

Analytical Continuity between one and Zero

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Analytical Continuity between one and Zero

Abstract

This paper will explain the reasoning for Analytical discontinuity in complex plane between values [0,1].

And getting a simpler formula that handle this discontinuity between [0,1]. And use this formula to give a proof for nontrivial Zeros at Re (0.5).

Finally, we will use this formula to find the sum of some infinite series.

Keywords: zeta function, Riemann hypothesis, complex plane, none-trivial zeros, critical strip, gamma function

1. Introduction

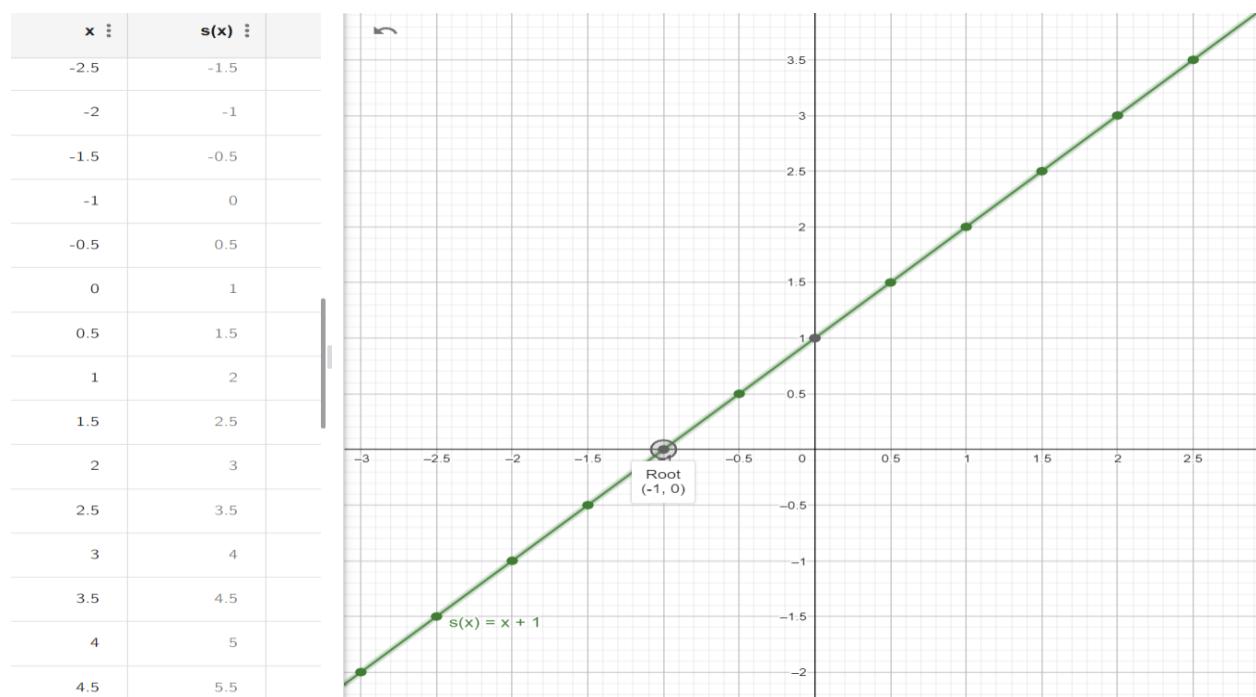
Understanding the reasoning behind Riemann hypothesis will give us better understanding for number distributions. In this paper we will cover a huge part of this number distributions. We will give a general formula for number distributions and graphs and how numbers are visualized on complex plane using this formula. Then we will explain why we have Zero at X = 0.5 and the reasoning behind this distribution

$\pi \rightarrow$ in radian = 3.1415926535897932384626433832795 ;

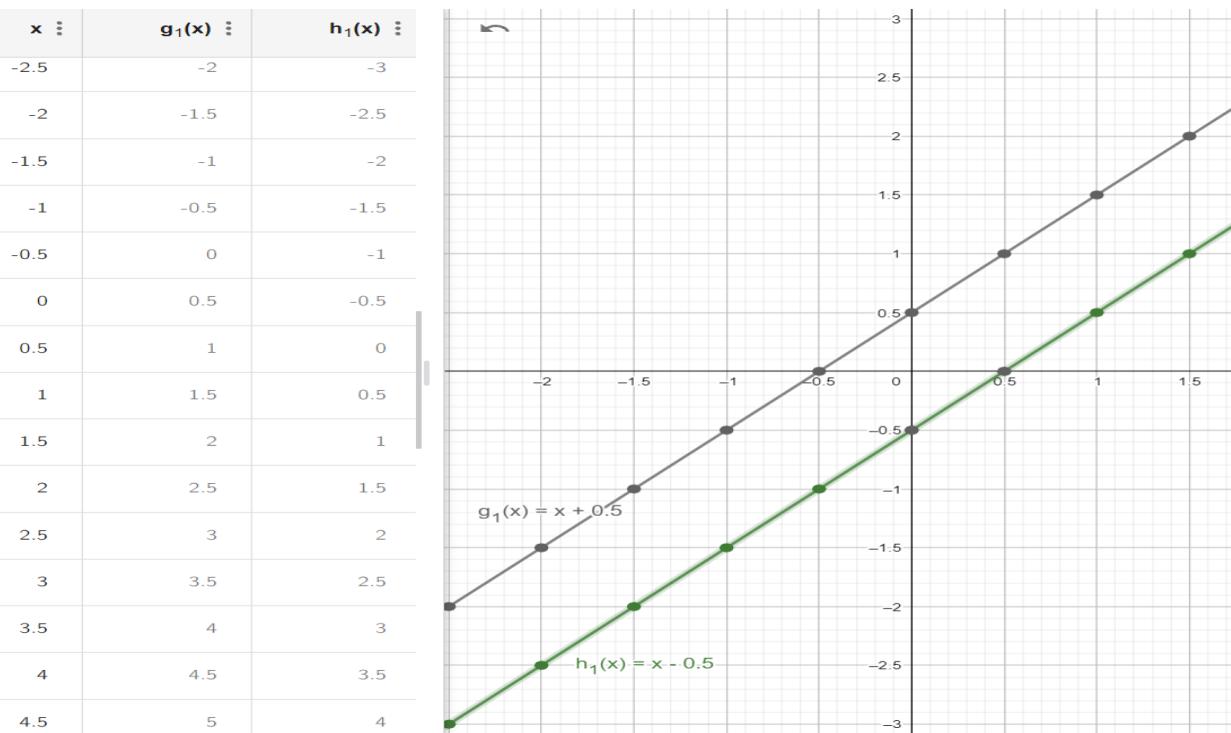
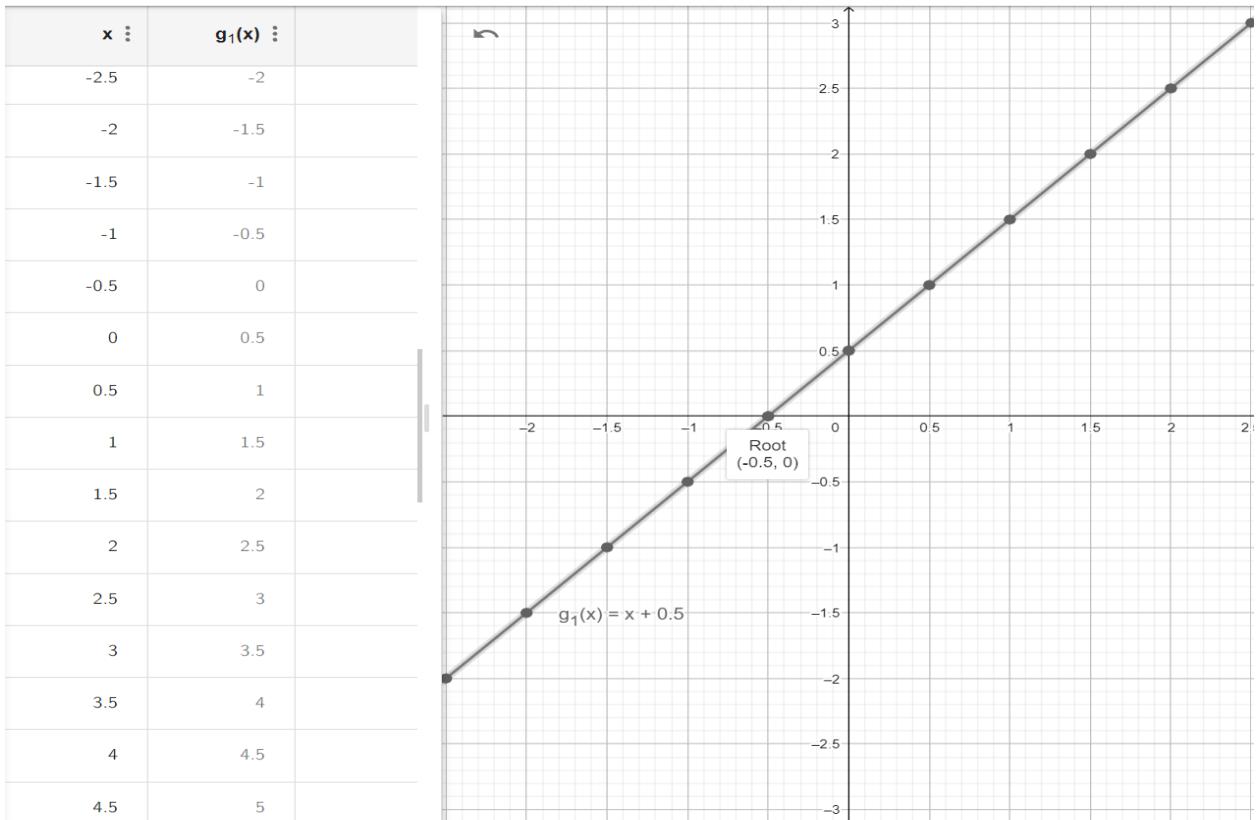
$\pi \rightarrow$ in Degrees = 180

In Degrees $\rightarrow f(X) = 360X + 180$; In Radian $\rightarrow f(X) = 2\pi X + \pi$

A) $f(X) = (X \pm 1)$

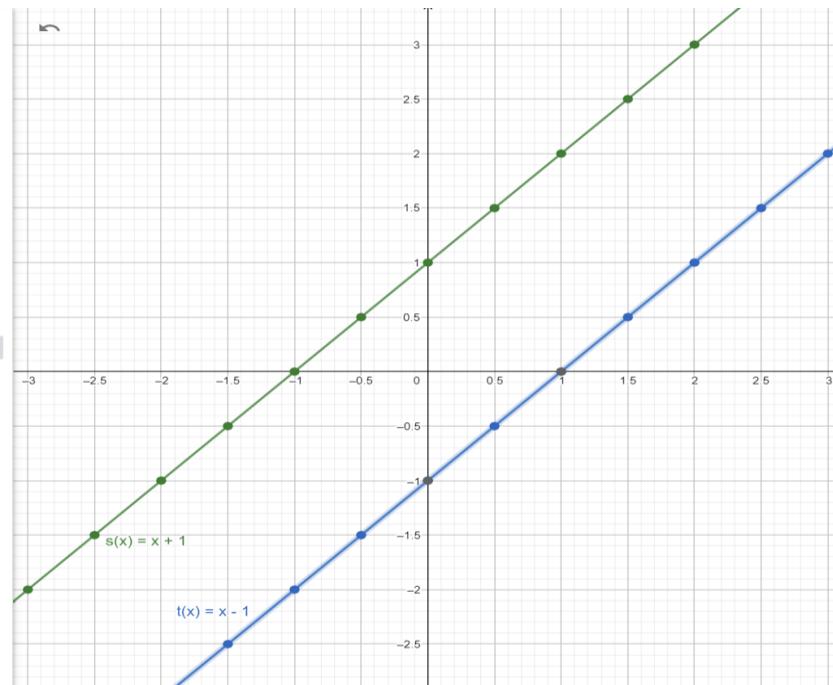


A) $f(X) = X \pm 0.5$



B)

x	$s(x)$	$t(x)$
-2.5	-1.5	-3.5
-2	-1	-3
-1.5	-0.5	-2.5
-1	0	-2
-0.5	0.5	-1.5
0	1	-1
0.5	1.5	-0.5
1	2	0
1.5	2.5	0.5
2	3	1
2.5	3.5	1.5
3	4	2
3.5	4.5	2.5
4	5	3
4.5	5.5	3.5

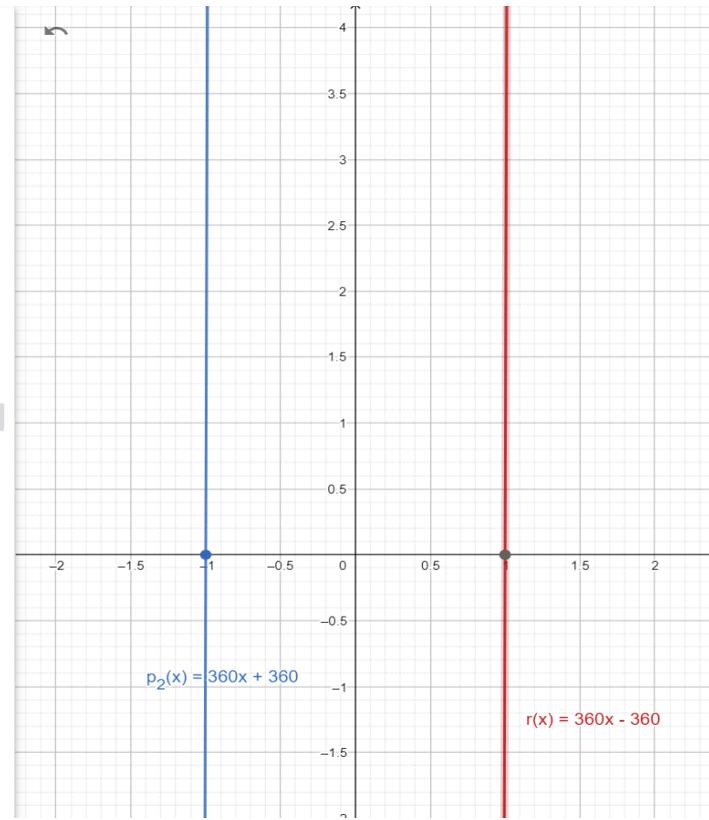


C) Multiply $f(X) = X \pm 1$ by (360) $\rightarrow f(X) = 360X \pm 360$

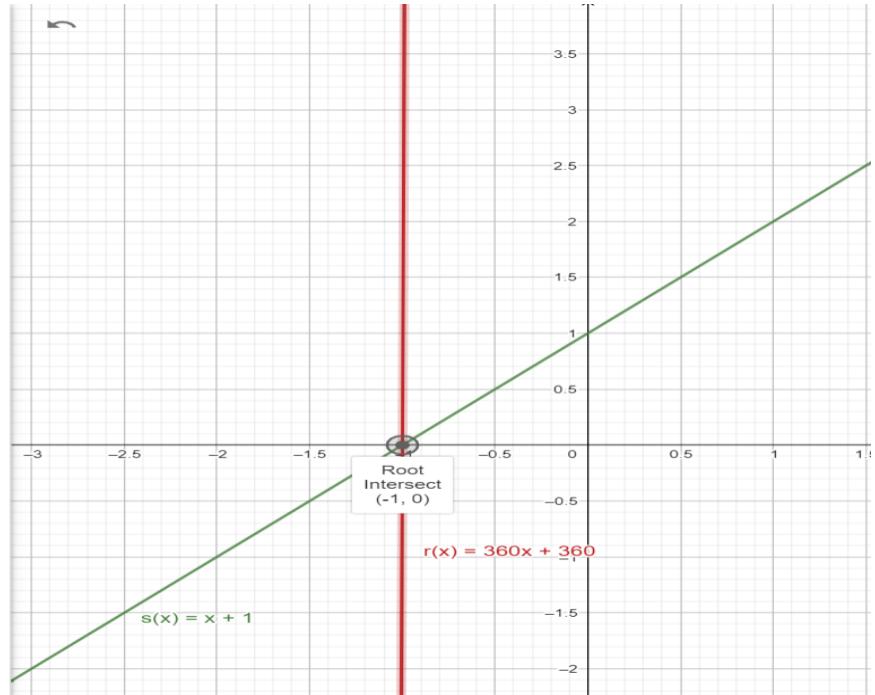
Then ($-0.5 = 180 = \pi$) and ($0.5 = 540 = 360 + 180 = 3\pi$)

$$f(X) = 360X + 360 = \cos(180) + i \sin(540)$$

x	$p_2(x)$	$r(x)$
-2.5	-540	-1260
-2	-360	-1080
-1.5	-180	-900
-1	0	-720
-0.5	180	-540
0	360	-360
0.5	540	-180
1	720	0
1.5	900	180
2	1080	360
2.5	1260	540
3	1440	720
3.5	1620	900
4	1800	1080
4.5	1980	1260

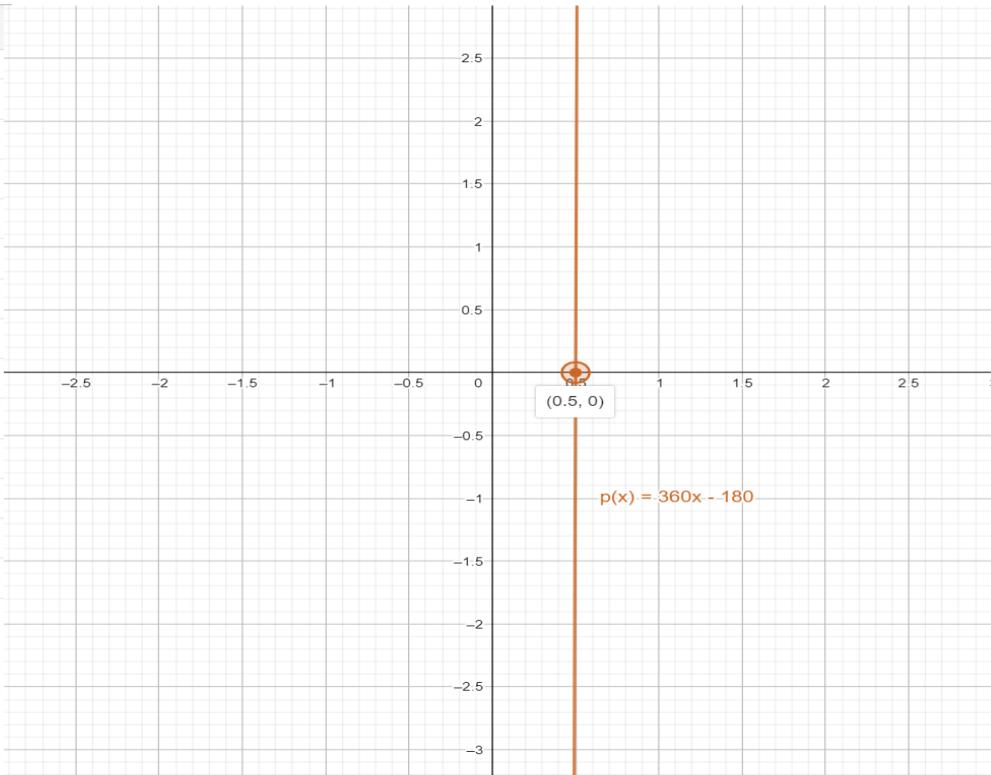


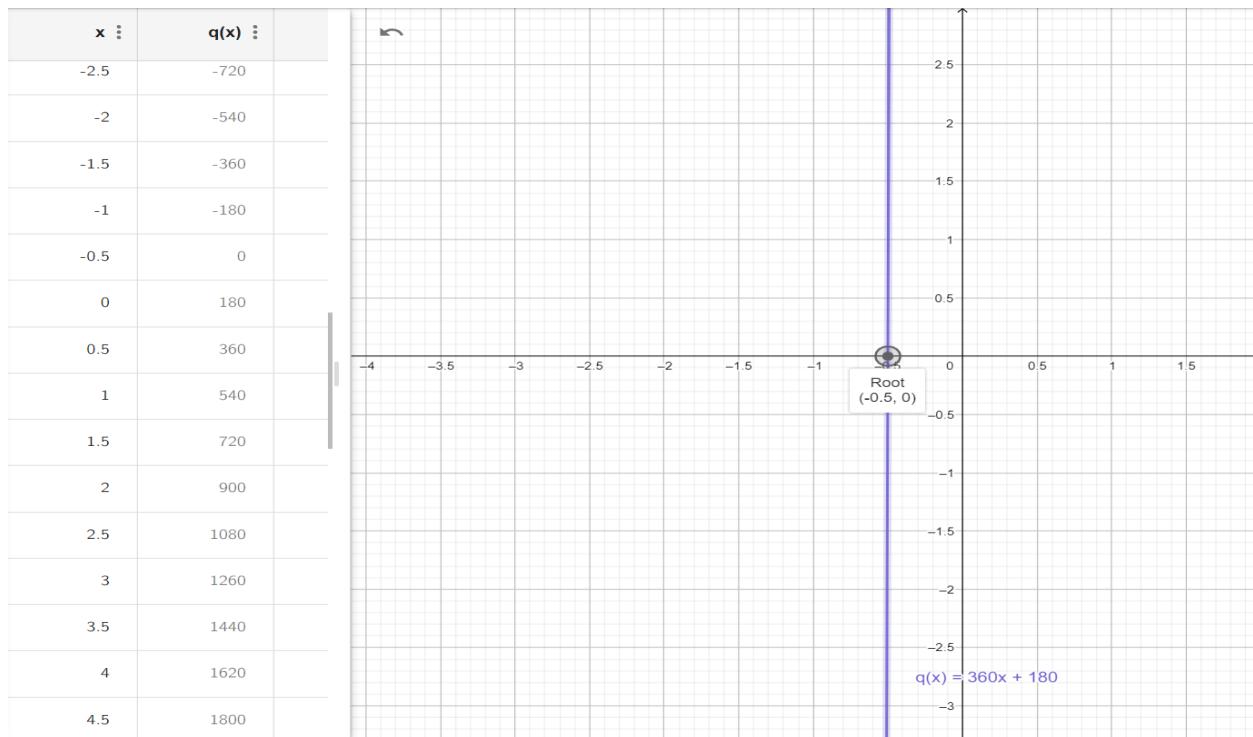
x	$r(x)$
-2.5	-540
-2	-360
-1.5	-180
-1	0
-0.5	180
0	360
0.5	540
1	720
1.5	900
2	1080
2.5	1260
3	1440
3.5	1620
4	1800
4.5	1980



1) Multiply $f(X) = X \pm 0.5$ by (360) $\rightarrow f(X) = 360X \pm 180$

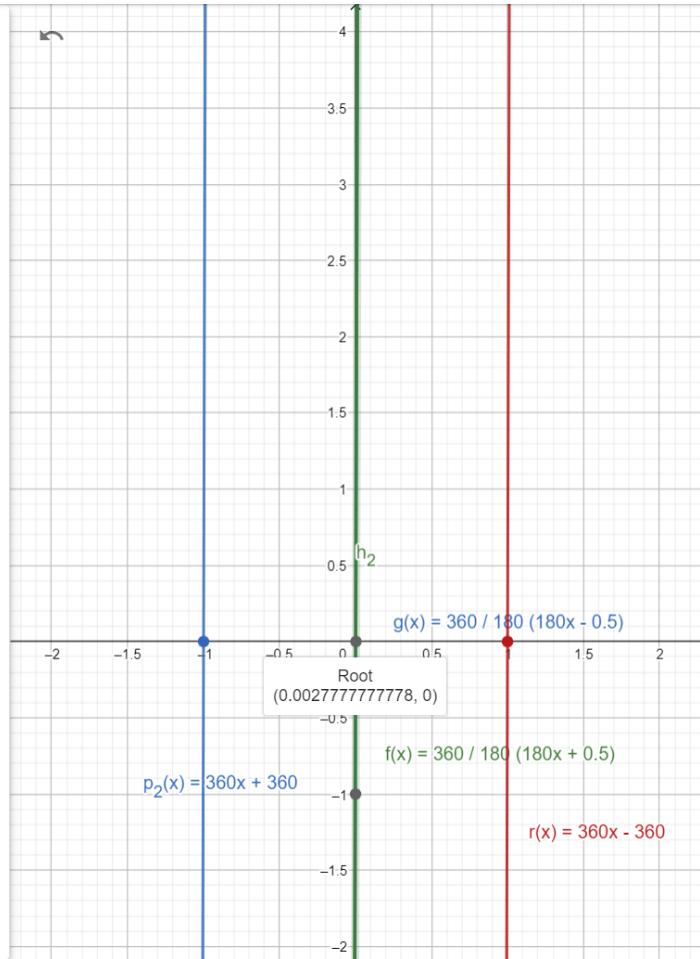
x	$p(x)$
-2.5	-1080
-2	-900
-1.5	-720
-1	-540
-0.5	-360
0	-180
0.5	0
1	180
1.5	360
2	540
2.5	720
3	900
3.5	1080
4	1260
4.5	1440





2) $f(X) = 360 X \pm 1 \rightarrow$ one degree difference and $Y=0$ when $X = 1/360$

x :	p ₂ (x) :	r(x) :	h ₂ (x) :
-2.5	-540	-1260	-901
-2	-360	-1080	-721
-1.5	-180	-900	-541
-1	0	-720	-361
-0.5	180	-540	-181
0	360	-360	-1
0.5	540	-180	179
1	720	0	359
1.5	900	180	539
2	1080	360	719
2.5	1260	540	899
3	1440	720	1079
3.5	1620	900	1259
4	1800	1080	1439
4.5	1980	1260	1619



$$1 = \frac{360}{360} = \frac{360}{180 * 2}$$

$$f(X) = 360 X \pm 1 = 360 X \pm \frac{360}{180 * 2}$$

$$f(X) = 360 X \pm 1 = \frac{360}{180} (180 X \pm \frac{1}{2})$$

$$f(X) = 360 X \pm 180 ; \text{when } X = 0.5 Y = 0$$

When X = 1/360

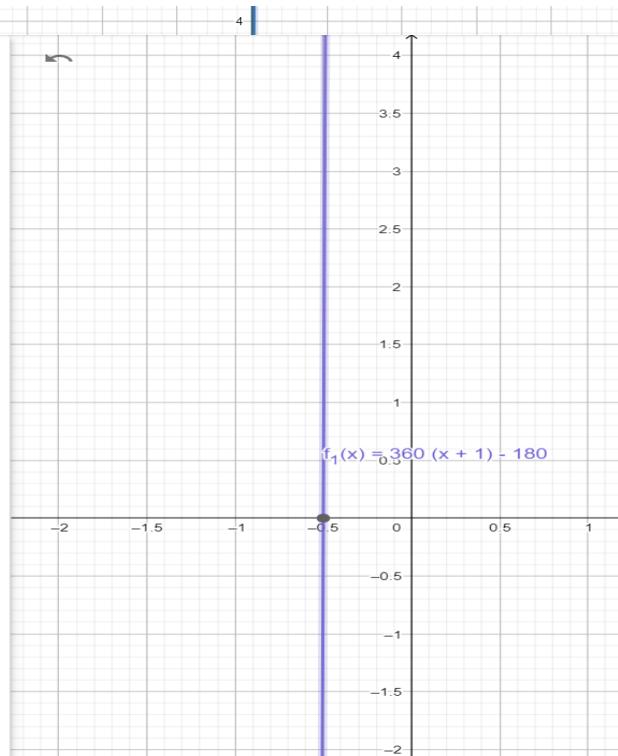
$$f(X) = 360 X \pm 180 = 1 \pm 180$$

$$f(X) = 360 X - 1 = 1 - 1 = 0 = \frac{360}{180} \left(180 \frac{1}{360} - \frac{1}{2} \right) = \frac{360}{180} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

3) $f(X) = 360X \pm 1 = \frac{360}{180} (180X \pm \frac{1}{2}) \rightarrow$ one degree difference and Y=0 when X =

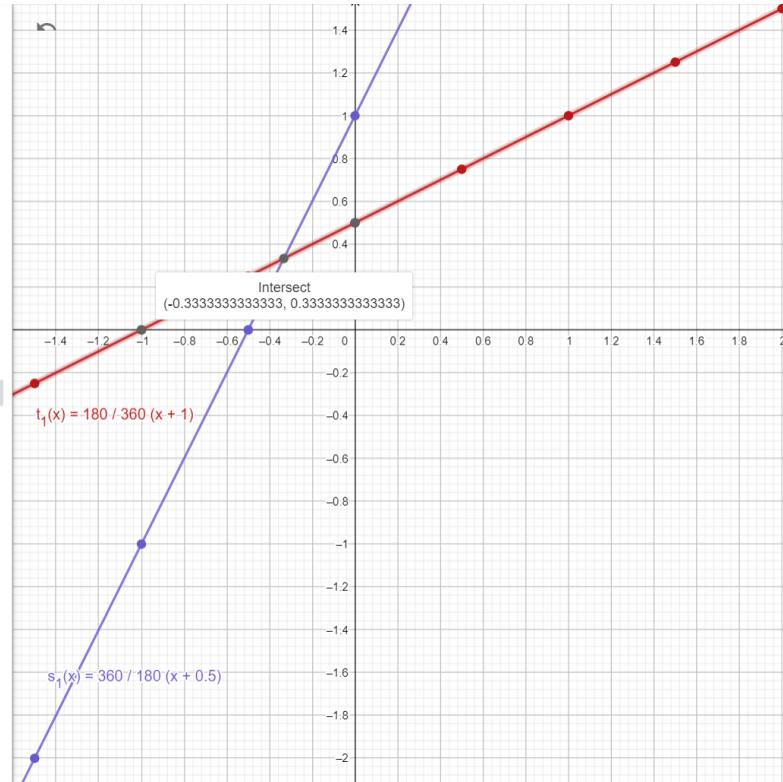
x ::	f(x) ::	g(x) ::	
x ::	f(x) ::	g(x) ::	f ₁ (x) ::
-2.5	-899	-901	-720
-2	-719	-721	-540
-1.5	-539	-541	-360
-1	-359	-361	-180
-0.5	-179	-181	0
0	1	-1	180
0.5	181	179	360
1	361	359	540
1.5	541	539	720
2	721	719	900
2.5	901	899	1080
3	1081	1079	1260
3.5	1261	1259	1440
4	1441	1439	1620
4.5	1621	1619	1800

1/360



4) $S(X) = \frac{360}{180} (X \pm 0.5)$ and $t(X) = \frac{180}{360} (X \pm 1)$

x	f ₁ (x)	s ₁ (x)	t ₁ (x)
-3.5	-1080	-6	-1.25
-3	-900	-5	-1
-2.5	-720	-4	-0.75
-2	-540	-3	-0.5
-1.5	-360	-2	-0.25
-1	-180	-1	0
-0.5	0	0	0.25
0	180	1	0.5
0.5	360	2	0.75
1	540	3	1
1.5	720	4	1.25
2	900	5	1.5
2.5	1080	6	1.75
3	1260	7	2



$S(X)$ intersect with $t(X)$ at point $(\frac{1}{3}, \frac{1}{3})$; Why?

1- Simplify $S(X)$ and $t(X)$

$$S(X) = (2X \pm 1) \text{ and } t(X) = \left(\frac{X}{2} \pm \frac{1}{2}\right)$$

For $t(X)$ liner relation with Y intercept = $\frac{1}{2}$

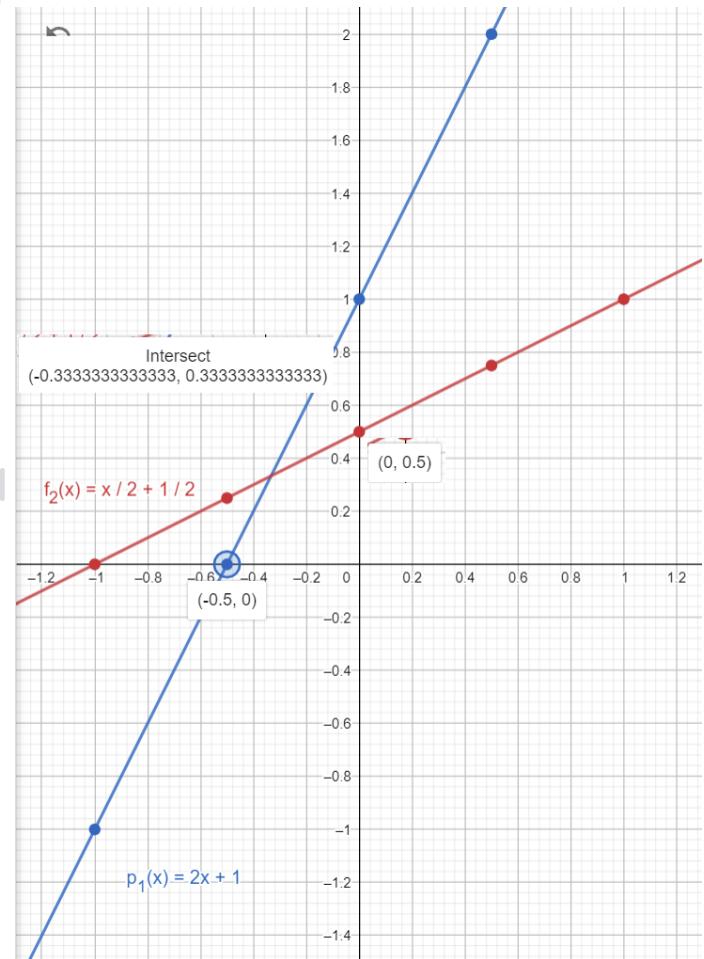
$$P(X) = (2X \pm 1) \text{ and } f(X) = \left(\frac{X}{2} \pm \frac{1}{2}\right)$$

both are the same exact traingel but with diffrent rotation

To keep the X intercept point at X = -1 for f(X) to be = 0

It must intercept Y at $\frac{1}{N}$ For $f(X) = \left(\frac{X}{N} \pm \frac{1}{N}\right)$

x ::	$f_1(x) ::$	$p_1(x) ::$	$f_2(x) ::$
-3.5	-1080	-6	-1.25
-3	-900	-5	-1
-2.5	-720	-4	-0.75
-2	-540	-3	-0.5
-1.5	-360	-2	-0.25
-1	-180	-1	0
-0.5	0	0	0.25
0	180	1	0.5
0.5	360	2	0.75
1	540	3	1
1.5	720	4	1.25
2	900	5	1.5
2.5	1080	6	1.75
3	1260	7	2

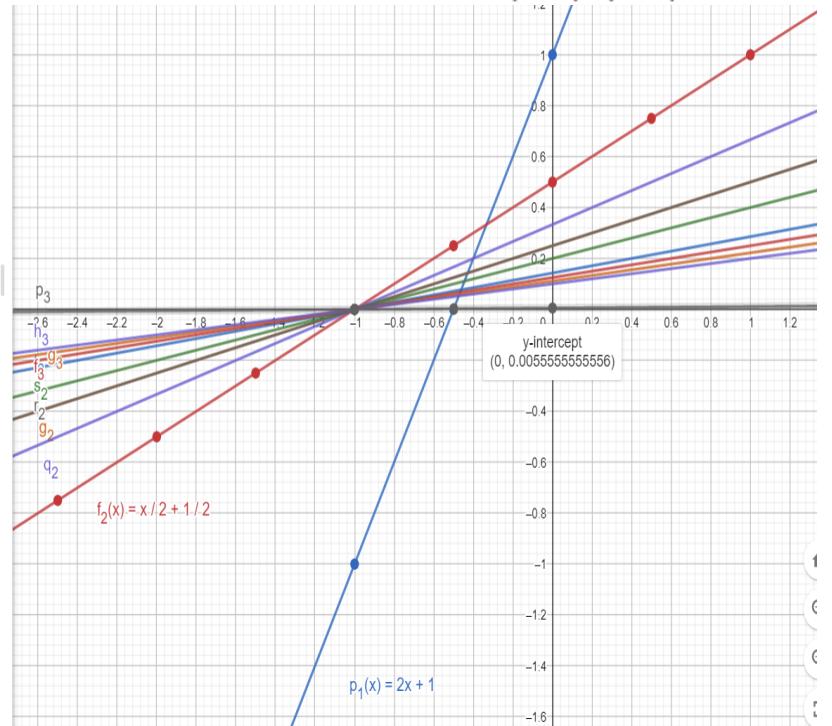


- 1- Each line $f(X) = \left(\frac{X}{N} \pm \frac{1}{N}\right)$ intercept Y at $\frac{1}{N}$

2- Each line $f(X) = (\frac{X}{N} \pm \frac{1}{N})$ intercept with line $f(X) = (2X \pm 1)$ at point $(\frac{N-1}{(2N-1)}, \frac{1}{(2N-1)})$

<input type="radio"/>	$q_2(x) = \frac{x}{3} + \frac{1}{3}$	⋮
<input checked="" type="radio"/>	$r_2(x) = \frac{x}{4} + \frac{1}{4}$	⋮
<input type="radio"/>	$s_2(x) = \frac{x}{5} + \frac{1}{5}$	⋮
<input type="radio"/>	$t_2(x) = \frac{x}{7} + \frac{1}{7}$	⋮
<input type="radio"/>	$f_3(x) = \frac{x}{8} + \frac{1}{8}$	⋮
<input type="radio"/>	$g_3(x) = \frac{x}{9} + \frac{1}{9}$	⋮
<input type="radio"/>	$h_3(x) = \frac{x}{10} + \frac{1}{10}$	⋮
<input checked="" type="radio"/>	$p_3(x) = \frac{x}{180} + \frac{1}{180}$	⋮
+ Input...		

GeoGebra Graphing Calculator

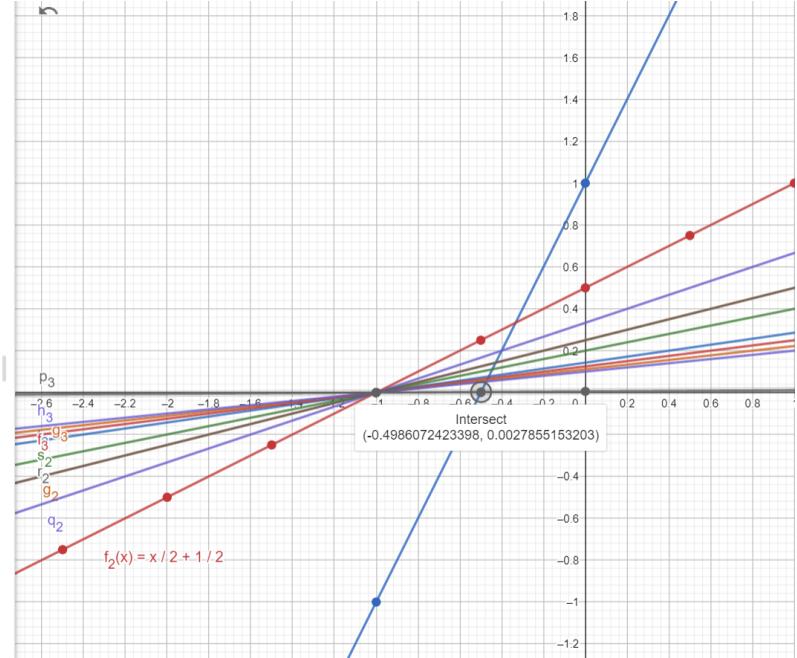


At N = 180

3- Each line $f(X) = (\frac{X}{180} \pm \frac{1}{180})$ intercept Y at $\frac{1}{180}$

4- Each line $f(X) = (\frac{X}{180} \pm \frac{1}{180})$ intercept with line $f(X) = (2X \pm 1)$ at point $(\frac{179}{359}, \frac{1}{359})$

<input type="radio"/>	$q_2(x) = \frac{x}{3} + \frac{1}{3}$	⋮
<input checked="" type="radio"/>	$r_2(x) = \frac{x}{4} + \frac{1}{4}$	⋮
<input type="radio"/>	$s_2(x) = \frac{x}{5} + \frac{1}{5}$	⋮
<input type="radio"/>	$t_2(x) = \frac{x}{7} + \frac{1}{7}$	⋮
<input type="radio"/>	$f_3(x) = \frac{x}{8} + \frac{1}{8}$	⋮
<input type="radio"/>	$g_3(x) = \frac{x}{9} + \frac{1}{9}$	⋮
<input type="radio"/>	$h_3(x) = \frac{x}{10} + \frac{1}{10}$	⋮
<input checked="" type="radio"/>	$p_3(x) = \frac{x}{180} + \frac{1}{180}$	⋮
	$I = \frac{1}{359}$ ≈ 0.0027855153203	⋮
	$m = \frac{179}{359}$ ≈ 0.4986072423398	⋮

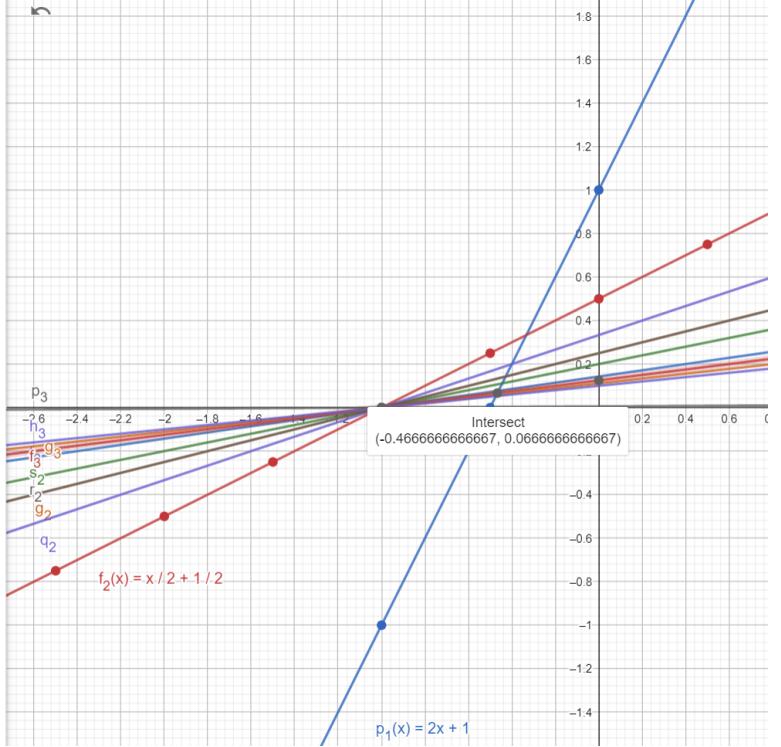


At X = 8

5- Each line $f(X) = (\frac{X}{8} \pm \frac{1}{8})$ intercept Y at $\frac{1}{8}$

6- Each line $f(X) = (\frac{X}{8} \pm \frac{1}{8})$ intercept with line $f(X) = (2X \pm 1)$ at point $(\frac{7}{15}, \frac{1}{15})$

	$q_2(x) = \frac{x}{3} + \frac{1}{3}$	⋮
	$r_2(x) = \frac{x}{4} + \frac{1}{4}$	⋮
	$s_2(x) = \frac{x}{5} + \frac{1}{5}$	⋮
	$t_2(x) = \frac{x}{7} + \frac{1}{7}$	⋮
	$f_3(x) = \frac{x}{8} + \frac{1}{8}$	⋮
	$g_3(x) = \frac{x}{9} + \frac{1}{9}$	⋮
	$h_3(x) = \frac{x}{10} + \frac{1}{10}$	⋮
	$p_3(x) = \frac{x}{180} + \frac{1}{180}$	⋮
	$l = \frac{1}{15}$ ≈ 0.06666666666667	⋮
	$m = \frac{7}{15}$ ≈ 0.46666666666667	⋮
+	Input...	



We can write this relation as

$$f(X) = \frac{X}{2N-1} + \frac{1}{2N-1}; \text{ at } X = N - 1.5; F(X) = 0.5$$

$$\text{Y intercept} = \frac{1}{2N-1}$$

$$\frac{(N-1.5)}{2N-1} + \frac{1}{2N-1} = \frac{1}{2}; \text{ for any } N; \text{ because it is ratio which} = 0.5 \text{ for any } N$$

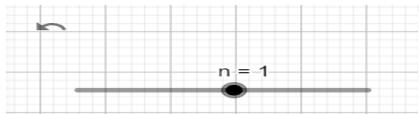
If $X = N - 1.5$

Then

$$f(X) = \frac{X}{2N-1} + \frac{1}{2N-1}; \text{ at } X = N - 1.5; F(X) = 0.5$$

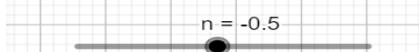
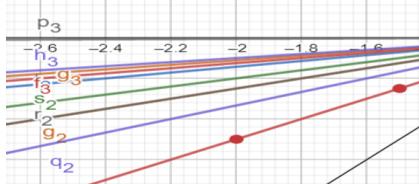
Which is a linear relation rotates 360 degrees around point $(-1, 0)$ for any value for N

$$i(X) = \frac{X}{2N-1} + \frac{1}{2N-1}$$



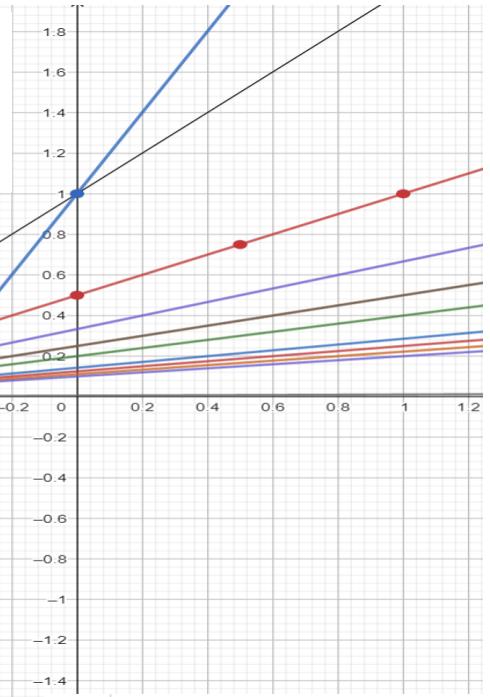
$$e = \frac{n-1}{2n-1} + \frac{1}{2n-1}$$

$\rightarrow 1$



$$i(x) = \frac{x}{2(-0.5) - 1} + \frac{1}{2(-0.5) - 1}$$

$$P_1(x) = 2x + 1$$

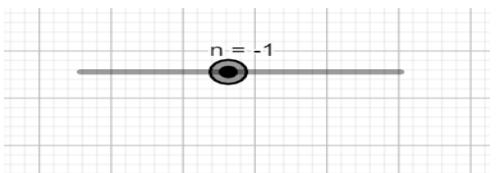


$$\rightarrow \frac{1}{4}$$

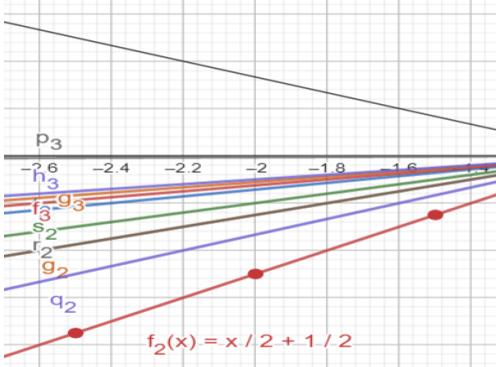


$$P_1(x) = 2x + 1$$





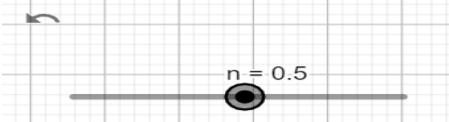
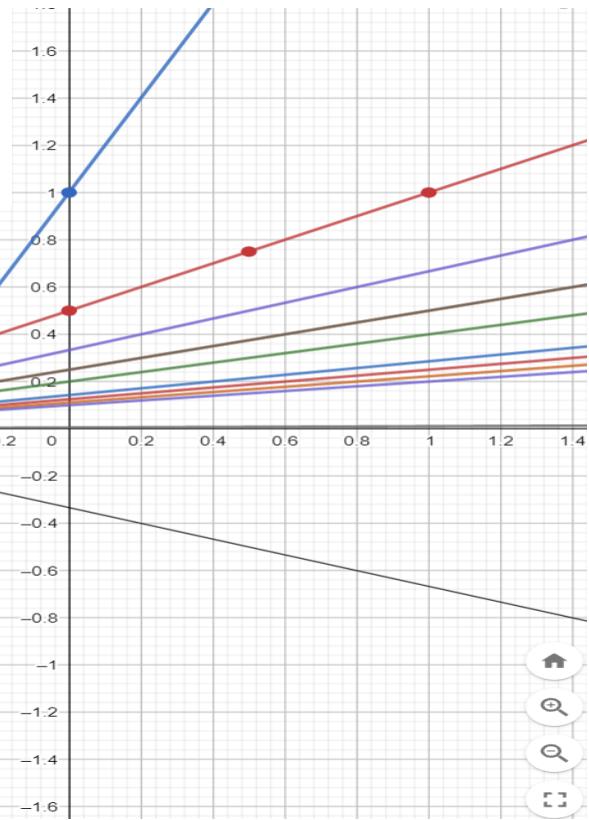
$$i(x) = \frac{x}{2(-1)-1} + \frac{1}{2(-1)-1}$$



$$e = \frac{n-1}{2n-1} + \frac{1}{2n-1}$$

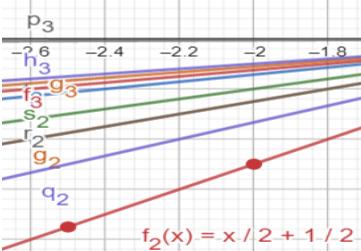
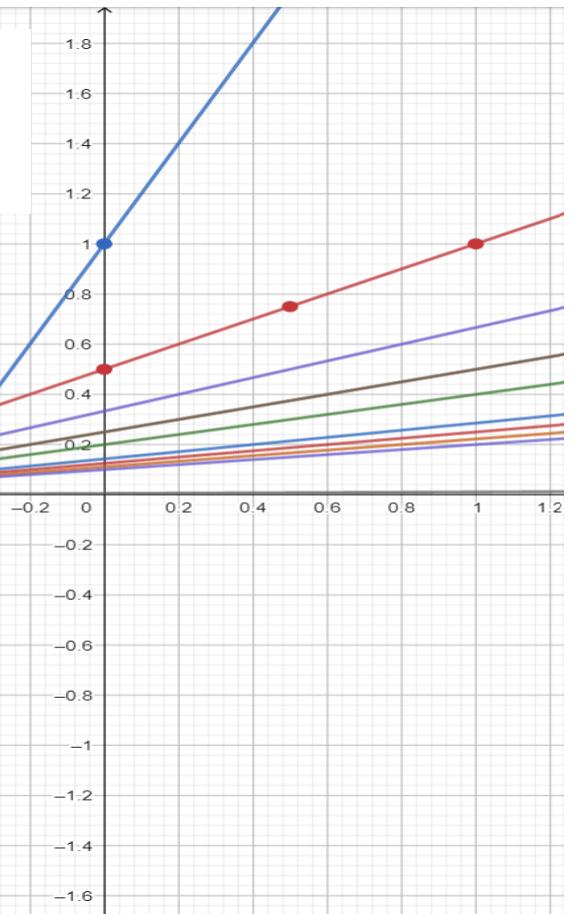
$$\rightarrow \frac{1}{3}$$

$$p_1(x) = 2x + 1$$



$$e = \frac{n-1}{2n-1} + \frac{1}{2n-1}$$

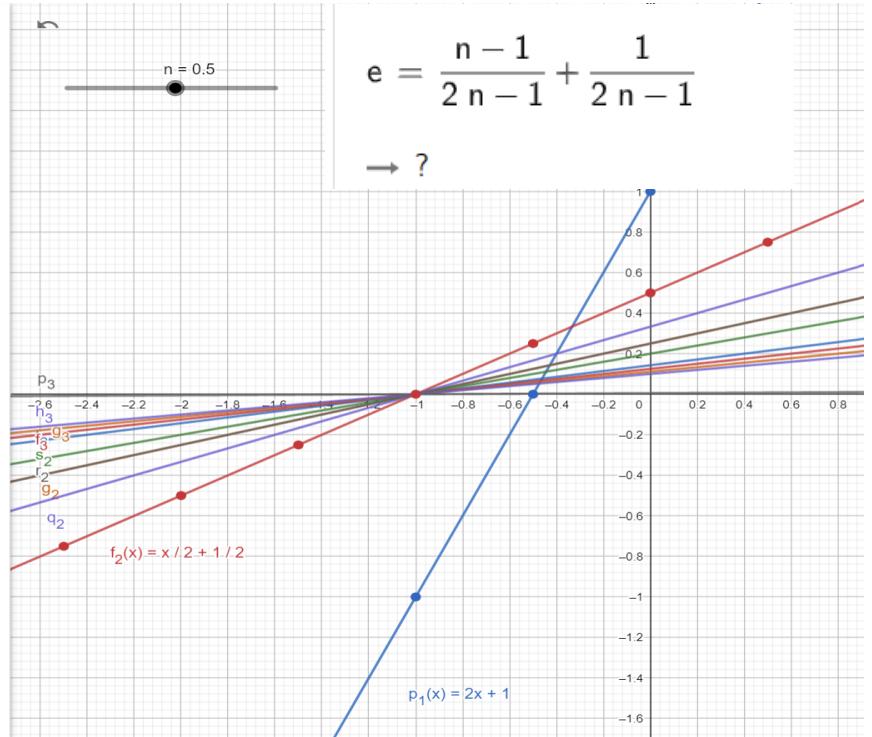
$\rightarrow ?$



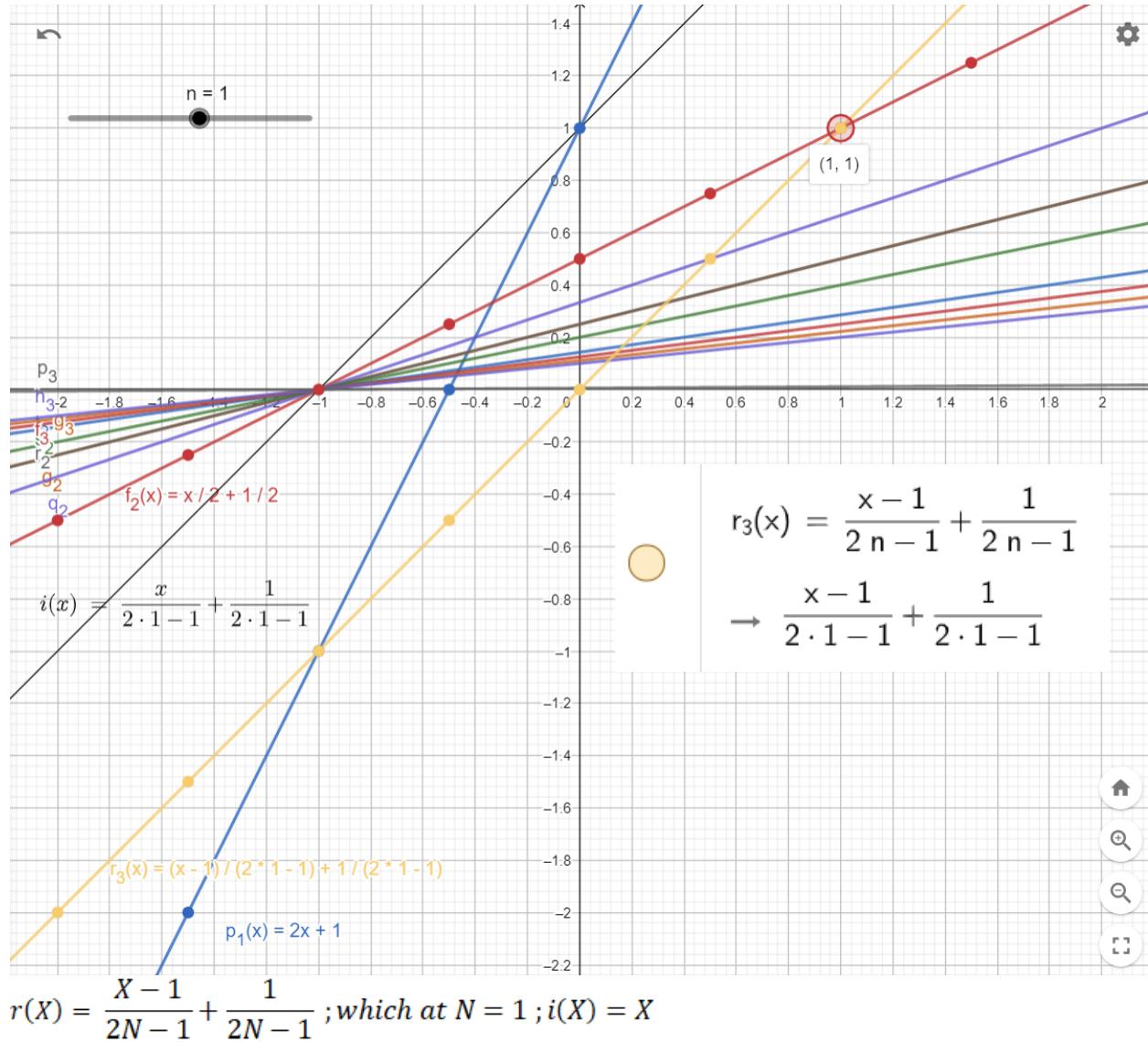
$$p_1(x) = 2x + 1$$

when N = 0.5 denominator is Zero so no value for i(X) or infinity for any value for X.

x :	$f_1(x)$:	$p_1(x)$:	$f_2(x)$:	$i(x)$:
-3.5	-1080	-6	-1.25	
-3	-900	-5	-1	
-2.5	-720	-4	-0.75	
-2	-540	-3	-0.5	
-1.5	-360	-2	-0.25	
-1	-180	-1	0	
-0.5	0	0	0.25	
0	180	1	0.5	
0.5	360	2	0.75	∞
1	540	3	1	∞
1.5	720	4	1.25	∞
2	900	5	1.5	∞
2.5	1080	6	1.75	∞
3	1260	7	2	∞

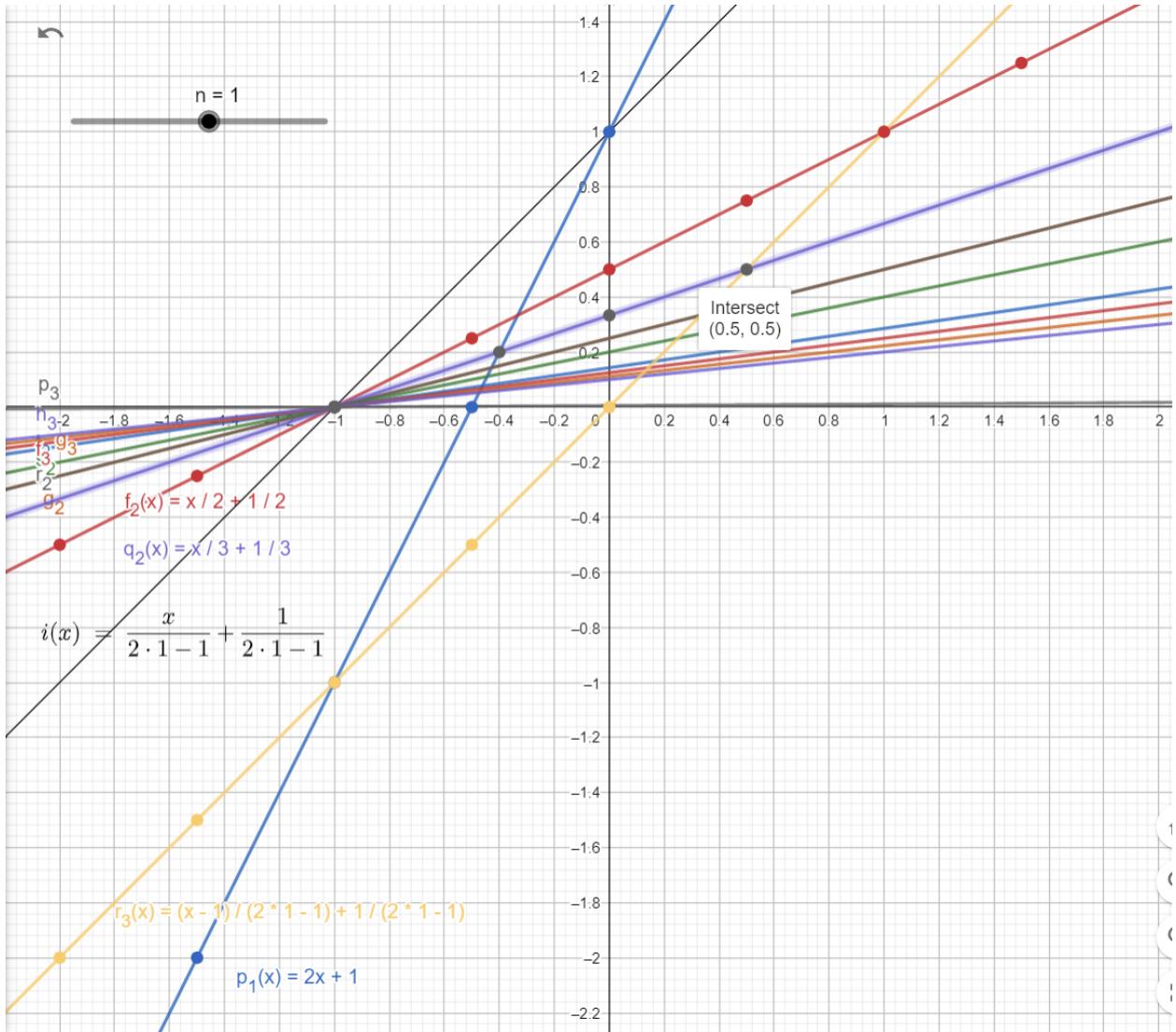


Let $X = X-1$ THEN

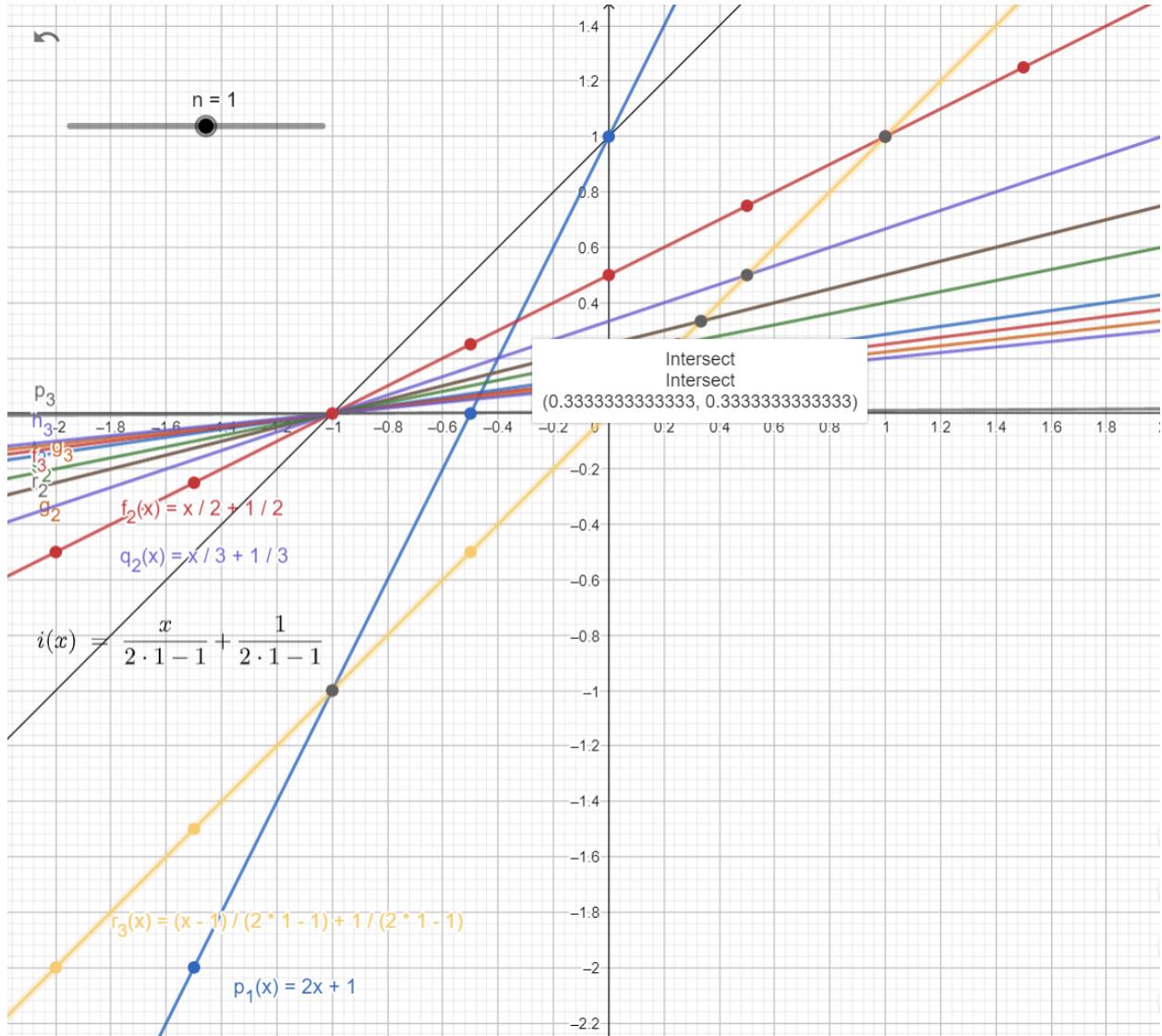


$$r(X) = \frac{X-1}{2N-1} + \frac{1}{2N-1}; \text{ Intersect with each Line } f(X) = \frac{x}{N} \pm \frac{1}{N} \text{ at point } \frac{1}{N-1}$$

$$r(X) = \frac{X-1}{2N-1} + \frac{1}{2N-1}; \text{ at } N=1; \text{ Intersect with each Line } f(X) = \frac{X}{3} \pm \frac{1}{3} \text{ at point } (\frac{1}{2}, \frac{1}{2})$$



$$r(X) = \frac{x-1}{2N-1} + \frac{1}{2N-1}; \text{ at } N=1; \text{ Intersect with each Line } f(X) = \frac{x}{4} \pm \frac{1}{4} \text{ at point } (\frac{1}{3}, \frac{1}{3})$$



$$r(X) = \frac{X-1}{2N-1} + \frac{1}{2N-1} - \frac{1}{4N-2};$$

$$r(X) = \frac{X-1}{2N-1} + \frac{1}{2N-1} - \frac{1}{4N-2} = \frac{X-1.5}{2N-1} + \frac{1}{2N-1}$$

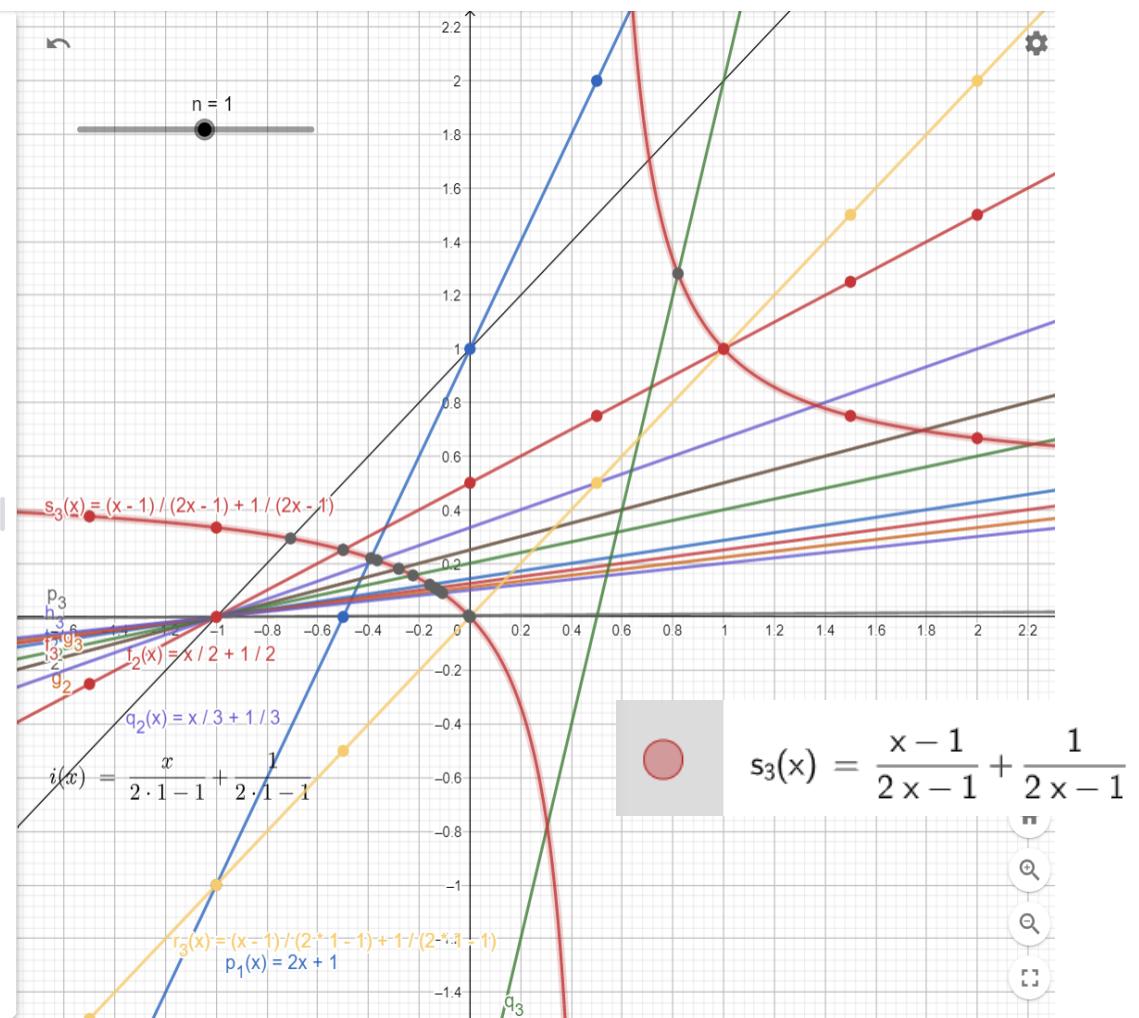
$$r(X) = \frac{X-1.5}{2N-1} + \frac{1}{2N-1}$$

$$\frac{X-1.5}{2X-1} + \frac{1}{2X-1} = \frac{X-0.5}{2X-1} = \frac{X-0.5}{2(X-0.5)} = \frac{1}{2}$$

$$r(X) = \frac{X-1.5}{2X-1} + \frac{1}{2X-1} = \frac{1}{2}$$

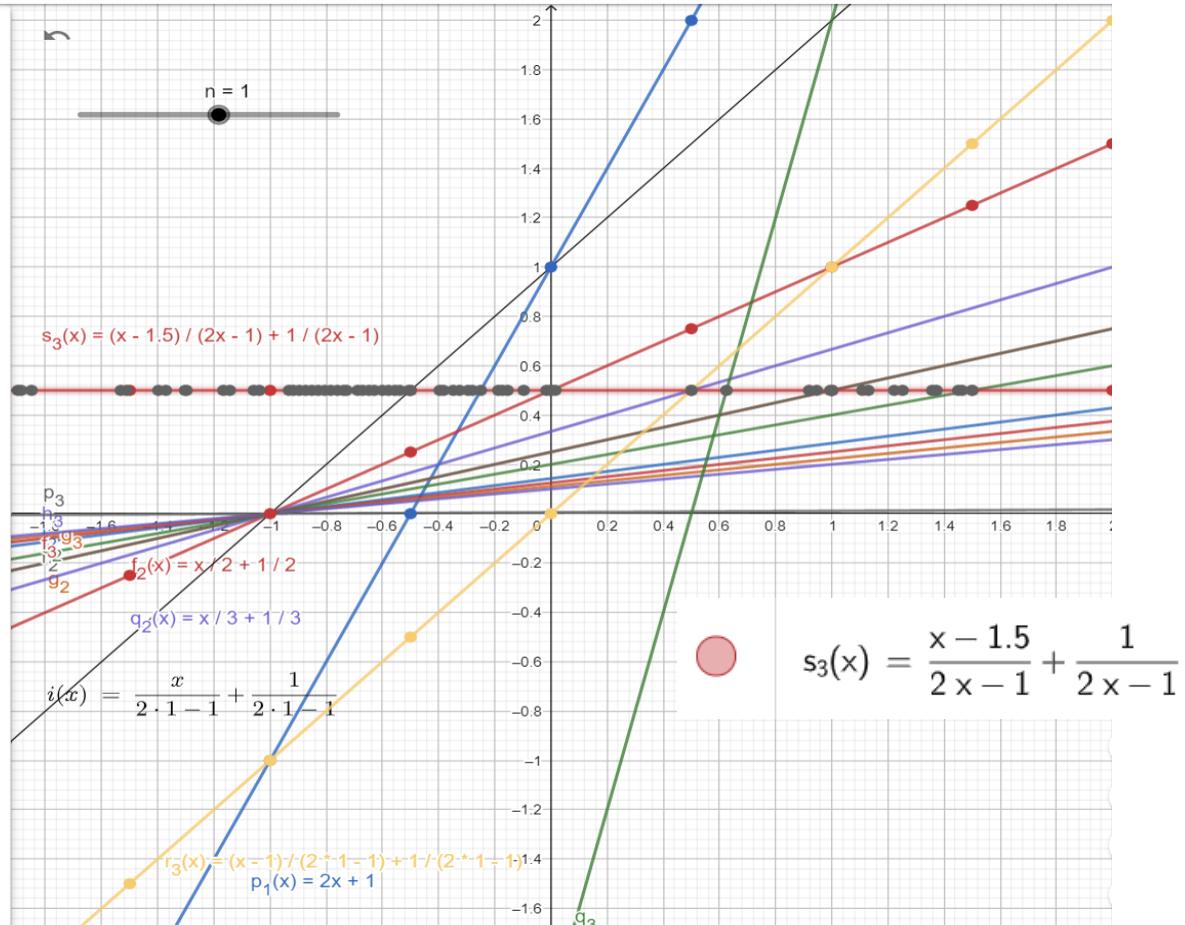
1) $S(X) = \frac{X-1}{2X-1} + \frac{1}{2X-1}$

$r_3(x)$	$s_3(x)$
-2.5	0.416666666...
-2	0.4
-1.5	0.375
-1	0.333333333...
-0.5	0.25
0	0
0.5	
1	1
1.5	0.75
2	0.666666666...
2.5	0.625
3	0.6
3.5	0.583333333...
4	0.571428571...
4.5	0.5625



$$2) \quad S(X) = \frac{X-1.5}{2X-1} + \frac{1}{2X-1}$$

$r_3(x)$	$s_3(x)$
-2.5	0.5
-2	0.5
-1.5	0.5
-1	0.5
-0.5	0.5
0	0.5
0.5	
1	0.5
1.5	0.5
2	0.5
2.5	0.5
3	0.5
3.5	0.5
4	0.5
4.5	0.5



3)

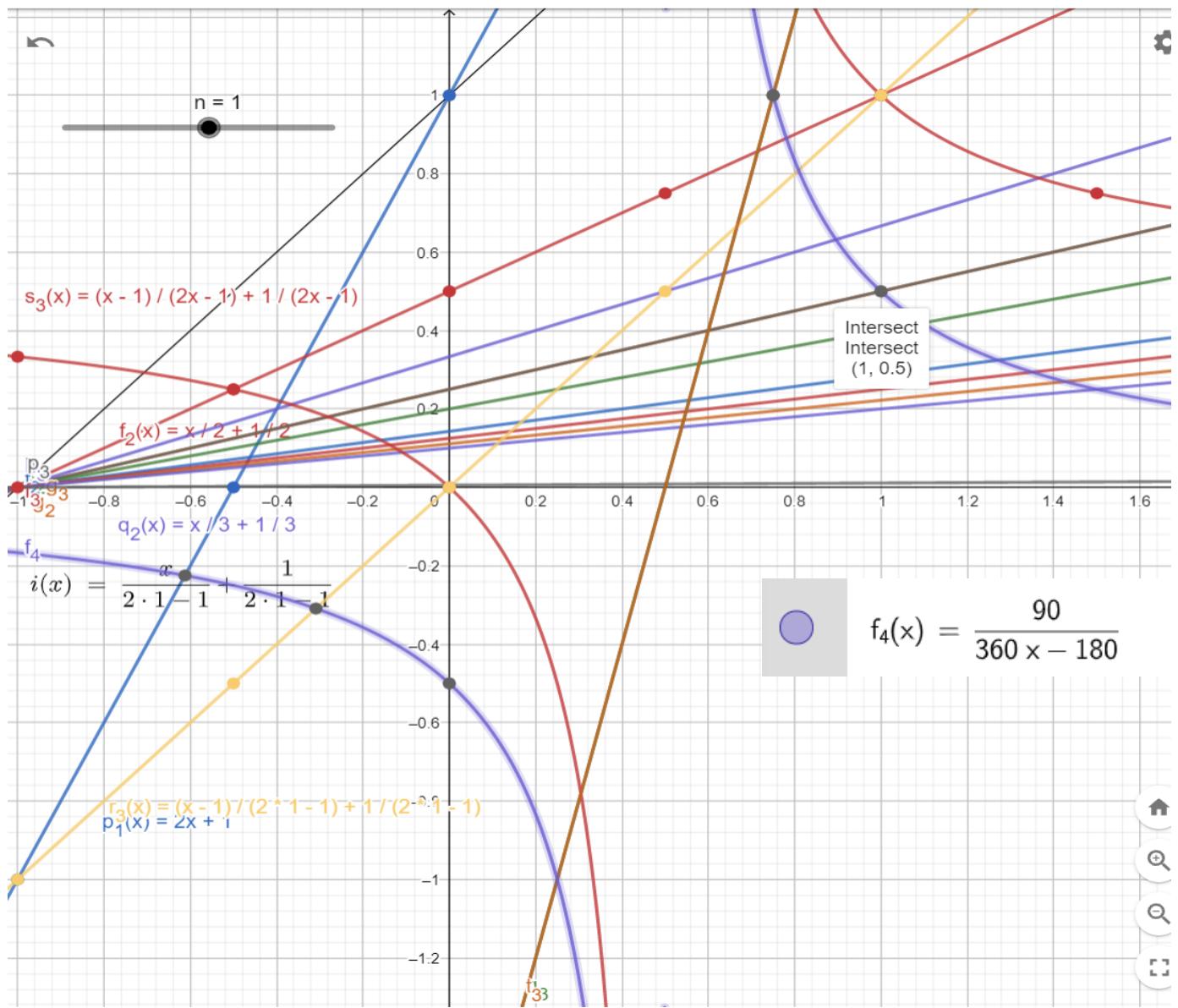
$$S(X) = \frac{X - 1.5}{2X - 1} + \frac{1}{2X - 1} = \frac{X - 1}{2X - 1} + \frac{1}{2X - 1} - \frac{1}{4X - 2}$$

This 0.5 is due to subtract this term $\frac{1}{4x-2}$; Change this term back to degrees, TEHN

$$S(X) = 4X - 2 = \frac{360}{90} (X - 0.5)$$

$$S(X) = \frac{1}{4X - 2} = \frac{90}{360(x - 0.5)}$$

$$f4(x) = \frac{360}{90} (X - 0.5)$$



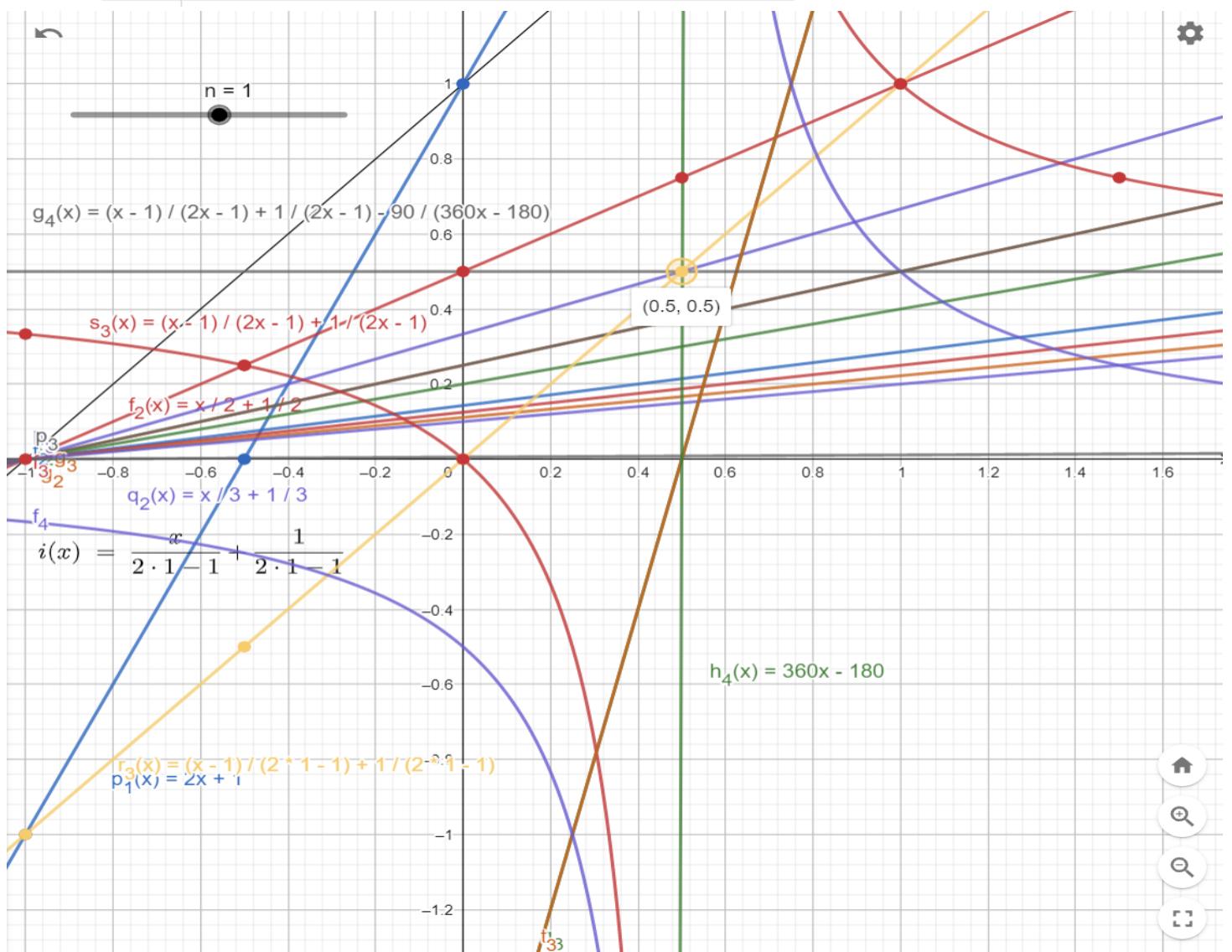
● $s_3(x) = \frac{x-1}{2x-1} + \frac{1}{2x-1}$

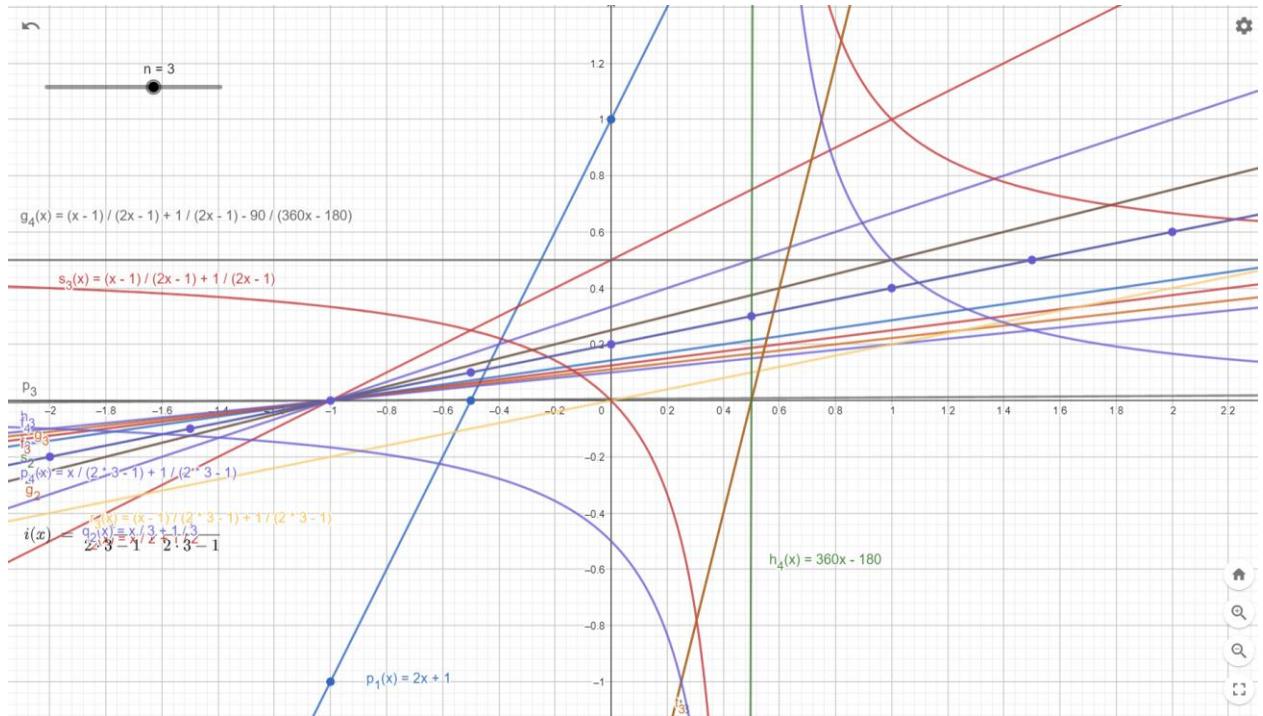
● $t_3(x) = \frac{360}{90} (x - 0.5)$

● $f_4(x) = \frac{90}{360x - 180}$

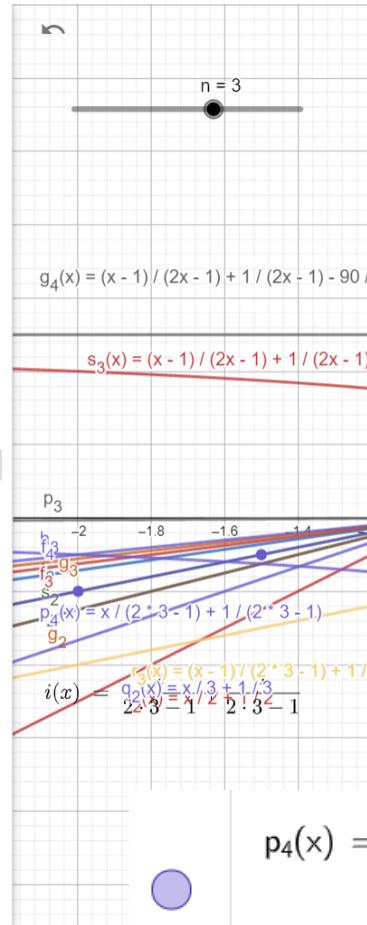
● $g_4(x) = \frac{x-1}{2x-1} + \frac{1}{2x-1} - \frac{90}{360x - 180}$

● $h_4(x) = 360x - 180$





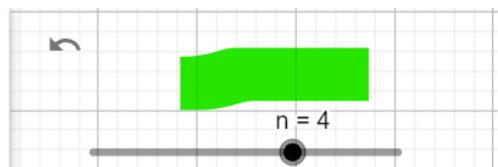
x ::	f ₁ (x) ::	p ₁ (x) ::	p ₄ (x) ::
-4	-1260	-7	-0.6
-3.5	-1080	-6	-0.5
-3	-900	-5	-0.4
-2.5	-720	-4	-0.3
-2	-540	-3	-0.2
-1.5	-360	-2	-0.1
-1	-180	-1	0
-0.5	0	0	0.1
0	180	1	0.2
0.5	360	2	0.3
1	540	3	0.4
1.5	720	4	0.5
2	900	5	0.6
2.5	1080	6	0.7
3	1260	7	0.8



$$\begin{aligned}
p_4(x) &= \frac{x}{2n-1} + \frac{1}{2n-1} \\
&\rightarrow \frac{x}{2 \cdot 3 - 1} + \frac{1}{2 \cdot 3 - 1}
\end{aligned}$$

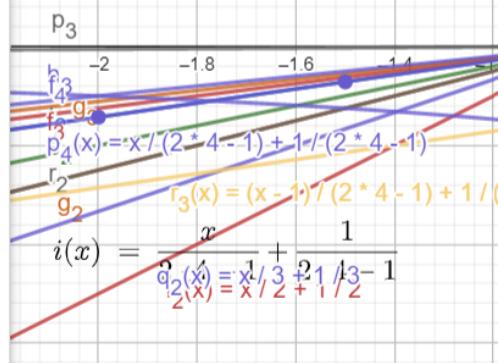
$y = 0.5$
 at
 $x = n - 1.5$

x	$f_1(x)$	$p_1(x)$	$p_4(x)$
-4	-1260	-7	-0.428571428...
-3.5	-1080	-6	-0.357142857...
-3	-900	-5	-0.285714285...
-2.5	-720	-4	-0.214285714...
-2	-540	-3	-0.142857142...
-1.5	-360	-2	-0.071428571...
-1	-180	-1	0
-0.5	0	0	0.0714285714...
0	180	1	0.1428571428...
0.5	360	2	0.2142857142...
1	540	3	0.2857142857...
1.5	720	4	0.3571428571...
2	900	5	0.4285714285...
2.5	1080	6	0.5
3	1260	7	0.5714285714...



$$g_4(x) = (x - 1) / (2x - 1) + 1 / (2x - 1) - 90 /$$

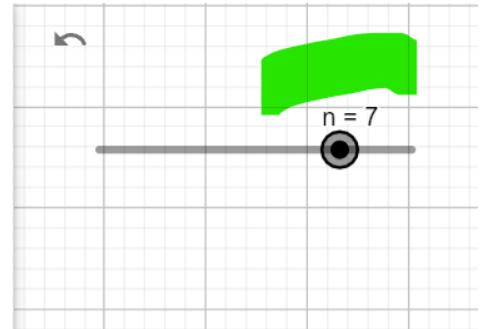
$$s_3(x) = (x - 1) / (2x - 1) + 1 / (2x - 1)$$



$$p_4(x) = \frac{x}{2n-1} + \frac{1}{2n-1}$$

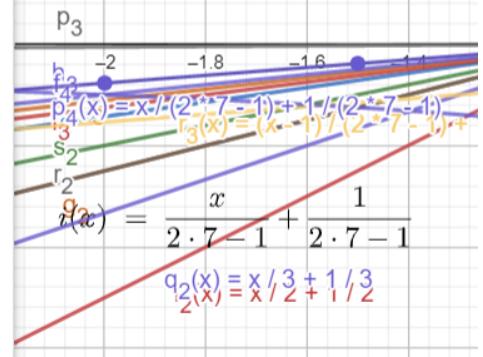
$$\rightarrow \frac{x}{2 \cdot 4 - 1} + \frac{1}{2 \cdot 4 - 1}$$

x	$f_1(x)$	$p_1(x)$	$p_4(x)$
1.5	720	4	0.1923076923...
2	900	5	0.2307692307...
2.5	1080	6	0.2692307692...
3	1260	7	0.3076923076...
3.5	1440	8	0.3461538461...
4	1620	9	0.3846153846...
4.5	1800	10	0.4230769230...
5	1980	11	0.4615384615...
5.5	2160	12	0.5
6	2340	13	0.5384615384...
6.5	2520	14	0.5769230769...
7	2700	15	0.6153846153...
7.5	2880	16	0.6538461538...
8	3060	17	0.6923076923...



$$g_4(x) = (x - 1) / (2x - 1) + 1 / (2x - 1) - 9$$

$$s_3(x) = (x - 1) / (2x - 1) + 1 / (2x - 1)$$

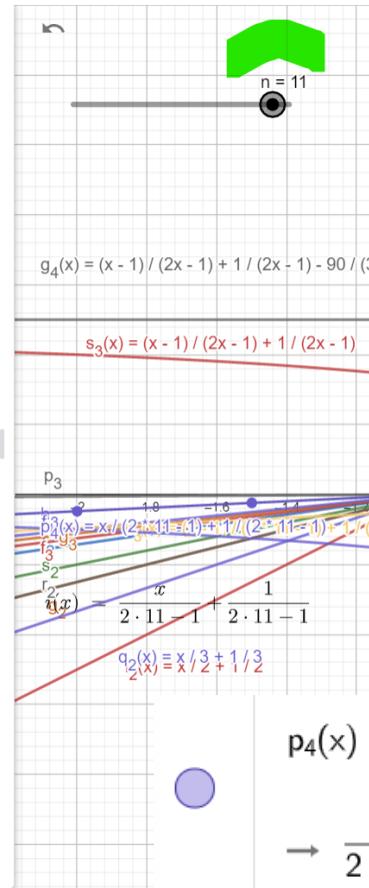


Handwritten notes on the right side:

$$p_4(x) = \frac{x}{2 \cdot 7 - 1} + \frac{1}{2 \cdot 7 - 1}$$

$$\rightarrow \frac{x}{2 \cdot 7 - 1} + \frac{1}{2 \cdot 7 - 1}$$

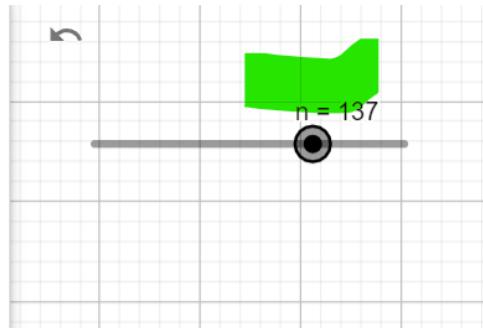
x	f ₁ (x)	p ₁ (x)	p ₄ (x)
7	2700	15	0.3809523809...
7.5	2880	16	0.4047619047...
8	3060	17	0.4285714285...
8.5	3240	18	0.4523809523...
9	3420	19	0.4761904761...
9.5	3600	20	0.5
10	3780	21	0.5238095238...
10.5	3960	22	0.5476190476...
11	4140	23	0.5714285714...
11.5	4320	24	0.5952380952...
12	4500	25	0.6190476190...
12.5	4680	26	0.6428571428...
13	4860	27	0.6666666666...
13.5	5040	28	0.6904761904...



$$p_4(x) = \frac{x}{2 \cdot 11 - 1} + \frac{1}{2 \cdot 11 - 1}$$

$$\rightarrow \frac{x}{2 \cdot 11 - 1} + \frac{1}{2 \cdot 11 - 1}$$

x	$f_1(x)$	$p_1(x)$	$p_4(x)$
132.5	47880	266	0.489010989...
133	48060	267	0.490842490...
133.5	48240	268	0.492673992...
134	48420	269	0.494505494...
134.5	48600	270	0.496336996...
135	48780	271	0.498168498...
135.5	48960	272	0.5
136	49140	273	0.501831501...
136.5	49320	274	0.503663003...
137	49500	275	0.505494505...
137.5	49680	276	0.507326007...
138	49860	277	0.509157509...
138.5	50040	278	0.510989010...
139	50220	279	0.512820512...
139.5	50400	280	0.514652014...



$$g_4(x) = (x - 1) / (2x - 1) + 1 / (2x - 1) - 90$$

$$s_3(x) = (x - 1) / (2x - 1) + 1 / (2x - 1)$$

p_3

$$p_4(x) = x / (2 \cdot 137 - 1) + 1 / (2 \cdot 137 - 1)$$

$$f_3(x) = g_3(x)$$

$$s_2(x) = r_2(x)$$

$$q_2(x) = x / 2 + 1 / 2$$

\bullet

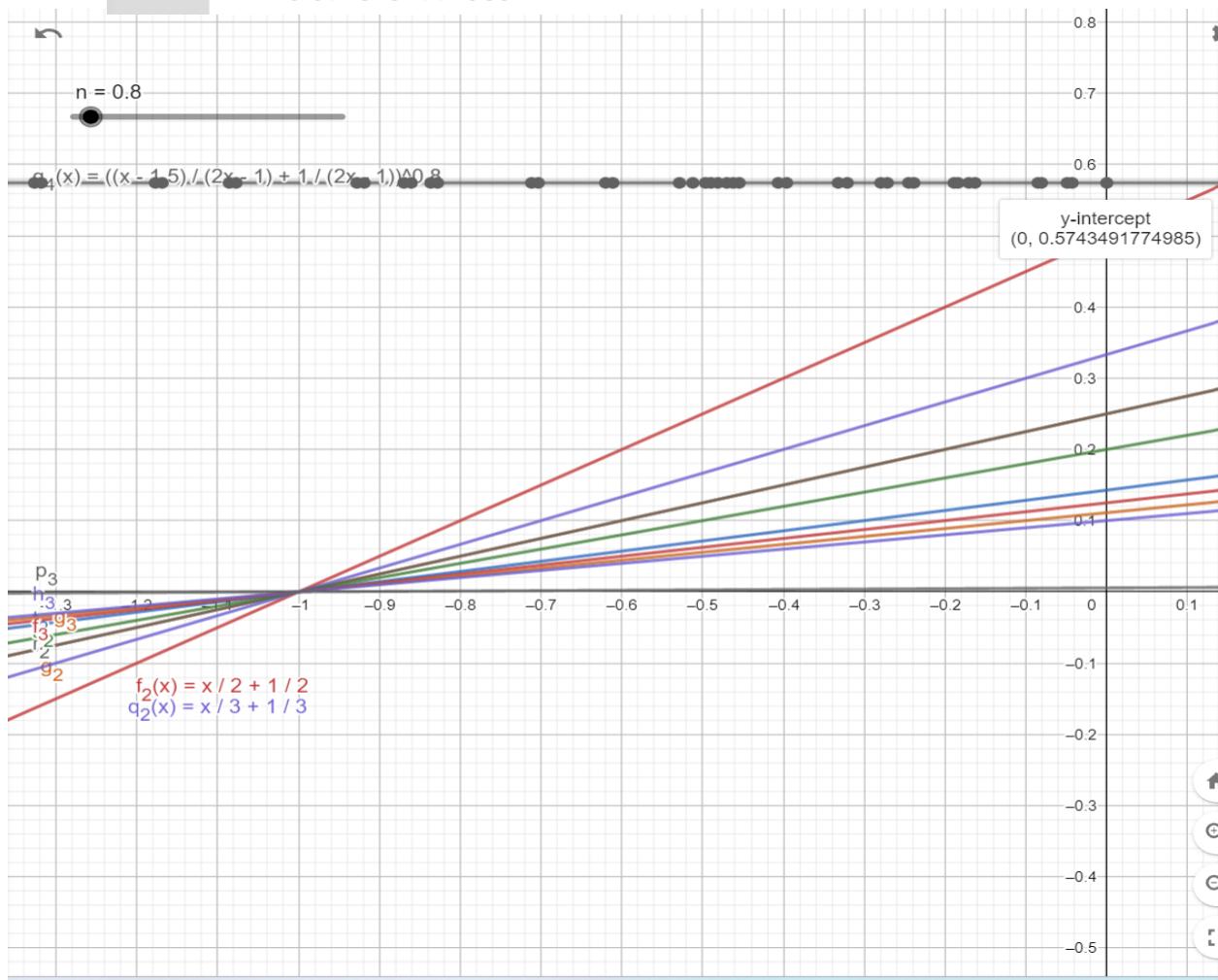
$$p_4(x) = \frac{x}{2 \cdot 137 - 1} + \frac{1}{2 \cdot 137 - 1}$$

$$\rightarrow \frac{x}{2 \cdot 137 - 1} + \frac{1}{2 \cdot 137 - 1}$$

$$Q(X) = \left(\frac{X - 1.5}{2X - 1} + \frac{1}{2X - 1} \right)^N = \left(\frac{1}{2} \right)^N ; \text{for any } N$$

q₄(x) = $\left(\frac{x - 1.5}{2x - 1} + \frac{1}{2x - 1} \right)^n$
 $\rightarrow \left(\frac{x - 1.5}{2x - 1} + \frac{1}{2x - 1} \right)^{0.8}$

$I = \left(\frac{1}{2} \right)^n$
 $\rightarrow 0.5743491774985$

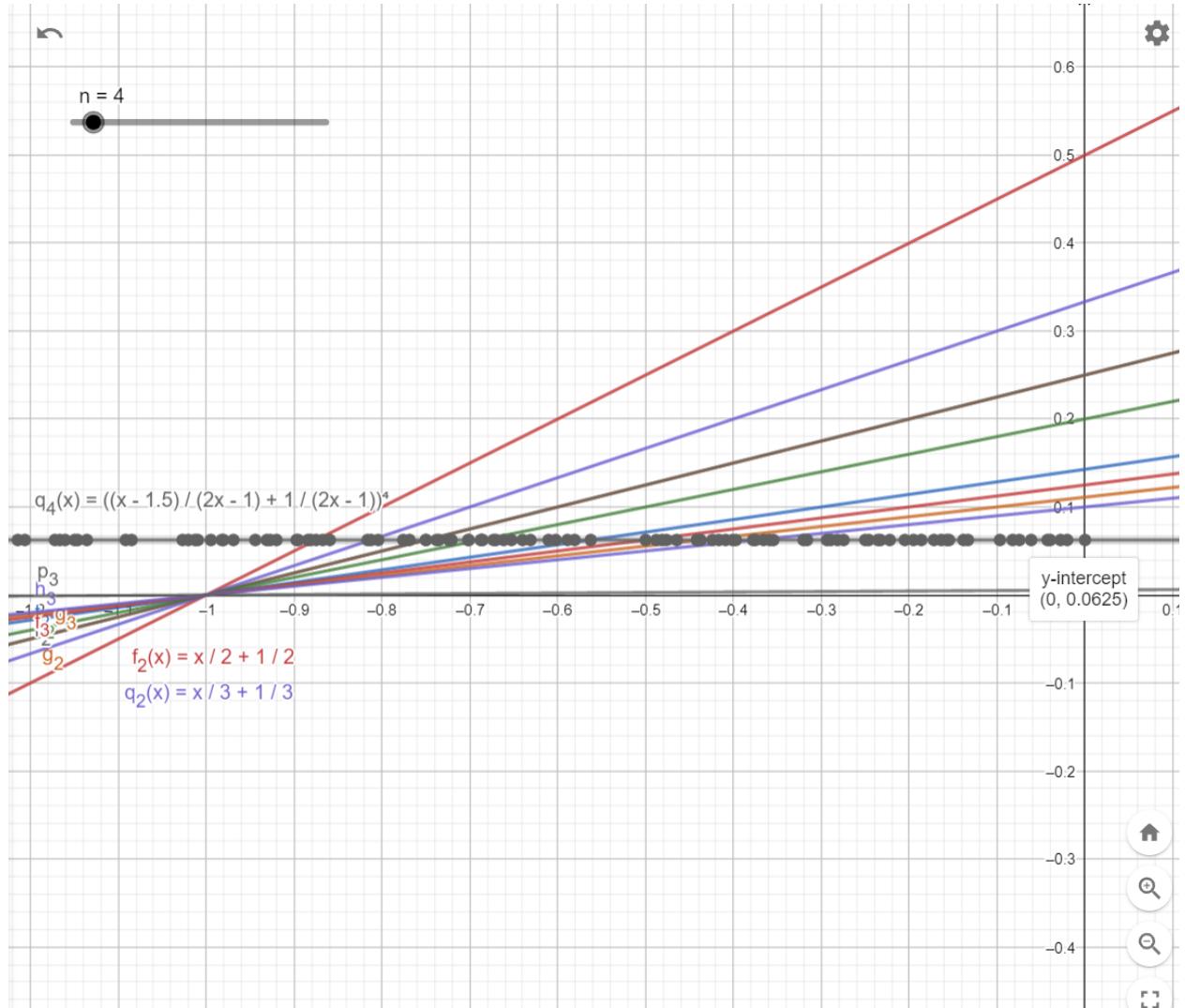


$$Q(X) = \left(\frac{X - 1.5}{2X - 1} + \frac{1}{2X - 1} \right)^N = \left(\frac{1}{2} \right)^N ; \text{for any } N$$

$$\begin{aligned} q_4(x) &= \left(\frac{x - 1.5}{2x - 1} + \frac{1}{2x - 1} \right)^n \\ &\rightarrow \left(\frac{x - 1.5}{2x - 1} + \frac{1}{2x - 1} \right)^4 \end{aligned}$$

$$l = \left(\frac{1}{2} \right)^n$$

$$\approx 0.0625$$

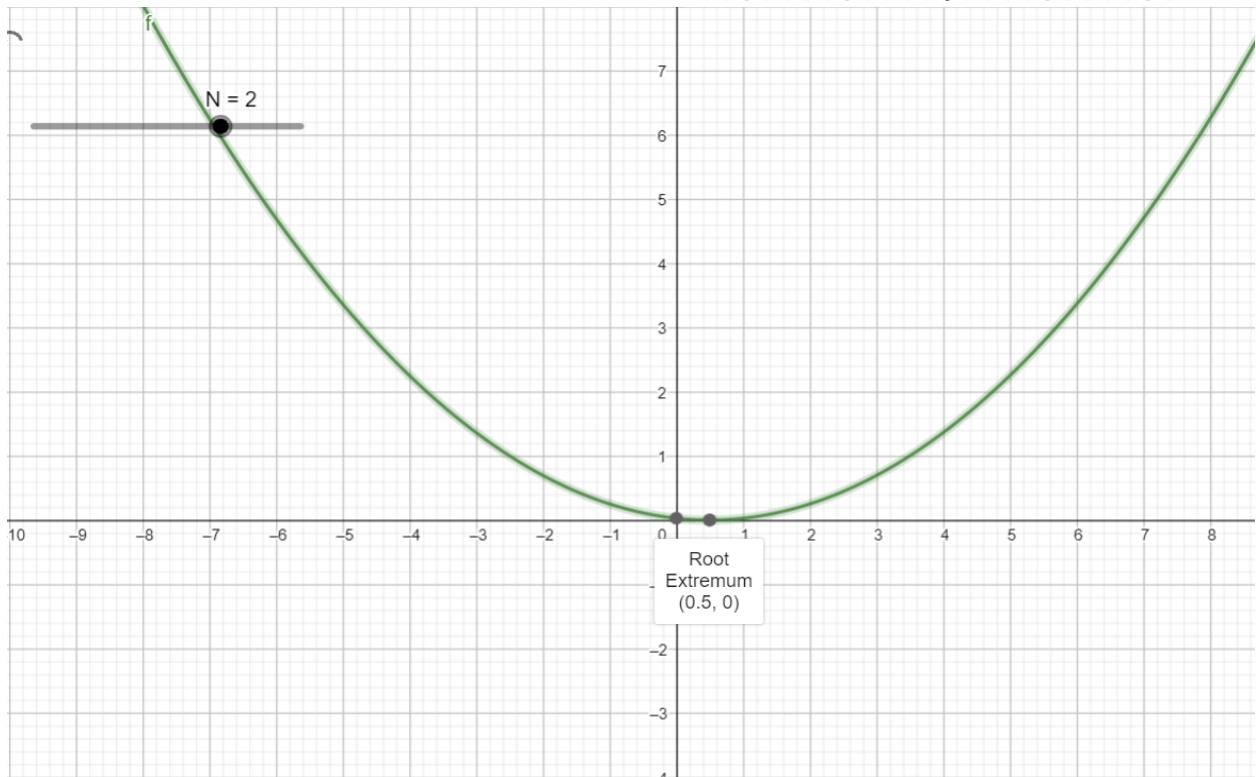
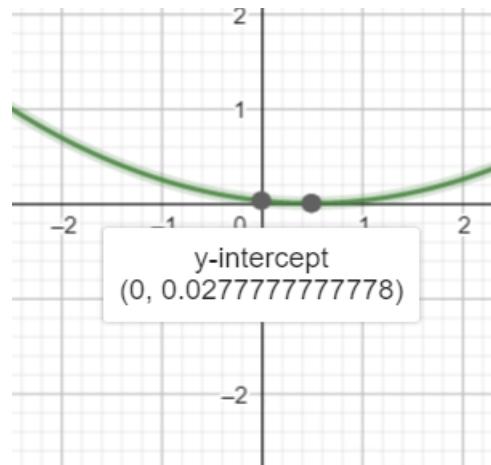


$$d = \frac{1}{360}$$

≈ 0.0027777777778

$$f(x) = \left(\frac{x - 1.5}{2N - 1} + \frac{1}{2N - 1} \right)^N$$

$$\rightarrow \left(\frac{x - 1.5}{2 \cdot 2 - 1} + \frac{1}{2 \cdot 2 - 1} \right)^2$$



For $X = 0.5$; for any $N \neq 0.5$; $F(X) = 0$; all the time; $F(X) = 0$ if $0.5 < X < 0.5$

(green circle)

$$f(x) = \left(\frac{x - 1.5}{2N - 1} + \frac{1}{2N - 1} \right)^N$$

$$\rightarrow \left(\frac{x - 1.5}{2 \cdot 2 - 1} + \frac{1}{2 \cdot 2 - 1} \right)^2$$

$$a = \frac{\frac{1}{2} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$

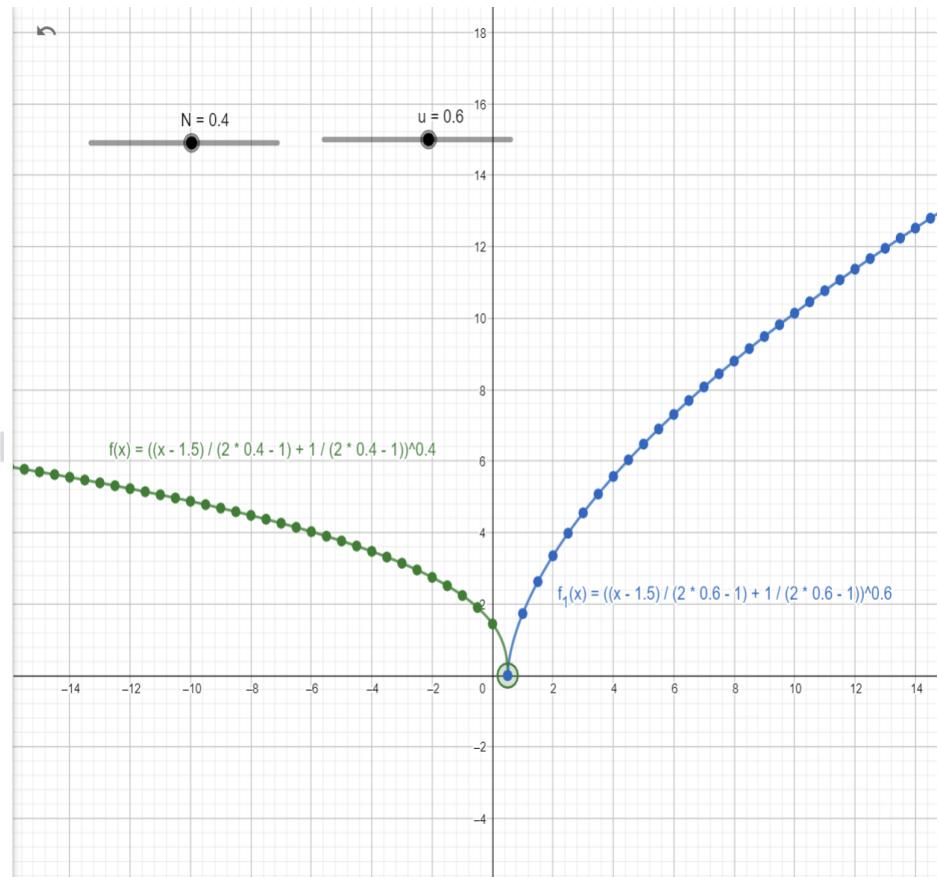
$$\rightarrow 0$$

IF $N = 0.4$ and $X = 0.5$ THEN $F(X) = 0$

IF $N = 0.6$ and $X = 0.5$ THEN $F(X) = 0$

IF $N = 0.5$ THEN $F(X)$ unknow

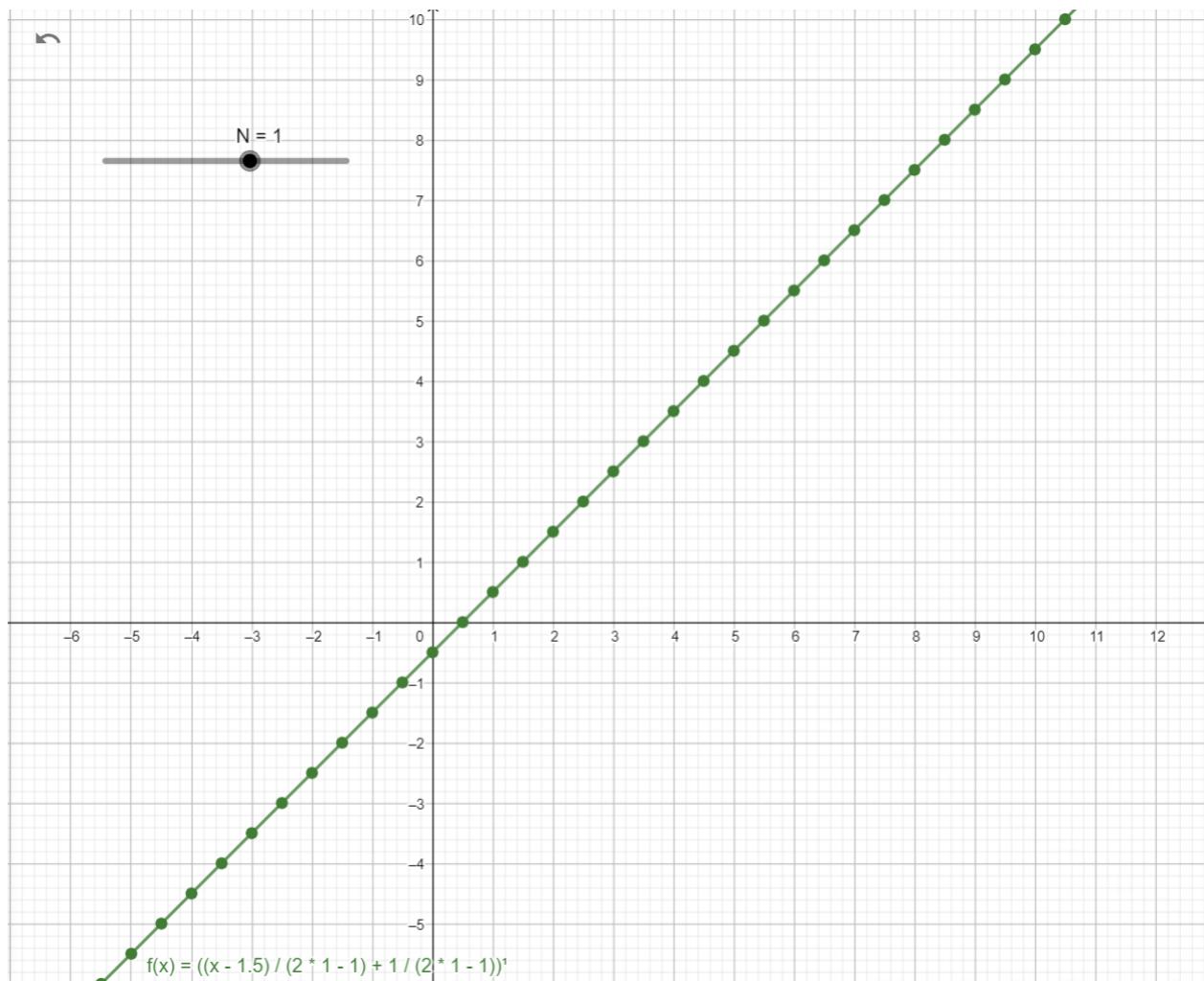
x :	f(x) :	$f_1(x)$
-2.5	2.954176939...	
-2	2.746401358...	
-1.5	2.511886431...	
-1	2.238847463...	
-0.5	1.903653938...	
0	1.442699905...	
0.5	0	
1	1.7328621078...	
1.5	2.6265278044...	
2	3.3499379133...	
2.5	3.98107170553	
3	4.5514105075...	
3.5	5.0775563919...	
4	5.5695849090...	



$$a = \frac{\frac{1}{1} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$

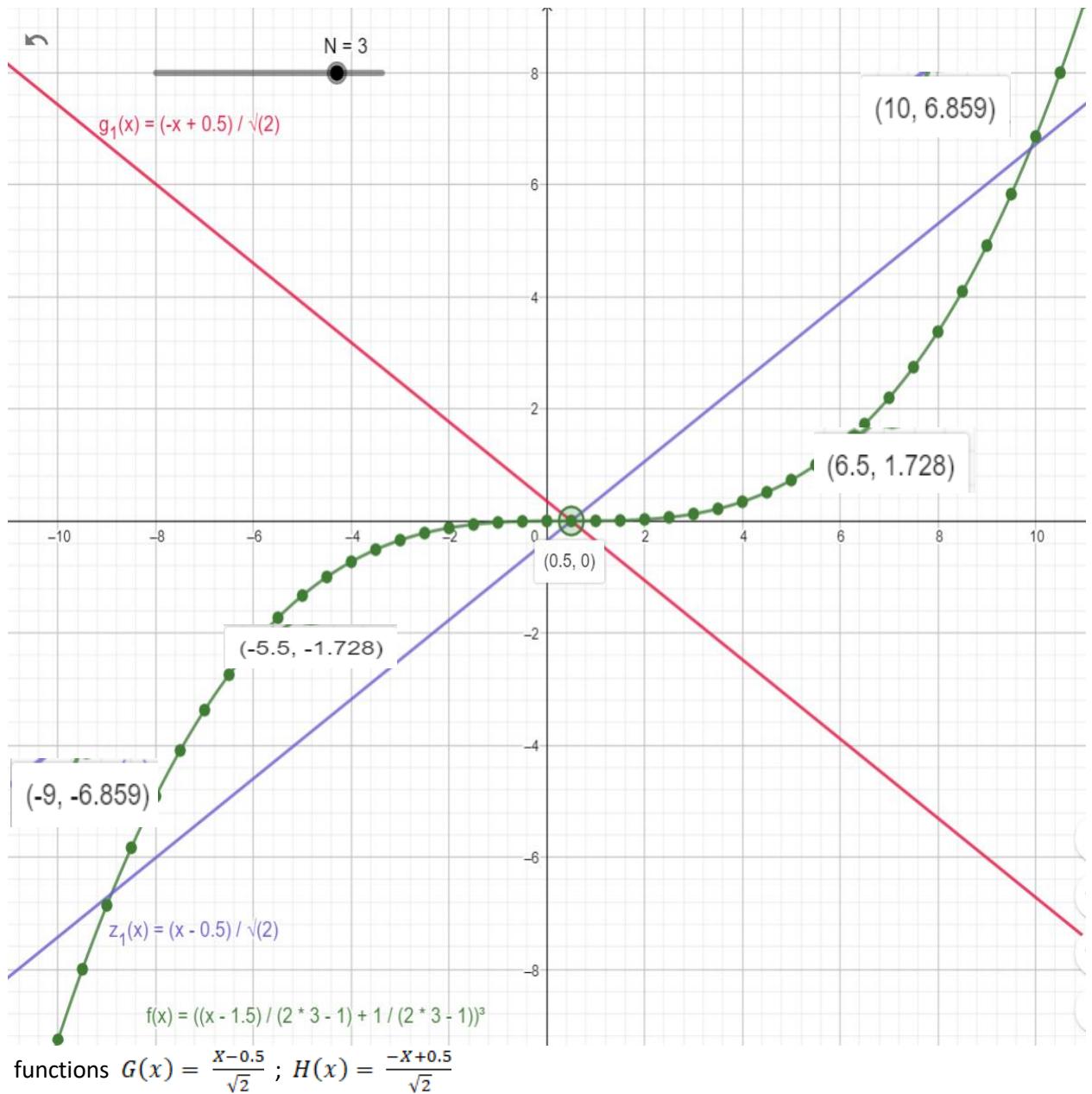
IF X = 1/1; N = 1

→ 0.5

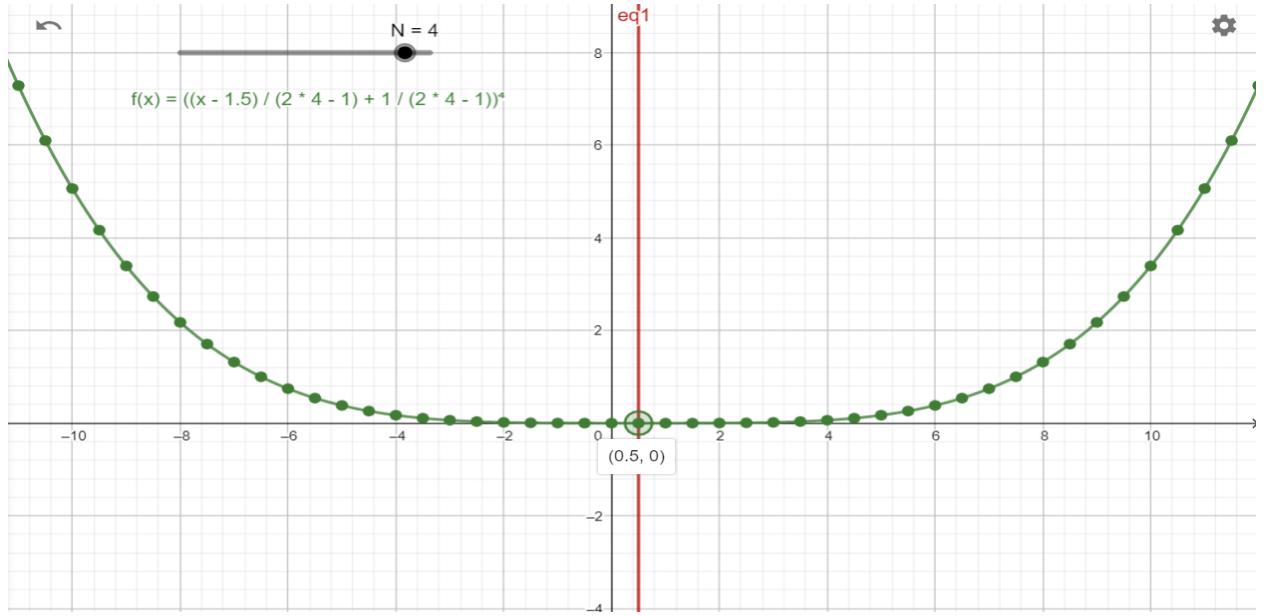


Symmetry Properties for N ODD numbers

Main property for N = Odd numbers they are symmetric over new axis intersection of two



But, For N = Even numbers the graphs are symmetric over Y axis. Or more specific over X = 0.5



But, because $F(X) = \left(\frac{X-1.5}{2N-1} + \frac{1}{2N-1}\right)^N$ denominator = $(2N-1)$ THEN at $N = X = 0.5$
 $F(X) =$ is unknown at $X = 0.5$.

So, we are going to use the symmetry property over axis; we will use small interval $X = [0, 1]$
And will get $F(X)$ at one of the edges of this interval.

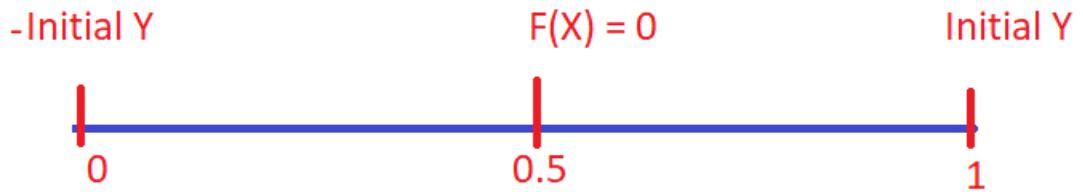
One of the values will be with opposite sign because $F(X)$ Nominator $(X-1.5+1)$; if $X = 0$ THEN $F(0) = (0-1.5+1 = -0.5)$ all the time = -0.5 and if raised to odd power will be negative for any $N = \text{ODD}$.

Same will be for the other Edge because $F(X)$ Nominator $(X-1.5+1)$; if $X = 1$ THEN $F(1) = (1-1.5+1 = 0.5)$ all the time = 0.5 and if raised to odd power will be Positive for any $N = \text{ODD}$.

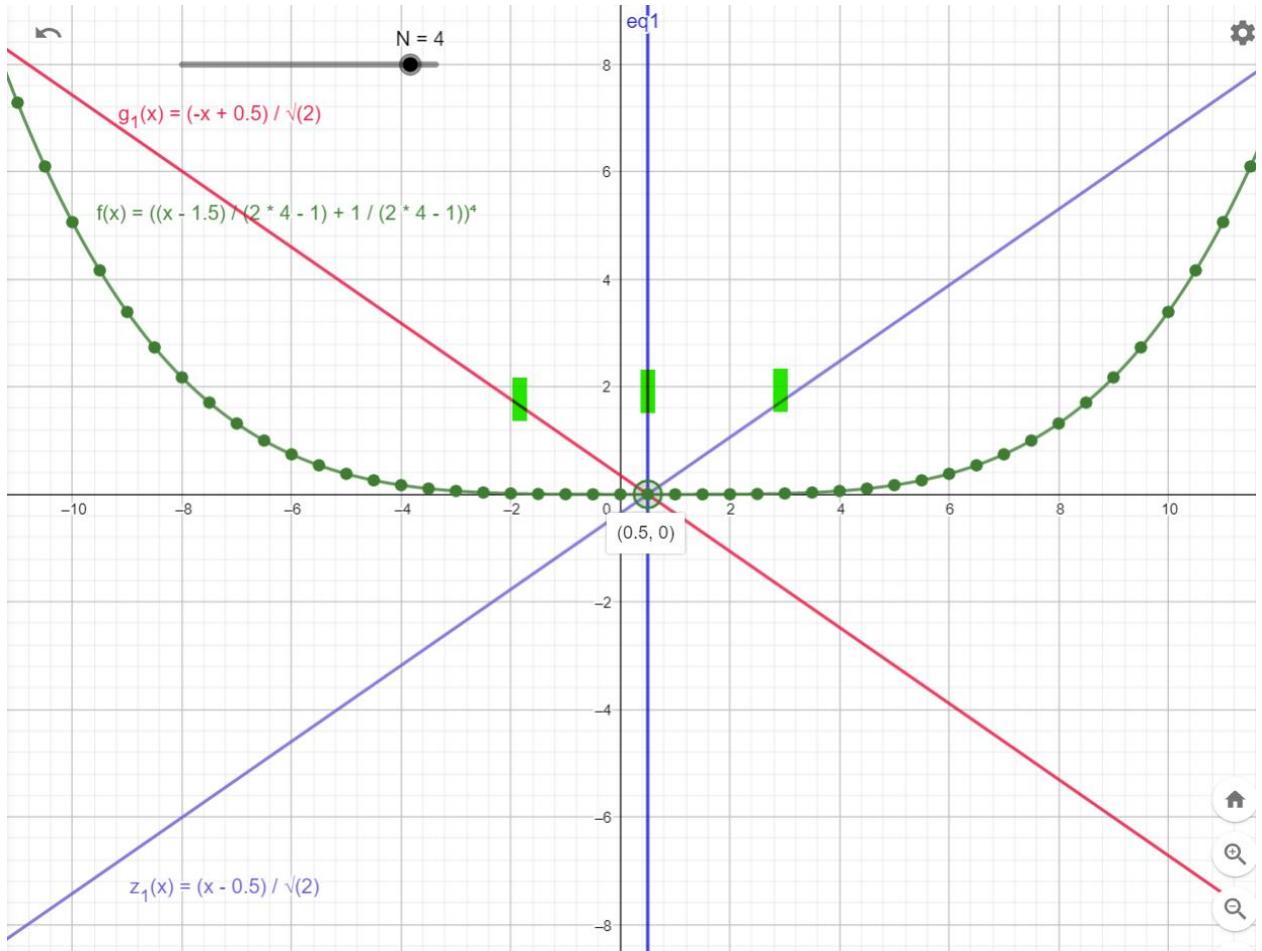
And as the values between these two sides of interval $X = [0, 1]$ one $F(0) = \text{Negative}$ and $F(1) = \text{Positive}$ then for sure one Zero will be between this interval $[0, 1]$ and because it is symmetric over axis $G(x) = \frac{x-0.5}{\sqrt{2}}$; $H(x) = \frac{-x+0.5}{\sqrt{2}}$ so for any $N = X = \text{ODD}$ a Zero will be at point $(0.5, 0)$.

IFF $X=1$; $F(X) = \left(\frac{X-1.5}{2N-1} + \frac{1}{2N-1}\right)^N$; for any N , will be a value $F(X)$ at $N=0.5$ that we was not able to get if we substitute by $N=0.5$, instead of use the symmetry property for interval $[0, 1]$

because value at $X = 0.5$ is exact mid point between $[0, 1]$; so, we get value for point $(0, \text{initial } Y)$ from $F(1)$ or $F(0)$, by getting value at one of the interval edges so we will get the interval edge $X=1$ and both points at $X = 1$ and $X=0$ both will have same value initial Y . [point1 = $(0, -\text{Initial } Y)$ and Point2 = $(1, \text{Initial } Y)$]



$$\text{Initial } Y = \left(\frac{1 - 1.5}{2N - 1} + \frac{1}{2N - 1} \right)^N ; \text{ for all } N \neq 0.5$$

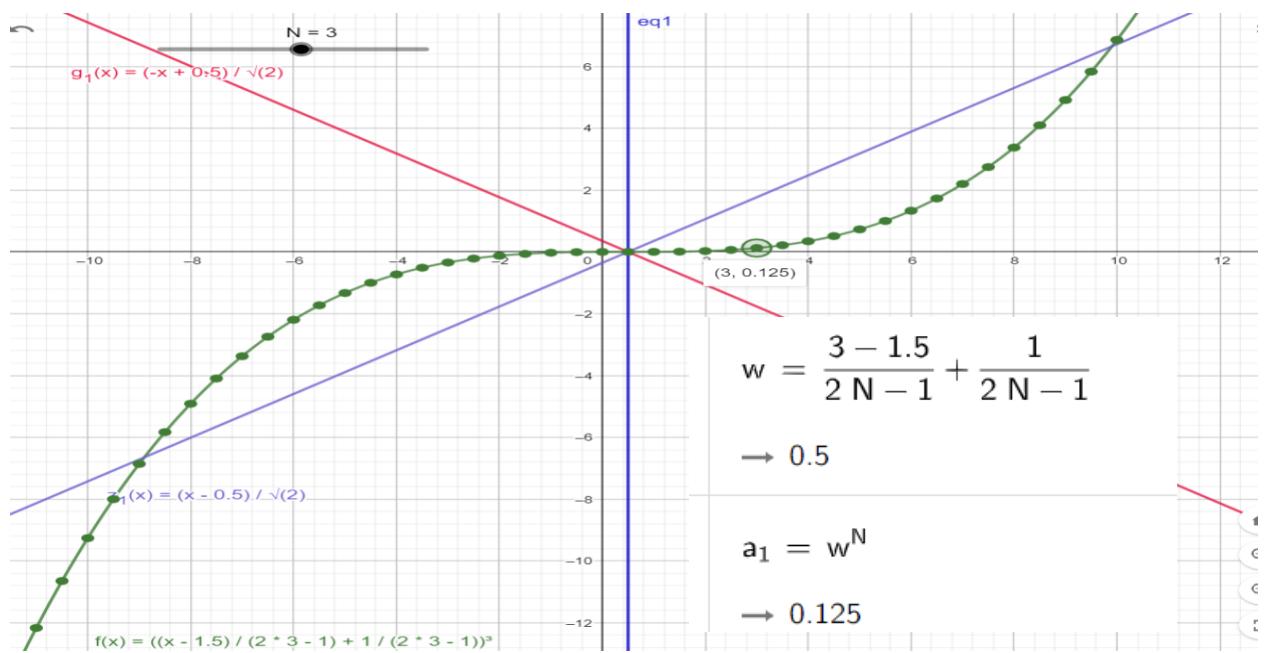
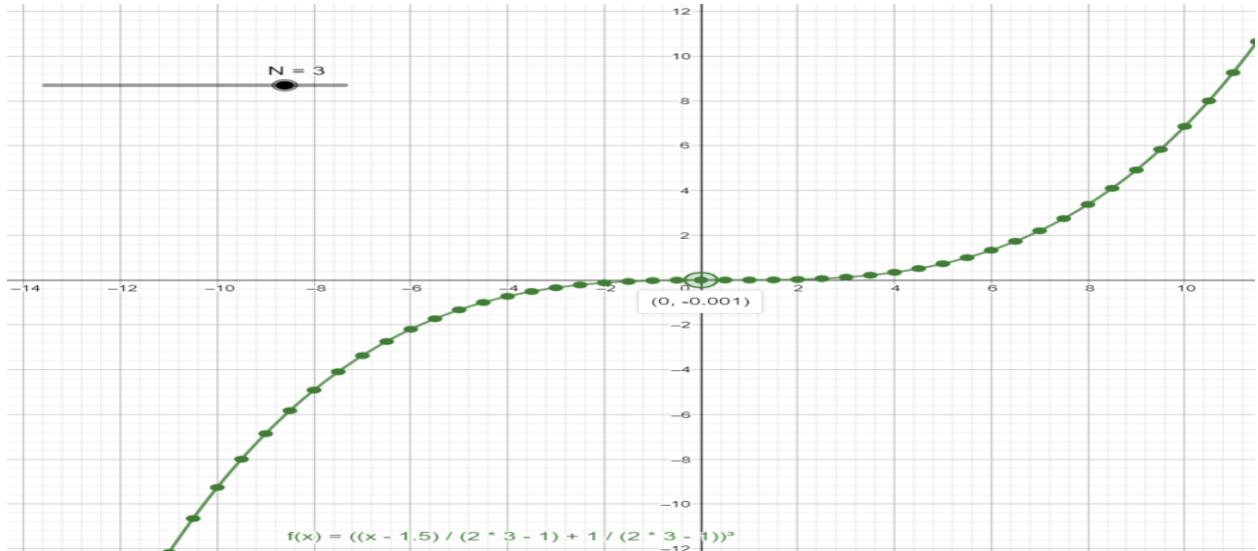


$$\text{FOR } X = 0 \text{ and } N = 3 \text{ THEN } F(X) = \left(\frac{0-1.5}{2*3-1} + \frac{1}{2*3-1}\right)^3 = (-0.1)^3 = -0.001$$

$$\text{FOR } X = 1 \text{ and } N = 3 \text{ THEN } F(X) = \left(\frac{1-1.5}{2*3-1} + \frac{1}{2*3-1}\right)^3 = (0.1)^3 = 0.001$$

$$\text{FOR } X = \frac{1}{2} \text{ and } N = 3 \text{ THEN } F(X) = 0$$

$$\text{FOR } X = 3 \text{ and } N = 3 \text{ THEN } F(X) = \left(\frac{3-1.5}{2*3-1} + \frac{1}{2*3-1}\right)^3 = (0.5)^3 = 0.125$$



IFF $N = N + 0.5$

$$F(X) = \left(\frac{0.5}{2N}\right)^{N+0.5} = \left(\frac{1}{4N}\right)^{N+0.5}; \text{where } N \text{ in } \{1, 2, 3, 4, 5, 6, \dots\} \text{ and } X = 1$$

We only get half the graph because we basically move by step $X = X - 1$. So, we only have half the graph but the same properties. And no symmetry on interval $[0,1]$. So now we have only half interval $[0.5, 1]$

At $X = 0.5$ and N in $\{1, 2, 3, 4, 5, \dots\}$ THEN $F(X) = \left(\frac{X-1.5}{2N-1} + \frac{1}{2N-1}\right)^N = 0$ for all N because $F(X)$ Nominator = 0.

At $X = 1$ and N in $\{1, 2, 3, 4, 5, \dots\}$ and $N = N + 0.5$ THEN

$$F(X) = \left(\frac{1-1.5}{2(N+0.5)-1} + \frac{1}{2(N+0.5)-1}\right)^{N+0.5}$$

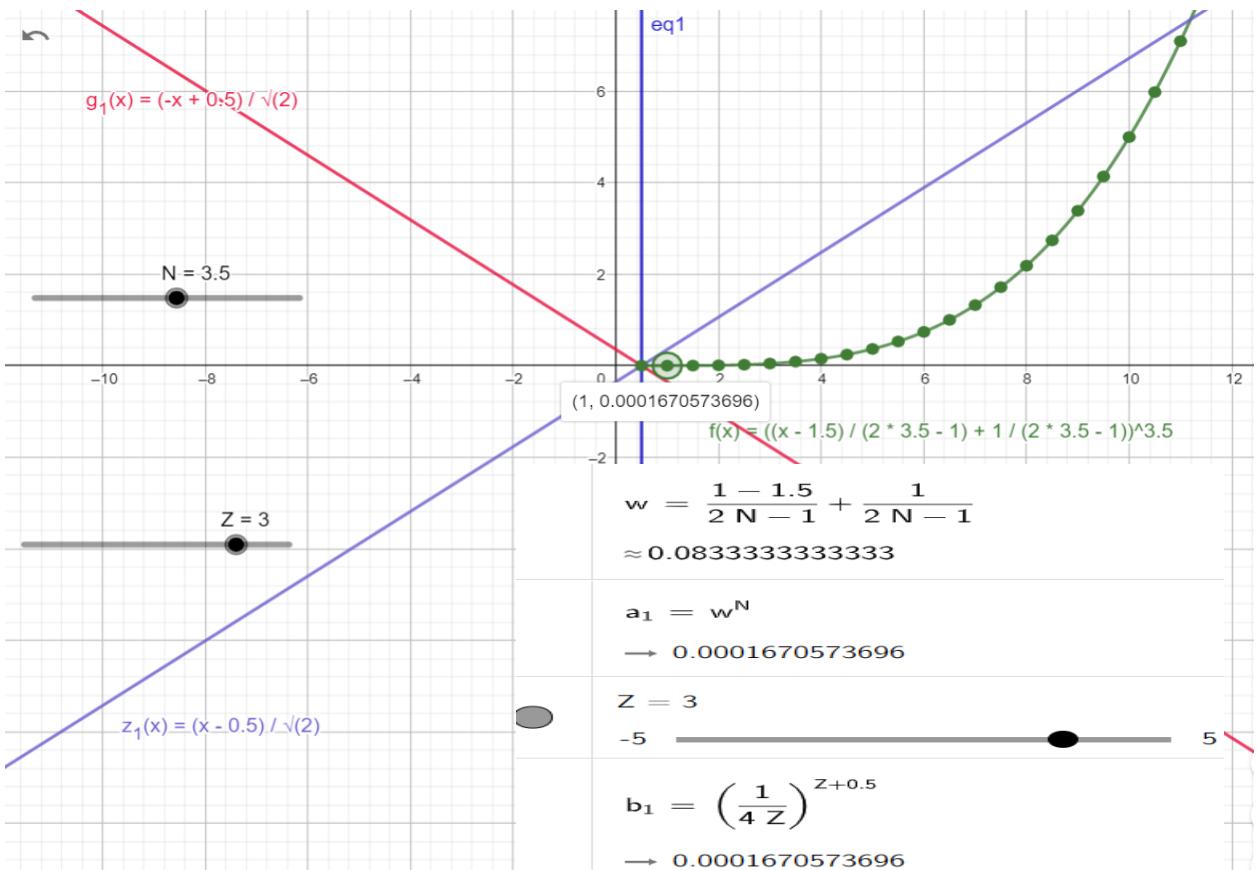
$$F(X) = \left(\frac{-0.5}{2N+1-1} + \frac{1}{2N+1-1}\right)^{N+0.5}$$

$$F(X) = \left(\frac{0.5}{2N}\right)^{N+0.5} = \left(\frac{1}{4N}\right)^{N+0.5}; \text{where } N \text{ in } \{1, 2, 3, 4, 5, 6, \dots\} \text{ and } X = 1$$

To get $F(1)$: at $X = 1$

we can use $N = \{1.5, 2.5, 3.5, 4.5, 5.5, 6.5, \dots\}$ and $F(X) = \left(\frac{1-1.5}{2N-1} + \frac{1}{2N-1}\right)^N$

Or use $N = \{1, 2, 3, 4, 5, 6, \dots\}$ and $F(X) = \left(\frac{1}{4N}\right)^{N+0.5}$



IF $X = 1/1; N = 1.5$

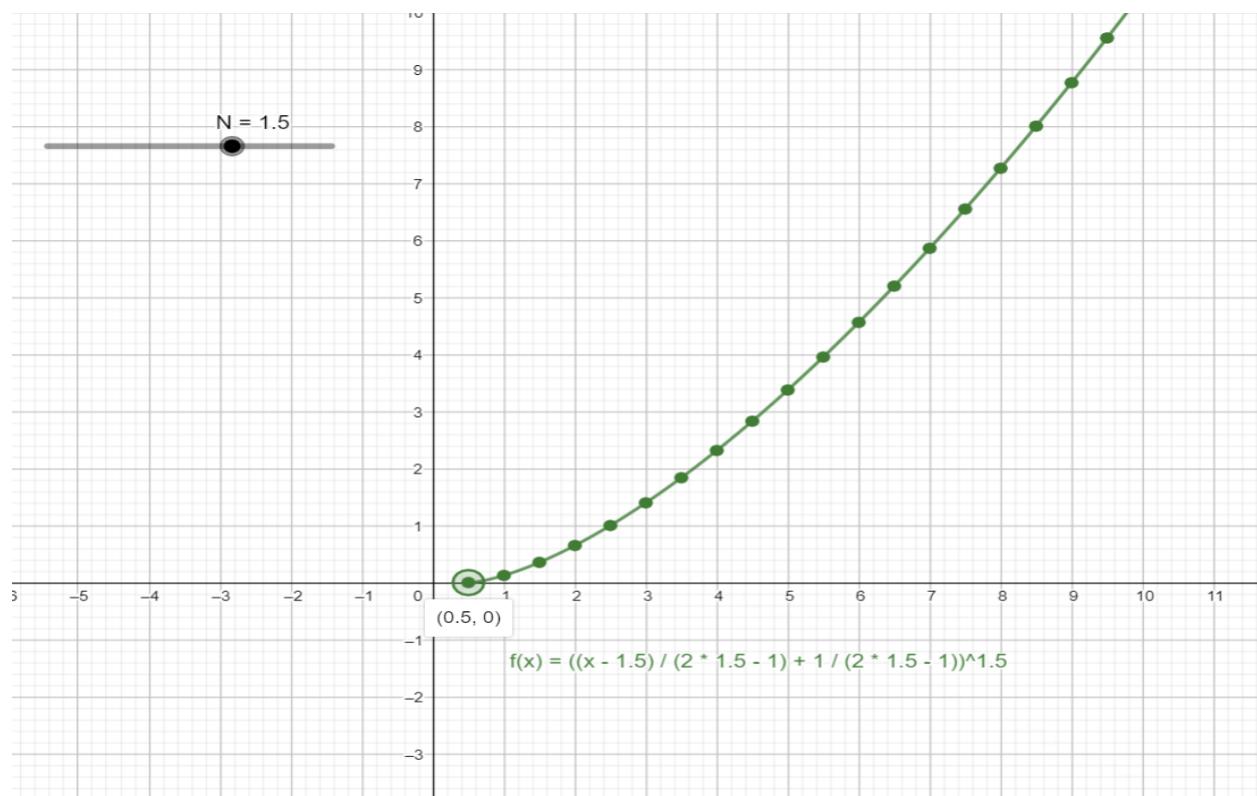
$$a = \frac{\frac{1}{1} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$
$$\rightarrow \frac{1}{4}$$

IF $X = 1/2; N = 1.5$

$$a = \frac{\frac{1}{2} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$
$$\rightarrow 0$$

IF $X = 1/3; N = 1.5$

$$a = \frac{\frac{1}{3} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$
$$\rightarrow -\frac{1}{12}$$

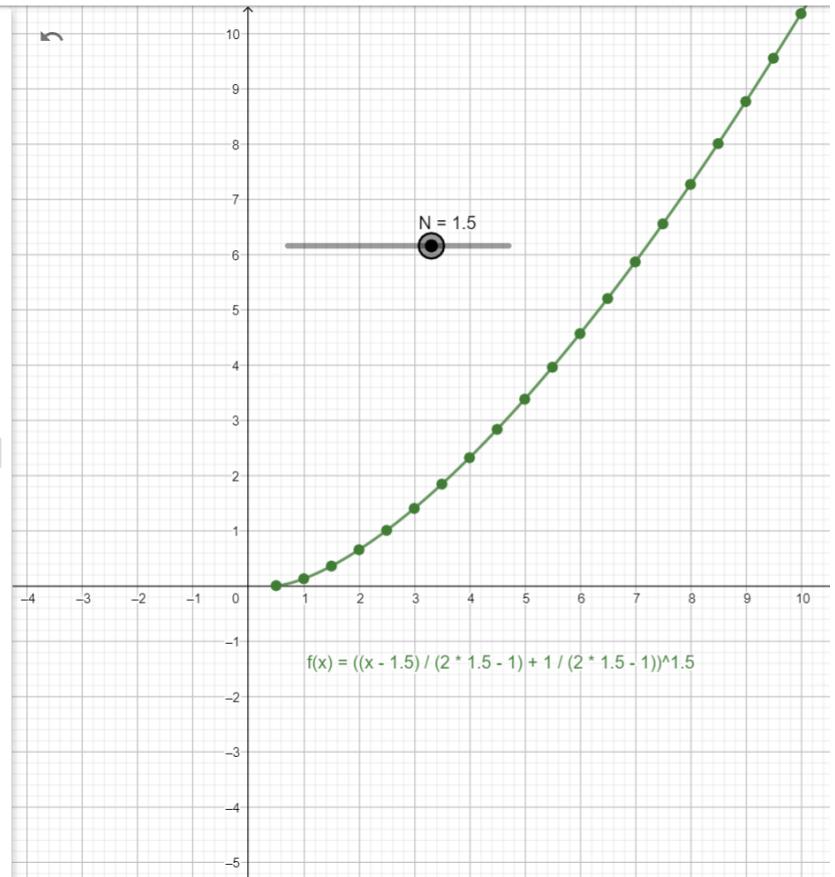


At X = 0.5 THEN F(X) Nominator = 0 for any N.

$$f(x) = \left(\frac{x - 1.5}{2N - 1} + \frac{1}{2N - 1} \right)^N$$

$$\rightarrow \left(\frac{x - 1.5}{2 \cdot 1.5 - 1} + \frac{1}{2 \cdot 1.5 - 1} \right)^{1.5}$$

x ::	f(x) ::
-0.5	
0	
0.5	0
1	0.125
1.5	0.3535533905...
2	0.6495190528...
2.5	1
3	1.3975424859...
3.5	1.8371173070...
4	2.3150323971...
4.5	2.8284271247...
5	3.375
5.5	3.9528470752...
6	4.5603590867...

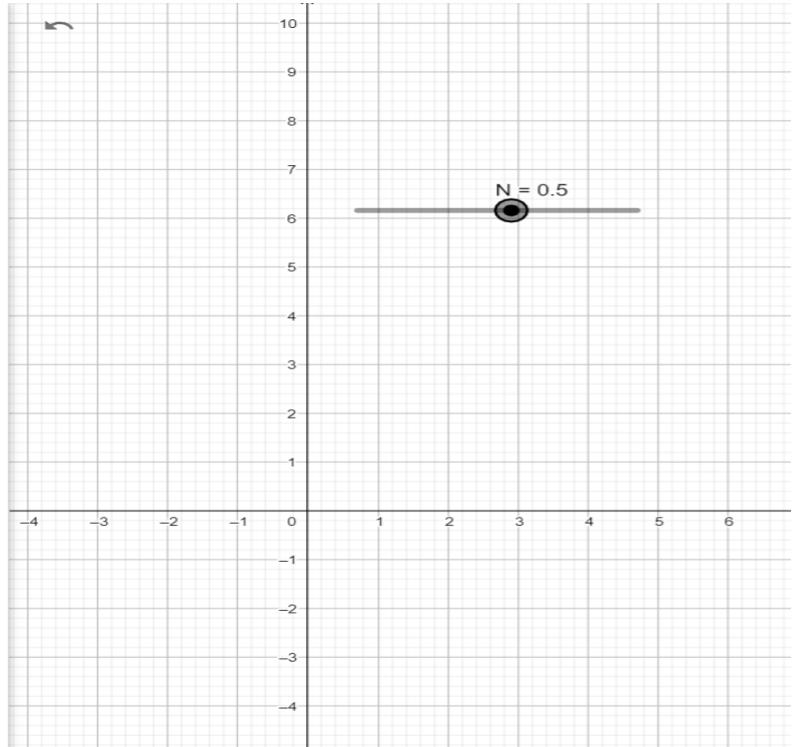


At $N = 0.5$ $F(X)$ undefined because denominator = 0

$$a = \frac{\frac{1}{1} - 1.5}{2N - 1} + \frac{1}{2N - 1}$$

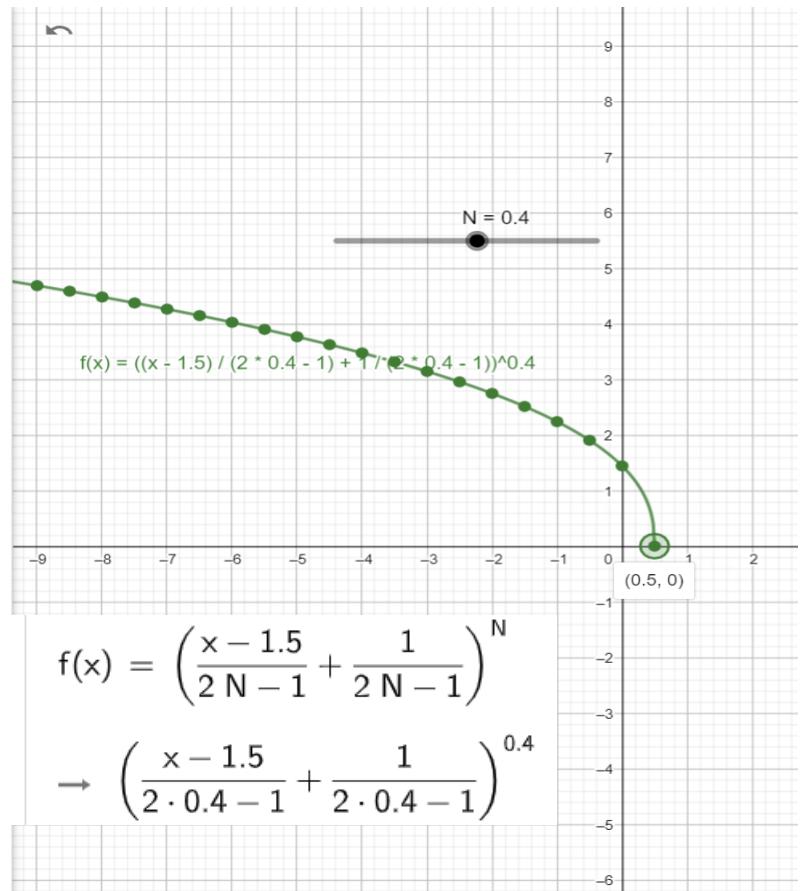
→ ?

x ::	f(x) ::
-0.5	
0	
0.5	
1	
1.5	
2	∞
2.5	∞
3	∞
3.5	∞
4	∞
4.5	∞
5	∞
5.5	∞
...	...

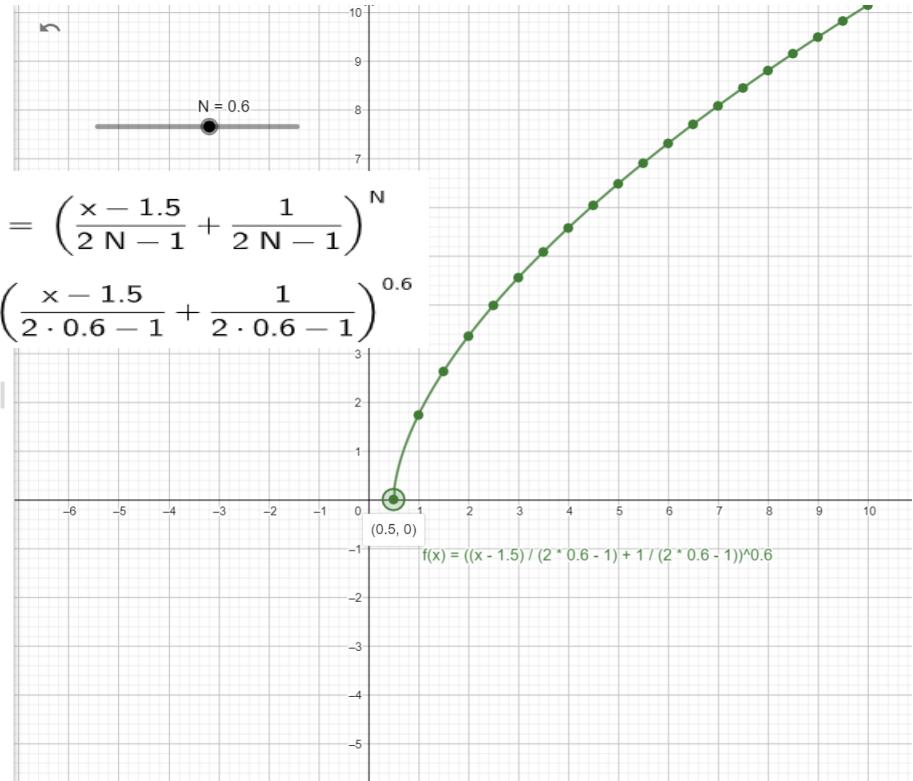


But at $X = 0.5$ and $0.5 < N < 0.5$ Then $F(x) = 0$ because nominator = 0.

x ::	f(x) ::
-4.5	3.623898318...
-4	3.474345526...
-3.5	3.31445401734
-3	3.142065393...
-2.5	2.954176939...
-2	2.746401358...
-1.5	2.511886431...
-1	2.238847463...
-0.5	1.903653938...
0	1.442699905...
0.5	0
1	
1.5	
2	



x ::	f(x) ::
-3	
-2.5	
-2	
-1.5	
-1	
-0.5	
0	0
0.5	
1	1.7328621078...
1.5	2.6265278044...
2	3.3499379133...
2.5	3.981071705535
3	4.5514105075...
3.5	5.0775563919...



A)

$$F(X) = \left(\frac{X - 1.5}{2X - 1} + \frac{1}{2X - 1}\right)^X$$

$$F(X) = \left(\frac{X - 0.5}{2X - 1}\right)^X$$

$$\sum_{X=0}^{\infty} F(X) = \sum_{X=0}^{\infty} \left(\frac{X - 0.5}{2X - 1}\right)^X \rightarrow 1 + \frac{0.5}{1} + \left(\frac{1.5}{3}\right)^2 + \left(\frac{2.5}{5}\right)^3 + \left(\frac{3.5}{7}\right)^3 + \left(\frac{4.5}{9}\right)^4 + \dots$$

$$F(0) + F(1) + F(2) + F(3) + F(4) + F(5) + \dots = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$$

$$\sum_{X=0}^{\infty} F(X) = \sum_{N=0}^{\infty} \frac{1}{2^N} = 2$$

B)

$$F(X) = \left(\frac{X - 1.5}{2X - 1} + \frac{1}{2X - 1}\right)^X$$

$$F(X) = \left(\frac{X - 0.5}{2X - 1}\right)^X$$

$$\sum_{X=0}^{\infty} F(X + 0.5) = \sum_{X=0}^{\infty} \left(\frac{(X + 0.5) - 0.5}{2(X + 0.5) - 1}\right)^{X+0.5} = \sum_{X=0}^{\infty} \left(\frac{X}{2X}\right)^{X+0.5} = \sum_{X=0}^{\infty} \left(\frac{1}{2}\right)^{X+0.5}$$

$$\sum_{X=0}^{\infty} F(X + 0.5) = \frac{1}{\sqrt{2}} \sum_{X=0}^{\infty} \left(\frac{1}{2}\right)^X = \frac{2}{\sqrt{2}} = \sqrt{2}$$

From A & B

$$\sum_{X=0}^{\infty} F(X) = 2 \text{ and } \sum_{X=0}^{\infty} F(X + 0.5) = \sqrt{2}$$

Conclusion

To get the reasoning of why we have none-trivial Zeros, and how number are distributed and how to get the exact value location for a Zero.

First, we got through the steps to introduce the new formula the new used this formula and its usage.

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