

Riemannian geometry manifold and unfolding using complex plane imaginary unit i as a frame of reference using axiomatic method

Shaimaa said soltan¹

¹ Computer Engineer, Toronto, Canada

Correspondence: Shaimaa Soltan, 3050 Constitution Blvd, Mississauga, ON., L4Y 3X1, Canada. Tel: 1-647-801-6063 E-mail: shaimaasultan@hotmail.com

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Abstract

In this paper, we will study manifold transformation effect by applying a combination of a basic mathematical operation, like $(+, -, *, /)$ on a frame of reference, which by itself is one of the transformations for the complex plane imaginary unit $[i]$. Then will introduce another concept to represent Euler's Identity equation using only the frame of reference and a basic mathematical operation.

In second part, we will study a complex plane folding and unfolding for a frame of reference using a set of basic mathematic operations. Then we will visualize a harmony relation between $[e]$ and $[\pi]$ and its formula.

Finally, we will present a simpler transformation formula for the frame of reference that helps in understanding the complex plane manifold at the strip line of zeta function, and how using this new formula can give us natural number results for all odd natural numbers including prime numbers at [0.5], which is known as strip line conjecture between $[0,1]$.

Keywords: Euler's Identity, Prime Number Distribution, Zeta function, complex plane manifold

1. Introduction

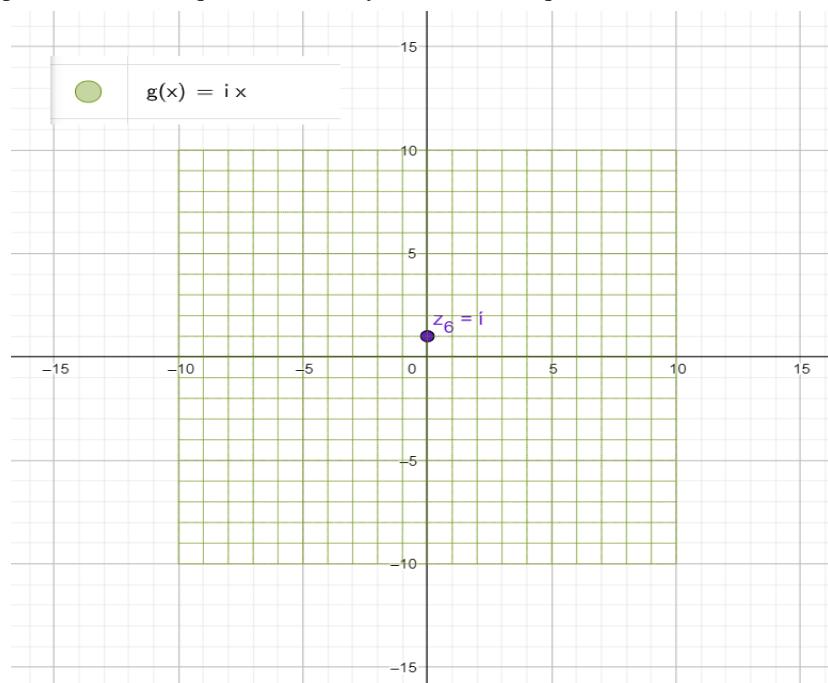
1.1 Introduce the Problem

Complex plane representation is based on two irrational numbers $[i \text{ and } \pi]$ and any basic mathematic operations using these two irrational numbers mostly will give us another decimal number in the best case, which makes most mathematical formulas complex and calculation consuming.

In this paper we will use an axiomatic method and the complex plane imaginary unit $[i]$ transformations as a frame of reference to explain manifold and unfolding of Riemannian geometry in complex plane.

First let us introduce our frame of reference in this axiomatic method. We will multiply the imaginary unit by variable $[X]$ which give us the discrete advantage in our complex plane.

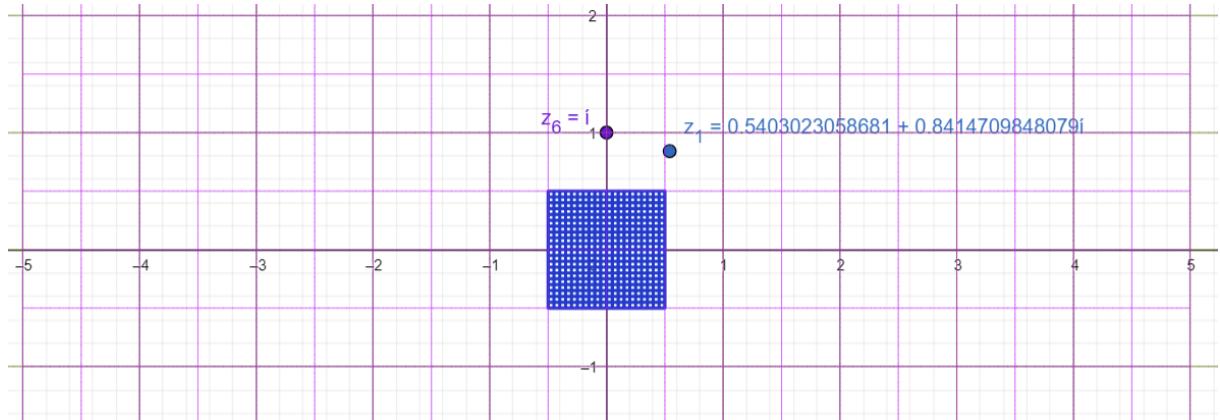
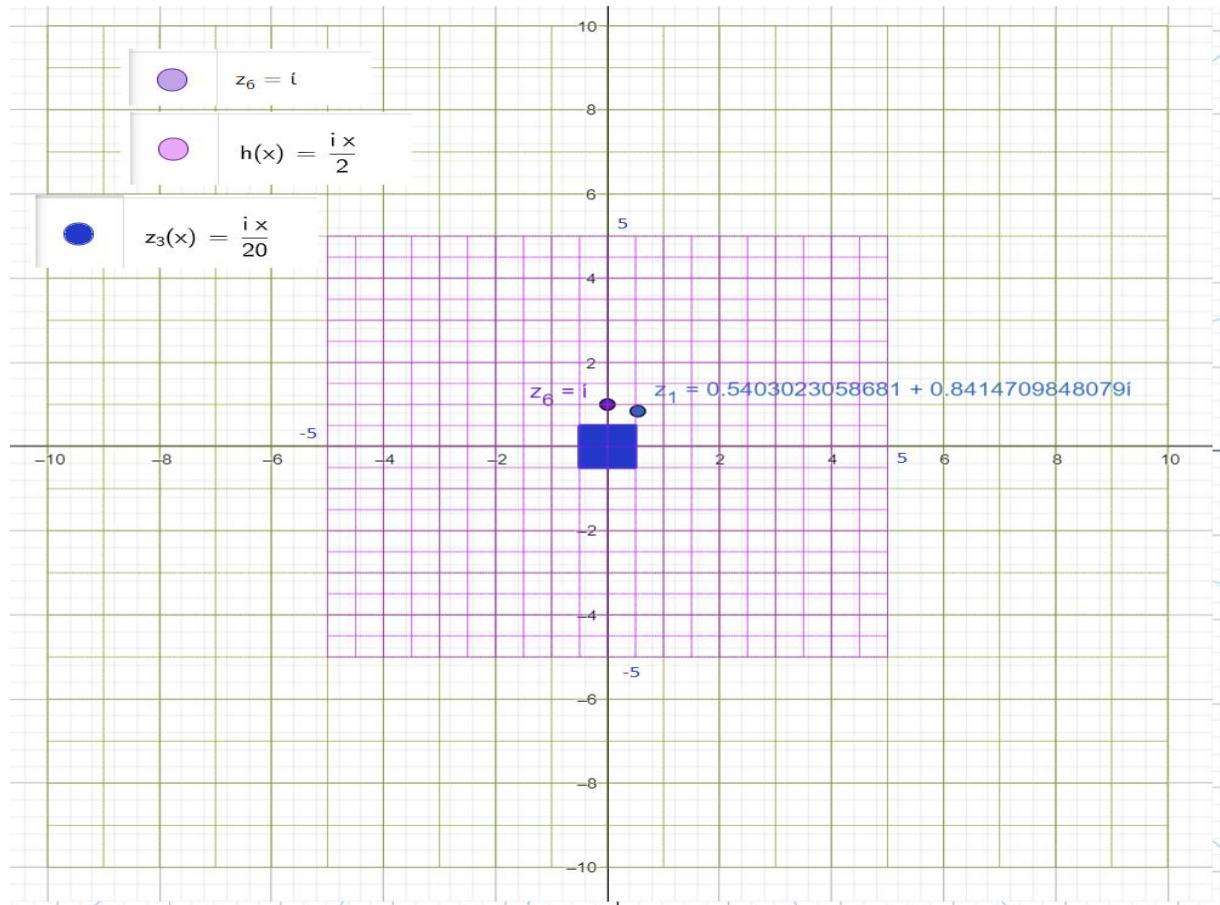
The complex plane works on a base 10 number system, so the value for multiplication of $[X]$ by $[i]$ will give us a square $[10 \times 10]$ and the plane origin point $[0,0]$ is at the center of the square. and as in figure 1; $[i]$ will be represented as a point inside this square located at y axis at a unit square $[U] = [I * X * 1/400]$



1.2 basic mathematic operations on imaginary unit [i] as a frame of reference

1.2.1 Scaling frame of reference [$g(x) = I * X$]

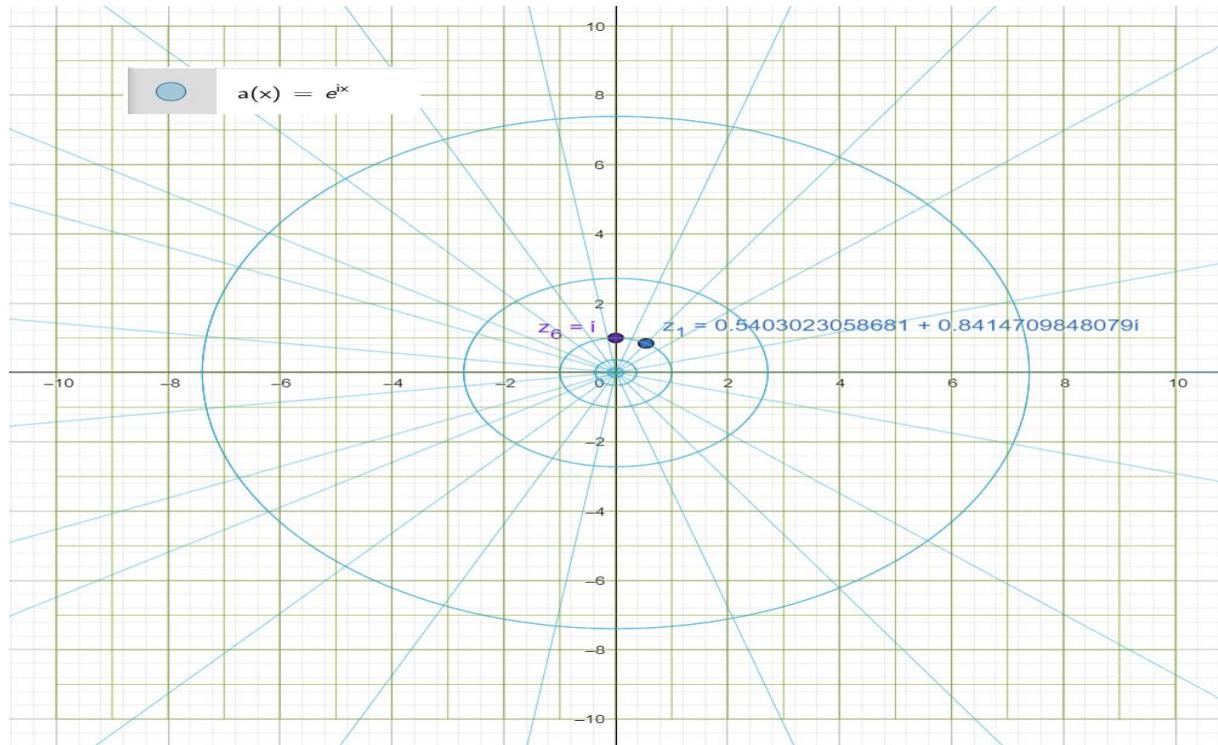
In next Figure, we scaled our frame of reference by a scale of $\frac{1}{2}$ and $\frac{1}{20}$. As you can see scaling by natural number simulate an increase and scale by fraction less than one simulate shrink in size (zoom in or out), without any distortion in square unit shape keeping shape ratio unchanged (resize with a fixed ratio for width and height).



1.2.2 Raise [e] to the power of frame of reference [$g(x) = I * X$]

In next figure, we used [e] representation of a complex plane as our mathematical transformation for our frame of reference [ix]. As we see it can be seen as a cone representation in a 2-D space with an infinity point at the original of the complex plane at point [0, 0] if we looked on the cone as if we are inside the cone at bottom and look top to the head point of the cone. Or we can look at it as 2-D Cone in the opposite way we are at the Top of the cone and look down on a cone from top.

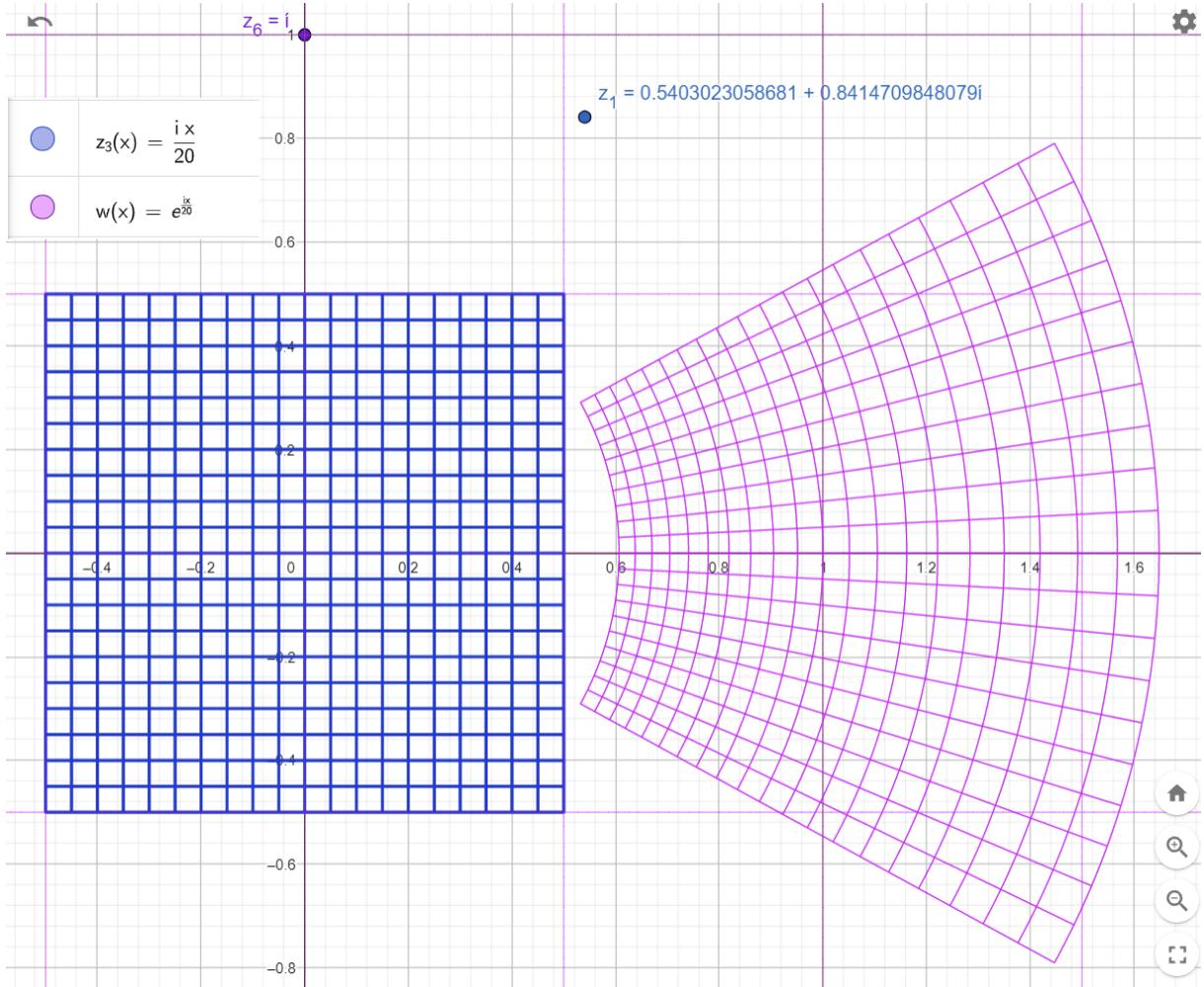
Also, this Cone 2-D representation in the complex plane have exactly 21 lines coming outside from the origin point [0,0]. These 21 lines are classified as exactly 5 groups, and each group have 4 lines except one group will have 5 lines and this extra line (line number 21) will be moving (rotating) between these 5 groups based on the applied operation. We will get through this in details with an example later.



1.2.3 Apply both operations together, Raise [e] to the power of the scaled frame of reference.

As we see in figure 5; the result of this both operations, [e] raised to the power of this scaled frame of reference, is nothing more than a sub folded representation for this scaled frame of reference square (a transformation for the frame of reference without keeping a fixed ratio for the square width and height).

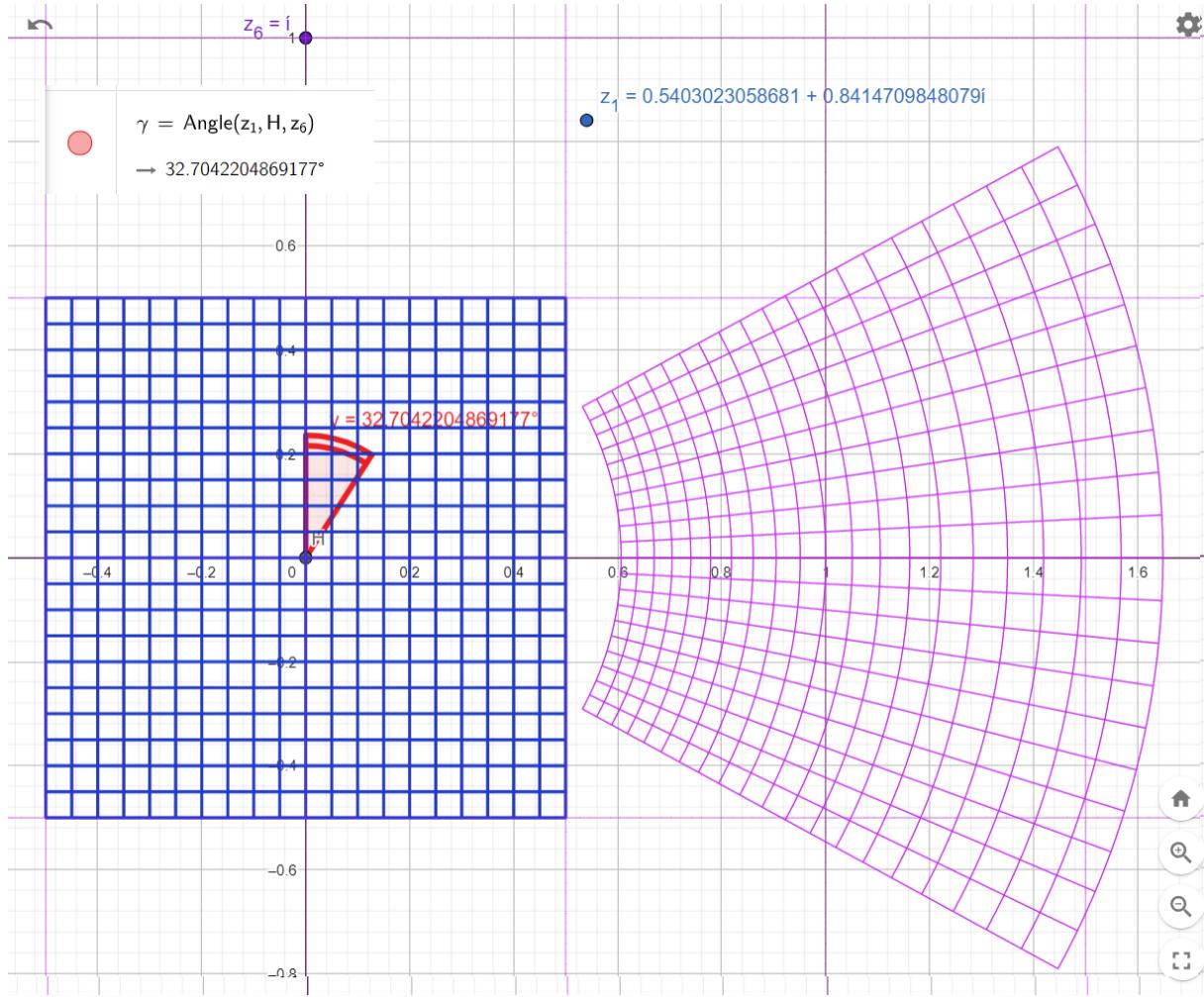
in the unfolding part in this paper, we will explore more the effect of the scaling value on unfolding in complex plane.



In next figure we will get the transformation angle for the frame of reference after applying these two operations. As you see the angel between point $[z_6=i]$ location on the frame of reference, and the location of $z_1 = [e^{ix}]$ on complex plane, $\Theta = 32.7042204869177$ degrees

And based on this we can represent each point on our frame of reference to a point in the complex plane using this formula. Which looks like the opposite of the gematric representation of Euler's Identity

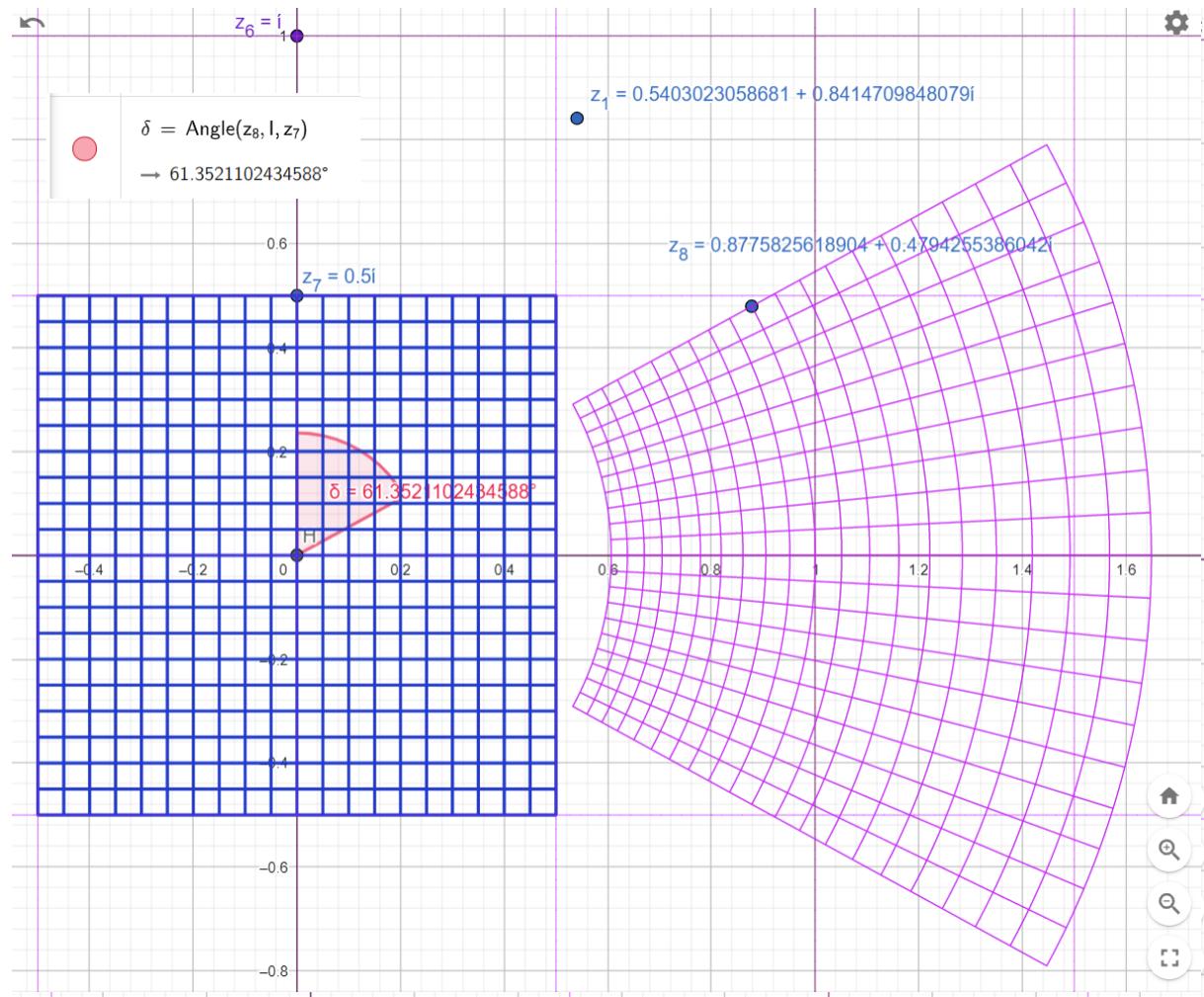
$$Z = \sin(\theta) \pm i \cos(\theta)$$



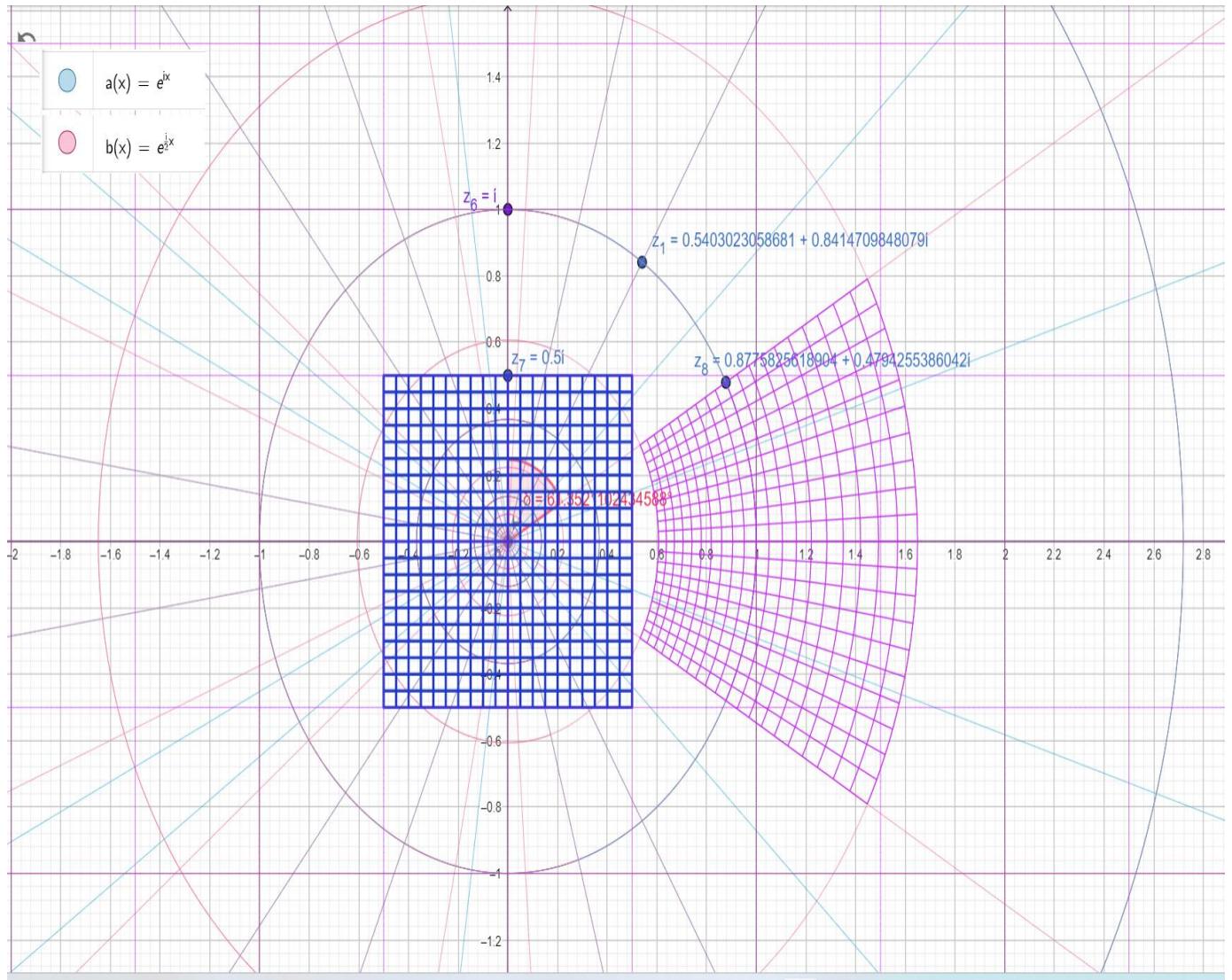
In figure 7; same as figure 6 but now we are getting angle for a point [Z8] on the final transformed frame of reference. As you see the angle $\theta = 61.3521102434588$ degrees.

And the transformed point on the complex plane will be also using the formula.

$$Z = \sin(\theta) \pm i \cos(\theta)$$

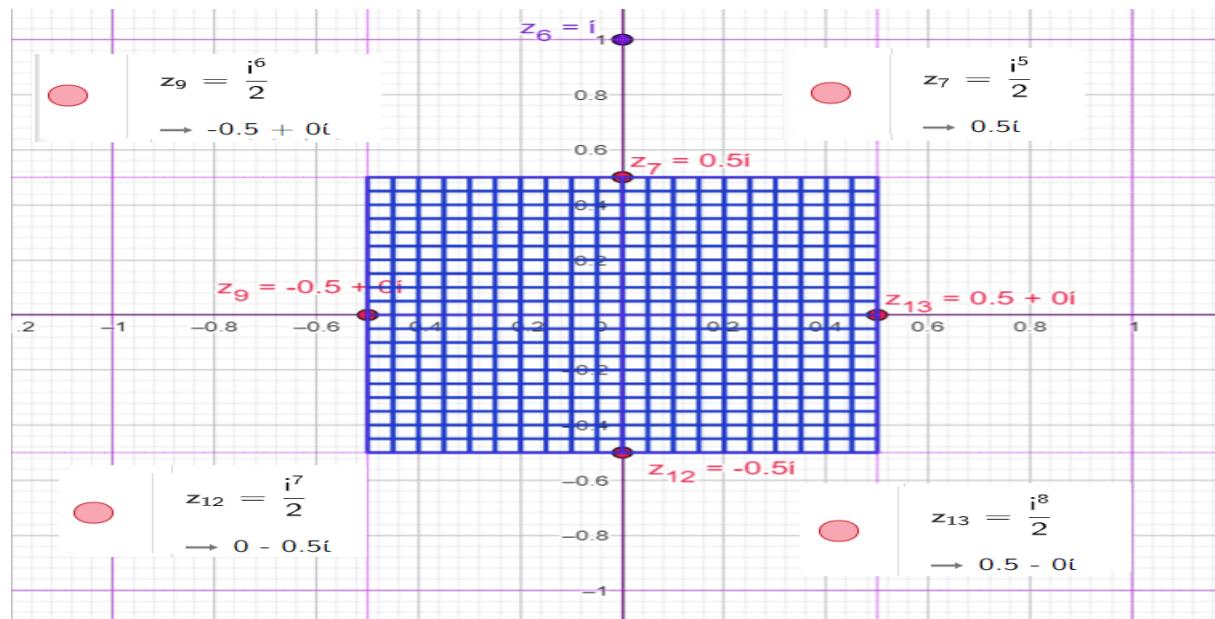
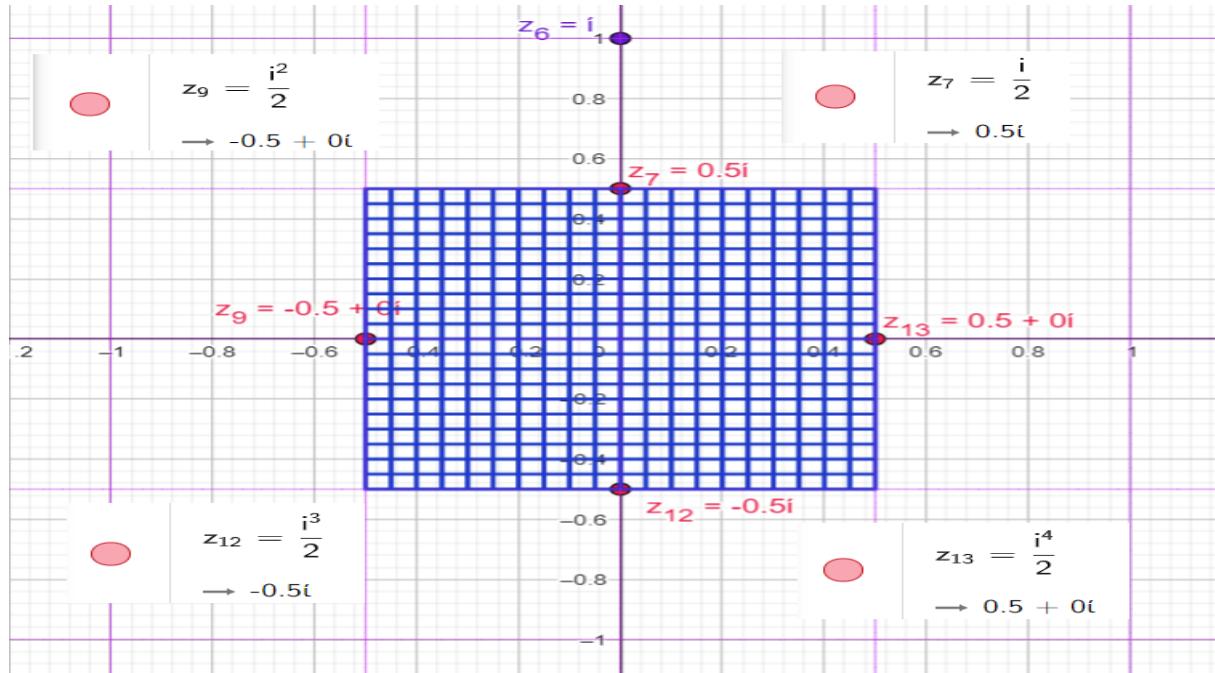


In figure 8; we combined all representations for operations in point 1, 2 ,3 altogether. As you see pints [Z6, Z1, Z8] all are located on the unit Circle of the complex plane or unit Circle of Euler's Identity.



1.2.4 similarity between a frame of reference and Euler's identity unit circle cycle.

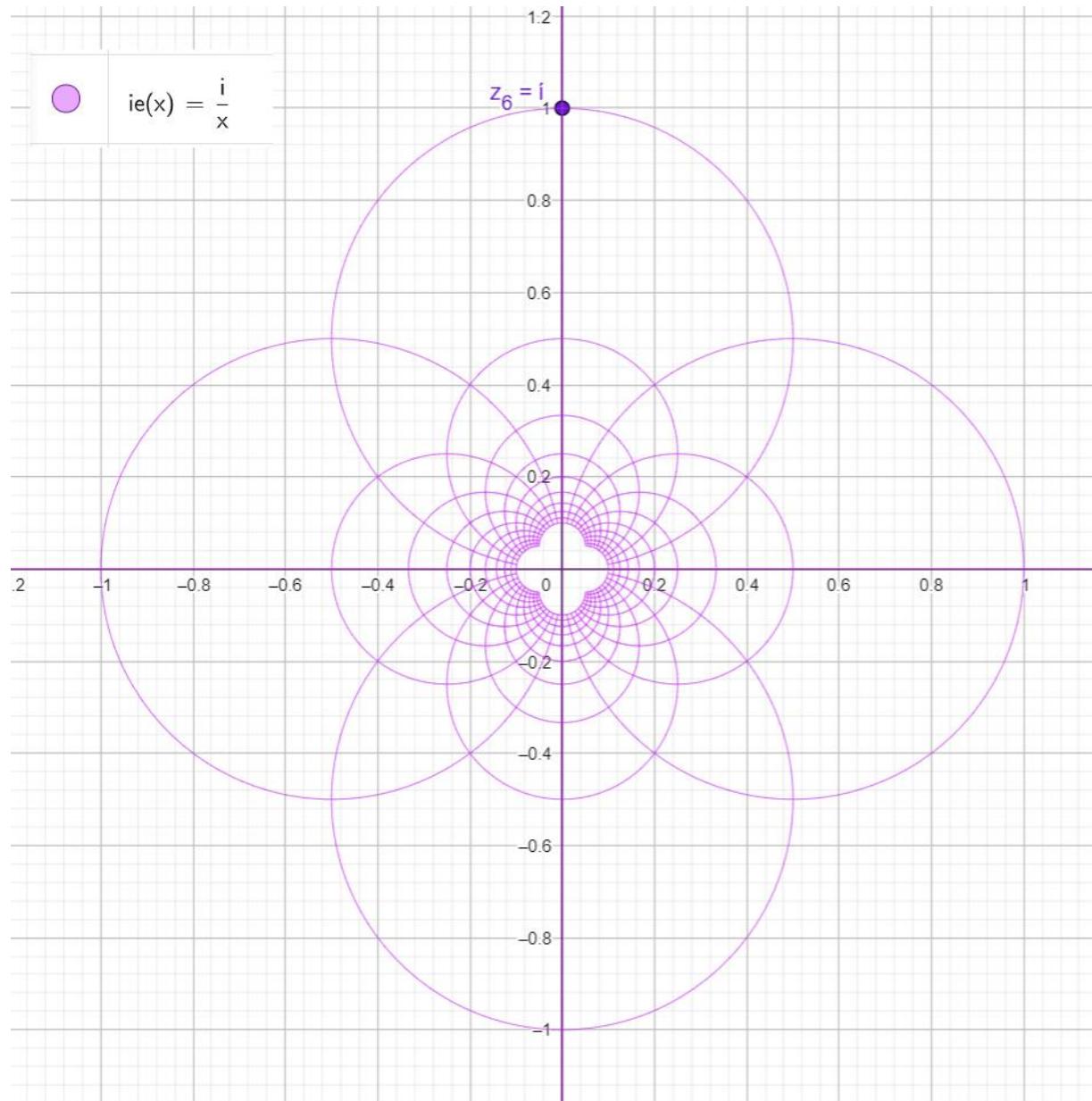
Frame of reference has the same cyclic concept in Euler's identity unit circle π cyclic, but frame of reference uses $[i]$, $[i^2]$, $[i^3]$, $[i^4]$,



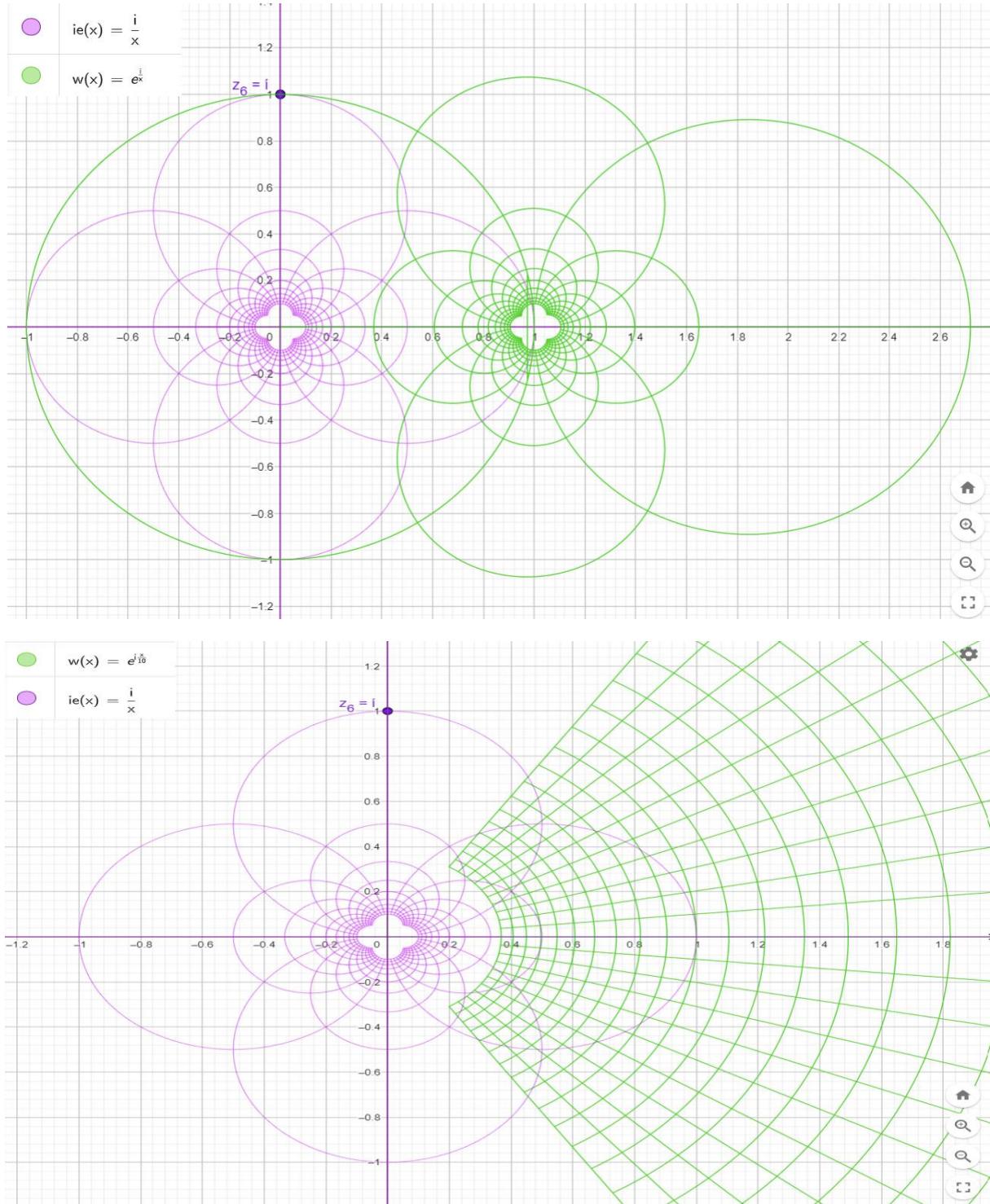
1.2.5 multiply imaginary unit [i] by [1/X].

In figure 9; as you see the result of this transformations give us 4 intersected Cones at the origin of the complex plane and the transformation is exactly binding each strait line the frame of reference to the center of the complex plane. An leaving the main points at [1, -1, i, -i] not affected by this transformation.

One note here; all 4 cones are identical in this frame of reference transformation.

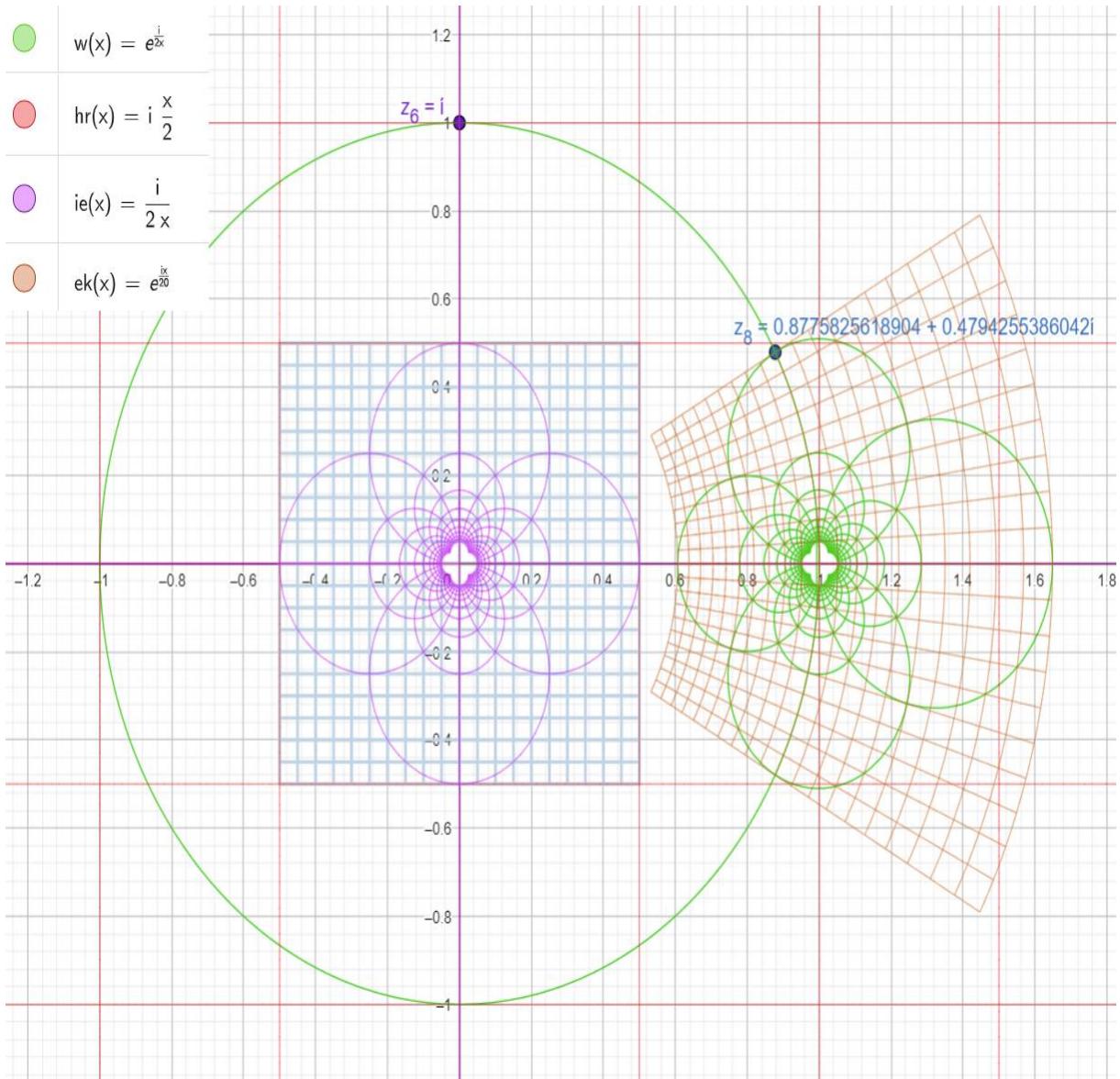


In figure 10; we combined both operations in one figure raise [e] to the power of a multiplication of frame of reference by $[1/x]$. same as we saw in previous transformation there are some distortions in the binding for $[e^{ix}/x]$ the transformation ration for the shape sides is not fixed. While the frame of reference transformation $[i/x]$ keeps a fixed ratio for the shape sides.

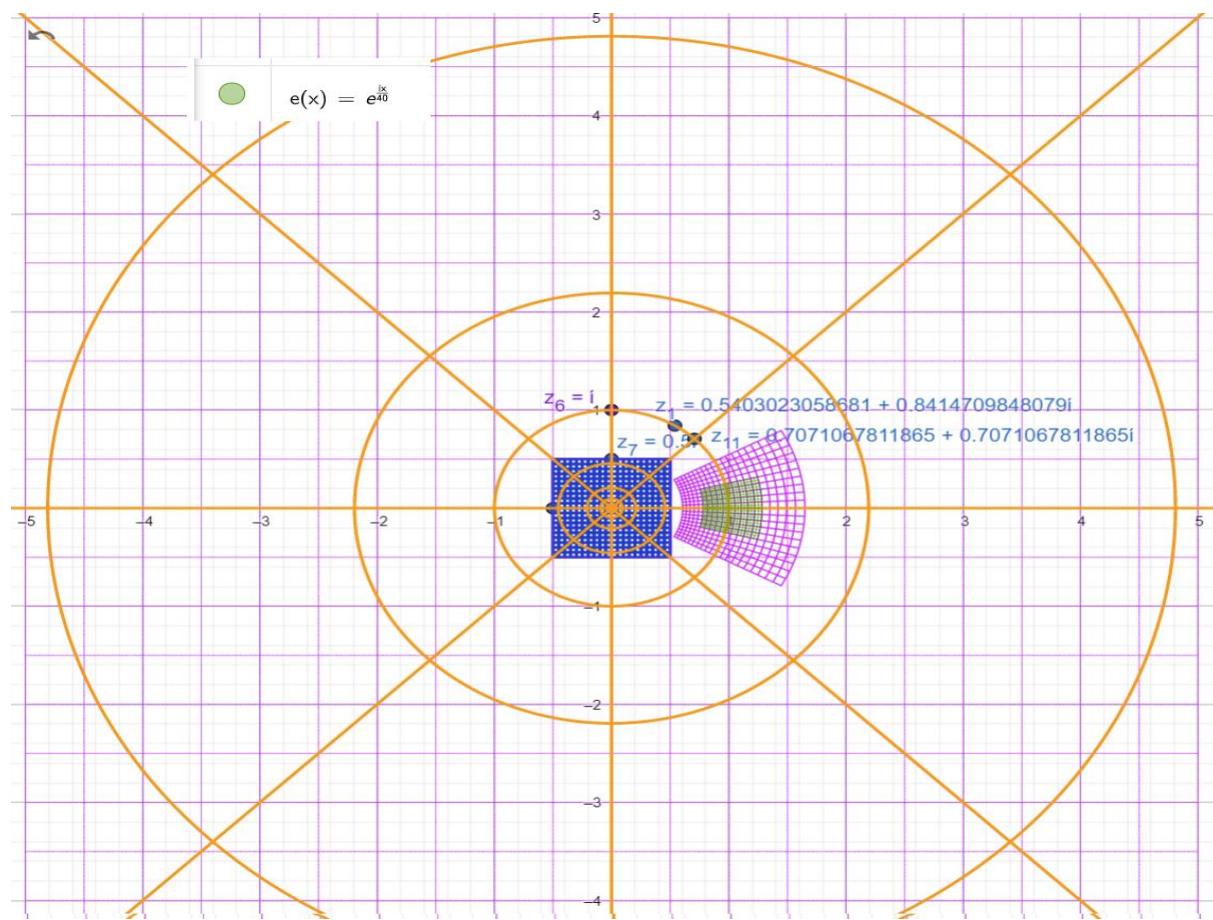
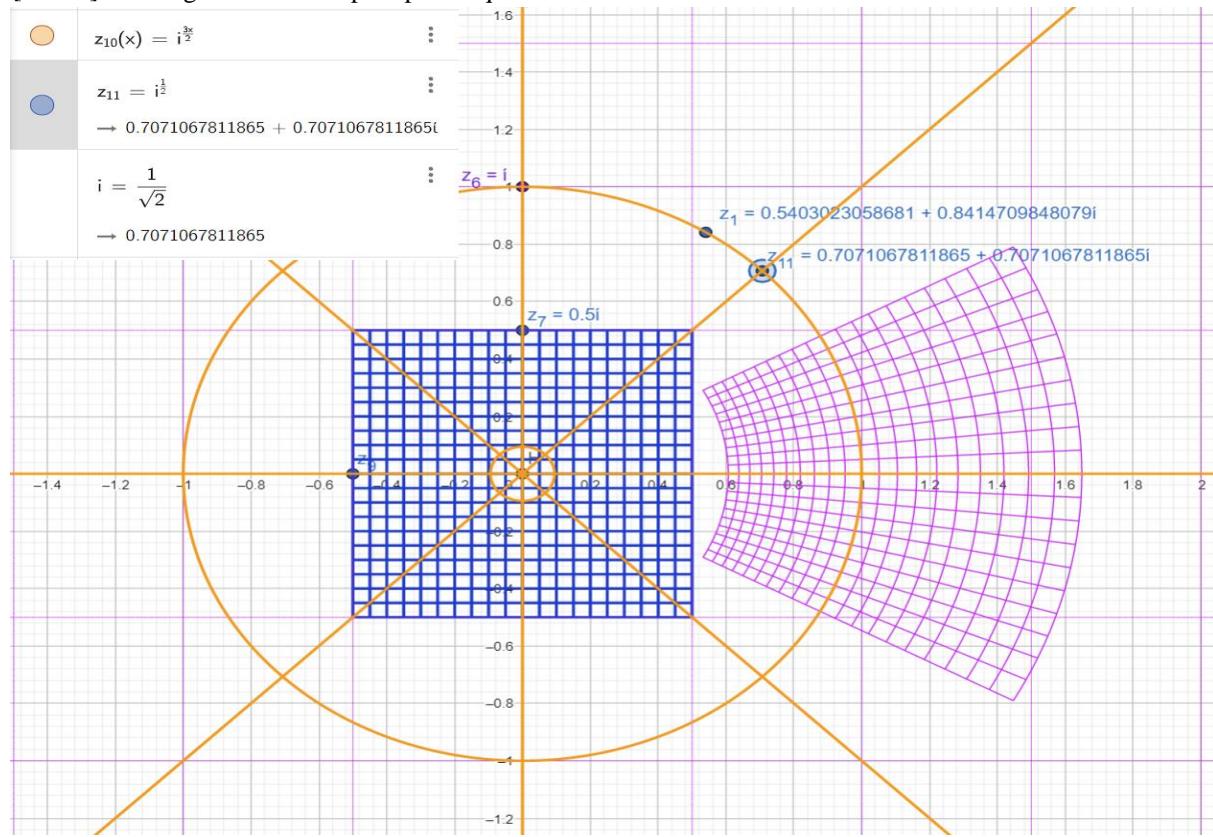


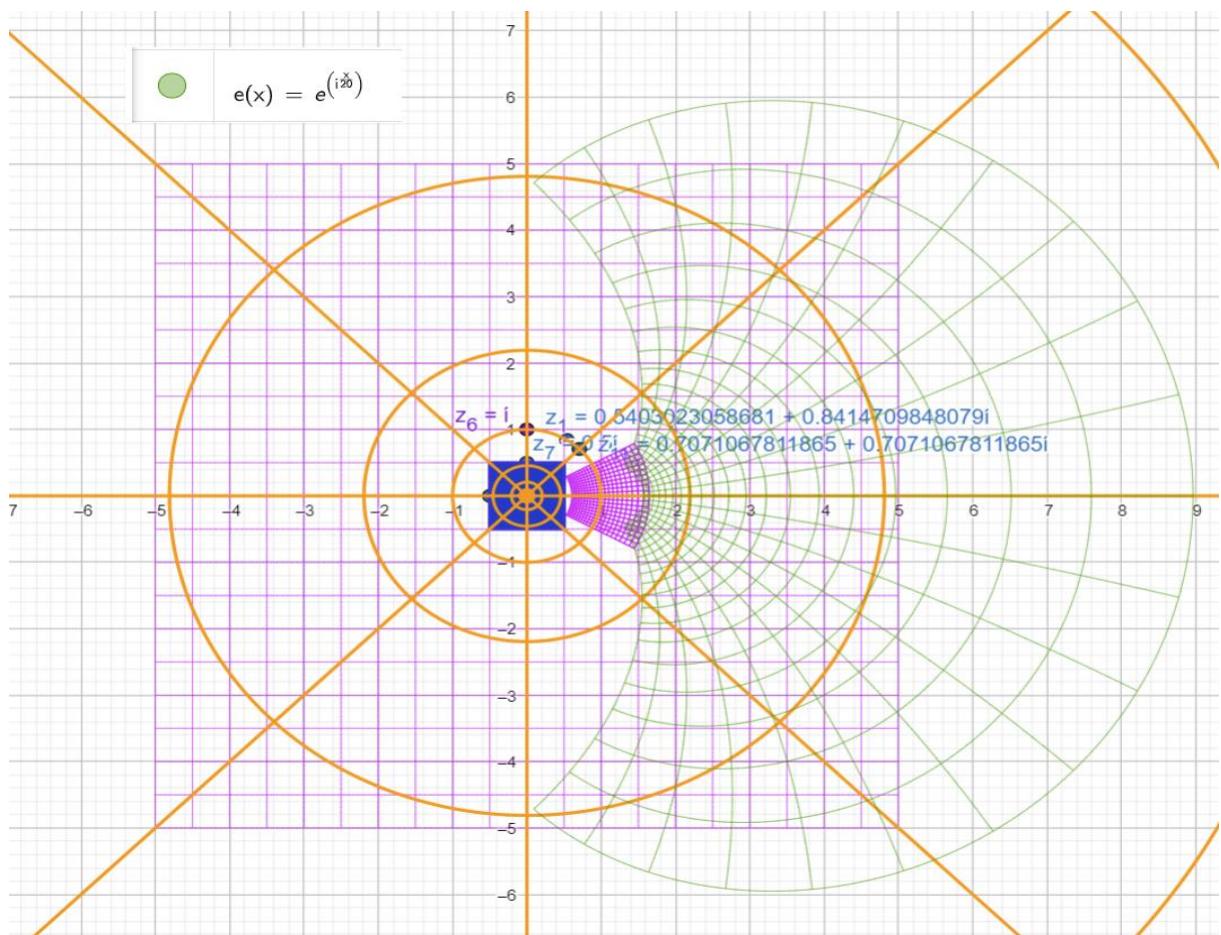
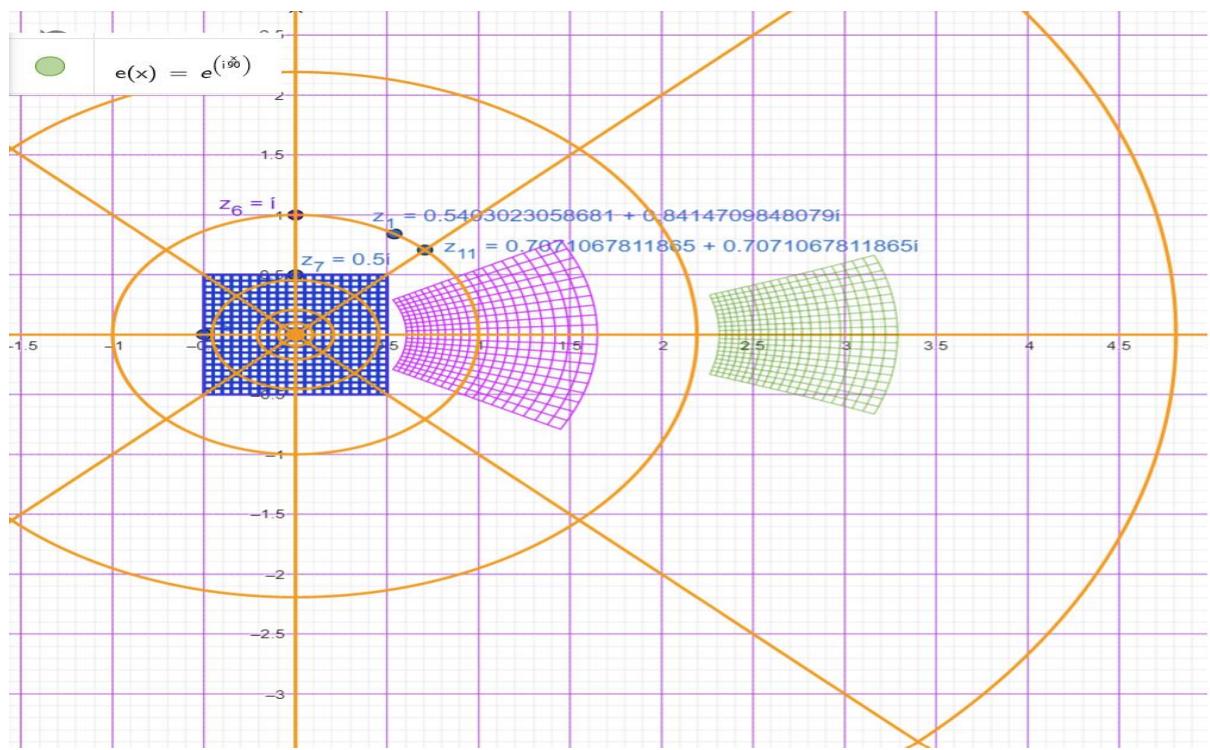
in figure 11; we combined more than one transformation in one graph.

The main note here, frame of reference transformation all time keeps the shape sides ratio fixed without any distortion due to the transformation which gives us the privilege of discreteness values in a continues space



In Figure 12; Figure 13; Figure 14; Figure 15; another series of transformations for square root of [i] for example [$i^{3x/2}$]. which give us the complex plane square root of 2.





2. Unfolding and manifold of frame of reference

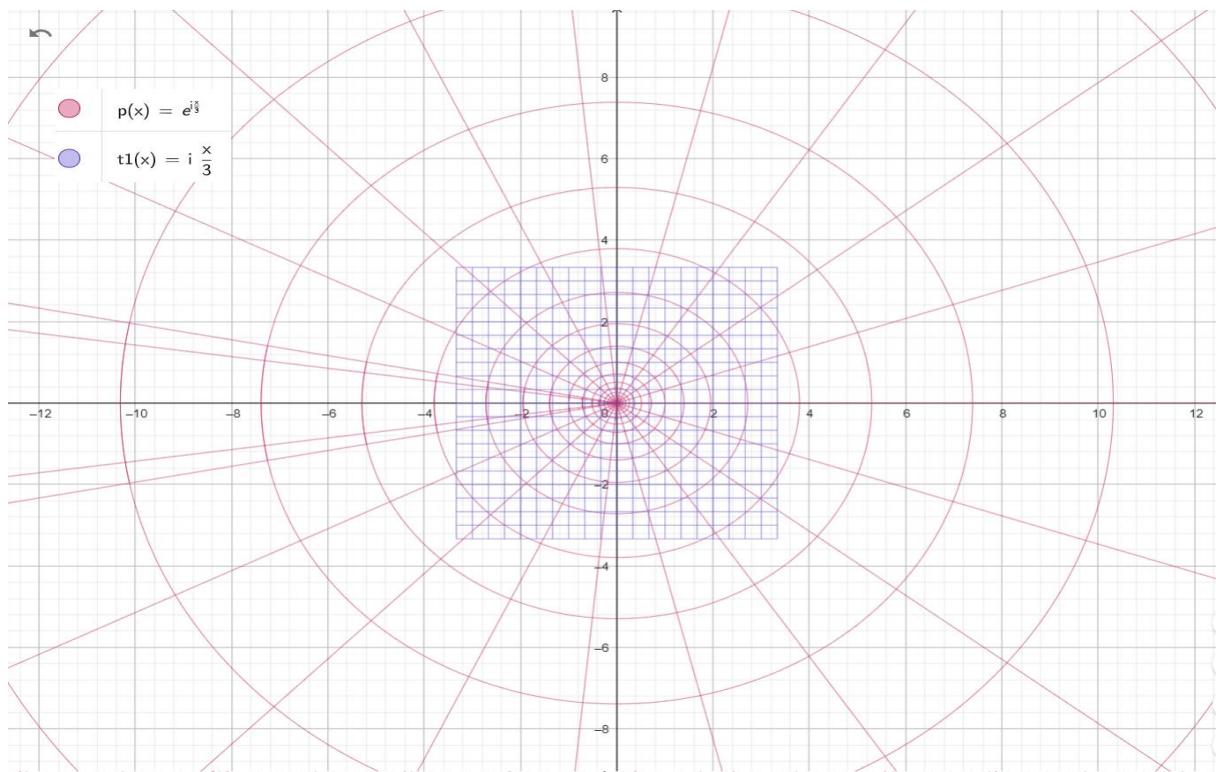
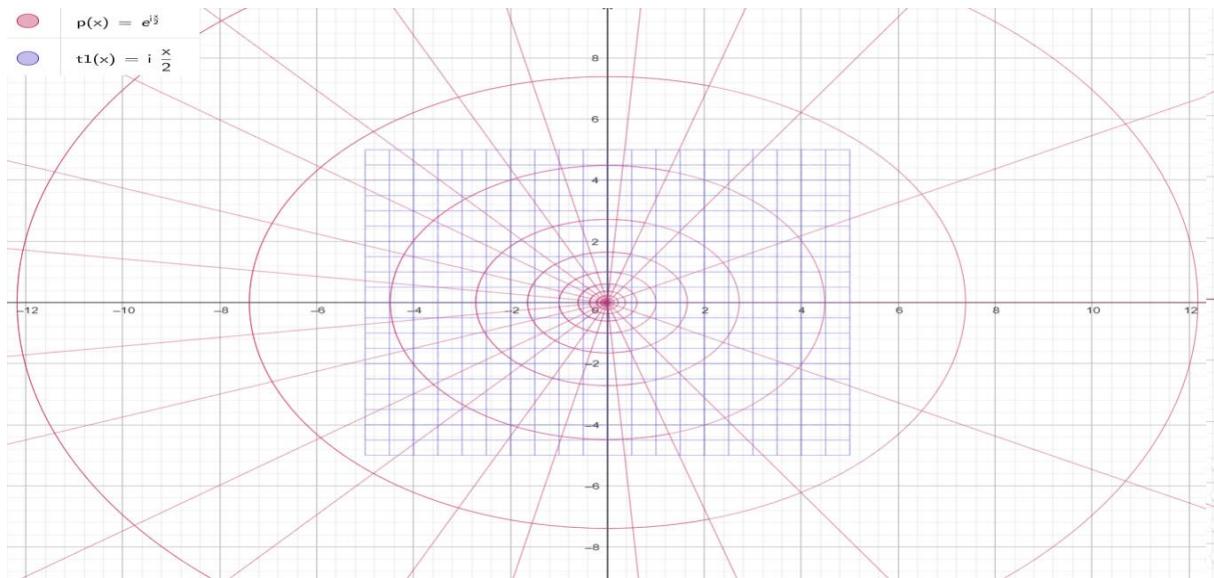
1.2 unfolding frame of reference transformation in complex plane

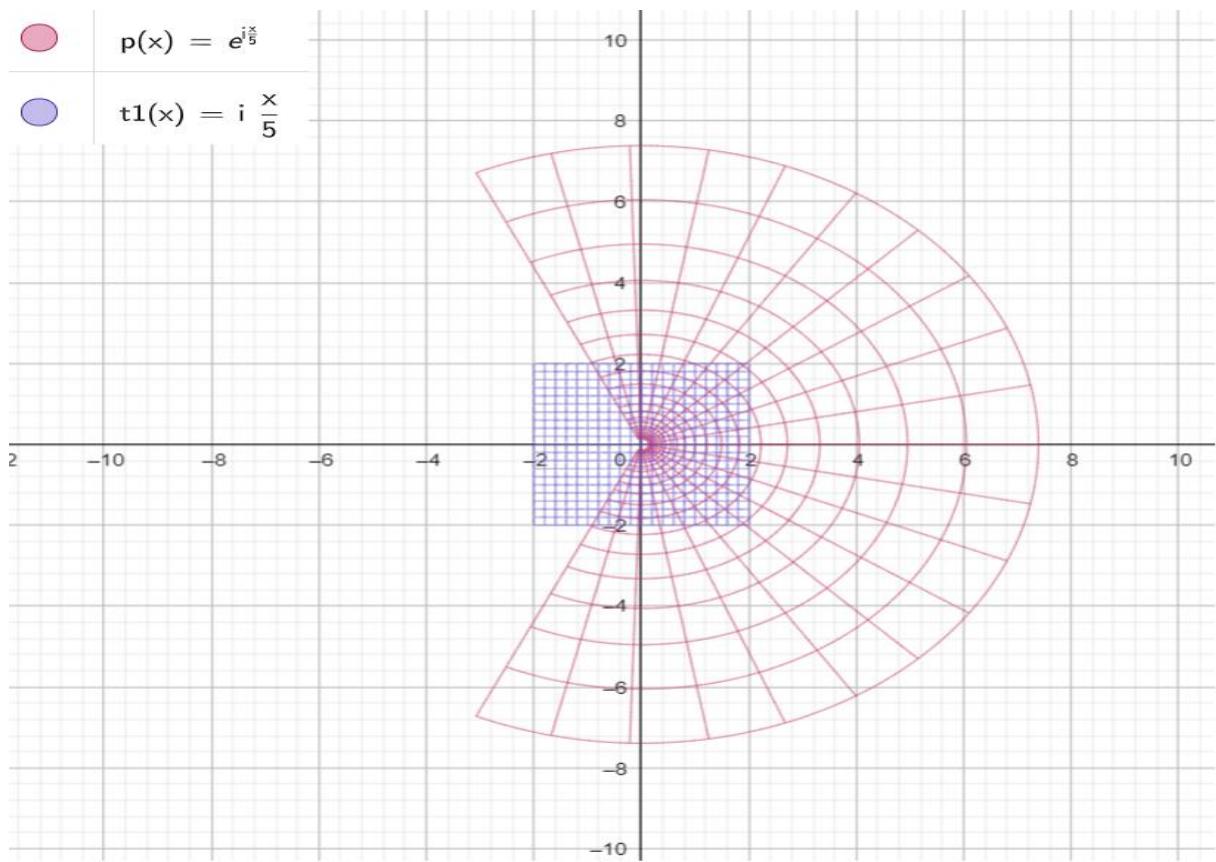
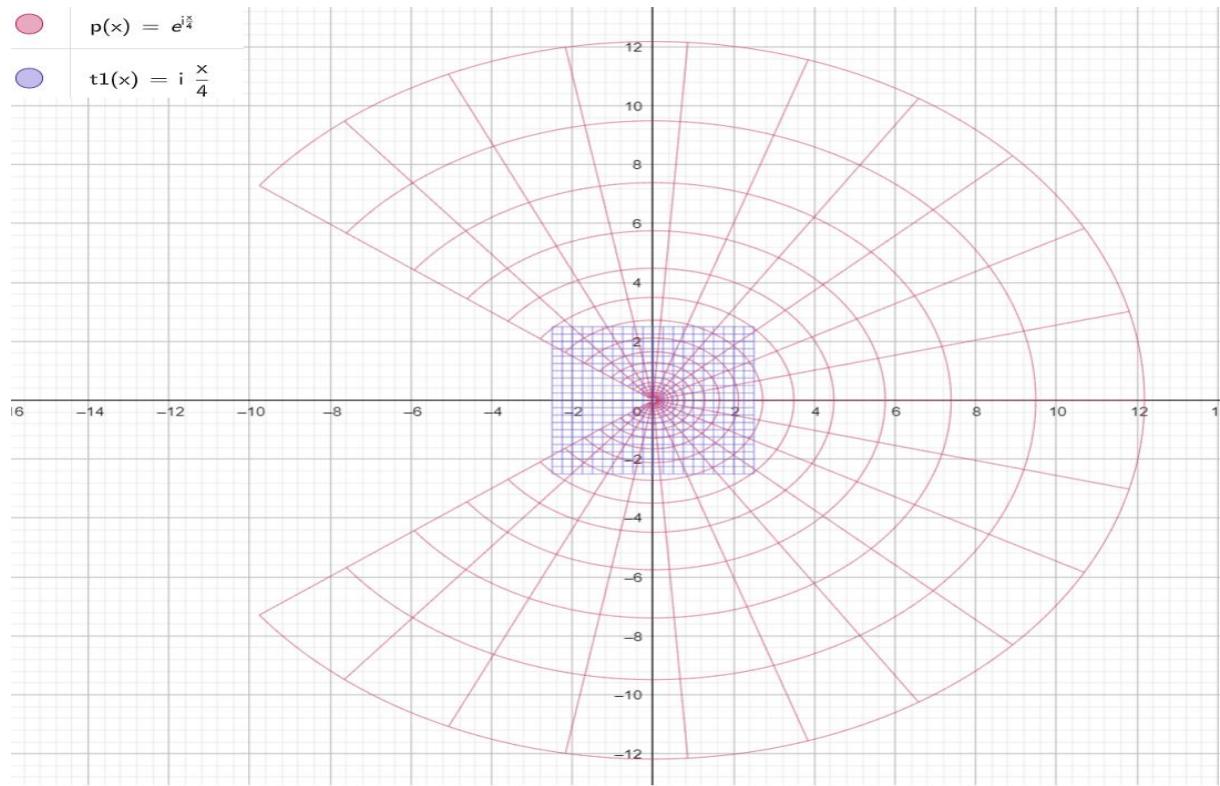
For unfolding we are going to scale frame of reference by a fraction less than 1. Like scale frame of reference by [1/2 or 1/3 or 1/4 or 1/5 Or 1/100]. In the next Figures in this section, we will see how decreasing the scale fraction will unfold our frame of reference transformation.

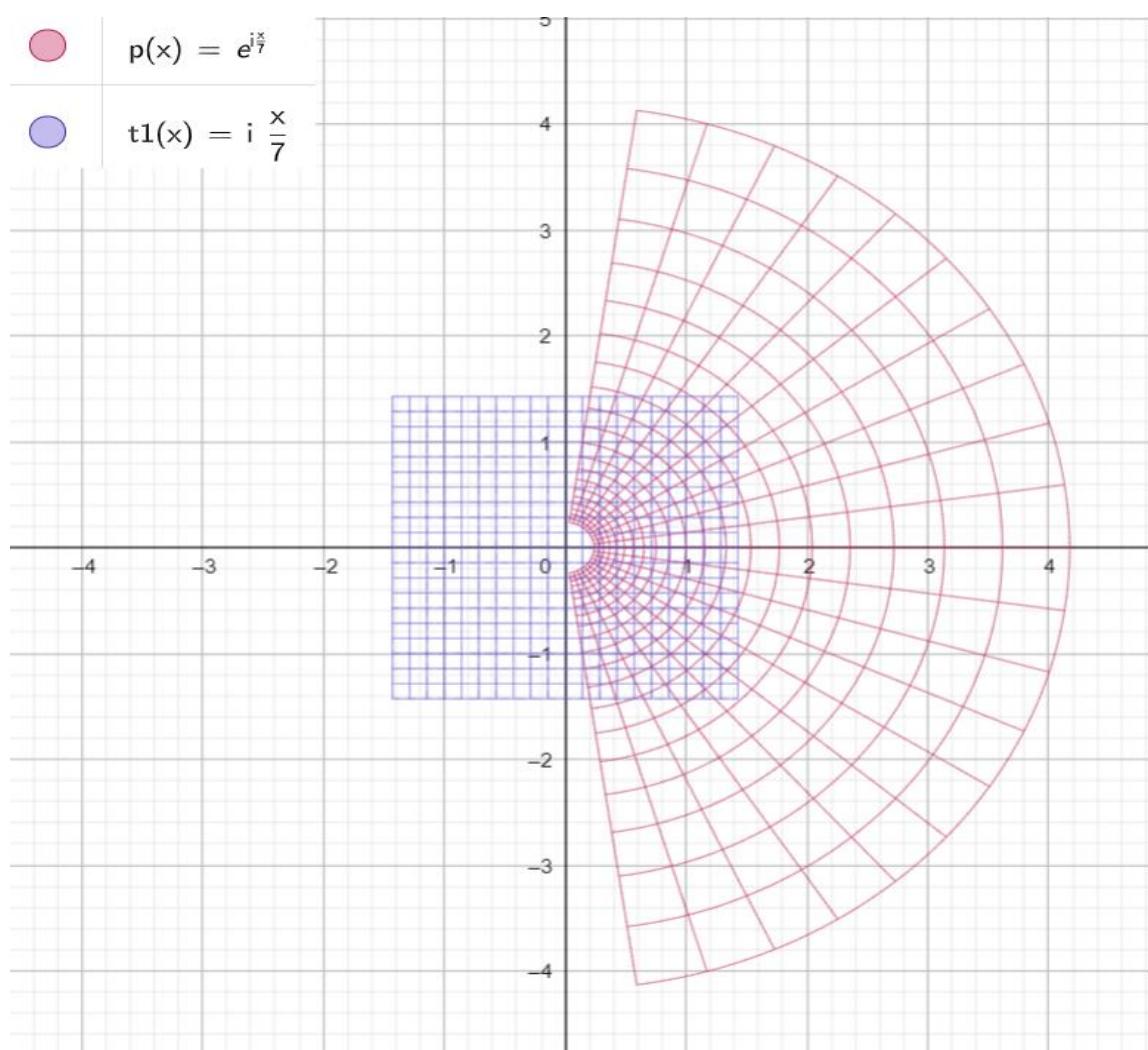
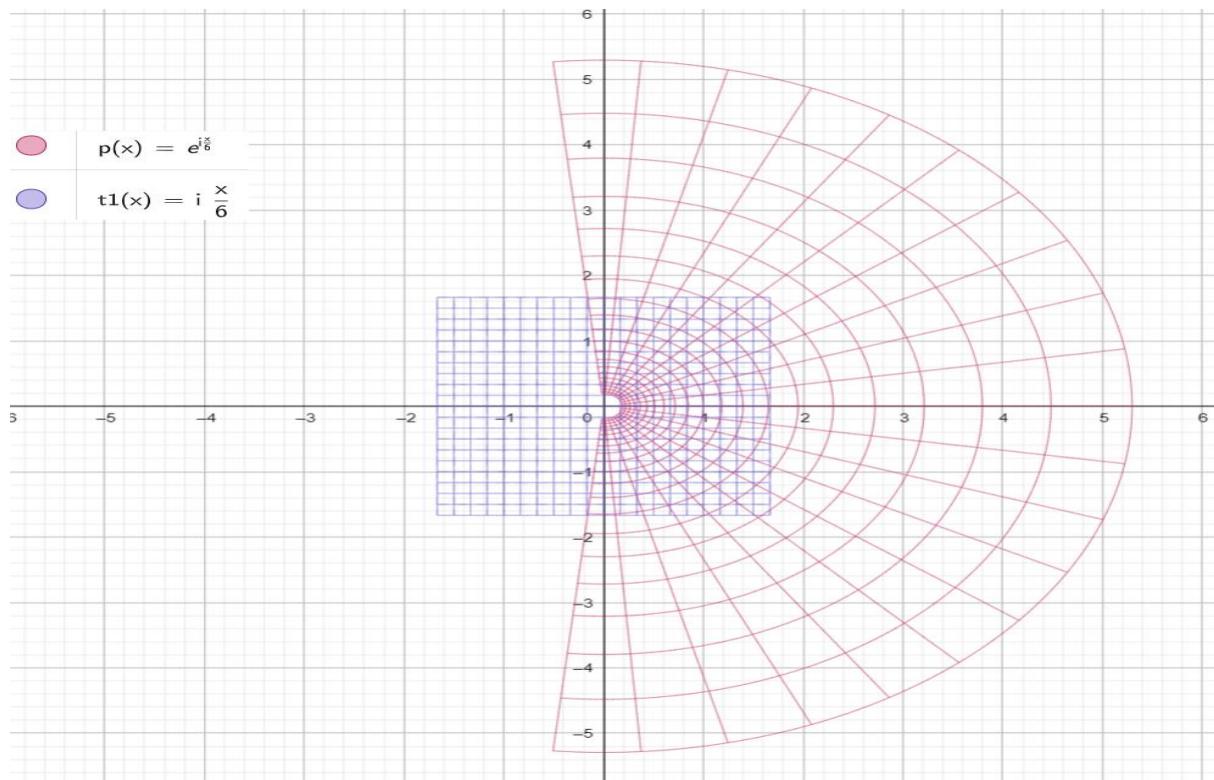
We can see the distortion in the frame of reference transformation decreases as we scale by smaller fraction. This can be interpreted as we move towards infinity the distortion in frame of reference transformations will be decreases.

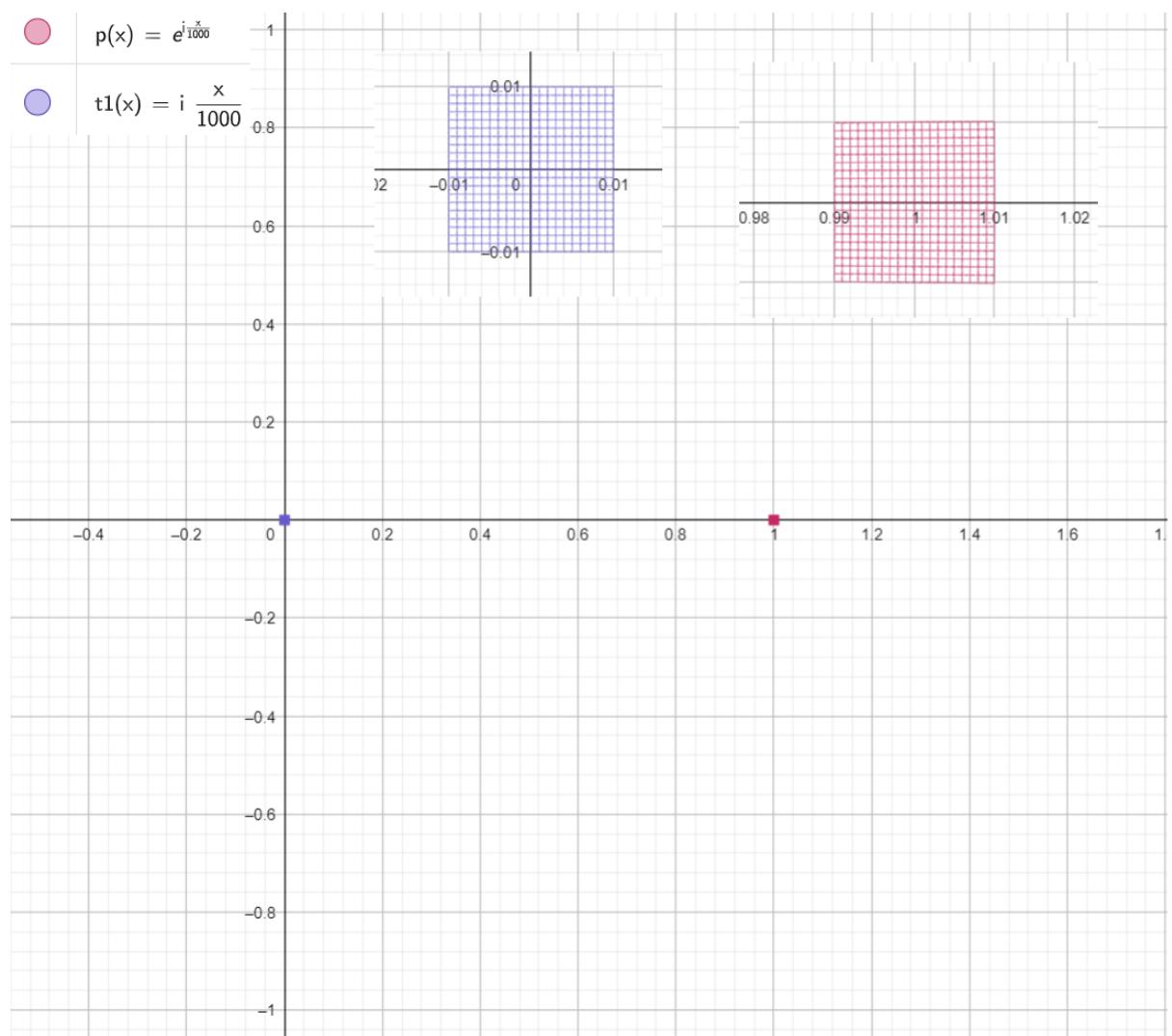
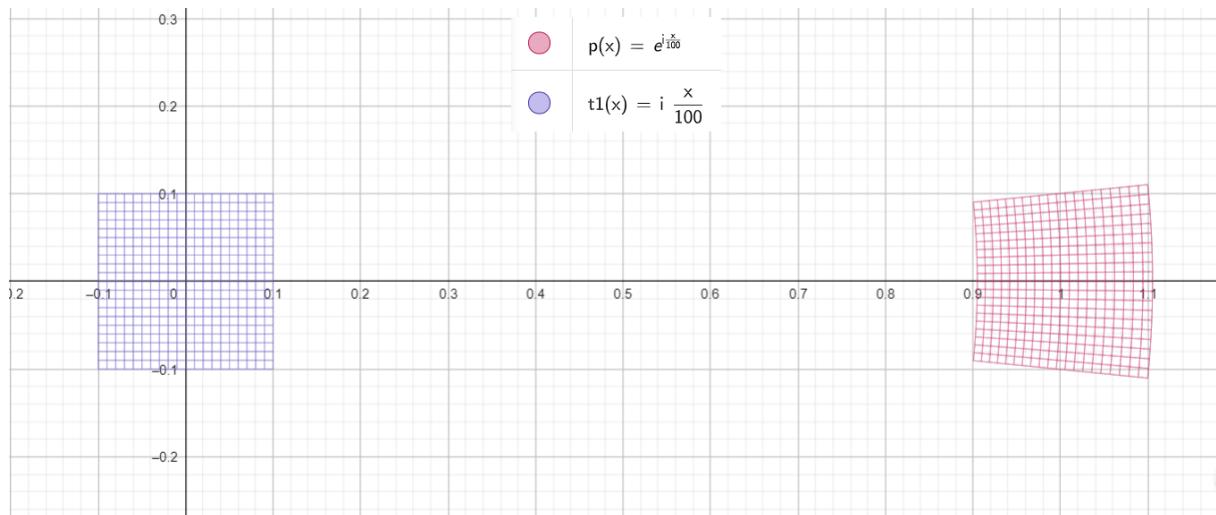
Strat from Figure 17; for scaling by $[x/3]$ we will start to see a clear unfolding. And as the scale fraction decreases the transformed frame of reference moves away from the original frame of reference until it reaches the unit [1] it stops, mostly because the limitation of the canvas of imaginary unit circle in the complex plane.

As we see Figures 16; the transformation is $[X/2]$ and $[e^X/2]$





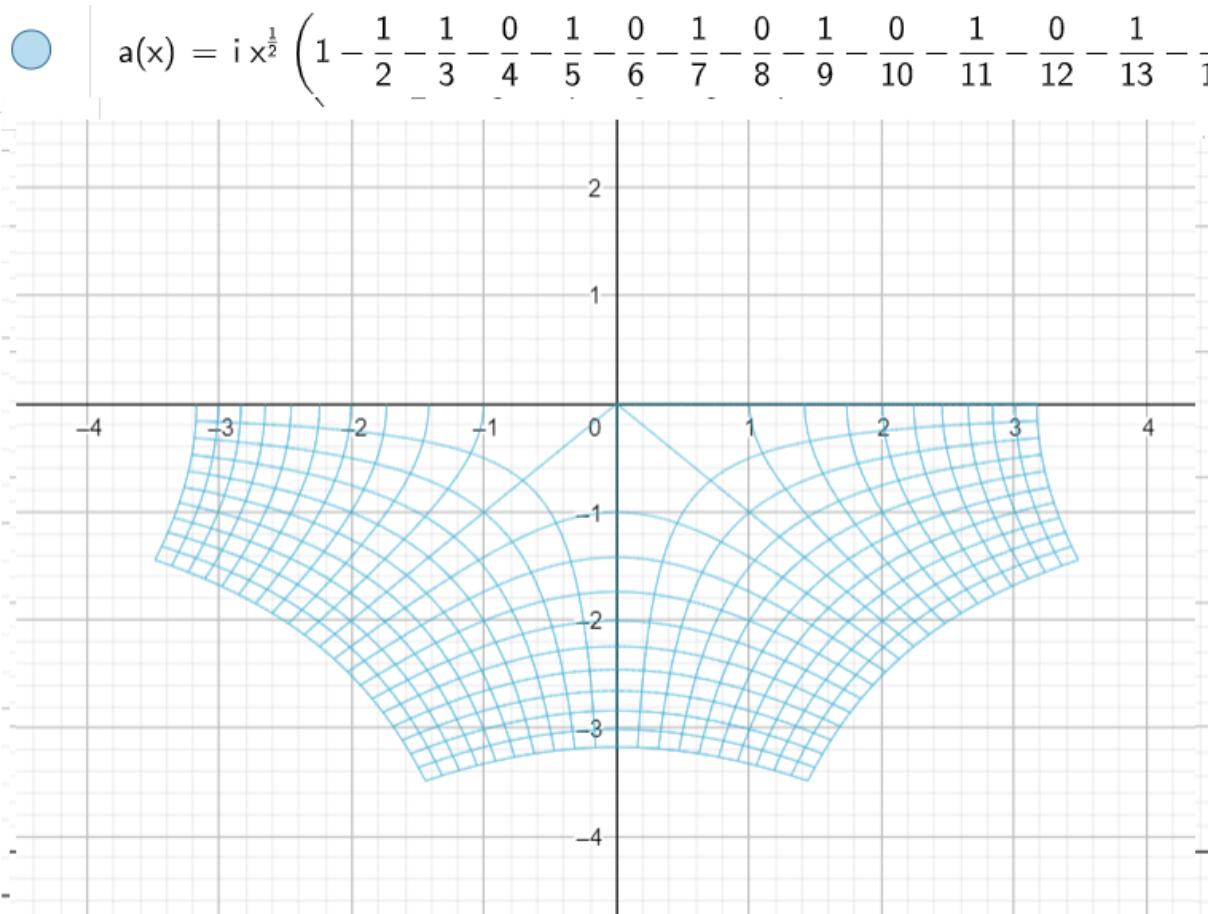




2.2 manifold frame of reference transformation in complex plane

Raising [X] to a power will give us the effect of manifold in complex plane. As the power increases

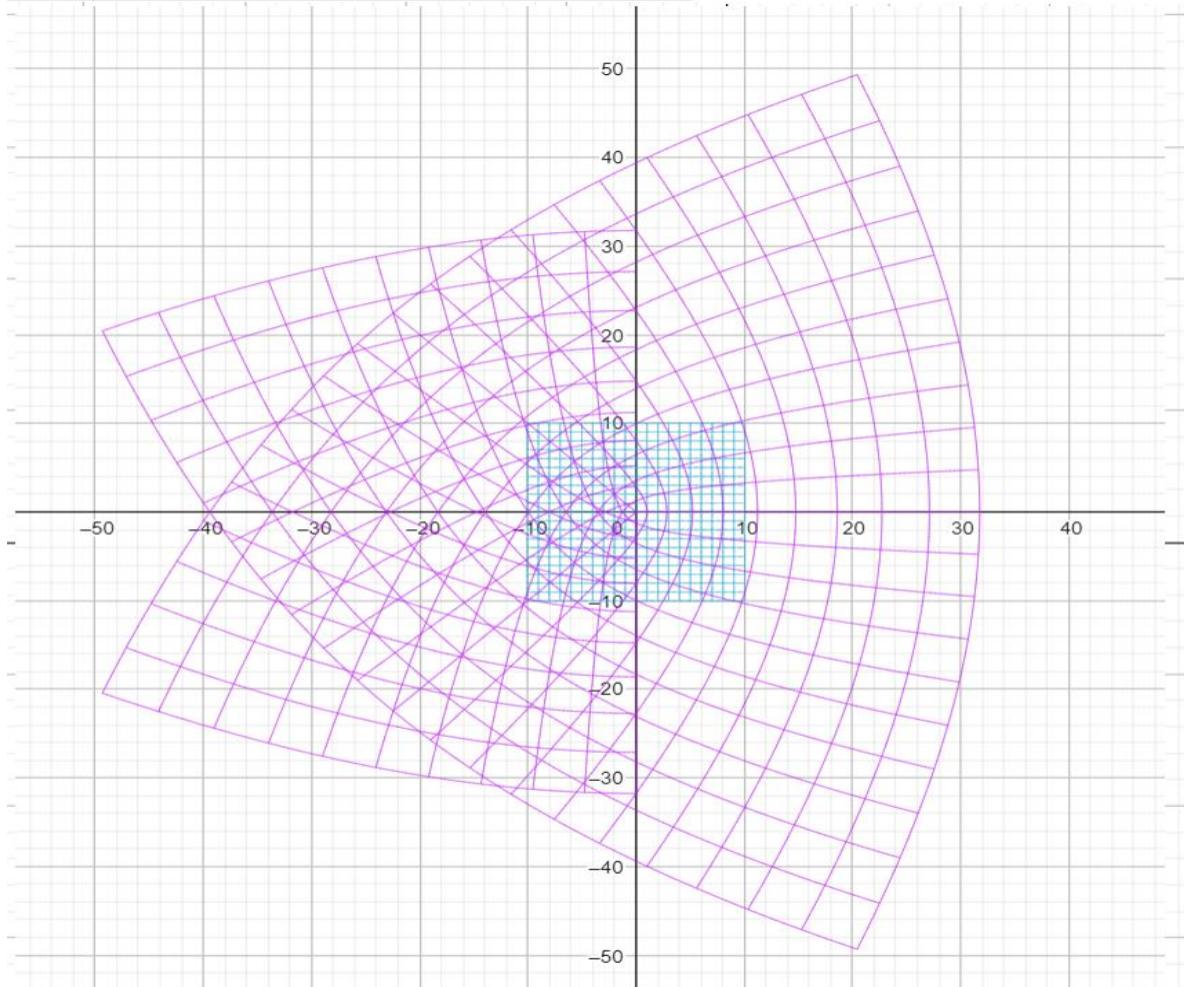
In Figure 24; the result of [sqrt(X)] and with only odd numbers. As you see it looks like cut the frame of reference in center and shift along the x axis



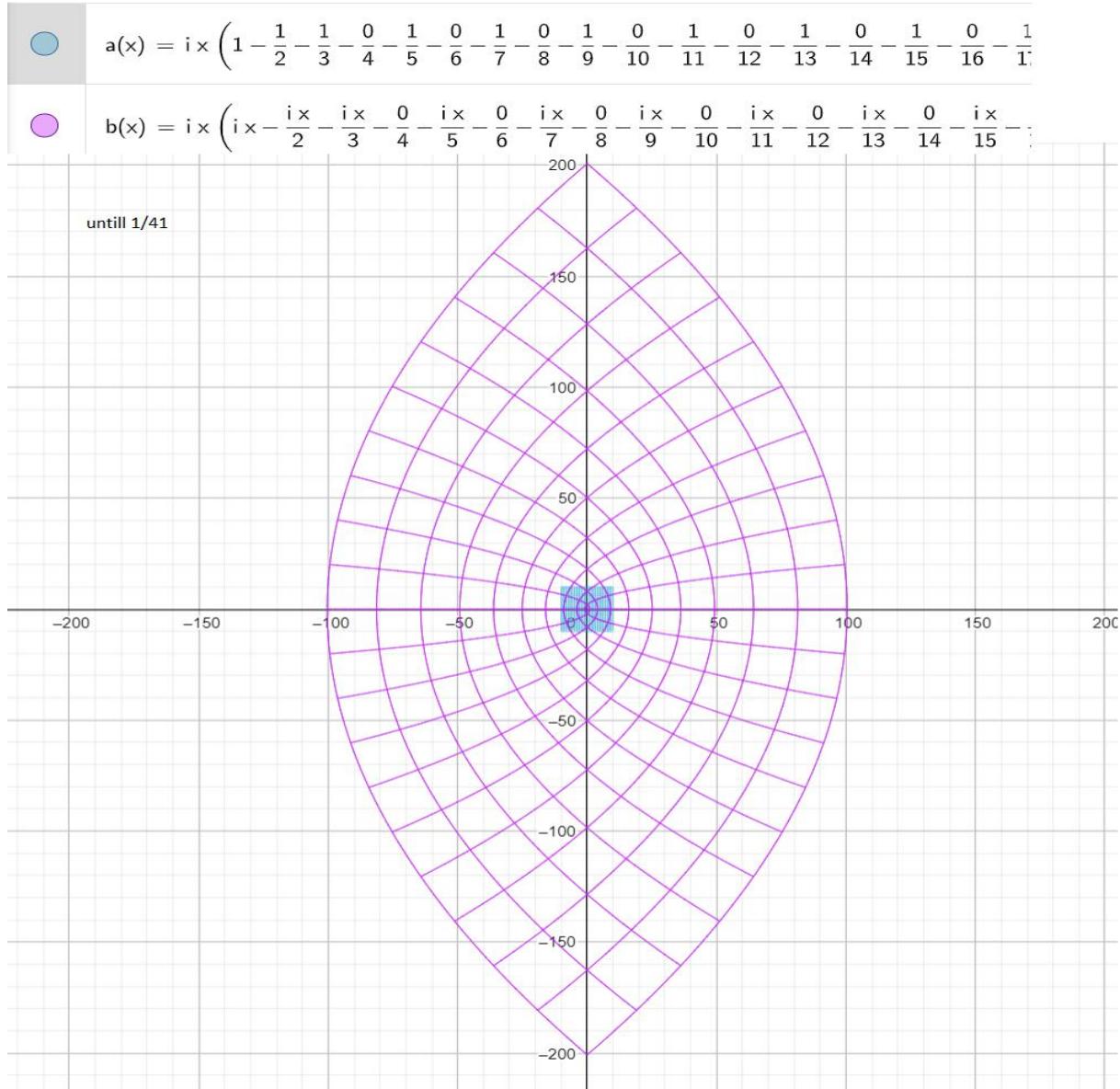
In Figure 25; the result of $[1/\sqrt{X}]$ transformation with only odd numbers. As you see it looks like we start to fold the frame of reference square

● $a(x) = i \times \left(1 - \frac{1}{2} - \frac{1}{3} - \frac{0}{4} - \frac{1}{5} - \frac{0}{6} - \frac{1}{7} - \frac{0}{8} - \frac{1}{9} - \frac{0}{10} - \frac{1}{11} - \frac{0}{12} - \frac{1}{13} - \frac{0}{14} - \frac{1}{15} - \frac{0}{16} \right)$

● $b(x) = i \times^{\frac{1}{2}} \left(i \times -\frac{i \times}{2} - \frac{i \times}{3} - \frac{0}{4} - \frac{i \times}{5} - \frac{0}{6} - \frac{i \times}{7} - \frac{0}{8} - \frac{i \times}{9} - \frac{0}{10} - \frac{i \times}{11} - \frac{0}{12} - \frac{i \times}{13} - \frac{0}{14} - \frac{i}{1} \right)$

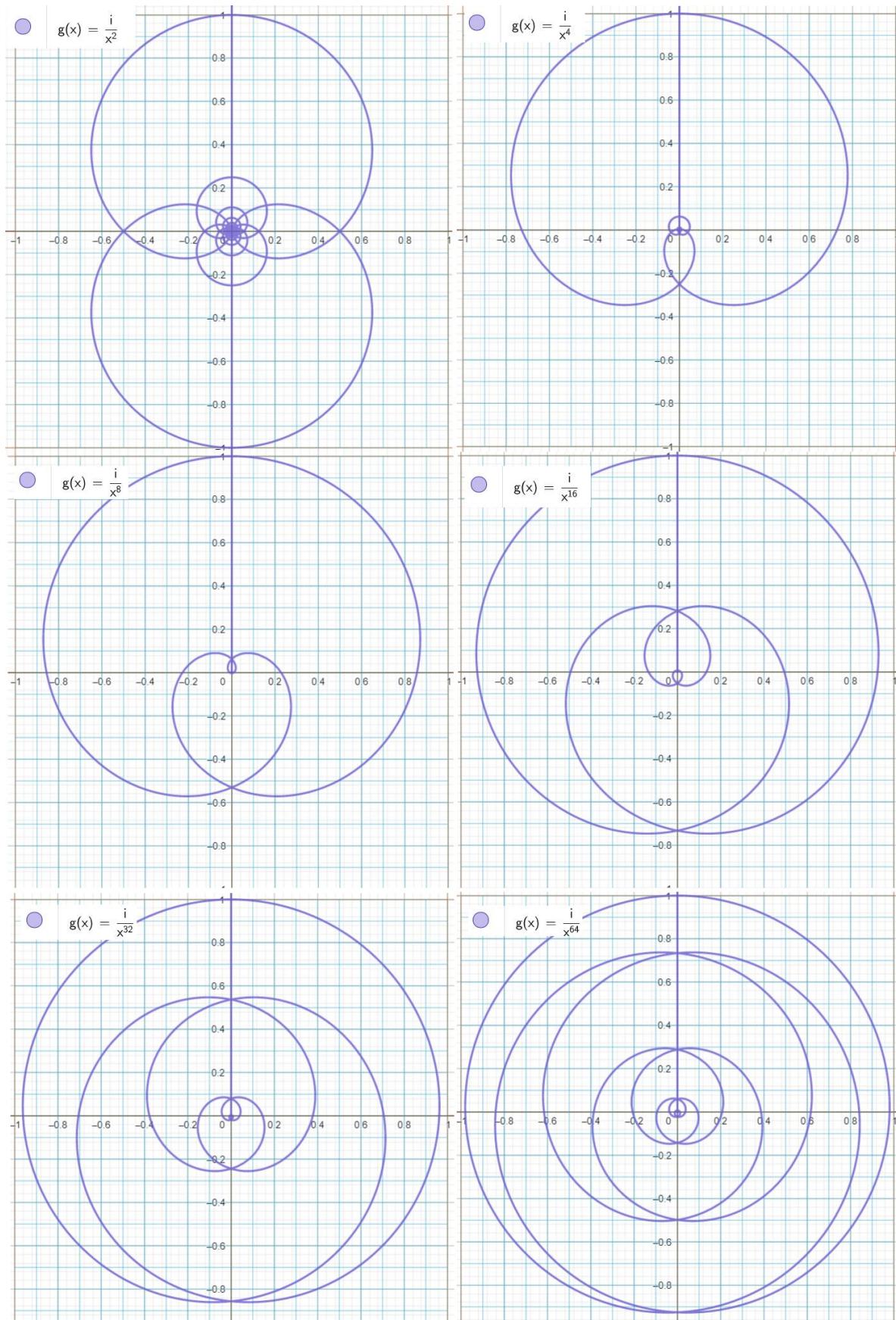


In Figure 26; the sum of $[x^2]$ transformation for frame of reference for odd numbers from 1 until [1/41]
 And because we in base 10 system. The fold shape is bounded by $[-x^2, x^2]$ i.e. $[-100, 100]$

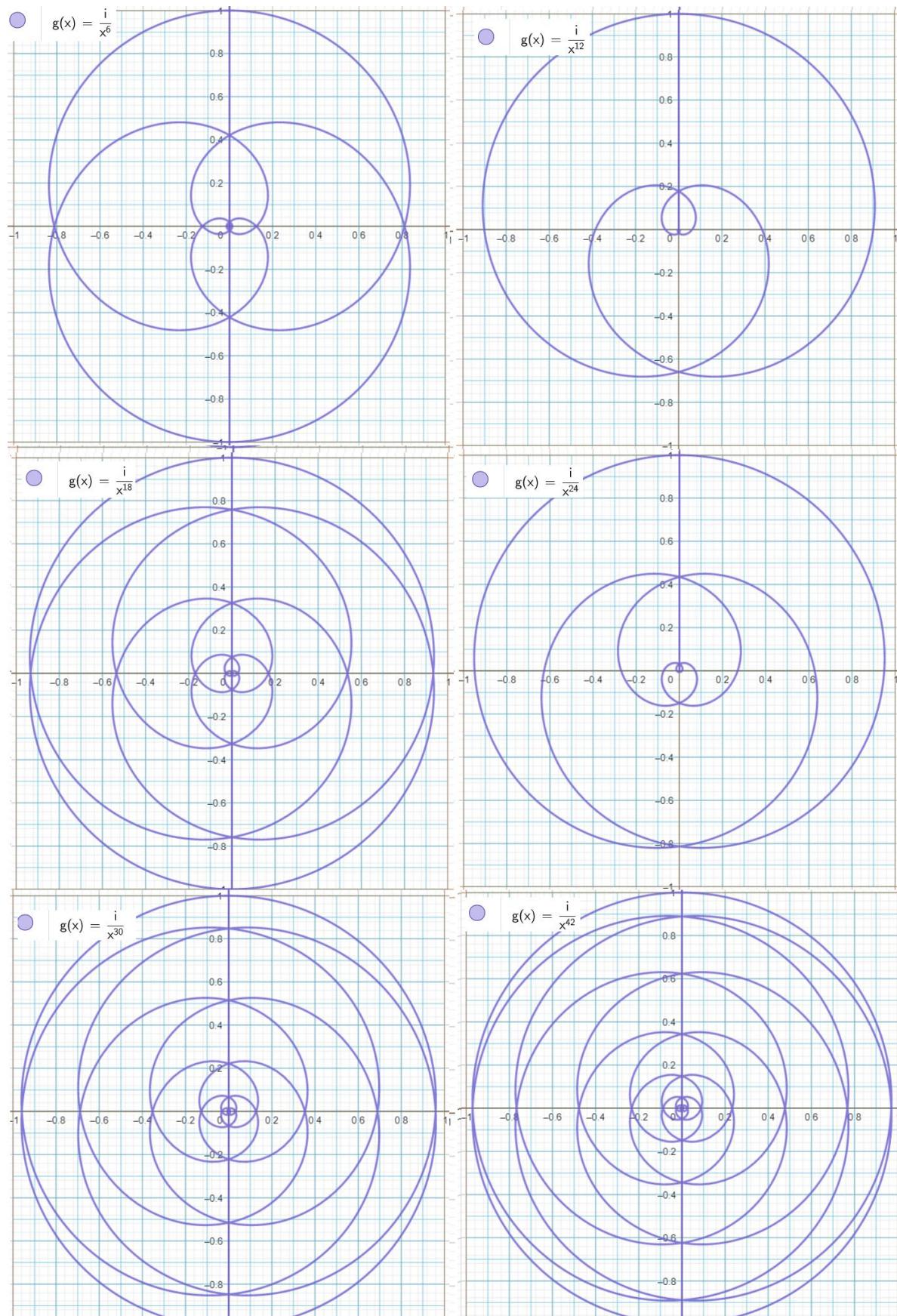


2.3 patterns in complex plane manifold of a frame of reference transformation

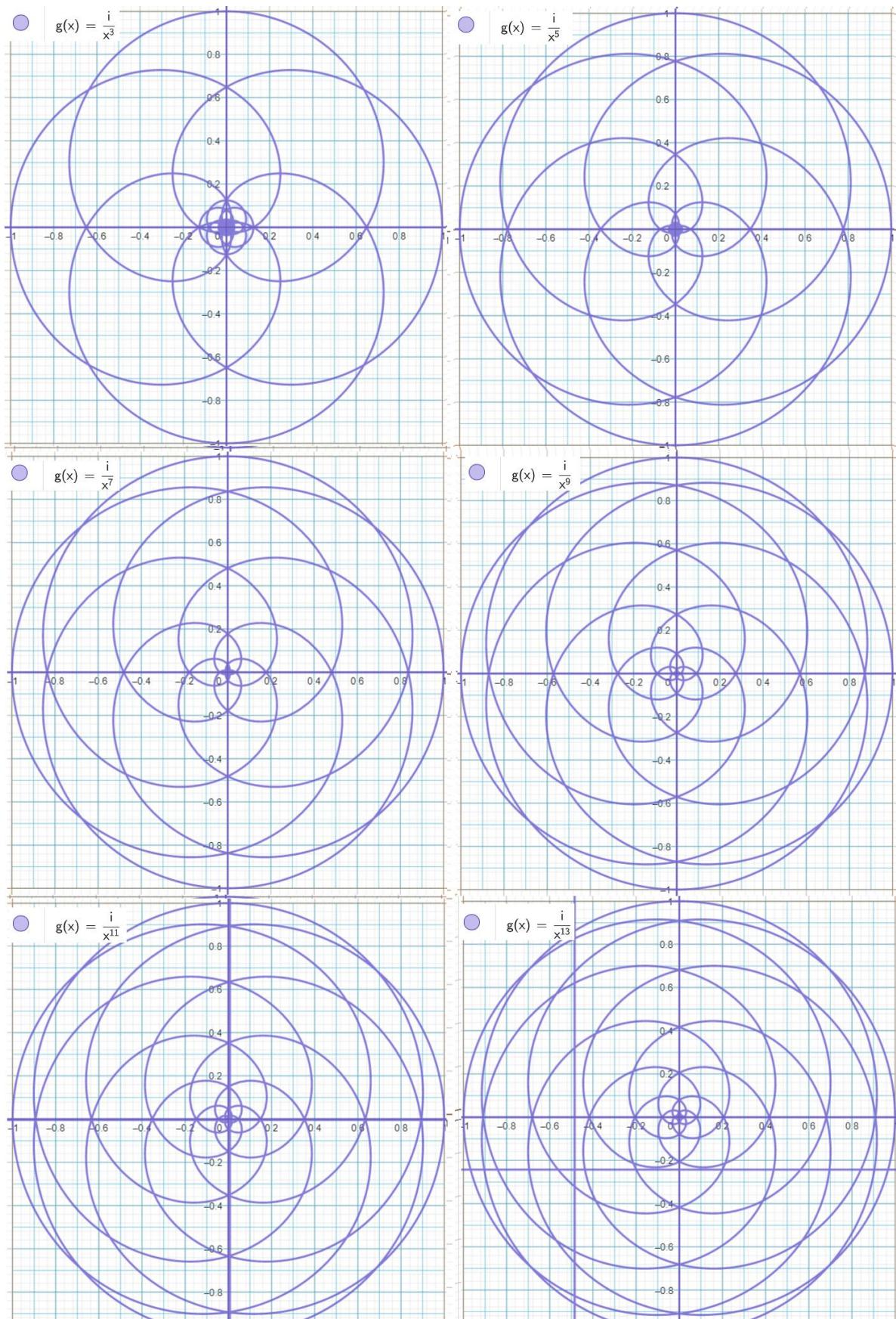
2.3.1 pattern for $[1/x^{2^n}]$



2.3.2 pattern for $[1/x^6*n]$

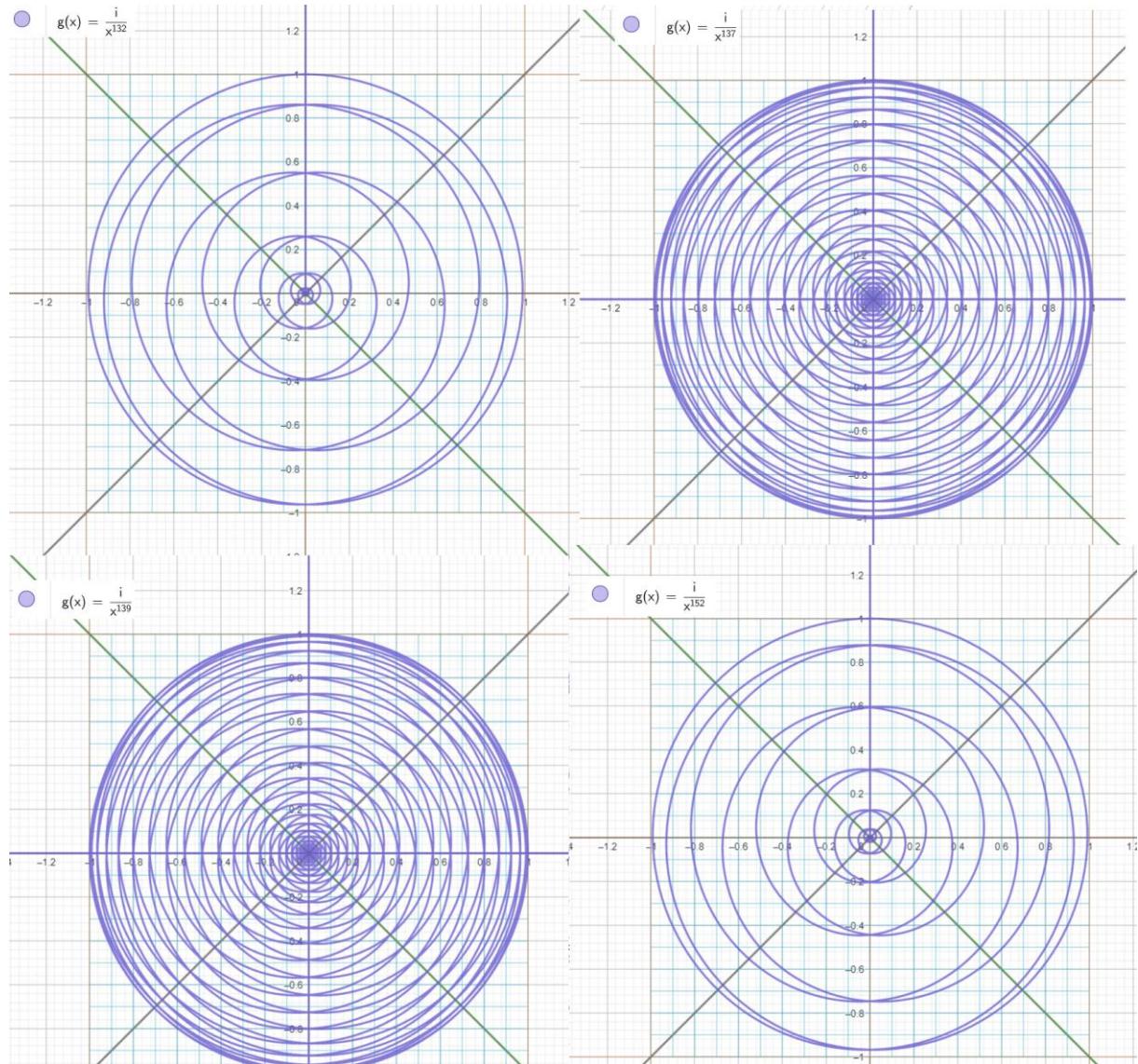


2.3.3 pattern for $[1/X^n]$ where n is odd number

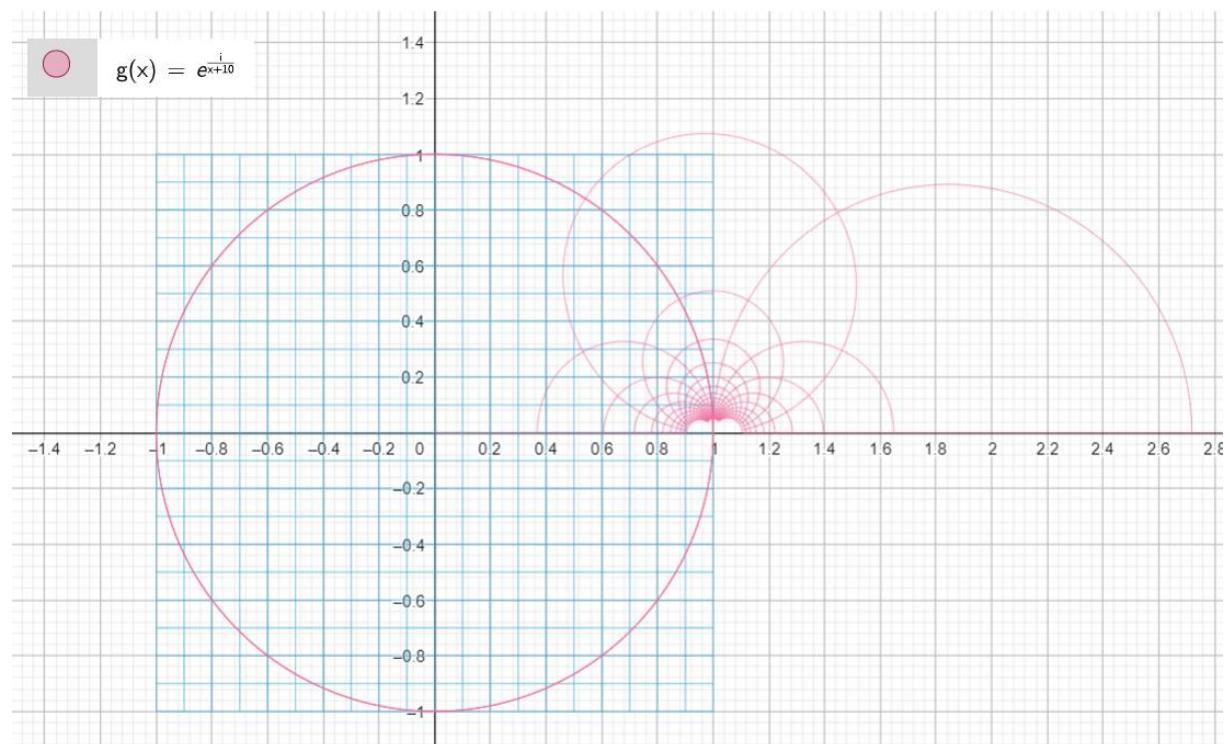
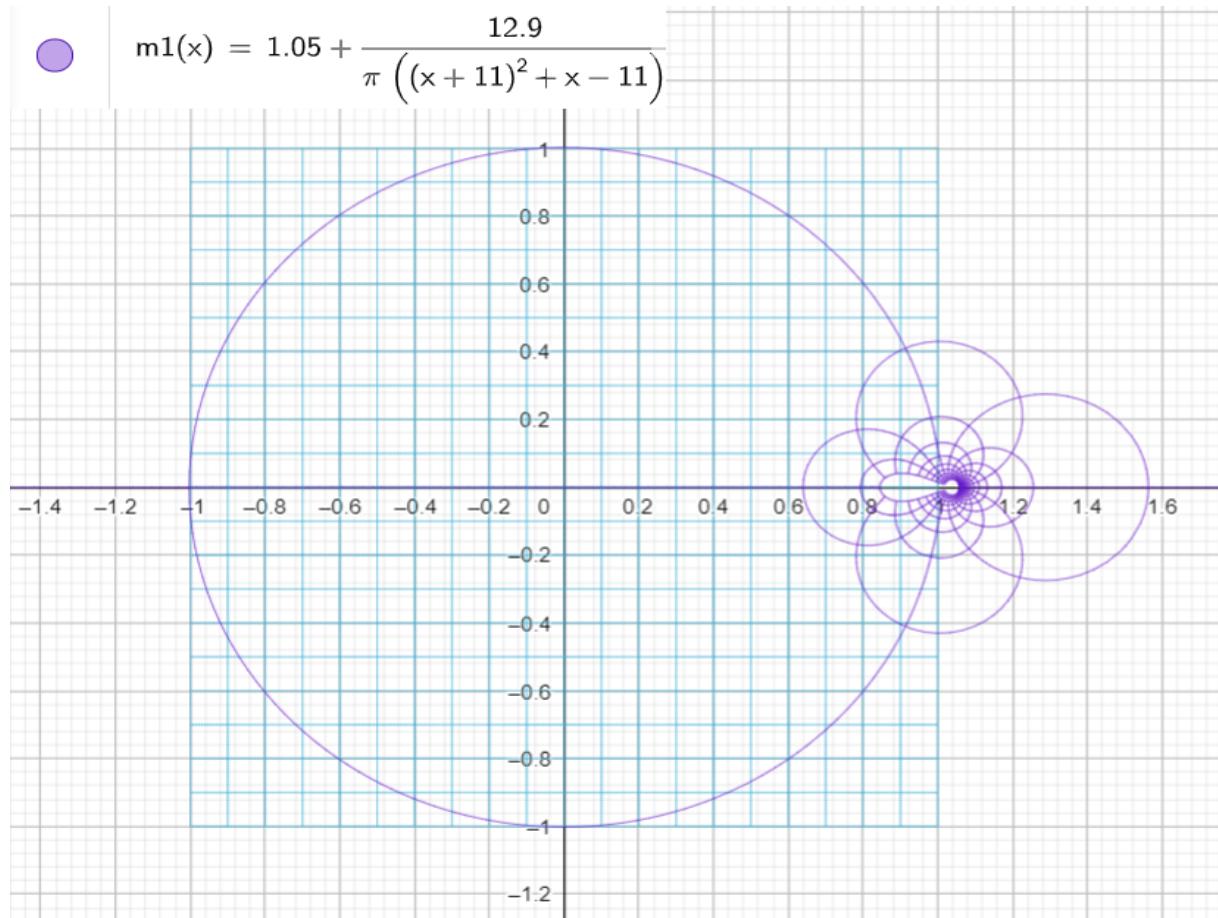


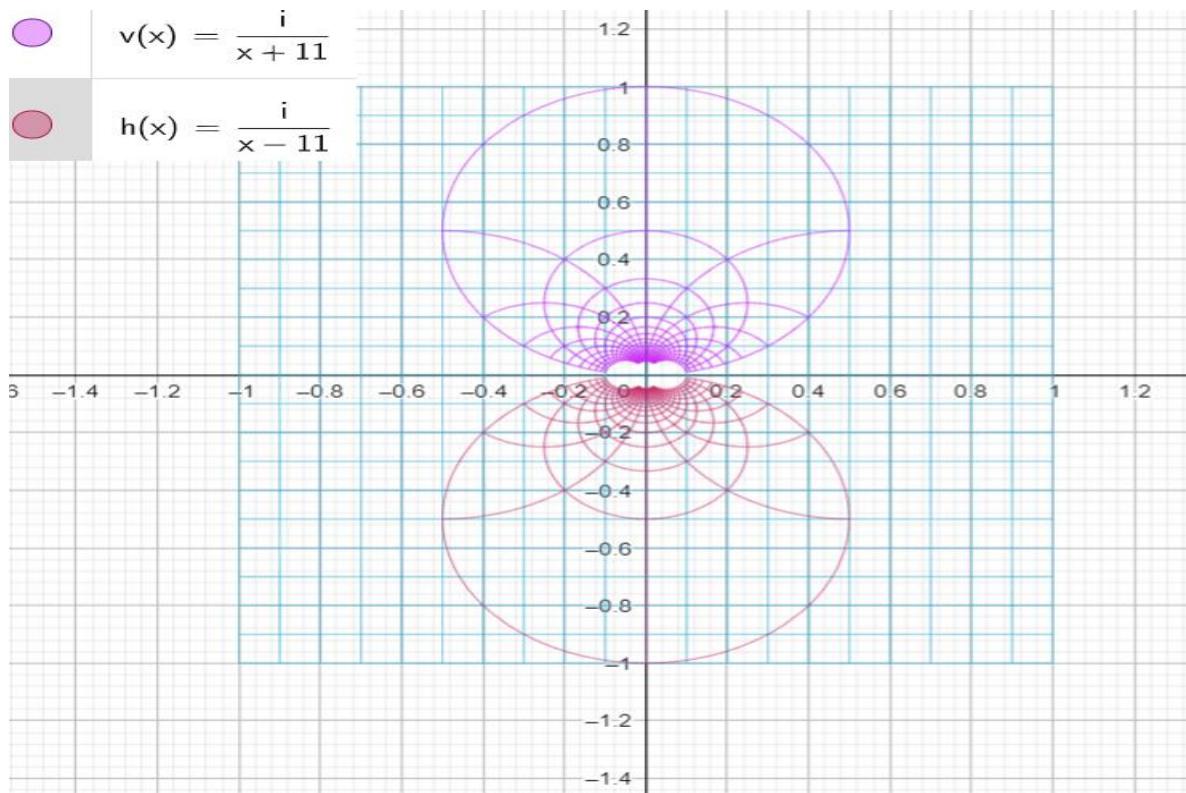
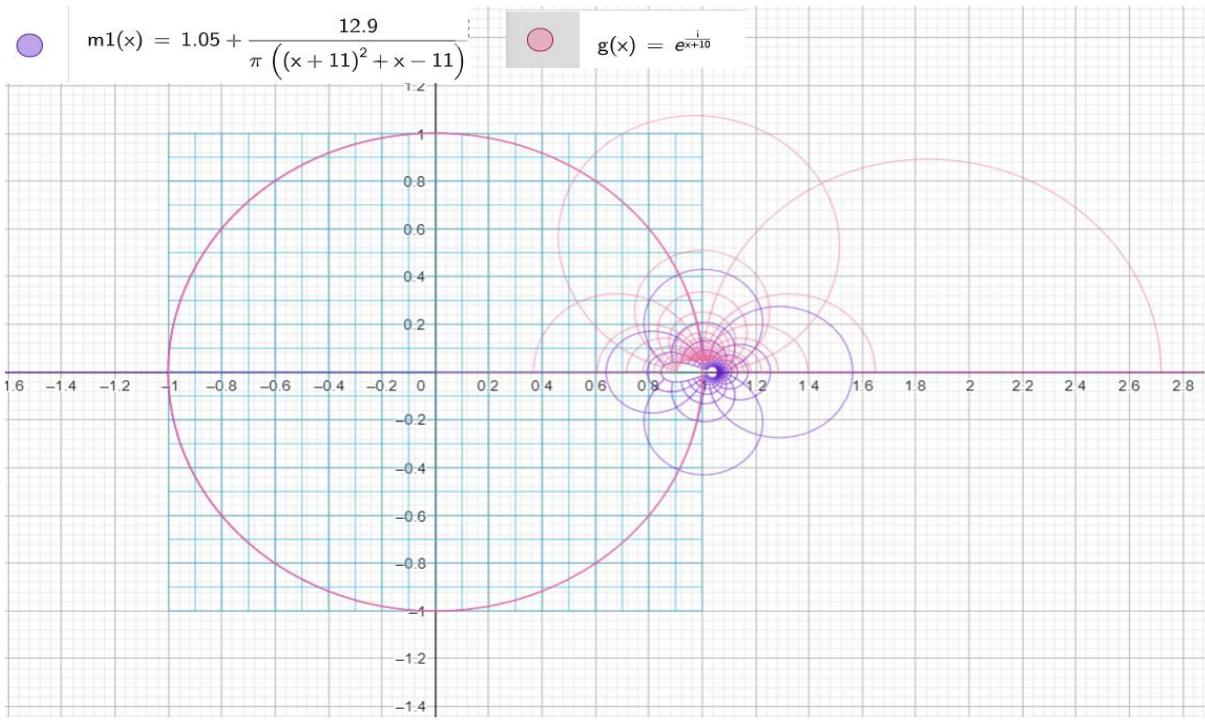
2.3.2 difference in pattern for $[1/x^n]$ when n is even and when n is odd.

When the power to raise n is an odd number it will intersects at $\theta = 45$ degree. but even powers only intersect at $\theta = 0$ or 90 or 180 or 360 .

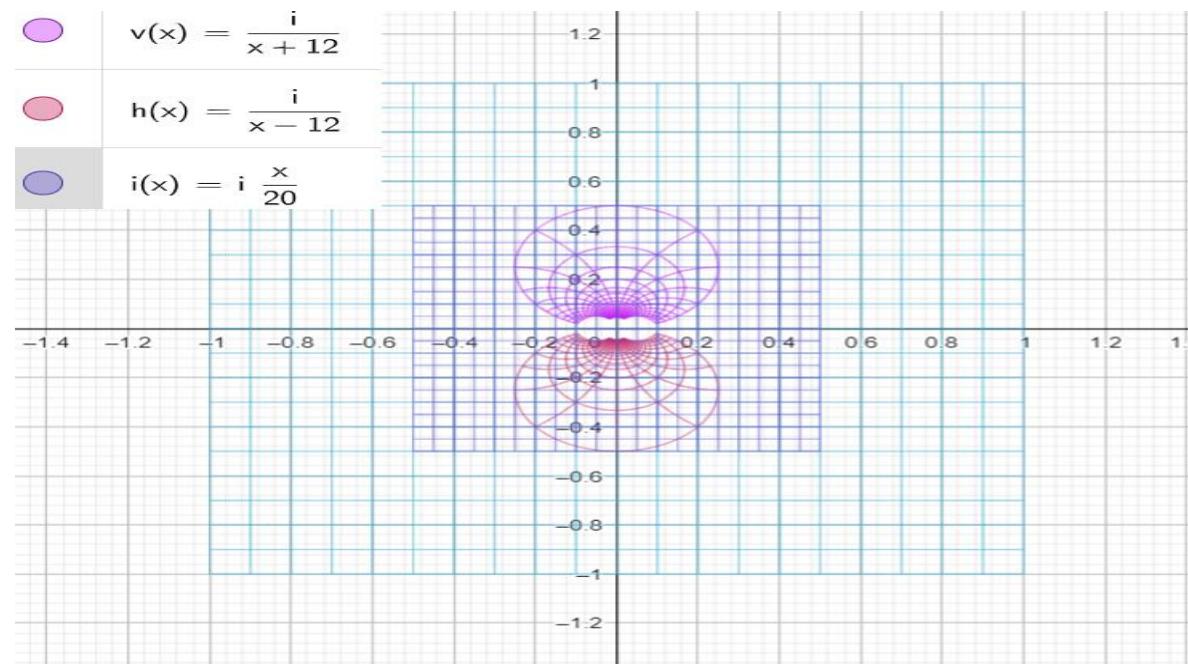


2.3.2 harmonization relation between $[\pi]$ and $[e]$ on imaginary unit circle and its formula

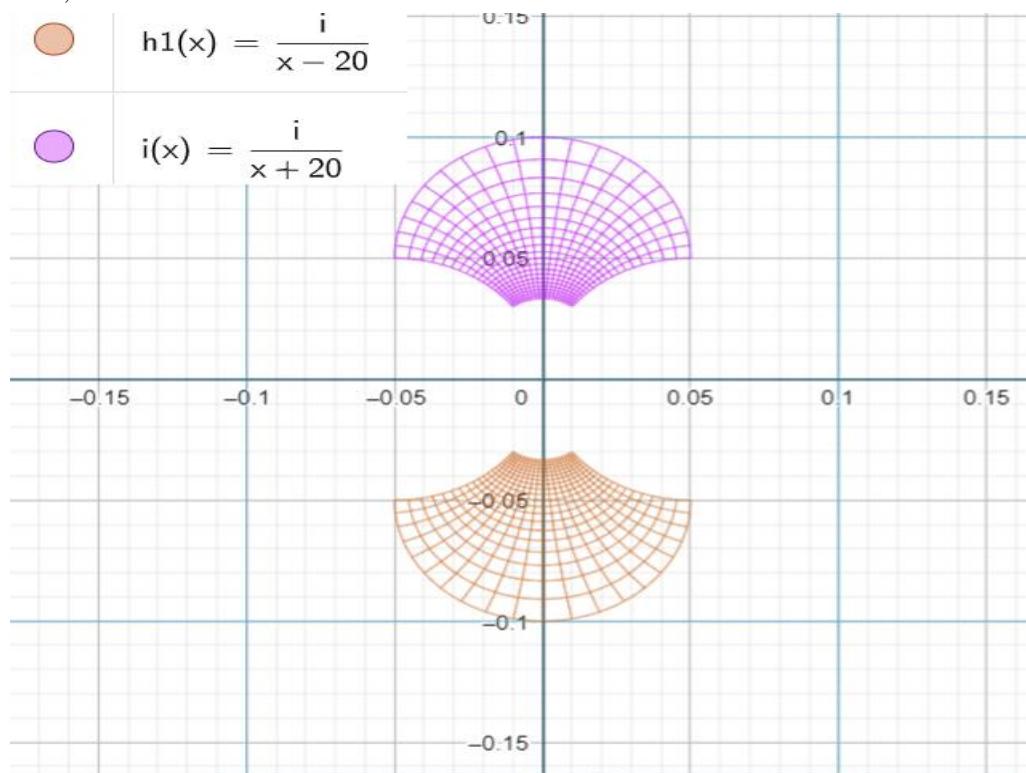




at $X \pm 12$ we will be moved (shrinking) to 0.5 size of our 10 base system numbers unit frame of reference square size.

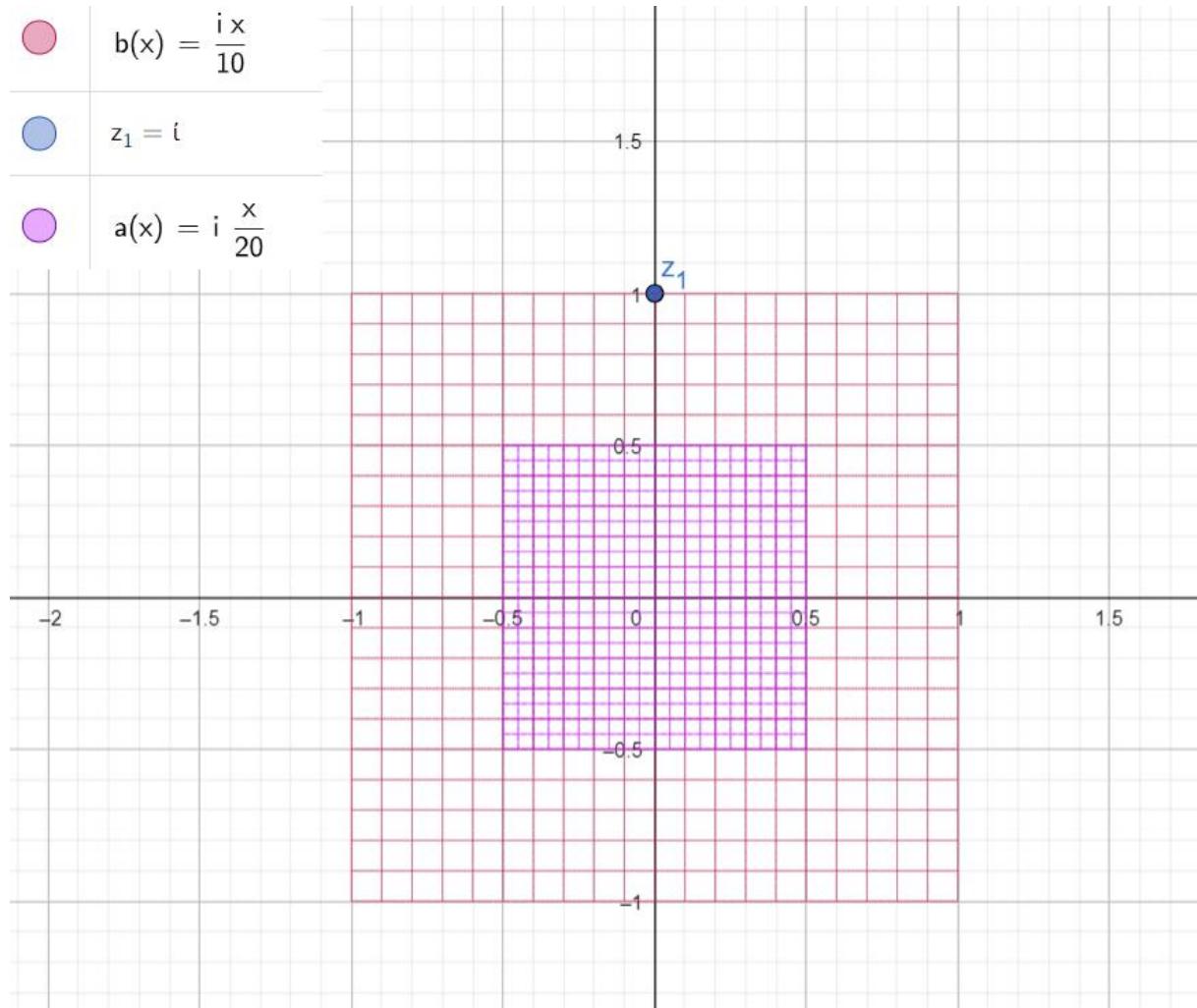


Also, we can do transformation on Y axis



3. Using the concept of Frame of reference as a replacement for analytic continuity

1- use a frame of reference but scaled to a 10 based number system and its scale to 0.5. $[a(x) = i \frac{x}{20}]$



2- use a simple principle $[X - 1 = X * (1 - 1/X)]$

$$c = 2 \left(1 - \frac{1}{2}\right)$$

→ 1

$$d = 3 \left(1 - \frac{1}{3}\right)$$

→ 2

$$e = 4 \left(1 - \frac{1}{4}\right)$$

→ 3

$$f = 31 \left(1 - \frac{1}{31}\right)$$

→ 30

$$g = 2 - 1$$

→ 1

$$h = 3 - 1$$

→ 2

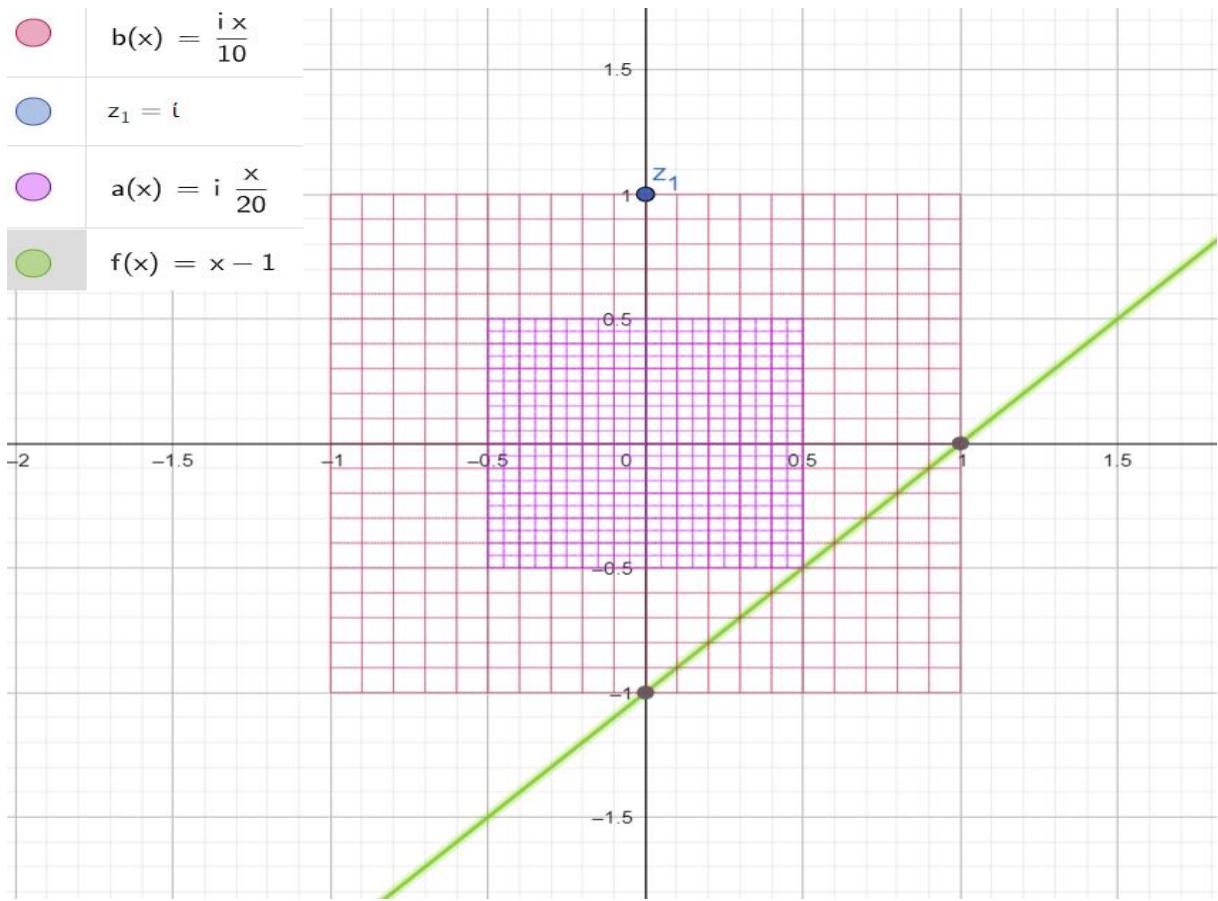
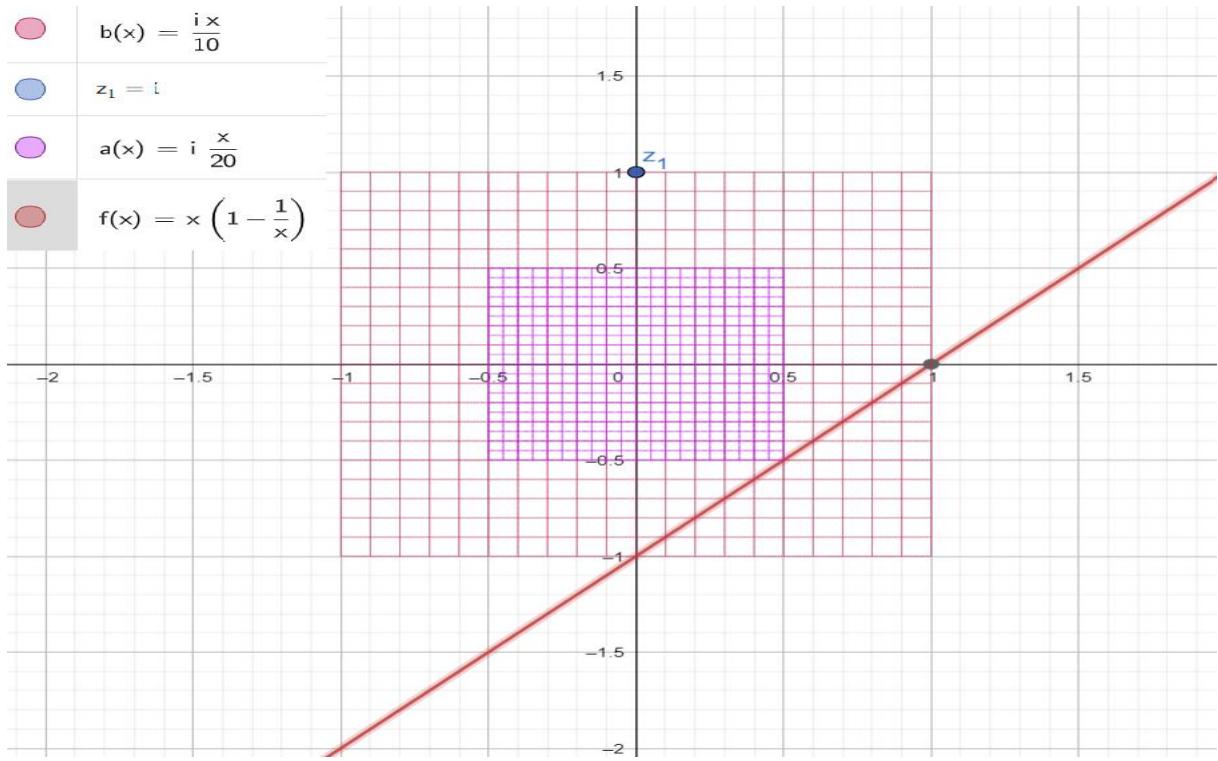
$$i = 4 - 1$$

→ 3

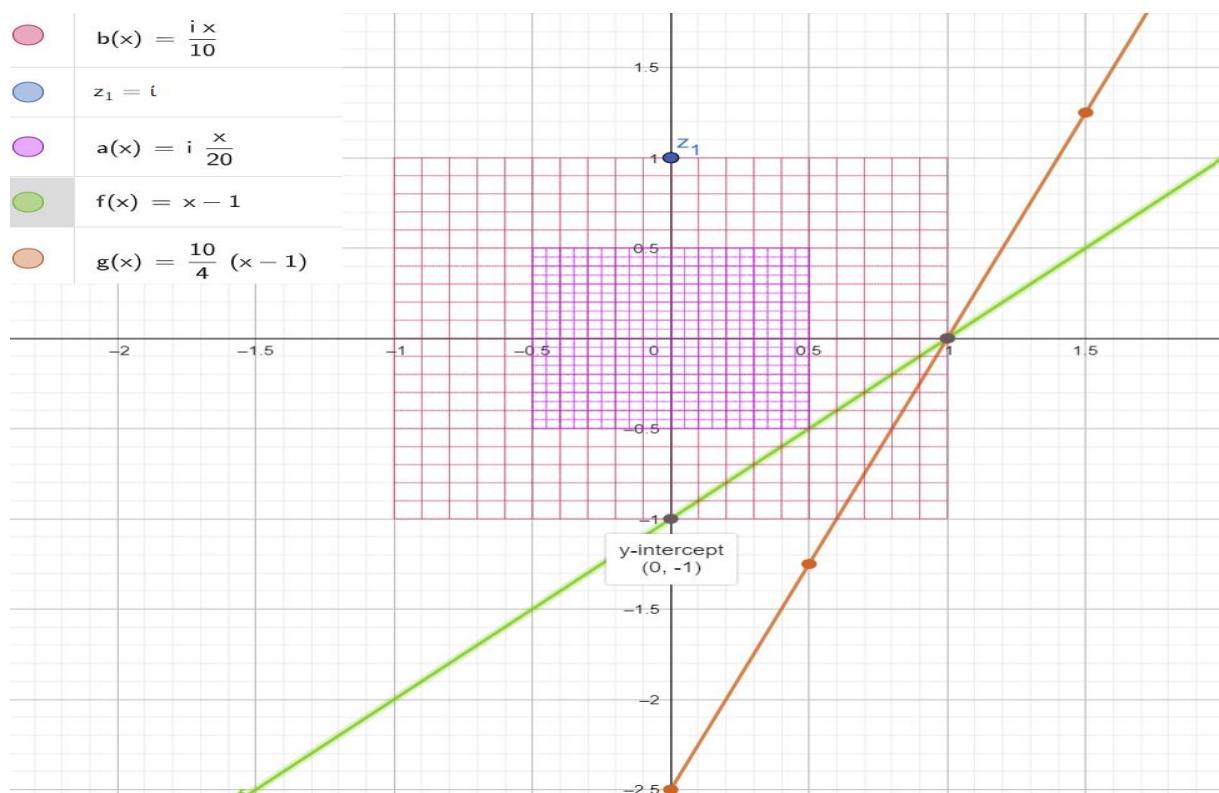
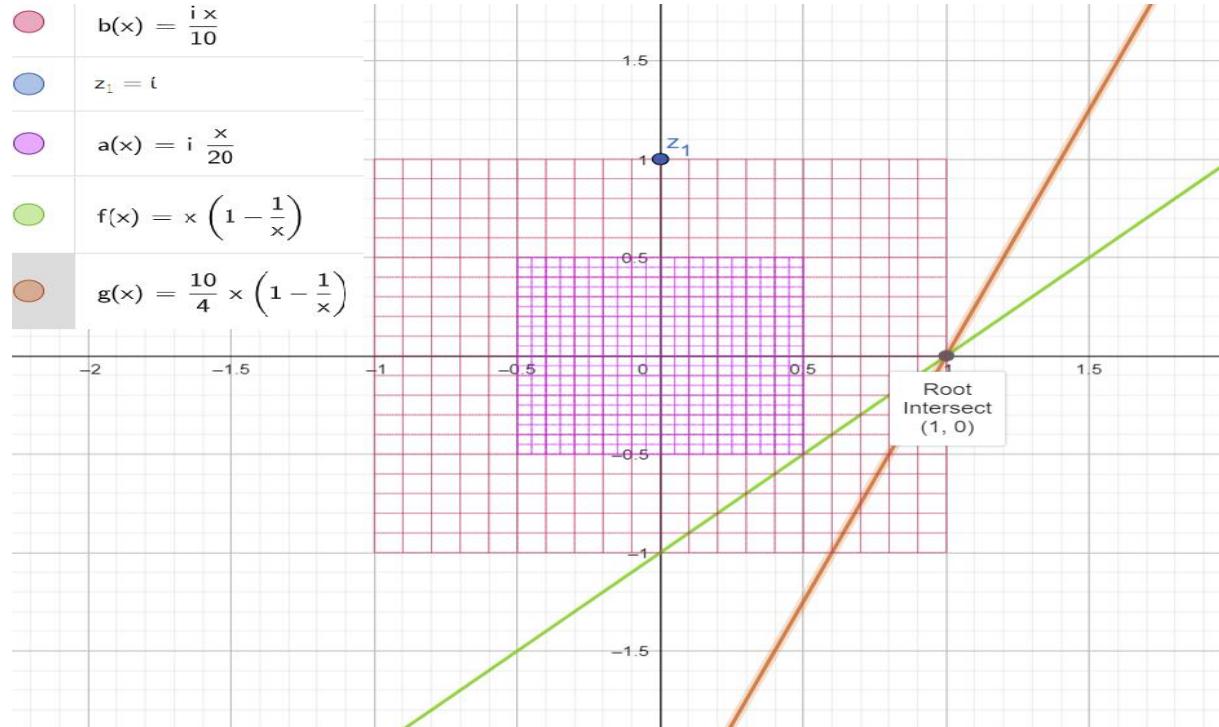
$$j = 31 - 1$$

→ 30

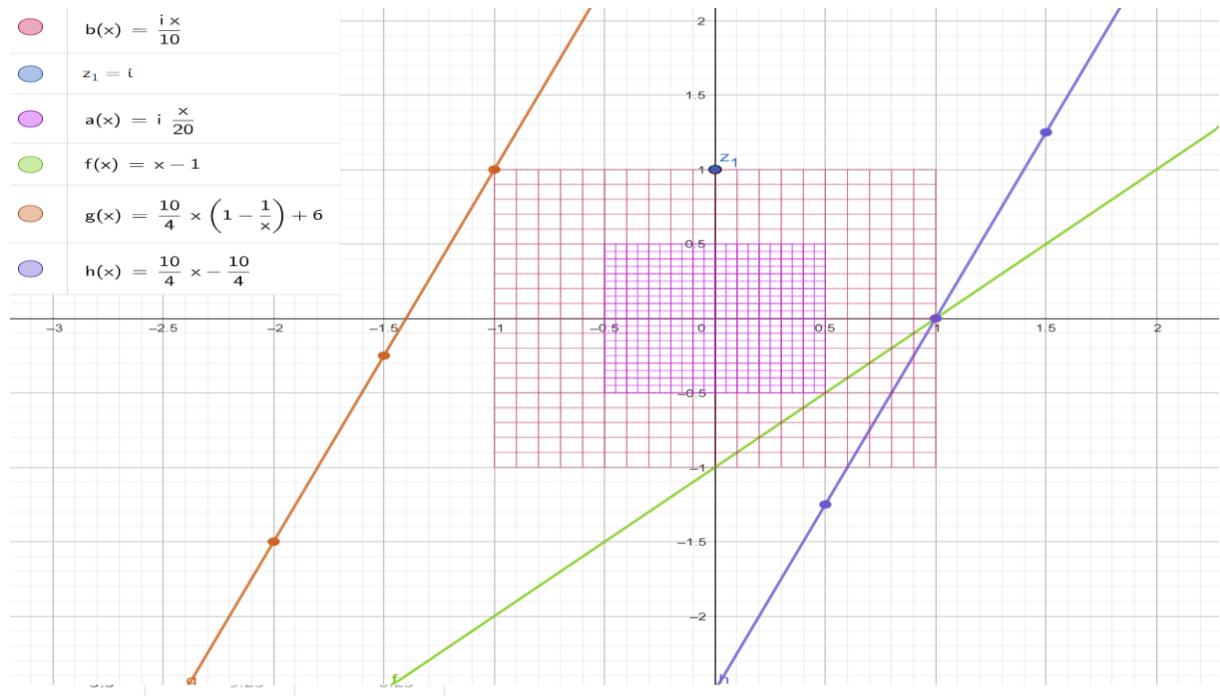
3- plotting $X * (1 - 1/X)$ intersects X axis at (1 , 0) and intersects Y axis at (0,-1), same as plotting ($X - 1$)



4- as we are using base 10 number system, we are going to scale our calculations by * 10. And, as the frame of reference is symmetrical along X axis and Y axis and origin in Center then any point in the frame of reference will be in a range of 4 values (1, -1, +i, -i), and any transformation for this frame of reference will keep the shape sides ratio the same, then we can use only one fourth from this frame of reference in our calculations. So, we are going to scale [X * (1 - 1/X)] and [X - 1] by multiply it by [10/4].



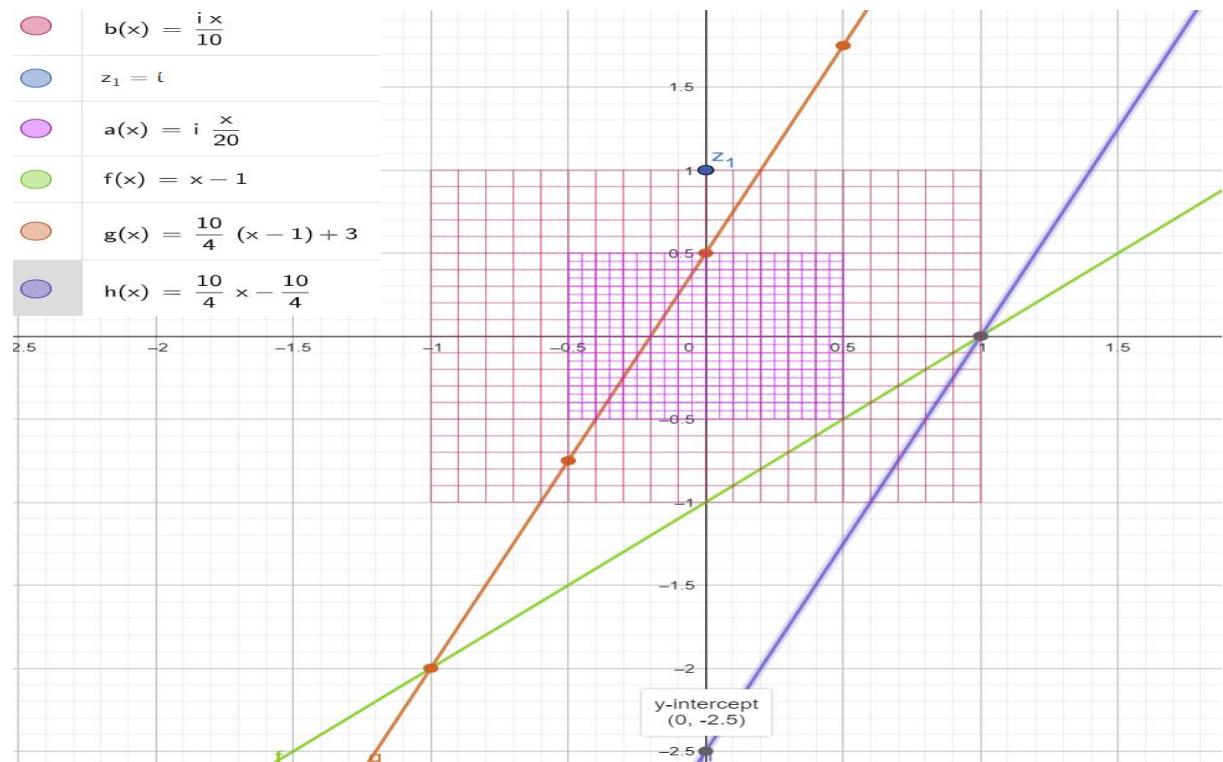
5- if we add 6 to this transformation, we will get $g(x)$ at exactly upper left corner of the frame of reference.

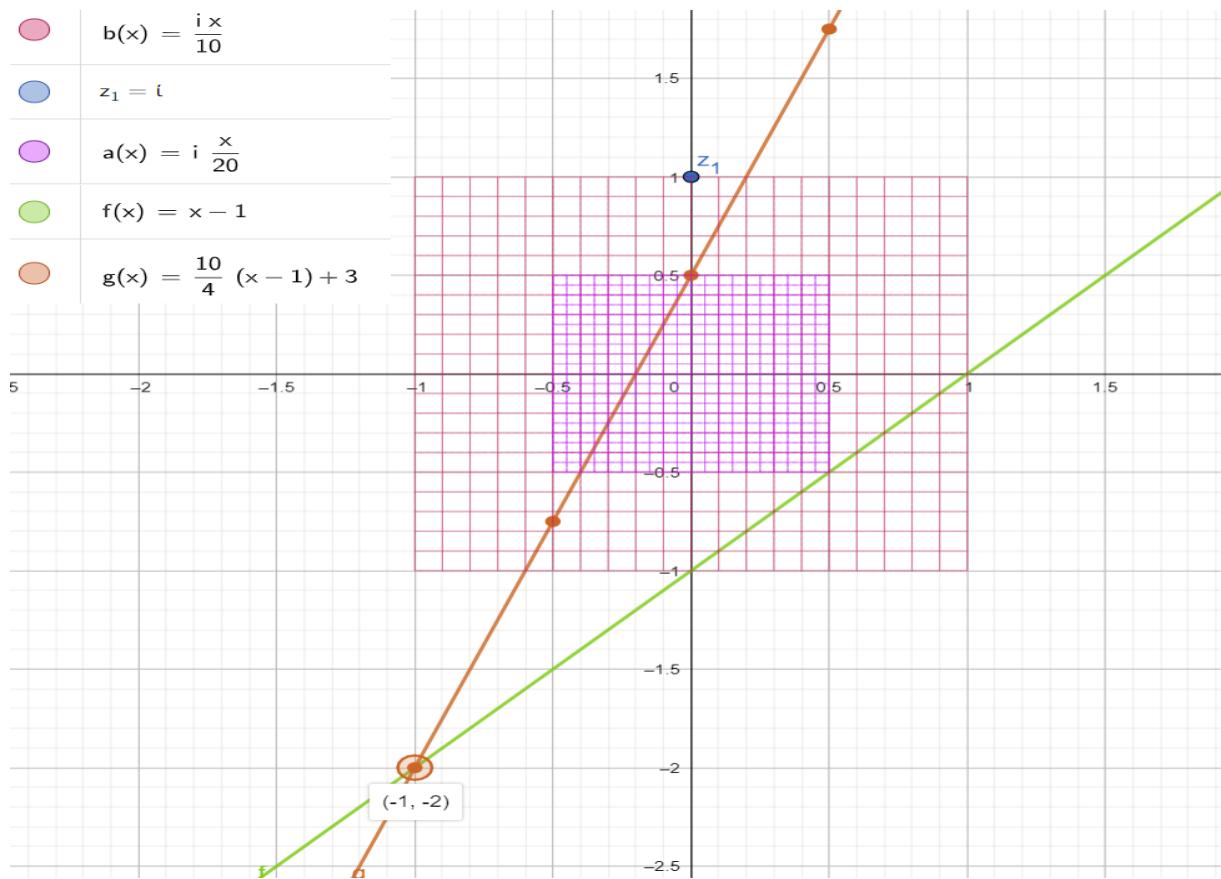
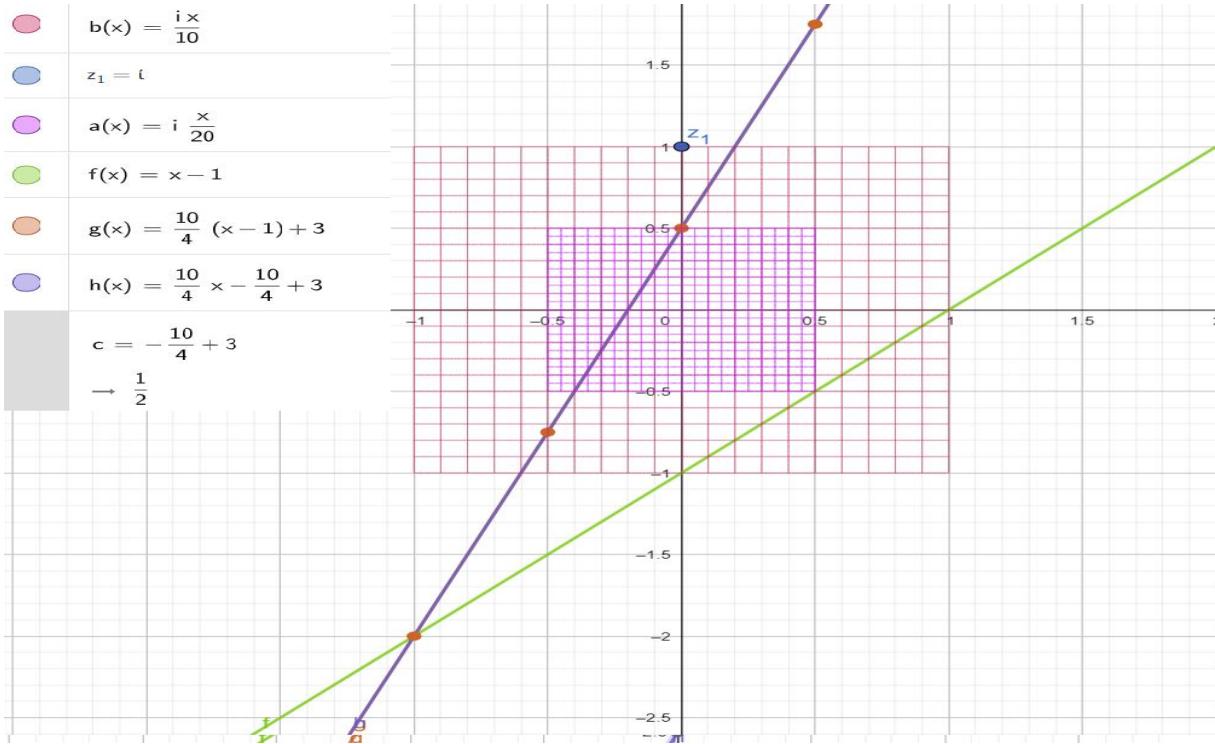


6- using 3 instead of 6 will give us a line intersects with Y axis at point $Z = 0 + 0.5i$. an as we explained before in frame of reference the Euler's Identity transformation is

$$Z = \sin(\theta) \pm i \cos(\theta)$$

Then this Transformation is $[10/4 * X]$ then add $[-10/4 + 3 = 1/2]$ intersects Y at 0.5.





7- now we are going to see how this transformation solve the infinity problem between [0 ,1] for primes at stripe line. Using this transformation, all odd numbers of values for X will give us a natural number for Y. for any odd value of X.

$x \equiv$	$g(x) \equiv$	$h(x) \equiv$		$b(x) = \frac{i x}{10}$
-3	-7	-10		$z_1 = i$
-2.5	-5.75	-8.75		
-2	-4.5	-7.5		$a(x) = i \frac{x}{20}$
-1.5	-3.25	-6.25		$f(x) = x - 1$
-1	-2	-5		
-0.5	-0.75	-3.75		$g(x) = \frac{10}{4} (x - 1) + 3$
0	0.5	-2.5		$h(x) = \frac{10}{4} x - \frac{10}{4}$
0.5	1.75	-1.25		
1	3	0		$c = -\frac{10}{4} + 3$
1.5	4.25	1.25		$\rightarrow \frac{1}{2}$
2	5.5	2.5		
2.5	6.75	3.75		
3	8	5		
3.5	9.25	6.25		

$x \equiv$	$g(x) \equiv$	$h(x) \equiv$		$b(x) = \frac{i x}{10}$
-3.5	-8.25	-11.25		
-3	-7	-10		$z_1 = i$
-2.5	-5.75	-8.75		
-2	-4.5	-7.5		$a(x) = i \frac{x}{20}$
-1.5	-3.25	-6.25		$f(x) = x - 1$
-1	-2	-5		
-0.5	-0.75	-3.75		$g(x) = \frac{10}{4} \times \left(1 - \frac{1}{x}\right) + 3$
0		-2.5		$h(x) = \frac{10}{4} x - \frac{10}{4}$
0.5	1.75	-1.25		
1	3	0		$c = -\frac{10}{4} + 3$
1.5	4.25	1.25		$\rightarrow \frac{1}{2}$
2	5.5	2.5		
2.5	6.75	3.75		
3	8	5		
3.5	9.25	6.25		

A) Coordinate Systems

First let us talk about coordinate Systems

- 1- Number Line System which is our X axis, and we can move on it back and forward using addition and subtract operations. And it is mainly used in one dimension ($D = 1$)
- 2- Cartesian Coordinate system, which have two axis X and Y, which gave us a way to add two more operations (multiplications and divisions), so we can represent the operations for $(+,-,\cdot, /)$. Which is mainly used in two dimensions ($D = 2$)

Then we moved to third dimension ($D = 3$), so used another extra axis Z, and to this gave us the pliability to represent more complex operations and visualize more complex shapes. Which is called spherical coordinate systems, which has three parameters one for each axis, represented by (r, θ, ϕ) , where r is the distance from the origin $(0,0,0)$.

And this extension in dimensionality can continue to N-Dimension, and because of the complexity that comes with the increasing in dimensionality because we will need N parameter. Then came the concept of Euclidean Space, to reduce the complexity again and reduce it to 2-D again using the complex plane to represent more complex dimensions in 2-D plane and do not lose the dimensionally as well.

Complex plane has two axis X and Y, but they are totally different from Cartesian coordinate systems X axis and Y axis.

Complex plane X, Y axis are used to represent more complex dimensions but flatten in 2-D. so basically, we use Y axis for folding Dimensions. Every time the dimension increased by 1, we use Y axis to fold the dimensions to reduce our dimensions again to 2-D which our complex plane.

And we still going to use complex plane X axis to get our operations Zeros, as we do in the Number Line System.

In complex plane we use imaginary unit number $[i]$ which it is an imaginary unit in the folded higher dimension. And in our 2-D plane (complex plane) we only see the current folded 2-D from this higher dimension. And because we have two axis (X, Y) we need two parameters as well, these two parameters are (r, θ) , where r is the vector length from the origin $(0,0)$ and θ is the angle for this Vector on 2-D (complex plane).

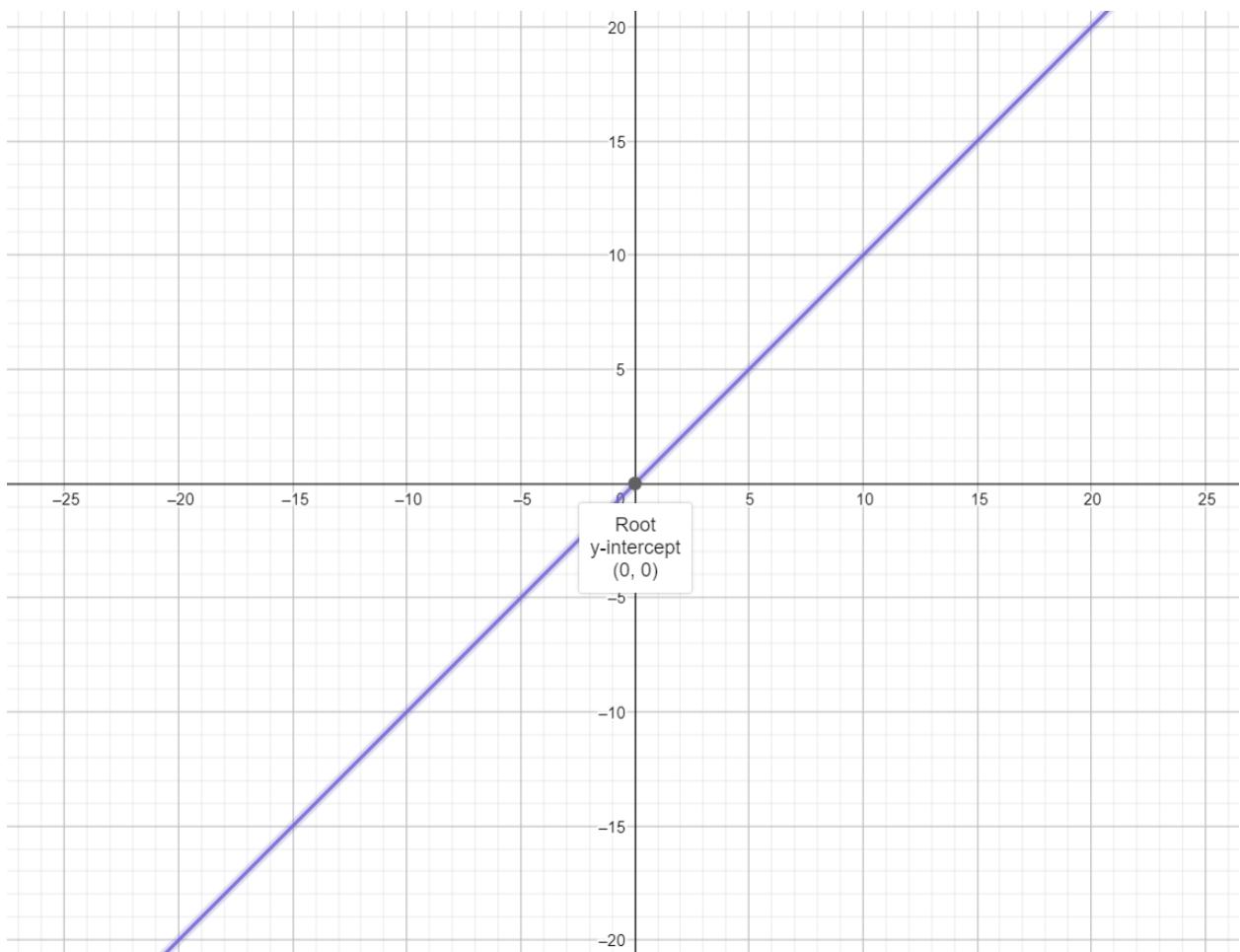
B) Operations in Complex Plane

Based on the introduction complex plane folding, this means that in complex plane we can represent 1-D without any issue in shapes because it is less dimension than the complex plane itself.

Figure (1): we can represent $f(X) = X$ without any issue.



$$Y(x) = x$$



What about if we go one dimension higher to 2-D.

To represent $Y = X$ as 2-D in complex plane. Like the cartesian plane we used same operations (+, -, *, /).

And in complex plane we have imaginary unit [i] so we will use ($Y = i X$) as multiplication operations

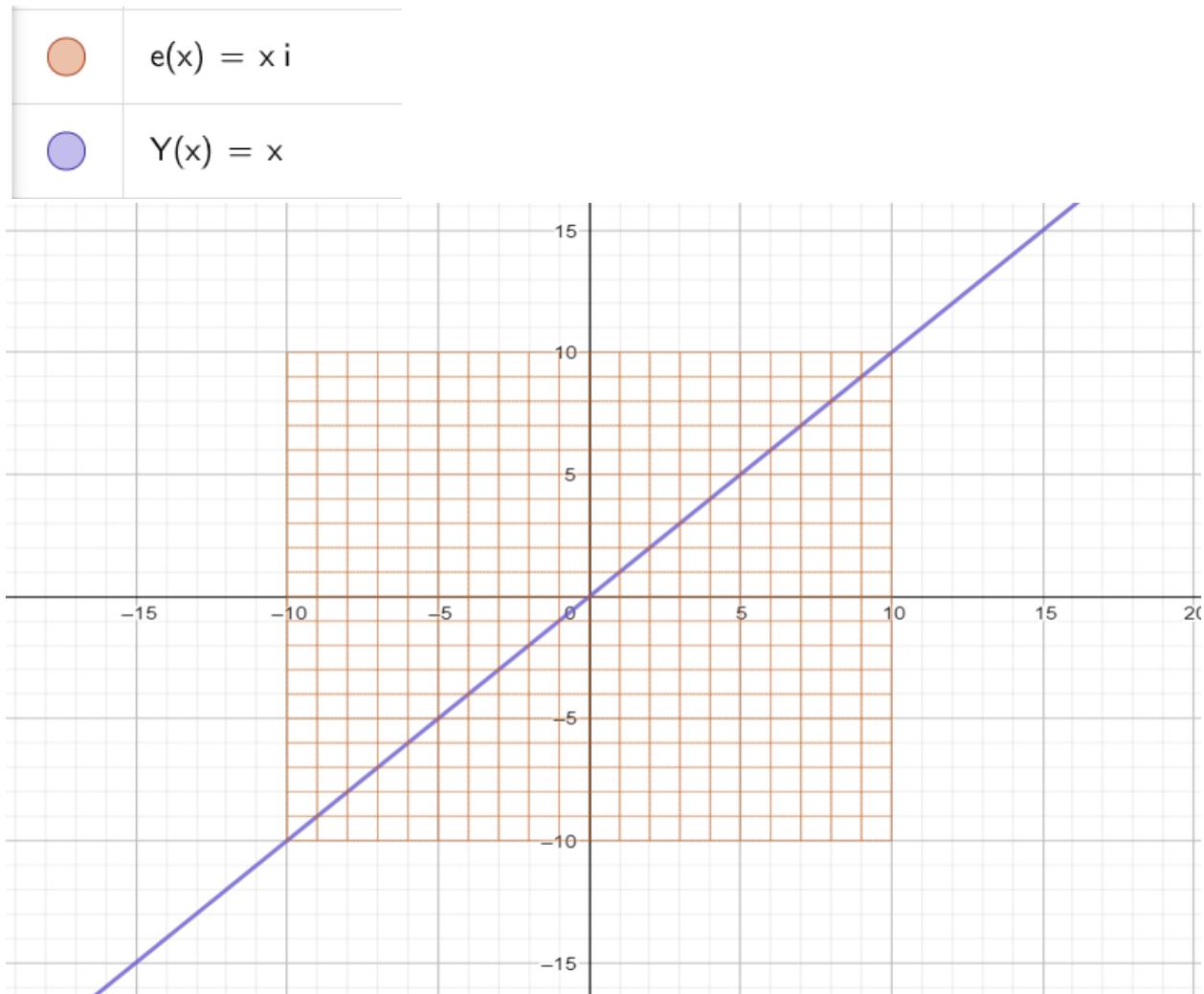
Or ($Y = i + X$) as addition operations. We get square because it is 2-D, and the size of the square is 10 by 10 because our number system is base 10 number system.

We get symmetric square over both X and Y axis. Intersects Y at (10, -10) and intersect X at (10, -10). And square have 400-unit squares. With 420 interstation points. 20 of them will be on X axis itself which are our Zeros.

And this symmetry will allow us to only study one quarter of this square and for sure with some operations

we can mirror this quarter to the rest of the plane with the same simple operations (+, -, *, /).

Figure (2): multiplication operations; multiply X by imaginary unit to upgrade from 1-D to 2-D.



To elaborate more on the operations on the complex plane. We will see the addition operations on complex plane

It is basically shifted the square up by one imaginary unit, now the square intersects Y axis at (-9, 11) and X axis still at (10, -10). But now $Y = X$ is not our square diagonal.

To realign our unit square back to $Y = X$ as diagonal we need to shift X by 1 to align with the new Y intersect at (-9,11).

Figure (3): Shift only in Y direction ($Y = X + i$).

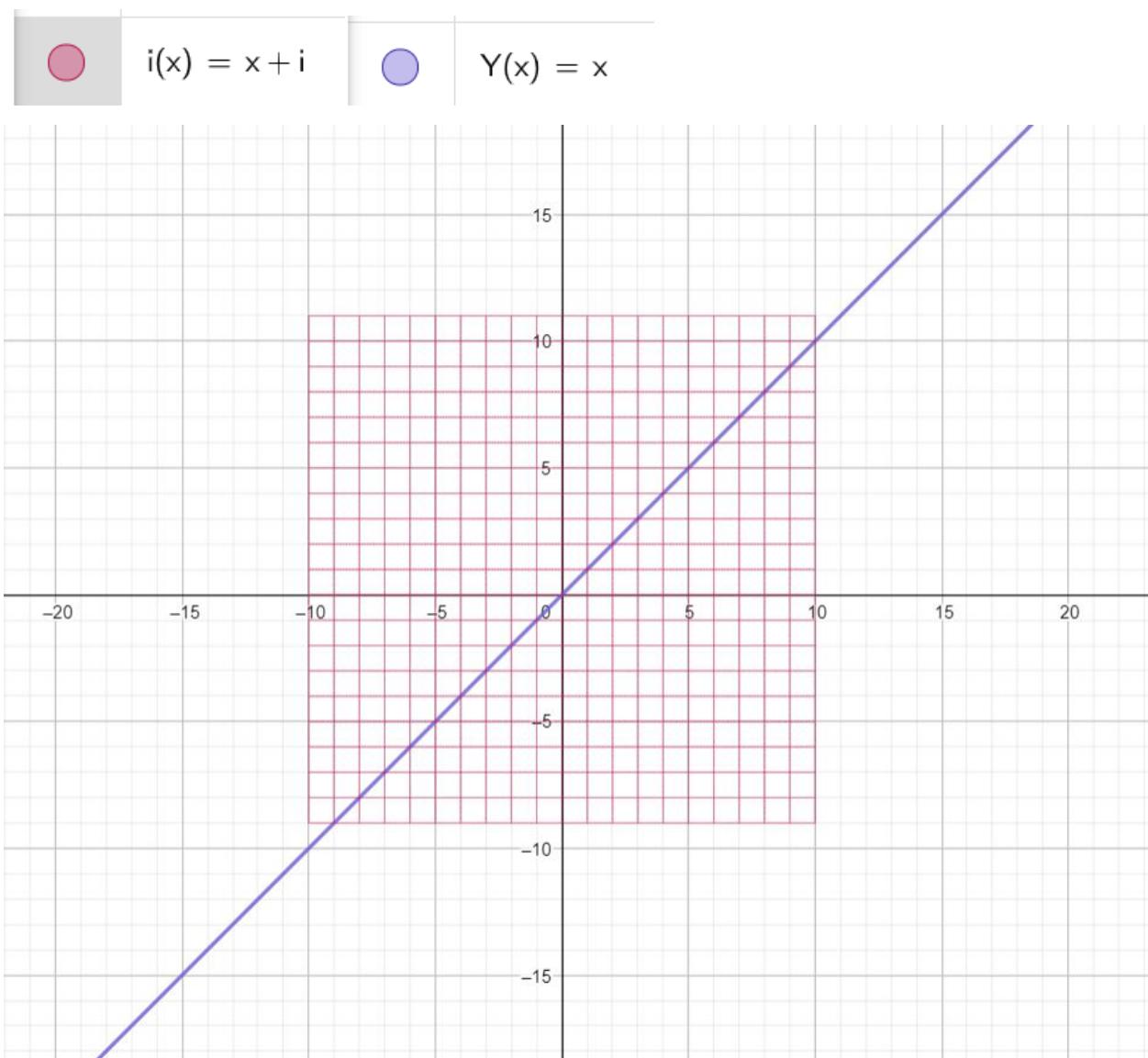
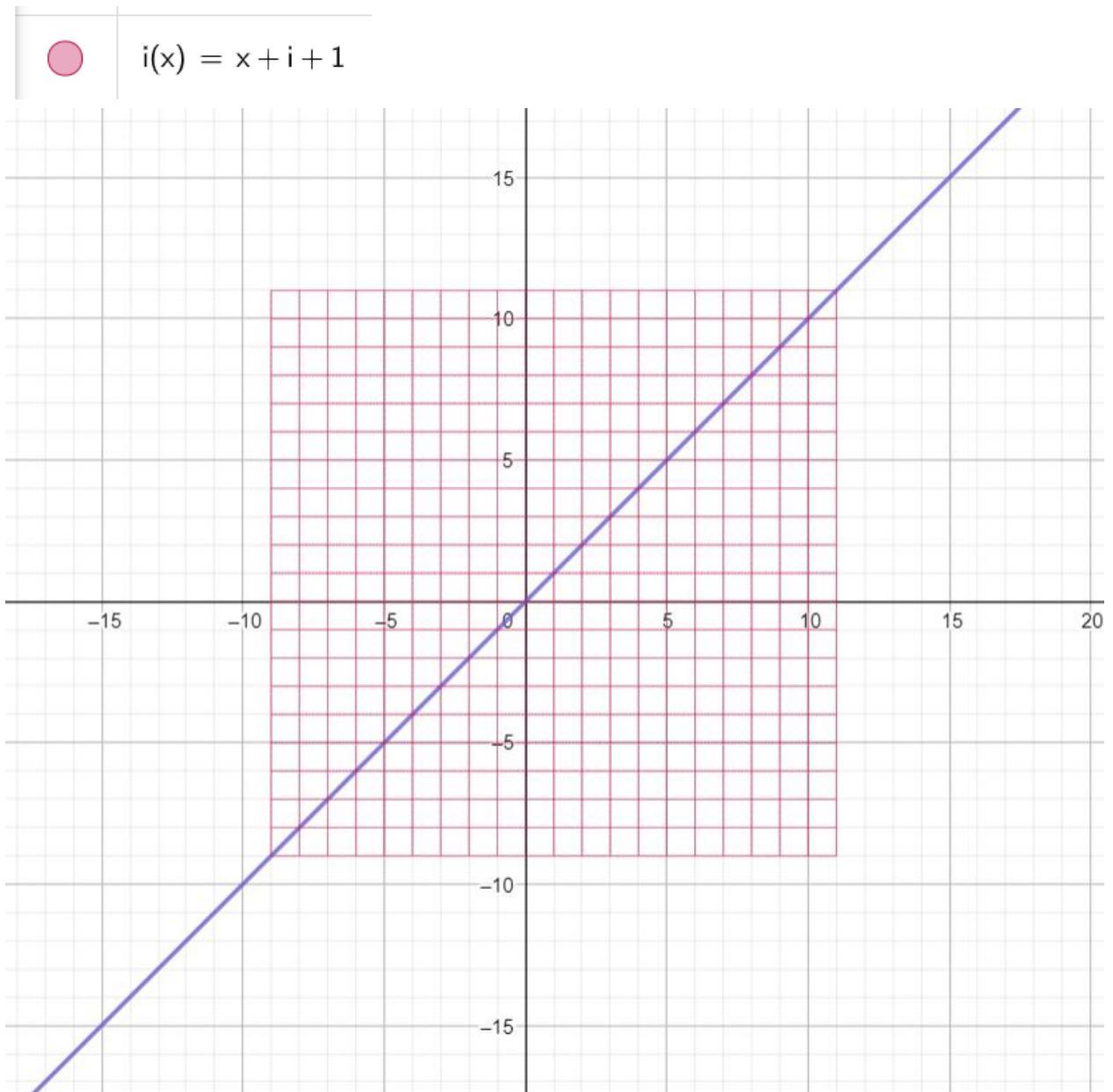


Figure (4): Shift in Y direction ($Y = X + i$) and in X direction ($Y = X + 1 + i$).

As a result, $Y=X$ is again at square diagonal, but our square now intersects Y axis at (-9,11) and intersects X axis at (-9,11).



We are going to use the addition operations only because it did not change the X axis or did not shift in it. So, we are going to use this transformation ($Y = X + i$). but we still have will have the issue of the shifted square diagonal one unit from $Y = X$. square intersects Y axis at (-9,11) and intersects X axis at (-10, 10).

Figure (5): shows upgrade $Y = X$ to 2-D using ($g(x) = X + i$) and how the square diagonal is shifted by one.



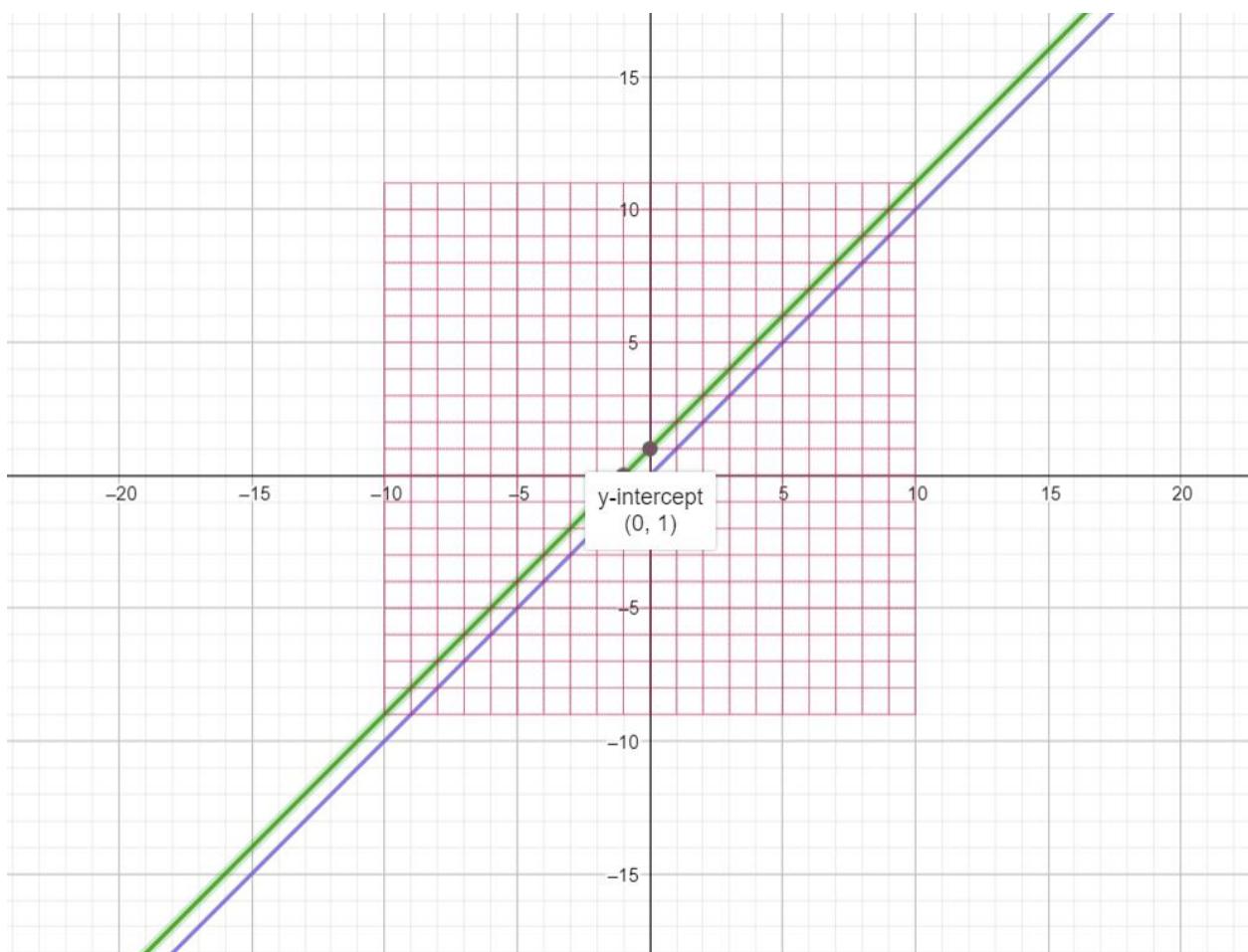
$$i(x) = x + i$$



$$Y(x) = x$$



$$g_1(x) = x + 1$$



As we now agreed on what operations we are going to use to upgrade from 1-D to 2-D and what issues we need to take care of in our moving between dimensions while we are folding and unfolding dimensions in complex plane.

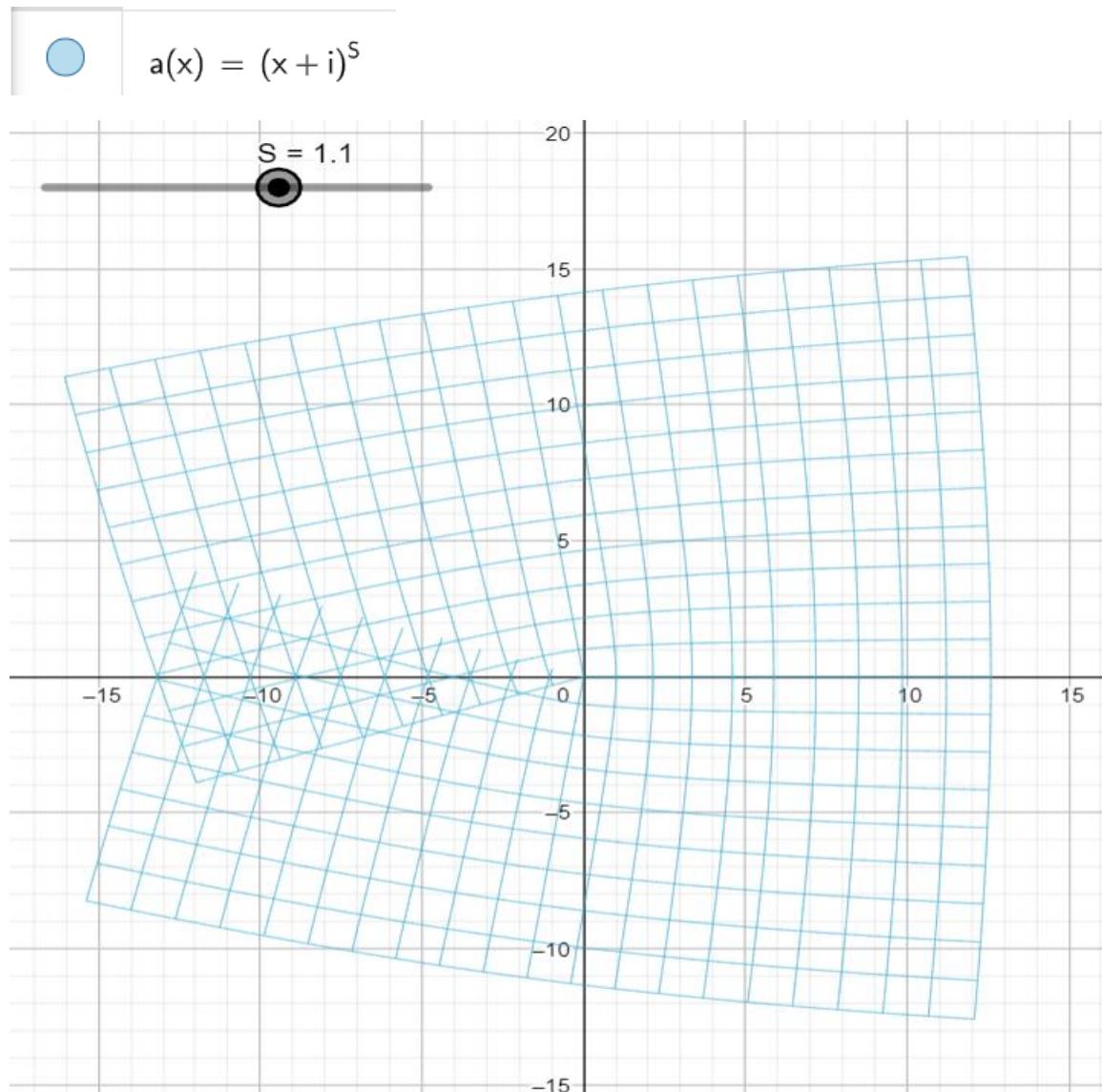
C) Manifolding in complex Plane

As we explained in the introduction for complex plane, Y axis is used for folding the higher dimensions to reduce it back to 2-D, the dimensionality of complex plane.

So, we are going to introduce one parameter in our complex plane to represent the folding for this 2-D square in complex plane. Which is basically we are doing increase in dimensionality from 2-D into higher dimension.

Our new parameter S will be the folding parameter or in other words will be the dimensionality variable in our complex plane.

Figure (6): folding in complex plane.



$[S]$ is our folding parameter takes any real values +ev or -ev. Increase in S means going up in dimensionality and means folding in complex plane to be reduced to 2-D. decrease in S means unfolding in complex plane.

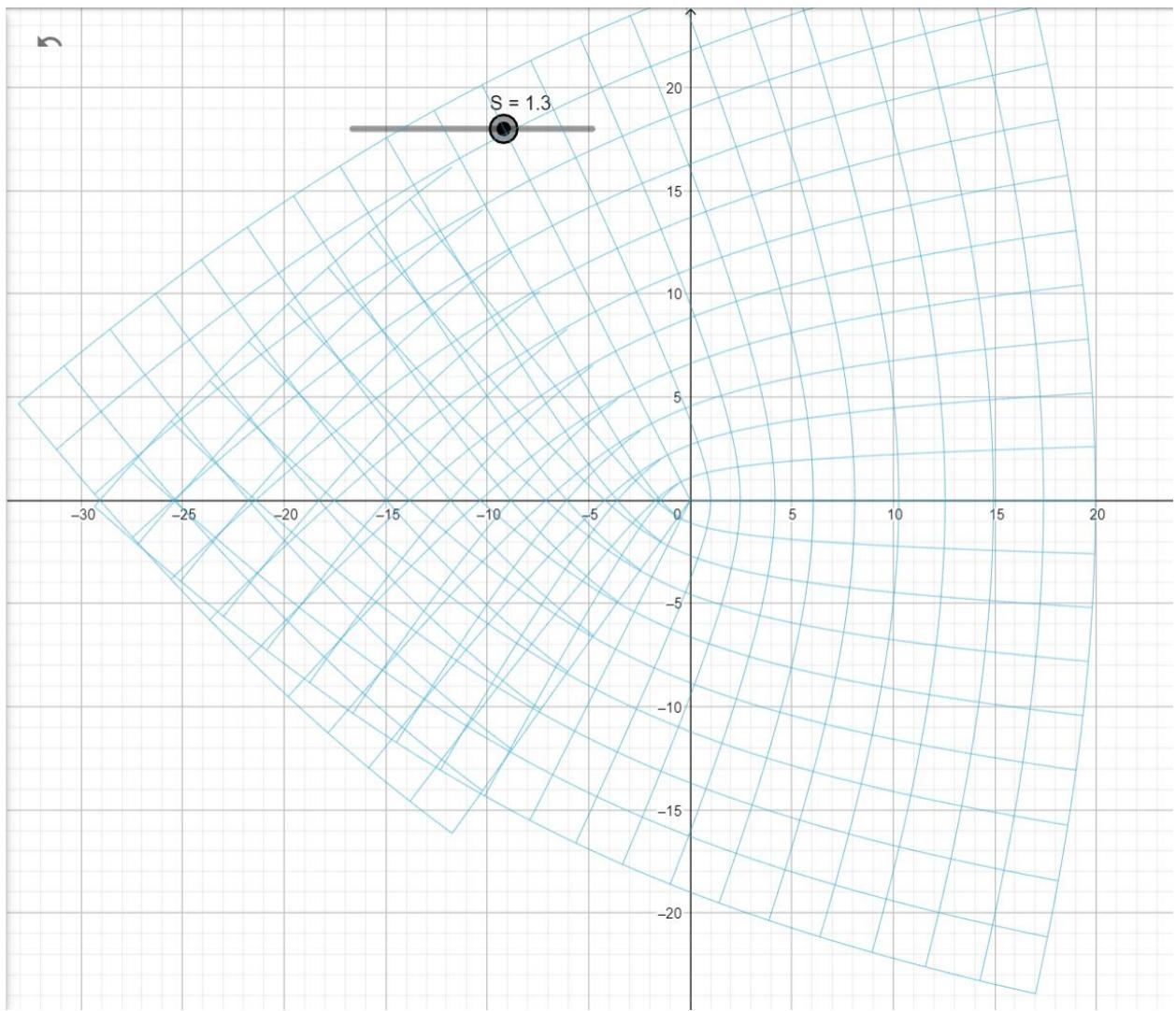
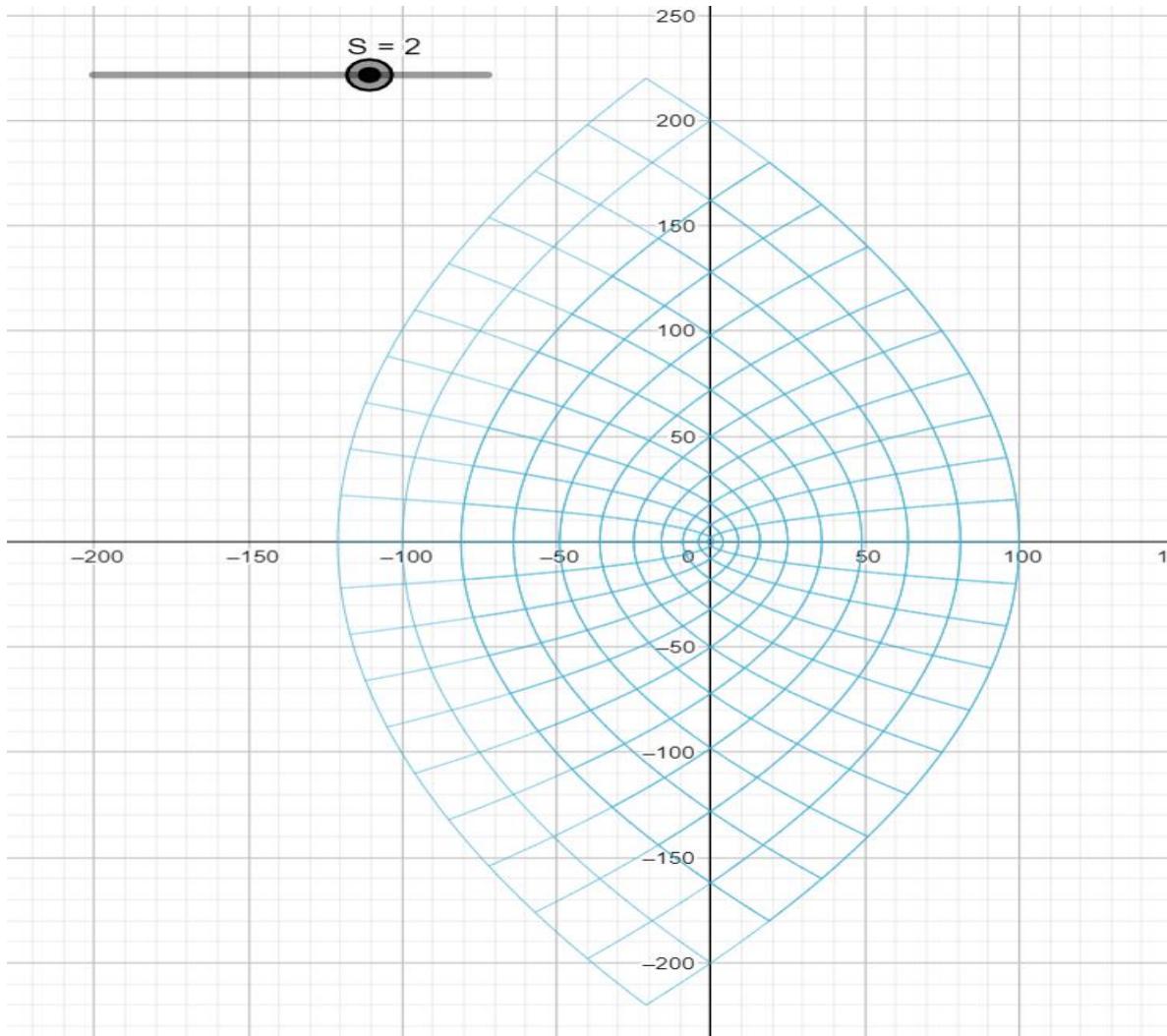


Figure (7): increase dimensionality by increasing [S = 2] and folding our frame of reference from higher dimension back to 2-D in complex plane.

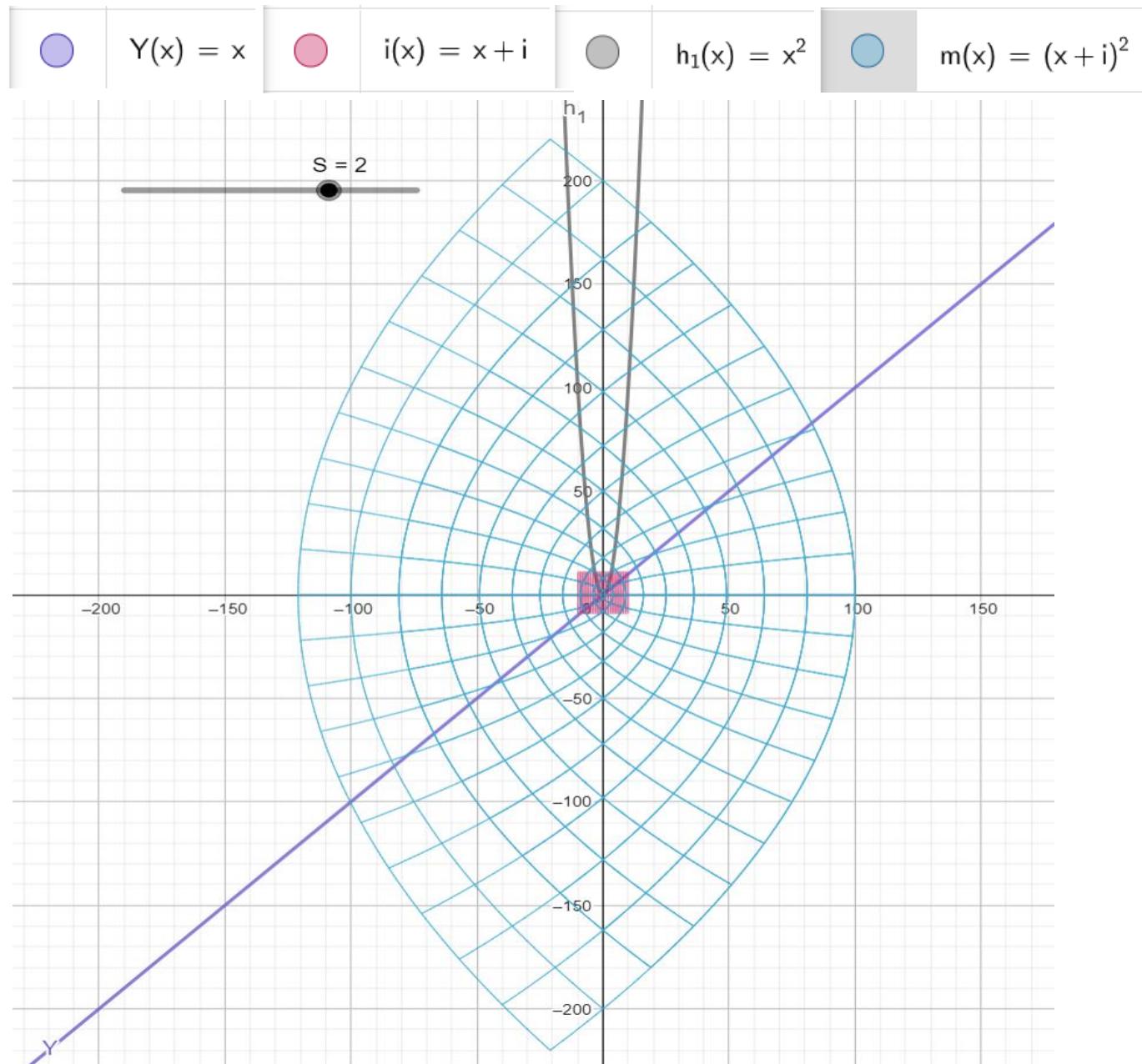
	S = 2	
	-5 5	
	$a(x) = (x + i)^s$	



As we see in Figure (8) we still have the shift issue we faced for the square diagonal in in 2-D. because Y-axis is not on the tip point of the folded shape.

And as this is for value $S = 2$ then this means in one axis it is equivalent to $\textcolor{brown}{Y} = \textcolor{teal}{X}^2$

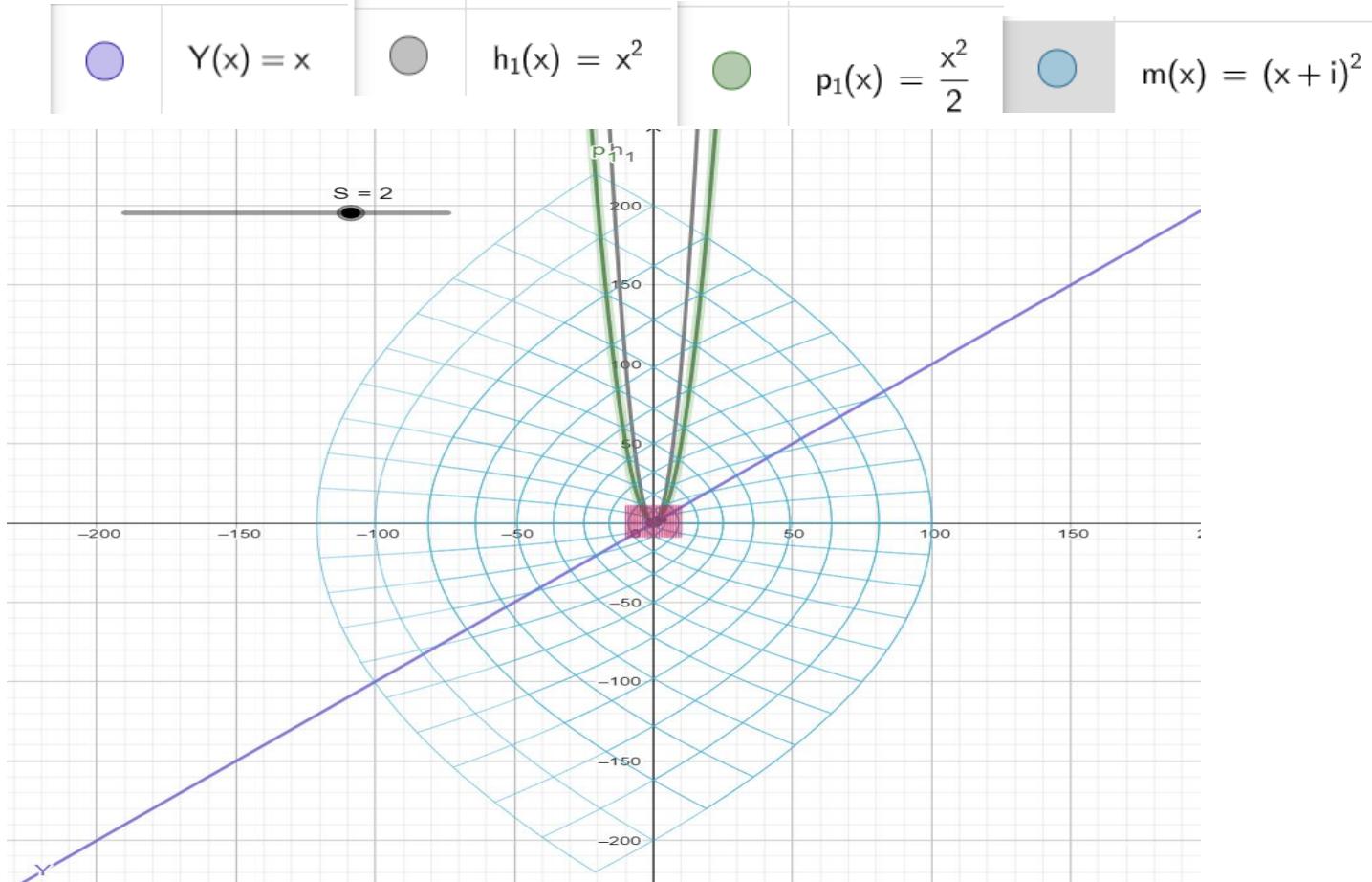
Figure (8): include both versions 1-D shapes and 2-D shapes for our simple function $Y = X$ and its transformation in complex plane.

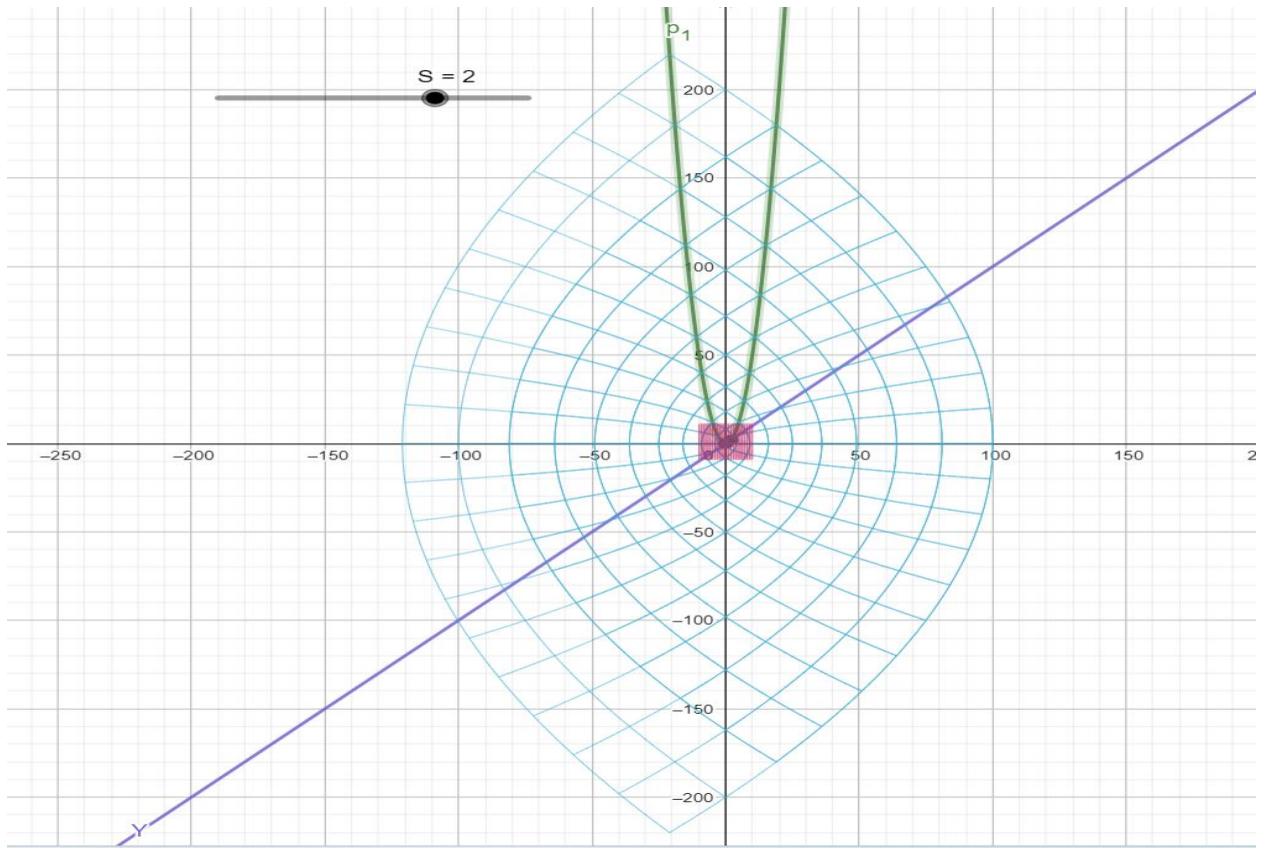


Ok looks like $Y = X^2$ is still have the same shift issue that was with $Y = X$ when we raised it from 1-D to 2-D it is will not shifted one unit from the tip of the folded shape. In order to make the $Y = X$ which will make

our transformation one-to-one. Is to fold $Y = X$ in higher dimension with this transformation $\textcolor{blue}{Y} = \frac{x^2}{2}$

Figure (8): shows the correct transformation to keep one-to-one relation in higher dimension after doing one fold extra. To keep $Y=X$ at the tip of the transformed shape, Which fixes the one unit shift issue we explained before for our square 2-D diagonal.





Next, we are going to move on to explore more higher folding (higher dimensions) in complex plane. And what transformation will be needed to keep the one-to-one transformation between dimensions.

Figure (9): increase in [S =3] folding and dimensionality in 2-D. the full view of the folding is in zoom range on 10 to the power of S because we are at base-10 system. So will be in X range values 1000.

In this Figure it is clear also that the same shift unit still exists even in [S=3] the one-to-one function Y=X is one unit square away from the shape tip point in top right corner.

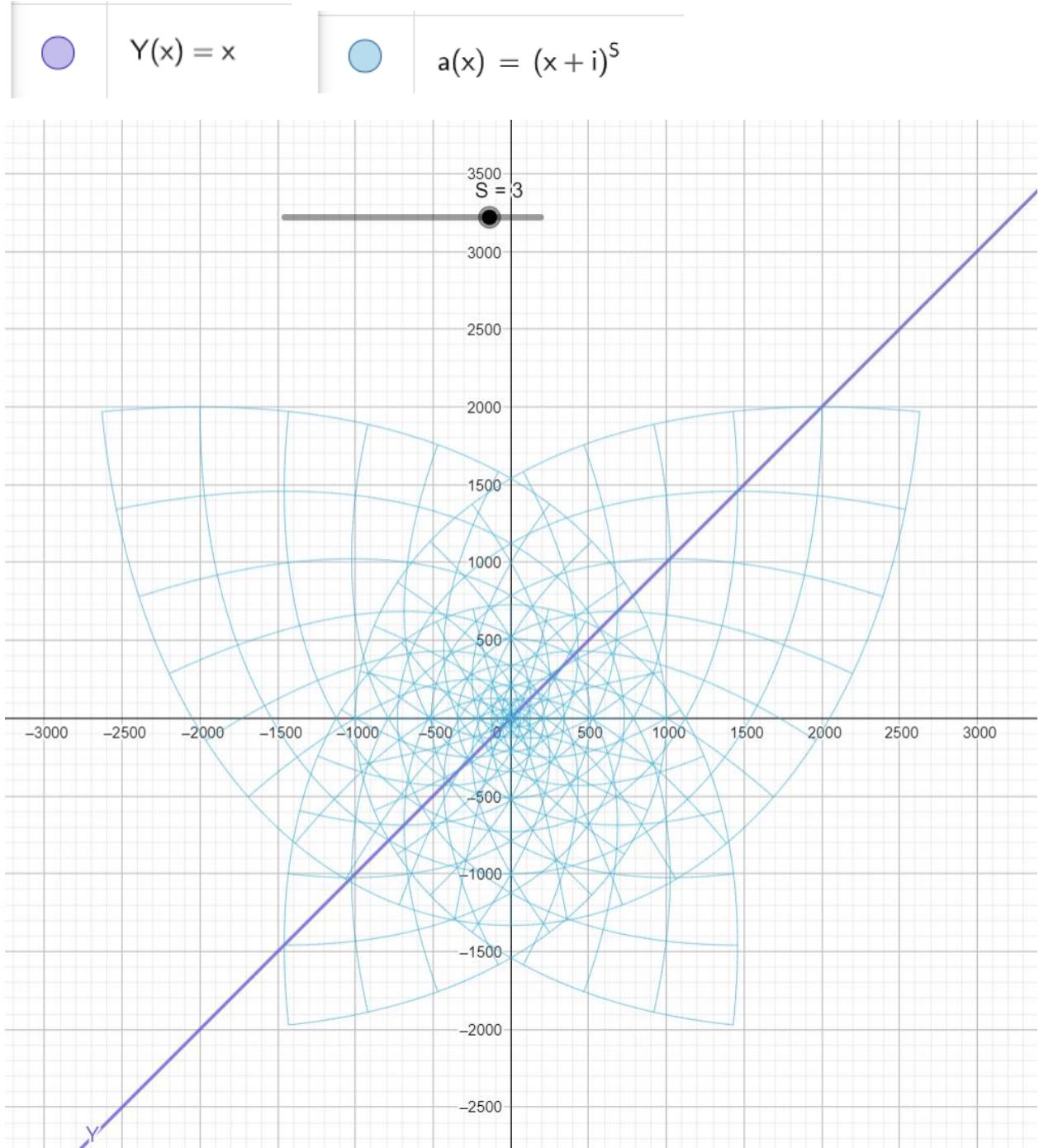
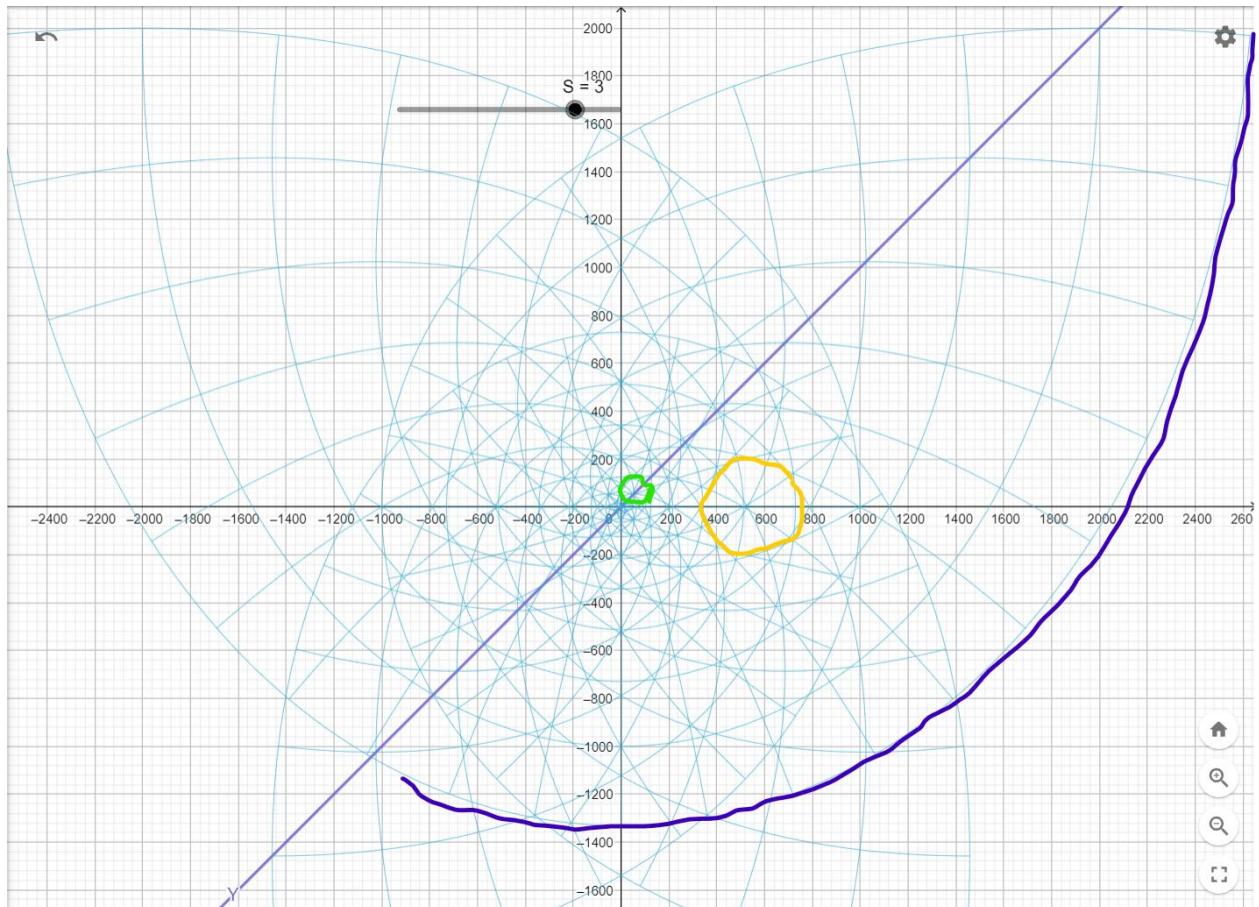


Figure (10): As we zoom in (going towards less values in X) folding the line binding is more aggressive if the line is close to the Y axis and lines away from Y axis are less banded. And we can see some circles starts to emerge due to lines binding and intersections during the folding.



As we said before the zeros will be on X axis all the time for any function Y., so each point intersects with X axis is a Zero for one function if we are keeping the one-to-one relationship in our transformation.

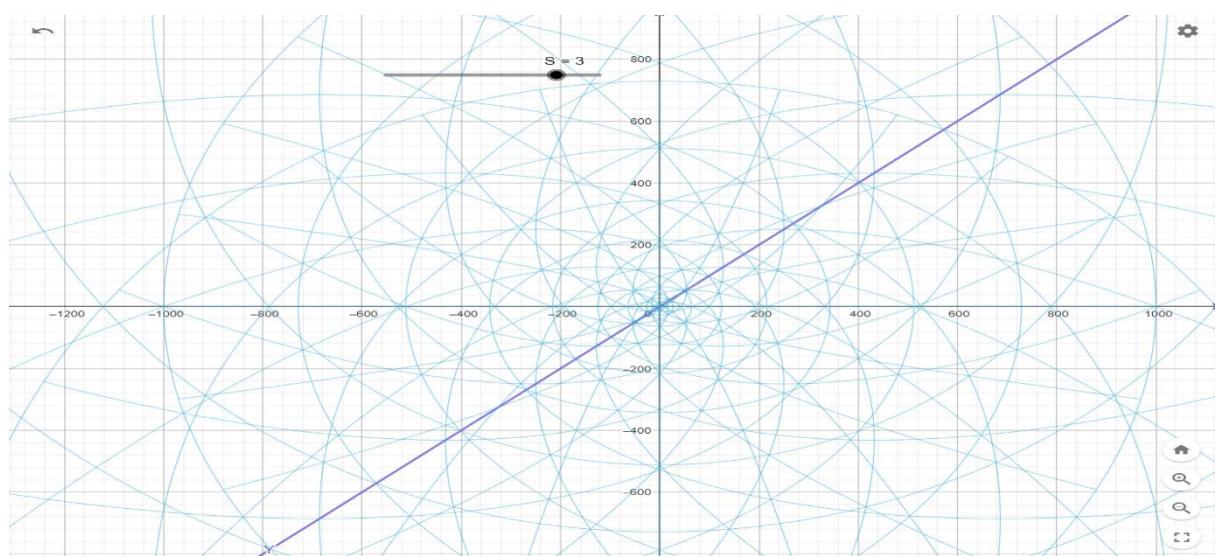


Figure (11): $S = 3$ we will have one circle for each line in our square intersect with X axis at X to the power of S .

So, at $x=1$ we will have one circle intersects with X at $(1, -1)$. At $X=2$ we will have one circle intersects with X axis at $(8, -8)$ and at $X=8$ we will have a circle intersects with X axis at $(512, 512)$ and at $X = 9$ we will have a circle intersects with X axis at $(729, 729)$.

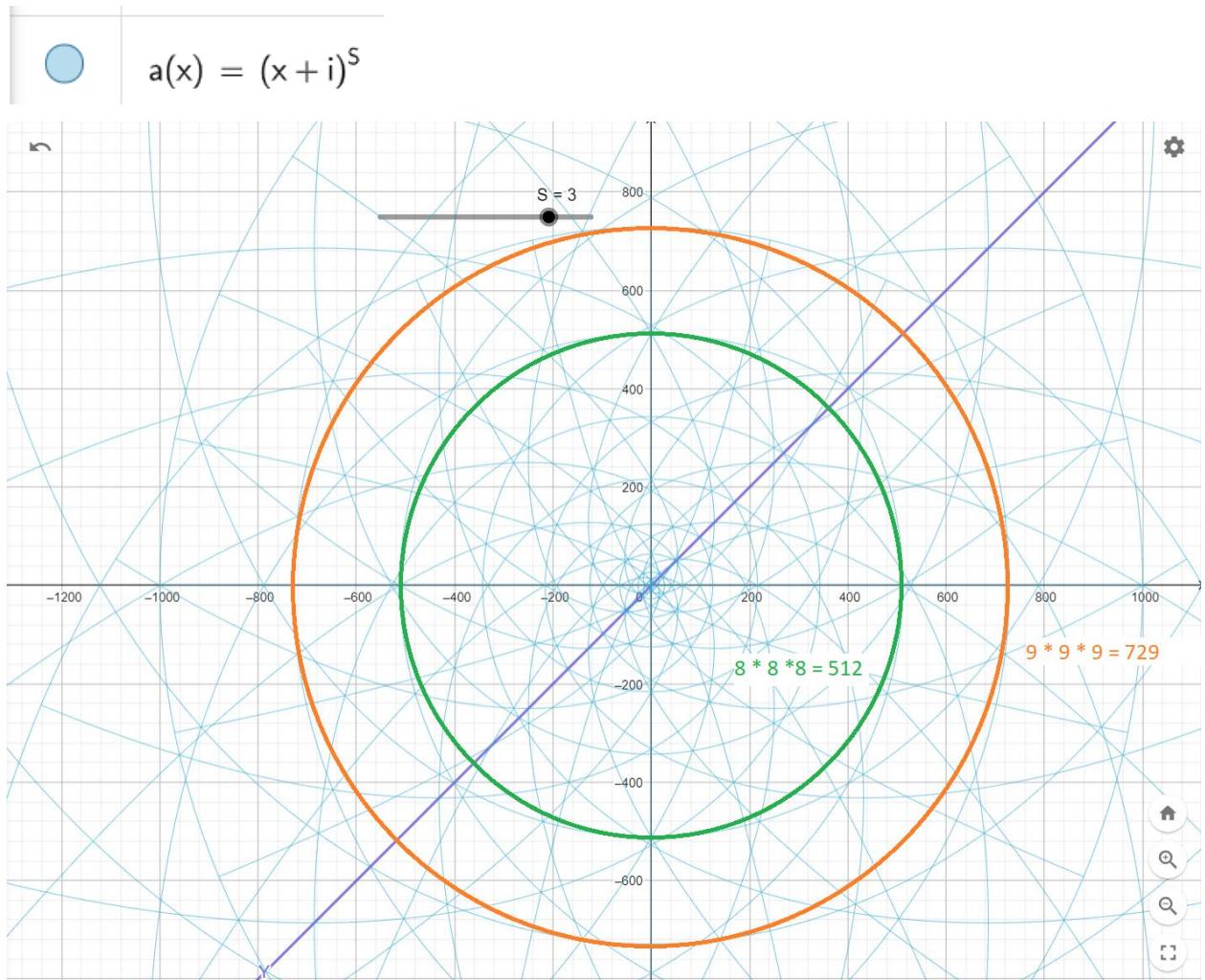


Figure (12): As we go lower and lower in X value i.e., goes towards $X = 0$, we will go to $X = 1$ the first line in our square and because it is the closest one to Y axis it is will be the most bounded line due to folding. And max shape for folding you can get is a circle in 2-D so for sure for any value for S we will have one circle at the center at origin $(0,0)$ this circle is a unit circle for line $X = 1$ and for any folding for any real value $S \geq 3$.

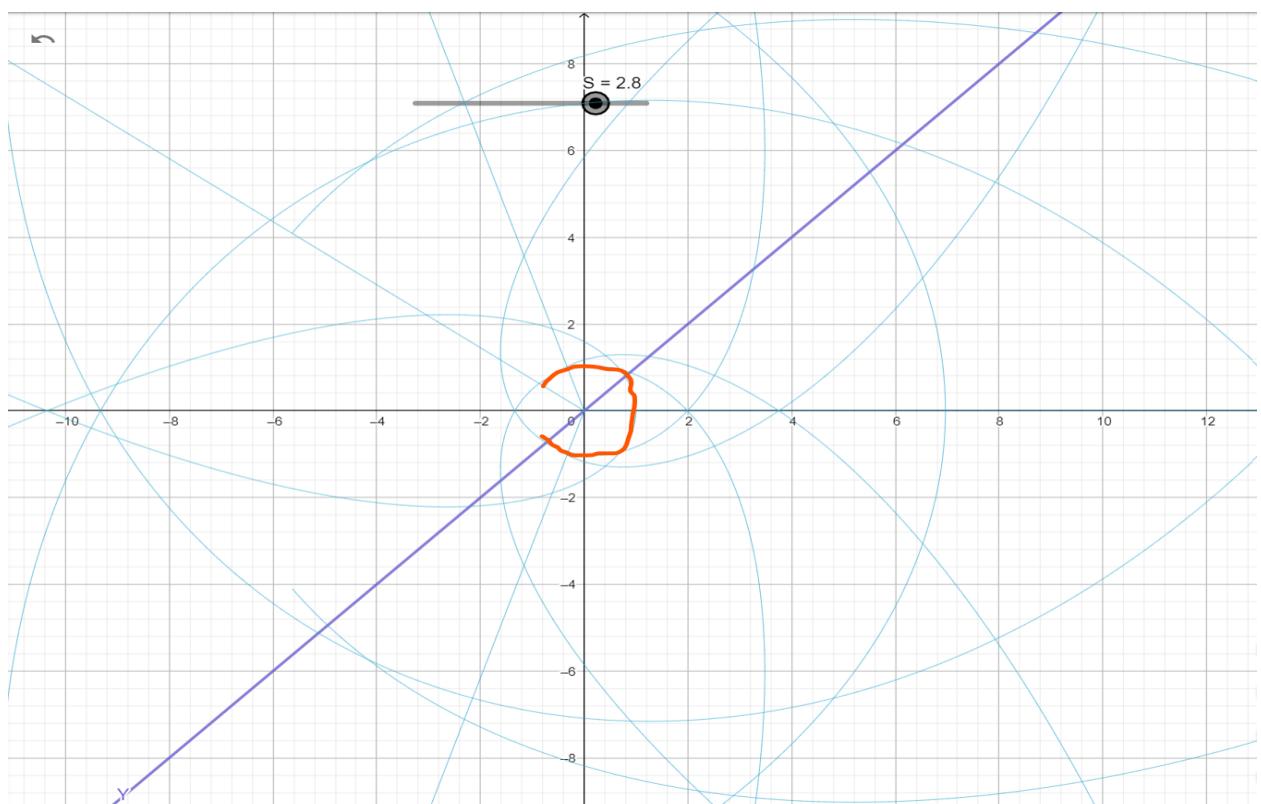
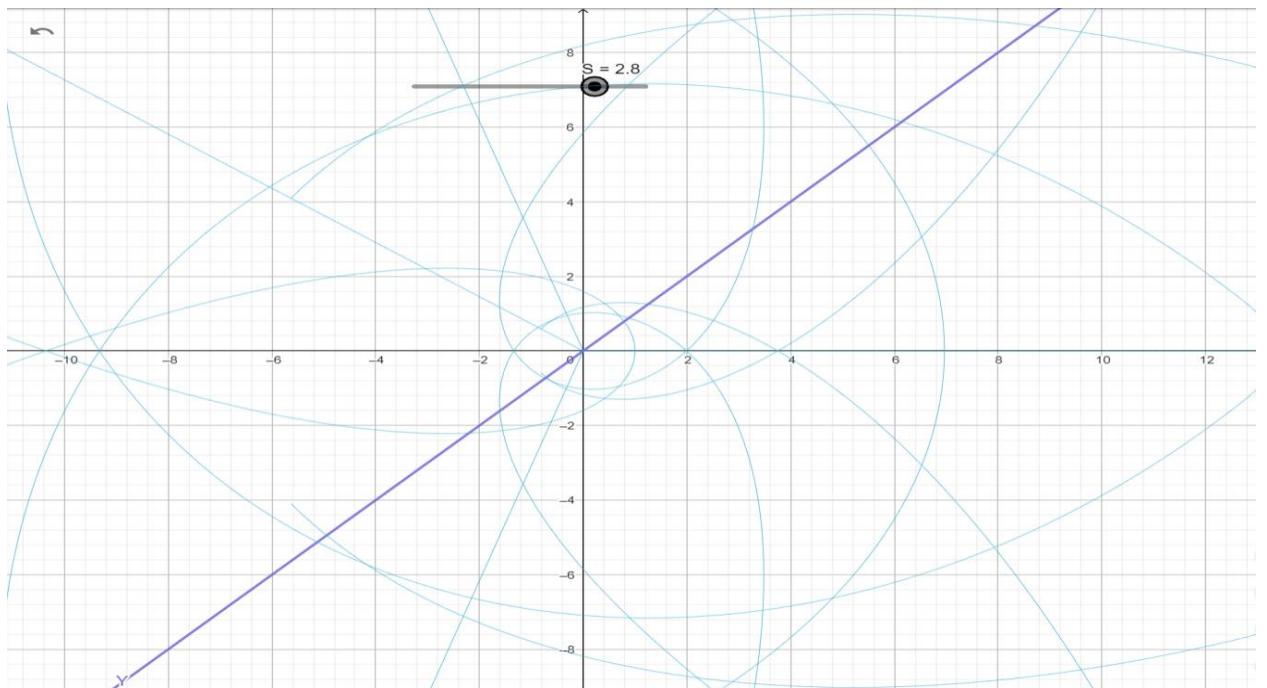
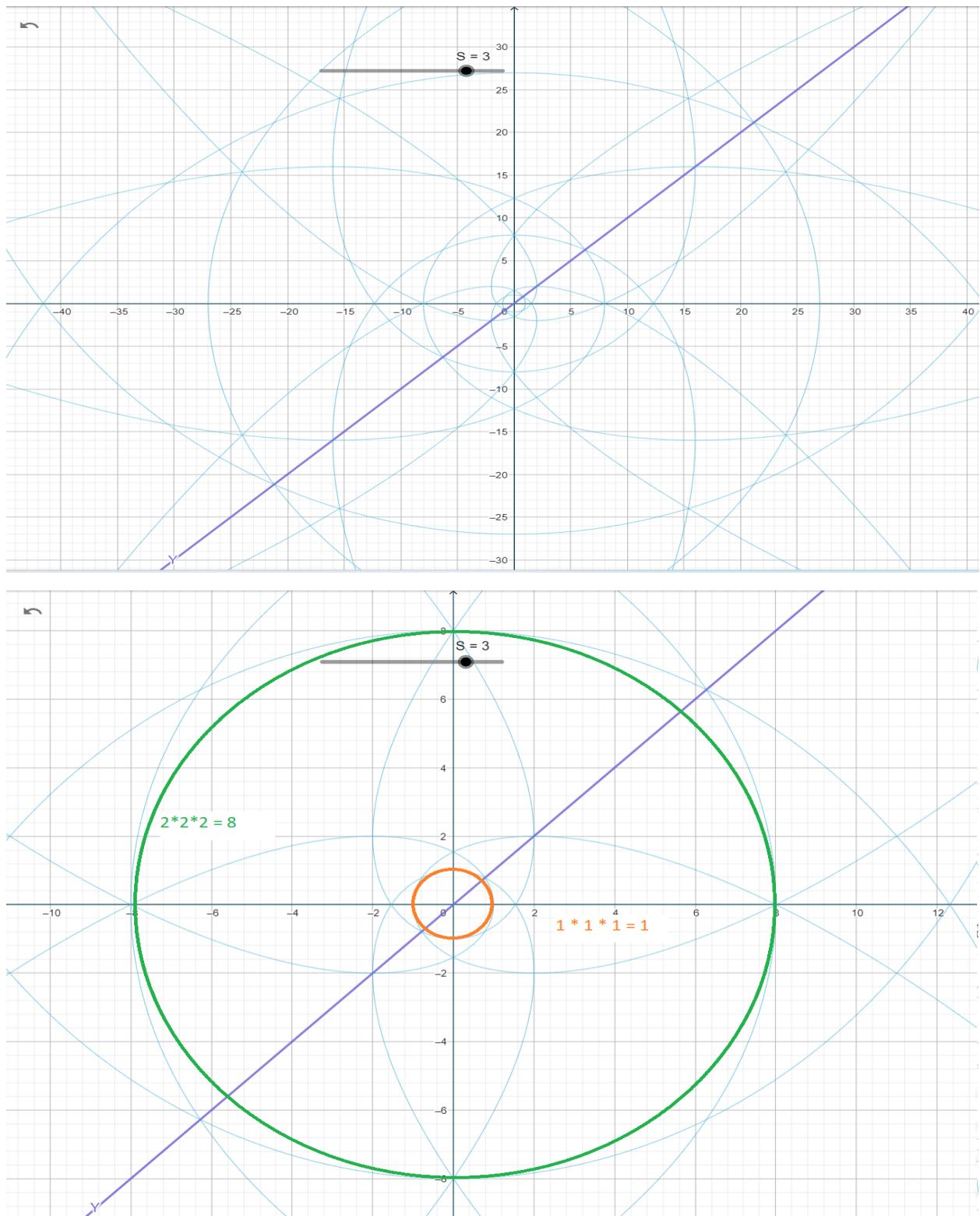


Figure (13): once $S = 3$ all 4 parts of one quarter of square complete a full spine completing the unit circle



of complex plane.

Figure (14): shows the same effect of S but with $1/S$ and how line $X=1$ still banded to intersect with X axis at $X=1$ on the unit circle.

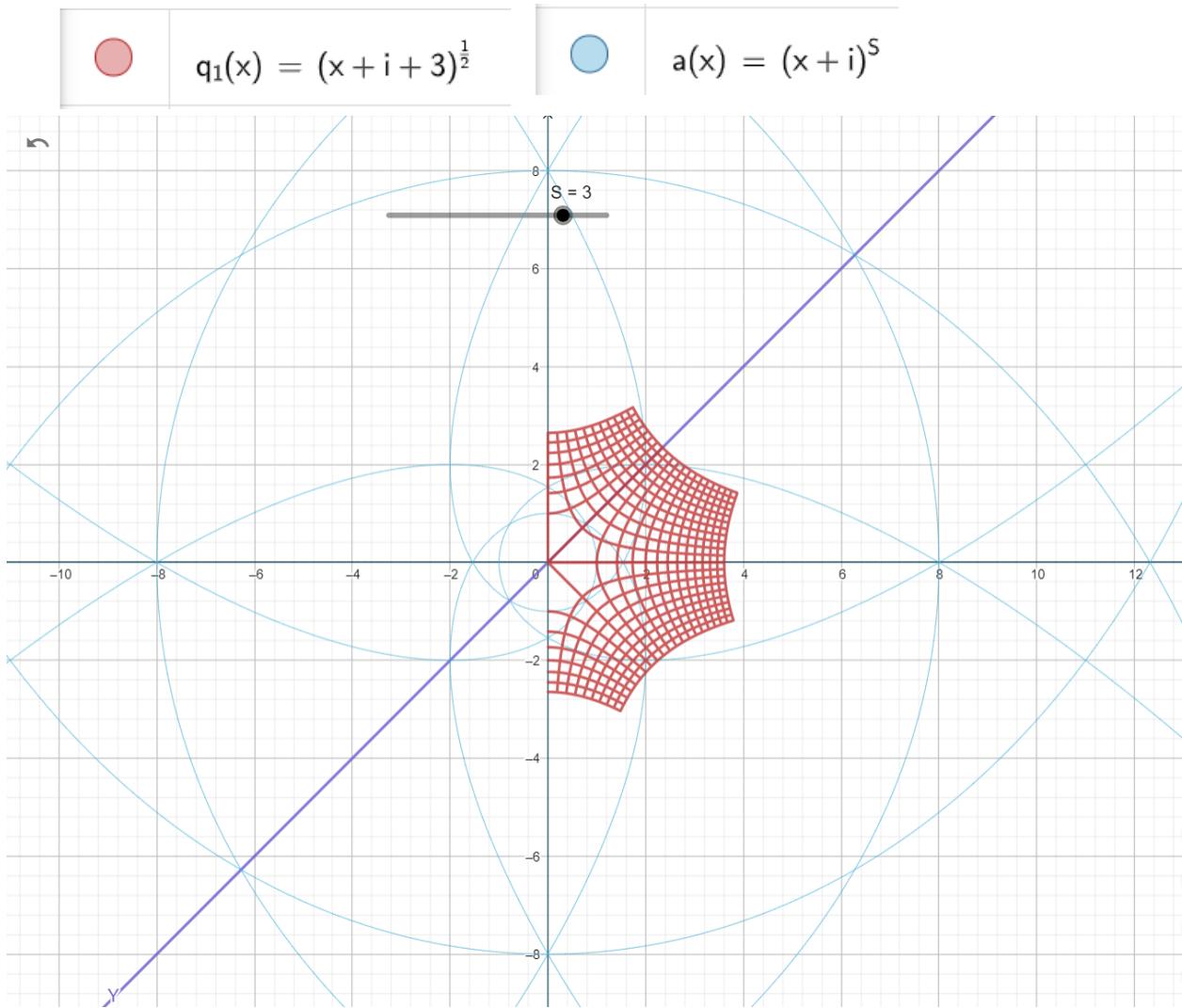


Figure (15): S = 5 still have the center unit circle for line X=1 fully bounded emerging the unit circle with origin (0.0) and radius r = 1.

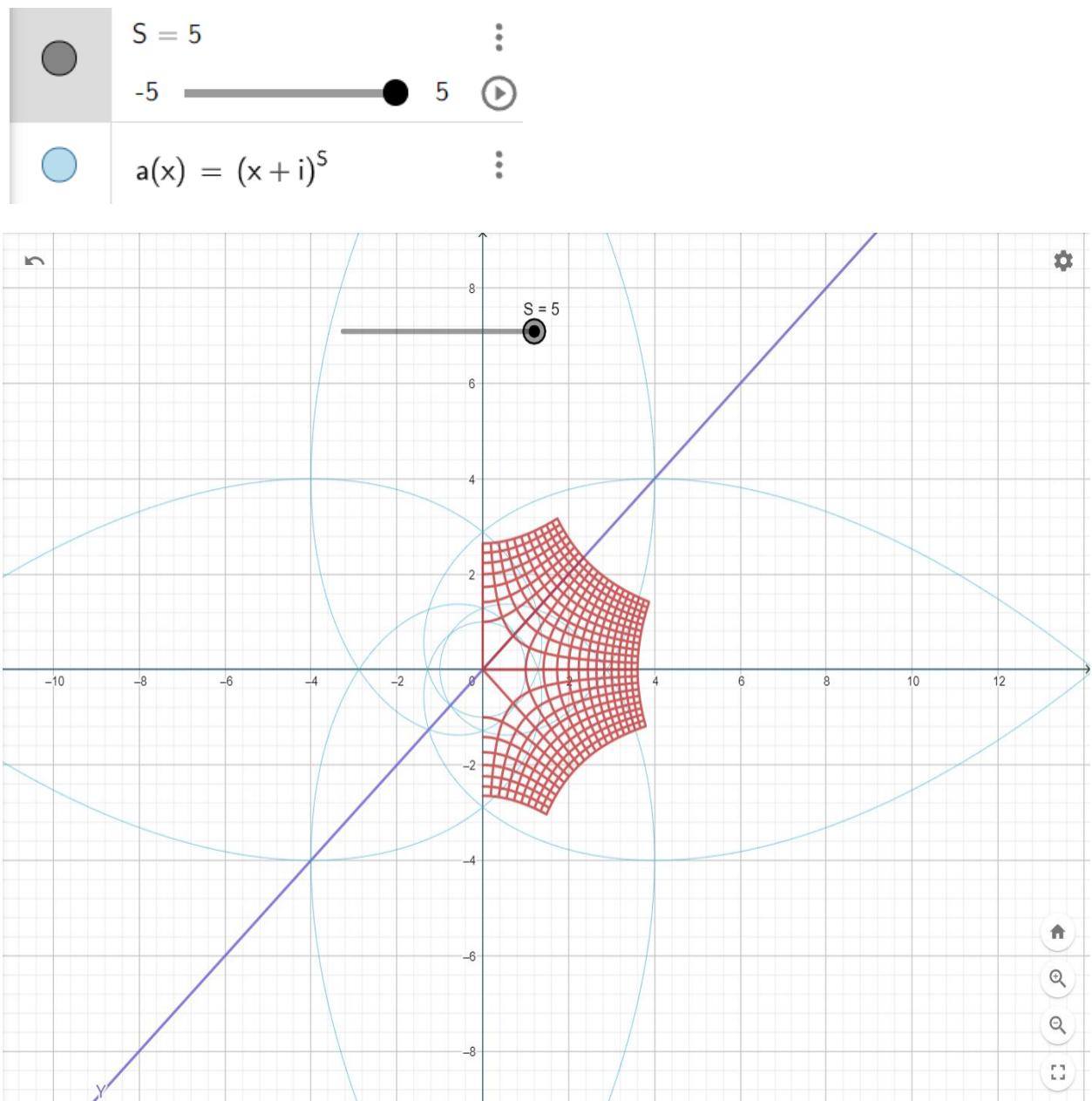
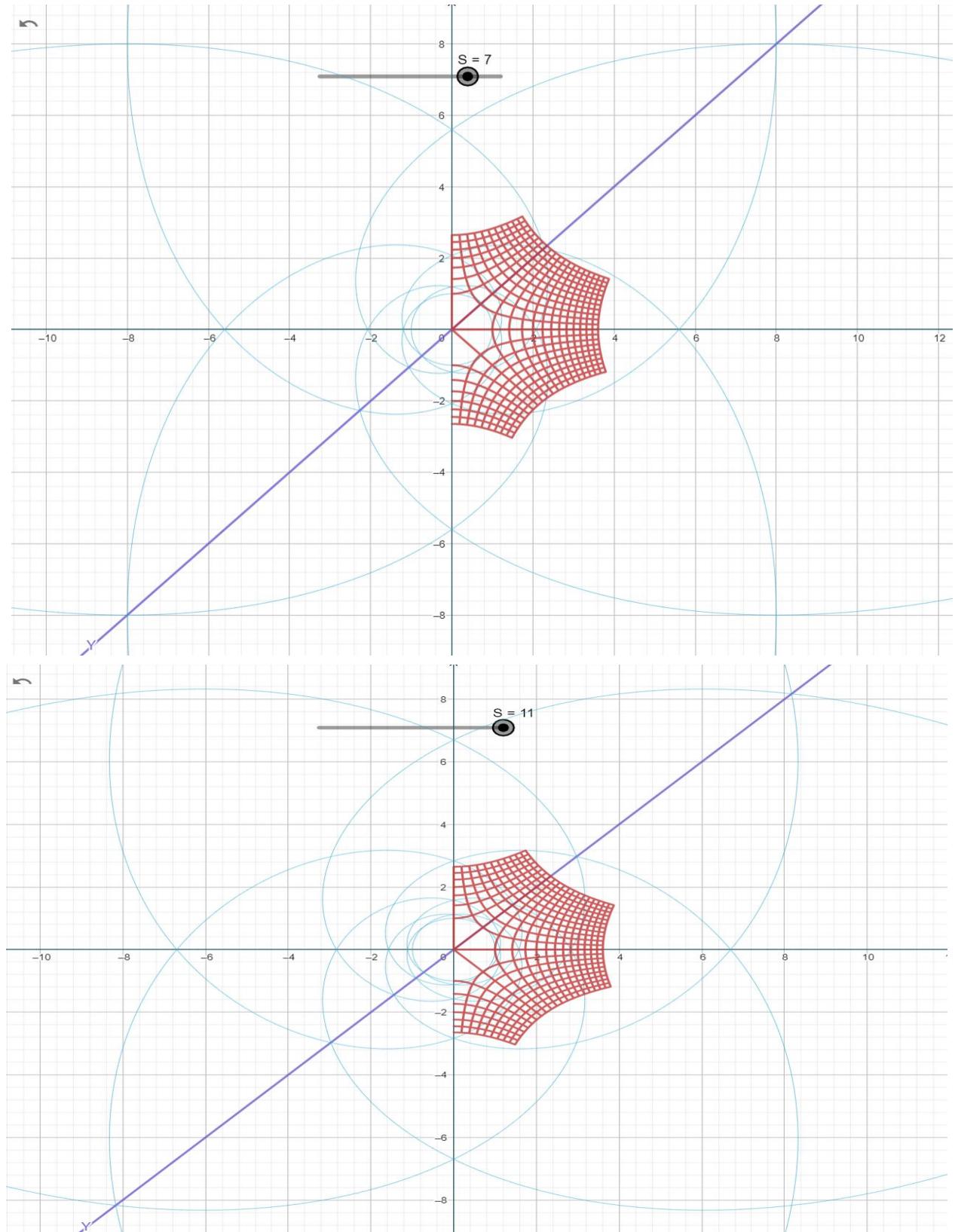
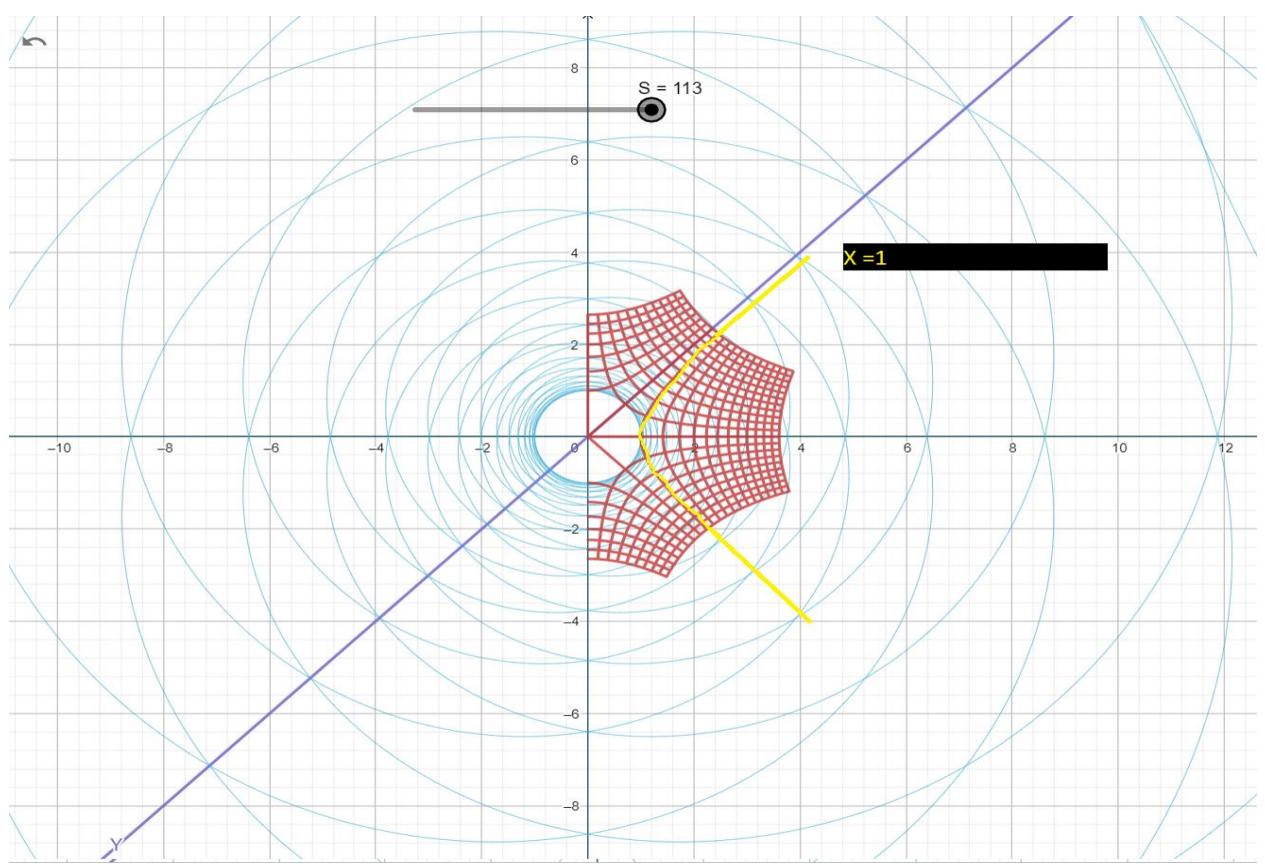
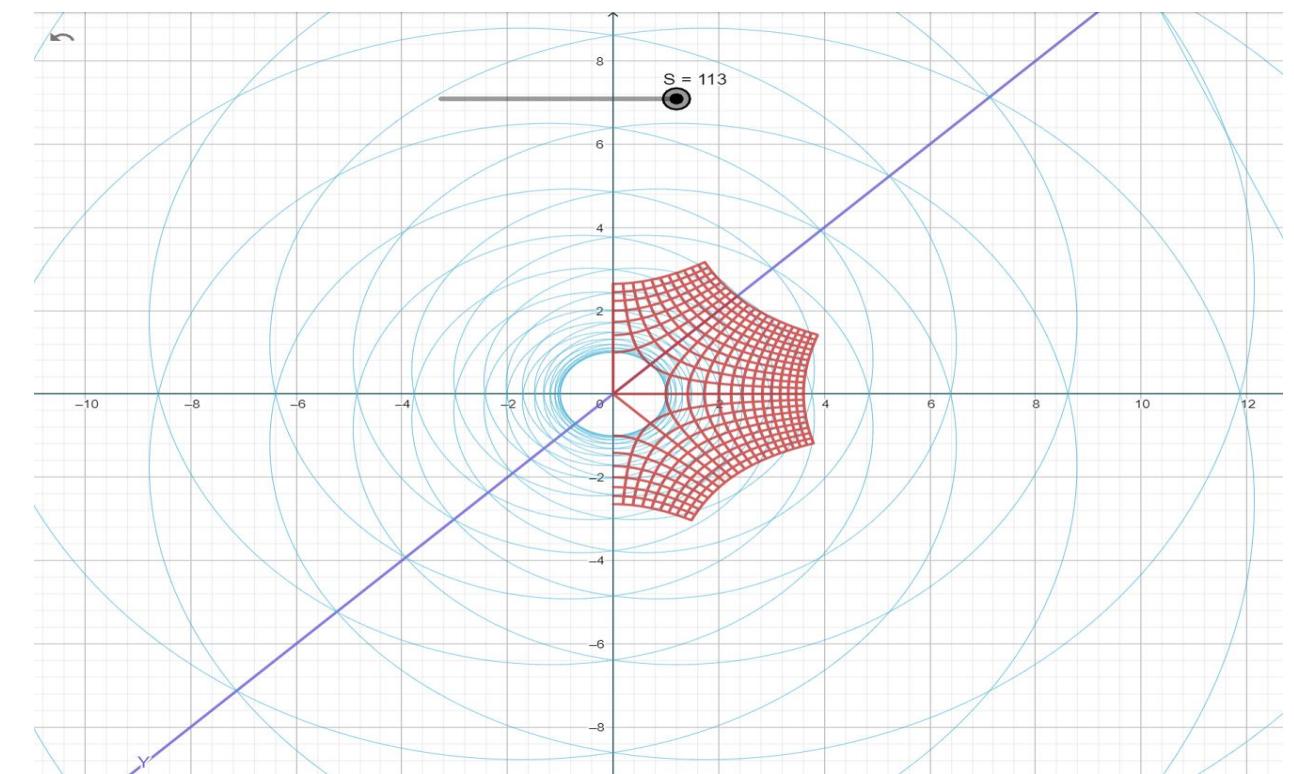


Figure (16): $S = 7$ and $S = 11$ and $S = 113$ all still have the same unit Circle at center with origin $(0,0)$ and radius $= 1$.

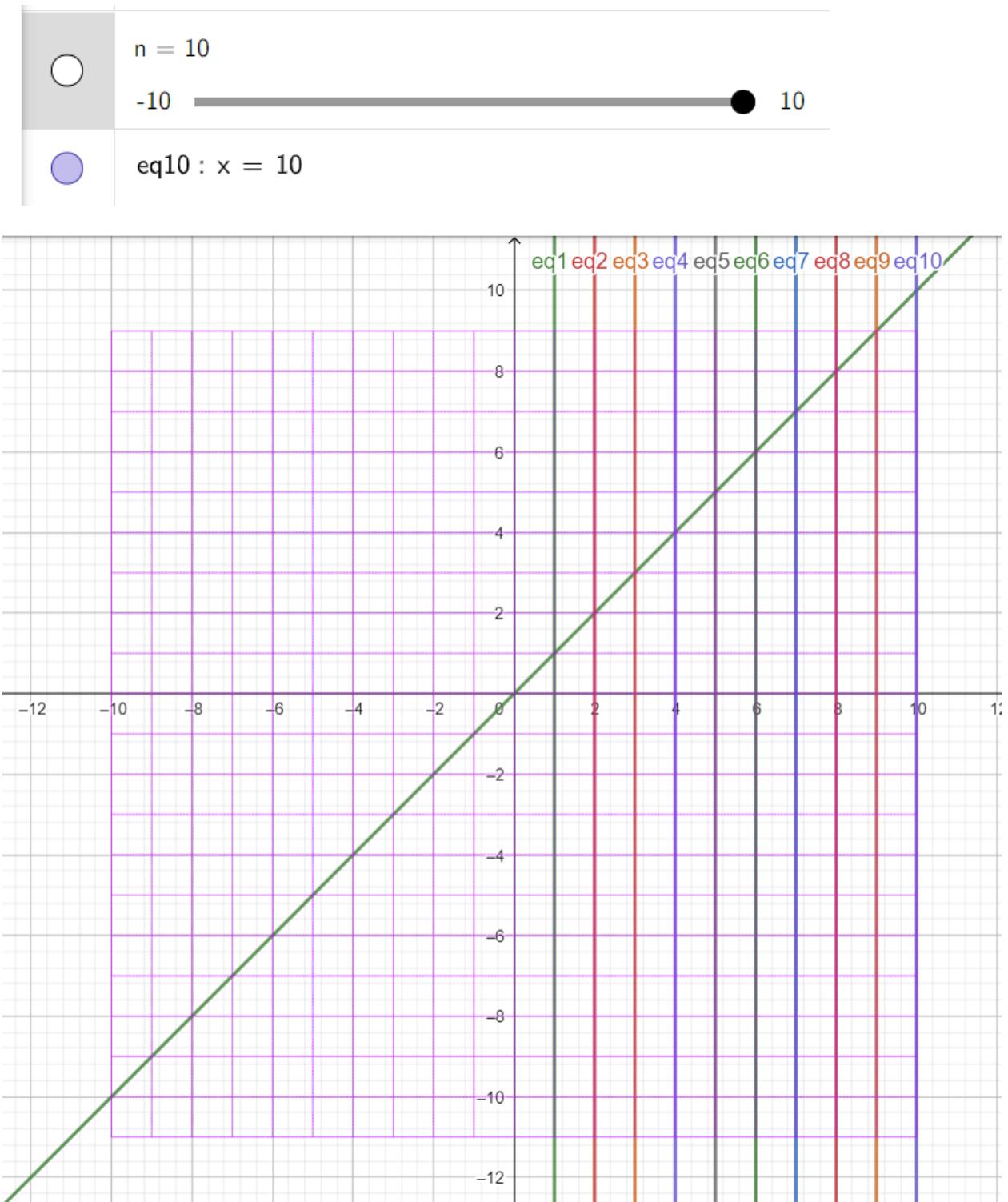




D) Base 10 number System folding in Complex plane

In base 10 we have base natural number set $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Figure (17): base 10 number system have 10 lines ($X=1, X=2, X=3, X=4, X=5, X=6, X=7, X=8, X=9, X=10$)



These 10 lines in base-10 number set for natural numbers set $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ if we do one-fold in complex plane by setting $S = 2$. We basically doing $Y = X^2$
 Each line will be bounded on each fold using this transformation formula

$$Y = 2 * N (X + N^2)^{\frac{1}{S}}$$

Where $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and S any real Number.

Figure (18): shows that this transformation keeps the bounded lines in one-to-one relations even in higher dimension as [$S=2$].

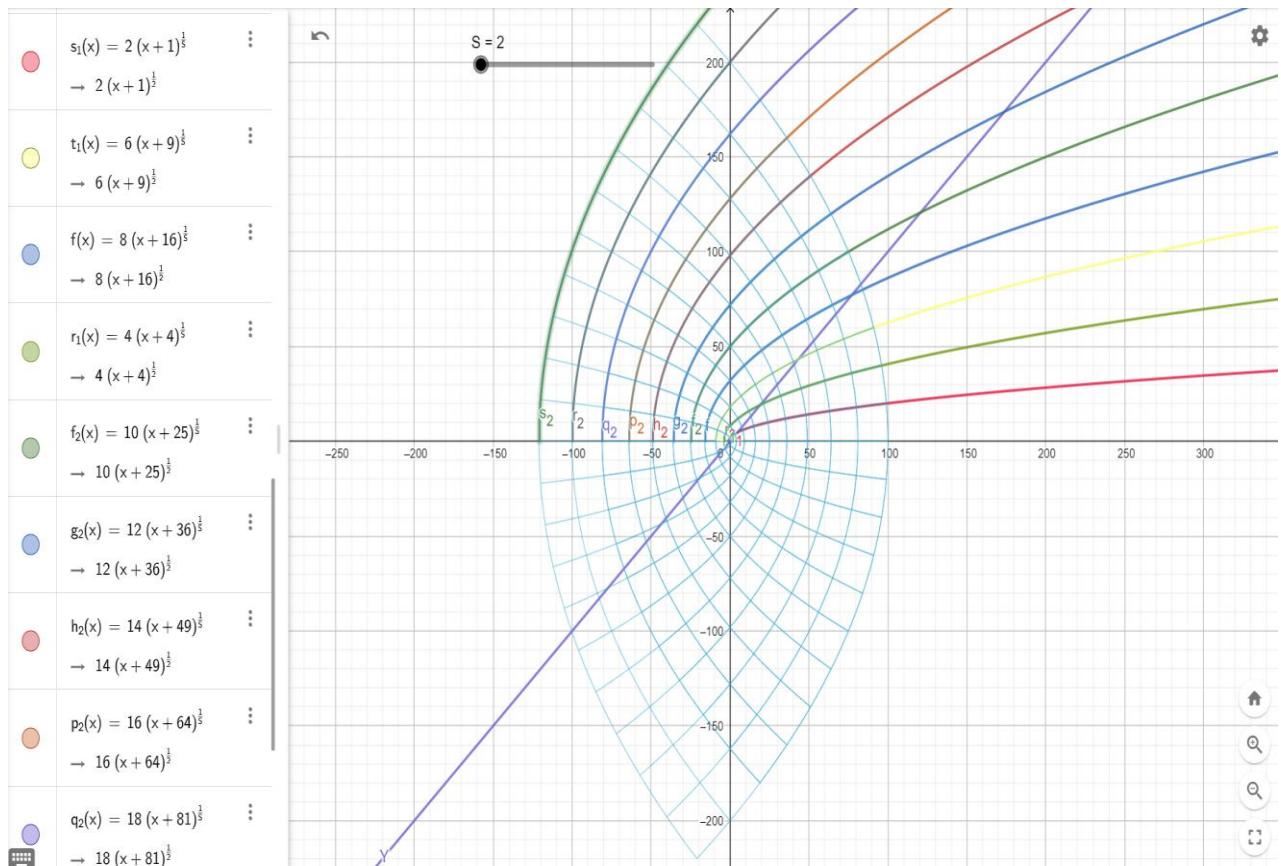


Figure (19): folding at [S=1]

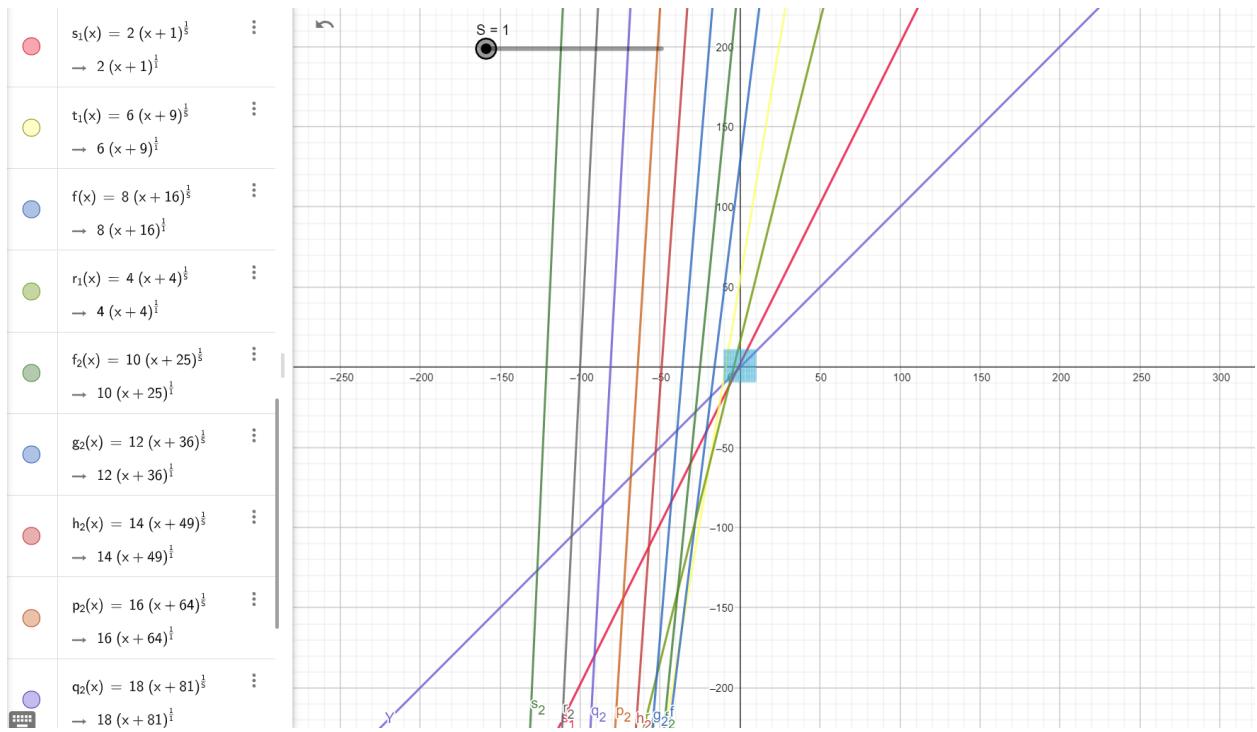
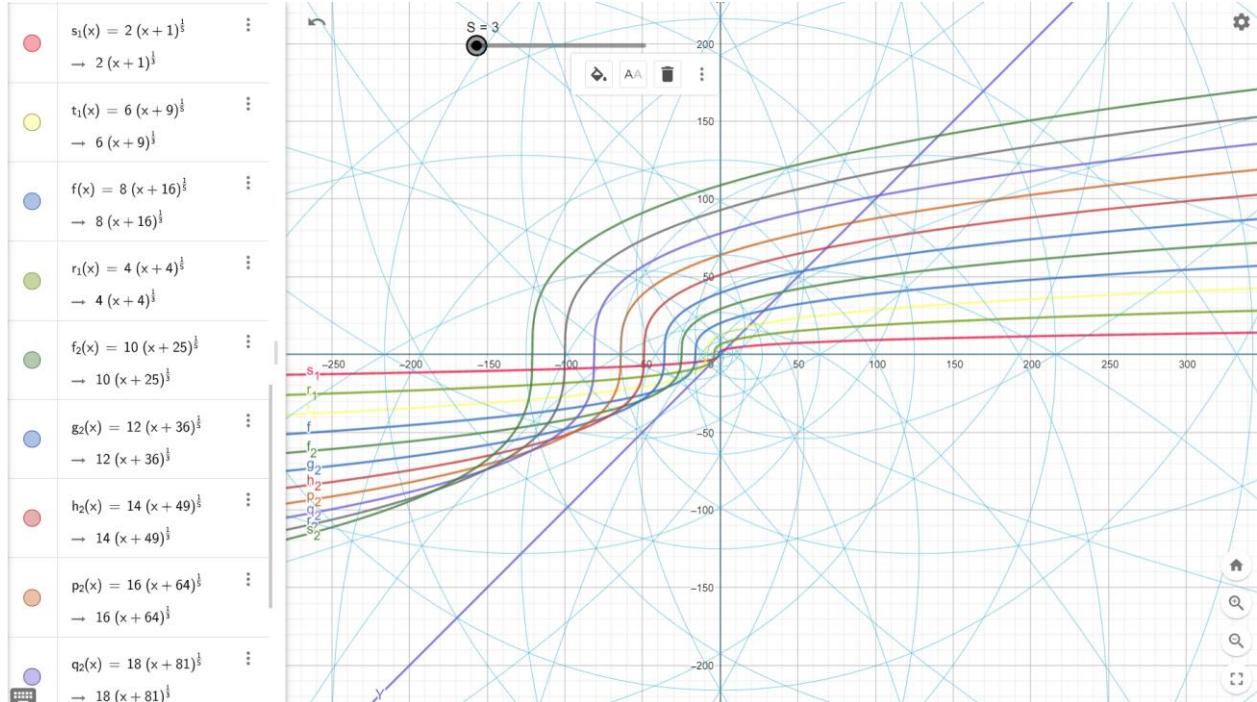
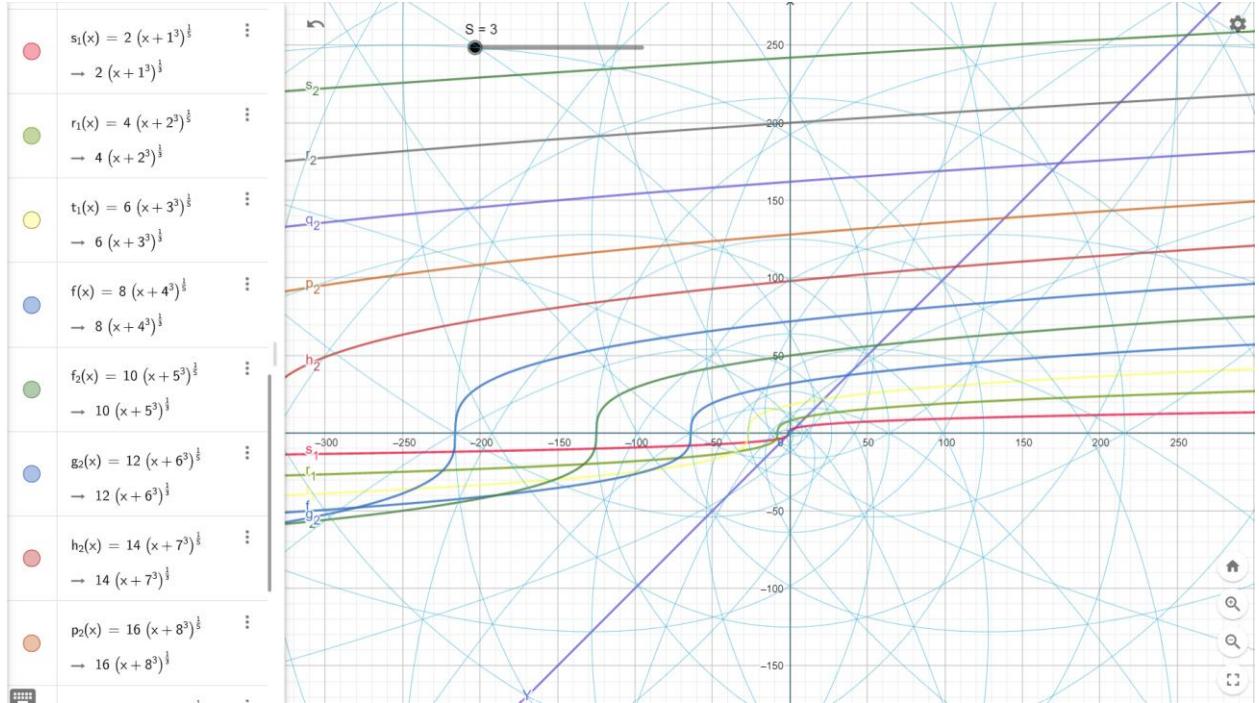


Figure (20) folding at [S=3]



As we see in Figure (20) the folding formula is not matching exactly folded shape at the higher dimension because we missed upgrading one of the parameters in the formula to S dimension instead of S=2. So, our transformation formula should be

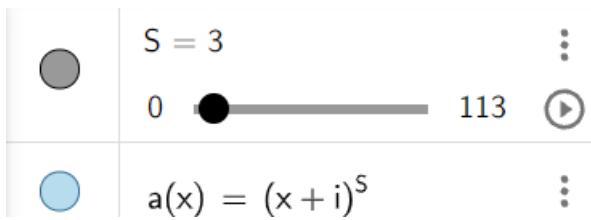
$$Y = 2 * N (X + N^S)^{\frac{1}{S}}$$



now $(X+i)$ square lines matching exactly the points that these lines for base-10 number set intersects with X axis at N raised to the power of S .

for example: - for $S = 3$ in this folding $X=1$ will intersect X axis at $X = 1*1*1 = 1 = 1$ to the power of S where S is any real number

and $X=2$ will intersect with X axis at $X = 2 * 2 * 2 = 8$ or 2 to the power of S , and $X=3$ will intersect with X axis at $X = 3 * 3 * 3 = 27$ or 3 to the power of S .



	$s_1(x) = 2(x+1^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 2(x+1^3)^{\frac{1}{5}}$	
	$r_1(x) = 4(x+2^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 4(x+2^3)^{\frac{1}{5}}$	
	$t_1(x) = 6(x+3^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 6(x+3^3)^{\frac{1}{5}}$	
	$f(x) = 8(x+4^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 8(x+4^3)^{\frac{1}{5}}$	
	$f_2(x) = 10(x+5^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 10(x+5^3)^{\frac{1}{5}}$	
	$g_2(x) = 12(x+6^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 12(x+6^3)^{\frac{1}{5}}$	
	$h_2(x) = 14(x+7^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 14(x+7^3)^{\frac{1}{5}}$	
	$p_2(x) = 16(x+8^5)^{\frac{1}{5}}$	\vdots
	$\rightarrow 16(x+8^3)^{\frac{1}{5}}$	

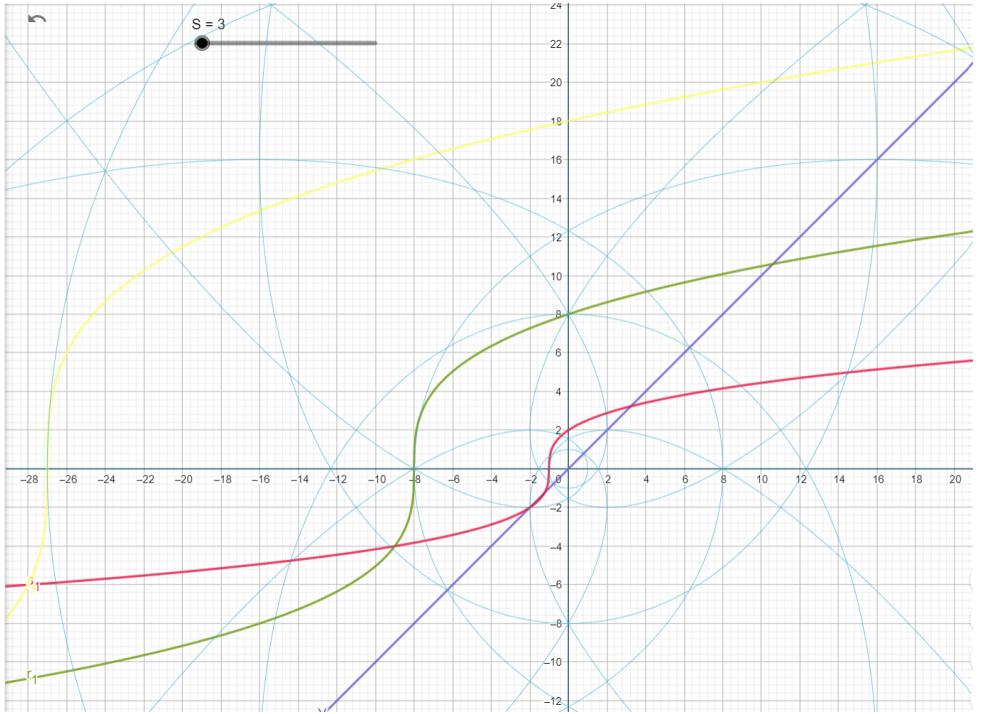
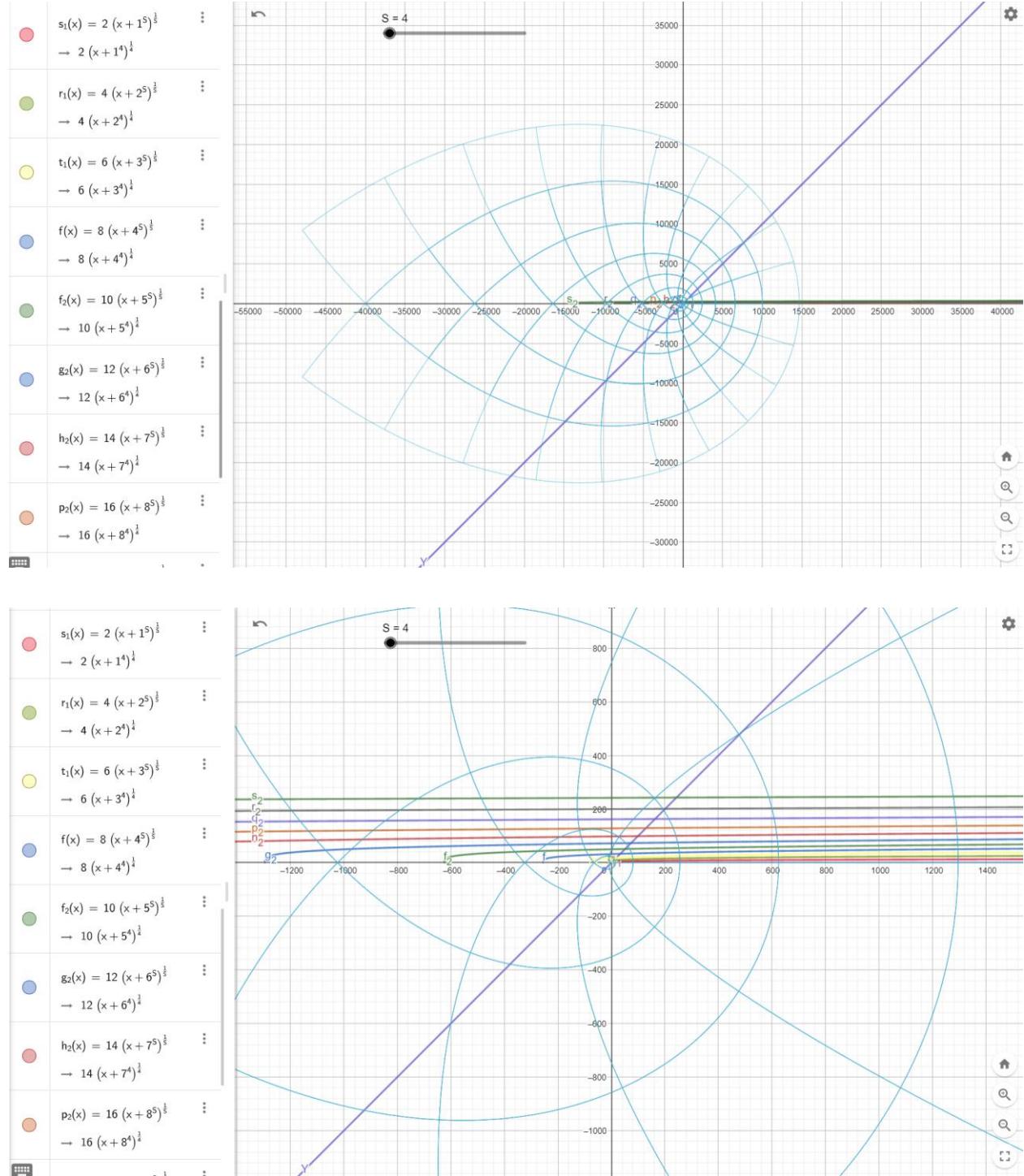
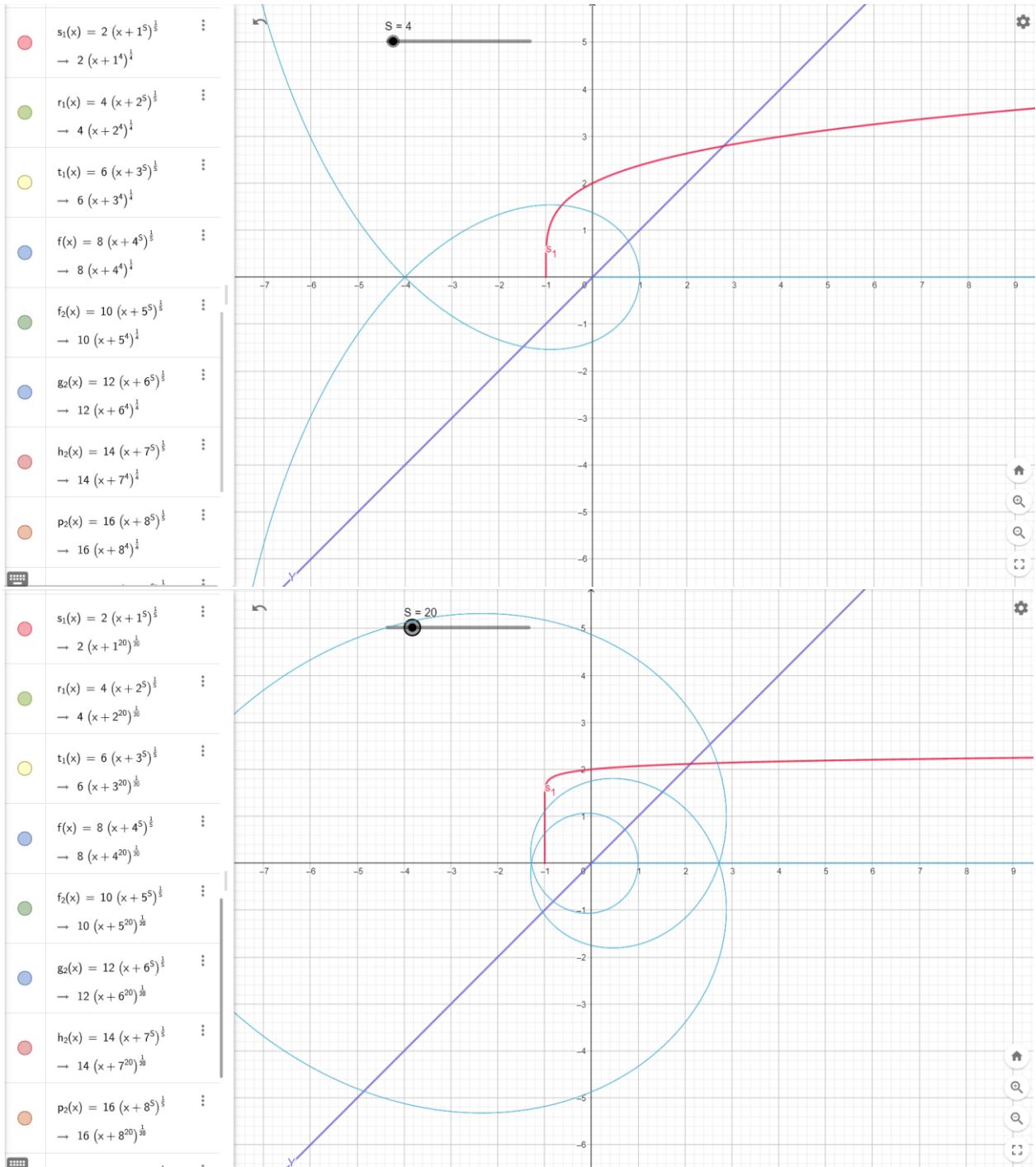


Figure (21): [S = 4] even real number, full folding shape will be at ranges of X = 10000. i.e., base-10 raised to power of S.



We still see the folding for square lines, but they only intersect X axis at X to the power of S and not matching the folding lines for the folded shape with small even numbers they lose the symmetry property on Y axis. The only point that meets the imaginary unit Circle at X axis is at X = 1. While odd numbers keep its symmetry on both X and Y axis.



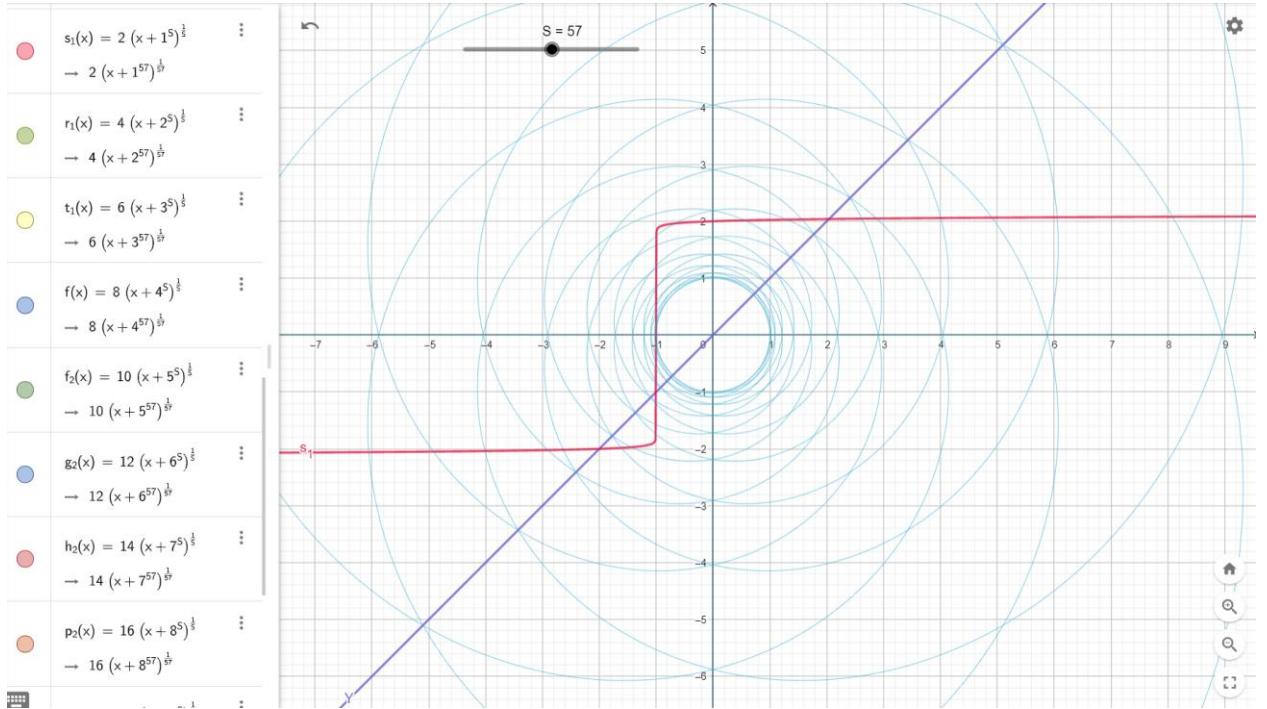
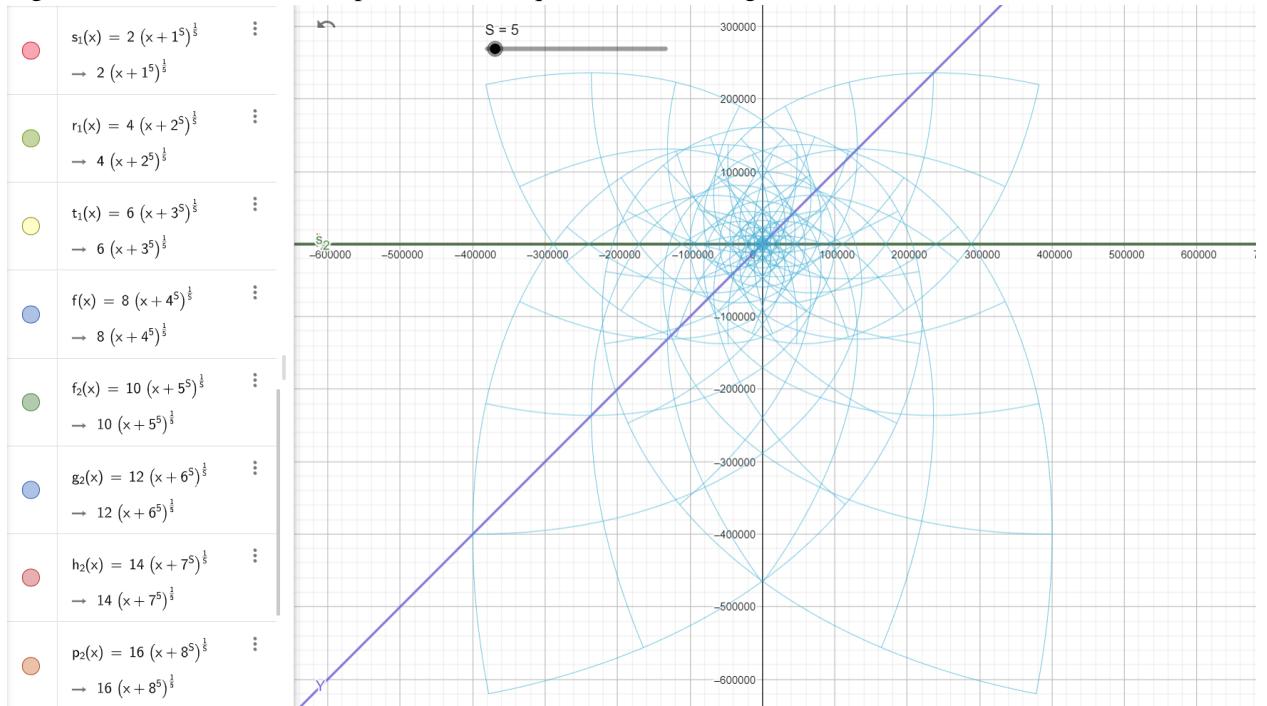
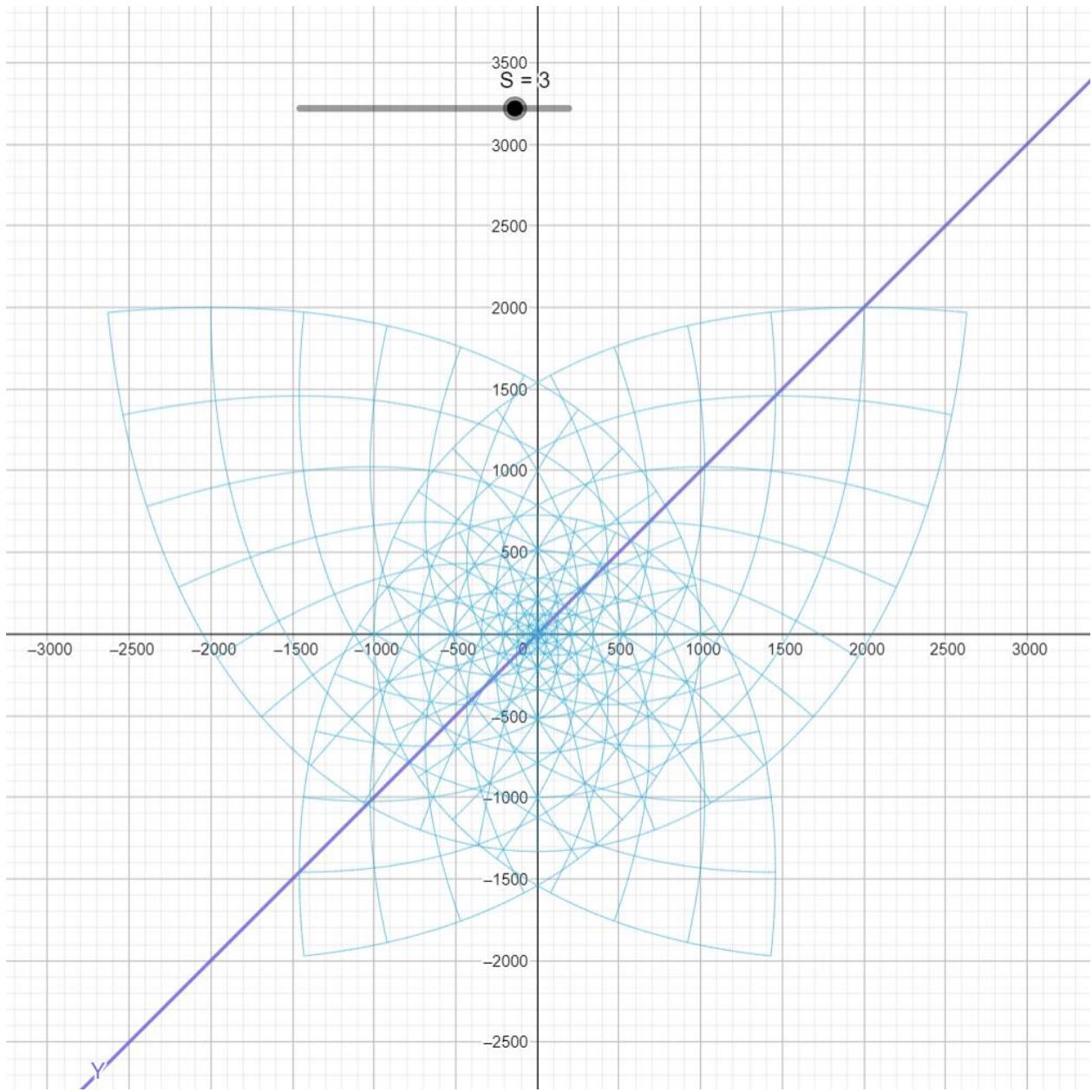


Figure (22): [S=5] full 2-D shape folded unit square will be at range of X = 100000.



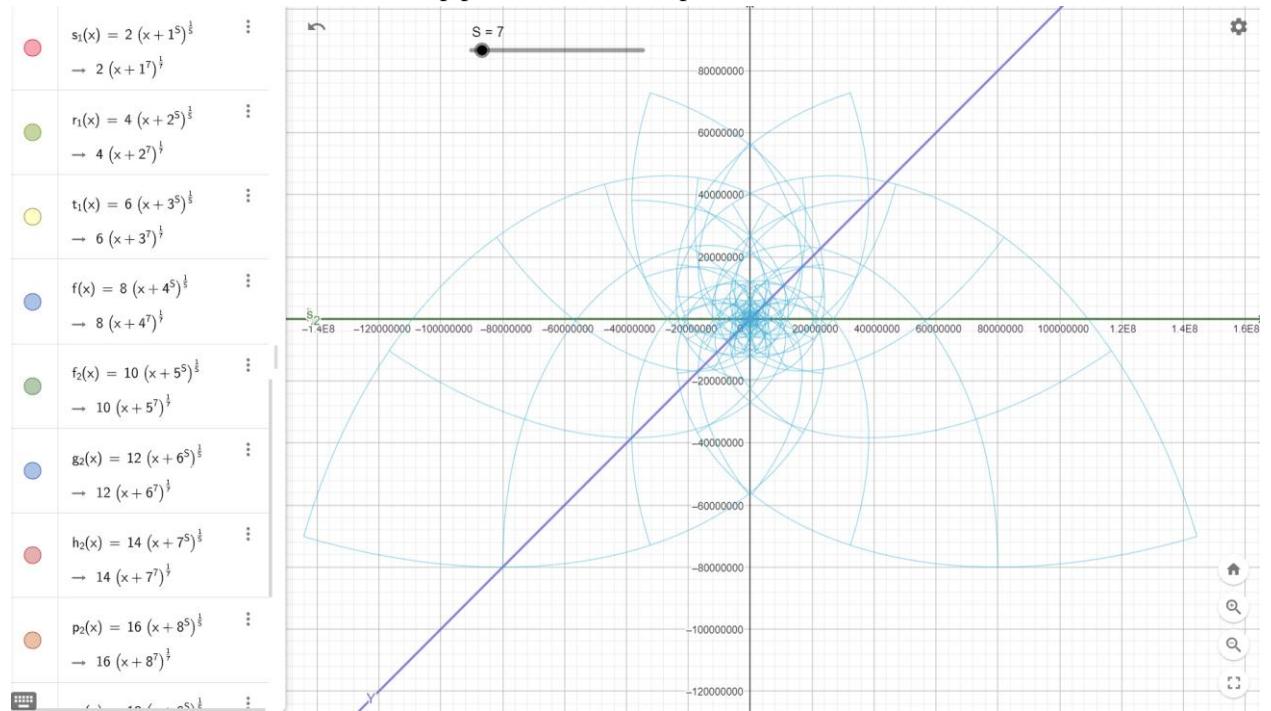
Still, we have the one unit at the tip of the shifted from Y=X the one-to-one relation

As we saw in figure (9) the full folded shape for [S=3] is the same as the full folded [S =5] but flipped on X axis

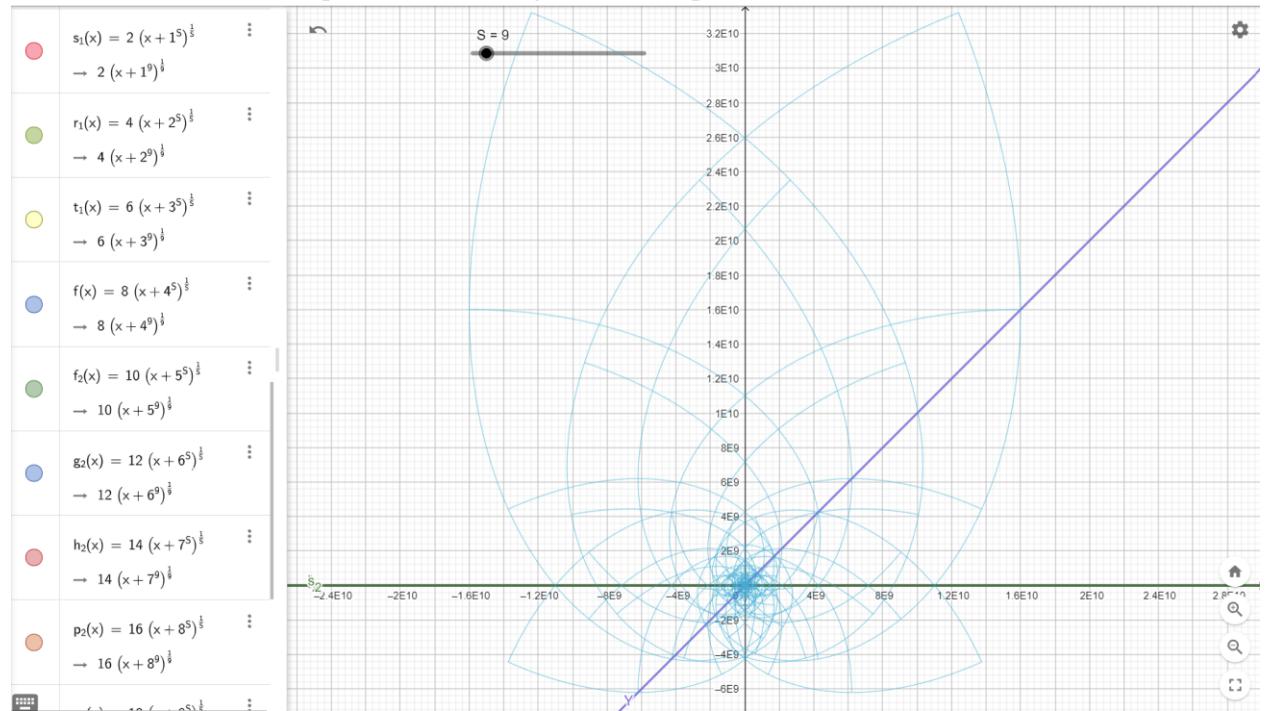


Which means odd numbers will have two folds one on Y axis and another one on the X axis all the time which gave us the symmetry on Y axis and X axis and give us the full shape symmetry and flipping and emerges the unit circle at origin (0,0).

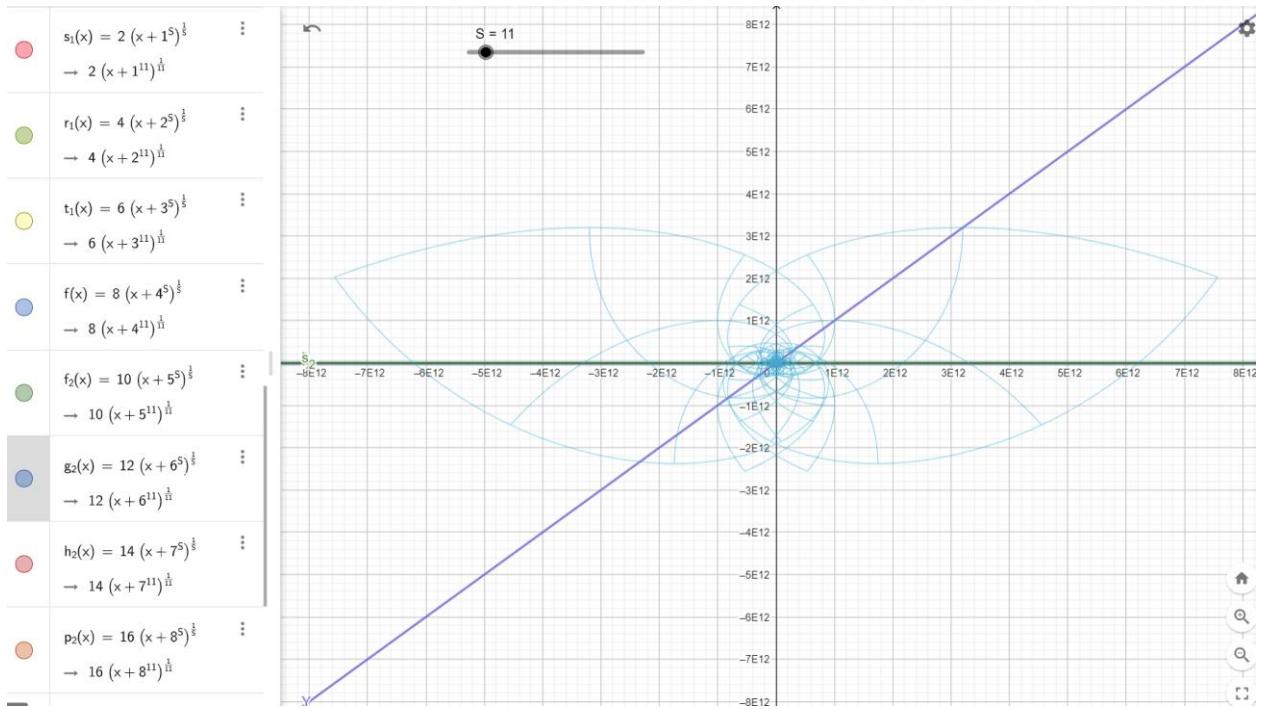
Figure (23): [S = 7] full 2-D fold shape will be at range X = 10000000. Or 10 to the power of S. Same one unit shift still exists at the tip point of the full shape to meet Y = X line.



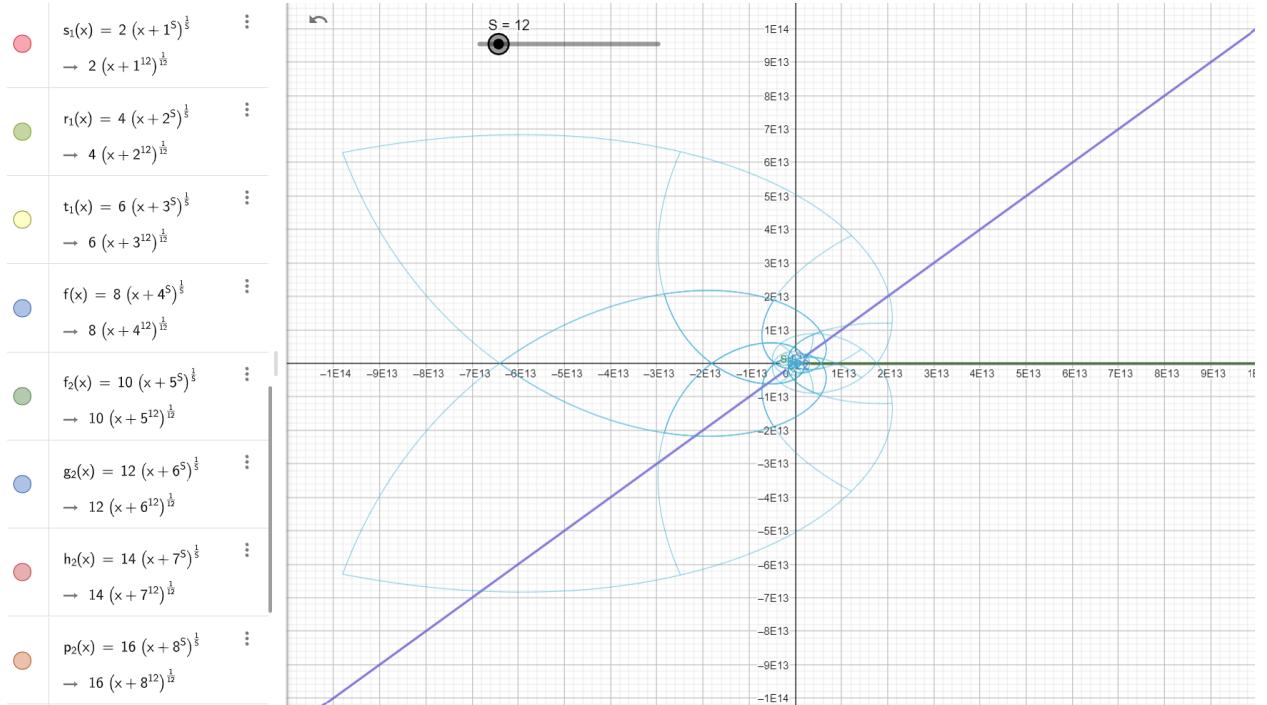
For [S = 9] full fold 2-D shape will be in X range = 10 to the power of 9.



For [S = 11] full 2-D shape will be in X range = 10 to the power of 11



For [S = 12] full 2-d shape will be in X rang = 10 to the power of 12.



E) Y value when X =0 for any folding order and for any real value for S at any dimension N.

As we saw at [S=2] transformation formula

$$Y = 2 * N (X + N^S)^{\frac{1}{S}}$$

This transformation will set us on the unit Circle in complex plane for all points N =1 line X=1 in unit square. Intersect Y axis at Y = 1 for X =0.

Same for the invers of this transformation for any real value for S

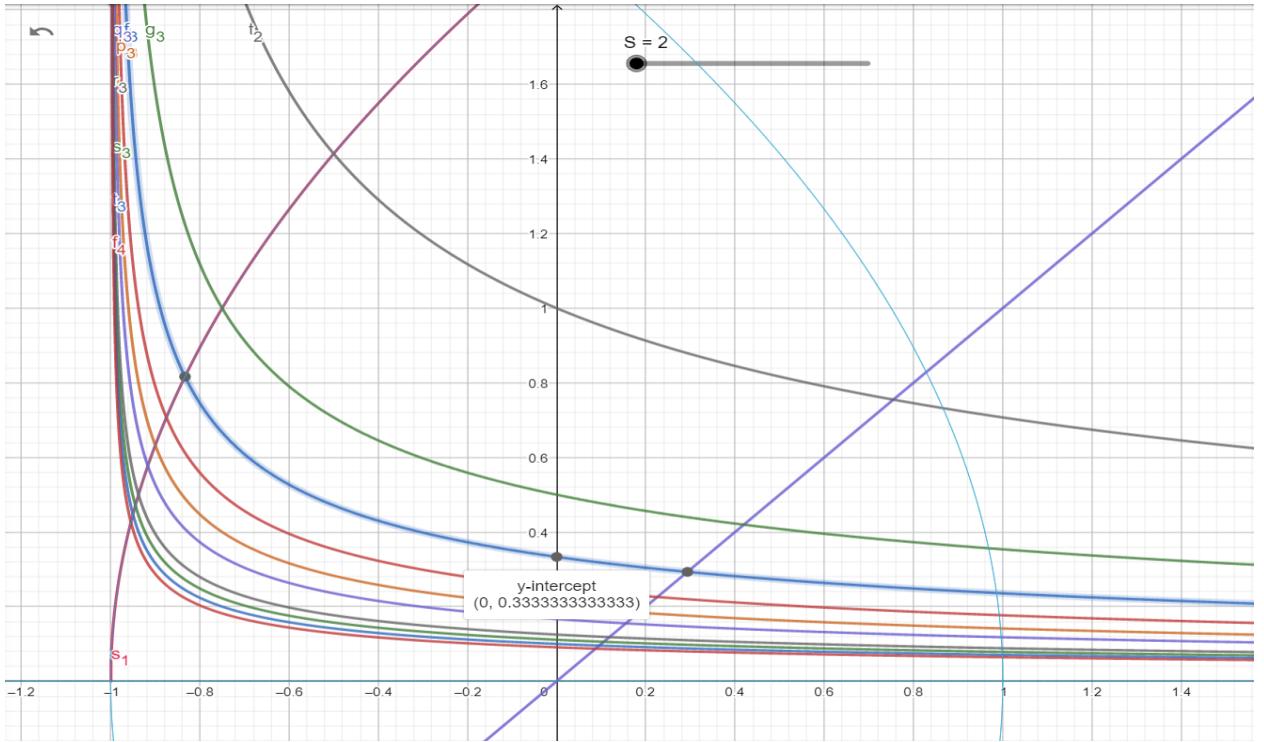
$$Y = (N^S * X + N^S)^{\frac{-1}{S}} = \frac{1}{N} (X + 1)^{\frac{-1}{S}}$$

This is basically the same transformation but for 1/S instead of S.

This transformation for X = 0 will give us Y = 1/N all the time. So all time the intersection with Y axis will be at point (0, 1/N) for any natural number N.

Figure (24): all lines X =N , with this transformation will intersect Y axis at point 1/N

	$t_2(x) = (x + 1)^{-\frac{1}{2}}$		$g_3(x) = (4x + 4)^{-\frac{1}{2}}$
	$f_3(x) = (9x + 9)^{-\frac{1}{2}}$		
	$h_3(x) = (16x + 16)^{-\frac{1}{2}}$		$p_3(x) = (25x + 25)^{-\frac{1}{2}}$
	$q_3(x) = (36x + 36)^{-\frac{1}{2}}$		
	$r_3(x) = (64x + 64)^{-\frac{1}{2}}$		$s_3(x) = (81x + 81)^{-\frac{1}{2}}$
	$t_3(x) = (100x + 100)^{-\frac{1}{2}}$		



This means for each new fold we have $1/N$ for each line.

So instead, we are going to use this transformation remove this $1/N$ for any N as $S < N$.

$$Y = (N^S * X + 2^S)^{\frac{-1}{S}}$$

Figure (25): all N lines $X = N$ and $S = 2$; all will intersect with Y axis at $1/S$ instead so intersect point $(0,0.5)$



$$t_2(x) = (x + 4)^{-\frac{1}{2}}$$



$$g_3(x) = (4x + 4)^{-\frac{1}{2}}$$



$$f_3(x) = (9x + 4)^{-\frac{1}{2}}$$



$$h_3(x) = (16x + 4)^{-\frac{1}{2}}$$



$$p_3(x) = (25x + 4)^{-\frac{1}{2}}$$



$$q_3(x) = (36x + 4)^{-\frac{1}{2}}$$



$$r_3(x) = (64x + 4)^{-\frac{1}{2}}$$



$$s_3(x) = (81x + 4)^{-\frac{1}{2}}$$



$$t_3(x) = (100x + 4)^{-\frac{1}{2}}$$

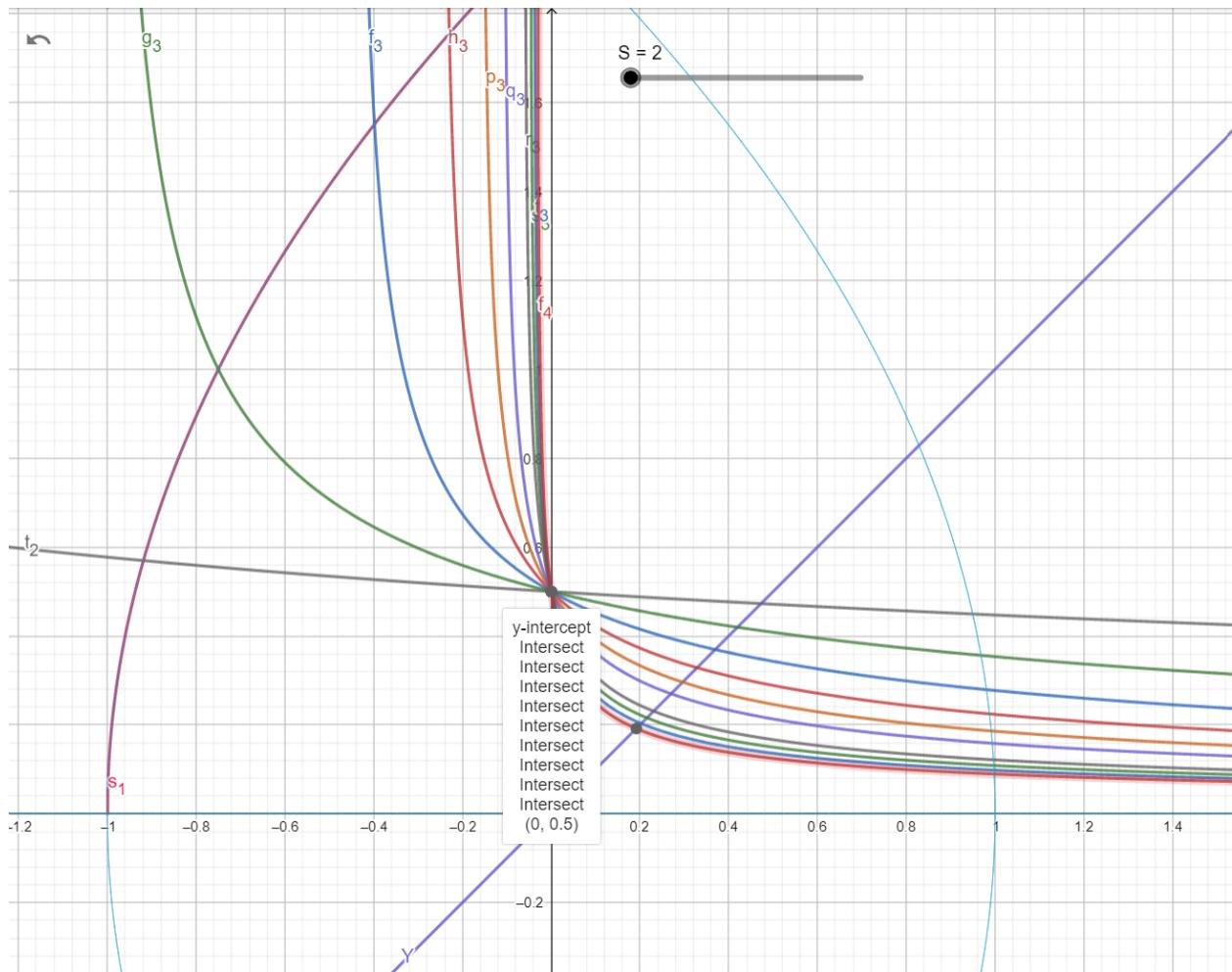


Figure (26): [S = 3]

all line numbers X = N

intersect with Y axis at point (0, 0.5)

Figure (27): [S =5] all line numbers X =N intersect with Y axis at point (0,0.5)

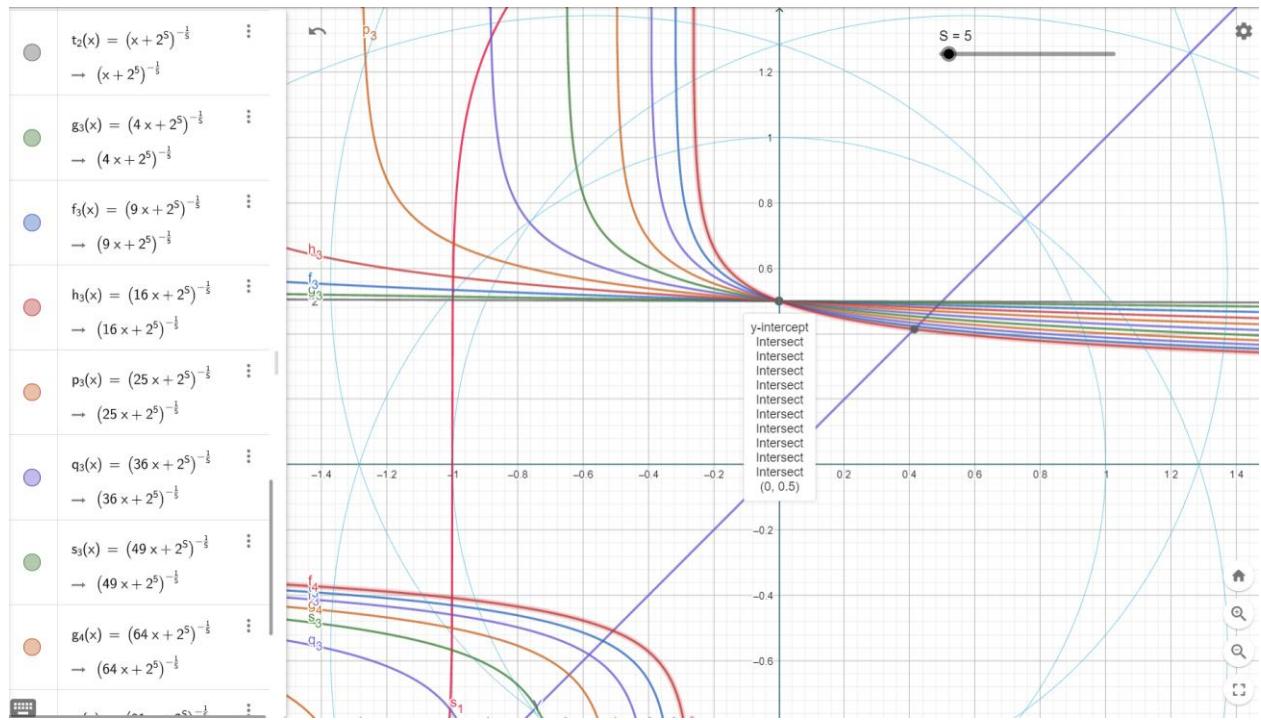
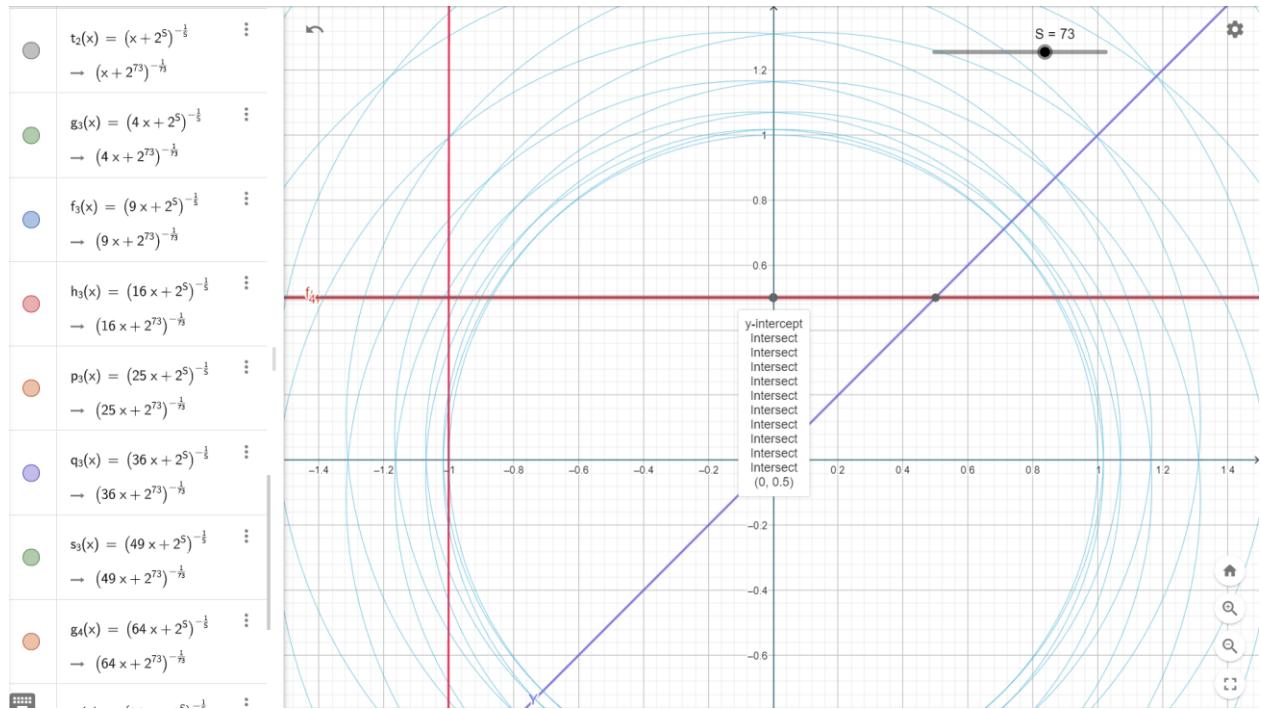
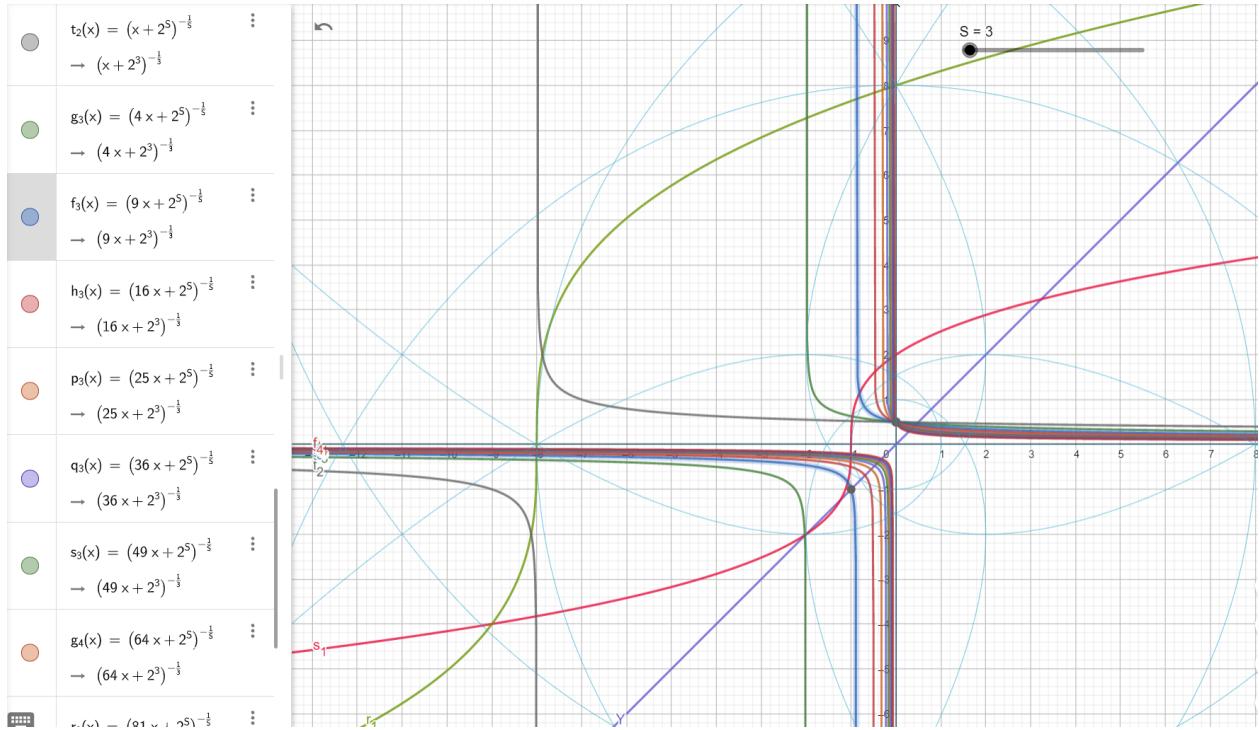


Figure (28)): [S =73] all line numbers X =N intersect with Y axis at point (0,0.5)





We can do the same transformation

$$Y = (N^S * X + 2^S)^{\frac{-1}{S}}$$

Using this formula as well

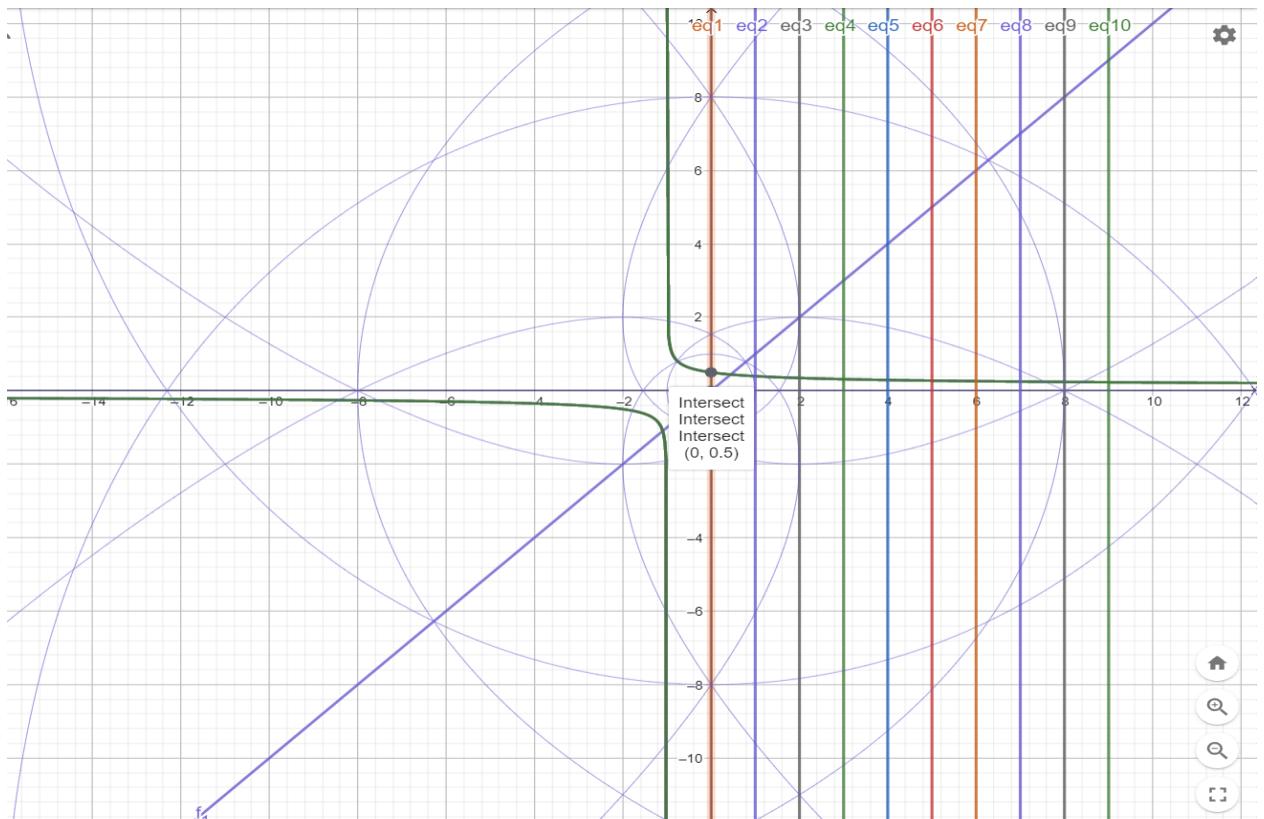
$$Y = \frac{N}{2} (N^S * X + N^S)^{\frac{-1}{S}} = \frac{1}{2} (X + 1)^{\frac{-1}{S}}$$

And this is the same transformation as

$$Y = 2 * N (X + N^S)^{\frac{1}{S}}$$

	$s_2(x) = \frac{2}{2} (8x + 8)^{-\frac{1}{3}}$
	$t_2(x) = \frac{3}{2} (27x + 27)^{-\frac{1}{3}}$
	$f_3(x) = \frac{4}{2} (16 \cdot 4x + 16 \cdot 4)^{-\frac{1}{3}}$

$i(x) = (x + i)^3$



For $N=1$ and $S=2$; $Y = 2 * N (X + N^S)^{\frac{1}{S}}$

$$Y = 2 \sqrt{(x + 1)}$$

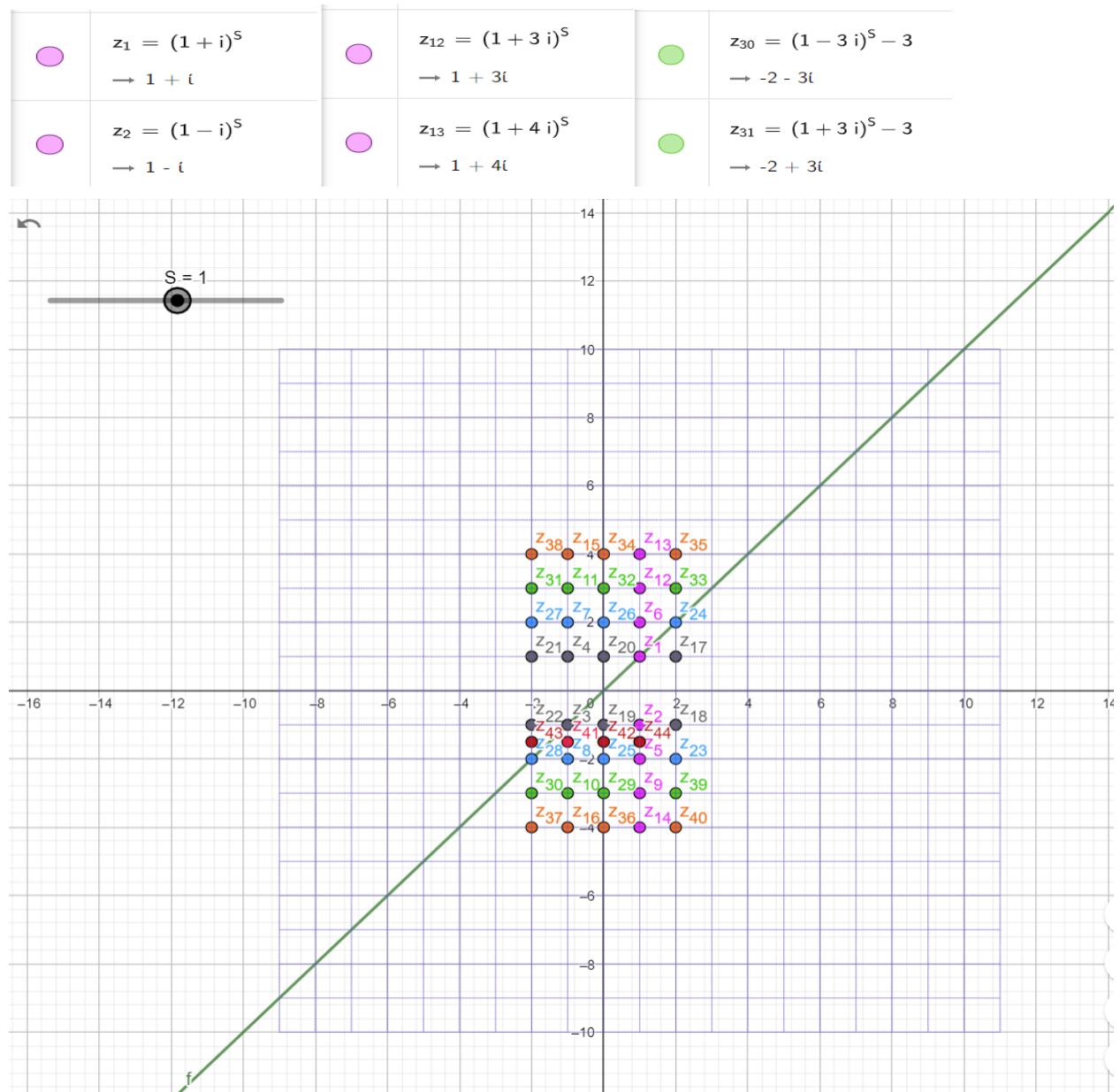
For $N=1$ and $S=2$; $Y = (N^S * X + 2^S)^{\frac{-1}{S}}$

F) Zeta Function have only one pole at one N =1

X=1 line in our square will be the only line that will be in sync with all points in the complex plane for all real values of S.

$$Y = (1 + ix)^s$$

Figure (29): all pink intersection points are results of $Y = (1+Xi)^s$ where $X = 1$ from set $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ for all other lines $X=N$.



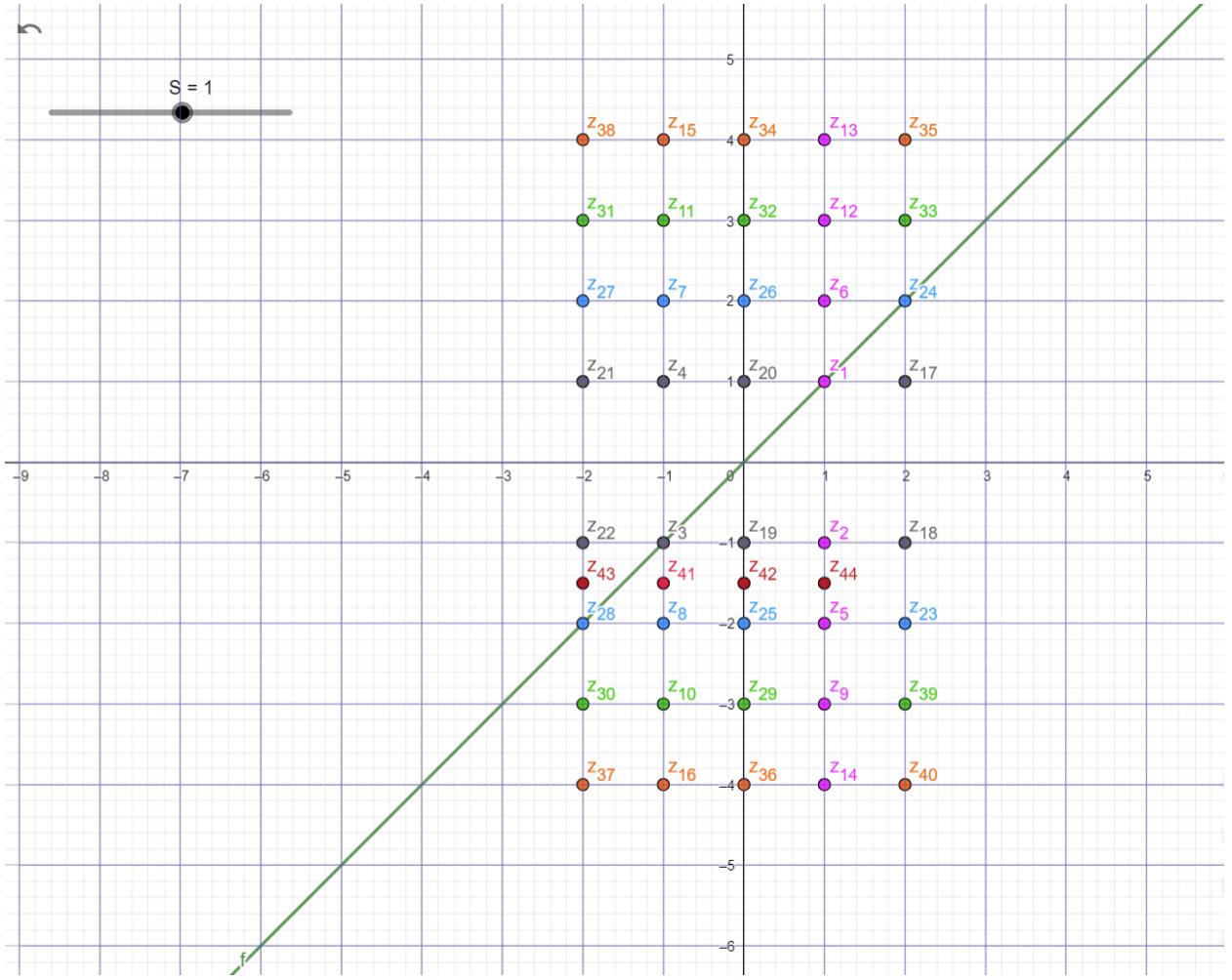
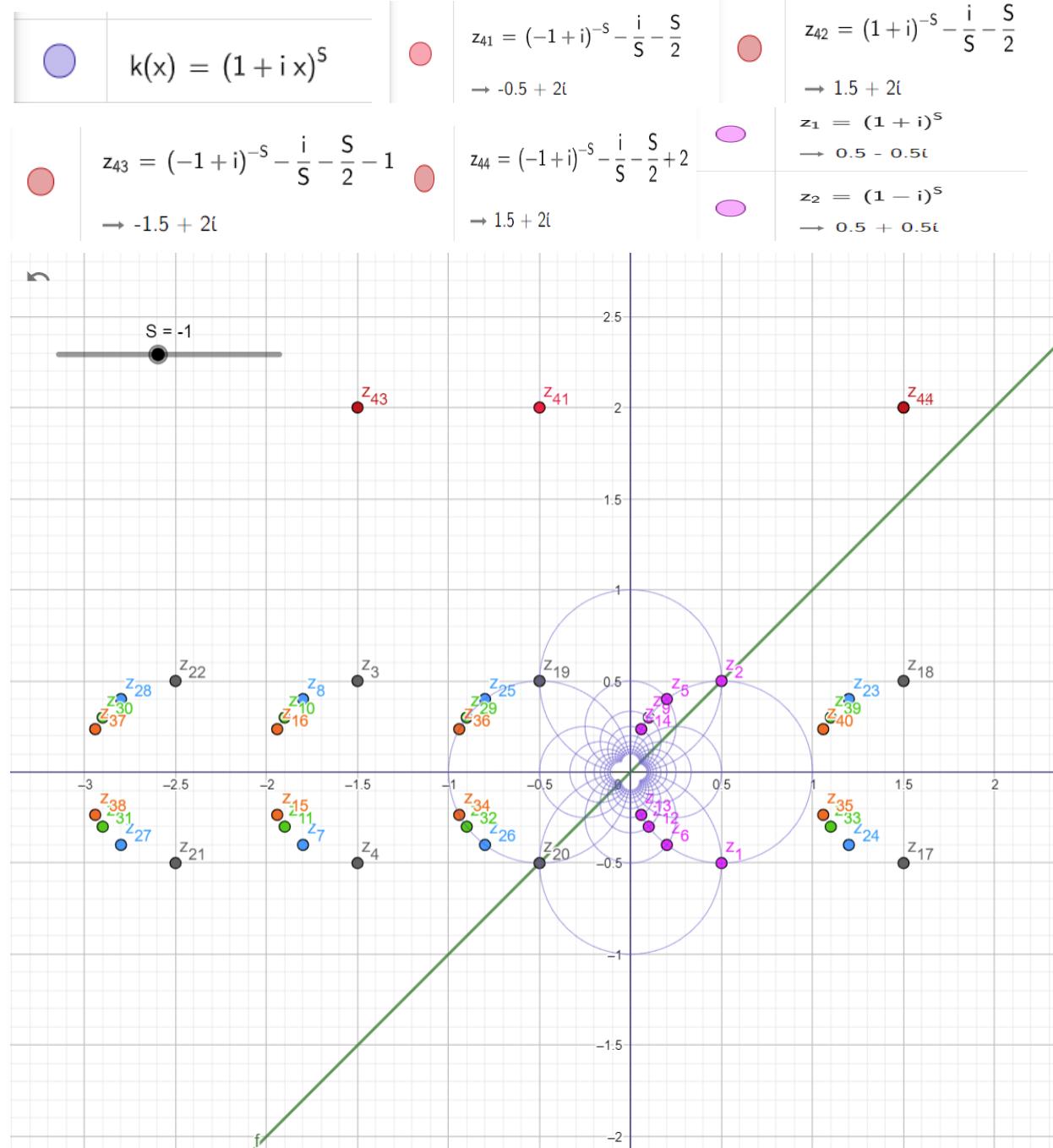


Figure (30): shows how the intersection points movement with the transformation shape as S increase. Each set of intersection points will land on its dimension transformed Circle. If we looked only on one dimension the other points will look like they move exactly at the same group without transformations.

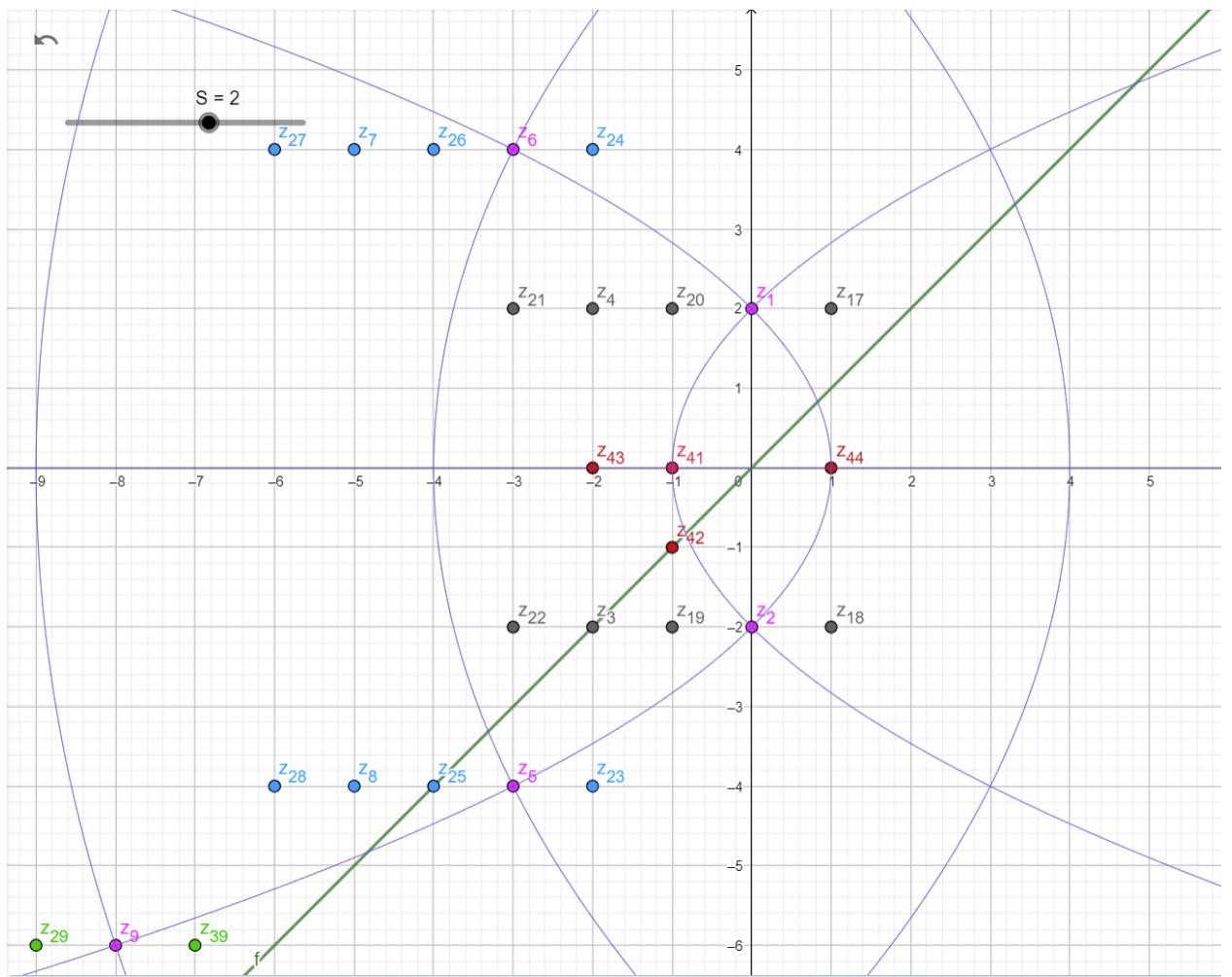
Looks like the spooky effect but it is not it only other time dimension line of $X=N$.

1- all z points (pink points) for line number $X = 1$ will be represented on the outer circle of the transformed square.

2- all red points which are all points that are at line number (0.5-ix)



$$z_{57} = S(S+1)D \left((-1+i)^{-S} - \frac{i}{S+1} - \frac{S+1}{2S} \right) + \frac{(S+1)D}{4}$$





$$z_{42} = (1+i)^{-s} - \frac{i}{s} - \frac{s}{2}$$

$$\rightarrow -1.75 - 0.58333333333333i$$



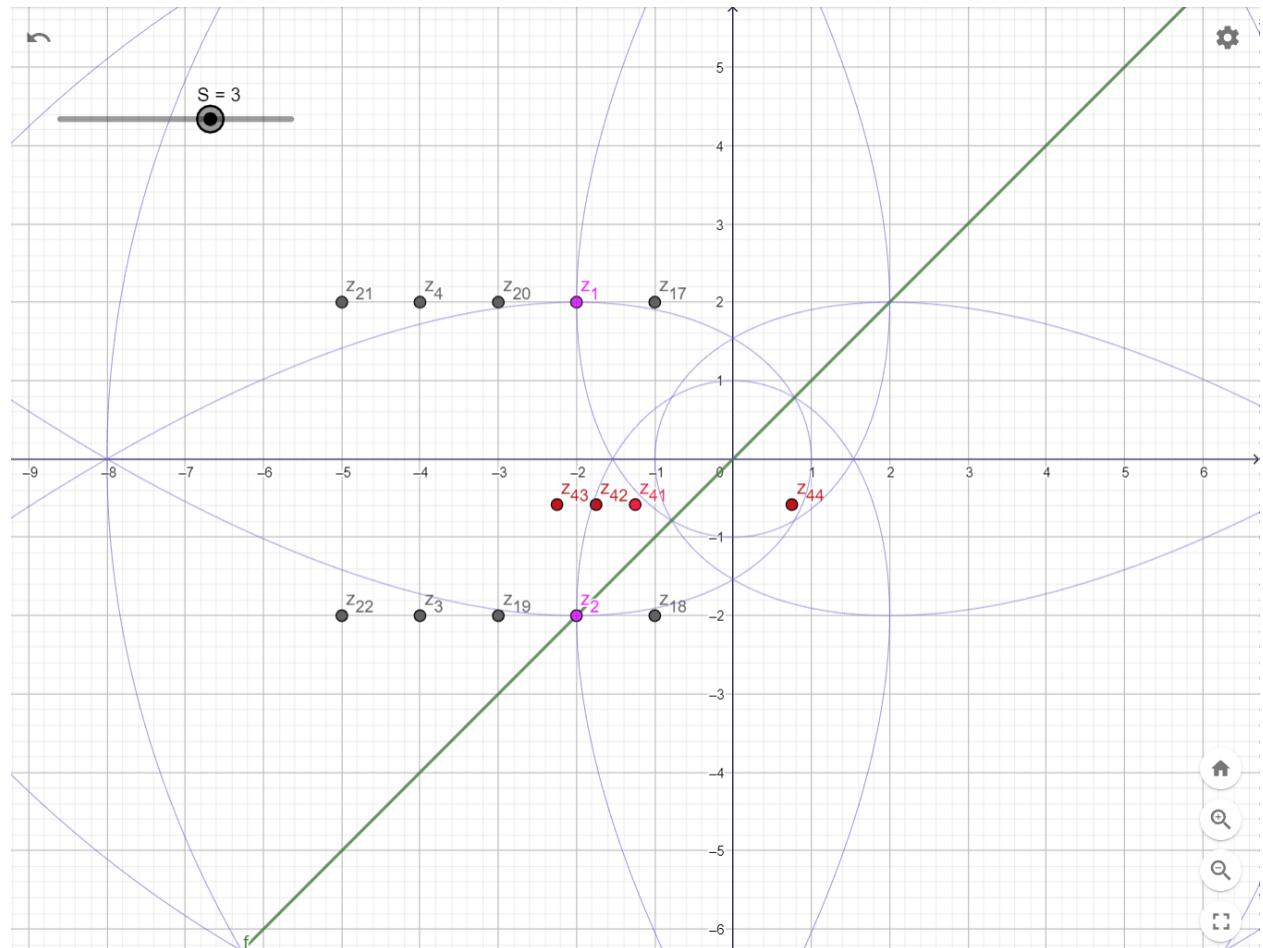
$$z_{43} = (-1+i)^{-s} - \frac{i}{s} - \frac{s}{2} - 1$$

$$\rightarrow -2.25 - 0.58333333333333i$$



$$z_{44} = (-1+i)^{-s} - \frac{i}{s} - \frac{s}{2} + 2$$

$$\rightarrow 0.75 - 0.58333333333333i$$





$$z_{42} = (1+i)^{-s} - \frac{i}{s} - \frac{s}{2}$$

$$\rightarrow -2.25 - 0.25i$$



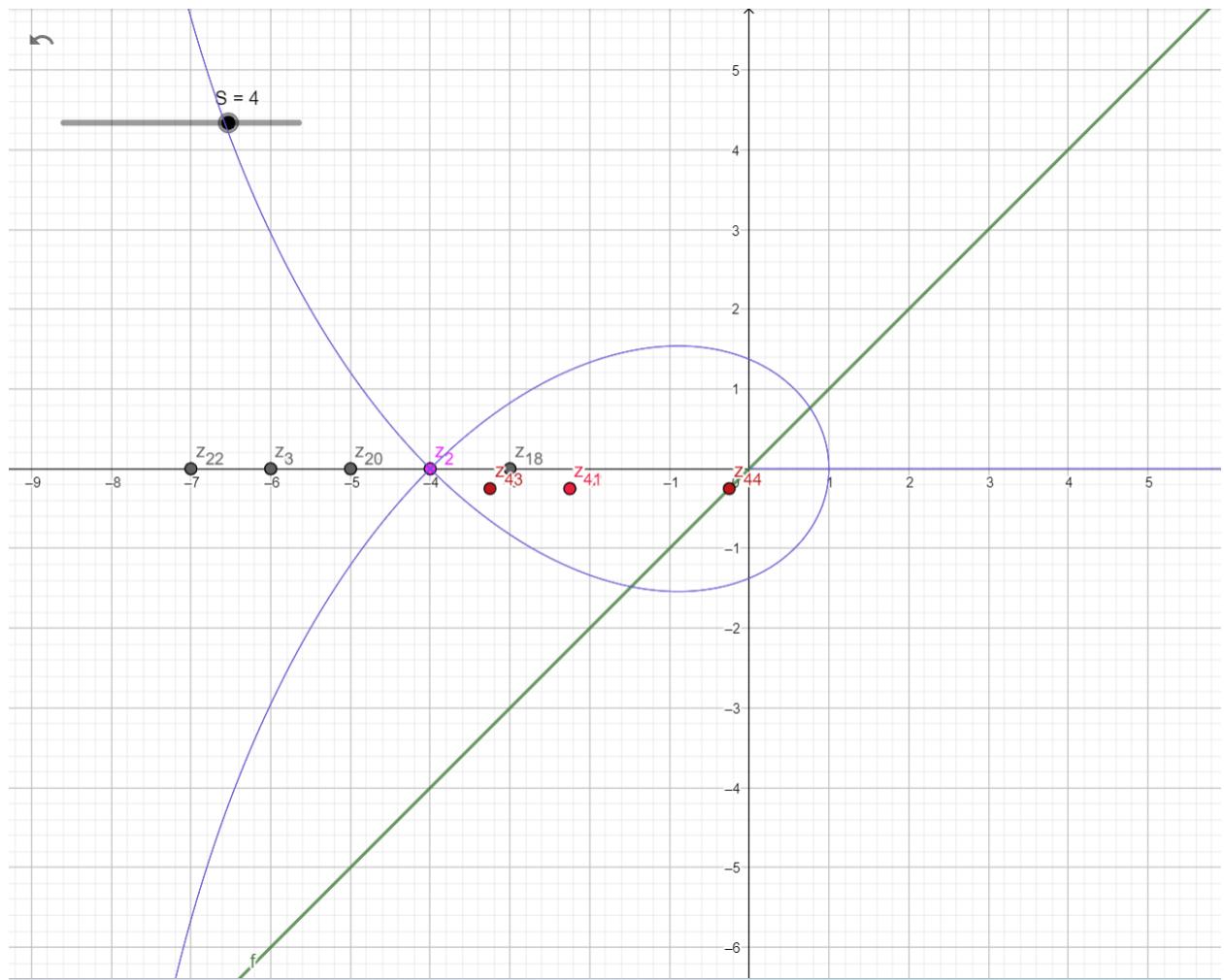
$$z_{43} = (-1+i)^{-s} - \frac{i}{s} - \frac{s}{2} - 1$$

$$\rightarrow -3.25 - 0.25i$$



$$z_{44} = (-1+i)^{-s} - \frac{i}{s} - \frac{s}{2} + 2$$

$$\rightarrow -0.25 - 0.25i$$



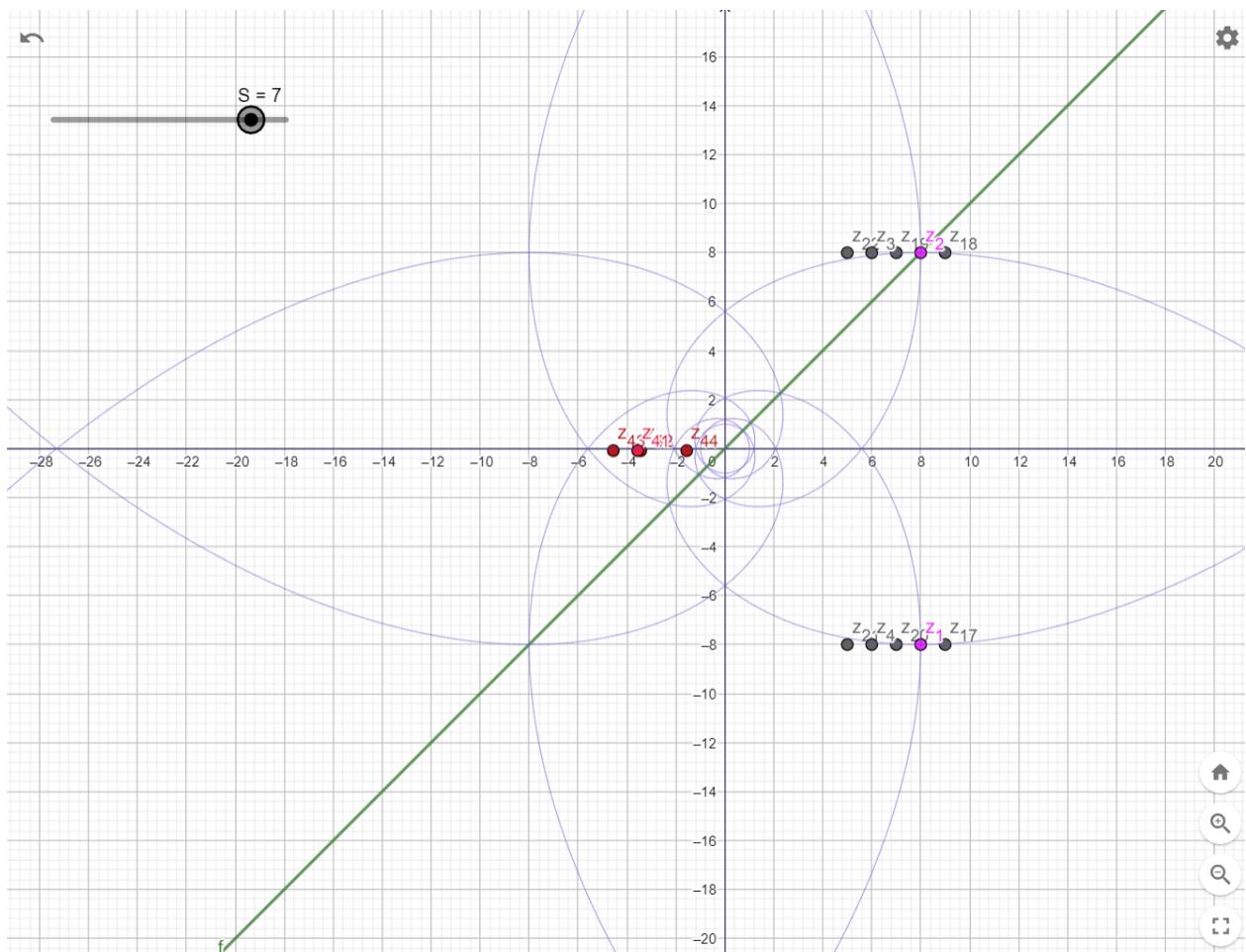
$$z_{42} = (1+i)^{-\frac{1}{5}} - \frac{1}{2}$$

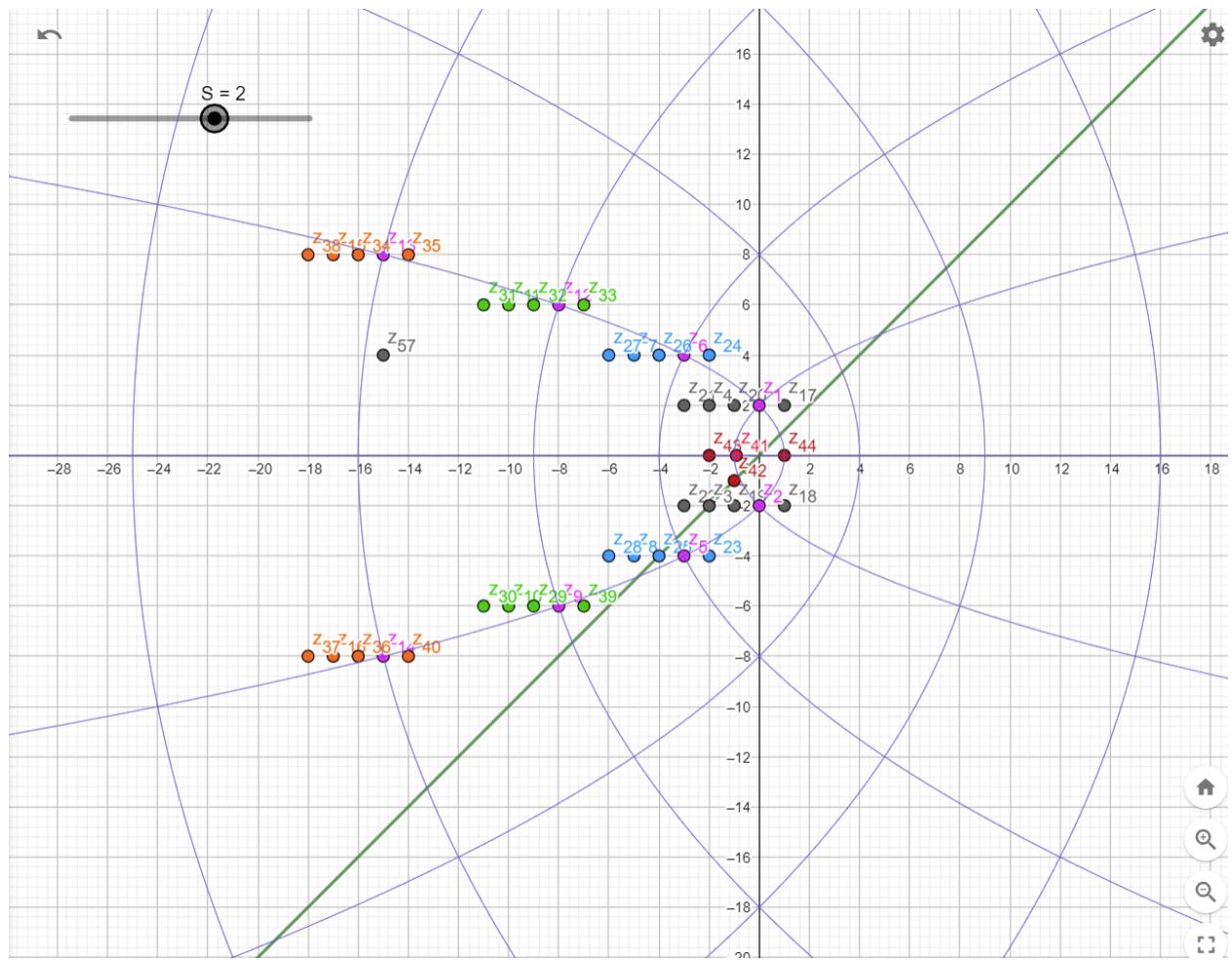
→ -3.4375 - 0.0803571428571i

$$z_{43} = (-1 + i)^{-s} - \frac{i}{s} - \frac{s}{2} - 1$$

$$z_{44} = (-1+i)^{-s} - \frac{i}{s} - \frac{s}{2} + 2$$

→ -1.5625 - 0.0803571428571i





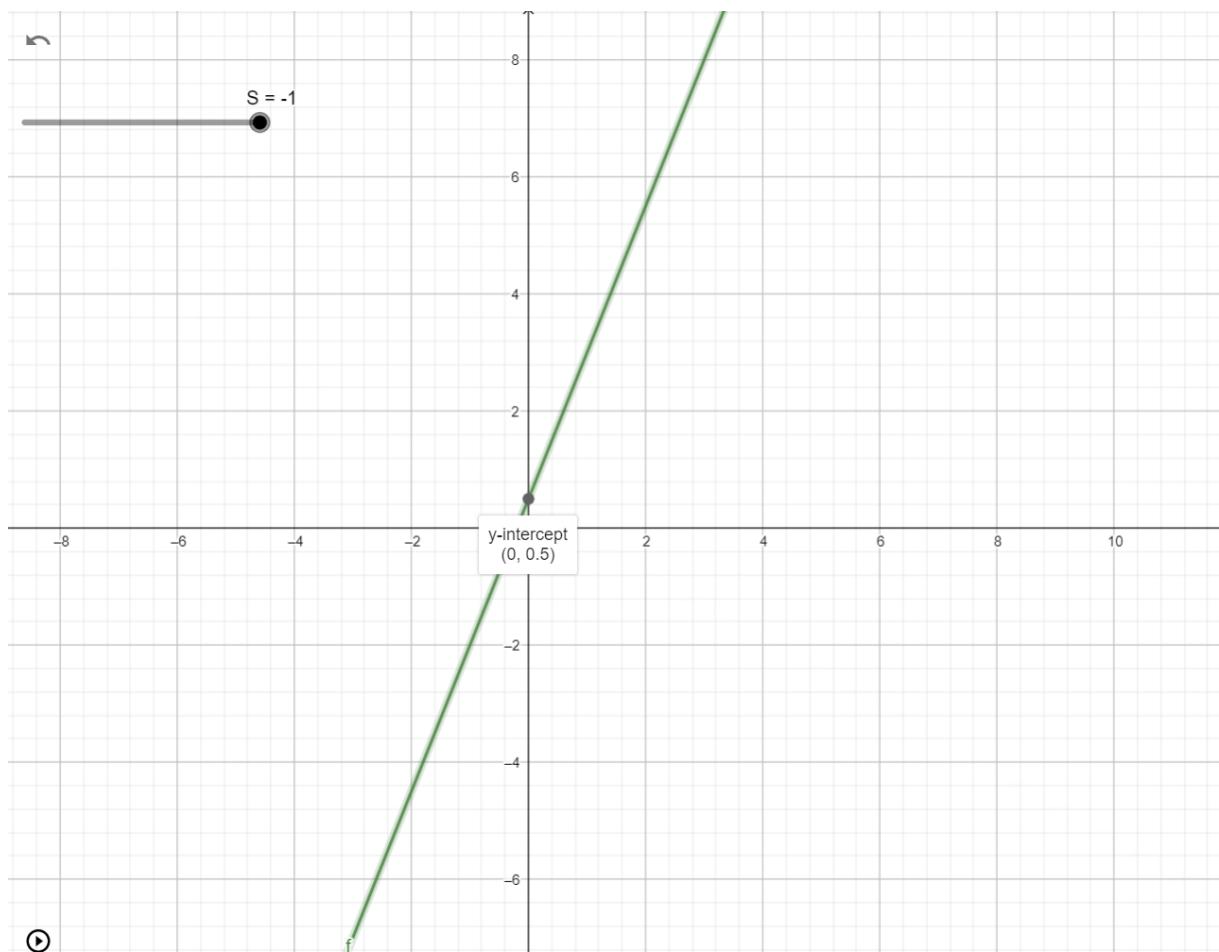
3.2 Axiomatic proof for zeta function zeros at strip line

Based on our explanation here in this document the complex plane doing folding for higher dimensions to be represented into 2-D plane on complex plane and this what we showed in the document. The functions keep folded over and over on X axis and on Y axis as we raised a transformation function for a frame of reference to another higher dimension in complex plane.



$$f(x) = \frac{10}{4} (x - 1) + 3$$

This transformation formula we already prove it before in this doement



$F(x)$ is the same as $t(x)$ at $S = -1$ and it is identical to $t(x)$ function for $S = -1$

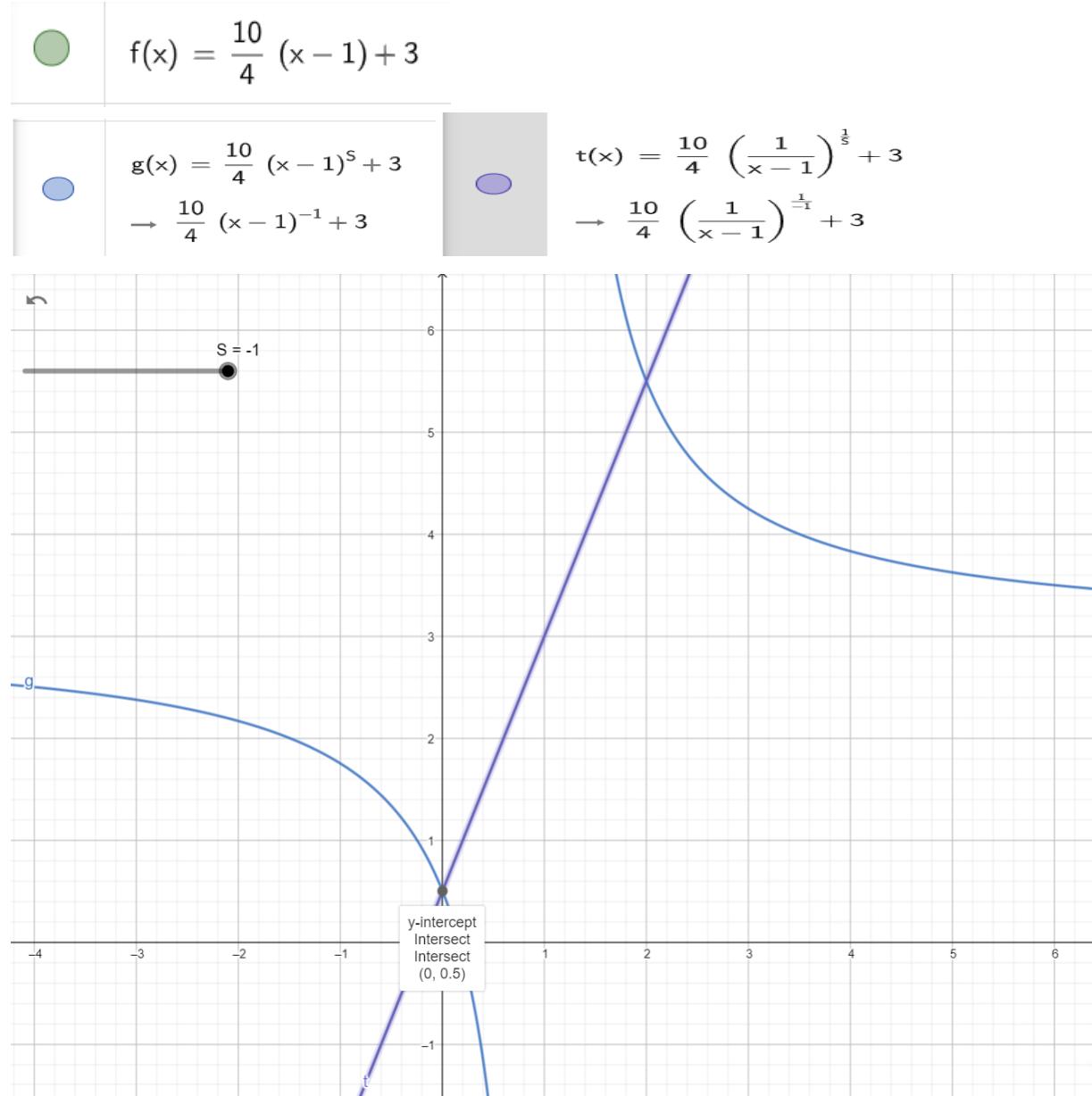
And $F(x)$ is the same as $g(x)$ at $s= 1$

Both are line intercept Y axis at $(0,0.5)$

If we took $(x-1)$ as whole as our variable Z , then one function has the reciprocal variable of the other function so $(x-1)$ the same as $1/(x-1)$ in the other function, so if $X = 3$ one function will work with 3 and the other function will work with $1/3$. So, the only value that could work for X that makes these two functions have the same solution is $X = 0$.

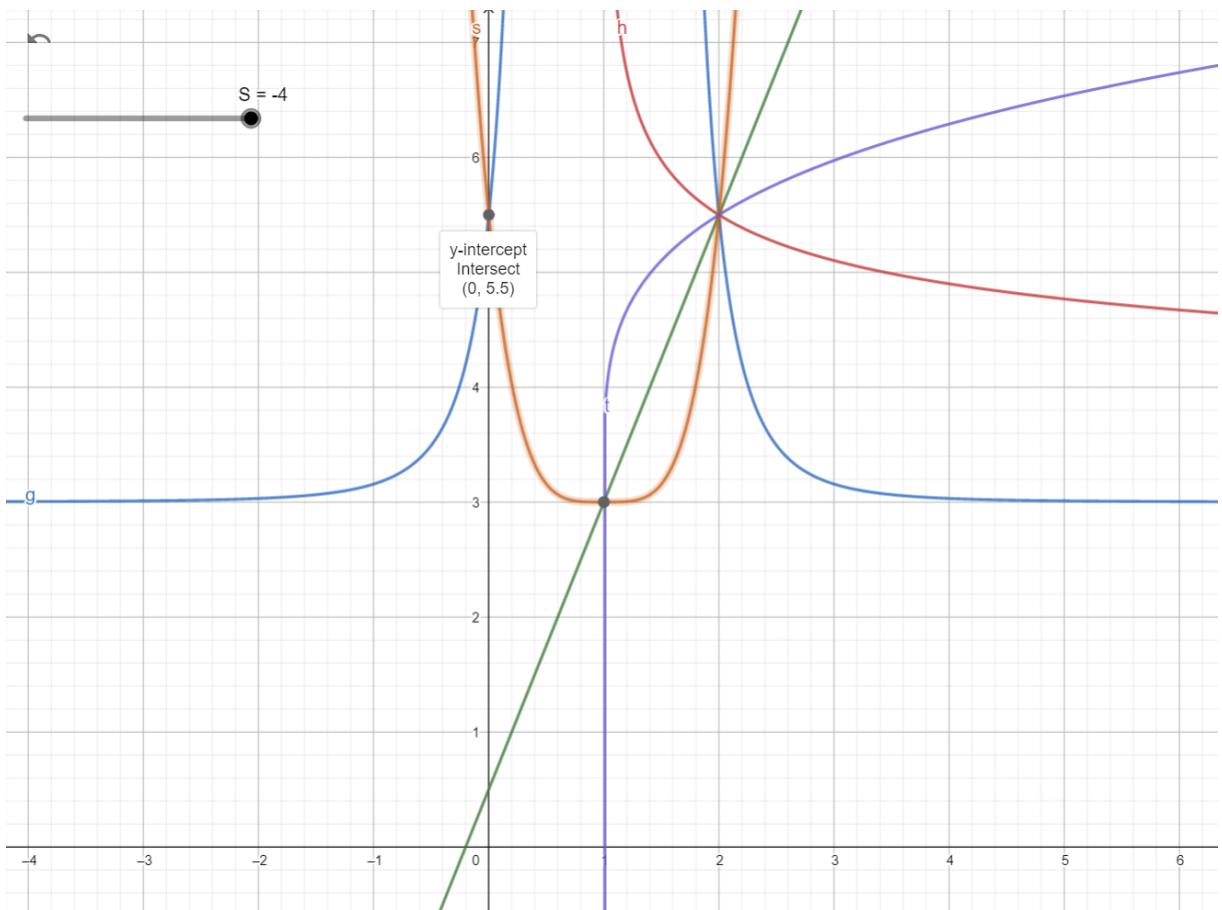
At $X = 0$ $f(x) = 0.5$ and $t(x) = 0.5$ for any real value for $S \geq 3$, and we show before why $S \geq 3$ in complex plane, it is the value when the imaginary unit Circle is emerging due to frame of reference folding.

As we go in the next graphs we will see that every $S =$ even number of folds or dimensions we actually folds the function over its symmetrical parts so we only see one part in 2-D, and for $S =$ odd values we see the full function symmetrical shape as we will see in the next examples and all the time the only solution for these two reciprocal functions will be at $X = 0$ and $Y = 0.5$ will intersects for S odd value at point $(0, 0.5)$



1- Even Values for S >= 3

●	$g(x) = \frac{10}{4} (x - 1)^s + 3$ $\rightarrow \frac{10}{4} (x - 1)^{-4} + 3$	●	$s(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^s + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{-4} + 3$
●	$h(x) = \frac{10}{4} (x - 1)^{\frac{1}{s}} + 3$ $\rightarrow \frac{10}{4} (x - 1)^{\frac{1}{-4}} + 3$	●	$t(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^{\frac{1}{s}} + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{\frac{1}{-4}} + 3$



$$g(x) = \frac{10}{4} (x - 1)^s + 3$$

$$\rightarrow \frac{10}{4} (x - 1)^{-6} + 3$$

$$h(x) = \frac{10}{4} (x - 1)^{\frac{1}{6}} + 3$$

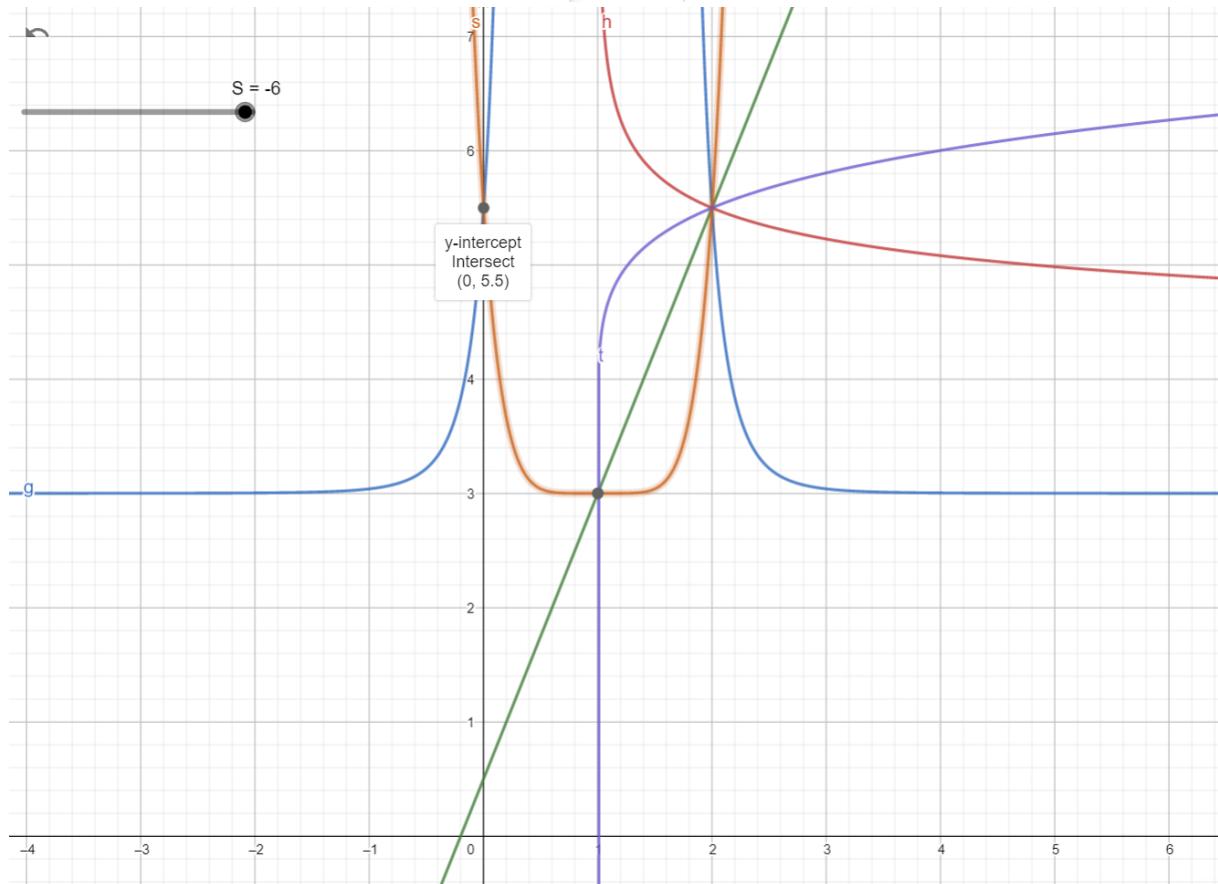
$$\rightarrow \frac{10}{4} (x - 1)^{\frac{1}{-6}} + 3$$

$$s(x) = \frac{10}{4} \left(\frac{1}{x-1} \right)^s + 3$$

$$\rightarrow \frac{10}{4} \left(\frac{1}{x-1} \right)^{-6} + 3$$

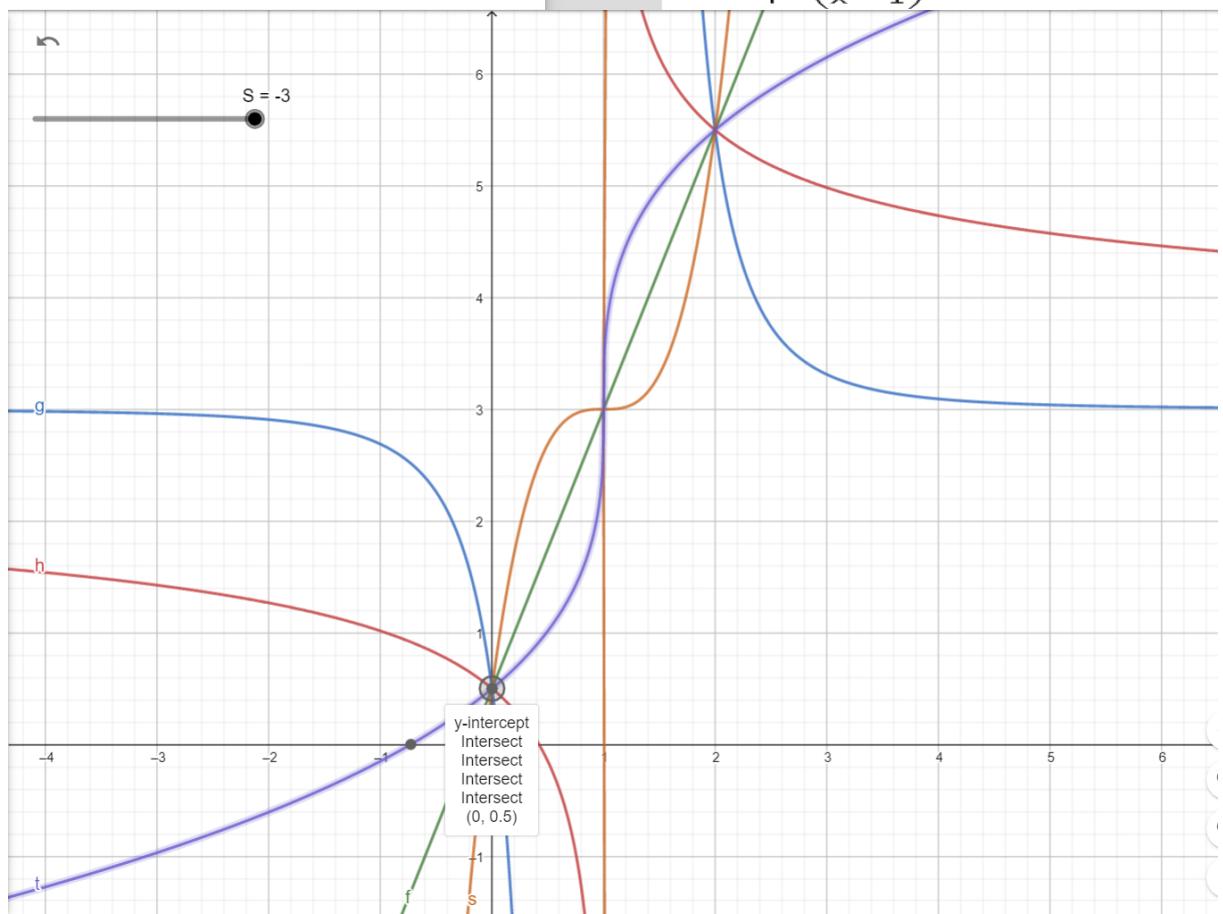
$$t(x) = \frac{10}{4} \left(\frac{1}{x-1} \right)^{\frac{1}{6}} + 3$$

$$\rightarrow \frac{10}{4} \left(\frac{1}{x-1} \right)^{\frac{1}{-6}} + 3$$



2- Odd values for $S \geq 3$

	$g(x) = \frac{10}{4} (x - 1)^S + 3$ $\rightarrow \frac{10}{4} (x - 1)^{-3} + 3$		$s(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^S + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{-3} + 3$
	$h(x) = \frac{10}{4} (x - 1)^{\frac{1}{S}} + 3$ $\rightarrow \frac{10}{4} (x - 1)^{\frac{1}{-3}} + 3$		$t(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^{\frac{1}{S}} + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{\frac{1}{-3}} + 3$

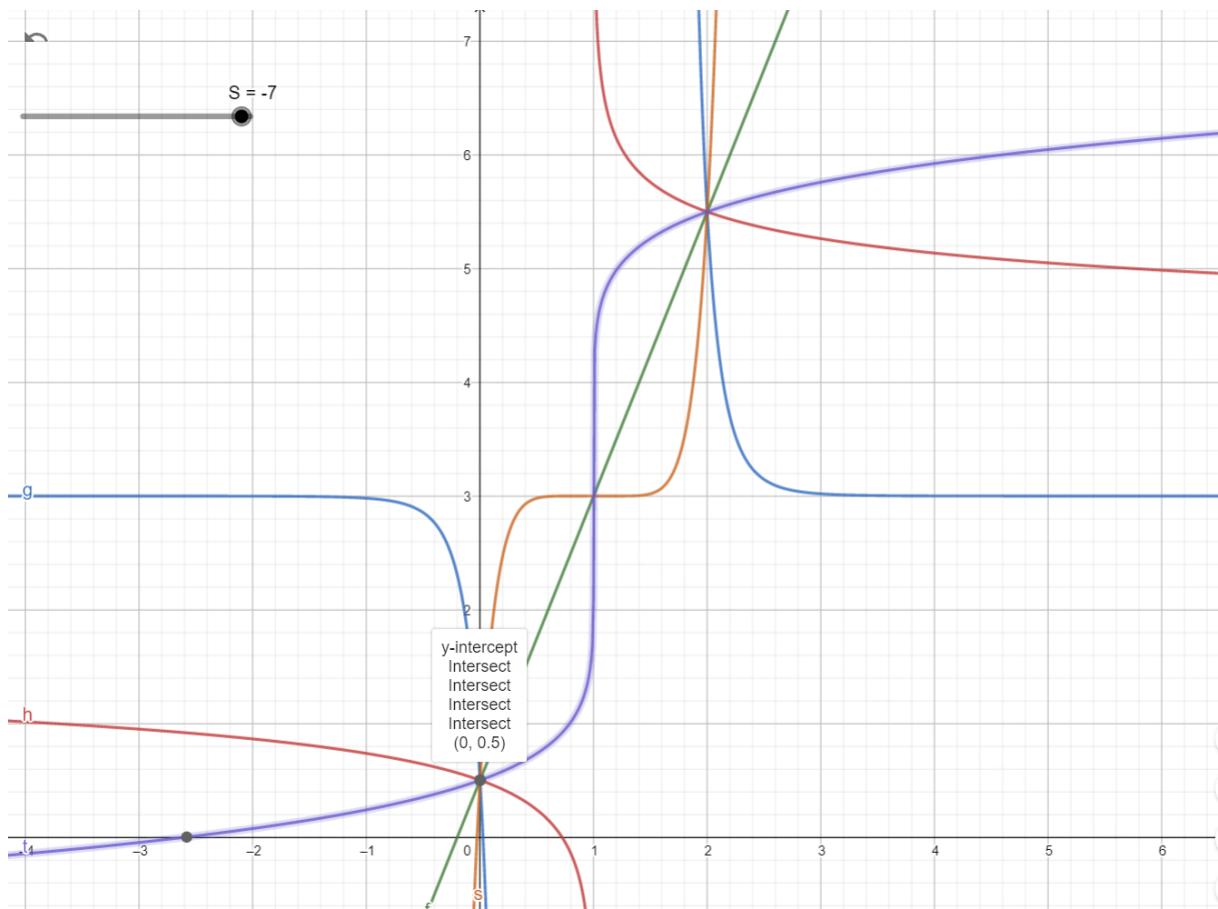


 $s(x) = \frac{10}{4} \left(\frac{1}{x-1} \right)^s + 3$
 $\rightarrow \frac{10}{4} \left(\frac{1}{x-1} \right)^{-7} + 3$

 $g(x) = \frac{10}{4} (x-1)^s + 3$
 $\rightarrow \frac{10}{4} (x-1)^{-7} + 3$

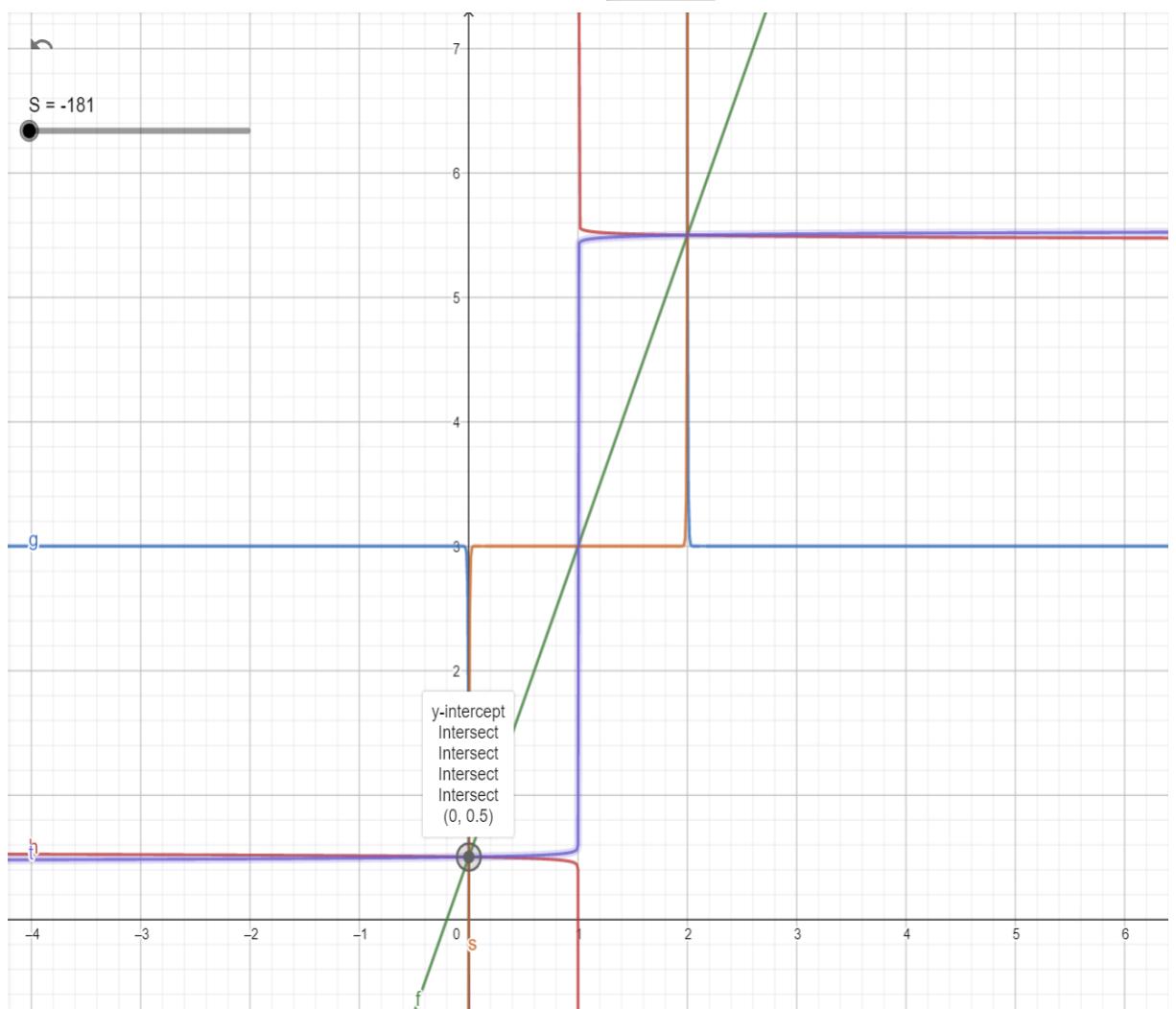
 $t(x) = \frac{10}{4} \left(\frac{1}{x-1} \right)^{\frac{1}{s}} + 3$
 $\rightarrow \frac{10}{4} \left(\frac{1}{x-1} \right)^{\frac{1}{-7}} + 3$

 $h(x) = \frac{10}{4} (x-1)^{\frac{1}{s}} + 3$
 $\rightarrow \frac{10}{4} (x-1)^{\frac{1}{-7}} + 3$



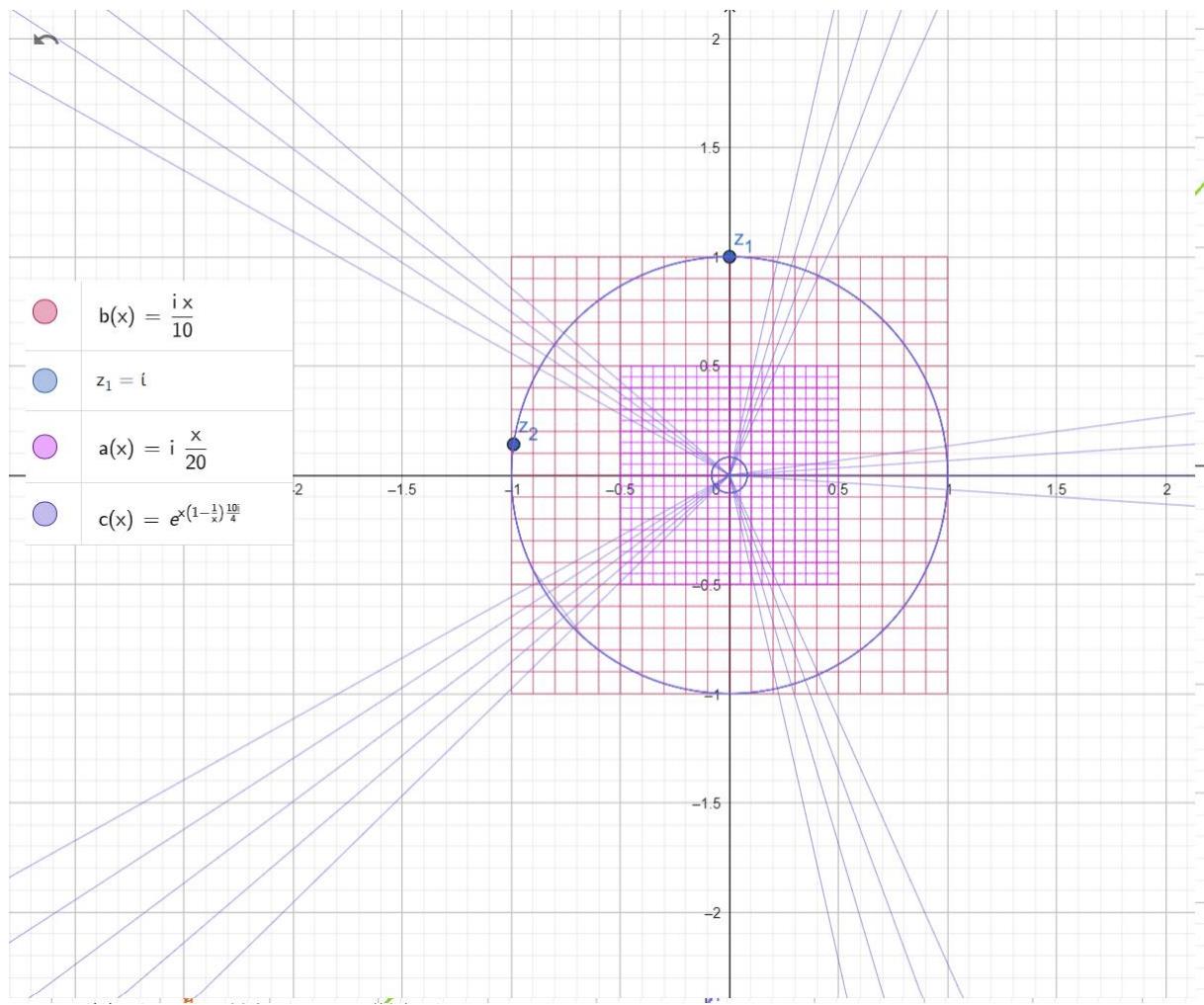
As S value increase this transformation tends to go to a STEP function with step at X = 1 (between 0.5 and 5.5). The next graph is for transformation at S = -181. And still intersects at (0, 0.5)

 $g(x) = \frac{10}{4} (x - 1)^s + 3$ $\rightarrow \frac{10}{4} (x - 1)^{-181} + 3$	 $s(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^s + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{-181} + 3$
 $h(x) = \frac{10}{4} (x - 1)^{\frac{1}{s}} + 3$ $\rightarrow \frac{10}{4} (x - 1)^{-\frac{1}{181}} + 3$	 $t(x) = \frac{10}{4} \left(\frac{1}{x-1}\right)^{\frac{1}{s}} + 3$ $\rightarrow \frac{10}{4} \left(\frac{1}{x-1}\right)^{-\frac{1}{181}} + 3$



3.3 How this transformation formula reflects on the imaginary unit Circle and its 21 lines.

We should only have 20 lines, because our number system is based 10 and the frame of reference is symmetric around Y and X axis, but we need 21 lines to create 20 smaller squares. when a frame of reference is manifold in complex plane, we get one more line form the edge of folding which will work like an indicator for the folding direction as a fold guide. (One of the line groups branches will going to have 5 lines).

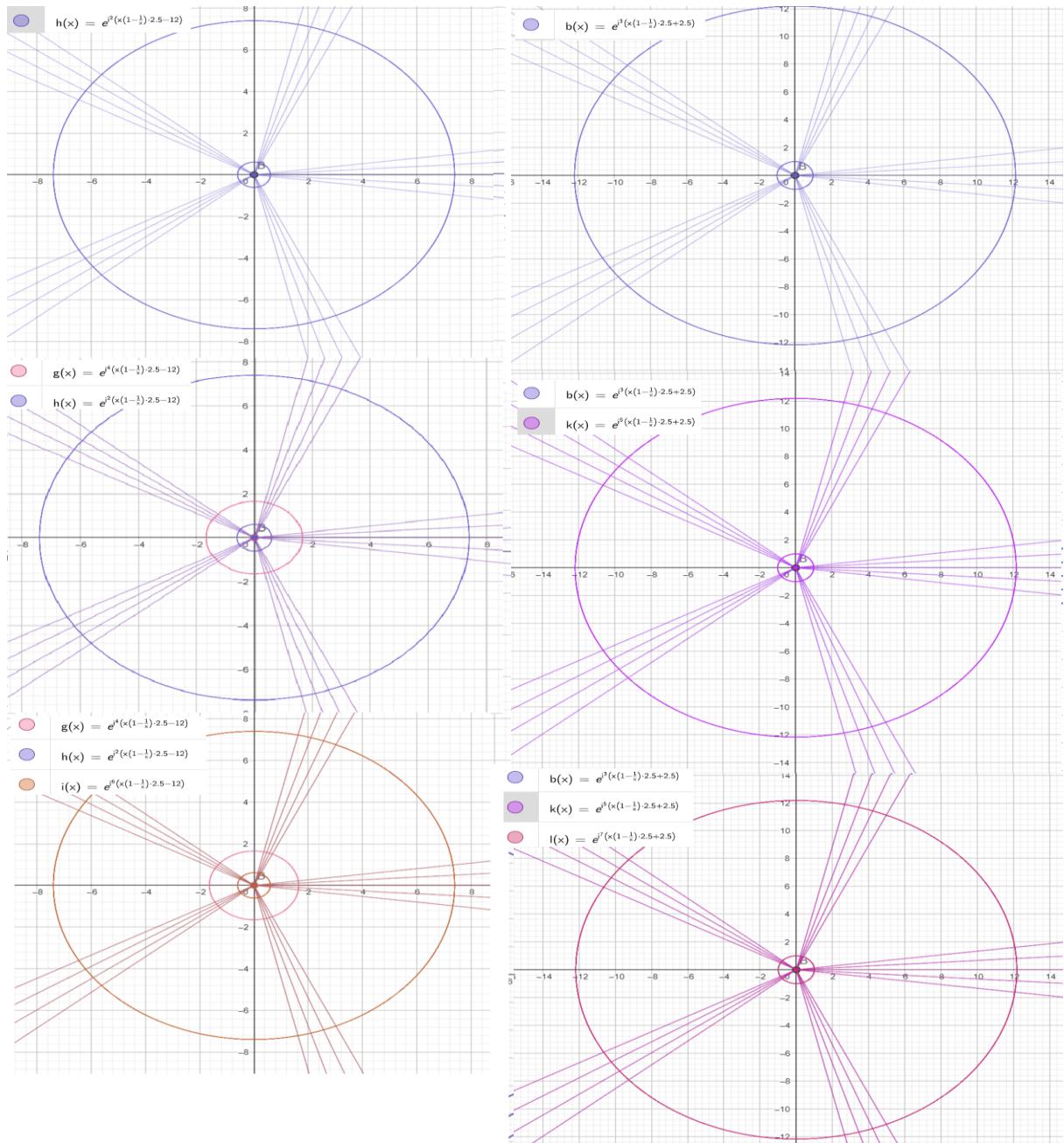


3.2.1 Transformation formula for even and odd powers of [i]

$$\text{Odd power formula} = \quad a(x) = e^{i^n(x(1-\frac{1}{x}) \cdot 2.5 + 2.5)}$$

$$\text{Even power formula} = \quad g(x) = e^{i^n(x(1-\frac{1}{x}) \cdot 2.5 - 12)}$$

Note: - that using these two formulas will make the positions of the folding lines do not change or rotate; no matter what value of n you are using as a power in the transformation.



4. Results

First, we introduced a frame of reference concept and the usage of mathematical transformation on this frame of reference for Euler's Identity manifold in a complex plane. Then we studied multiple mathematical transformations on the frame of reference and its visualizations. Then we got through the manifold and unfolding in complex plane using this frame of reference transformation. Then we presented another way to visualize Euler's Identity in a complex plane using the concept of a frame of reference transformation. Then we got an explanation for the strip line in the current analytical continuity visualization for zeta function in a complex plane. Finally, we presented a new formal for a frame of reference transformation that can work from both even and odd numbers.

As a conclusion, using a frame of reference is more efficient in term of the calculations and approximations. Instead of relying on two rational numbers (i , $i\pi$) in our transformation, which makes complex calculations and give us a distortion for transformed shape that make it harder for us to visualize and understand the complex plane manifold.

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<https://www.geogebra.org/>

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