# Riemannian geometry manifold and unfolding for imaginary frame of reference using axiomatic method in a complex plane

Shaimaa said soltan[1]

[1] Computer Engineer, Toronto, Canada

Correspondence: Shaimaa Soltan, 3050 Constitution Blvd, Mississauga, ON., L4Y 3X1, Canada. Tel: 1-647-801-6063 E-mail: shaimaasultan@hotmail.com

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# Riemannian geometry manifold and unfolding for imaginary frame of reference using axiomatic method in a complex plane

**Abstract**

In this paper, we will study manifold transformation effect by applying a combination of a basic mathematical operation, like (+, -, *, /) on a frame of reference, which by itself is one of the transformations for the complex plane imaginary unit [i]. Then will introduce another concept to represent Euler's Identity equation using only the frame of reference and a basic mathematical operation.

In second part, we will study a complex plane folding and unfolding for a frame of reference using a set of basic mathematic operations. Then we will visualize a harmony relation between [e] and [π] and its formula.

Finally, we will present a simpler transformation formula for the frame of reference that helps in understanding the complex plane manifold at the strip line of zeta function, and how using this new formula can give us natural number results for all odd natural numbers including prime numbers at [0.5], which is known as strip line conjecture between [0,1].

**Keywords:** Euler's Identity, Prime Number Distribution, Zeta function, complex plane manifold

## 1. Introduction

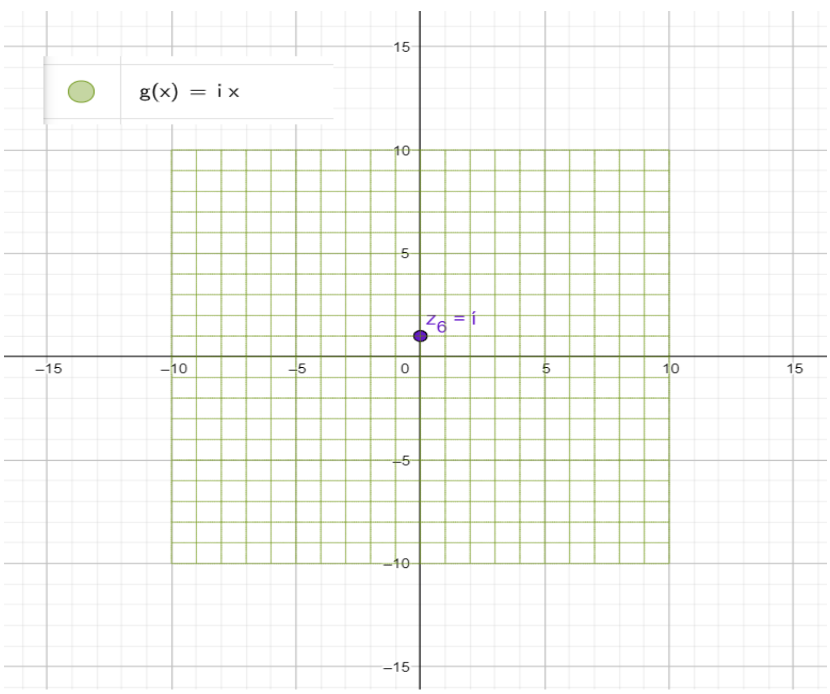
*1.1 Introduce the Problem*

Complex plane representation is based on two irrational numbers [ i and π] and any basic mathematic operations using these two irrational numbers mostly will give us another decimal number in the best case, which makes most mathematical formulas complex and calculation consuming.

In this paper we will use an axiomatic method and the complex plane imaginary unit [i] transformations as a frame of reference to explain manifold and unfolding of Riemannian geometry in complex plane.

First let us introduce our frame of reference in this axiomatic method. We will multiply the imaginary unit by variable [X] which give us the discrete advantage in our complex plane.

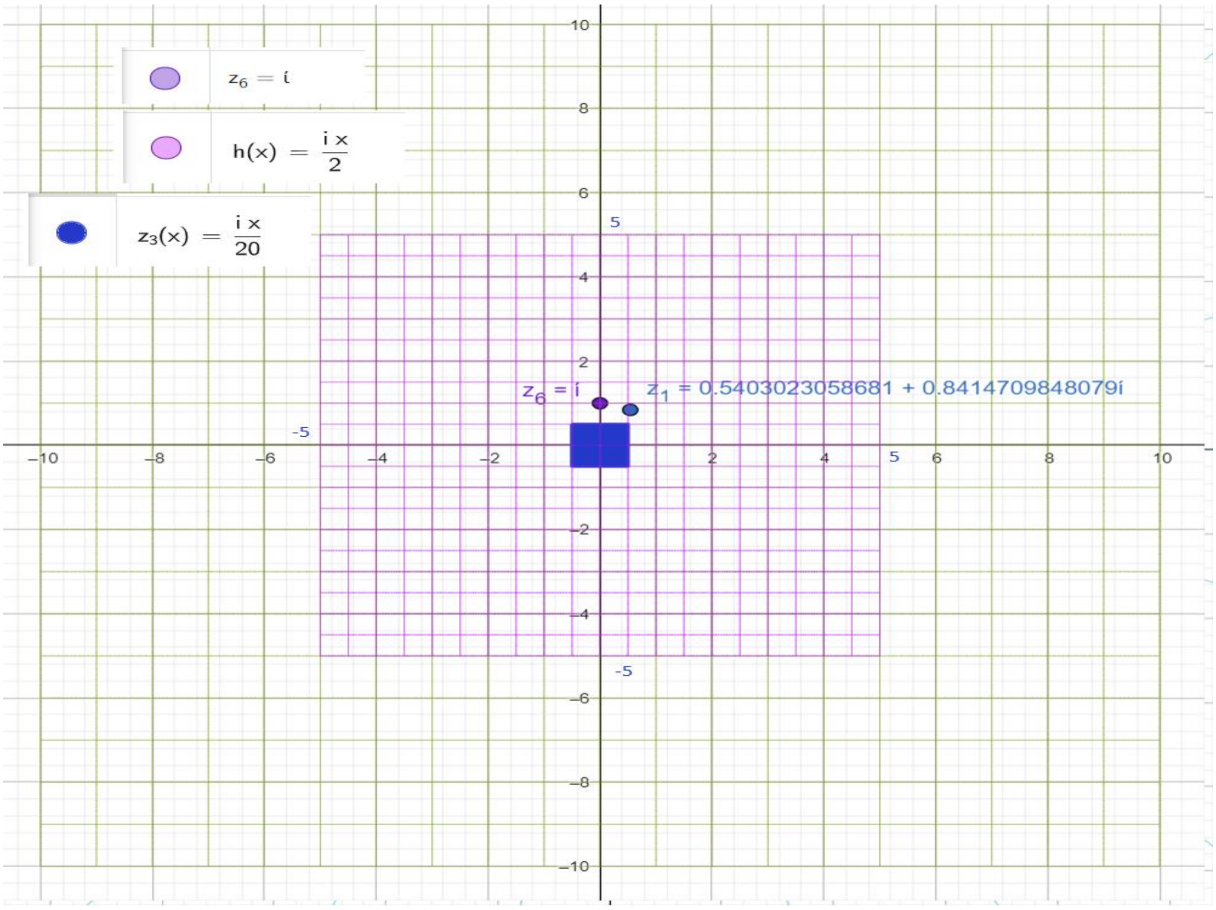
The complex plane works on a base 10 number system, so the value for multiplication of [X] by [i] will give us a square [10*10] and the plane origin point [0,0] is at the center of the square. and as in figure 1; [i] will be represented as a point inside this square located at y axis at a unit square [U] = [ I * X * 1/400]

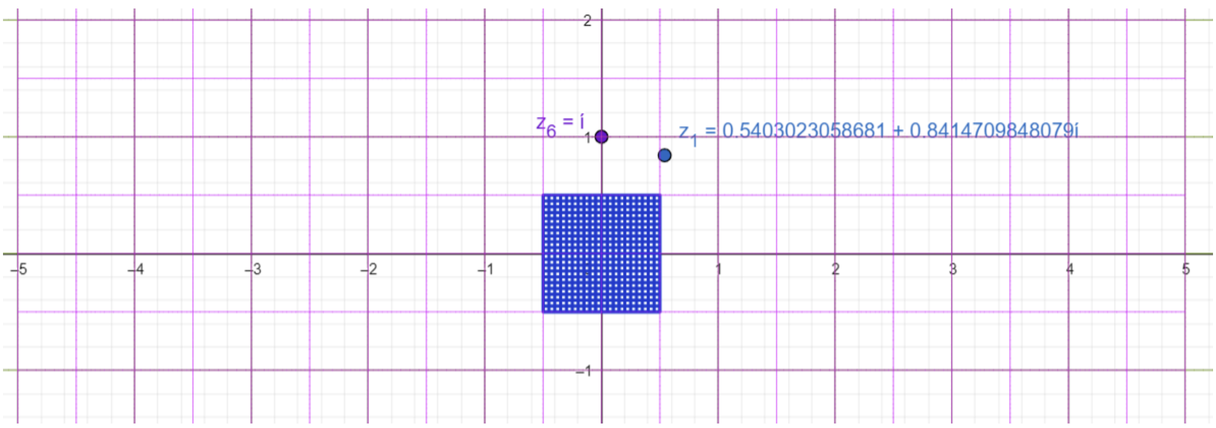


*1.2 basic mathematic operations on imaginary unit [i] as a frame of reference*

*1.2.1 Scaling frame of reference [g(x) = I * X]*

In next Figure, we scaled our frame of reference by a scale of ½ and 1/20. As you can see scaling by natural number simulate an increase and scale by fraction less than one simulate shrink in size (zoom in or out), without any distortion in square unit shape keeping shape ratio unchanged (resize with a fixed ratio for width and height).

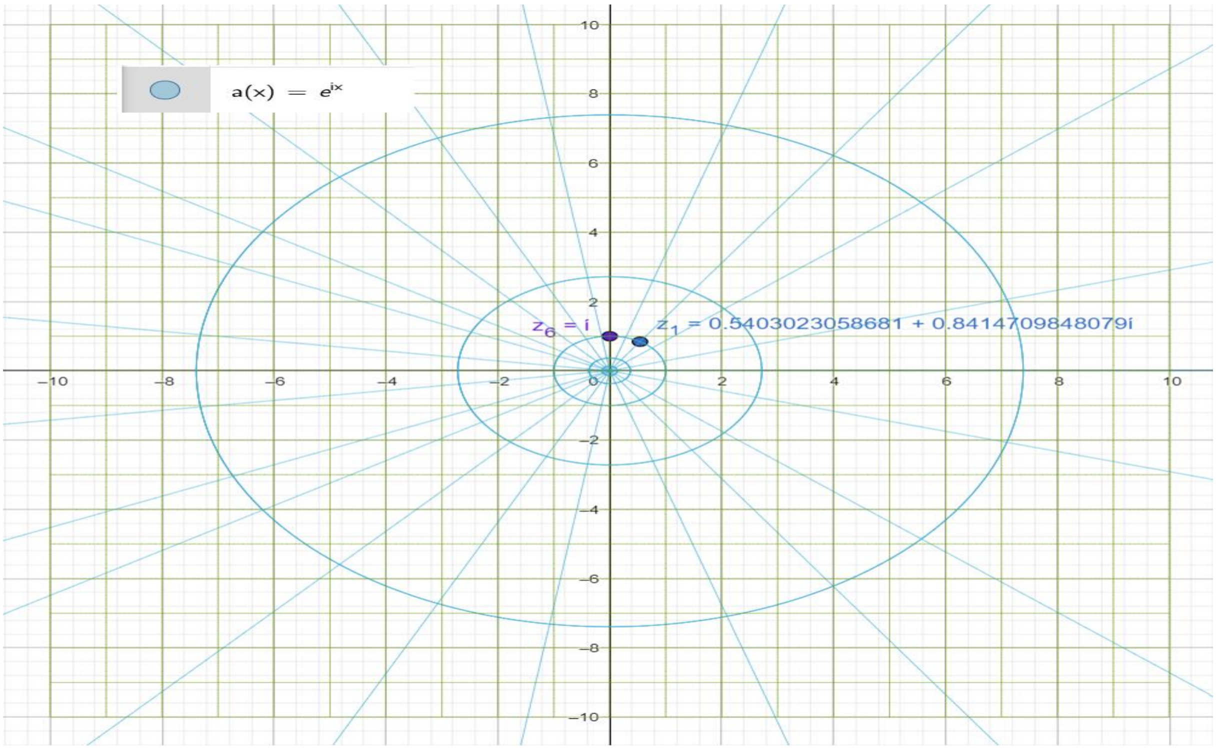


*1.2.2 Raise [e] to the power of frame of reference [g(x) = I * X]*

In next figure, we used [e] representation of a complex plane as our mathematical transformation for our frame of reference [ix]. As we see it can be seen as a cone representation in a 2-D space with an infinity point at the original of the complex plane at point [0, 0] if we looked on the cone as if we are inside the cone at bottom and look top to the head point of the cone. Or we can look at it as 2-D Cone in the opposite way we are at the Top of the cone and look down on a cone from top.

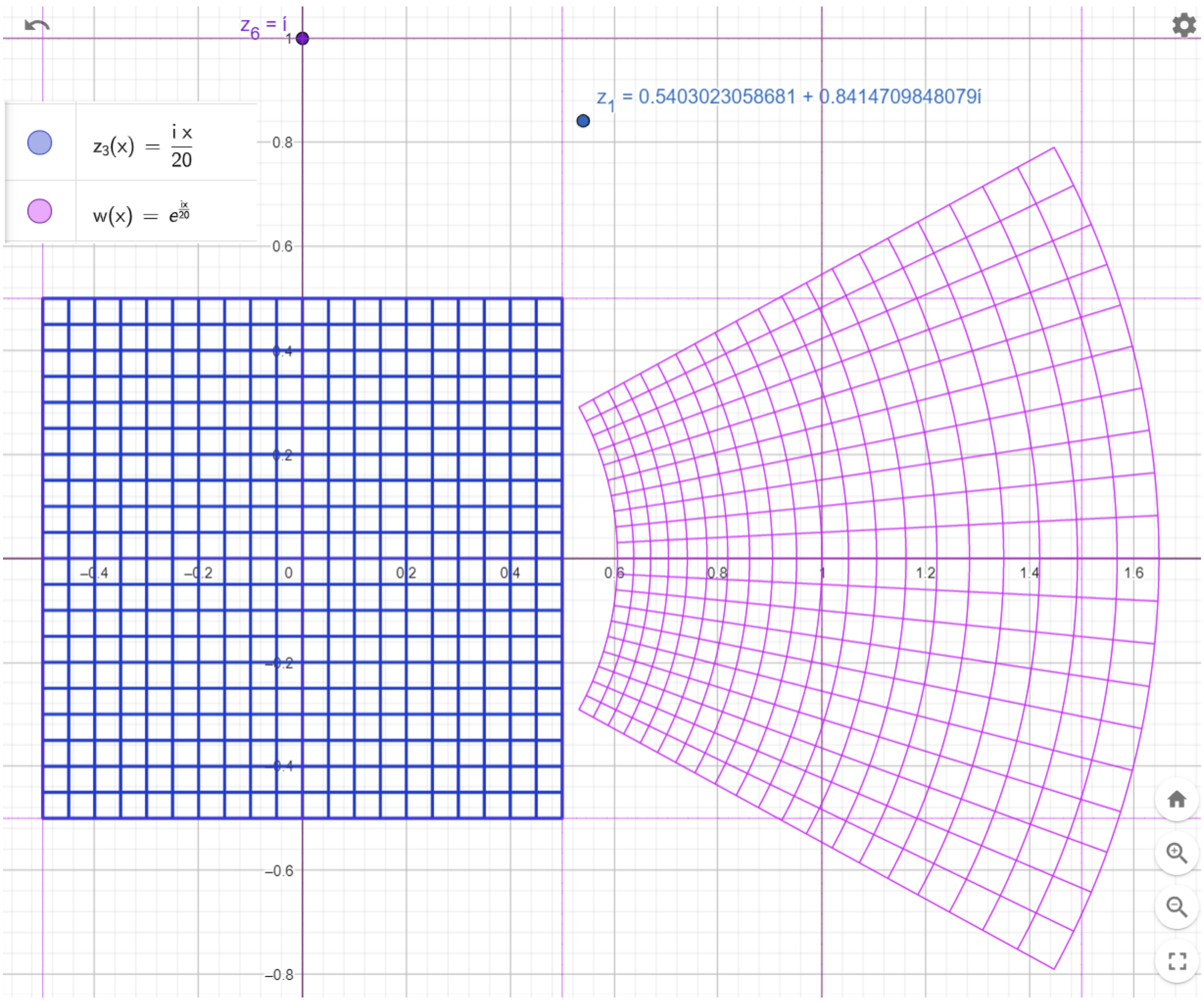
Also, this Cone 2-D representation in the complex plane have exactly 21 lines coming outside from the origin point [0,0]. These 21 lines are classified as exactly 5 groups, and each group have 4 lines except one group will have 5 lines and this extra line (line number 21) will be moving (rotating) between these 5 groups based on the applied operation. We will get through this in details with an example later.



*1.2.3 Apply both operations together, Raise [e] to the power of the scaled frame of reference.*

As we see in figure 5; the result of this both operations, [e] raised to the power of this scaled frame of reference, is nothing more than a sub folded representation for this scaled frame of reference square (a transformation for the frame of reference without keeping a fixed ratio for the square width and height).

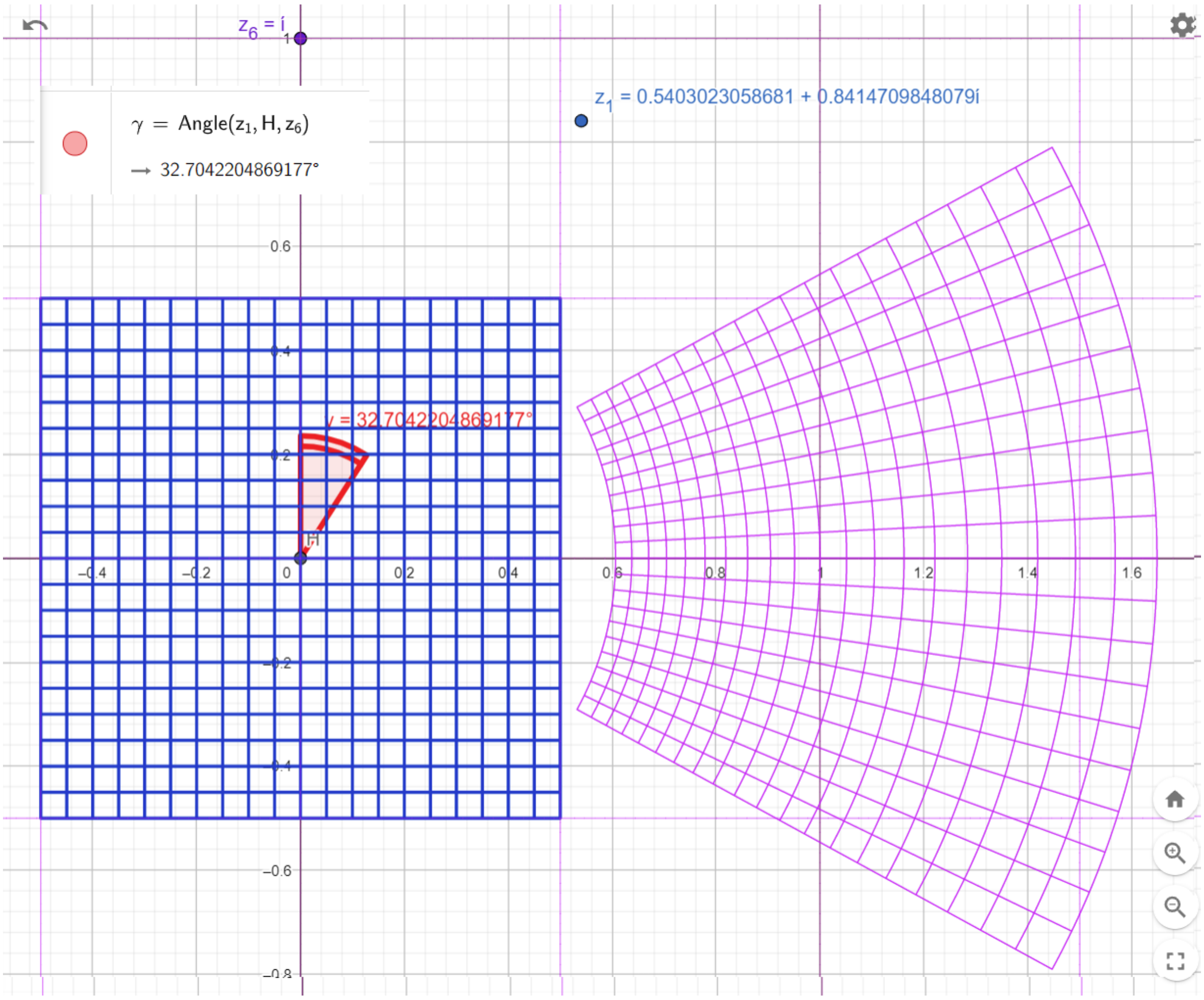
in the unfolding part in this paper, we will explore more the effect of the scaling value on unfolding in complex plane.



In next figure we will get the transformation angel for the frame of reference after applying these two operations. As you see the angel between point [z6=i] location on the frame of reference, and the location of z1= [e^ix] on complex plane, Θ = 32.7042204869177 degrees

And based on this we can represent each point on our frame of reference to a point in the complex plane using this formula. Which looks like the opposite of the gematric representation of Euler's Identity

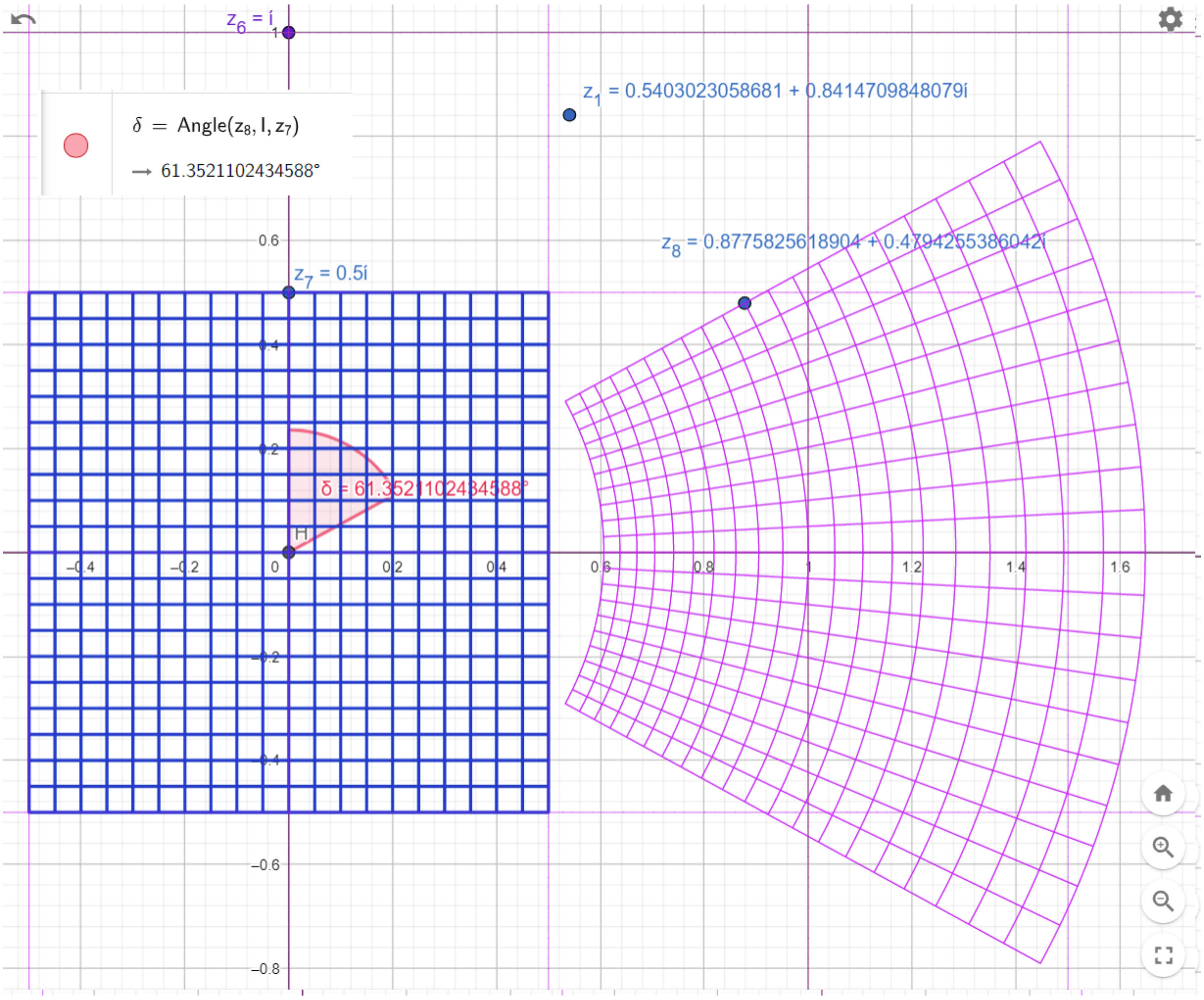
$$Z = \sin(\theta) \pm i \cos(\theta)$$



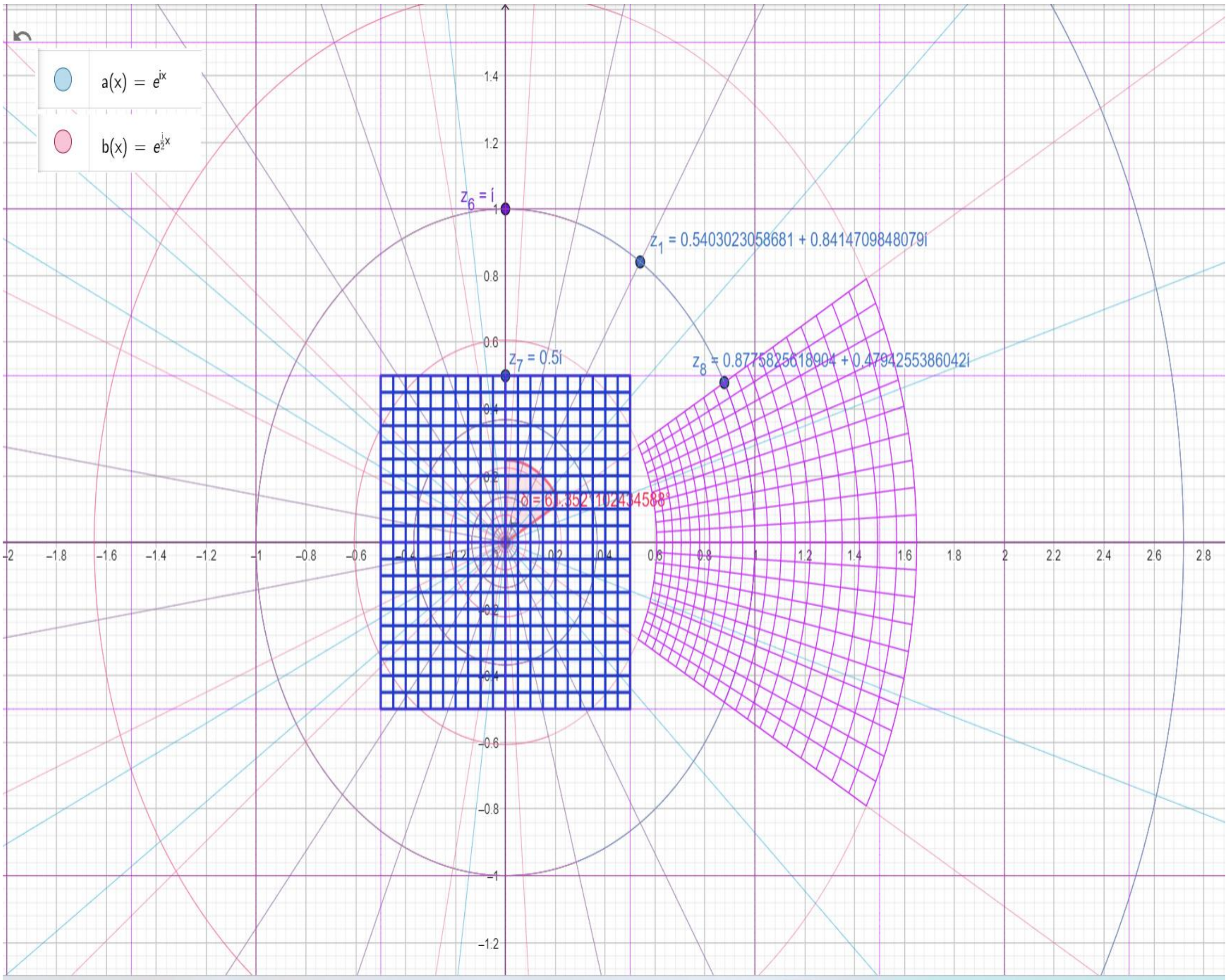
In figure 7; same as figure 6 but now we are getting angel for a point [Z8] on the final transformed frame of reference. As you see the angel θ = 61.3521102434588 degrees.

And the transformed point on the complex plane will be also using the formula.

$$Z = \mathrm{Sin}(\theta) \pm i\,\mathrm{Cos}(\theta)$$

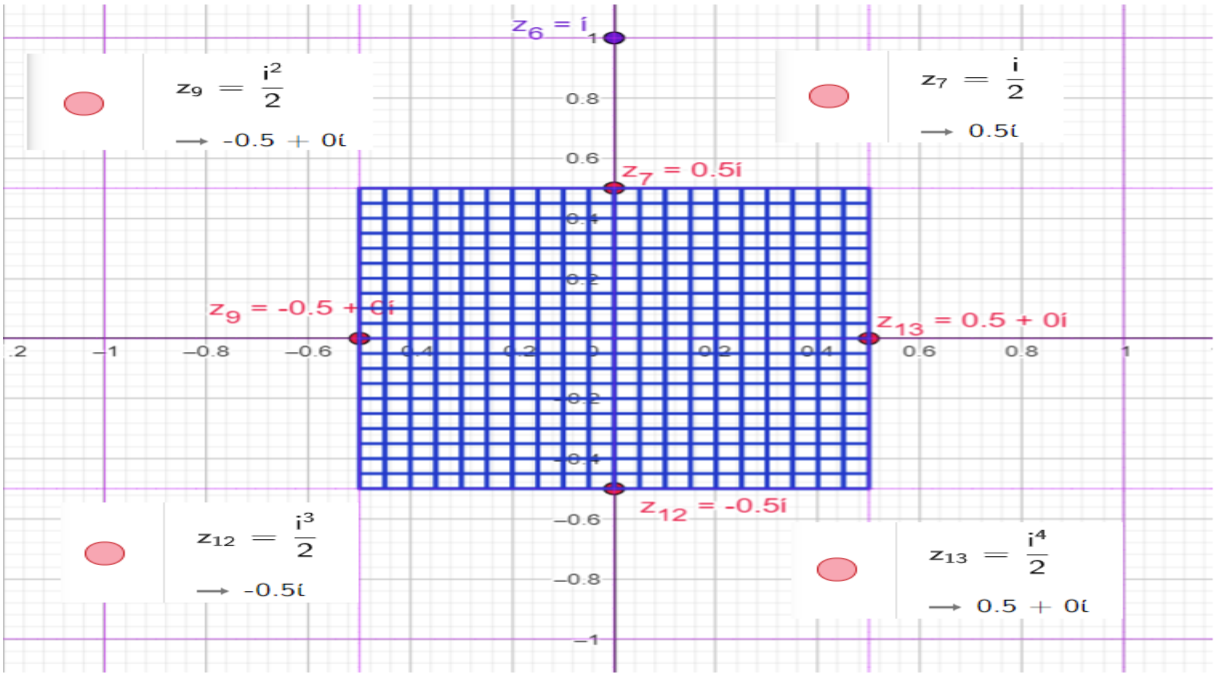


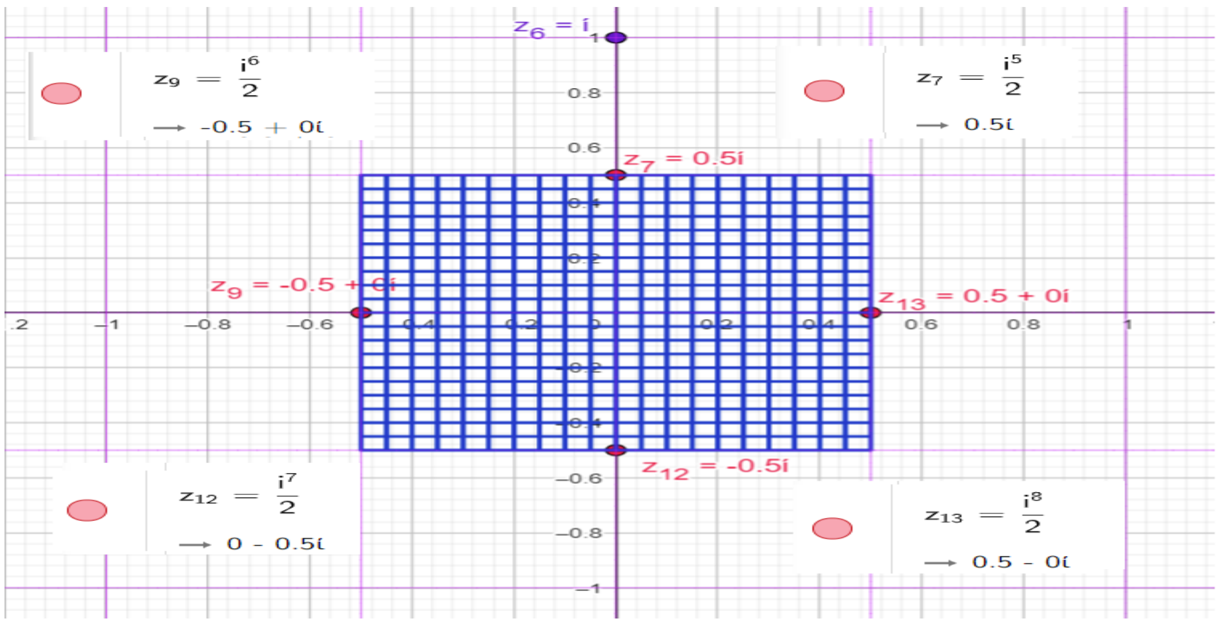
In figure 8; we combined all representations for operations in point 1, 2 ,3 altogether. As you see pints [Z6, Z1, Z8] all are located on the unit Circle of the complex plane or unit Circle of Euler's Identity.



*1.2.4 similarity between a frame of reference and Euler's identity unit circle cycle.*

*Frame of reference has the same cyclic concept in Euler's identity unit circle π cyclic, but frame of reference uses [i], [i^2], [i^3], [i^4] , ….*

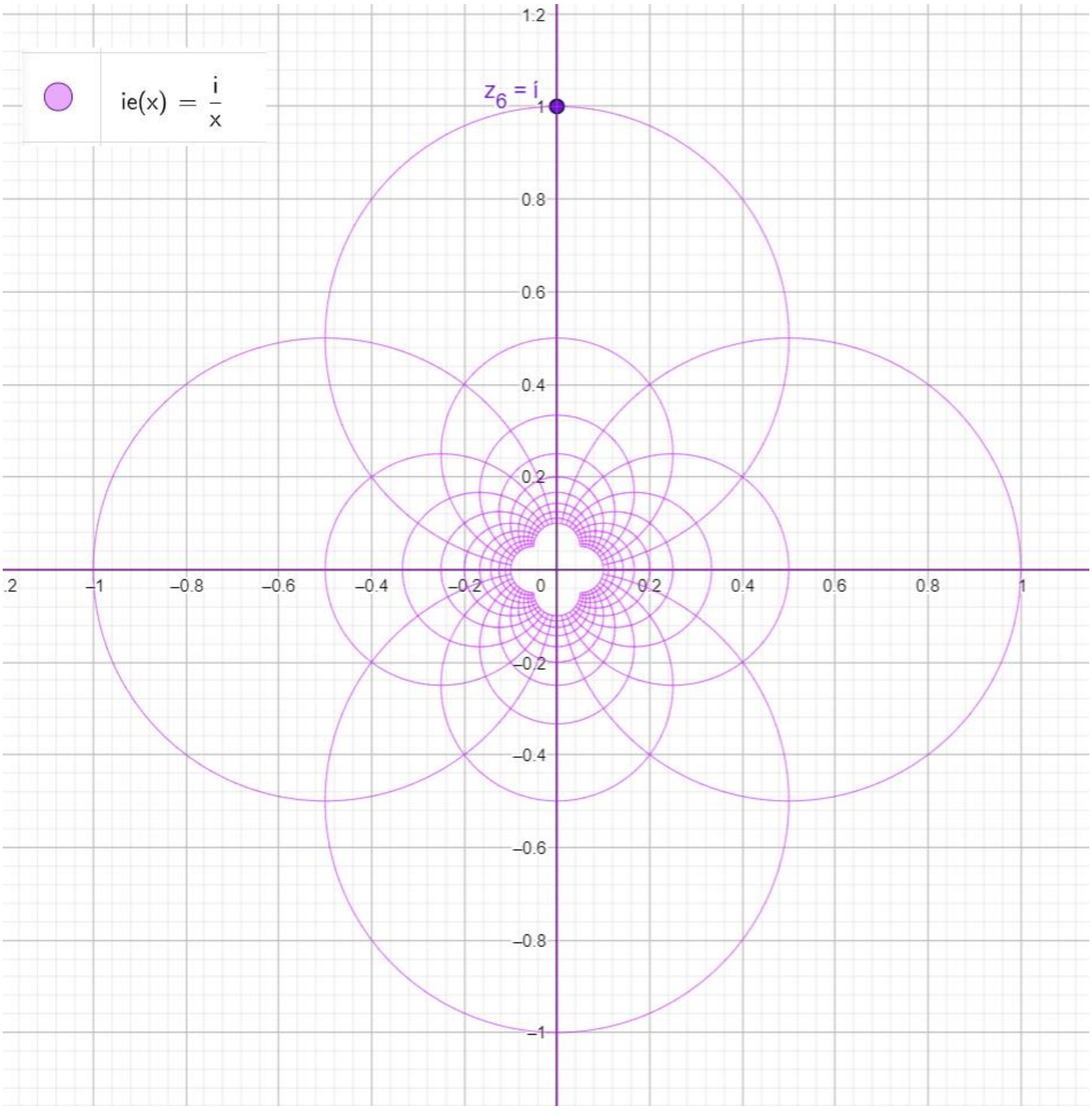


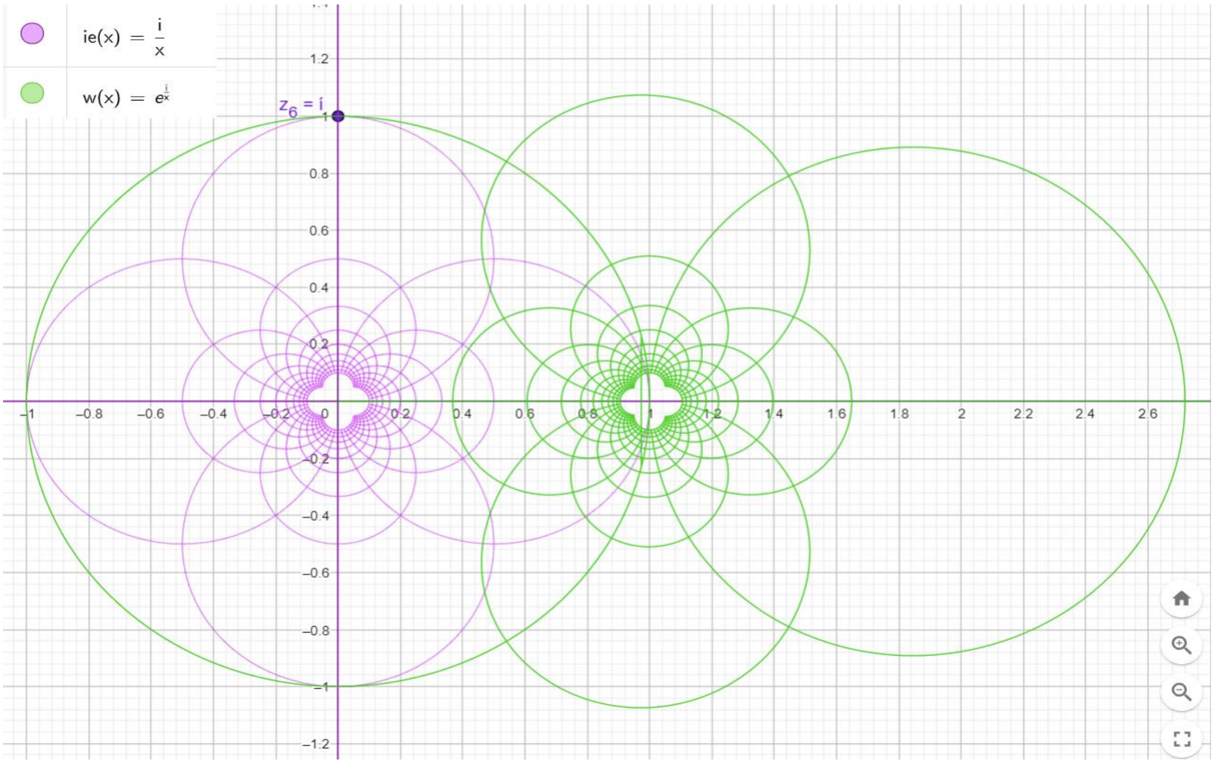
*1.2.5 multiply imaginary unit [i] by [1/X].*

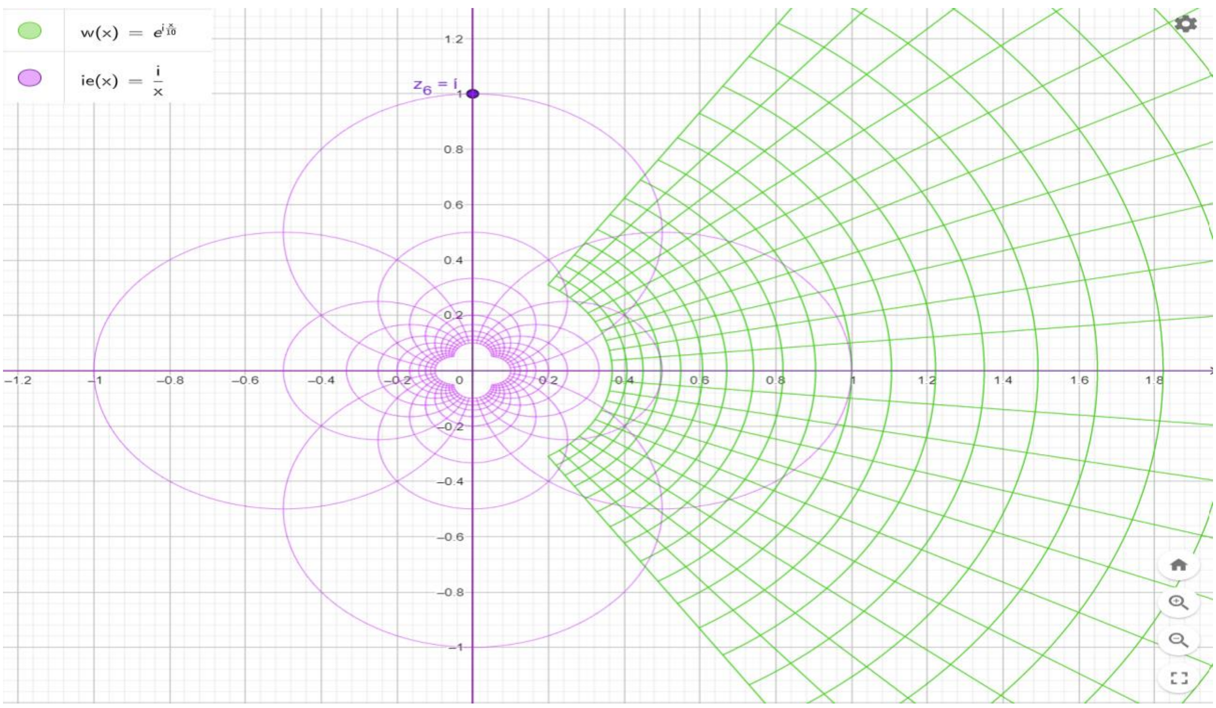
In figure 9; as you see the result of this transformations give us 4 intersected Cones at the origin of the complex plane and the transformation is exactly binding each strait line the frame of reference to the center of the complex plane. An leaving the main points at [ 1, -1, i, -i] not affected by this transformation.

One note here; all 4 cones are identical in this frame of reference transformation.



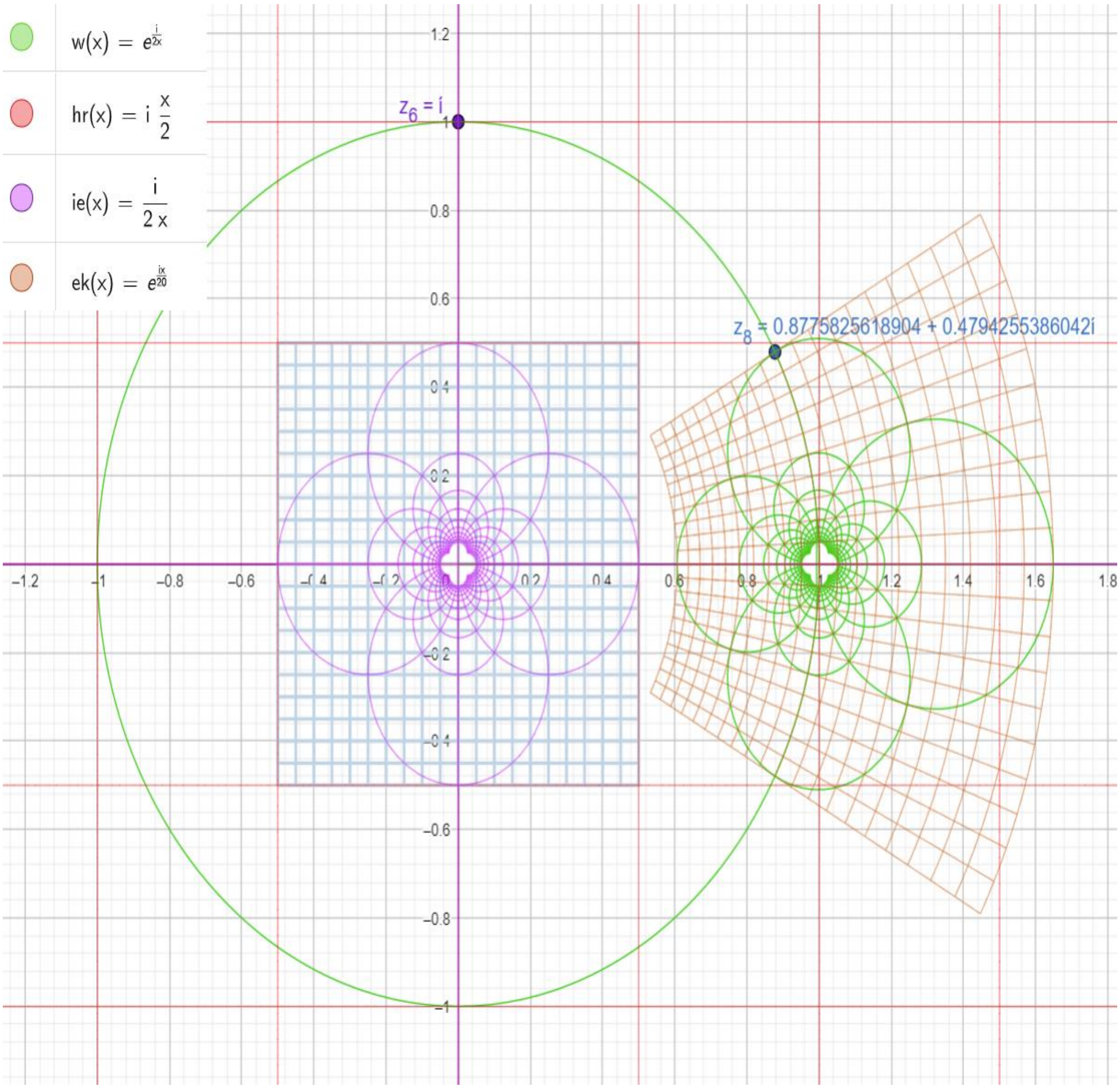
In figure 10; we combined both operations in one figure raise [e] to the power of a multiplication of frame of reference by [1/x]. same as we saw in previous transformation there are some distortions in the binding for [e^i/x] the transformation ration for the shape sides is not fixed. While the frame of reference transformation [i/x] keeps a fixed ratio for the shape sides.



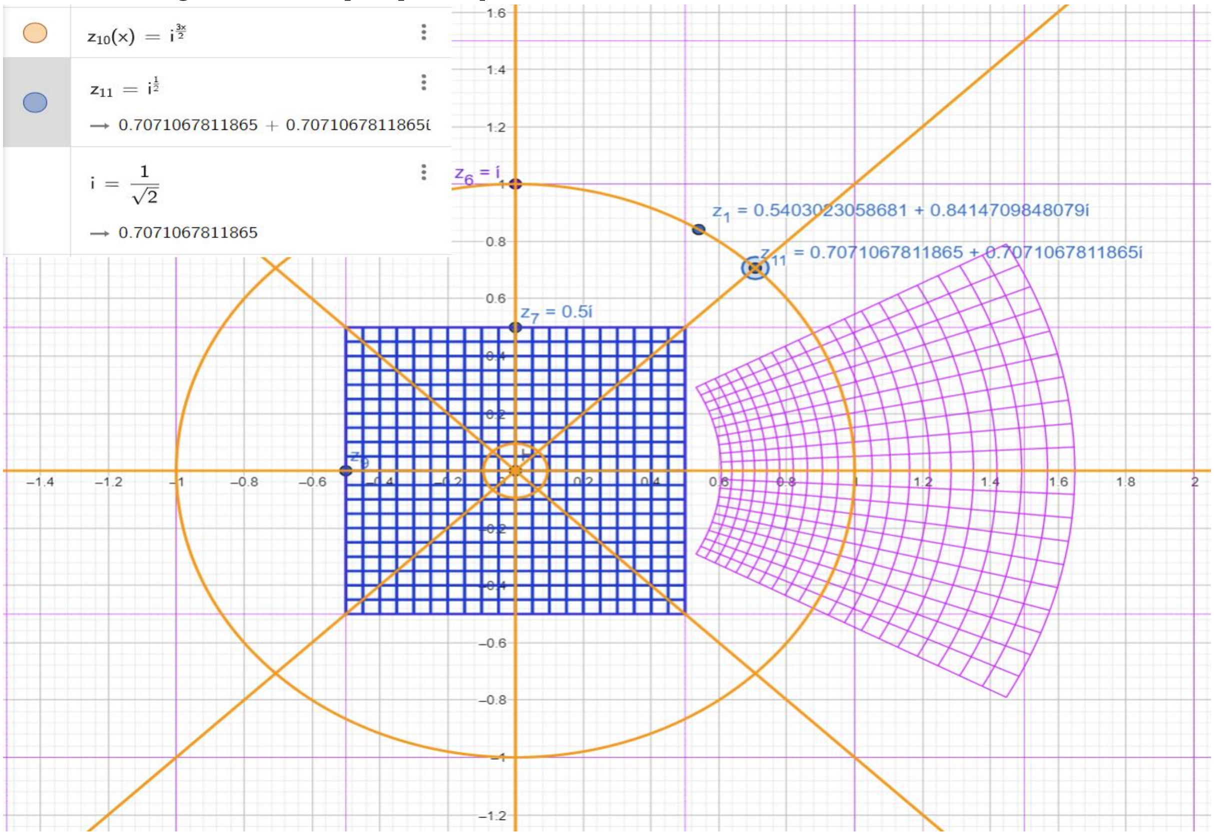



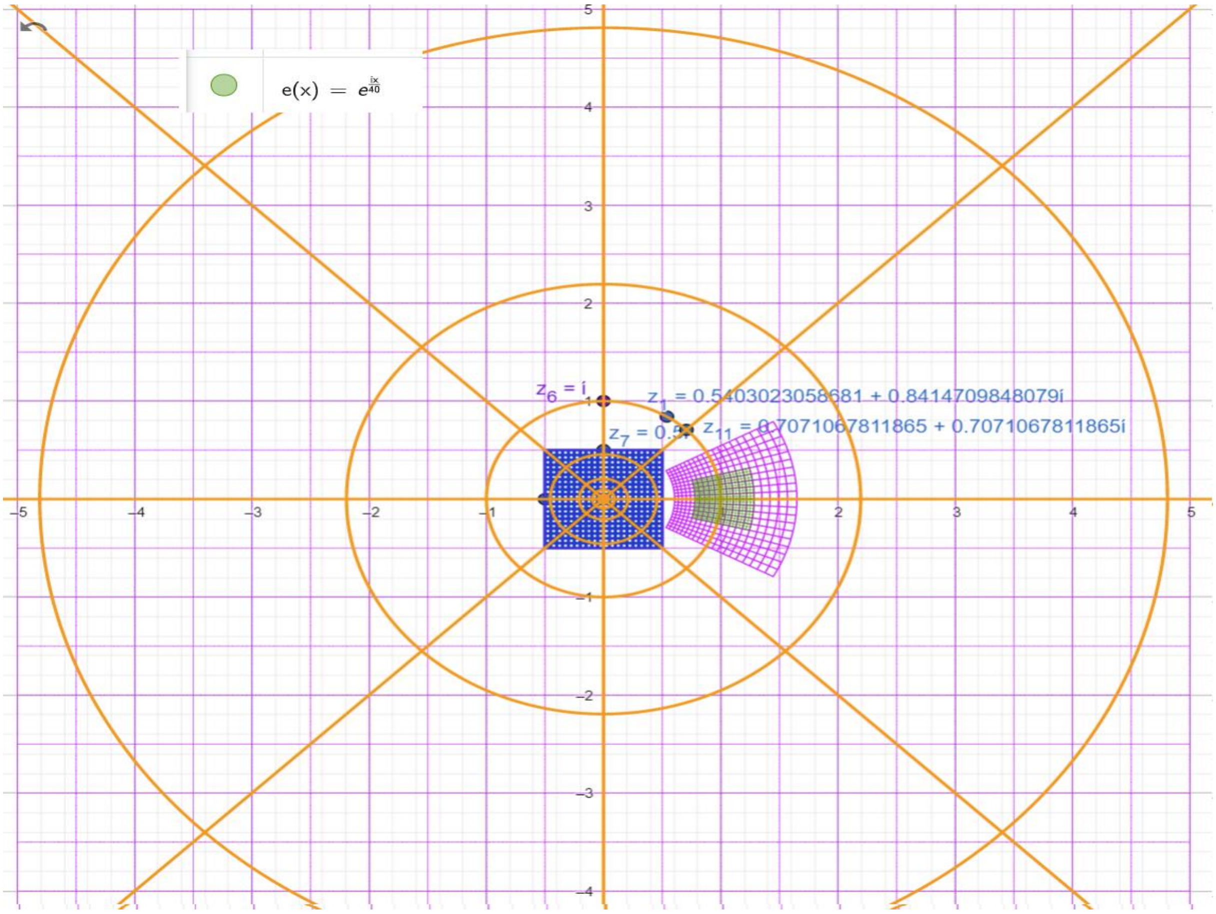
in figure 11; we combined more than one transformation in one graph.

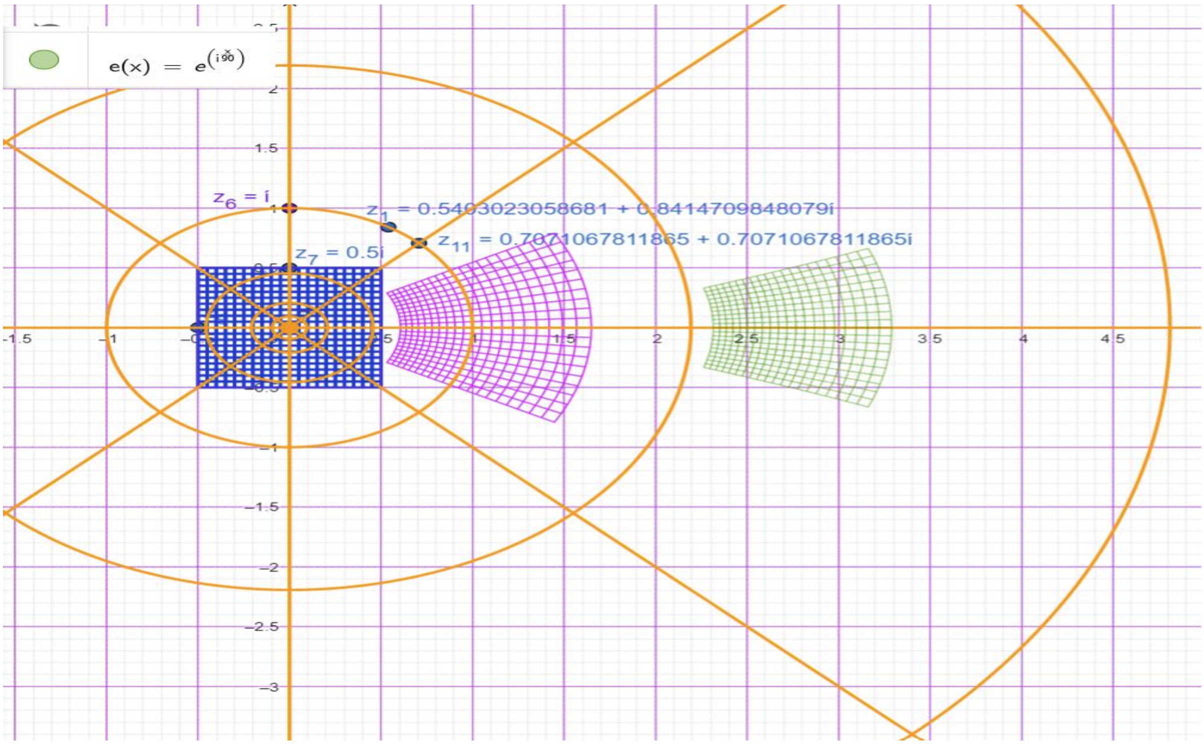
The main note here, frame of reference transformation all time keeps the shape sides ratio fixed without any distortion due to the transformation which gives us the privilege of discreteness values in a continues space



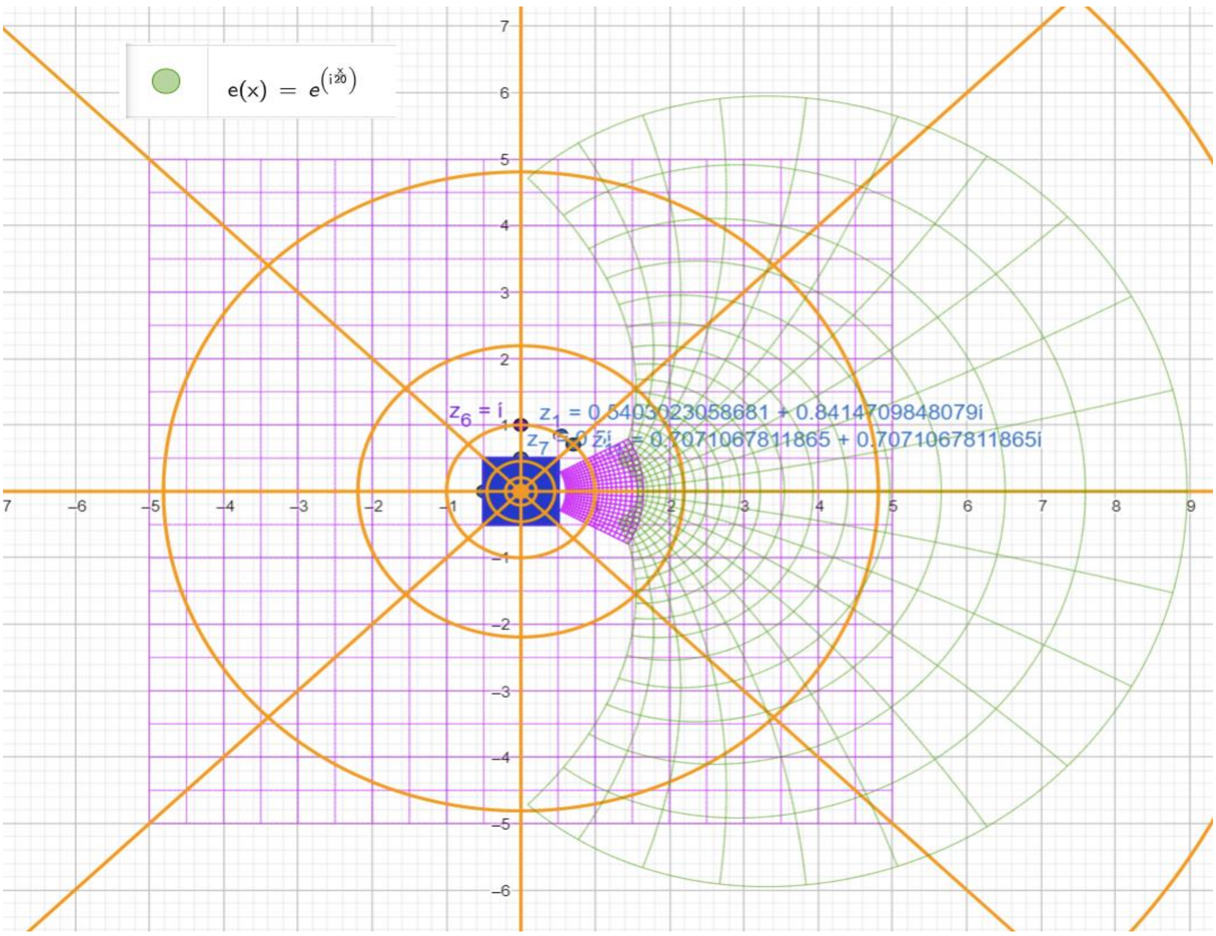
In Figure 12; Figure 13; Figure 14; Figure 15; another series of transformations for square root of [i] for example [i^3x/2]. which give us the complex plane square root of 2.





$e(x) = e^{\left(i\frac{x}{90}\right)}$
$z_6 = i$
$z_1 = 0.5403023058681 + 0.8414709848079i$
$z_{11} = 0.7071067811865 + 0.7071067811865i$
$z_7 = 0.5i$

$e(x) = e^{\left(i\frac{x}{20}\right)}$
$z_6 = i$
$z_1 = 0.5403023058681 + 0.8414709848079i$
$= 0.7071067811865 + 0.7071067811865i$

## 2. Unfolding and manifold of frame of reference

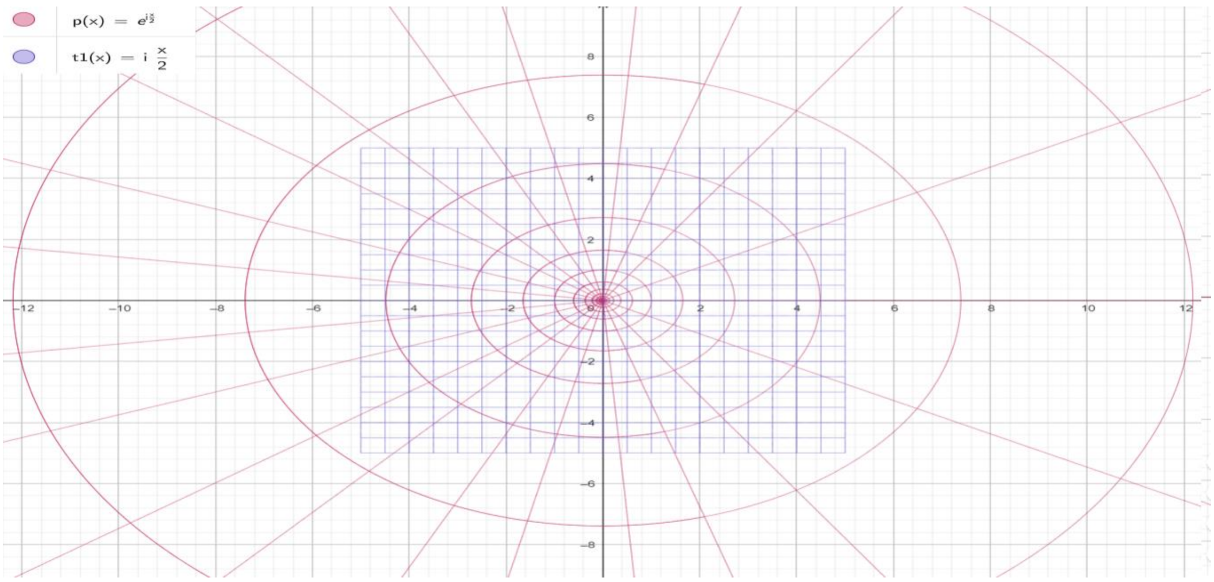
*1.2 unfolding frame of reference transformation in complex plane*

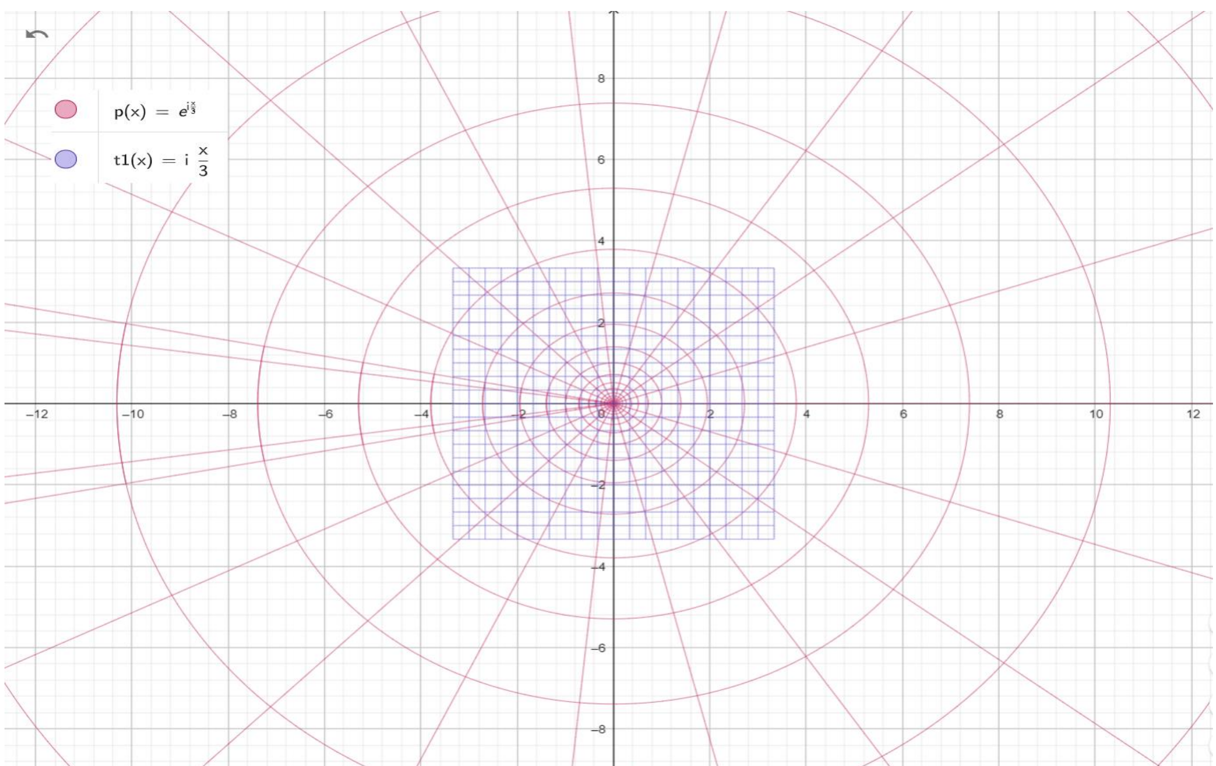
For unfolding we are going to scale frame of reference by a fraction less than 1. Like scale frame of reference by [1/2 or 1/3 or 1/4 or 1/5 …. Or 1/100]. In the next Figures in this section, we will see how decreasing the scale fraction will unfold our frame of reference transformation.

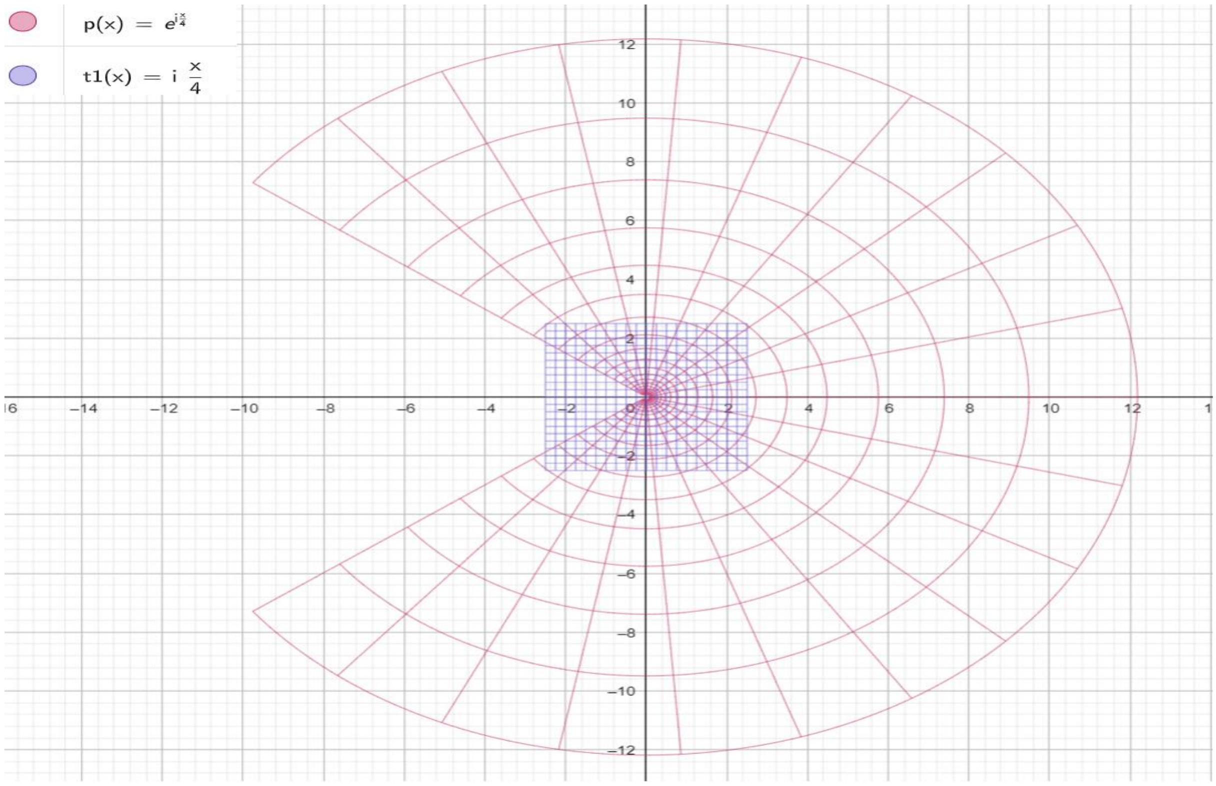
We can see the distortion in the frame of reference transformation decreases as we scale by smaller fraction. This can be interpreted as we move towards infinity the distortion in frame of reference transformations will be decreases.

Strat from Figure 17; for scaling by [x/3] we will start to see a clear unfolding. And as the scale fraction decreases the transformed frame of reference moves away from the original frame of reference until it reaches the unit [1] it stops, mostly because the limitation of the canvas of imaginary unit circle in the complex plane.

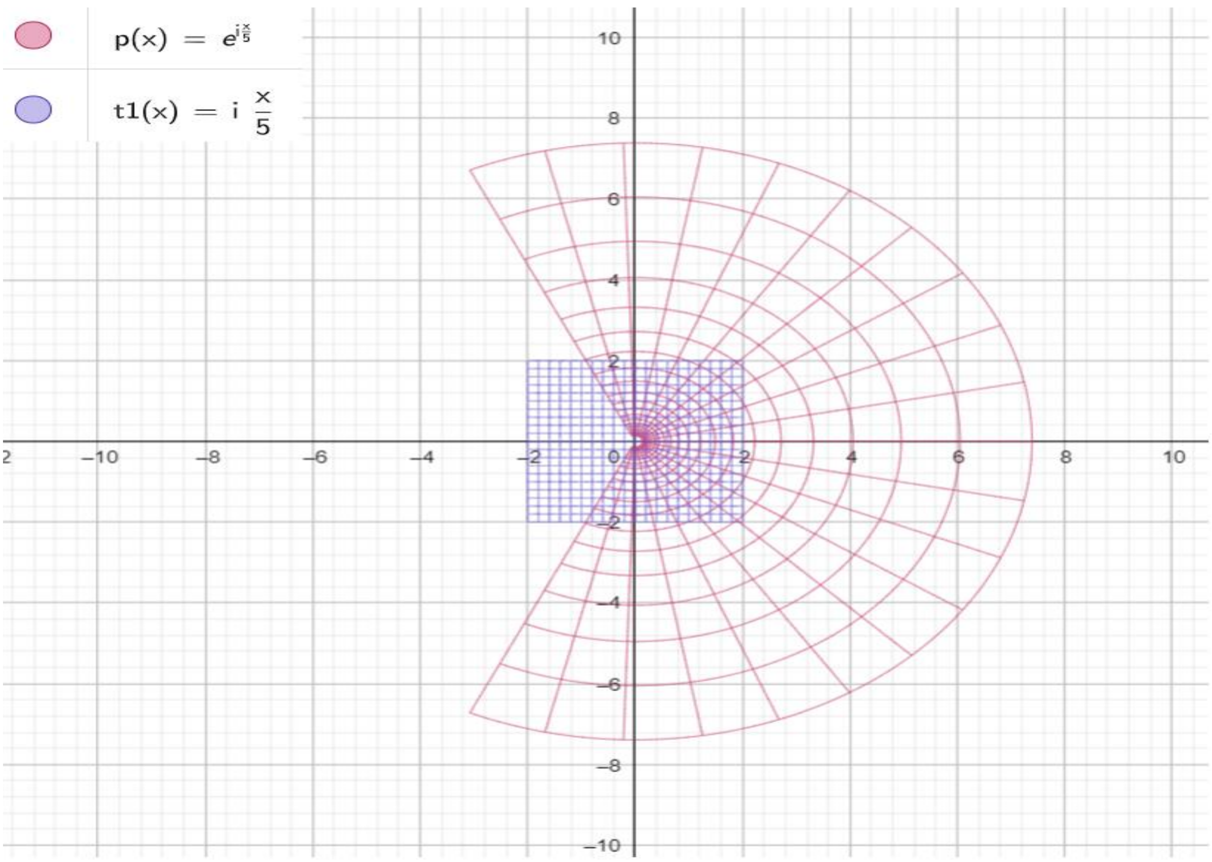
As we see Figures 16; the transformation is [X/2] and [e^X/2]



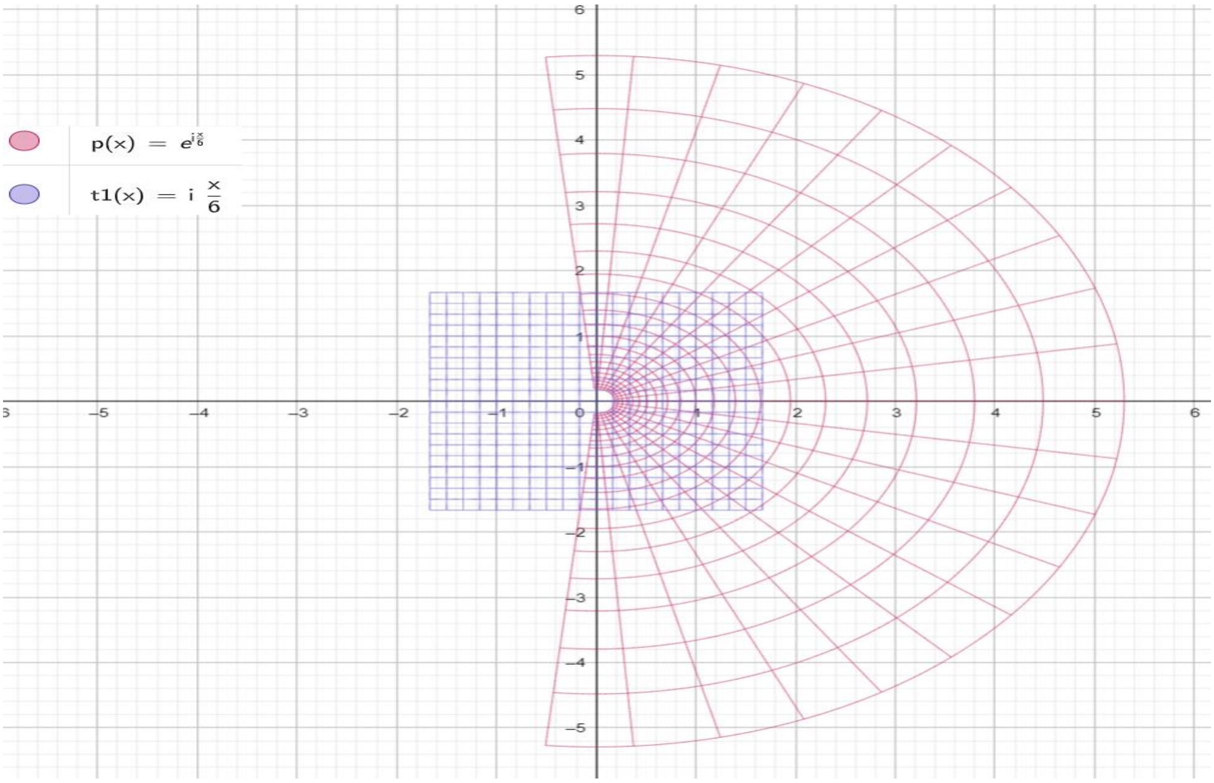





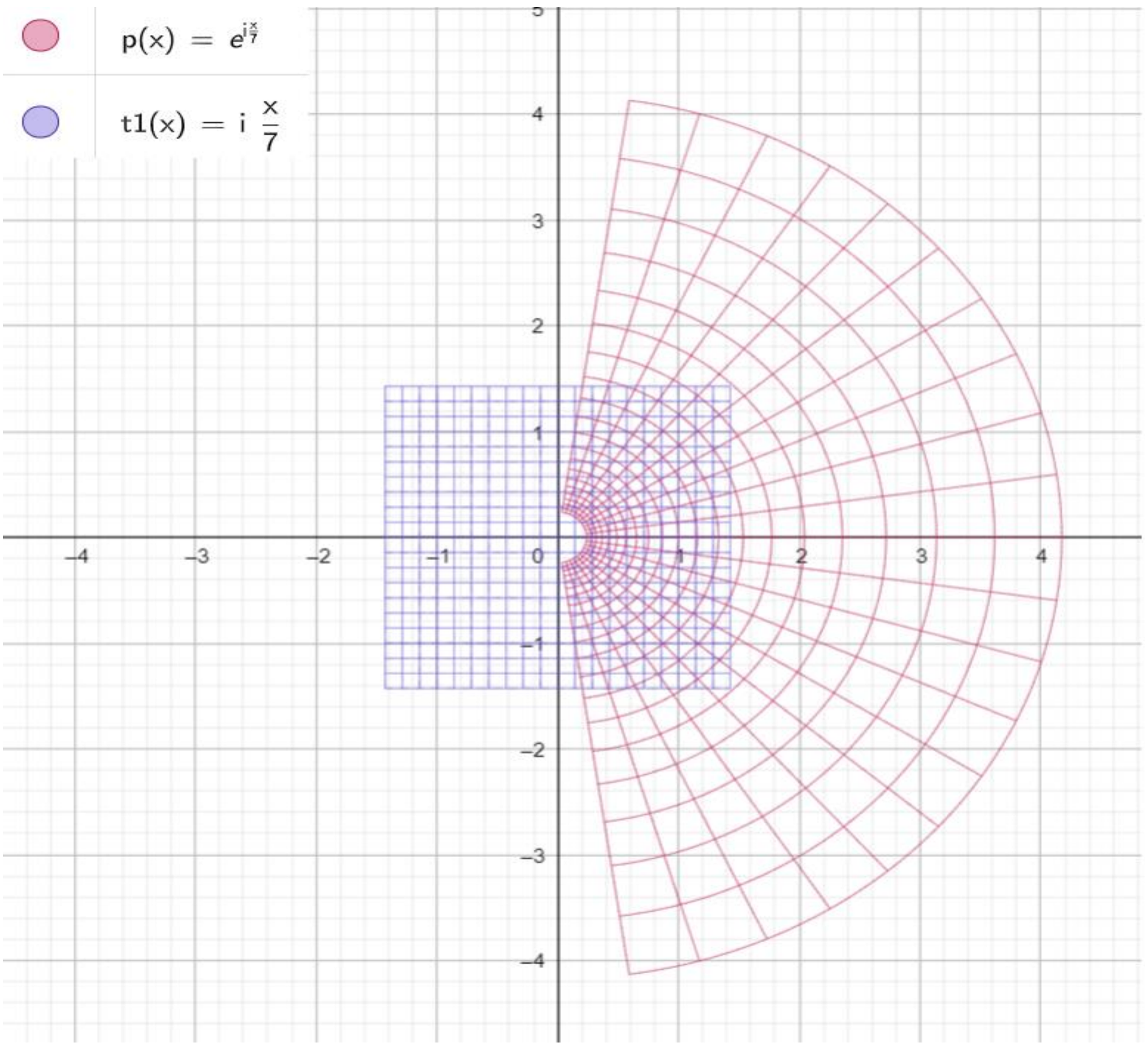
$p(x) = e^{i\frac{x}{4}}$
$t1(x) = i\frac{x}{4}$

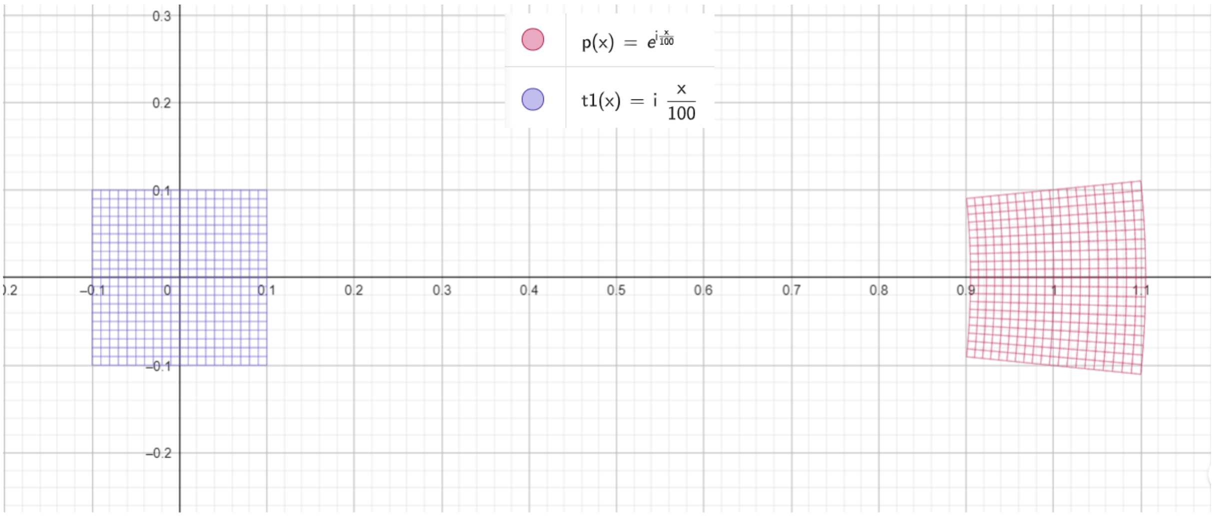
$p(x) = e^{i\frac{x}{5}}$
$t1(x) = i\frac{x}{5}$

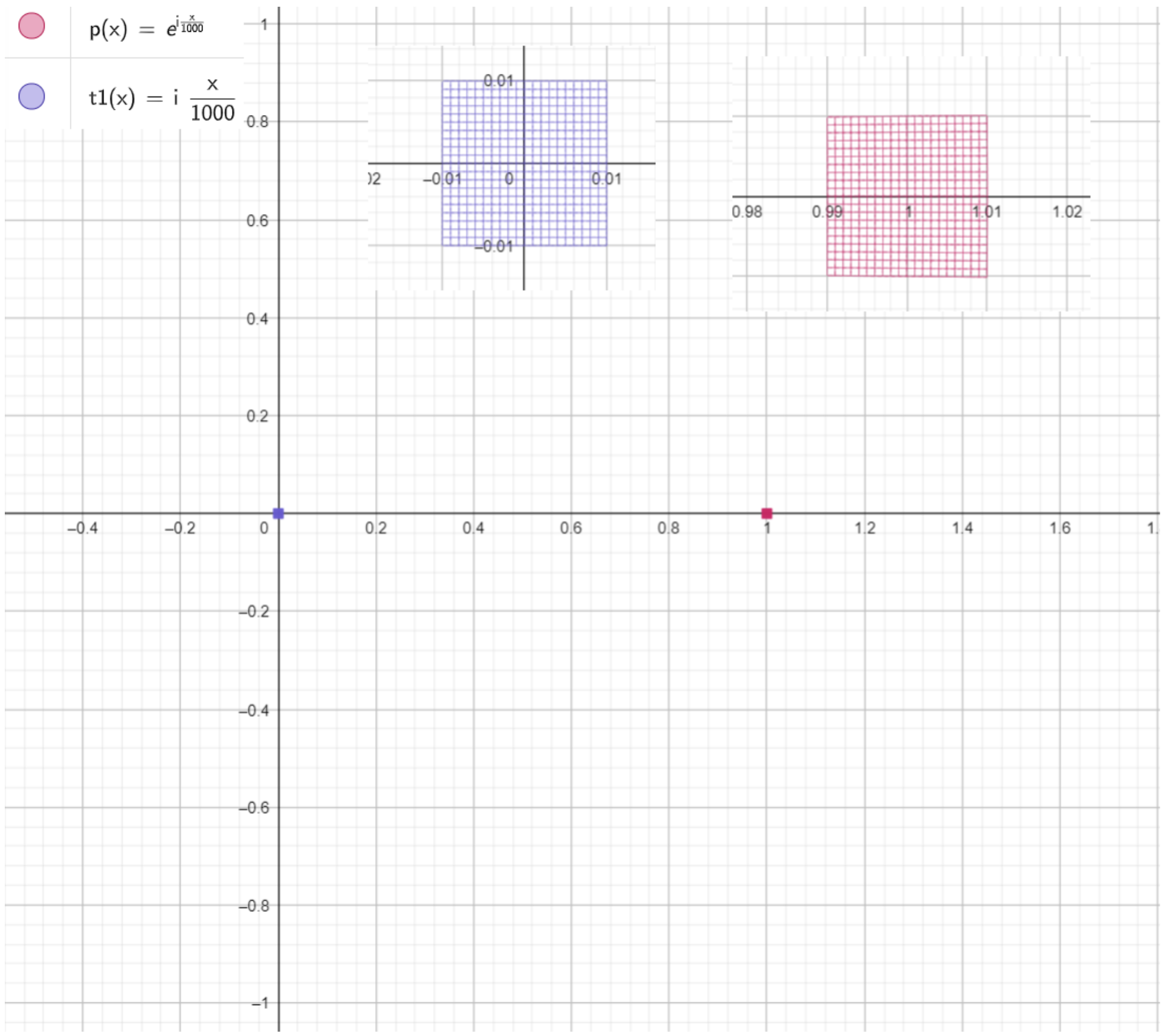


$p(x) = e^{i\frac{x}{6}}$
$t1(x) = i\ \frac{x}{6}$

$p(x) = e^{i\frac{x}{7}}$
$t1(x) = i\ \frac{x}{7}$

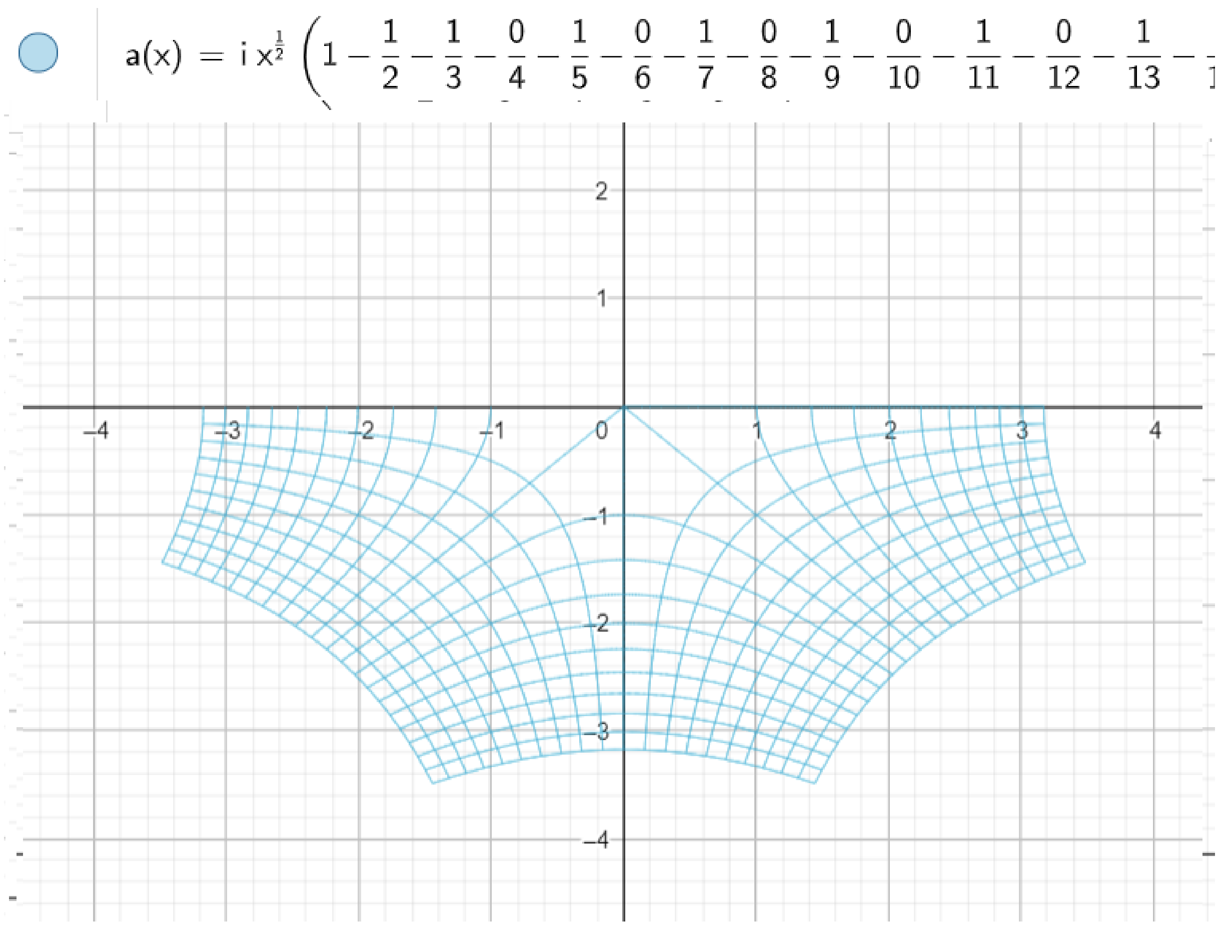

$p(x) = e^{i\frac{x}{100}}$
$t1(x) = i\frac{x}{100}$


$p(x) = e^{i\frac{x}{1000}}$
$t1(x) = i\frac{x}{1000}$

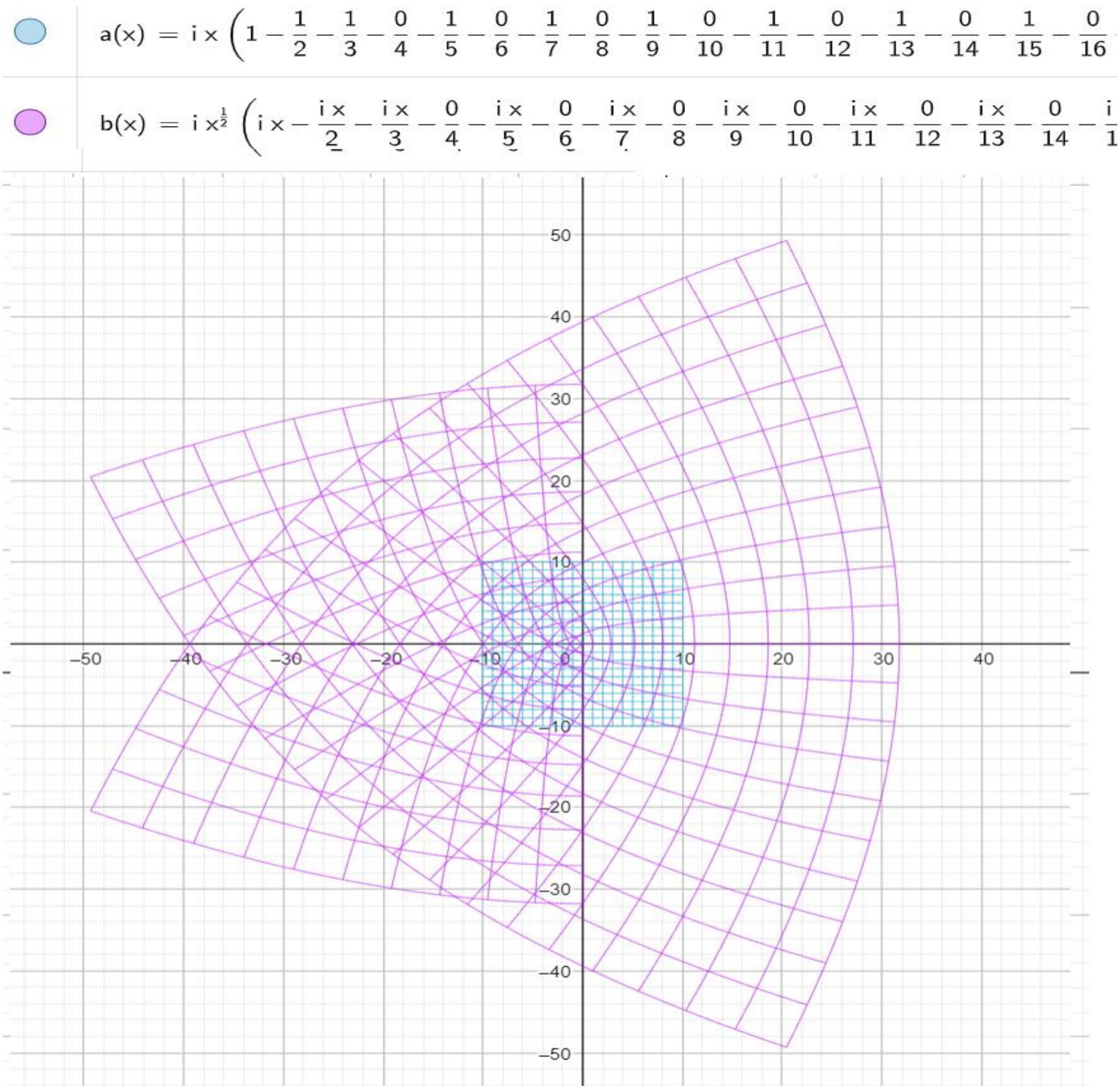
*2.2 manifold frame of reference transformation in complex plane*

*Raising [X] to a power will give us the effect of manifold in complex plane. As the power increases*

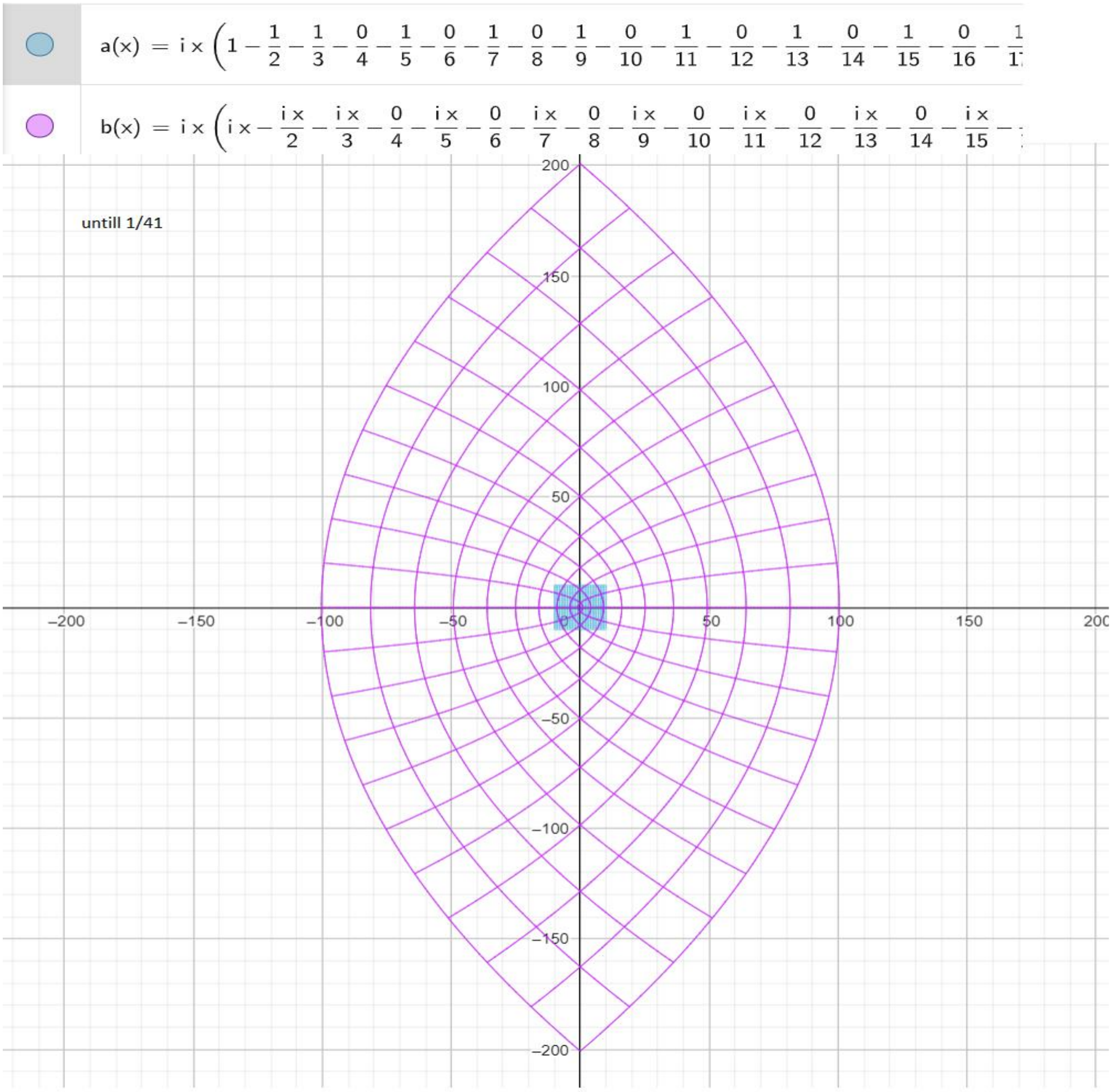
*In Figure 24; the result of [sqrt(X)] and with only odd numbers. As you see it looks like cut the frame of reference in center and shift along the x axis*



*In Figure 25; the result of [1/sqrt(X)] transformation with only odd numbers. As you see it looks like we start to fold the frame of reference square*

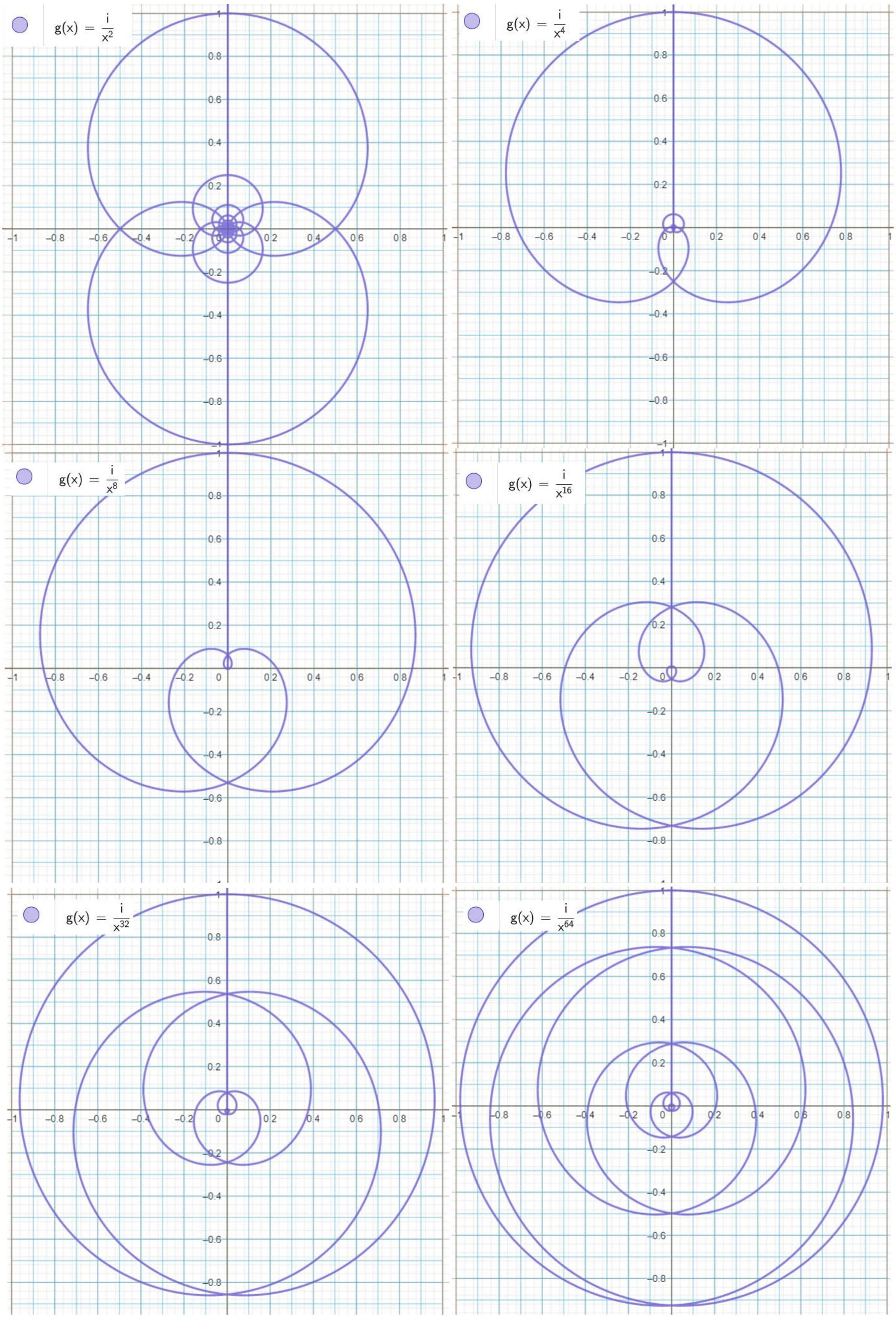


In Figure 26; the sum of [x^2] transformation for frame of reference for odd numbers from 1 until [1/41]
And because we in base 10 system. The fold shape is bounded by [-x^2, x^2] i.e. [-100 , 100]

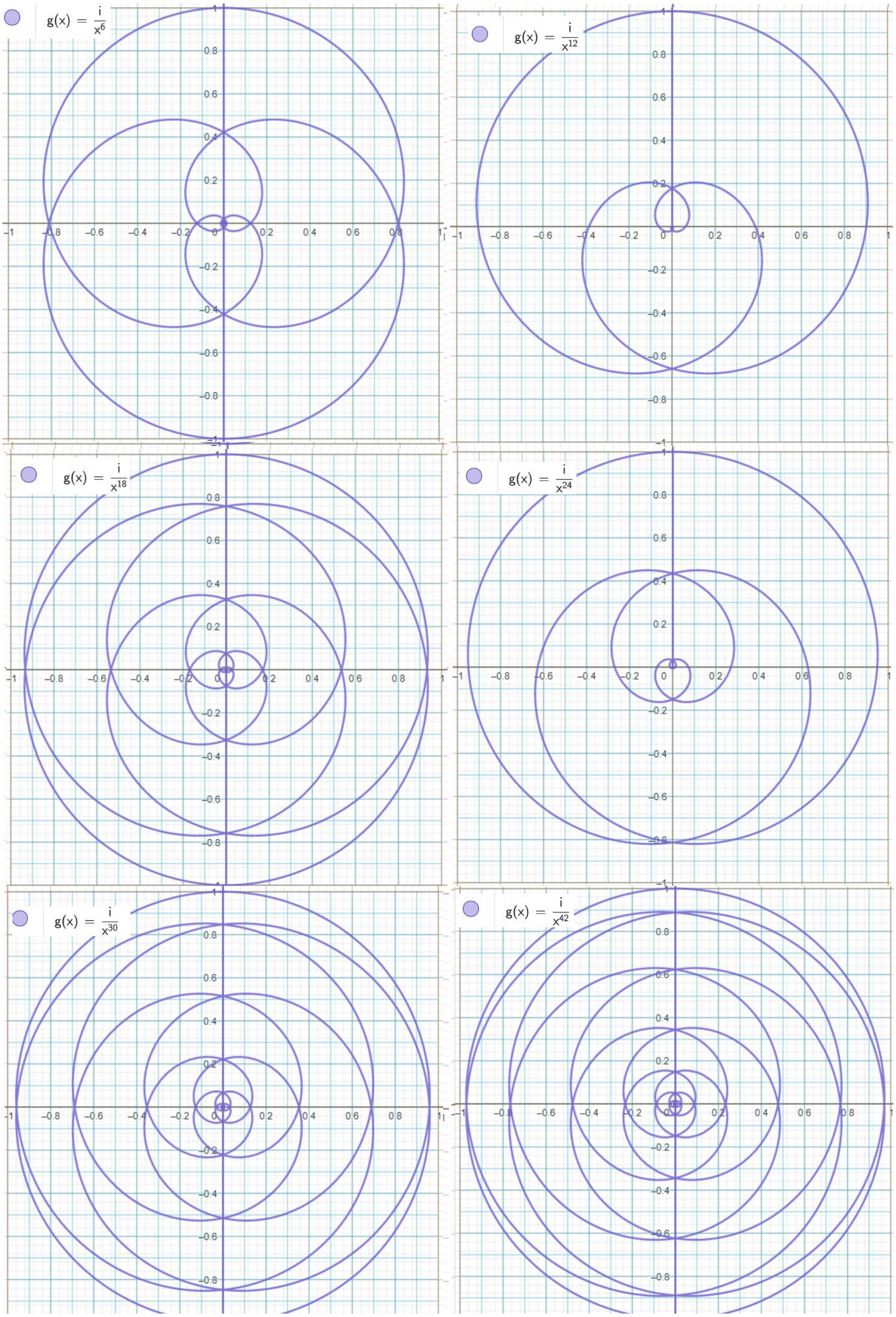


*2.3 patterns in complex plane manifold of a frame of reference transformation*

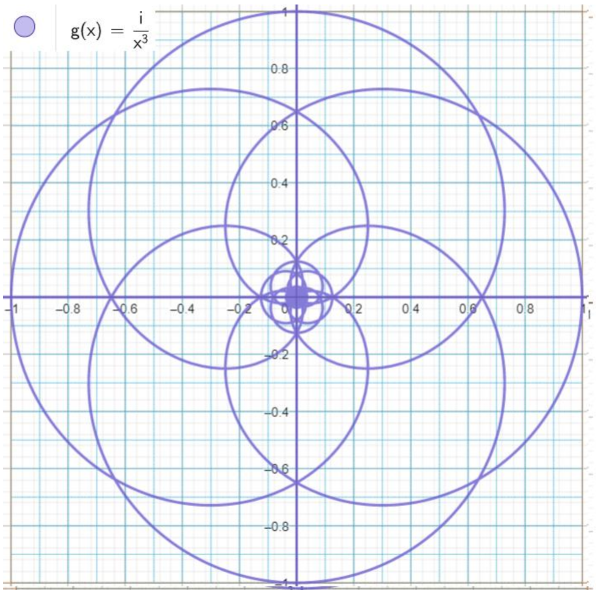
*2.3.1 pattern for [1/x^2^n]*

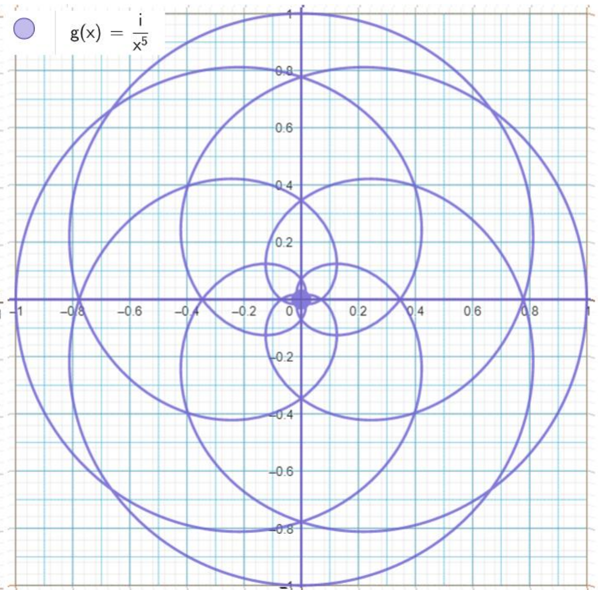


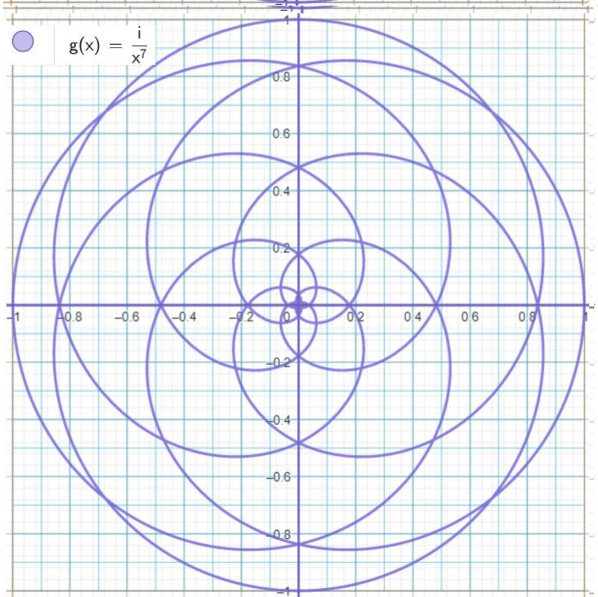
*2.3.2 pattern for [1/x^6*n]*

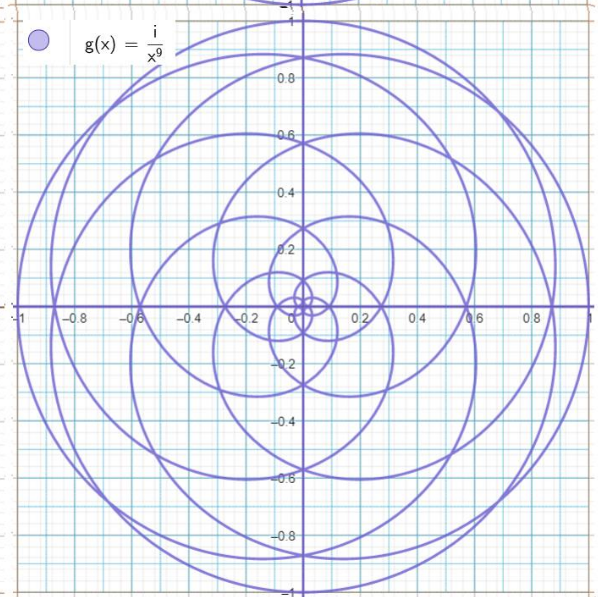


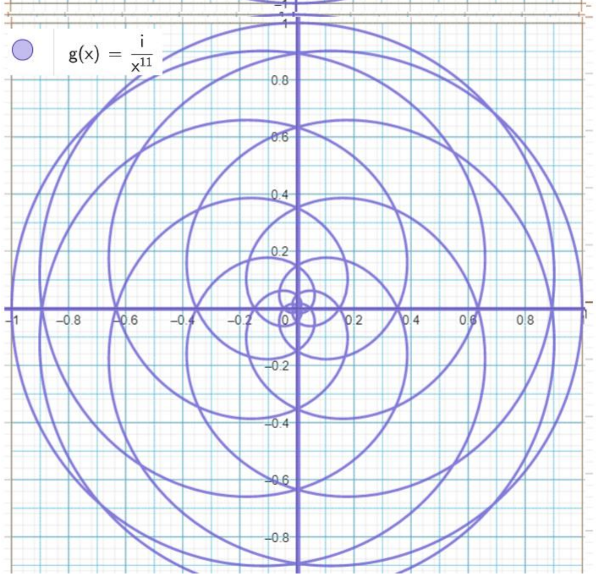
*2.3.3 pattern for [1/X ^ n] where n is odd number*

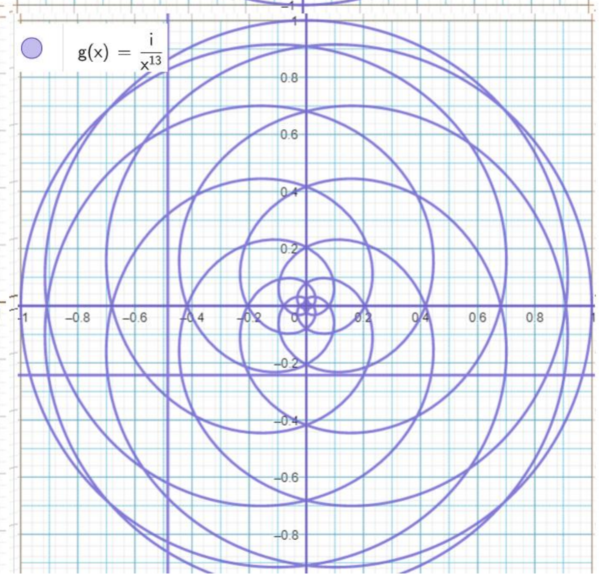


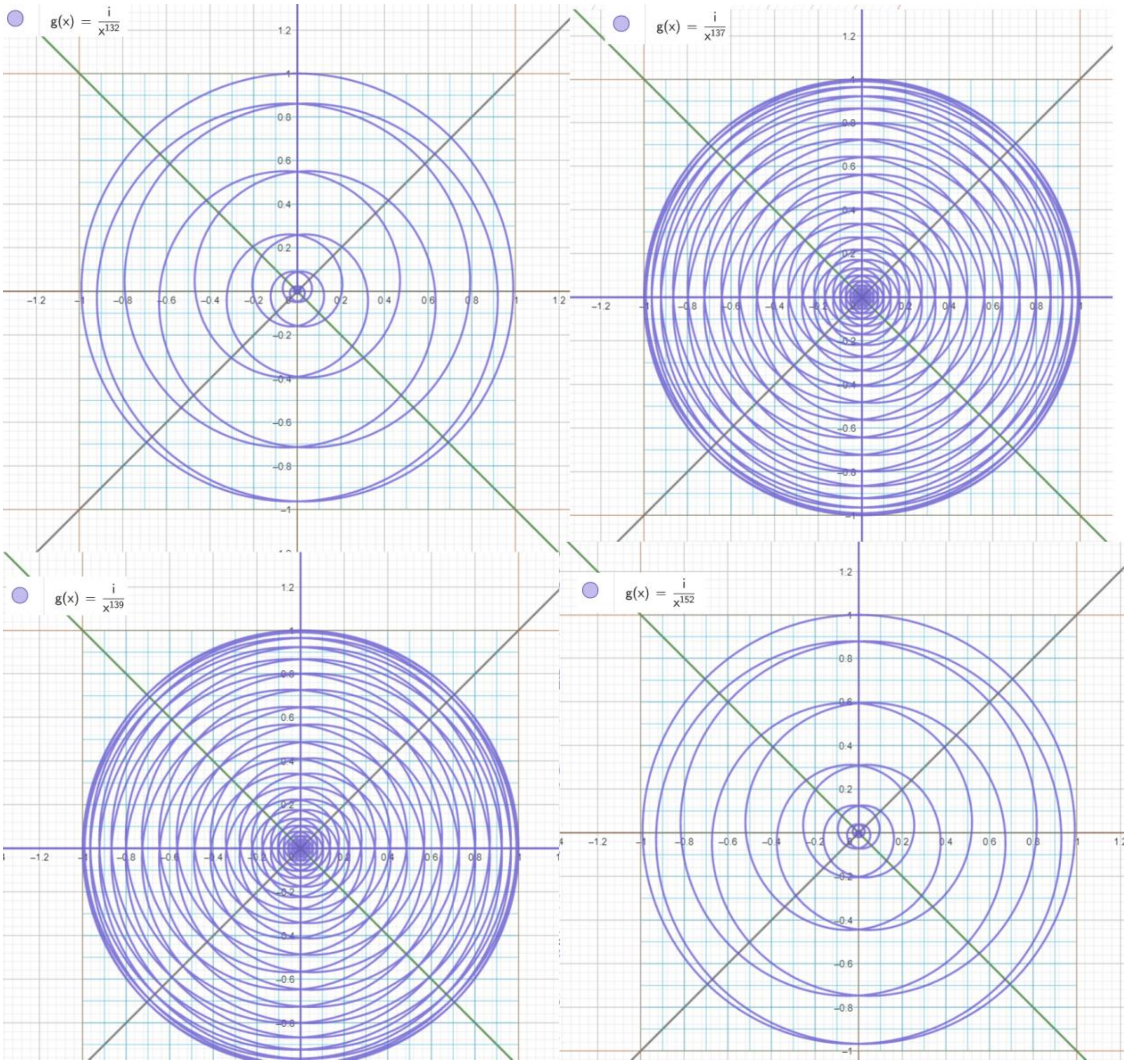



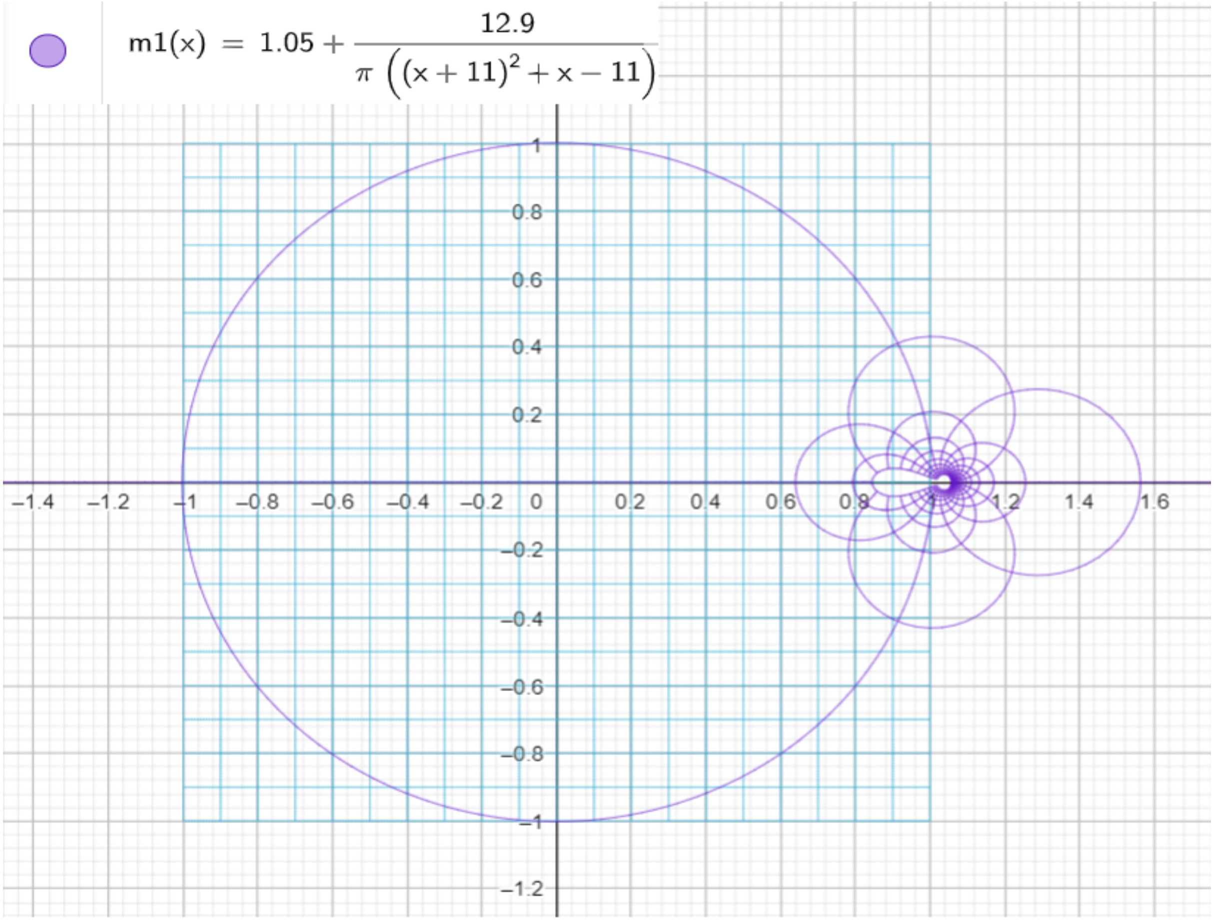



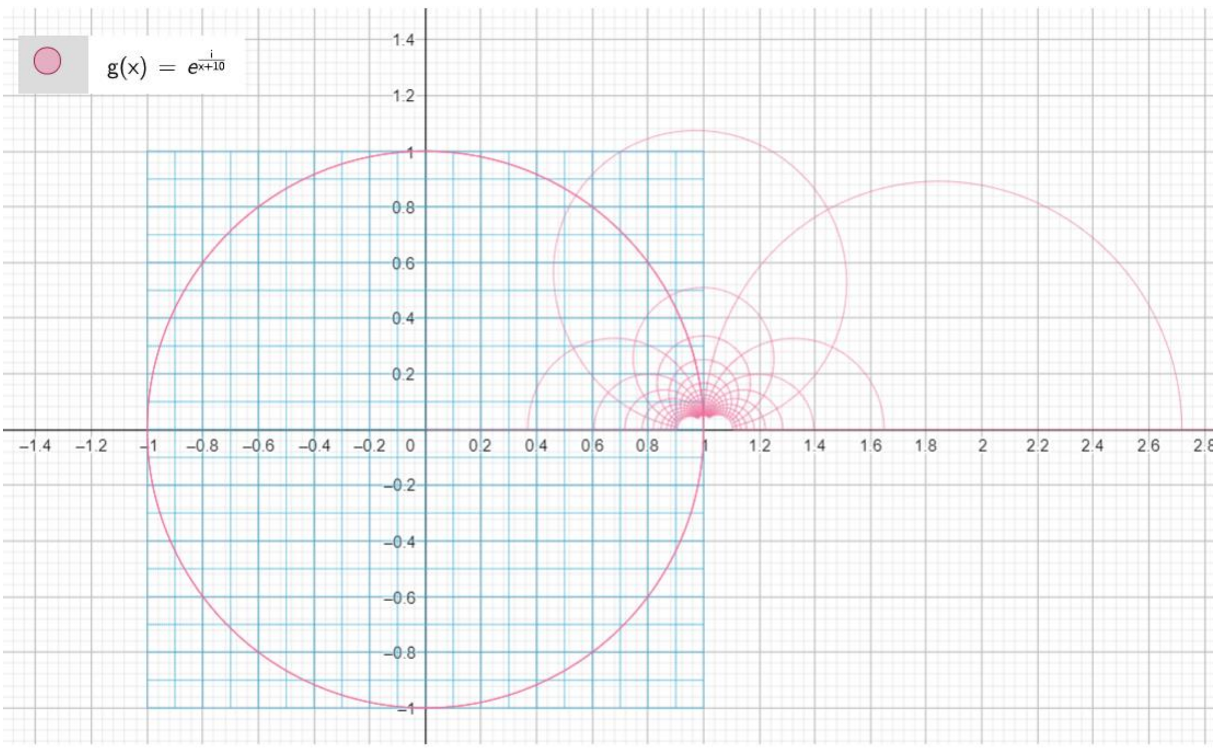
*2.3.2 difference in pattern for [1/x^n] when n is even and when n is odd.*

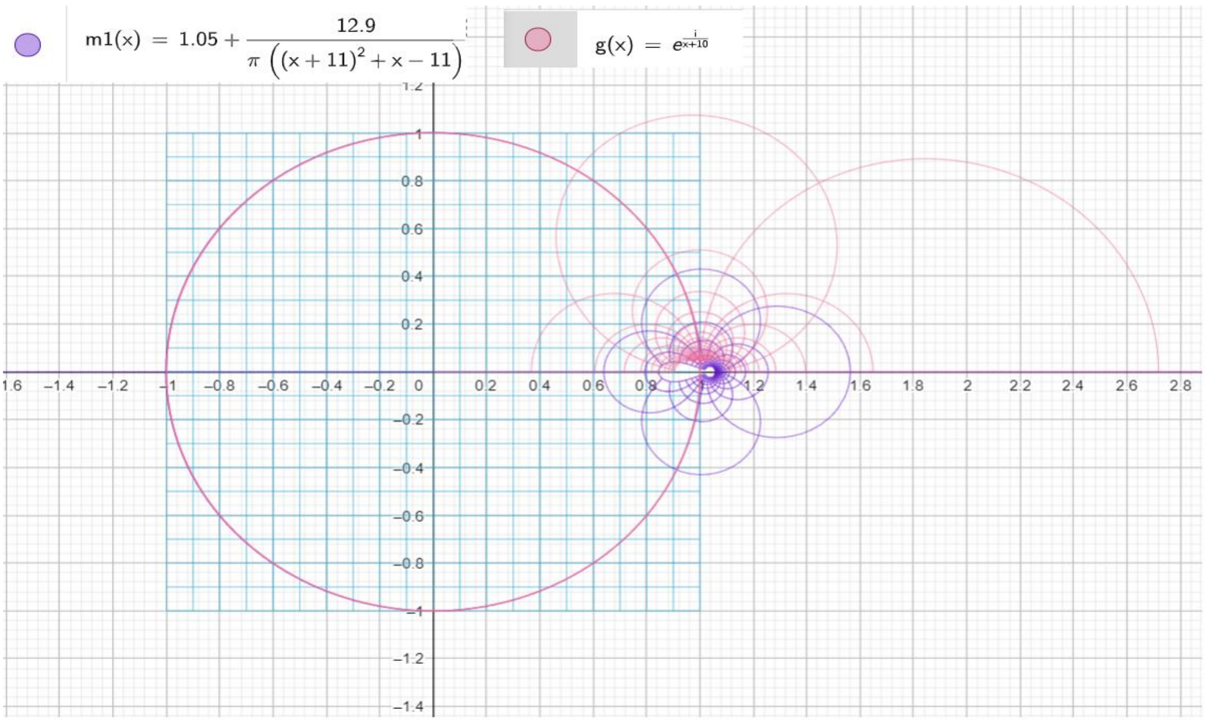
*When the power to raise n is an odd number it will intersects at $\theta$ = 45 degree. but even powers only intersect at $\theta$ = 0 or 90 or 180 or 360.*

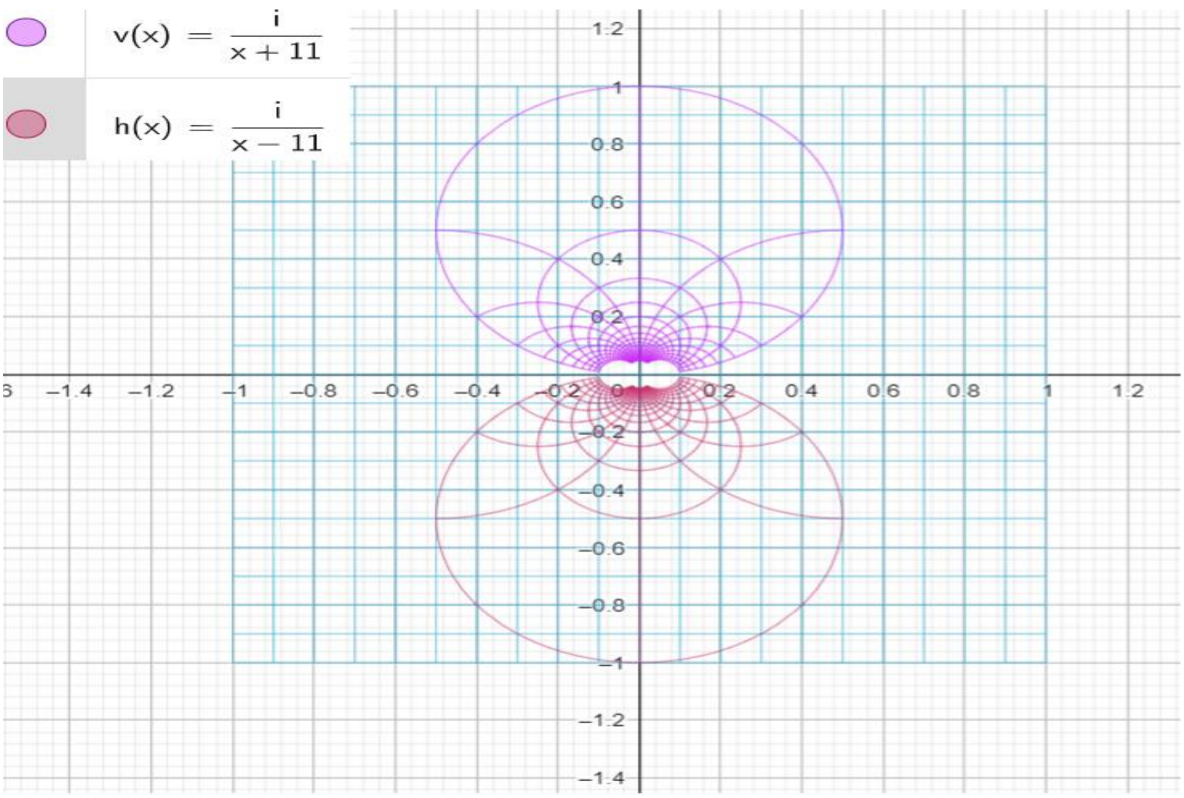


*2.3.2 harmonization relation between [π] and [e] on imaginary unit circle and its formula*

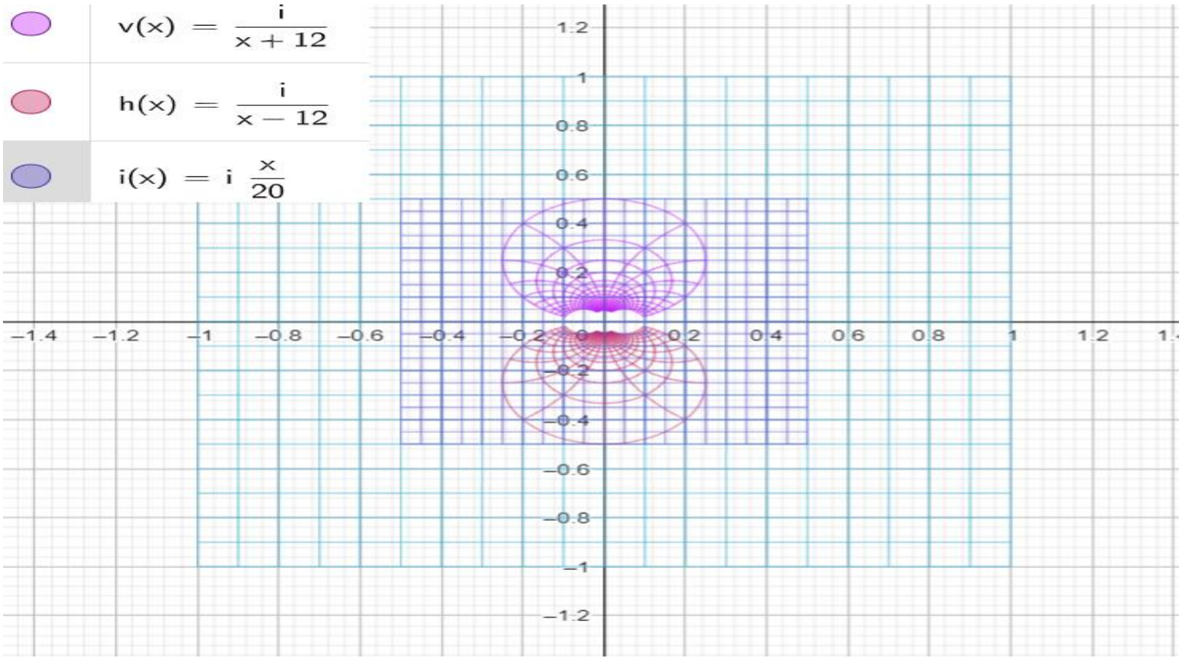


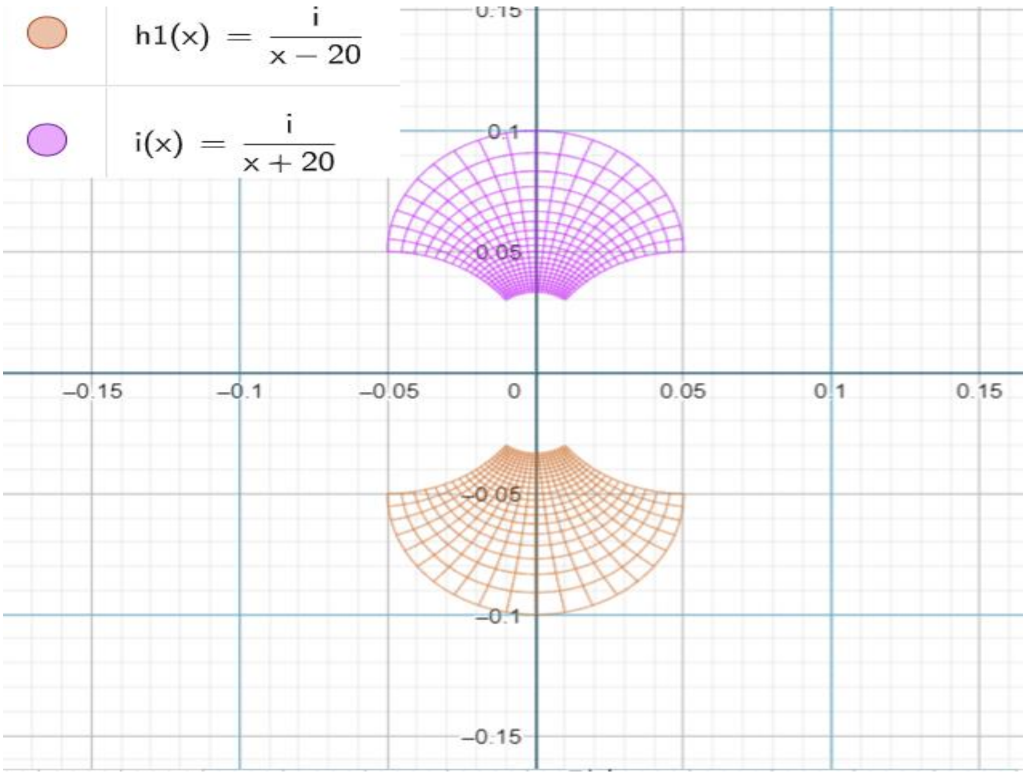

$m1(x) = 1.05 + \frac{12.9}{\pi\left((x+11)^2 + x - 11\right)}$
$g(x) = e^{\frac{i}{x+10}}$


$v(x) = \frac{i}{x+11}$
$h(x) = \frac{i}{x-11}$

at X ± 12 we will be moved (shrinking) to 0.5 size of our 10 base system numbers unit frame of reference square size.

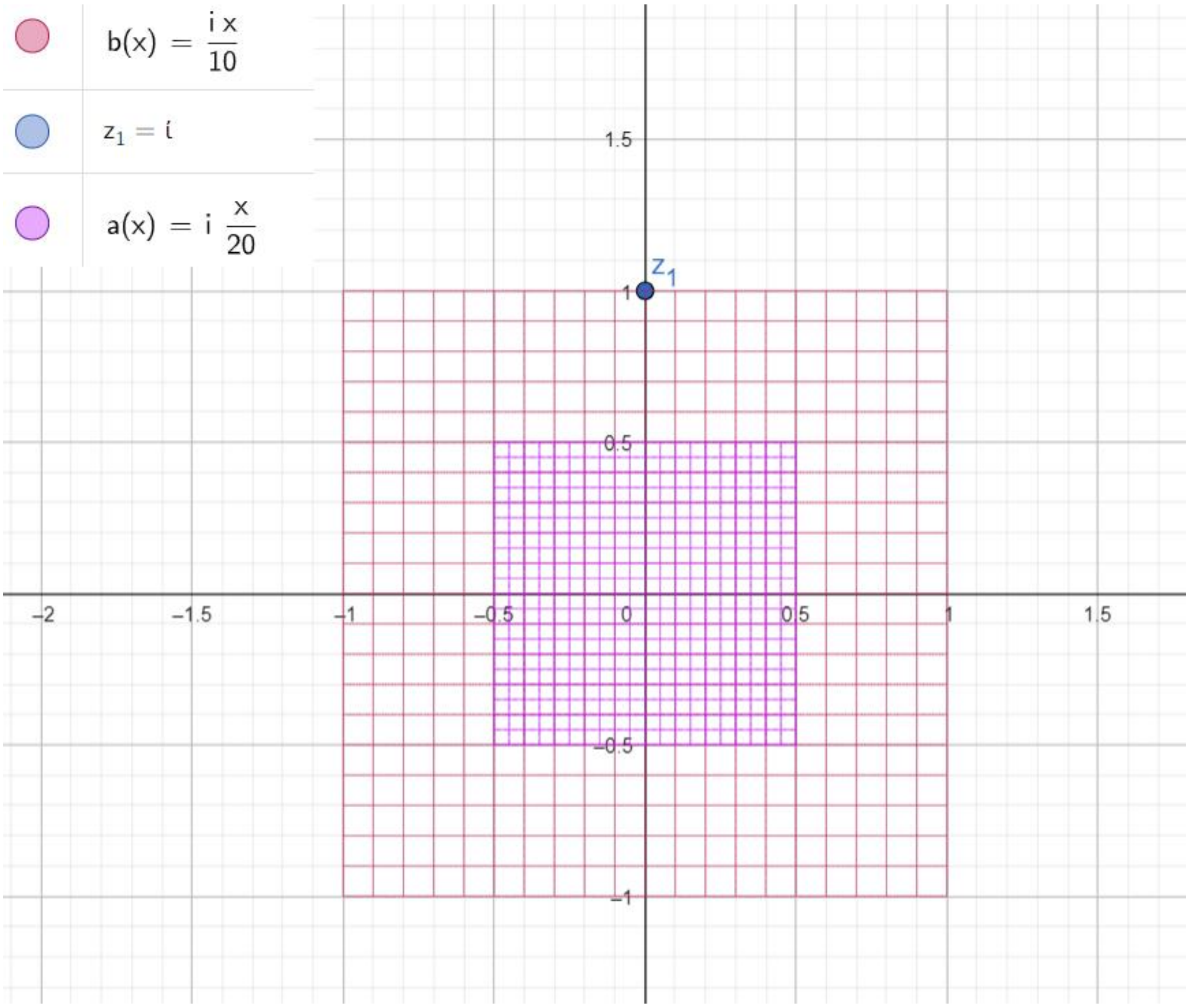



Also, we can do transformation on Y axis



### 3. Using the concept of Frame of reference as a replacement for analytic continuity

1- use a frame of reference but scaled to a 10 based number system and its scale to 0.5. [a(x) = i x/20]




2- use a simple principle [ X – 1 = X * (1-1/X)]

$$c = 2\left(1 - \frac{1}{2}\right) \rightarrow 1$$

$$d = 3\left(1 - \frac{1}{3}\right) \rightarrow 2$$

$$e = 4\left(1 - \frac{1}{4}\right) \rightarrow 3$$

$$f = 31\left(1 - \frac{1}{31}\right) \rightarrow 30$$

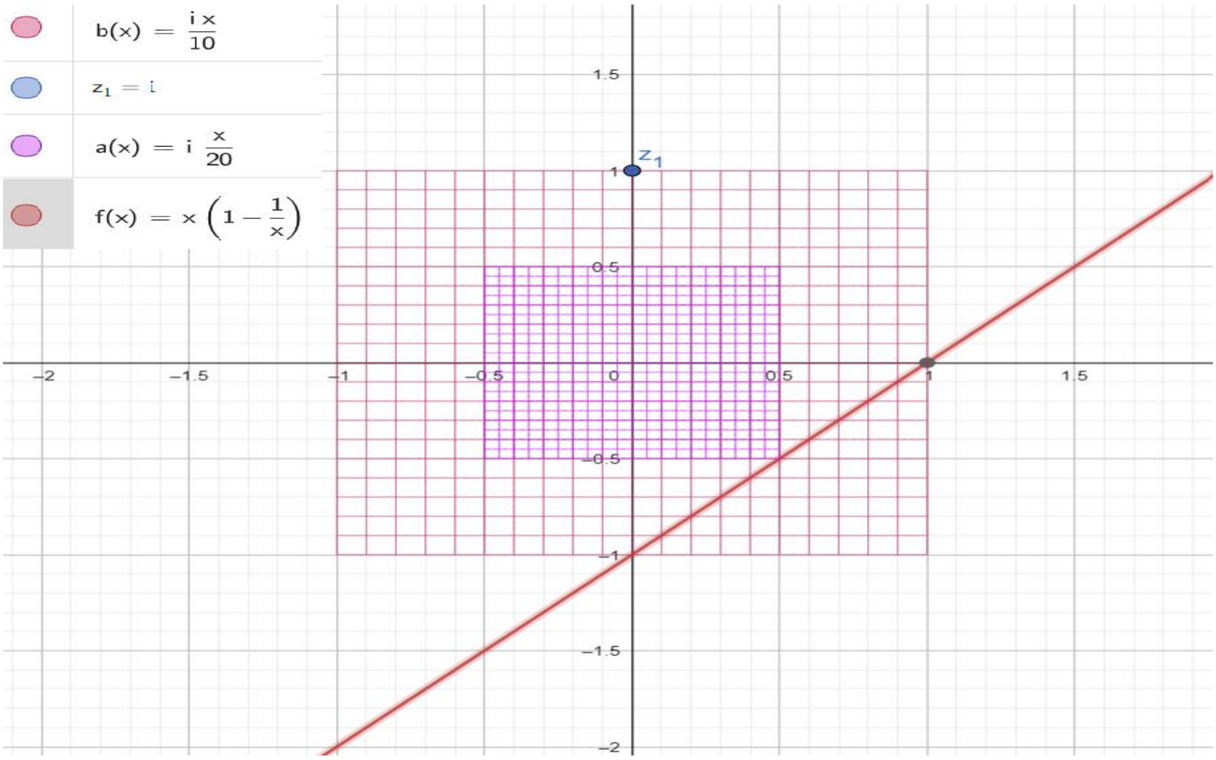
$$g = 2 - 1 \rightarrow 1$$

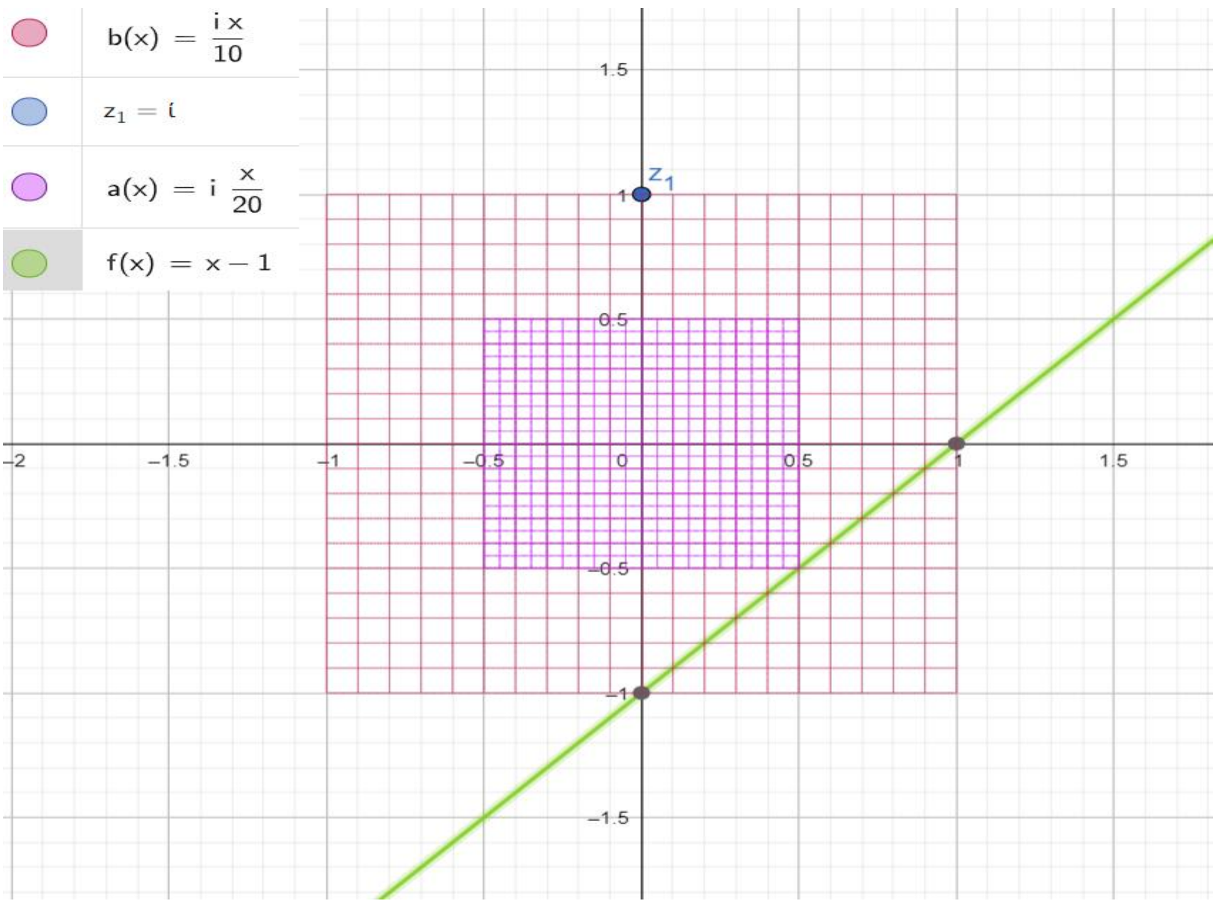
$$h = 3 - 1 \rightarrow 2$$

$$i = 4 - 1 \rightarrow 3$$

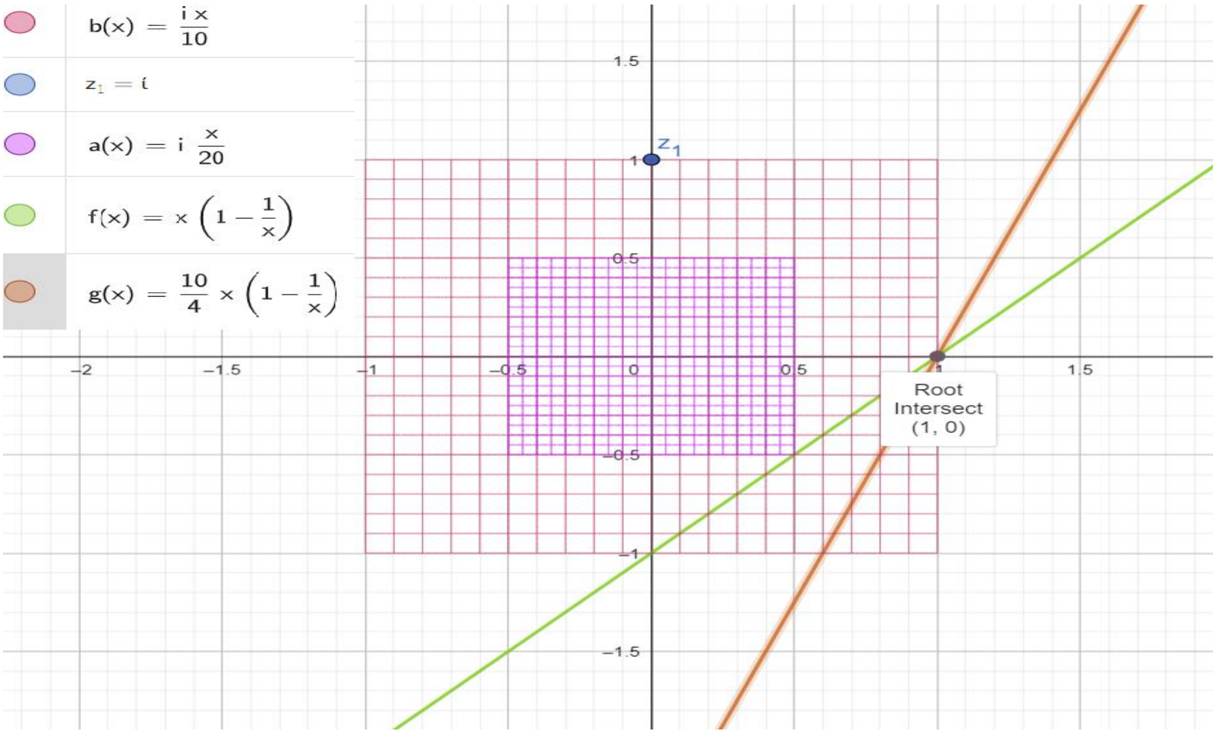
$$j = 31 - 1 \rightarrow 30$$

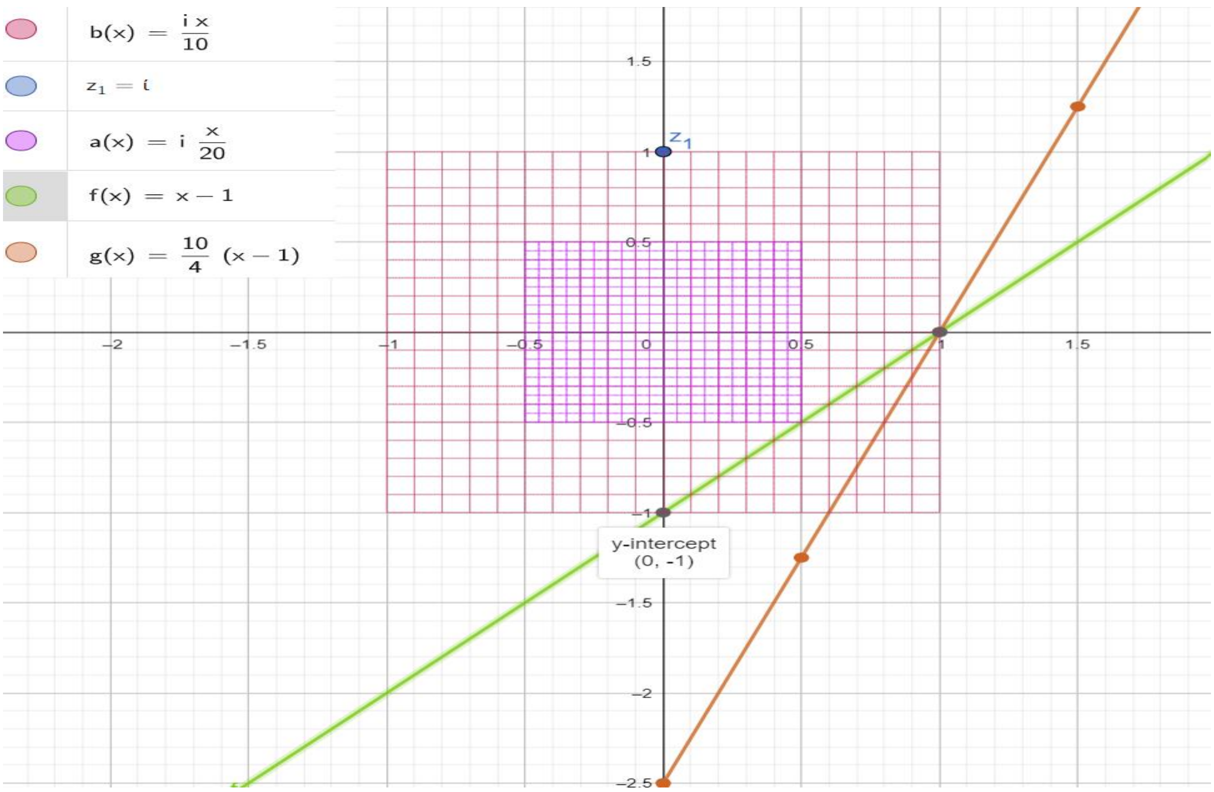
3- plotting X * (1 – 1/X) intersects X axis at (1 , 0) and intersects Y axis at (0,-1), same as plotting (X -1)



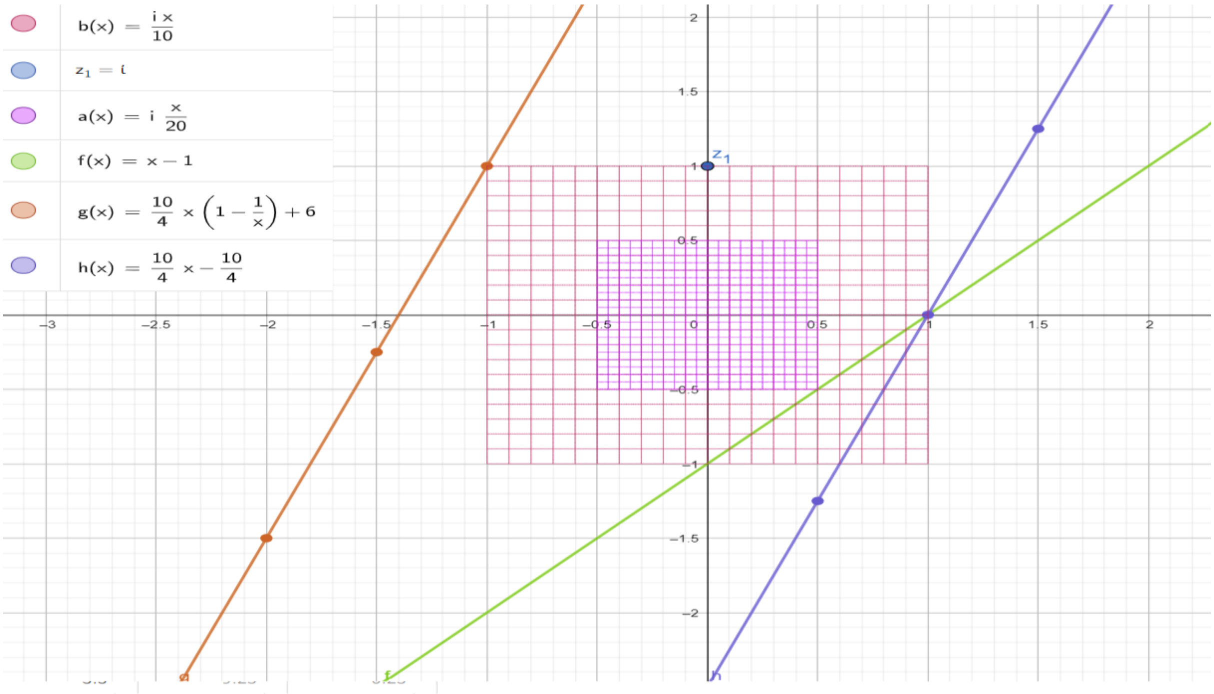



4- as we are using base 10 number system, we are going to scale our calculations by * 10. And, as the frame of reference is symmetrical along X axis and Y axis and origin in Center then any point in the frame of reference will be in a range of 4 values (1, -1, +i, -i), and any transformation for this frame of reference will keep the shape sides ratio the same, then we can use only one fourth from this frame of reference in our calculations. So, we are going to scale [ X * (1 – 1/X)] and [X -1] by multiply it by [10/4].



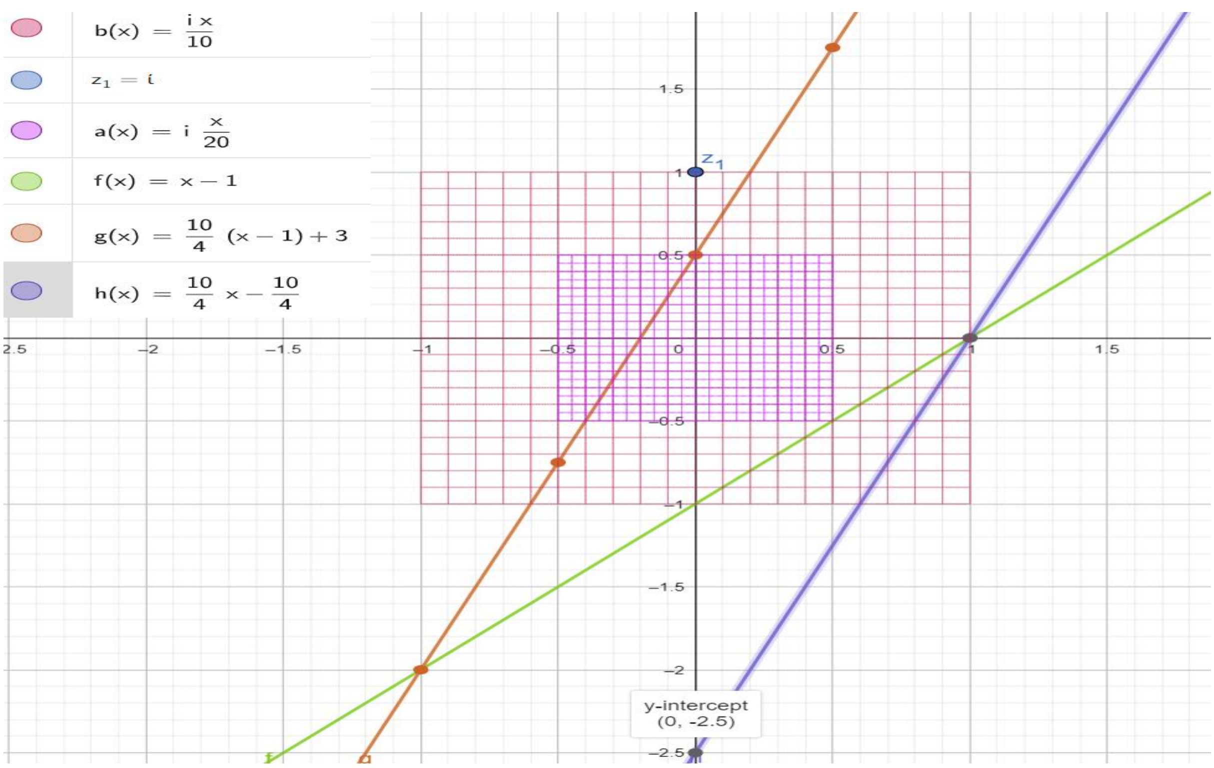
5- if we add 6 to this transformation, we will get g(x) at exactly upper left corner of the frame of reference.

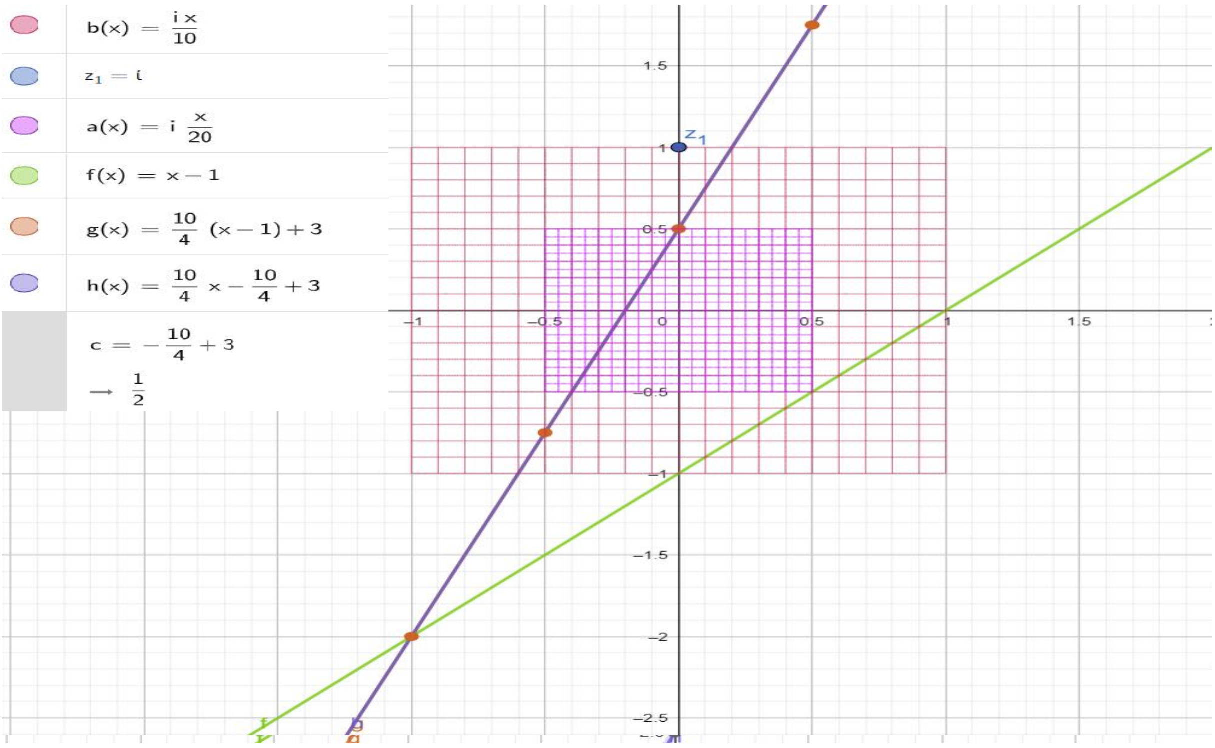


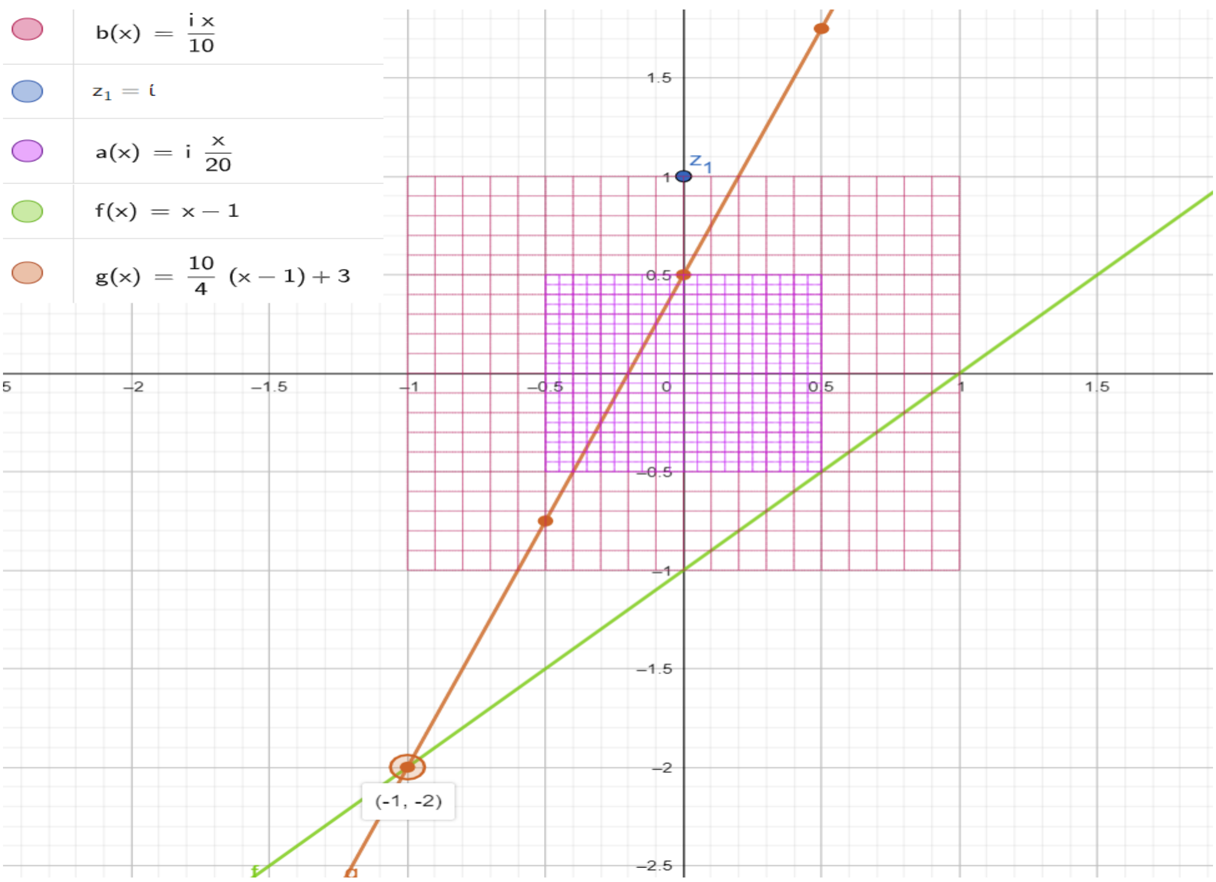

6- using 3 instead of 6 will give us a line intersects with Y axis at point Z = 0 + 0.5i. an as we explained before in frame of reference the Euler's Identity transformation is

$$Z = \text{Sin}(\theta) \pm i\ \text{Cos}(\theta)$$

Then this Transformation is [ 10/4 * X] then add [-10/4 + 3 = 1/2] intersects Y at 0.5.




$b(x) = \frac{i\,x}{10}$
$z_1 = i$
$a(x) = i\,\frac{x}{20}$
$f(x) = x - 1$
$g(x) = \frac{10}{4}\,(x - 1) + 3$
$h(x) = \frac{10}{4}\,x - \frac{10}{4} + 3$
$c = -\frac{10}{4} + 3$
$\rightarrow \frac{1}{2}$
$z_1$


$b(x) = \frac{i\,x}{10}$
$z_1 = i$
$a(x) = i\,\frac{x}{20}$
$f(x) = x - 1$
$g(x) = \frac{10}{4}\,(x - 1) + 3$
$z_1$
(-1, -2)

7- now we are going to see how this transformation solve the infinity problem between [0 ,1] for primes at stripe line. Using this transformation, all odd numbers of values for X will give us a natural number for Y. for any odd value of X.

| x | g(x) | h(x) |
|---|---|---|
| -3 | -7 | -10 |
| -2.5 | -5.75 | -8.75 |
| -2 | -4.5 | -7.5 |
| -1.5 | -3.25 | -6.25 |
| -1 | -2 | -5 |
| -0.5 | -0.75 | -3.75 |
| 0 | 0.5 | -2.5 |
| 0.5 | 1.75 | -1.25 |
| 1 | 3 | 0 |
| 1.5 | 4.25 | 1.25 |
| 2 | 5.5 | 2.5 |
| 2.5 | 6.75 | 3.75 |
| 3 | 8 | 5 |
| 3.5 | 9.25 | 6.25 |

$$b(x) = \frac{i\,x}{10}$$

$$z_1 = \imath$$

$$a(x) = i\,\frac{x}{20}$$

$$f(x) = x - 1$$

$$g(x) = \frac{10}{4}(x - 1) + 3$$

$$h(x) = \frac{10}{4}x - \frac{10}{4}$$

$$c = -\frac{10}{4} + 3 \rightarrow \frac{1}{2}$$

| x | g(x) | h(x) |
|---|---|---|
| -3.5 | -8.25 | -11.25 |
| -3 | -7 | -10 |
| -2.5 | -5.75 | -8.75 |
| -2 | -4.5 | -7.5 |
| -1.5 | -3.25 | -6.25 |
| -1 | -2 | -5 |
| -0.5 | -0.75 | -3.75 |
| 0 | | -2.5 |
| 0.5 | 1.75 | -1.25 |
| 1 | 3 | 0 |
| 1.5 | 4.25 | 1.25 |
| 2 | 5.5 | 2.5 |
| 2.5 | 6.75 | 3.75 |
| 3 | 8 | 5 |
| 3.5 | 9.25 | 6.25 |

$$b(x) = \frac{i\,x}{10}$$

$$z_1 = \imath$$

$$a(x) = i\,\frac{x}{20}$$

$$f(x) = x - 1$$

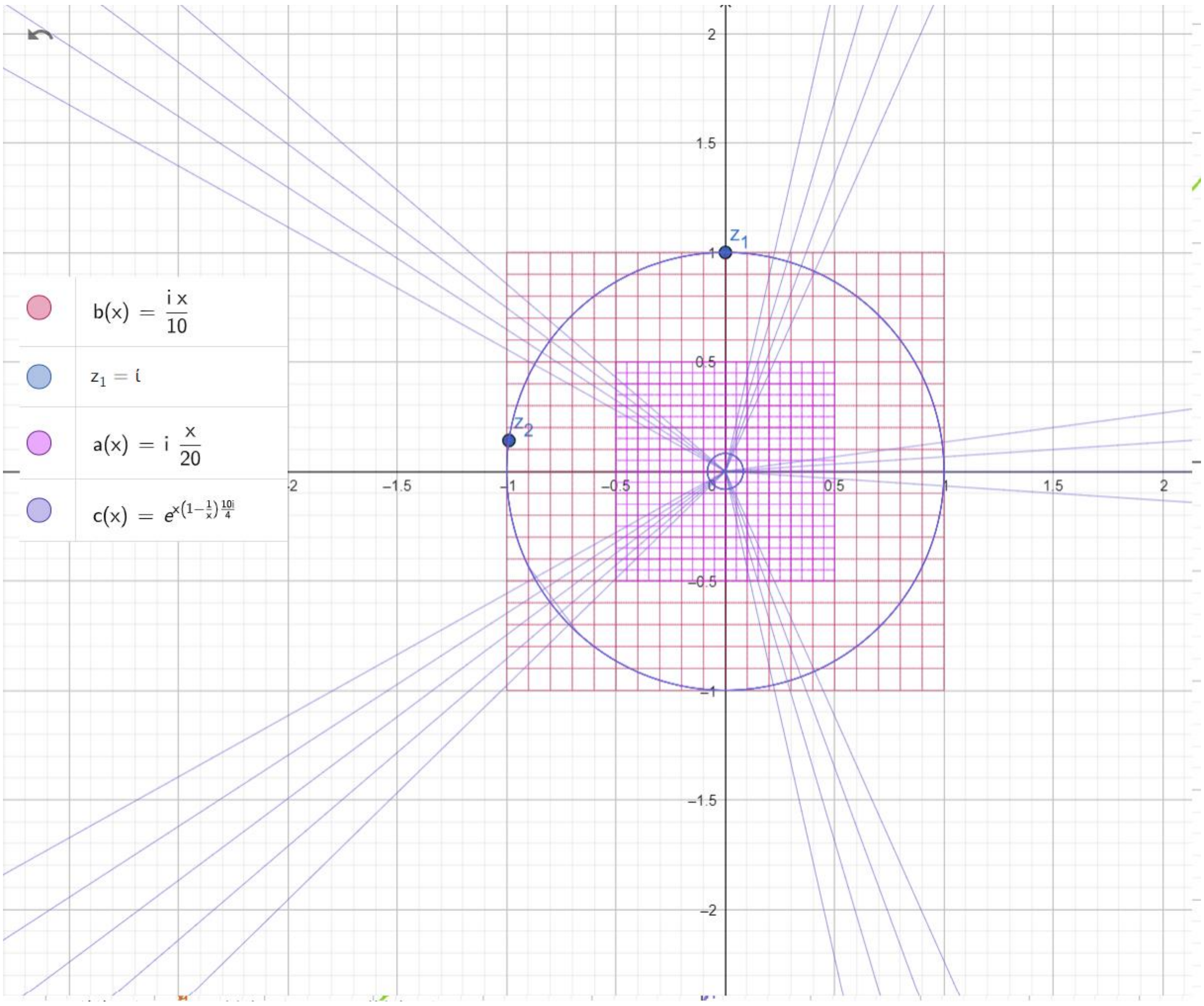
$$g(x) = \frac{10}{4}\,x\left(1 - \frac{1}{x}\right) + 3$$

$$h(x) = \frac{10}{4}x - \frac{10}{4}$$

$$c = -\frac{10}{4} + 3 \rightarrow \frac{1}{2}$$

*3.2 How this transformation formula reflects on the imaginary unit Circle and its 21 lines.*

We should only have 20 lines, because our number system is based 10 and the frame of reference is symmetric around Y and X axis, but we need 21 lines to create 20 smaller squares. when a frame of reference is manifold in complex plane, we get one more line form the edge of folding which will work like an indicator for the folding direction as a fold guide. (One of the line groups branches will going to have 5 lines).

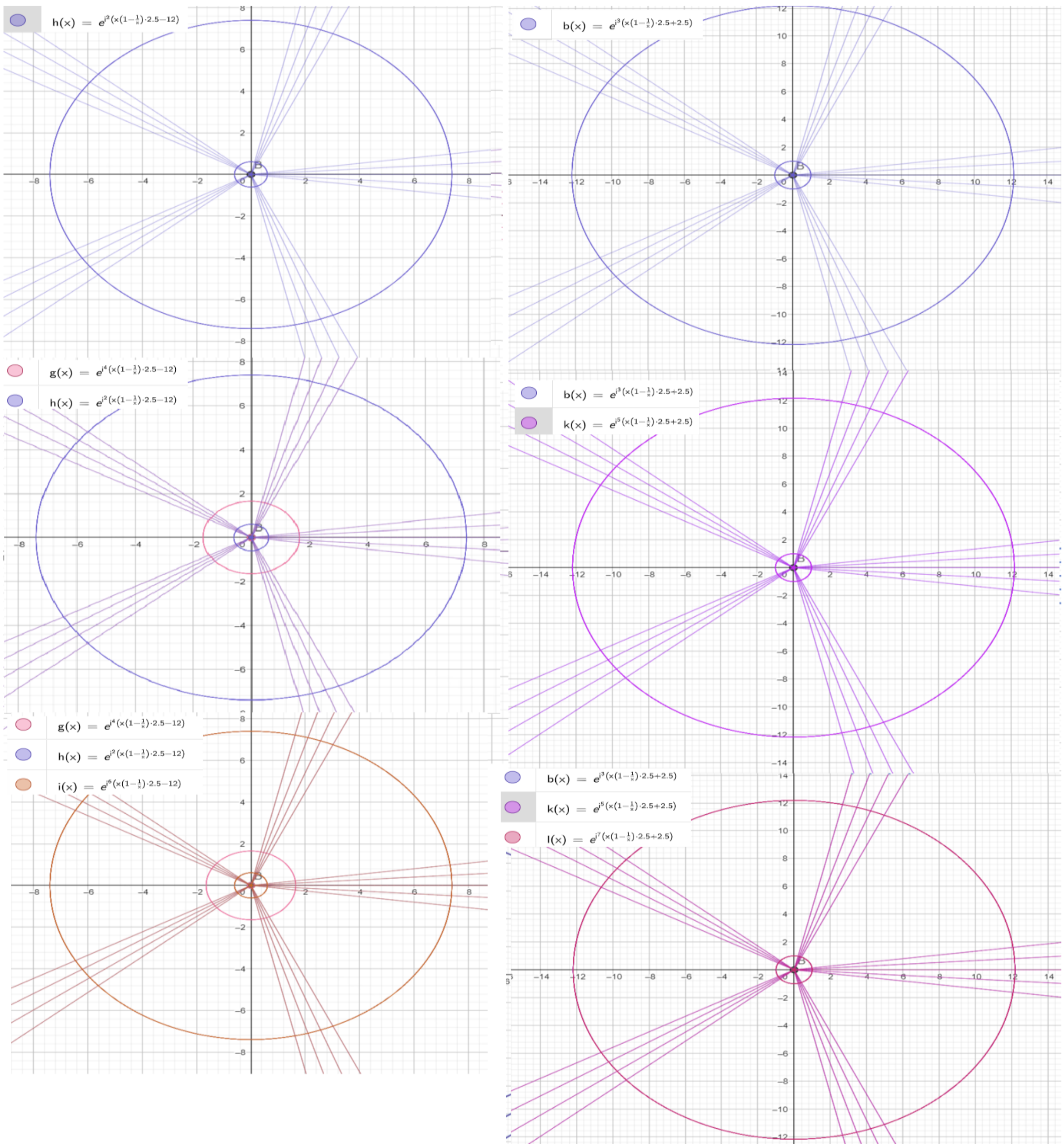


*3.2.1 Transformation formula for even and odd powers of [i]*

*Odd power formula =* $a(x) = e^{i^{n}\left(x\left(1-\frac{1}{x}\right)\cdot 2.5+2.5\right)}$

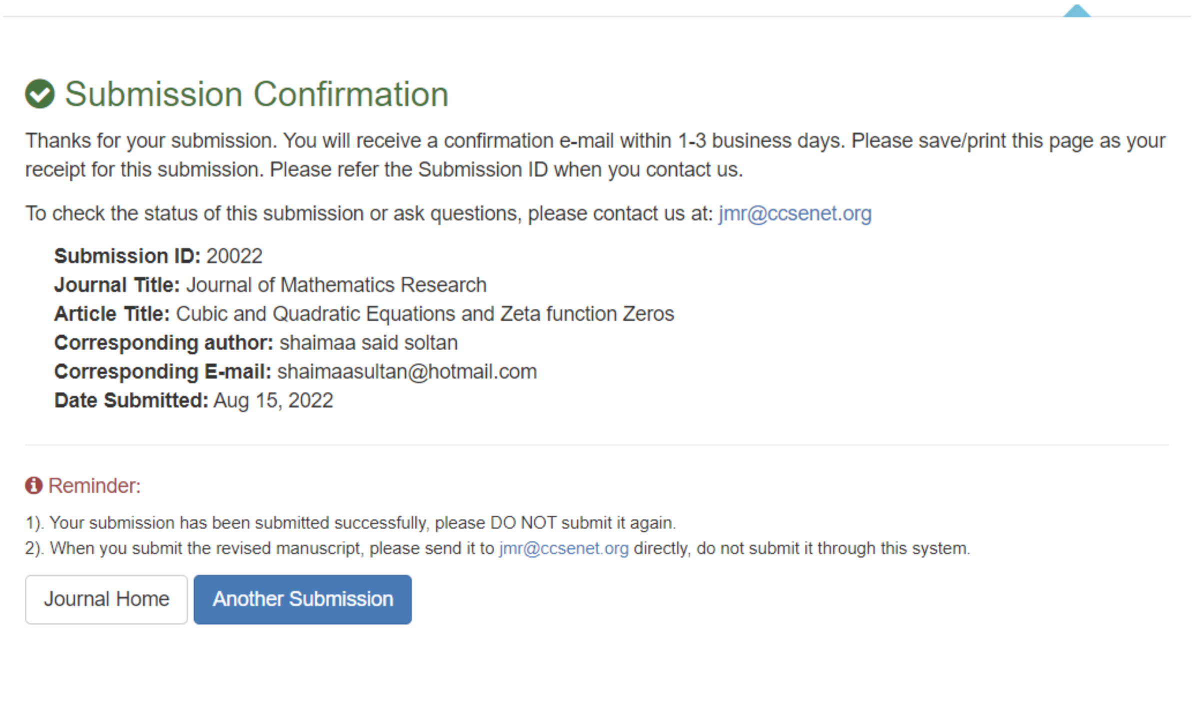
*Even power formula =* $g(x) = e^{i^{n}\left(x\left(1-\frac{1}{x}\right)\cdot 2.5-12\right)}$

*Note: - that using these two formulas will make the positions of the folding lines do not change or rotate; no matter what value of n you are using as a power in the transformation.*



## 4. Results

First, we introduced a frame of reference concept and the usage of mathematical transformation on this frame of reference for Euler's Identity manifold in a complex plane. Then we studied multiple mathematical transformations on the frame of reference and its visualizations. Then we got through the manifold and unfolding in complex plane using this frame of reference transformation. Then we precented another way to visualize



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