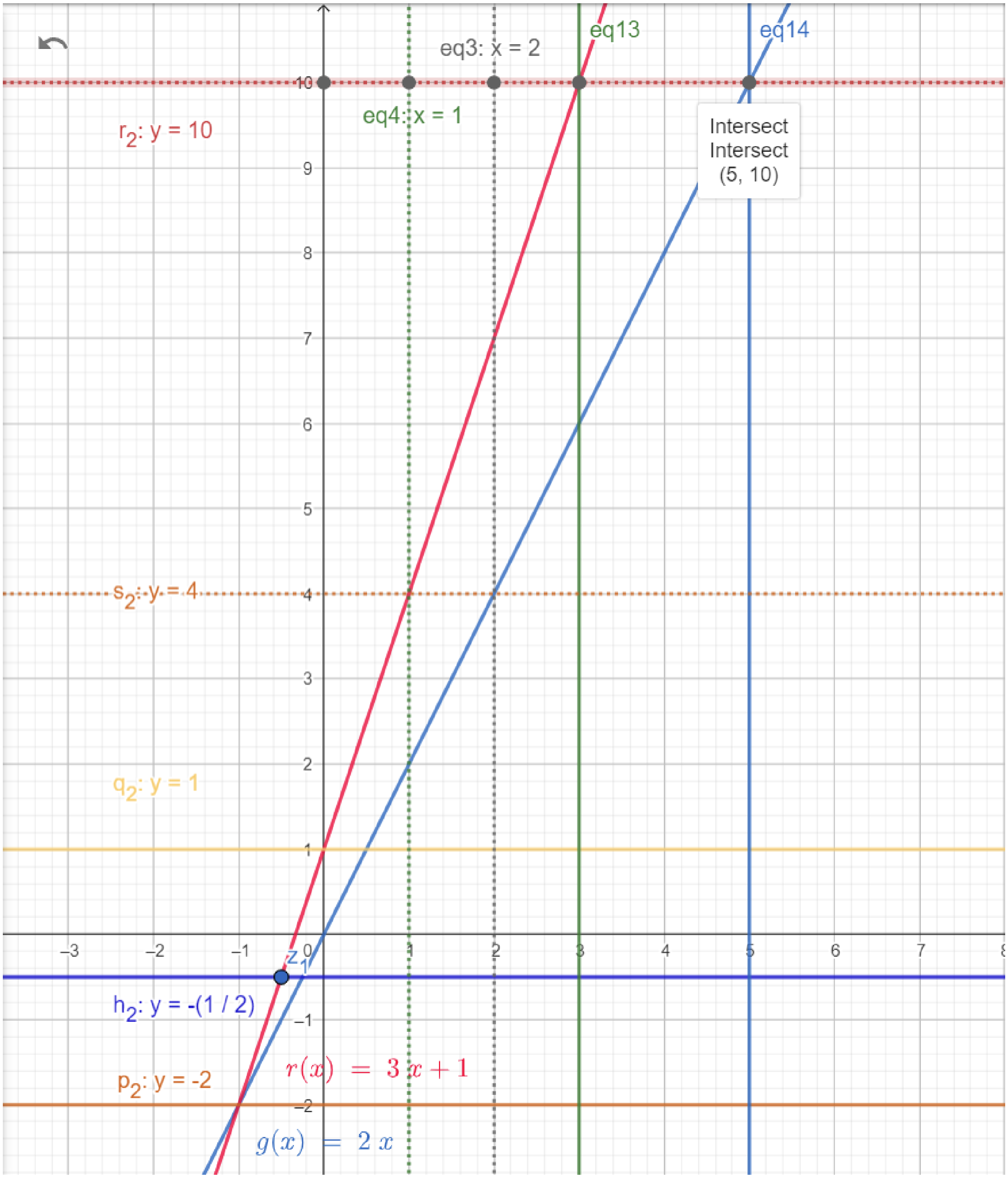

Figure 13. flipping points at each increase in Y by 6 every increase in the value of f(X) by 6 have a flipping value between even and odd domain. At Y = {-2, 4 , 10 , 16, 22, …}

The third flipping point to flip between the two domains at Y = 10 +6 = 16 at points (5,16) and (8 ,16)

Y = 16 then Y moves to even domain and the new Y = 8 and the new Y = 4 then 2 then 1 and remains in this cycle.

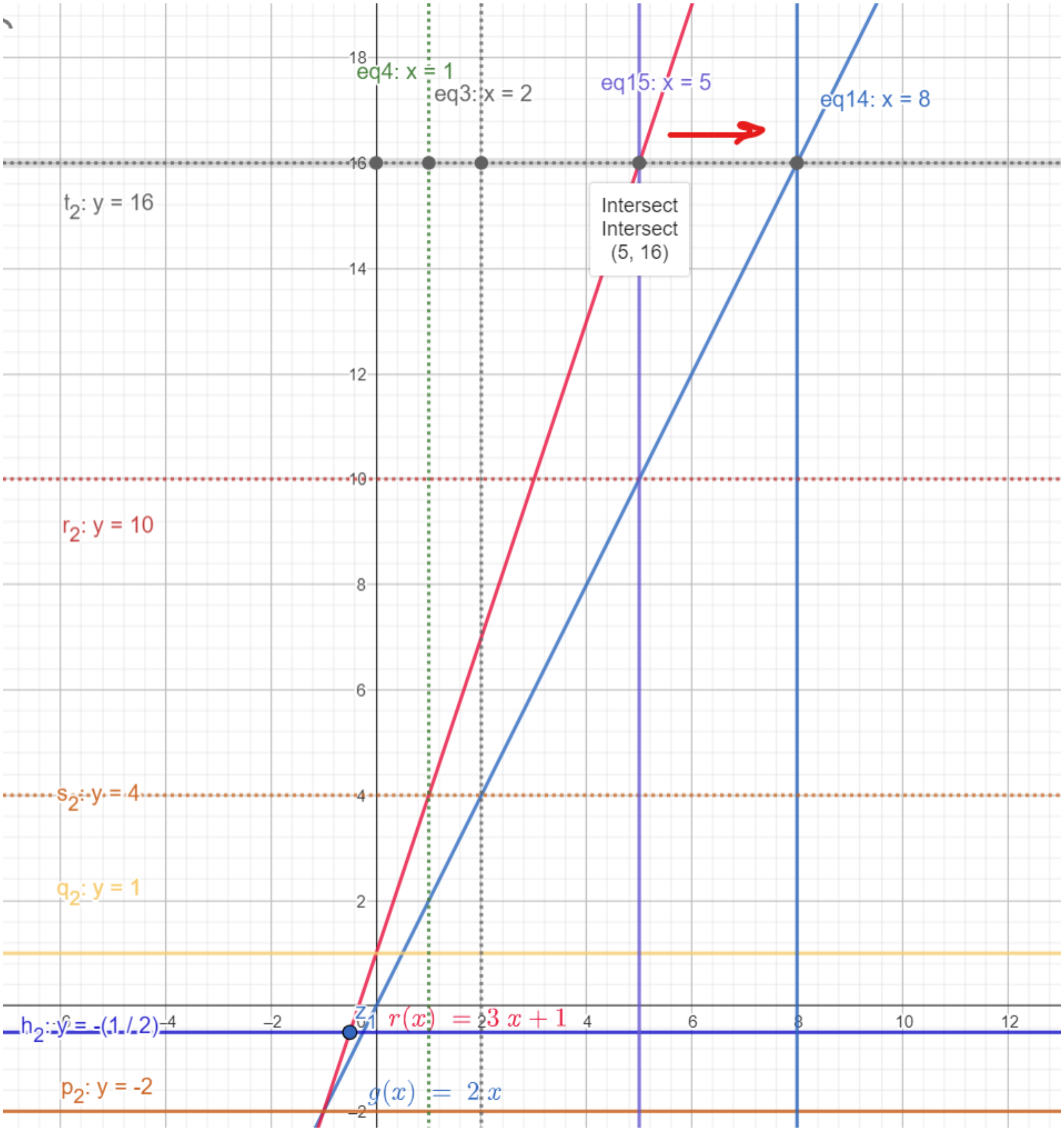

Figure 14. Third flipping point at Y = 16 after 3 flips starting from [-2].

The third flipping point to flip between the two domains at Y = 16 + 6 = 22 at points (7,22) and (11 ,22)

Y = 22 then Y moves to the even domain and the new Y = 11 so it moves back to the odd domain to the next flipping point

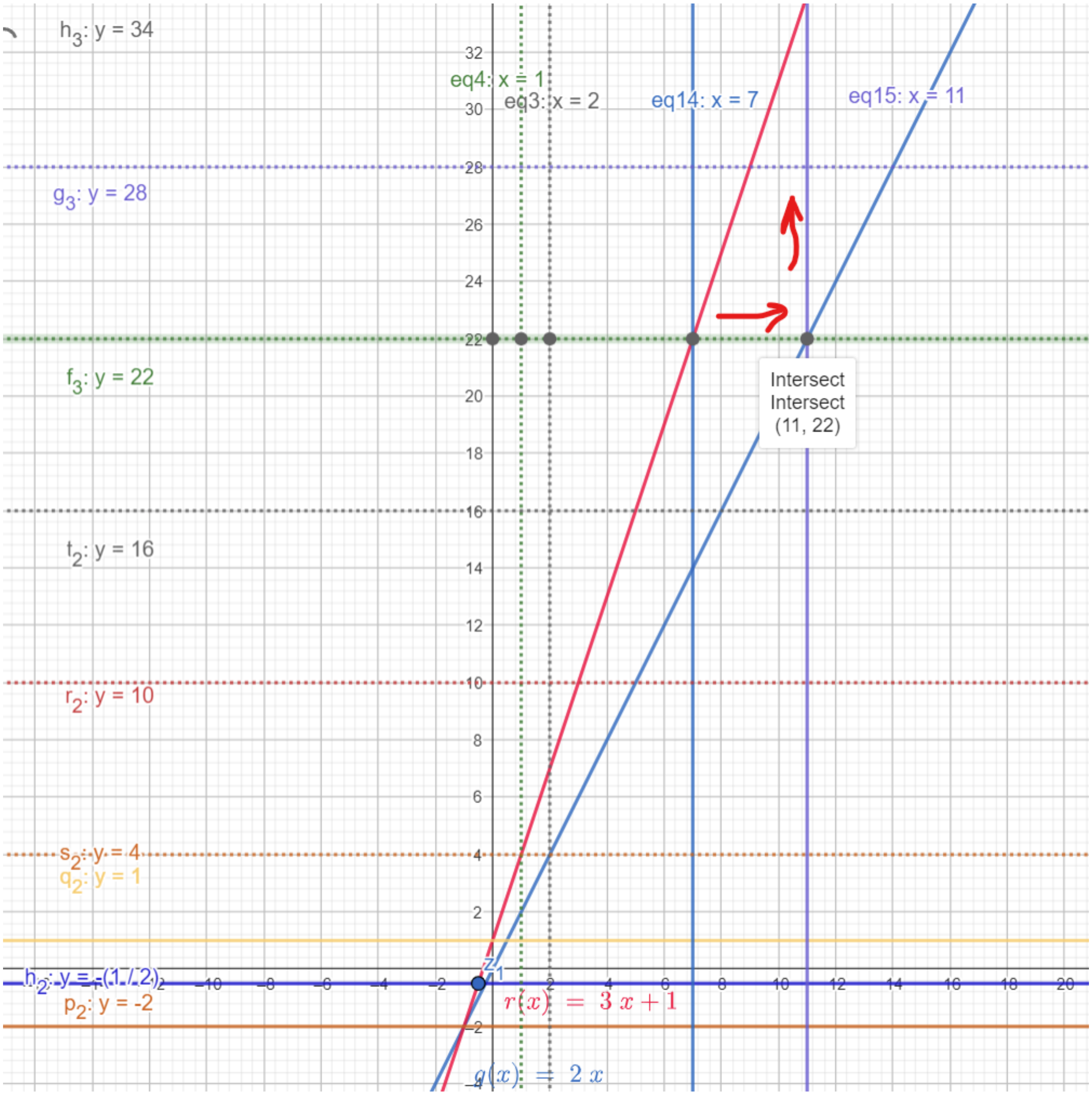

Figure 15. Fourth flipping point at Y = 22 after 4 flips starting from [-2].

The Fifth flipping point to flip between the two domains at Y = 28 + 6 = 34 at points (11,34) and (17, 34)

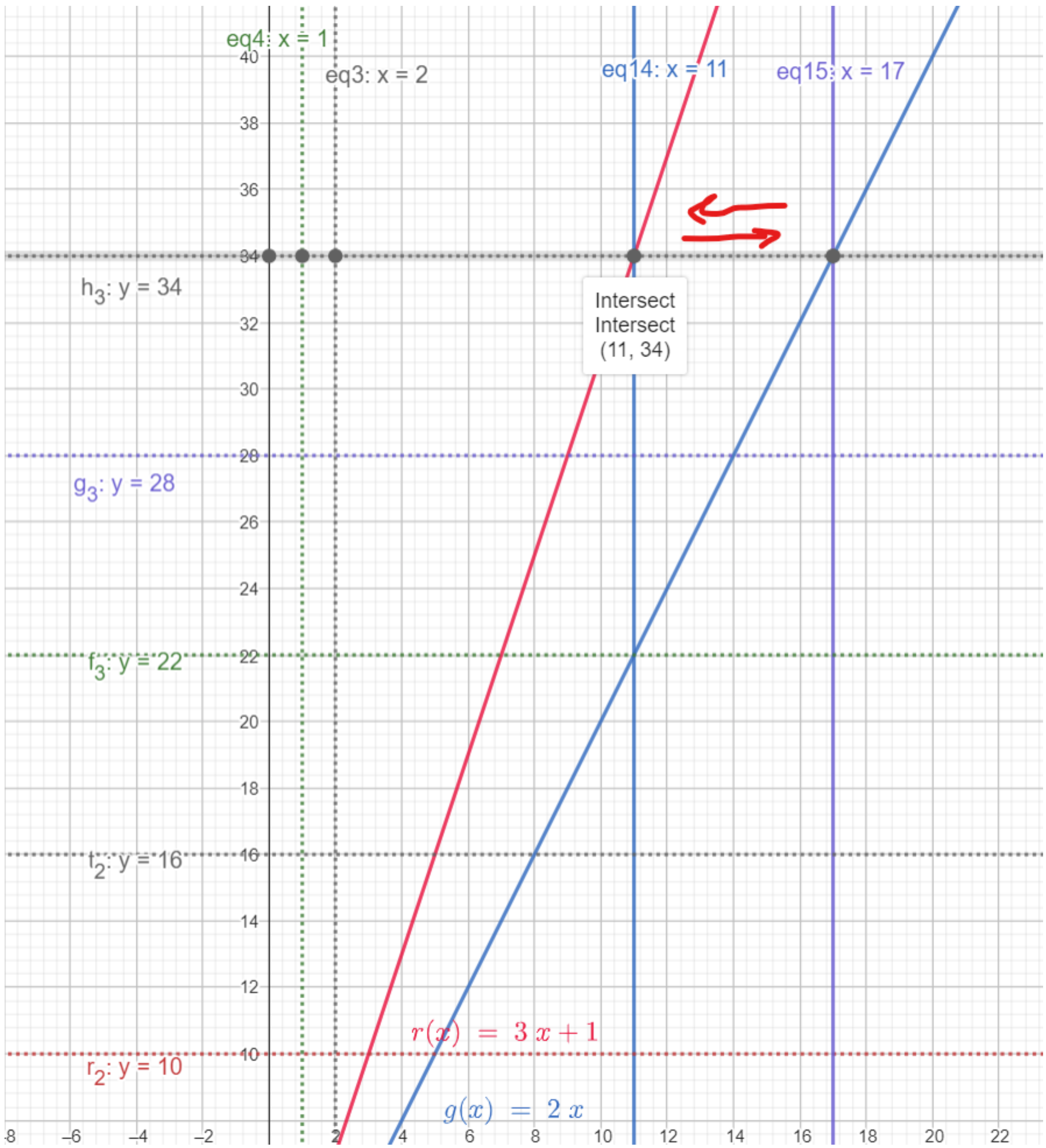

Figure 16. Six flipping point at Y = 22 after 6 flips starting from [-2].

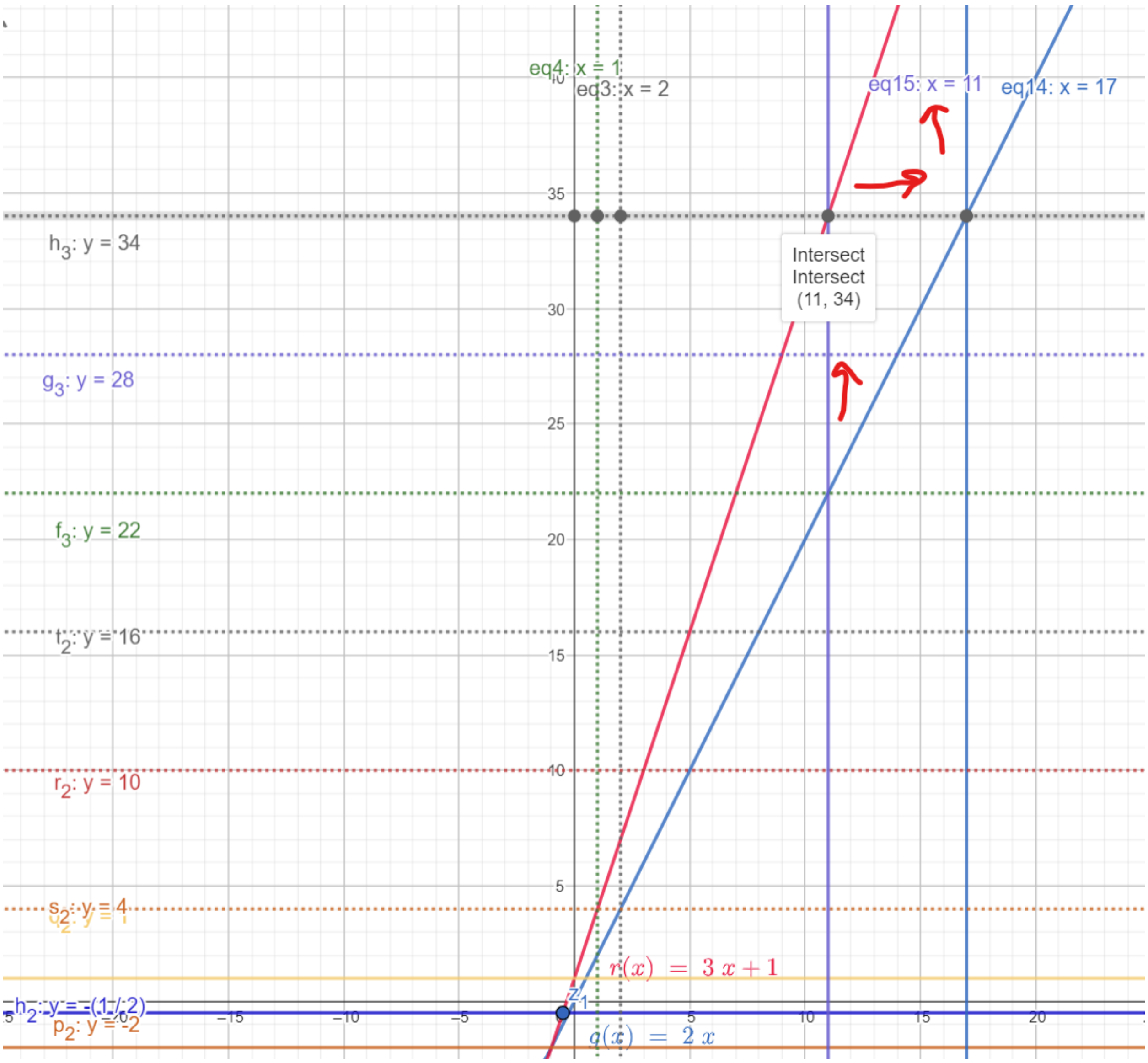

Figure 17. Six flipping point at Y = 22 after 6 flips starting from [-2]. At X =11 f(X) = 34 moves to even domain with X = 34 then f(X) = 17 so flip back to odd domain and see where at X = 17 will intersects with f(X) = 3 X +1.

Conclusion

In This paper we studied the two parallel domains the even and the odd domains and the Pure even domain f(X) = 2 X and the pure odd domain f(X) = 2 X +1. And explained the way to find the odd flipping domain f(X) = 3 X +1 which will give us the ability to alternate between the two domains.

Then we showed how this flipping between domains is happened in a steady frequency = 6. As each increase by 6 in the evaluation of the odd flipping function f(X) = 3 X +1 will force us to alternate to the other domain.

Then we showed how this odd flipping domain f(X) = 3 X +1 starts from -2 at X = -1 and at X = 4 it evaluates to 4 which halt us in a Cycle, an alternating cycle between even and odd domain, the Collatz Cycle (4,2,1).

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