New Odd Numbers Identity and The None-trivial Zeros of Zeta Function

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Abstract

This paper is going to introduce a new identity unit circle function for complex plane specific for odd numbers.

Second, we are going to show some properties of these new unit Identity function.

Third, use this new unit Identity function to study the distribution of odd roots for sin term in zeta function but using the new identity function not Euler Identity to explain Riemann conjunction about the critical strip line and the none-trivial zeros along Re(S) = 0.5.

Riemann's functional equation

$$\zeta(s) = 2^s \pi^{s-1} \ \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s).$$

The Riemann zeta function on the critical line can be written

$$\zeta\left(\frac{1}{2}+it\right) = e^{-i\theta(t)} Z(t),$$

$$Z(t) = e^{i\theta(t)} \zeta\left(\frac{1}{2}+it\right).$$

Then Zeta function will be zero

1- At $\sin\left(\frac{\pi s}{2}\right)$ is Zero for any complex number S.

2- If exponential term is zero also when S = S + 0.5 where S is any complex number.

Keywords: zeta function, Riemann hypothesis, complex plane, none-trivial zeros, critical strip, gamma function

1. Introduction

A) $f(x) = z^x \ = \ (\pm \frac{1}{\sqrt{2}} \pm i \ \frac{1}{\sqrt{2}})^x$ new Identity function for odd number in a complex plane.

our objective in this paper to show how this new Identity function f(x) shows odd number distribution, which is the same as imaginary unit Identity but with angel θ = π/4 = 180º/4.

$$\theta * 2 * x = \frac{\pi}{8} * 2 * \ x = 22.5° * 2\ x = 45° * x$$

$$f(x) = (\pm \frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^x \ = Cos(x\,\theta) + \ i\, Sin(\ x\,\theta)$$

first, we will explore eigen characteristics of Sin and Cos waves then we will visualize these characteristics more using this new introduced Identity for odd numbers in complex plane.

1.1 Eigen characteristics for geometric functions $Cos(\pi * \sqrt{X})$ *and* $Sin(\pi * \sqrt{X})$ *roots distributions.*

We will see here that each of these two geometric functions has its own characteristic in terms of, the distribution of its Zeros in its wave signals, regardless of scaling or shifting transformations.

1- Eigen characteristics for $Cos(\pi * \sqrt{X})$ roots distribution

These geometric function Cos wave have roots distributed in specific values for X.
Starting from a start point with specific jumping steps up and until the root.
{+2, +4, +6, +8, + 10, +12, +14, +16….}

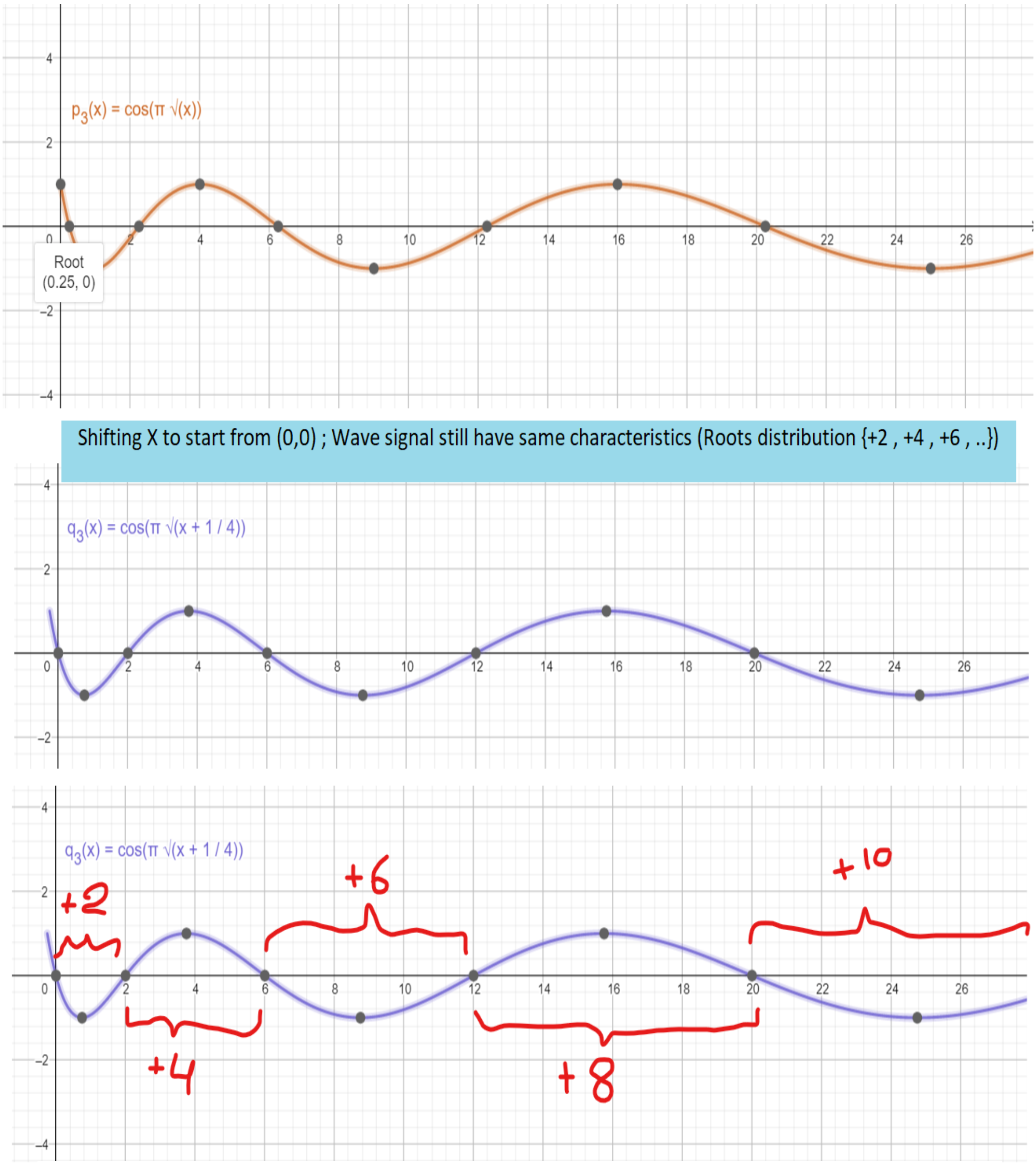


Figure (1): Roots distribution characteristics for $Cos(\pi * \sqrt{X})$ and $Cos\left(\pi * \sqrt{X + \frac{1}{4}}\right)$

2- Eigen characteristics for $Sin(\pi * \sqrt{X})$ roots distribution

These geometric function Sin wave have roots distributed in specific values for X.
Starting from a start point with specific jumping steps up and until to the root.
{+1, +3, +5, +7, +9, +11, +13, +16…….}

** For every natural value in {1, 2, 3, 4, 5, 6, 7, 8…}

***We get roots at its square {1, 4, 9, 16, 25, 36, 49, 64, 81…}

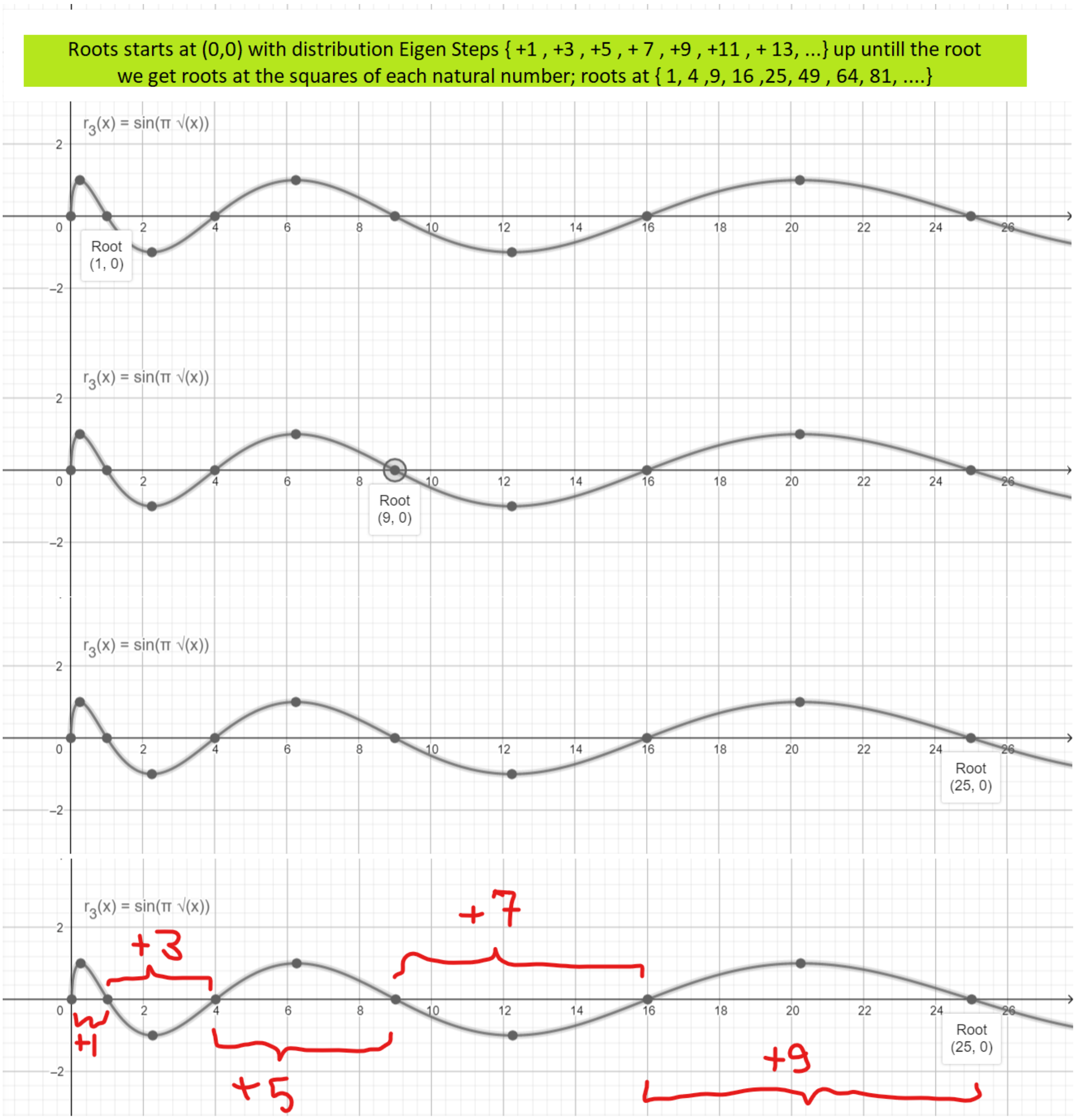


Figure (2): Roots distribution characteristics for $Sin(\pi * \sqrt{X})$

3- Eigen characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$ roots distribution

These geometric function Sin wave have roots distributed in specific values for X. Starting from a start point with specific jumping steps up and until to the root.
{+1, +3, +5, +7, +9, +11, +13, +16…….}

** For every natural value in {1π, 2π, 3π, 4, 5, 6, 7, 8…}

***We get roots at its square {1π, 4π, 9π, 16π, 25π, 36π, 49π, 64π, 81π…}

Roots distribution for [pi] have same charachteristics distributions {+1 ,+3, +5, +7..}

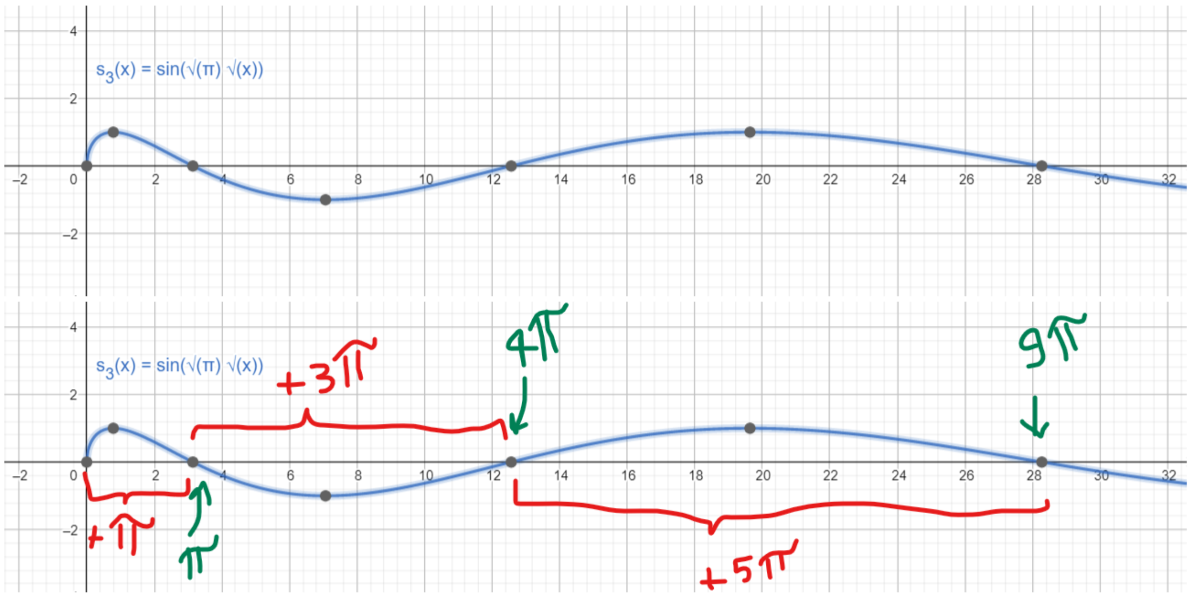
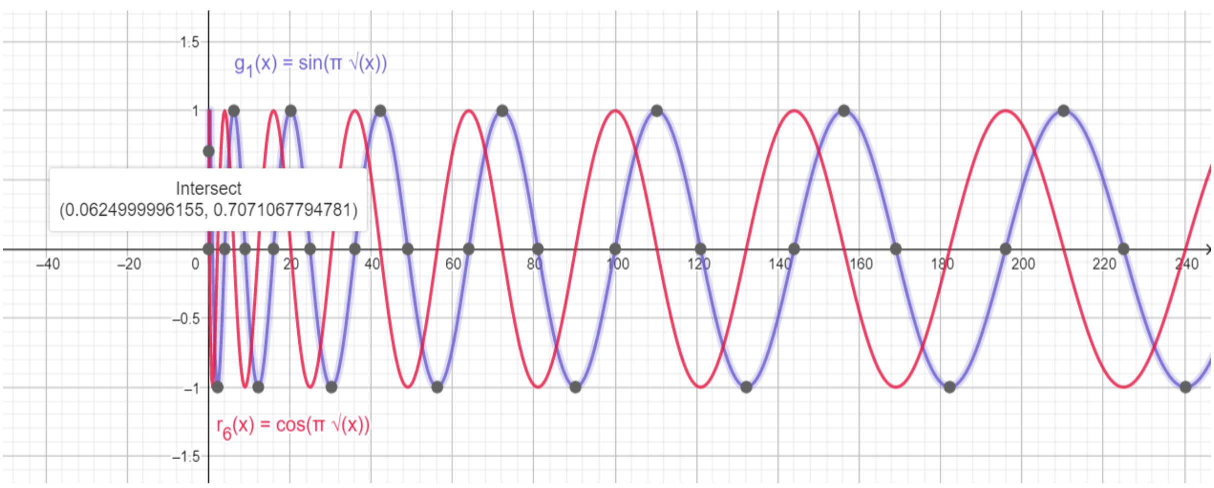


Figure (3): Root distribution characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$

4- characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$ and $Cos(\pi * \sqrt{x})$



$Cos(\pi * \sqrt{x})$ and $Sin(\pi * \sqrt{x})$ intersects at ($\frac{1}{16}$, $\frac{1}{\sqrt{2}}$)

Figure 4. $Cos(\pi * \sqrt{x})$ and $Sin(\pi * \sqrt{x})$ intersects at (1/16, $\frac{1}{\sqrt{2}}$)

5- Eigen characteristics for $Sin\left(\frac{\pi\, S}{2}\right)$ in Zeta function formula

$$\zeta(1-S) = \frac{2}{(2\,\pi)^S}\, Cos\left(\frac{\pi\, S}{2}\right)\, \Gamma(S)\, \zeta(S) \;\rightarrow \text{EQ [1]}$$

$$\zeta(S) = 2^S\, \pi^{S-1}\, Sin\left(\frac{\pi\, S}{2}\right)\, \Gamma(1-S)\, \zeta(1-S) \;\rightarrow \text{EQ [2]}$$

$$Cos\left(\frac{\pi}{8} * 2 * X\right) = Sin\left(\frac{\pi}{8} * (2 * X + 4)\right) \;\rightarrow \text{EQ [3]}$$

$$Cos\left(\frac{\pi}{2} * \frac{X}{2}\right) = Sin\left(\frac{\pi}{2} * \left(\frac{X}{2} + 1\right)\right) \;\rightarrow \text{EQ [4]}$$

$$Cos\left(\frac{\pi}{2} * S\right) = Sin\left(\frac{\pi}{2} * (S + 1)\right) \;\rightarrow \text{EQ [5]}$$

$$Cos\left(\frac{\pi}{2} * \left(S - \frac{1}{2}\right)\right) = Sin\left(\frac{\pi}{2} * \left(S + \frac{1}{2}\right)\right) \;\rightarrow \text{EQ [6]}$$

$$Cos\left(\frac{\pi}{2} * (S - 1)\right) = Sin\left(\frac{\pi}{2} * S\right) \;\rightarrow \text{EQ [7]}$$

$$Cos\left(\frac{\pi}{2} * \left(S - \frac{3}{2}\right)\right) = Sin\left(\frac{\pi}{2} * \left(S - \frac{1}{2}\right)\right) \;\rightarrow \text{EQ [8]}$$

$$\Gamma(S)\, \Gamma(1-S) = \frac{\pi}{Sin(\pi S)} \rightarrow \text{EQ [9]}$$

From EQ [1] and EQ [2]

$$\zeta(S) = 2^S\, \pi^{S-1}\, Sin\left(\frac{\pi\, S}{2}\right)\, \Gamma(1-S) * \frac{2}{(2\,\pi)^S}\, Cos\left(\frac{\pi\, S}{2}\right)\, \Gamma(S)\, \zeta(S)$$

$$1 = 2^S\, \pi^{S-1}\, Sin\left(\frac{\pi\, S}{2}\right)\, \Gamma(1-S) * \frac{2}{(2\,\pi)^S}\, Cos\left(\frac{\pi\, S}{2}\right)\, \Gamma(S)$$

$$1 = 2^S \pi^{S-1} Sin\left(\frac{\pi S}{2}\right) * \frac{2}{(2\pi)^S} Cos\left(\frac{\pi S}{2}\right) \Gamma(S) \Gamma(1-S)$$

From EQ [5]

$$1 = Sin\left(\frac{\pi S}{2}\right) * Cos\left(\frac{\pi S}{2}\right) * \frac{2^S \pi^{S-1} * 2}{(2\pi)^S} \Gamma(S) \Gamma(1-S)$$

$$1 = Sin\left(\frac{\pi S}{2}\right) * Sin(\frac{\pi}{2} * (S+1)) * \frac{2}{\pi} * \Gamma(S) \Gamma(1-S)$$

From EQ [9]

$$1 = Sin\left(\frac{\pi S}{2}\right) * Sin(\frac{\pi}{2} * (S+1)) * \frac{2}{\pi} * \Gamma(S) \Gamma(1-S)$$

$$1 = Sin\left(\frac{\pi S}{2}\right) * Sin\left(\frac{\pi}{2} * (S+1)\right) * \frac{2}{\pi} * \frac{\pi}{Sin(\pi S)}$$

$$\frac{Sin\left(\frac{\pi S}{2}\right) * Sin\left(\frac{\pi}{2} * (S+1)\right)}{Sin(\pi S)} = \frac{1}{2}$$

$$2 * Sin\left(\frac{\pi S}{2}\right) * Sin\left(\frac{\pi}{2} * (S+1)\right) = Sin(\pi S)$$

$2 * Sin\left(\frac{\pi S}{2}\right) * cos\left(\frac{\pi S}{2}\right) = Sin(\pi S)$ ➔ EQ [10]

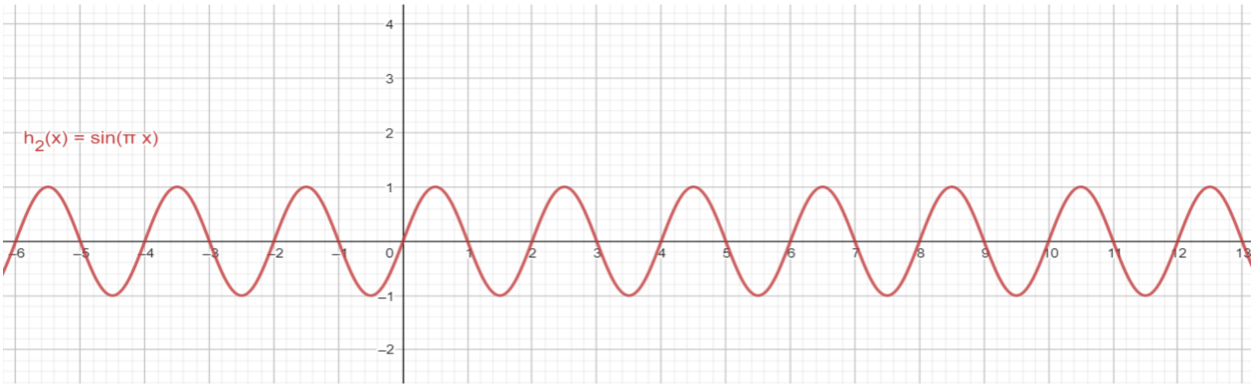


Figure 5. $Sin(\pi S)$➔ Have roots for any natural number value for ±S.

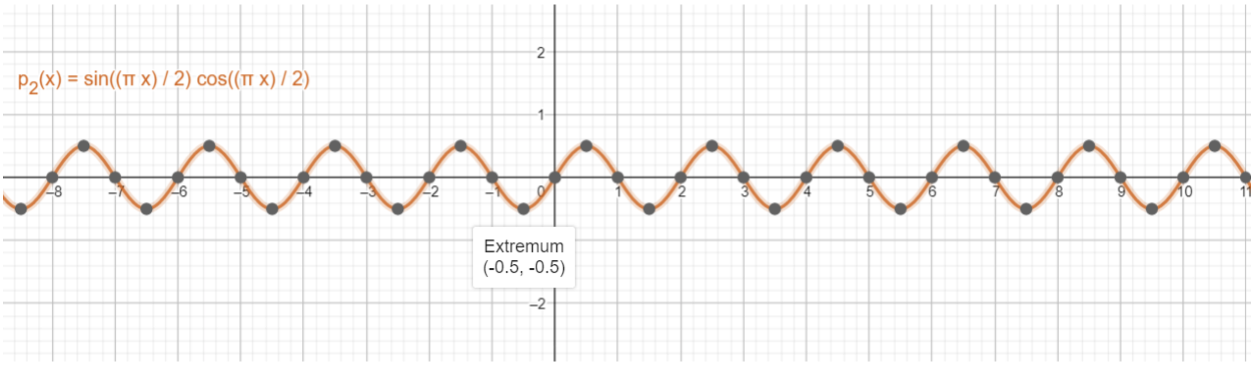


Figure 6. $Sin\left(\frac{\pi S}{2}\right) * \ cos\left(\frac{\pi S}{2}\right)$ ➔ have roots for any natural number value for ±S.

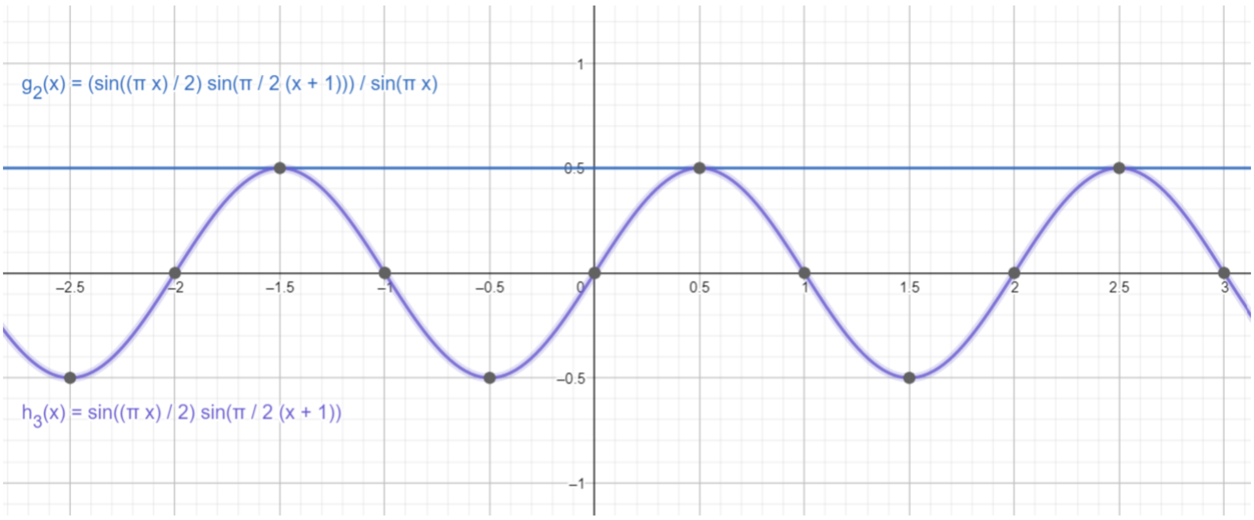


Figure 7. $\frac{Sin\left(\frac{\pi S}{2}\right) * \ Sin\left(\frac{\pi}{2} * (S+1)\right)}{Sin(\pi S)} = \frac{1}{2}$ ➔blue line; equal $\frac{1}{2}$ for any natural number value for ±S.

Back substitution from EQ [5]

$Sin\left(\frac{\pi S}{2}\right) = \frac{Sin(\pi S)}{2 * Cos(\frac{\pi S}{2})}$ ➔ EQ [11]

$Cos\left(\frac{\pi S}{2}\right) = \frac{Sin(\pi S)}{2 * Sin(\frac{\pi S}{2})}$ ➔ EQ [12]

From EQ [6] Substitute back in EQ [2]

$$\zeta(S) = 2^S \ \pi^{S-1} \ Cos\left(\frac{\pi}{2} \ (S-1)\right) \ \Gamma(1-S) \ \zeta(1-S)$$

Let S = S + 0.5

$$\zeta\left(S+\frac{1}{2}\right) = (2*\pi)^{\,S} * \sqrt{\frac{2}{\pi}} * \;\; Cos\left(\frac{\pi}{2}\,(S-\frac{1}{2})\right)\,\Gamma\left(\frac{1}{2}-S\right)\,\zeta\left(\frac{1}{2}-S\right)$$

From EQ [6]

$Cos\left(\frac{\pi}{2} * (S-\frac{1}{2})\right) = Sin(\frac{\pi}{2} * (S+\frac{1}{2}))$ → EQ [6]

$$\zeta\left(S+\frac{1}{2}\right) = (2*\pi)^{\,S} * \sqrt{\frac{2}{\pi}} * \;\; Sin\left(\frac{\pi}{2}\,(S+\frac{1}{2})\right)\,\Gamma\left(\frac{1}{2}-S\right)\,\zeta\left(\frac{1}{2}-S\right)$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} (2*\pi)^{\,S} * \sqrt{\frac{2}{\pi}} * \;\; Sin\left(\frac{\pi}{2}\,(S+\frac{1}{2})\right)\,\Gamma\left(\frac{1}{2}-S\right)\,\zeta\left(\frac{1}{2}-S\right) \\ (2*\pi)^{\,S} * \sqrt{\frac{2}{\pi}} * \;\; Cos\left(\frac{\pi}{2}\,(S-\frac{1}{2})\right)\,\Gamma\left(\frac{1}{2}-S\right)\,\zeta\left(\frac{1}{2}-S\right) \end{cases}$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} (2*\pi)^{S} * \sqrt{\frac{2}{\pi}} * \;\; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) = 0\,;\; when\ S\ odd \\ (2*\pi)^{S} * \sqrt{\frac{2}{\pi}} * \;\; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) = 0; when\ S\ odd \end{cases} \quad \rightarrow EQ(A)$$

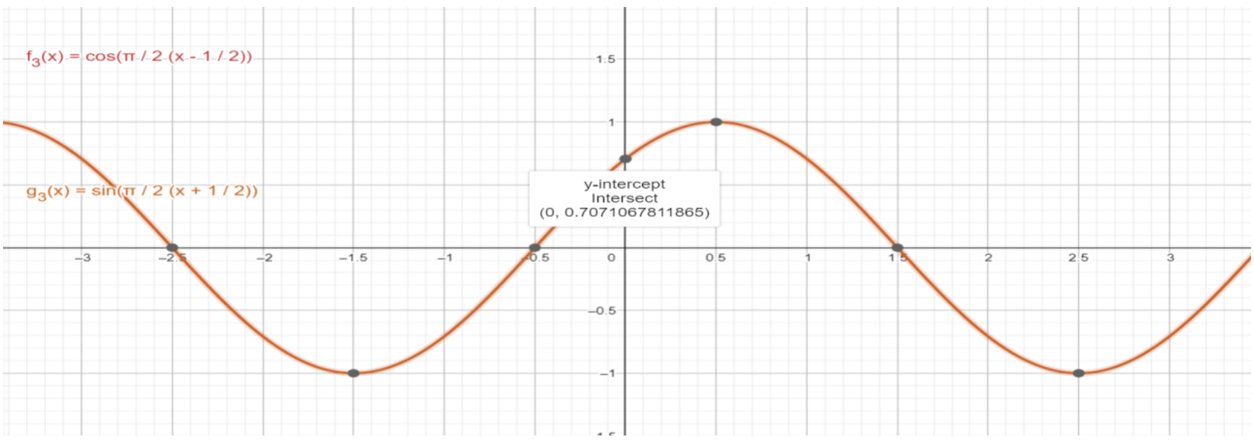


Figure 8. $Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)$ $and\ Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)$ → Identical with roots at $[\frac{-1}{2}]$ and Y-intercept = $\frac{1}{\sqrt{2}}$; for any ±S

For S = S -1

$$\zeta\left(S-\frac{1}{2}\right)=\begin{cases}(2*\pi)^{S-1}*\sqrt{\frac{2}{\pi}}*\ Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{3}{2}-S\right)\zeta\left(\frac{3}{2}-S\right)=0\ ;\ when\ S\ odd\\(2*\pi)^{S-1}*\sqrt{\frac{2}{\pi}}*\ Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right)\Gamma\left(\frac{3}{2}-S\right)\zeta\left(\frac{3}{2}-S\right)=0;when\ S\ odd\end{cases}\quad \rightarrow EQ(B)$$

From EQ [8]; $Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right)=\ Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)$; *Identical wave signals*

Until now from EQ (A) and EQ (B) we showed that Zeta function will have Root at (S= -0.5)
And equal Zero if (S = S ± 0.5) for any odd number S.

Next, we will show how all these Zeros will have imaginary unit value by introducing new Identity function for odd number in a complex plane.

B) $f(x)=z^x\ =\ (\pm\frac{1}{\sqrt{2}}\pm i\ \frac{1}{\sqrt{2}})^x$ new Identity function for odd number in a complex plane.

our objective in this part is to show how this new Identity function f(x) shows odd number distribution, which is the same as imaginary unit Identity but with angel = π/4 = 180º/4.

$$\theta*2*x=\frac{\pi}{8}*2*\ x=22.5°*2\ x=45°*x$$

$$e^{i\theta x}=Cos(\theta x)+i\ Sin(\theta x)\qquad \rightarrow EQ(13)$$

$$z=\frac{1}{\sqrt{2}}+i\ \frac{1}{\sqrt{2}}\qquad \rightarrow EQ(14)$$

$$f(x)=z^x\ =\ (\pm\frac{1}{\sqrt{2}}\pm i\ \frac{1}{\sqrt{2}})^x\qquad \rightarrow EQ(15)$$

$$Cos\left(\frac{\pi}{8}*2*X\right)=Sin\left(\frac{\pi}{8}*(2*X+4)\right)\qquad \rightarrow EQ(16)$$

$$f(x)=\begin{cases}(\pm\frac{1}{\sqrt{2}}\pm\frac{i}{\sqrt{2}})^x\\\pm Cos(\theta*2x)\pm i\ Sin(\theta*2\ x)\end{cases}\qquad \rightarrow EQ(18)$$

At X = ½

$$f(x)=(\pm\frac{1}{\sqrt{2}}\pm i\ \frac{1}{\sqrt{2}})^{0.5}=Cos(22.5)+\ i\ Sin(\ 22.5)$$
$$=\ 0.9238795325113\ +\ 0.38268343236512i$$

$$Cos\left(\frac{\pi}{2}*\frac{x}{2}\right) = Sin(\frac{\pi}{2}*(\frac{x}{2}+1)) \rightarrow \text{EQ [19]}$$

$$Cos\left(\frac{\pi}{2}*(S-\frac{1}{2})\right) = Sin(\frac{\pi}{2}*(S+\frac{1}{2})) \rightarrow \text{EQ [20]}$$

$$Cos\left(\frac{\pi}{8}\right) = Sin(\frac{\pi}{4}+\frac{\pi}{8})$$

$$Cos\left(\frac{\pi}{2}*\frac{x}{4}\right) = Sin\left(\frac{\pi}{2}\left(\frac{x}{4}+1\right)\right)$$

$$Cos\left(\frac{\pi}{2}*\frac{x+0.5}{2}\right) = Cos\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) = Sin\left(\frac{\pi}{4}\left(x+\frac{5}{2}\right)\right) \rightarrow EQ(22)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-0.5}{2}\right) = Cos\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right) = Sin\left(\frac{\pi}{4}\left(x+\frac{3}{2}\right)\right) \rightarrow EQ(21)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-1.5}{2}\right) = Cos\left(\frac{\pi}{4}*\left(x-\frac{3}{2}\right)\right) = Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right) \rightarrow EQ(23)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-2.5}{2}\right) = Cos\left(\frac{\pi}{4}*\left(x-\frac{5}{2}\right)\right) = Sin\left(\frac{\pi}{4}\left(x-\frac{1}{2}\right)\right) \rightarrow EQ(24)$$

$$Cos\left(\frac{\pi}{2}*\left(\frac{x}{4}+\frac{1}{2}+1\right)\right) = Sin\left(\frac{\pi}{2}\left(\frac{x}{4}-\frac{1}{2}-1\right)\right)$$

$$Cos\left(\frac{\pi}{2}*\left(\frac{x}{4}+\frac{3}{2}\right)\right) = Sin\left(\frac{\pi}{2}\left(\frac{x}{4}-\frac{3}{2}\right)\right) \rightarrow EQ(25)$$

$$Cos\left(\frac{\pi}{8}*(x+6)\right) = Sin\left(\frac{\pi}{8}(x-6)\right) \rightarrow EQ(26)$$

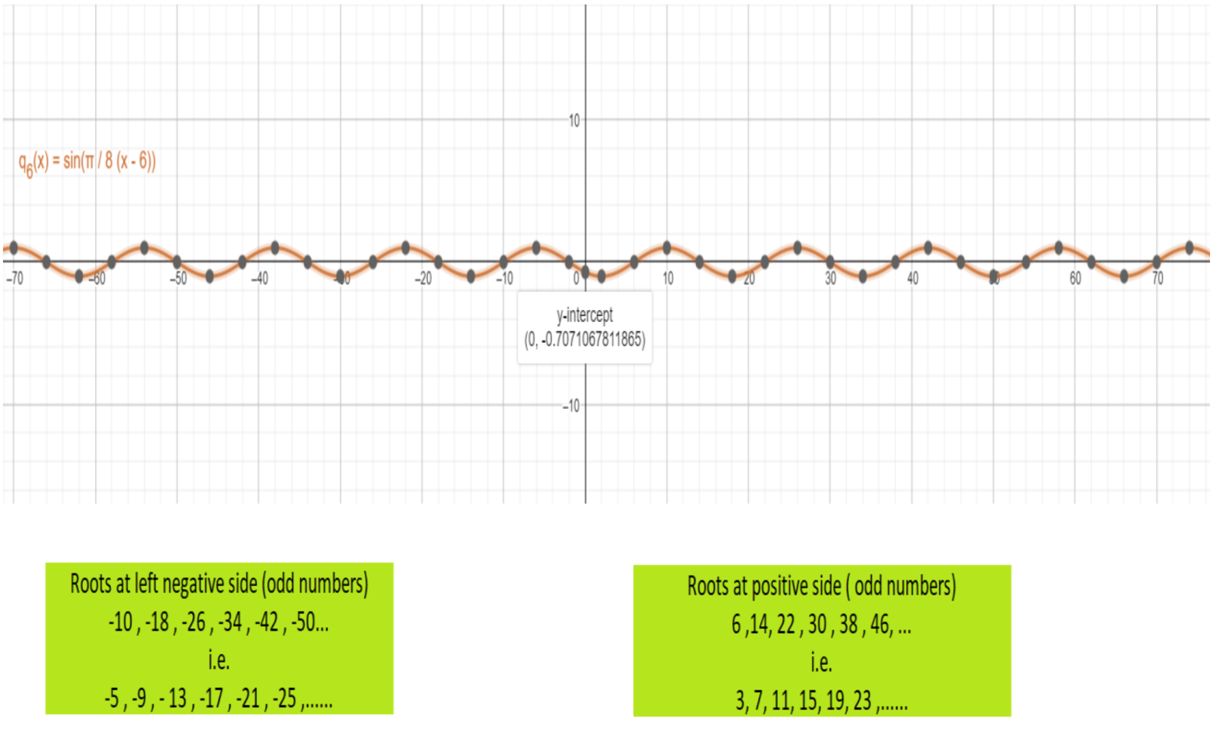


Figure 9. odd numbers Root distribution for $Sin\left(\frac{\pi}{8}\,(x-6)\right)$

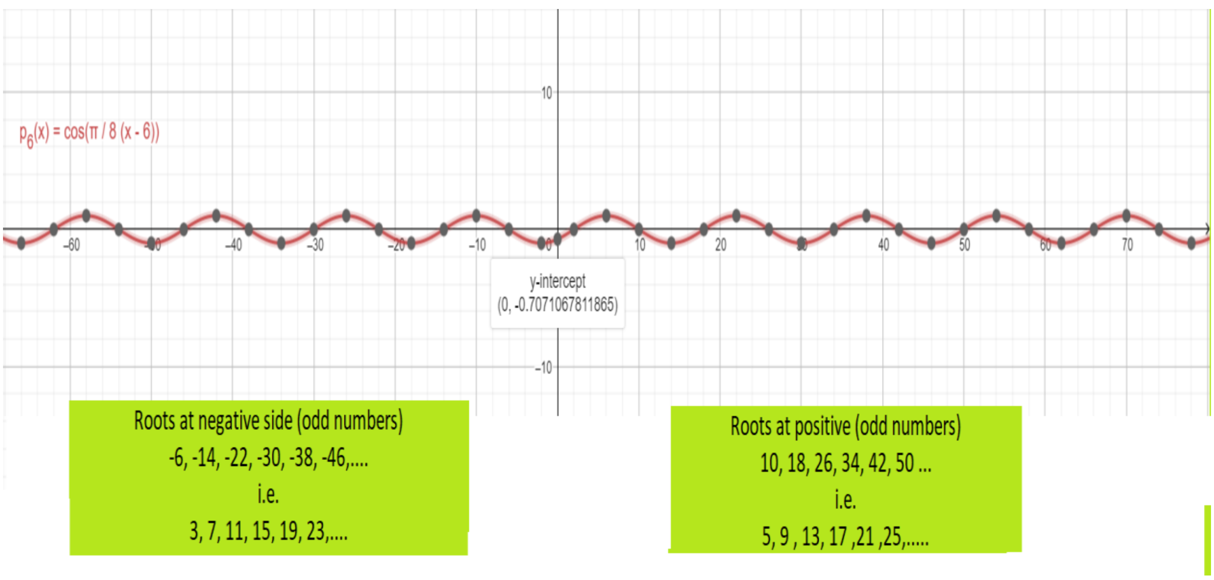


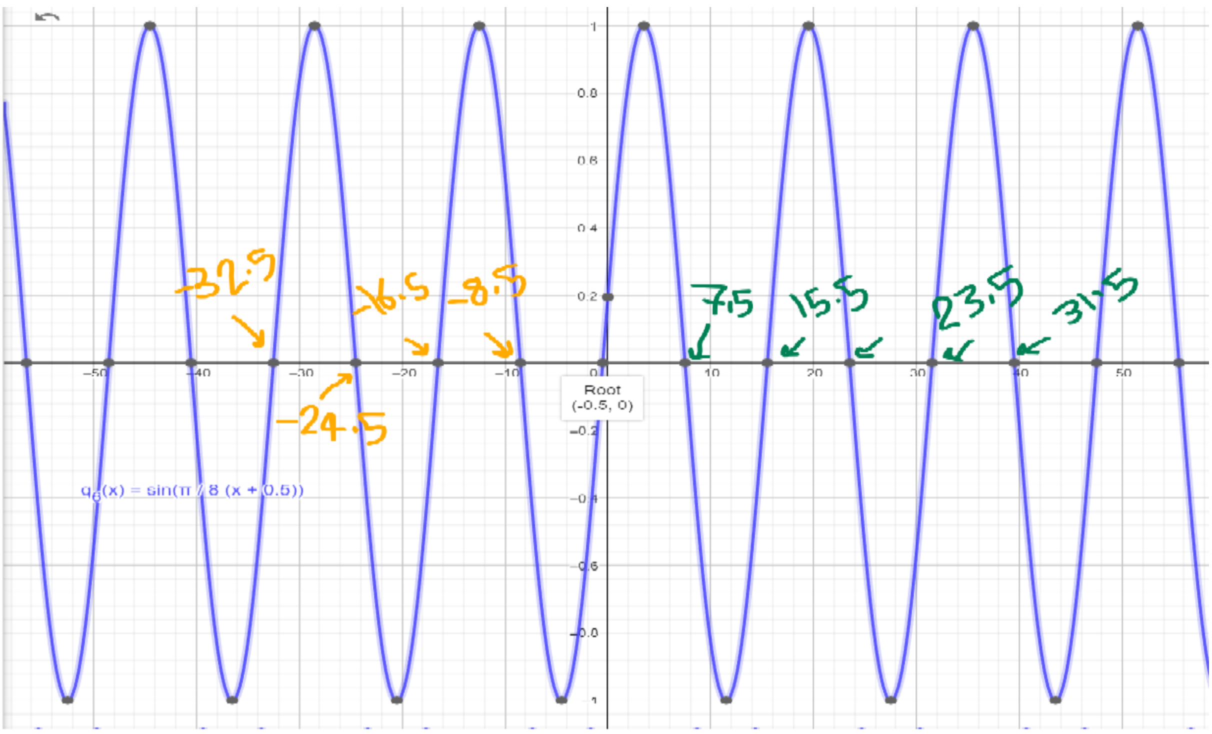
Figure 10. odd numbers Root distribution for $Cos\left(\frac{\pi}{8} * (x-6)\right)$

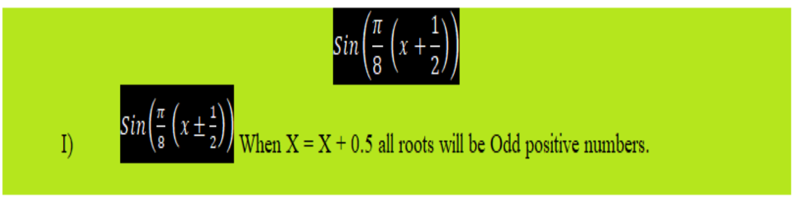
I) If we used degrees (π = 180º), and X = X ± 0.5 and θ = 22.5º THEN $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$; then wave signal will have Root at (±0.5,0)

II) $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$ When X = X - 0.5 all roots will be Odd negative numbers.

III) $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$ When X = X + 0.5 all roots will be Odd positive numbers.

IV) $For\ Sin(22.5 * \ (X + 0.5)\ );\ there\ will\ be\ Y = \frac{\pm 1}{\sqrt{2}}\ ;\ for\ X = \pm 0.5$




$Sin\left(\frac{\pi}{8}\left(x + \frac{1}{2}\right)\right)$

I) $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$ When X = X + 0.5 all roots will be Odd positive numbers.

Figure 11. using $\frac{\pi}{8}$ instead of 22.5 decrease the frequency so it is easier to see the roots.

Please note here we are using 22.5 as number not degrees. This will increase the sign wave frequency, but we will still have root at -0.5.

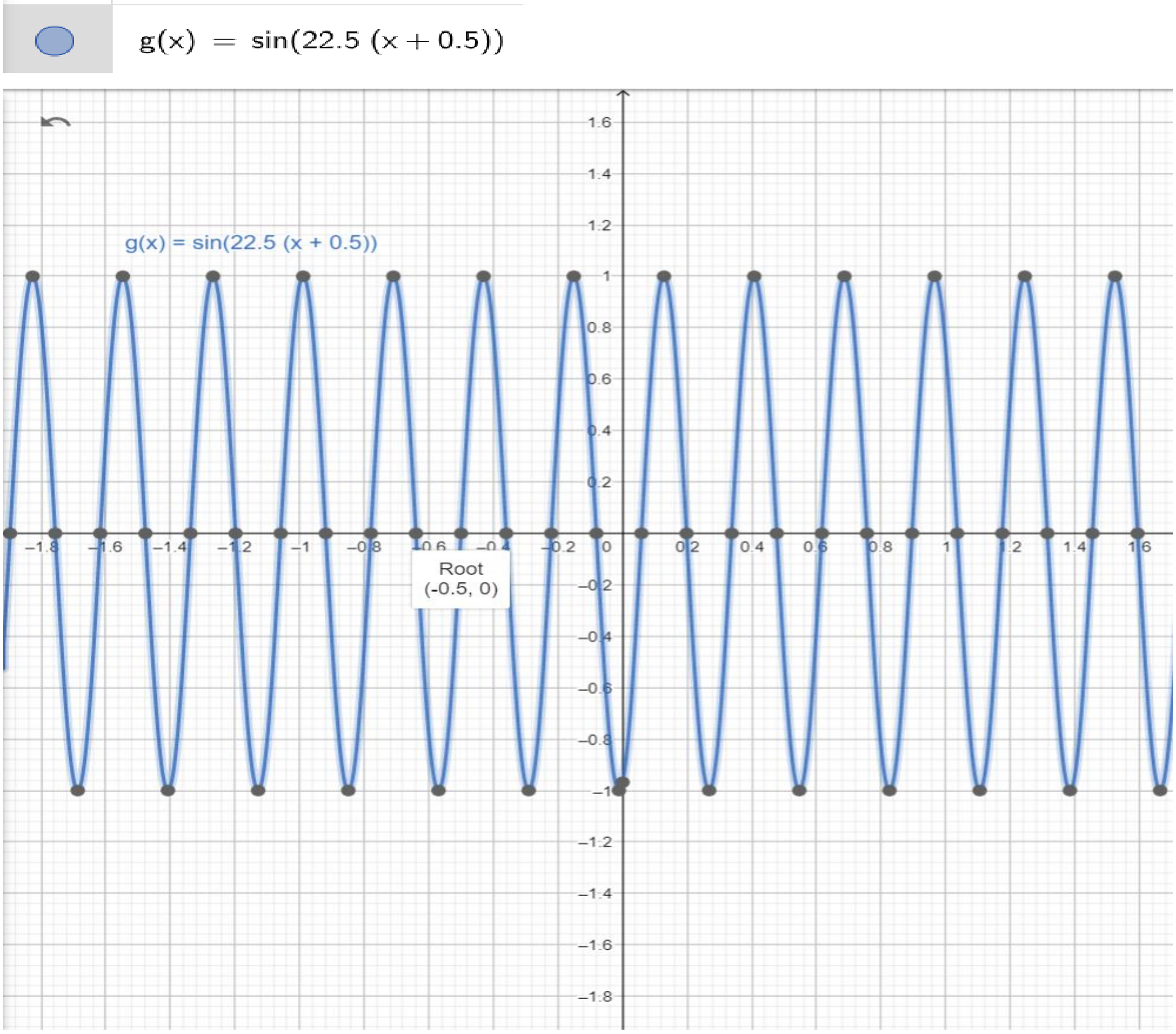


Figure 12. using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at -0.5.

$u_1 = \sin\left(\frac{45}{4}\right)$

$\rightarrow$ -0.9678079975113

$v_1 = \frac{45}{4}$

≈ 11.25

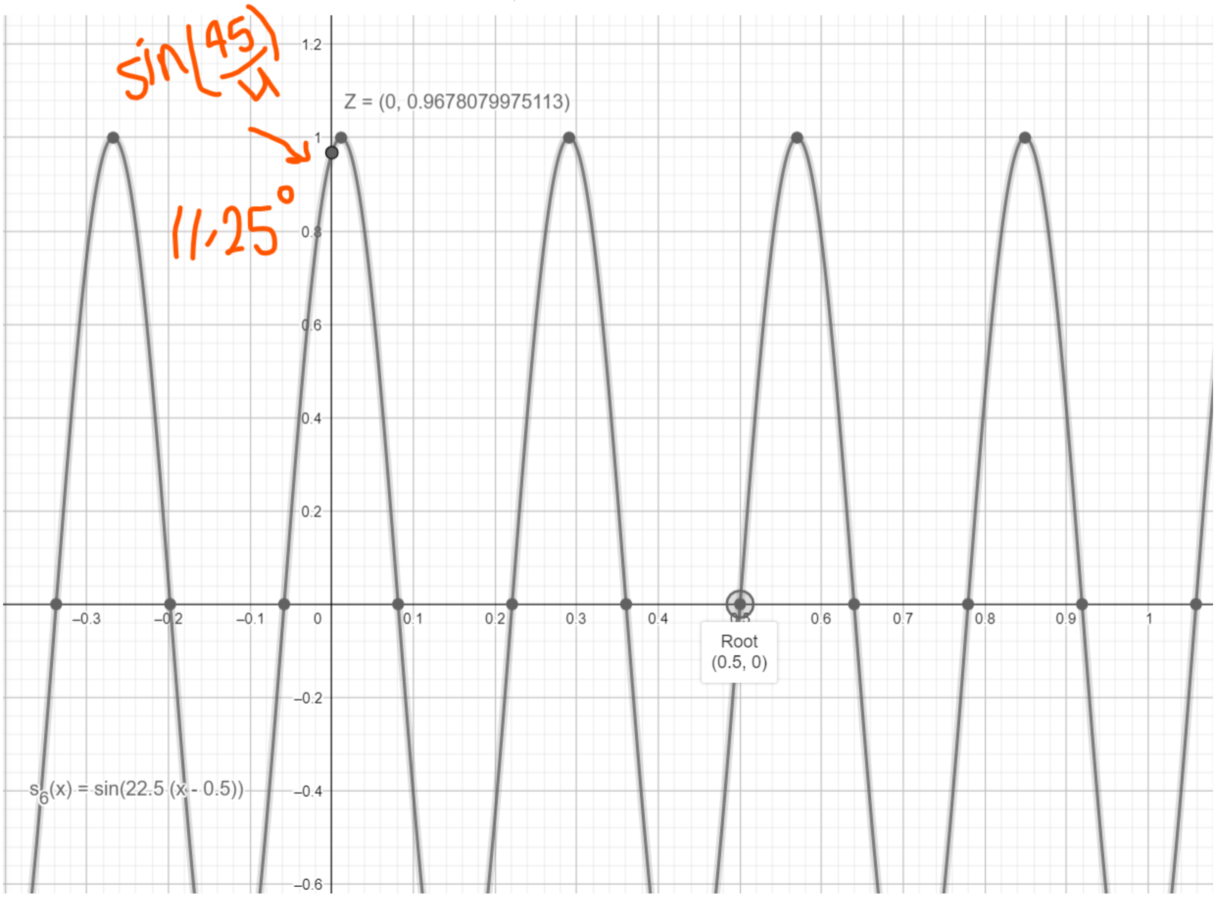
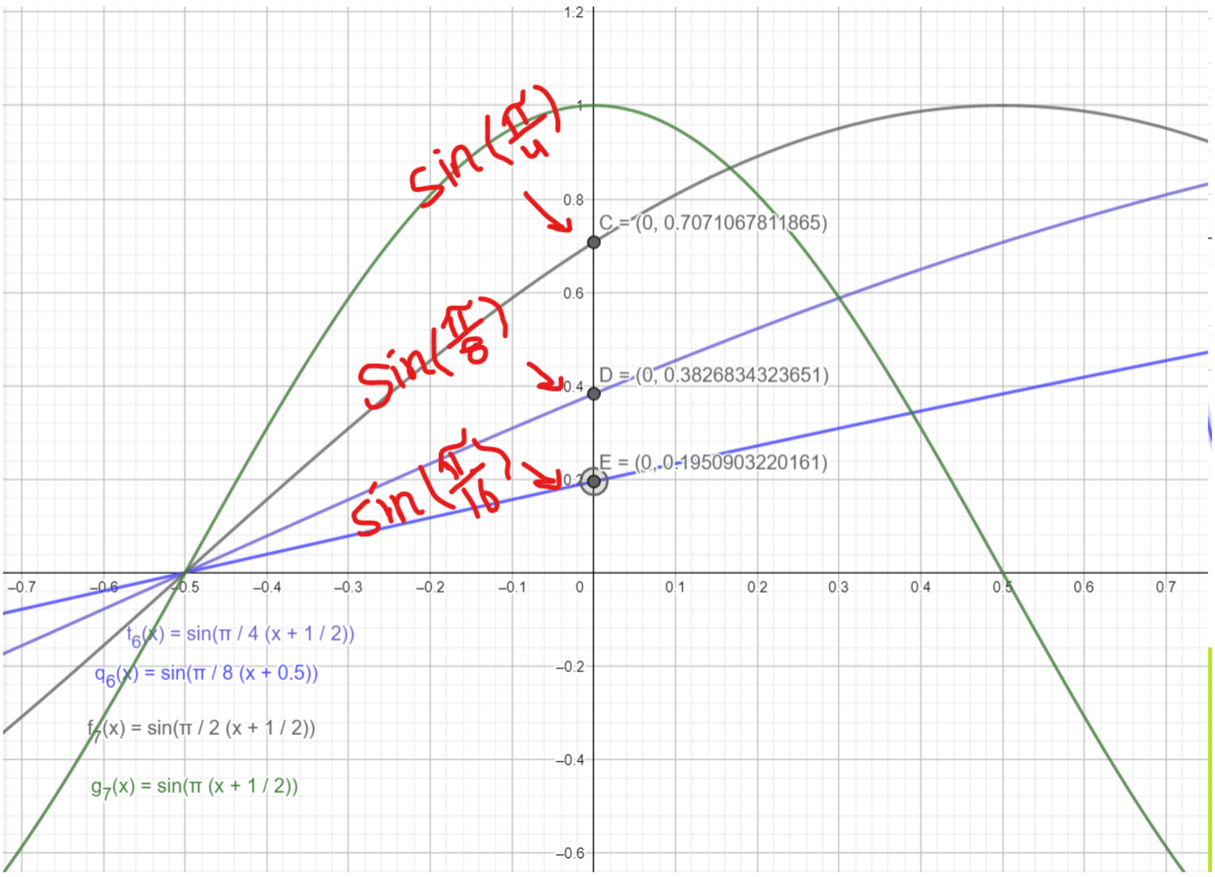
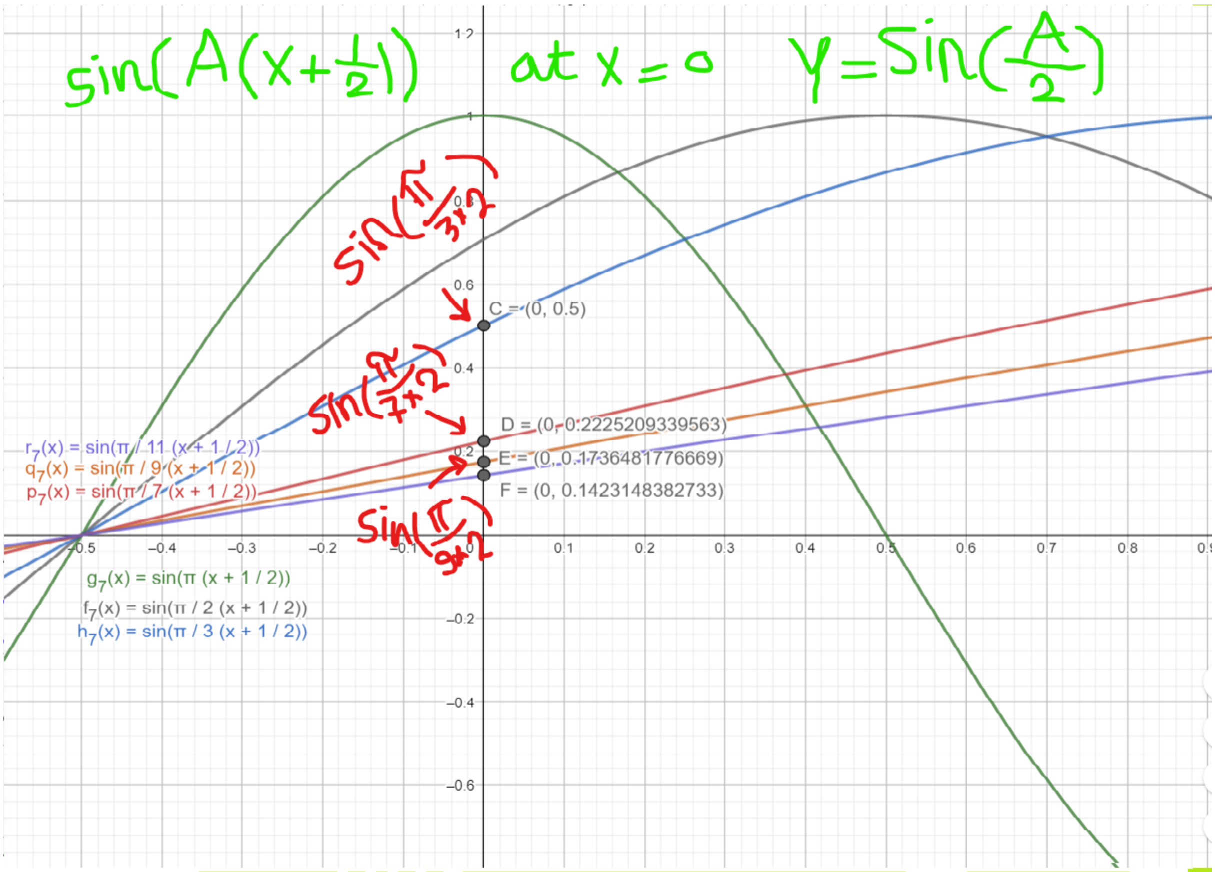


Figure 13. $Sin\left(22.5\left(x-\frac{1}{2}\right)\right)$ using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at 0.5.



$Sin\left(A*\left(x+\frac{1}{2}\right)\right)$ for any A = { $\frac{\pi}{1}$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, $\frac{\pi}{8}$, $\frac{\pi}{16}$,....} will have root at -0.5 and at X = 0 then Y = $Sin\left(\frac{A}{2}\right)$
in order to keep Sin wave characterstics, the width of the half Sin wave needs to be adjusted by the same ratio and this is why it keeps intersecting on Y axis because the slope keeps changing each time we change A each time we change A we change the number of partitions of pi by factor of 1/2.

Figure 14. $Sin\left(A*\left(x+\frac{1}{2}\right)\right)$ for any A even partitons for pi; the Sin wave will keep its charateriscts by adjusting its width and slope with the same ratio we partion pi with.



$Sin\left(A * \left(x+\frac{1}{2}\right)\right)$ for any A = { $\frac{\pi}{1}, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{8}, \frac{\pi}{16}$,....} will have root at -0.5 and at X = 0 then Y = $Sin\left(\frac{A}{2}\right)$
in order to keep Sin wave characterstics, the width of the half Sin wave needs to be adjusted by the same ratio and this is why it keeps intersecting on Y axis because the slope keeps changing each time we change A each time we change A we change the number of partitions of pi by factor of 1/2.

Figure 15. $Sin\left(A * \left(x+\frac{1}{2}\right)\right)$ for any A odd partitons for pi; the Sin wave will keep its charateriscts by adjusting its width and slope with the same ratio we partion pi with.

In the previous graph A can take any partition value $\{\frac{\pi}{1}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{5}, \frac{\pi}{6}, \frac{\pi}{7}, \ldots\ldots\}$

To keep Sin wave characteristics, changing A in $A * (x+\frac{1}{2})$ will adjust Sin wave width to keep root -1/2.

$$for\ A = \frac{\pi}{3}\ THEN\ A * \left(x+\frac{1}{2}\right) = \frac{\pi}{3}\left(x+\frac{1}{2}\right) = \frac{\pi}{3} * x + \frac{\pi}{6}$$

$$for\ A = \frac{\pi}{5}\ THEN\ A * \left(x+\frac{1}{2}\right) = \frac{\pi}{5}\left(x+\frac{1}{2}\right) = \frac{\pi}{5} * x + \frac{\pi}{10}$$

Y intercept will be = $\frac{A}{2}$ for any A. (A is partition segments for π)

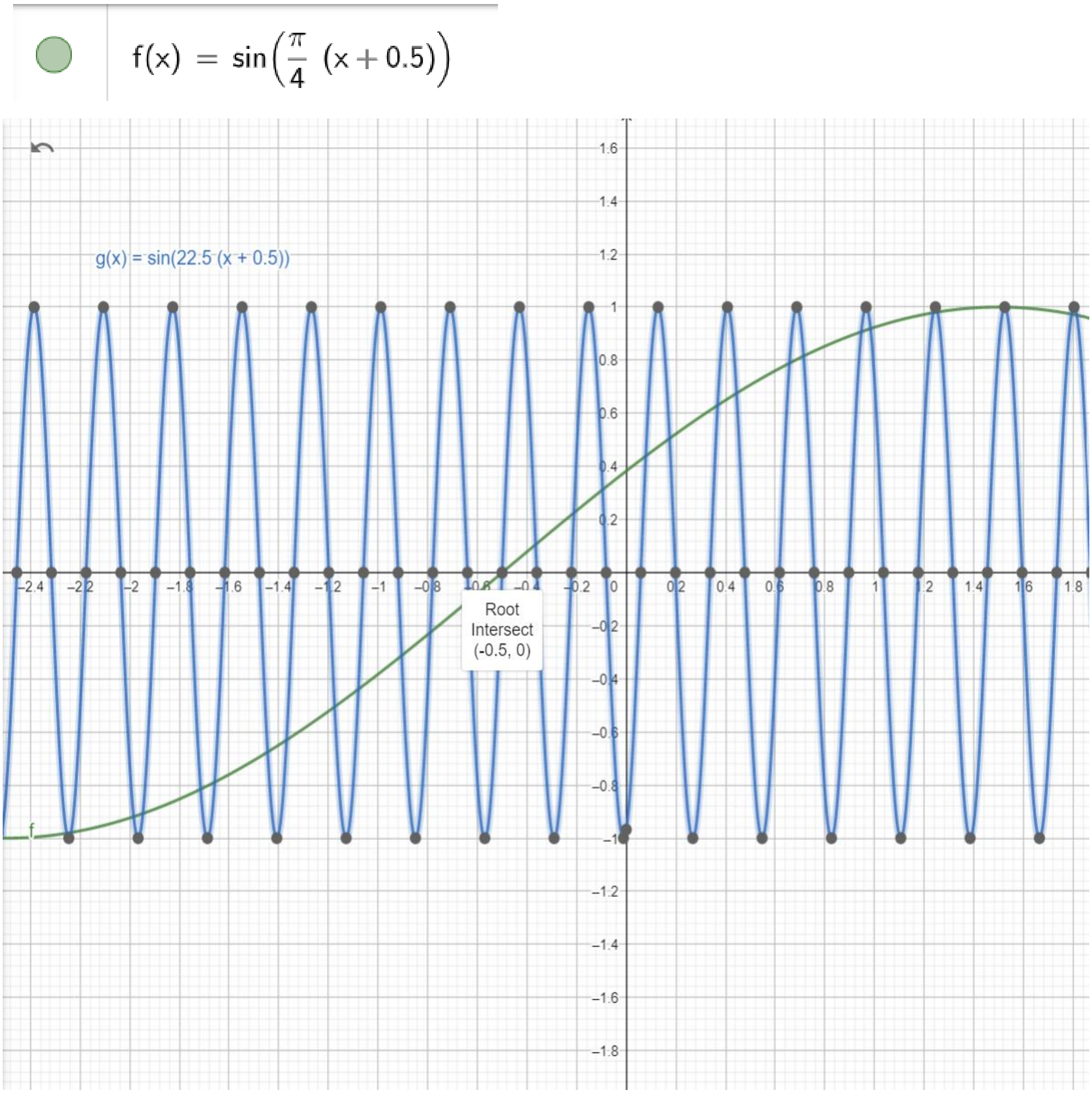


Figure 16. $Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right)$ and $Sin\left(22.5\left(x+\frac{1}{2}\right)\right)$ both intersects at same roots even with different frequency.

Table 1. f(x) = $Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right)$ = { $\frac{\pm 1}{\sqrt{2}}$} for any X = X + 0.5 as X={0,1,2,3,4,5,6,7…} increases distribution divide numbers into odd and even numbers on both sides of x = 0.5 for f(x) = 1/sqrt(2).

x	f(x)
-19.5	-0.707106781...
-17.5	-0.707106781...
-15.5	0.7071067811...
-13.5	0.7071067811...
-11.5	-0.707106781...
-9.5	-0.707106781...
-7.5	0.7071067811...
-5.5	0.7071067811...
-3.5	-0.707106781...
-1.5	-0.707106781...
0.5	0.7071067811...
2.5	0.7071067811...
4.5	-0.707106781...
6.5	-0.707106781...

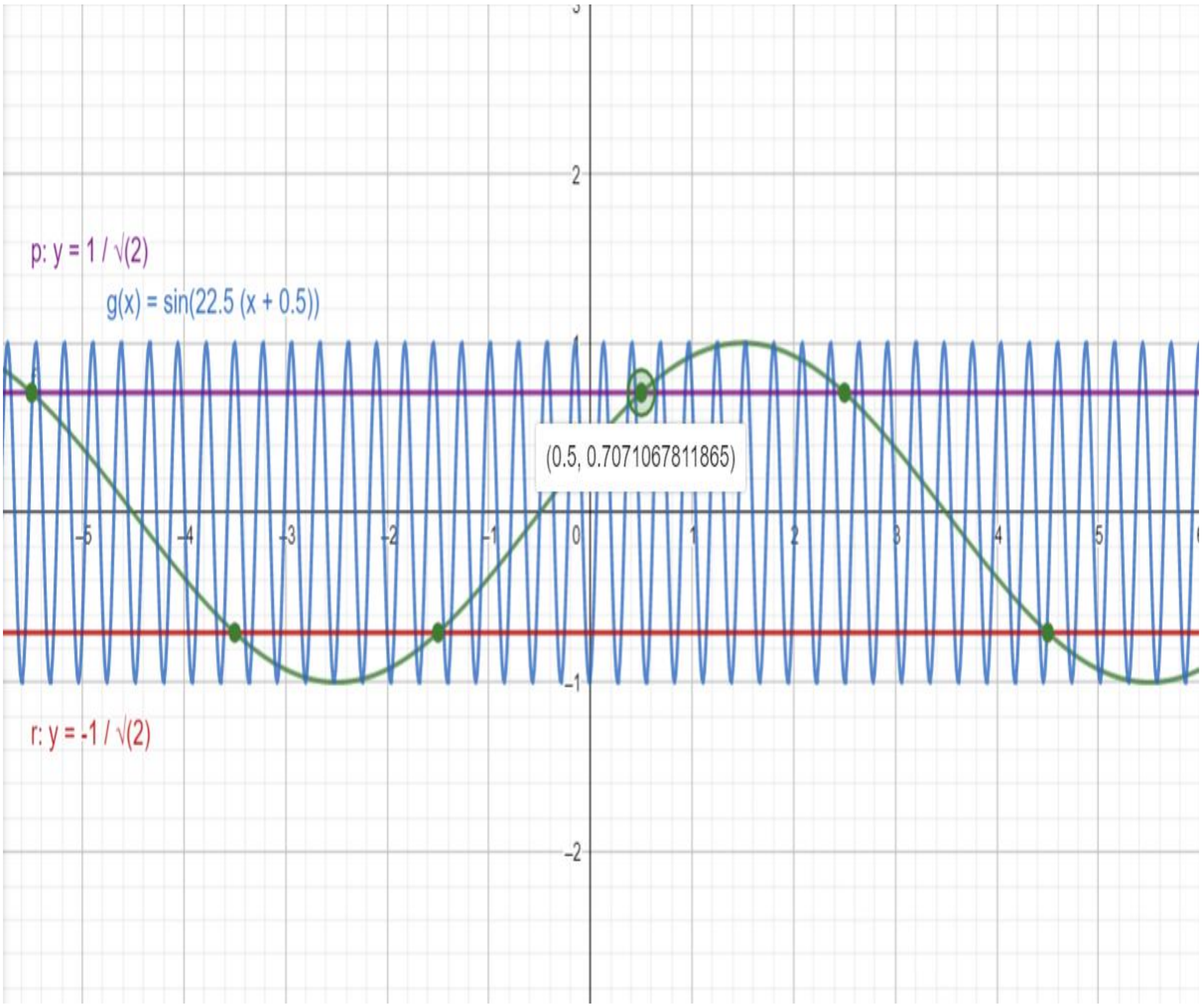


Figure 17. Shows how Y = 1/sqrt(2) intersects with $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ (green wave) at (X+0.5 , 1/sqrt(2)) and intersects with Y = -1/sqrt(2) at (X-1.5 , -1/sqrt(2))

Therefore; if we mulitply f(x)= $Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right)$ by $\pm\frac{1}{\sqrt{2}}$ i.e. multipy by Sin(45) or Sin(225); then Q(X)= {0.5 , -0.5} all the time for odd numbers.

$$Q(x) = Sin\left(\frac{\pi}{2}*\left(x+\frac{1}{2}\right)\right)*Sin\left(\frac{\pi}{4}\right) = Sin\left(\frac{\pi}{2}*\left(x+\frac{1}{2}\right)\right)*Sin(45°) = \pm\frac{1}{2}\ for\ any\ x$$

Table 2. shows $Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right)*Sin(45°) = \pm\frac{1}{2}$; for each negative odd natural number x = x+0.5.

And for each positive even natural numebr x = x + 0.5.

$$a = \sin(45°) - \frac{1}{\sqrt{2}} \quad \rightarrow 0$$

$$b = \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) \quad \rightarrow 0.5$$

$$q(x) = \sin\left(\frac{\pi}{4}(x+0.5)\right)\sin(45°)$$

$$f(x) = \sin\left(\frac{\pi}{4}(x+0.5)\right)$$

x	f(x)	s(x)	q(x)
-11.5	-0.707106781...	0	-0.5
-9.5	-0.707106781...	-1	-0.5
-7.5	0.7071067811...	0	0.5
-5.5	0.7071067811...	1	0.5
-3.5	-0.707106781...	0	-0.5
-1.5	-0.707106781...	-1	-0.5
0.5	0.7071067811...	0	0.5
2.5	0.7071067811...	1	0.5
4.5	-0.707106781...	0	-0.5
6.5	-0.707106781...	-1	-0.5
8.5	0.7071067811...	0	0.5
10.5	0.7071067811...	1	0.5
12.5	-0.707106781...	0	-0.5
14.5	-0.707106781...	-1	-0.5
16.5	0.7071067811...	0	0.5

1.2 Zeta Function none-trivial Zeros.

If we used the same trick we used here when we multiplied by Sin(45)
And do the same in Zeta function formula knowing that

$$\sqrt{2} = \frac{1}{Sin(45)} = \frac{1}{Sin(\frac{\pi}{4})} \quad \rightarrow EQ(C)$$

And from EQ(A)

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} (2*\pi)^S * \sqrt{\frac{2}{\pi}} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) = 0\,;\ when\ S\ odd \\ (2*\pi)^S * \sqrt{\frac{2}{\pi}} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) = 0;when\ S\ odd \end{cases} \quad \rightarrow EQ(A)$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \sqrt{2} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \sqrt{2} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \end{cases}$$

If we substitute from EQ(C) into EQ(A)

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \frac{1}{Sin(\frac{\pi}{4})} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \frac{1}{Sin(\frac{\pi}{4})} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right) \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \end{cases}$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \frac{Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} * \; \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \frac{Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} \; \Gamma\left(\frac{1}{2}-S\right) \zeta\left(\frac{1}{2}-S\right) \end{cases} \quad \rightarrow EQ(D)$$

This Equation EQ(D) have Sin wave that has Root at S = -0.5 and even negative Roots at S = S +0.5 and Odd positive Roots at S = S +0.5.

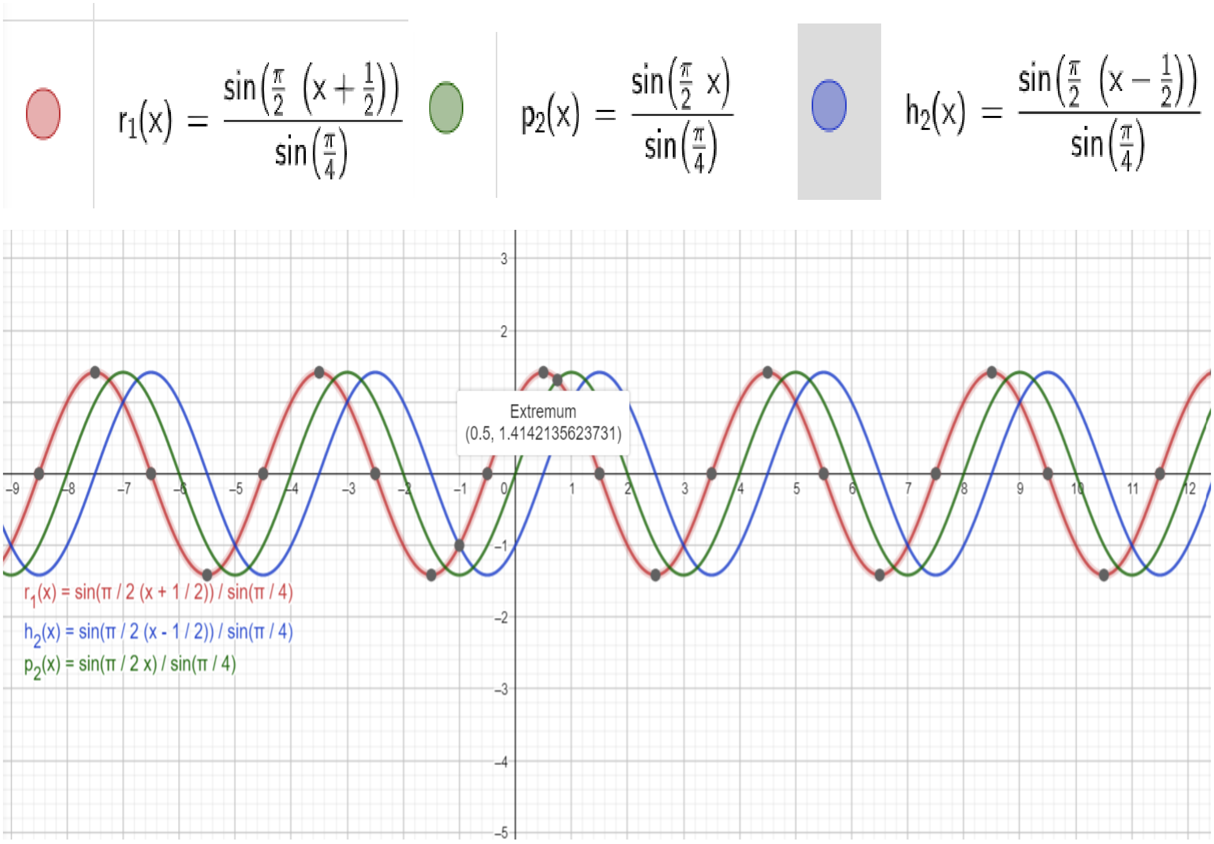


Figure 18. Green Sin wave (original Zeta Sin wave), have Roots only at even numbers and by adding or subtracting 0.5 from an even number we get { odd number ±0.5 , even number ±0.5}

Table 3. even and odd roots flipping bwteen positive side and negative side depends on adding 0.5 or subtractig 0.5. x= { ±odd number ±0.5 , ±even number ±0.5}

$$r_1(x) = \frac{\sin\left(\frac{\pi}{2}\ \left(x + \frac{1}{2}\right)\right)}{\sin\left(\frac{\pi}{4}\right)}$$

x	$r_1(x)$
-12.5	0
-10.5	0
-8.5	0
-6.5	0
-4.5	0
-2.5	0
-0.5	0
1.5	0
3.5	0
5.5	0
7.5	0
9.5	0
11.5	0
13.5	0

$$h_2(x) = \frac{\sin\left(\frac{\pi}{2}\ \left(x - \frac{1}{2}\right)\right)}{\sin\left(\frac{\pi}{4}\right)}$$

x	$h_2(x)$
-13.5	0
-11.5	0
-9.5	0
-7.5	0
-5.5	0
-3.5	0
-1.5	0
0.5	0
2.5	0
4.5	0
6.5	0
8.5	0
10.5	0
12.5	0

Table 4. shwos root shifting between $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ and $Sin\left(\frac{\pi}{4} * \left(x - \frac{1}{2}\right)\right)$ by 1 between both.

F(x) and S(x) = {$0, \pm 1, \pm 0.7071067811865, \pm 0.3826834323651, \pm 0.9238795325113$}

F(x) and S(x) = { $0, \pm 1, \pm \frac{1}{\sqrt{2}}, \pm Sin(\frac{\pi}{8}), \pm Sin(\frac{3 * \pi}{8})$ } , both functions are the same but thier roots are sliding by 1 for any x = x+0.5.

	$f(x) = \sin\left(\frac{\pi}{4}\ (x + 0.5)\right)$		$s(x) = \sin\left(\frac{\pi}{4}\ (x - 0.5)\right)$

x	f(x)	s(x)
-9.5	-0.707106781...	-1
-8.5	0	-0.707106781...
-7.5	0.7071067811...	0
-6.5	1	0.7071067811...
-5.5	0.7071067811...	1
-4.5	0	0.7071067811...
-3.5	-0.707106781...	0
-2.5	-1	-0.707106781...
-1.5	-0.707106781...	-1
-0.5	0	-0.707106781...
0.5	0.7071067811...	0
1.5	1	0.7071067811...
2.5	0.7071067811...	1
3.5	0	0.7071067811...
4.5	-0.707106781...	0

Roots set for these two functions.

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\{0,\pm 1,\pm 0.7071067811865,\pm 0.3826834323651,\pm 0.9238795325113\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{0,\pm 1,\pm\frac{1}{\sqrt{2}},\pm Sin\left(\frac{\pi}{8}\right),\pm Sin\left(\frac{3*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{0,\pm 1,\pm Sin\left(\frac{\pi}{4}\right),\pm Sin\left(\frac{\pi}{8}\right),\pm Sin\left(\frac{3*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{0,\pm 1,\pm Sin\left(\frac{5*\pi}{4}\right),\pm Sin\left(\frac{9*\pi}{8}\right),\pm Sin\left(\frac{11*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0,\pm 1,\pm Sin\left(\frac{1}{1}*\frac{\pi}{4}\right),\pm Sin\left(\frac{1}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{3}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(26)$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0,\pm 1,\pm Sin\left(\frac{2}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{7}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{5}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(27)$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0,\pm 1,\pm Sin\left(\frac{2}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{9}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{11}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(28)$$

From EQ(26) and EQ(27) and EQ(28) these two functions have 5 steady roots $\{0,\pm 1,\pm\frac{1}{\sqrt{2}}\}$ and 4 other

moving roots starting from $Sin\left(\frac{1}{2}*\frac{\pi}{4}\right)$

like { $\pm Sin\left(\frac{1}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{3}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{5}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{7}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{9}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{11}{2}*\frac{\pi}{4}\right)\ldots$ }

1.3 Similar reslts if X = X -0.5

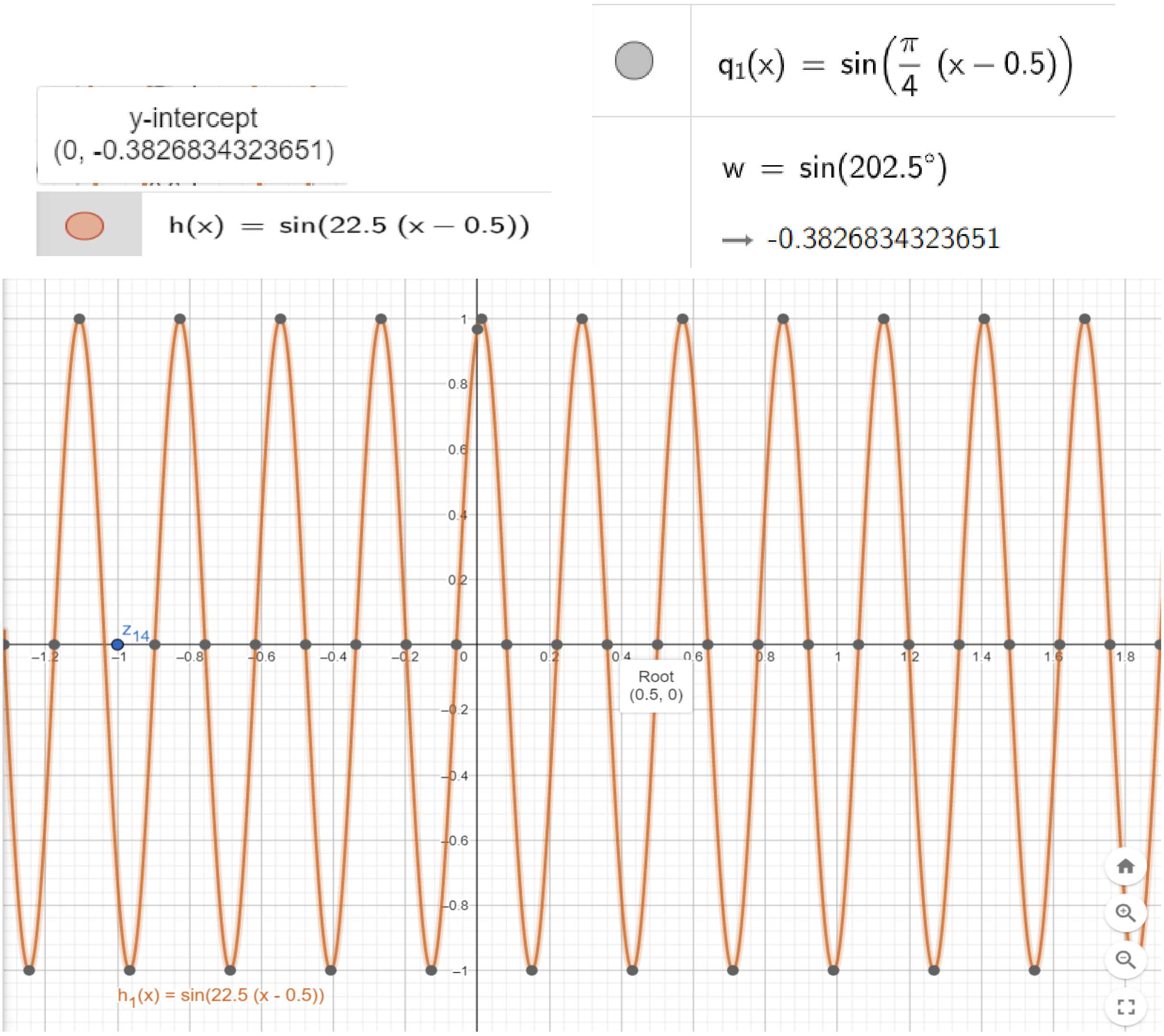
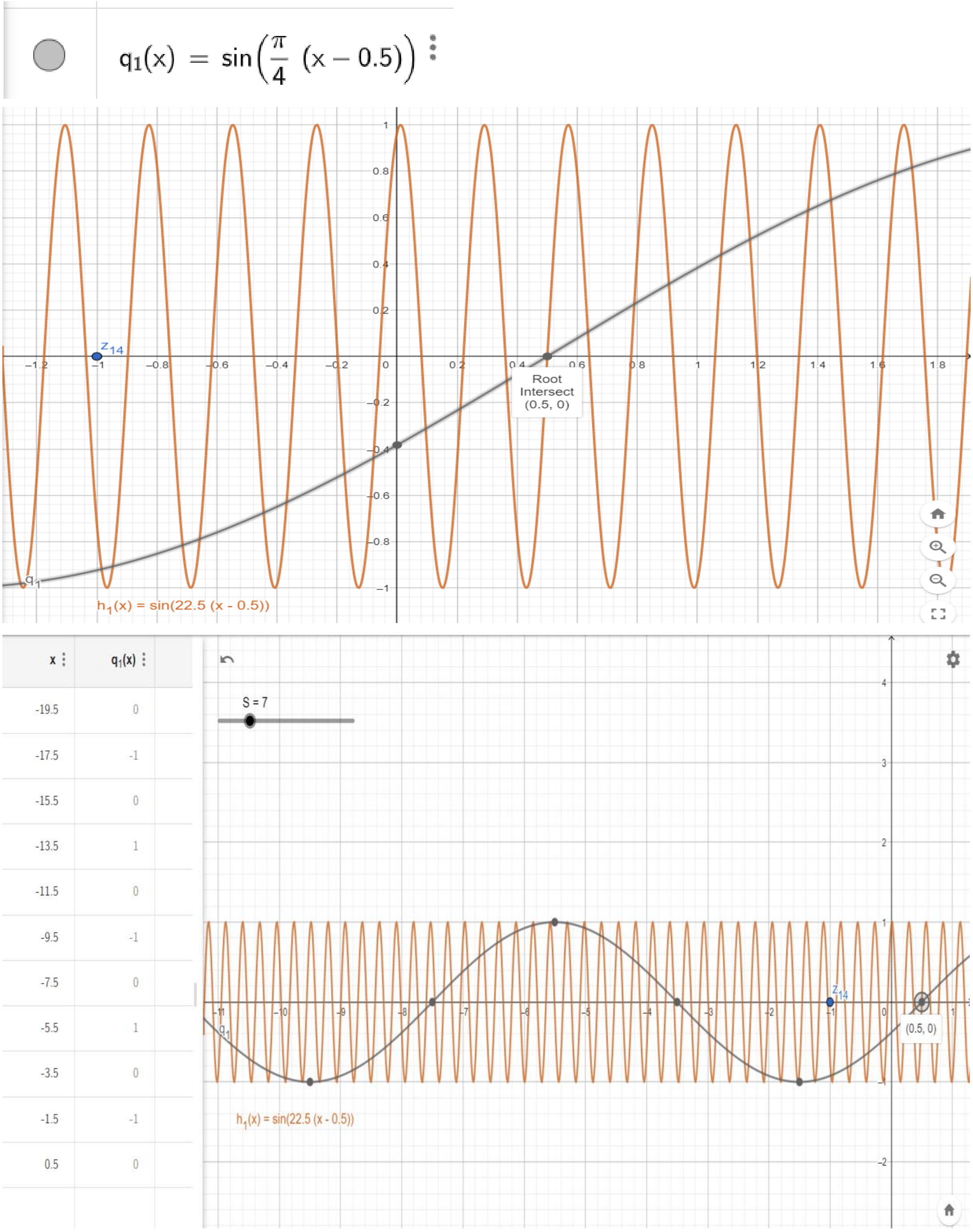


Figure 19. using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at 0.5.



x	$q_1(x)$
-19.5	0
-17.5	-1
-15.5	0
-13.5	1
-11.5	0
-9.5	-1
-7.5	0
-5.5	1
-3.5	0
-1.5	-1
0.5	0

Figure 20. using 22.5 or $\frac{\pi}{4}$ both intersects at same roots even with different frequency

- Q1(X) = 0 at X = { 0.5,-3.5 , -7.5 , -11.5, -15.4,…..} ; with step = 4
- And Q1(X) = { i , -i} at X={-1.5 , -5.5 , -9.5, -13.5, -17.5, …..} with step = 4

  We will see later how to make Q1(X) = 0 for all Odd numbers.

1- For any X = X ±0.5
Then zeta functional Sin() term, will be moving in term of θ = 22.5º and will hav rootse for all odd numbers with value = 1/sqrt(2)
Zeta functional sin() term will intersect withY at point sin(θ = 22.5º) = 0.38268343236509

2- Sin() term in zeta function at X = X + 0.5 will equal to = { 0.5 , -0.5} if we multiply it by by $= \frac{\pm 1}{\sqrt{2}}$ or Sin(45º). And in complex plane multiplication means rotation. Which means if we rorate our complex plane axis by 45º

3- Because sin() term in Zeta function have 90º with S; but we are going to replace it by S = S + 0.5 we converted the engel into 45º which made all odd numbers sin() are with value $= \frac{\pm 1}{\sqrt{2}}$ which is Acutally due to 45º rotation done when we tranfered S = S + 0.5. by this angel all odd numbers landed on rotated axis by 45º.

4- We can do the oposite transformation by normalizing this rotation back into the original complex plane axis by replaceing $\sqrt{2}$ by $\frac{1}{Sin(45)}$ then we will get all the roots back to the orginal complex plane axis. (this $\sqrt{2}$ came zeta formula when replacing each S = S +0.5 in 2^S term)

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} * \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases} \quad \rightarrow EQ(D)$$

$$Cos\left(\frac{\pi}{8} * 2 * X\right) = Sin\left(\frac{\pi}{8} * (2 * X + 4)\right)$$

$$Cos\left(\frac{\pi}{2} * \frac{X}{2}\right) = Sin(\frac{\pi}{2} * (\frac{X}{2} + 1))$$

$$Sin\left(\frac{\pi}{2} * \left(\frac{x+0.5}{2}\right)\right) = Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$$

In the rest of the document we are goin to see the distribution odd roots on our new odd Identity function and how it explaines the distribution of the roots for Sin() term in Zeta function.

2.1 Odd Identity unit Circle function Properties

$$f(x) = (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^{x} = Cos(x\,\theta) + \,i\,Sin(\,x\,\theta)$$

This f(x) is like Euler's Identity but for odd numbers. We will call it odd Identity unit Circle.

1- equivalent to the complex plane unit circle. And Euler's Identity $e^{i\pi} + 1 = 0$
2- odd Identity unit Circle f(X) axis, rotates 45 degrees from the original complex plane axis X, Y.
3- this f(Z) Odd Identity unit circle axis is Y =X and Y =-X, which means if X = e then Y = e or Y =-e
4- this odd Identity unit circle function intersects with Y = X and Y =-X at square root of two.
5- This Odd Identity unit circle function intersects with 4 axis (2 original and 2 rotated), in 8 points. {1, -1, i, -i, sqrt (2), -sqrt (2)}
6- every cycle of 8 integer values for x, we start new cycle of same values of f(x). first cycle starts at X =0, second cycle starts at X =8.

Table 5. Odd Identity Unit Circle Rotation Values at [x] Natural Numbers (Cycle every 8)

We have 4 axes with total 8 unique complex numbers $\left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right) and\ (\pm 1)\ and\ (\pm i)$.

x	$f(z) = z^{x} \;=\; (\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}})^{x}$
0	1+0i
1	$\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
2	i
3	$\frac{-1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
4	-1+0i
5	$\frac{-1}{\sqrt{2}} + i\,\frac{-1}{\sqrt{2}}$
6	-i
7	$\frac{1}{\sqrt{2}} + i\,\frac{-1}{\sqrt{2}}$
8 -→ end of one cycle	1+0i
9	$\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
10	….

7- For any even integer values for X; f(X) value will be on original complex plane axis. And any odd integer values of X; f(X) value will be a complex number on the odd Identity unit circle axis which are the new rotated axis (45 degrees).

$$f(x) = z^x \;=\; (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$$

$$for\ all\ odd\ values\ of\ x, \qquad f(x) = \pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}$$

Which means all values of f(X) will be on the new Odd Identity unit Circle; where Cos (45) = Sin (45) = 1/sqrt (2).

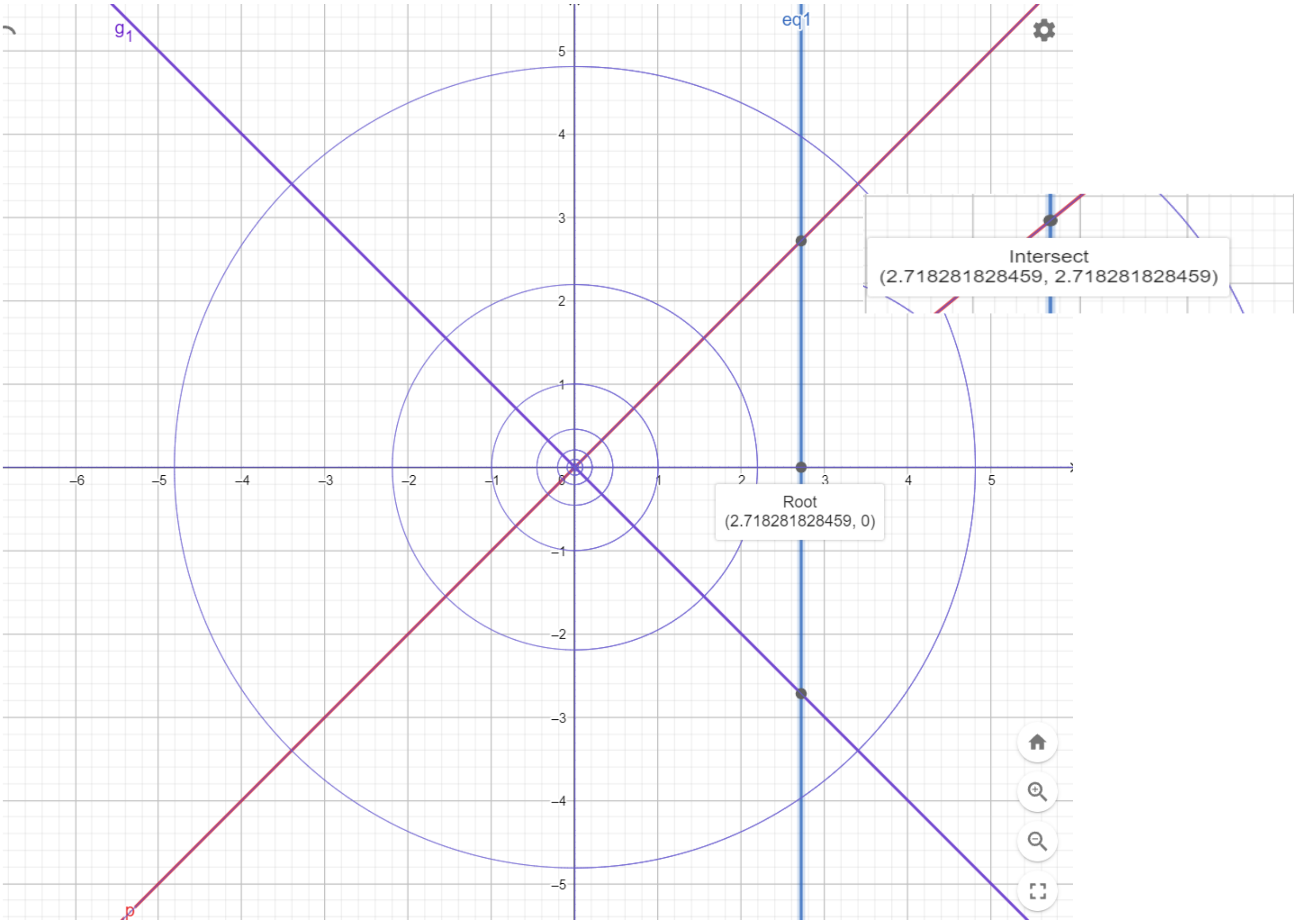


Figure 21. Shows our Odd Identity 4 axis and how [e] intersects with the New Odd Identity axis at [e].

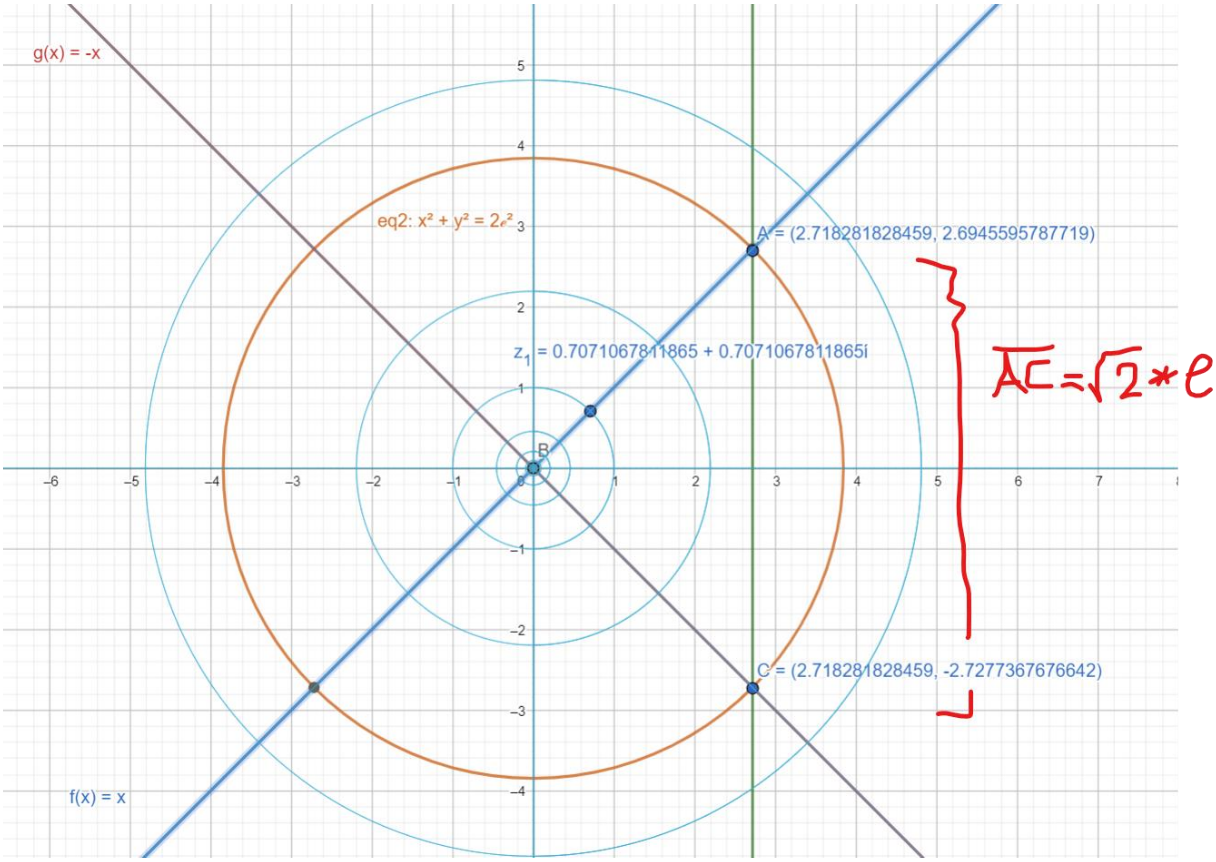


Figure 22. Δ(ABC) triangle with two equal sides =[e] intersects with $x^2 + y^2 = 2 * e^2$ at [e].

2.1.1 Odd Identity unit Circle function at natural number values for x

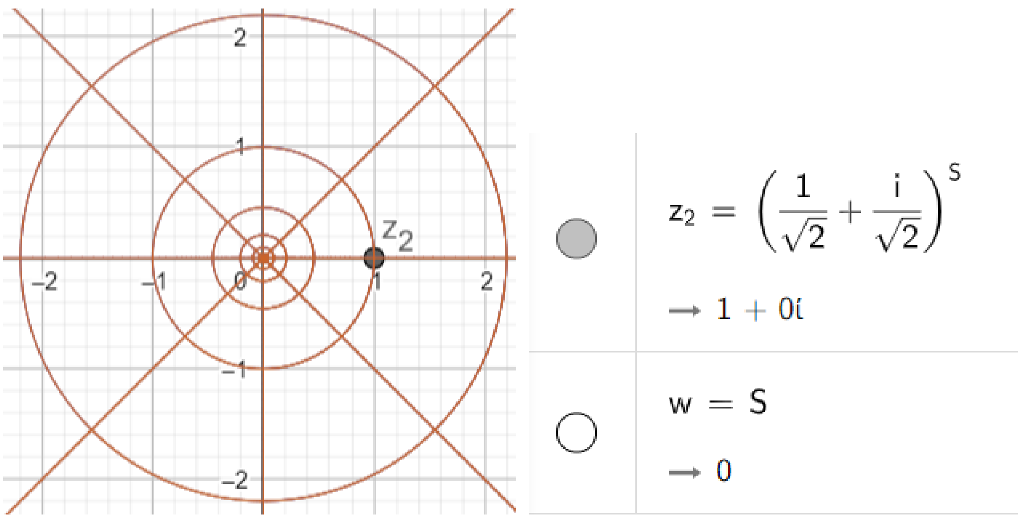


Figure 23. New F(x) Odd numbers unit identity, at x = S = 0, start point at noraml complex plane x axis.

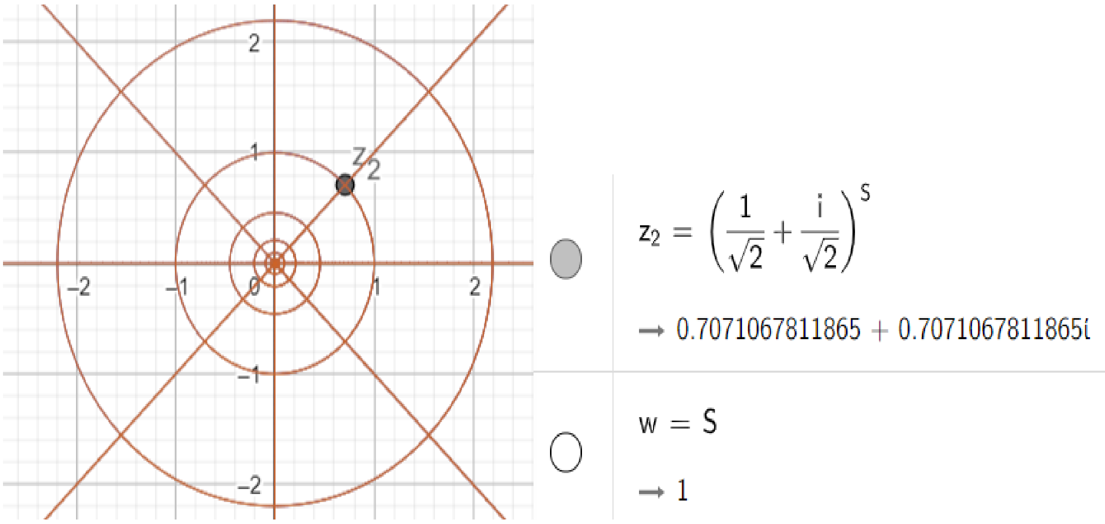


Figure 24. New F(x) Odd numbers unit identity, at x = S = 1, [Z2] at new Odd Idnetity unit axis at (45) degrees from start point at 1 on noraml complex plane X axis.

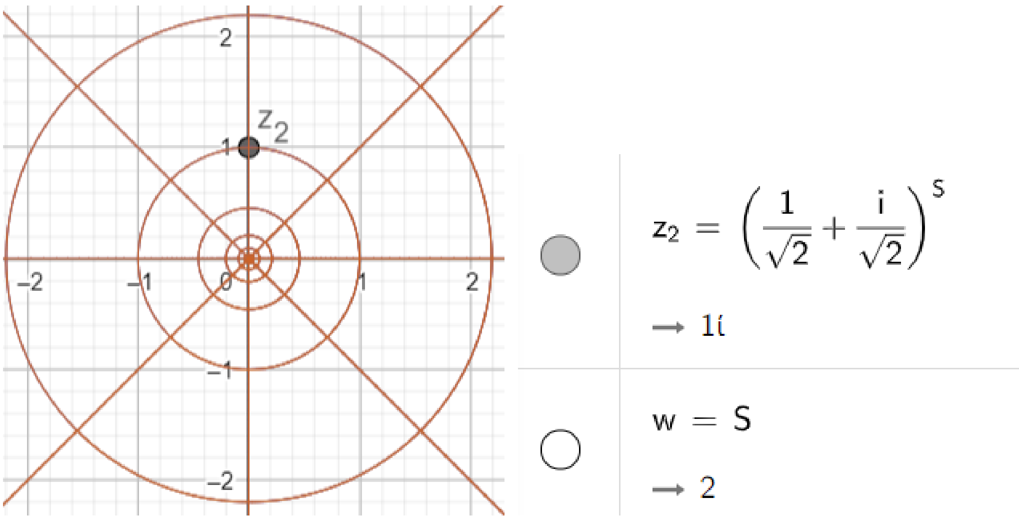


Figure 25. New F(x) Odd numbers unit identity, at x = S = 2, [Z2] at noraml complex plane Y axis.

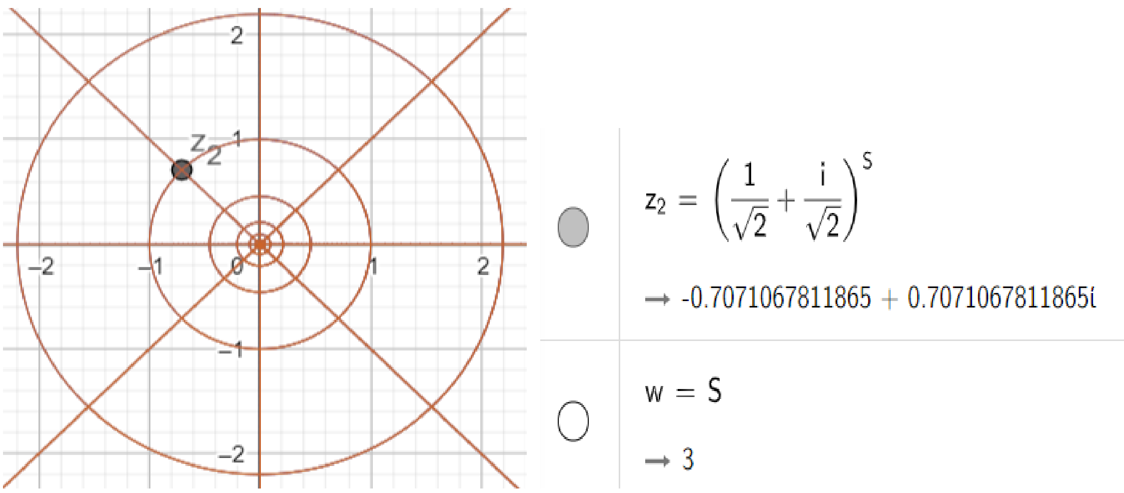


Figure 26. New F(x) Odd numbers unit identity, at x = S = 3, [Z2] at new Odd Idnetity unit axis at (135) degrees from start point at 1 on noraml complex plane X axis.

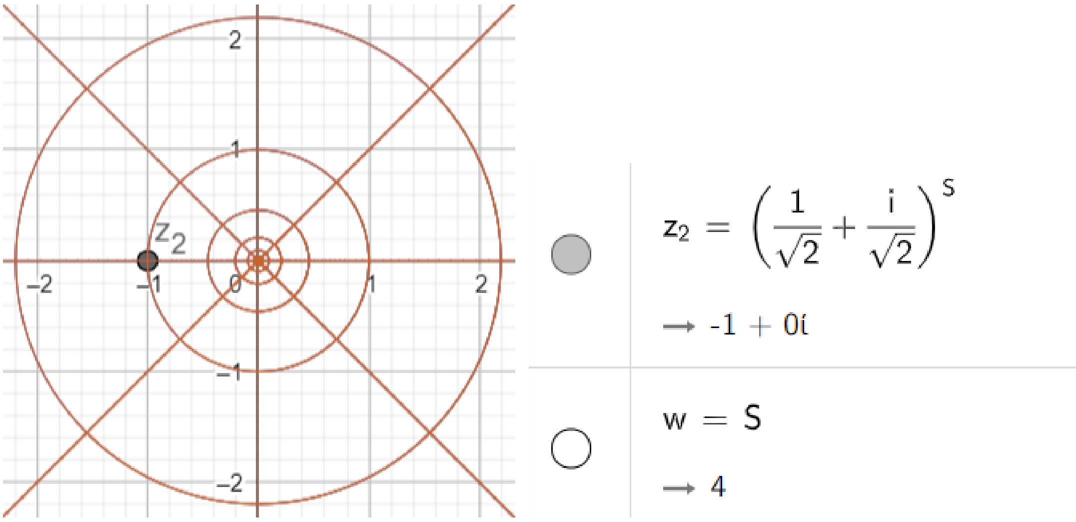


Figure 27. New F(x) Odd numbers unit identity, at x = S = 4, [Z2] at noraml complex plane X axis.

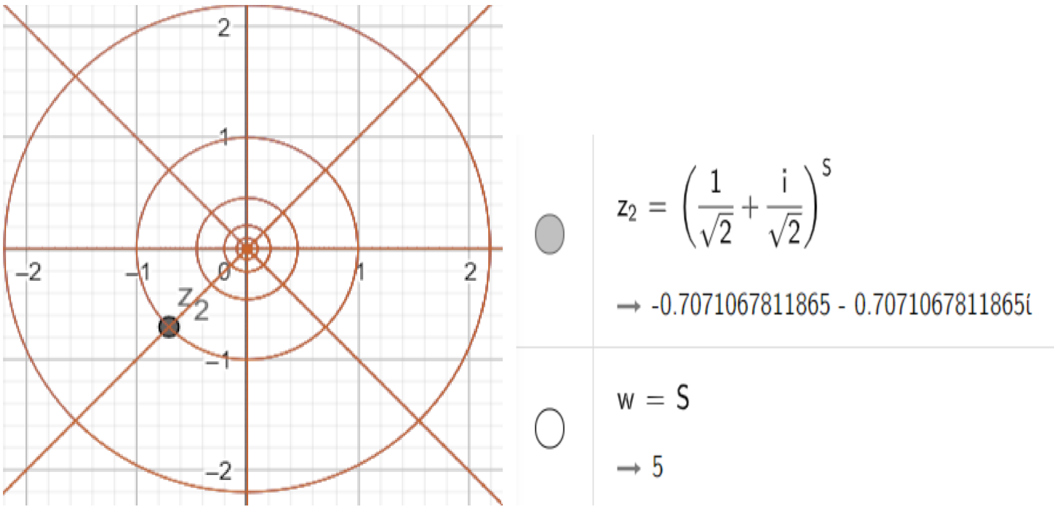


Figure 28. New F(x) Odd numbers unit identity, at x = S = 5, [Z2] at new Odd Idnetity unit axis at (225) degrees from start point at 1 on noraml complex plane X axis.

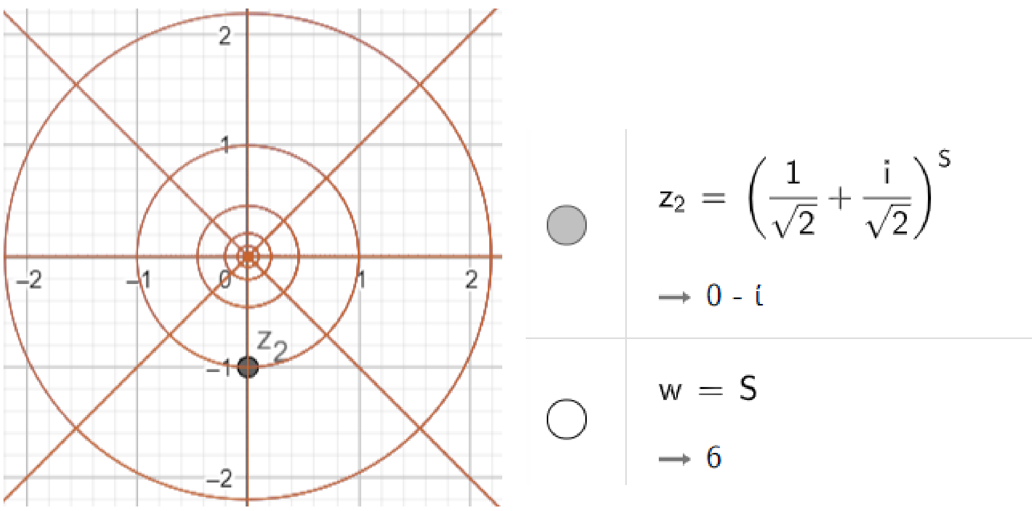


Figure 29. New F(x) Odd numbers unit identity, at x = S = 6, [Z2] at noraml complex plane Y axis.

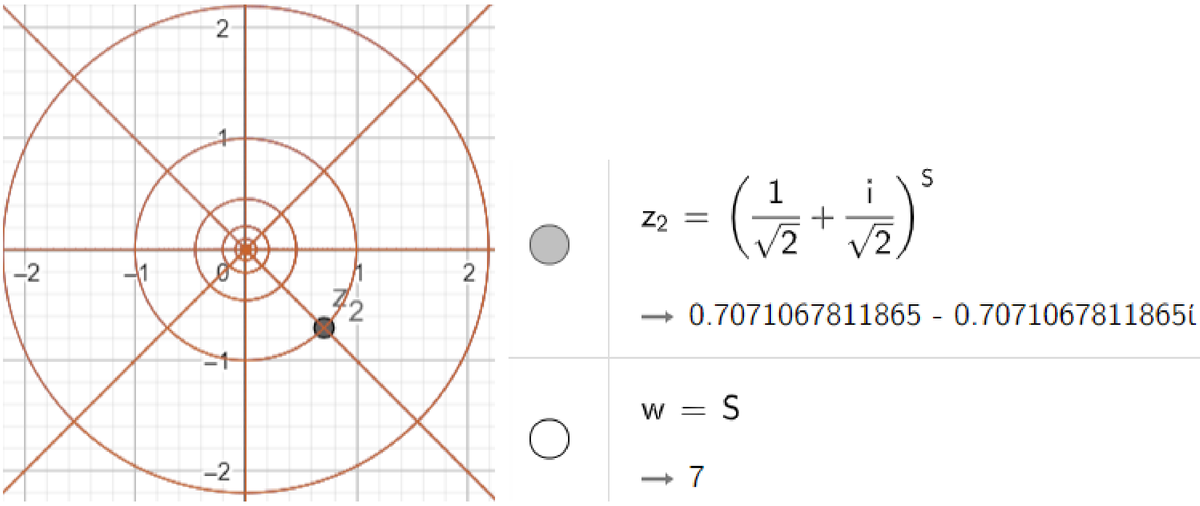


Figure 30. New F(x) Odd numbers unit identity, at x = S = 7, [Z2] at new Odd Idnetity unit axis at (315) degrees from start point at 1 on noraml complex plane X axis.

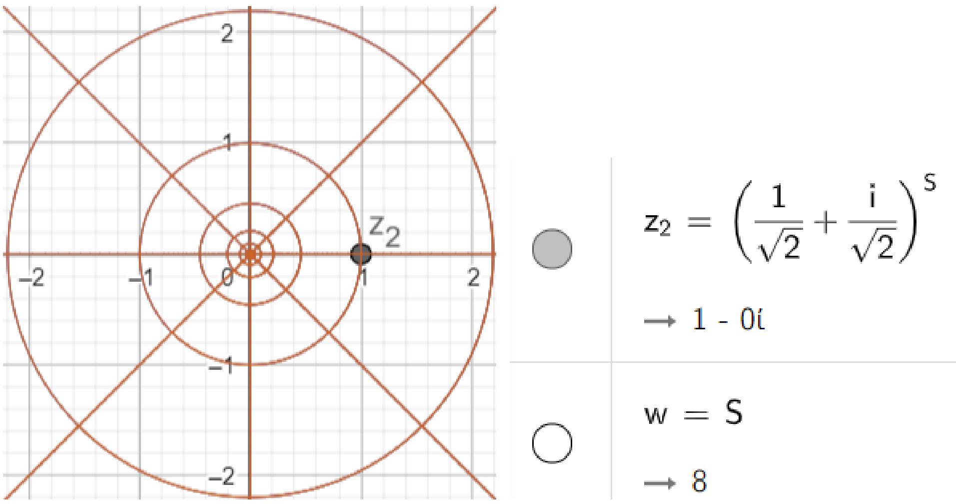


Figure 31. New F(x) Odd numbers unit identity, at x = S = 8, [Z2] completes a full cycle and goes back to start point on noraml complex plane x axis at 1.

2.1.2 Odd Identity unit Circle function at any real number values for x

$$f(x) = z^x \ = \ (\pm\frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^x$$

With natural number values for x we used (45) degrees, and we got full cycle every 8 values. Here in real values for x we are going to use (22.5) degrees, starting at square root at X = 0.5. Or at X = X+0.5 and X =0.

1- for all real values of x, then f(X) value will be any point on the odd Identity unit Circle. Where odd Identity unit Circle origin (0,0) but the axis is rotated by 22.5 degrees.

$$f(x) = (\pm\frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^x = Cos(x\,\theta) + \ i\,Sin(\,x\,\theta)$$

2- for x = S = 0.5, THEN θ = 22.5º, because at x = 1, was θ = 45º
so here with real numbers values we are going to use a cycle with start point at x = S = 0.5, THEN θ = 22.5º

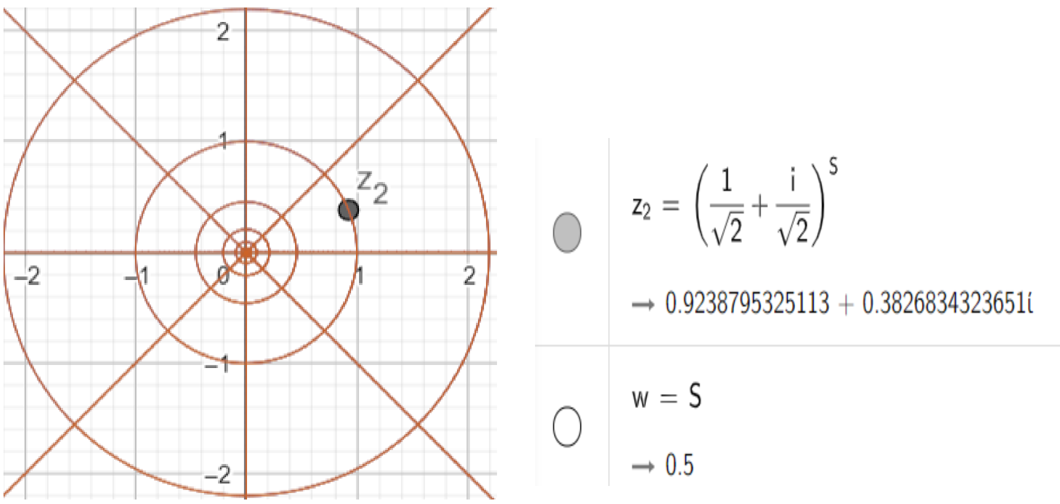


Figure 32. New F(x) Odd numbers unit identity, at x = S = 0.5, for real values for x, [Z2] is a start Cycle point, starts at (22.5) degrees.

$$f(x) = (\pm\frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^{0.5} = Cos(22.5) + \ i\,Sin(\,22.5)$$

3- one property for this θ = 22.5º and Sin (22.5) and Cos (22.5)

$$Cos\left(\frac{\pi}{8} * 2 * X\right) = Sin\left(\frac{\pi}{8} * (2 * X + 4)\right)$$

$$Cos\left(\frac{\pi}{2} * \frac{X}{2}\right) = Sin(\frac{\pi}{2} * (\frac{X}{2} + 1))$$

$Cos\left(\frac{\pi}{2} * \frac{x}{2}\right)$ *in complex number have same value as* $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$

in another complex number in the same cycle of 22.5 degree partitions

Table 6. shows how starting a Cycle at (22.5) degrees we will have $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ for x =2 will equal

$Cos\left(\frac{\pi}{2} * \frac{x}{2}\right)$ at x = 1 as it show in matching pair colours in the table {Cos(22.5) = Sin(112.5)}

x	θ	$f(x) = z^x = (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$
0.5	22.5	$Cos(22.5) + i\,Sin(22.5) = 0.9238795325113 + 0.3826834323651i$
1.5	67.5	$Cos(67.5) + i\,Sin(67.5) = 0.3826834323651 + 0.9238795325113i$
2.5	112.5	$Cos(112.5) + i\,Sin(112.5) = -0.3826834323651 + 0.9238795325113i$
3.5	157.5	$Cos(135) + i\,Sin(135) = -0.9238795325113 + 0.3826834323651i$
4.5	202.5	$Cos(202.5) + i\,Sin(202.5) = -0.9238795325113 - 0.3826834323651i$
5.5	247.5	$Cos(247.5) + i\,Sin(247.5) = -0.3826834323651 - 0.9238795325113i$
6.5	292.5	$Cos(292.5) + i\,Sin(292.5) = 0.3826834323651 - 0.9238795325113i$
7.5	337.5	$Cos(337.5) + i\,Sin(337.5) = 0.9238795325113 - 0.3826834323651\,i$
8	360	1-0 i
8.5	382.5	$Cos(382.5) + i\,Sin(382.5) = 0.9238795325113 + 0.3826834323651\,i$
9.5	427.5	…….

Table 7. every cycle cover 8 values for x. one cycle starts at X= 0.5 and θ=22.5 and ends at X =8, θ=360 new cycle of same values of f(x).

x	θ	$f(x) = z^x = (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$
0.5	22.5	$Cos(22.5) + i\,Sin(22.5) = 0.9238795325113 + 0.3826834323651i$
1.5	67.5	$Cos(67.5) + i\,Sin(67.5) = 0.3826834323651 + 0.9238795325113i$
2.5	112.5	$Cos(112.5) + i\,Sin(112.5) = -0.3826834323651 + 0.9238795325113i$
3.5	157.5	$Cos(135) + i\,Sin(135) = -0.9238795325113 + 0.3826834323651i$
4.5	202.5	$Cos(202.5) + i\,Sin(202.5) = -0.9238795325113 - 0.3826834323651i$
5.5	247.5	$Cos(247.5) + i\,Sin(247.5) = -0.3826834323651 - 0.9238795325113i$
6.5	292.5	$Cos(292.5) + i\,Sin(292.5) = 0.3826834323651 - 0.9238795325113i$
7.5	337.5	$Cos(337.5) + i\,Sin(337.5) = 0.9238795325113 - 0.3826834323651\,i$
8	360	1-0 i
8.5	382.5	$Cos(382.5) + i\,Sin(382.5) = 0.9238795325113 + 0.3826834323651\,i$
9.5	427.5	…….

$z_{13} = \cos(22.5°) + i\ \sin(22.5°)$

$\rightarrow 0.9238795325113 + 0.3826834323651i$

$z_{14} = \cos(67.5°) + i\ \sin(67.5°)$

$\rightarrow 0.3826834323651 + 0.9238795325113i$

$z_{15} = \cos(112.5°) + i\ \sin(112.5°)$

$\rightarrow -0.3826834323651 + 0.9238795325113i$

$z_{16} = \cos(157.5°) + i\ \sin(157.5°)$

$\rightarrow -0.9238795325113 + 0.3826834323651i$

$z_{17} = \cos(202.5°) + i\ \sin(202.5°)$

$\rightarrow -0.9238795325113 - 0.3826834323651i$

$z_{18} = \cos(247.5°) + i\ \sin(247.5°)$

$\rightarrow -0.3826834323651 - 0.9238795325113i$

$z_{19} = \cos(292.5°) + i\ \sin(292.5°)$

$\rightarrow 0.3826834323651 - 0.9238795325113i$

$z_{20} = \cos(337.5°) + i\ \sin(337.5°)$

$\rightarrow 0.9238795325113 - 0.3826834323651i$

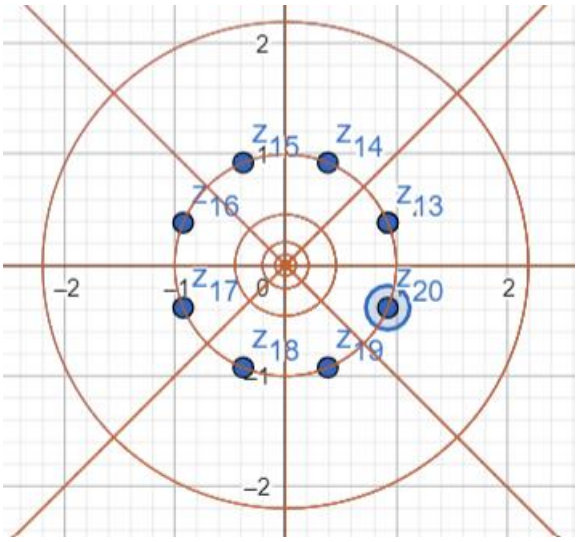


Figure 33. positions of complex numbers for first cycles of F(x) on Odd numbers unit identity circle, starts at (22.5) degrees.

3.1 Using e and θ = 22.5 to represent our odd new Identity function in a complex plane

1- $e^{\theta x} = e^{22.5\,x}$; intersect Y at point (0,1) and start from X =1.5 Y =0 for any X

And $e^{\theta x} = 2\,e^{22.5\,x}$; intersect Y at point (0,2) and start from X =1.5 Y =0 for any X

And $e^{\theta x} = 3\,e^{22.5\,x}$; intersect Y at point (0,3) and start from X =1.5 Y =0 for any X

THEN we are going to use 22.5 as a number in order explore residuals in limited operational machines. Because this will not be going to change the characteristics of the graph itself.

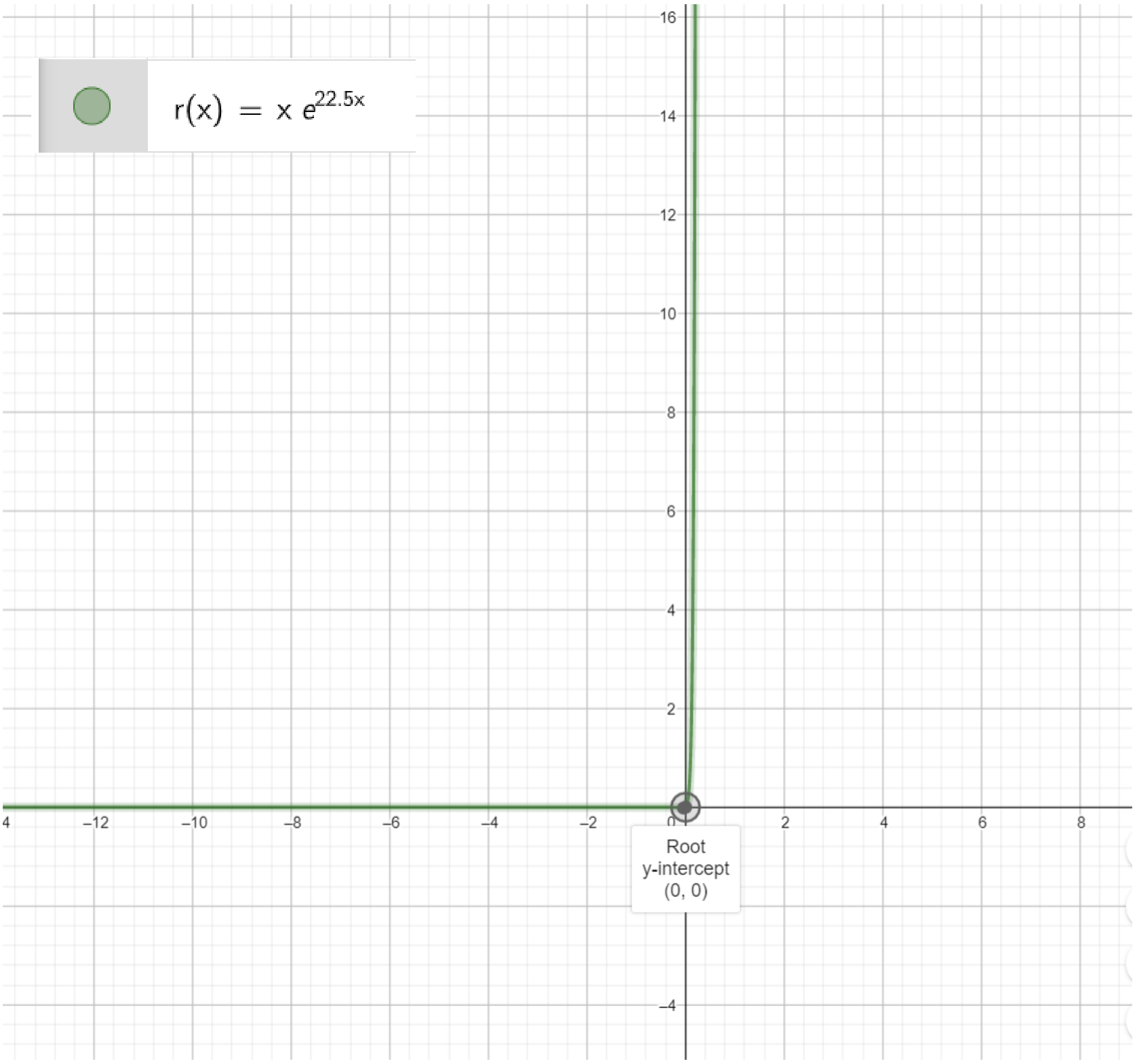


Figure 34. using (22.5) as number not as degree (pi/8), gave us scaled version of the graph.

2- Now we are going to remove this 22.5 degrees from f(x) = X

$$q(x) = x - e^{22.5x}$$

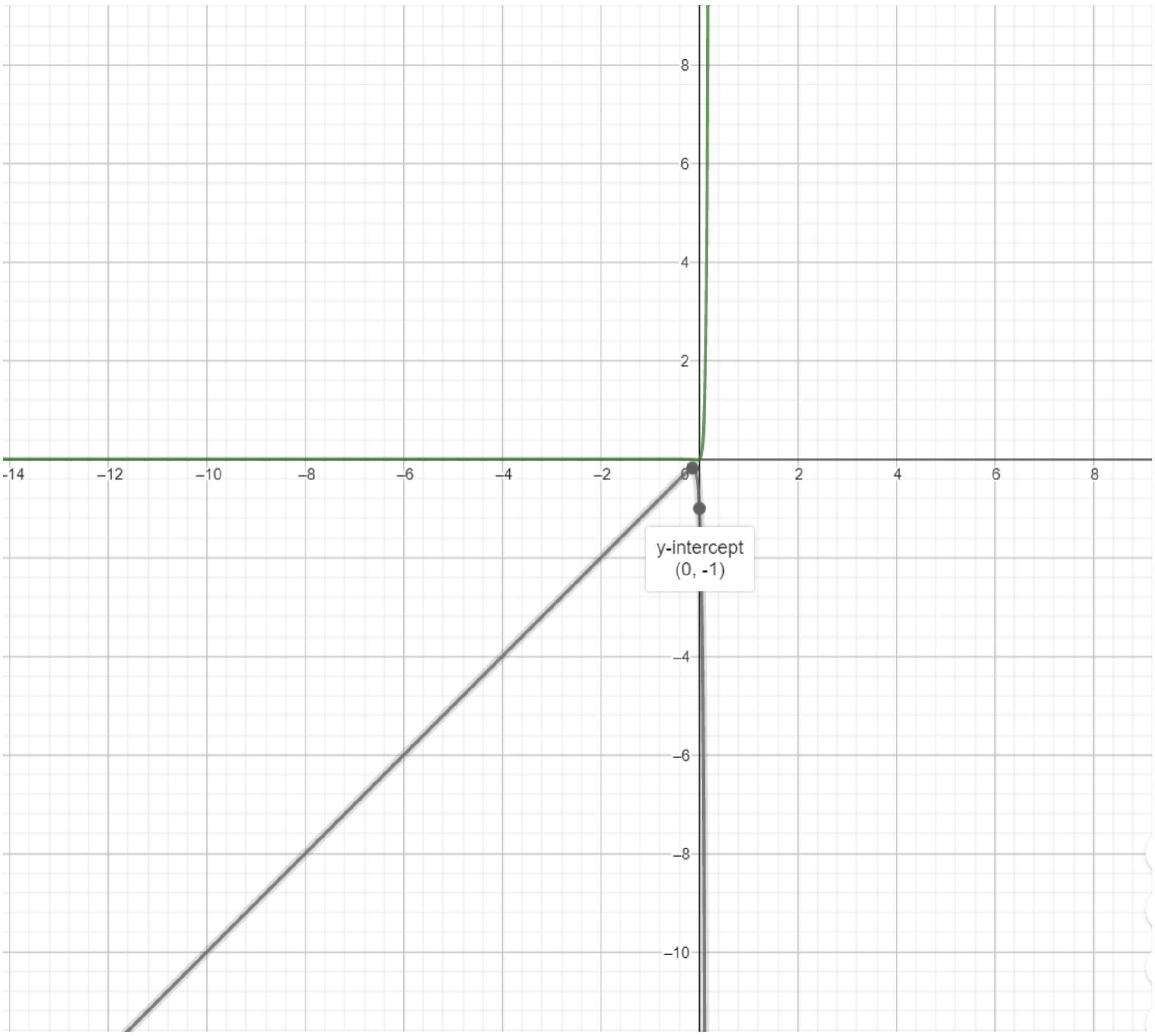


Figure 35. Removing 22.5 degrees from f(x) = X from scaled version of the graph with (pi/8).

Table 8. Q(X) = X for all x <= -1.5 and Q(X) = -1 at X = 0 AND R(X) = 0 for all X <= -1.5 and R(X) = 0 at X =0

$q(x) = x - e^{22.5x}$ $r(x) = x\,e^{22.5x}$

x	q(x)	r(x)
-4.5	-4.5	0
-4	-4	0
-3.5	-3.5	0
-3	-3	0
-2.5	-2.5	0
-2	-2	0
-1.5	-1.5	0
-1	-1.0000000001692	-0.0000000001692
-0.5	-0.5000130072977	-0.0000065036488
0	-1	0
0.5	-76879.41976467772	38439.95988233886
1	-5910522062.023283	5910522063.023283
1.5	-454400461972585...	681600692958881.1
2	-349342710574850...	698685421149700...
2.5	-268574395593695...	671435988984237...

3- We are going to add both functions together.
S(X) = Q(X) + R(X); this will make S(X) = 1 at X =1 and S(X) = -1 at X=0
Half the graph is at Y =X i.e., at 45 degrees

$$s(x) = q(x) + r(x)$$
$$\rightarrow x - e^{22.5x} + x\, e^{22.5x}$$

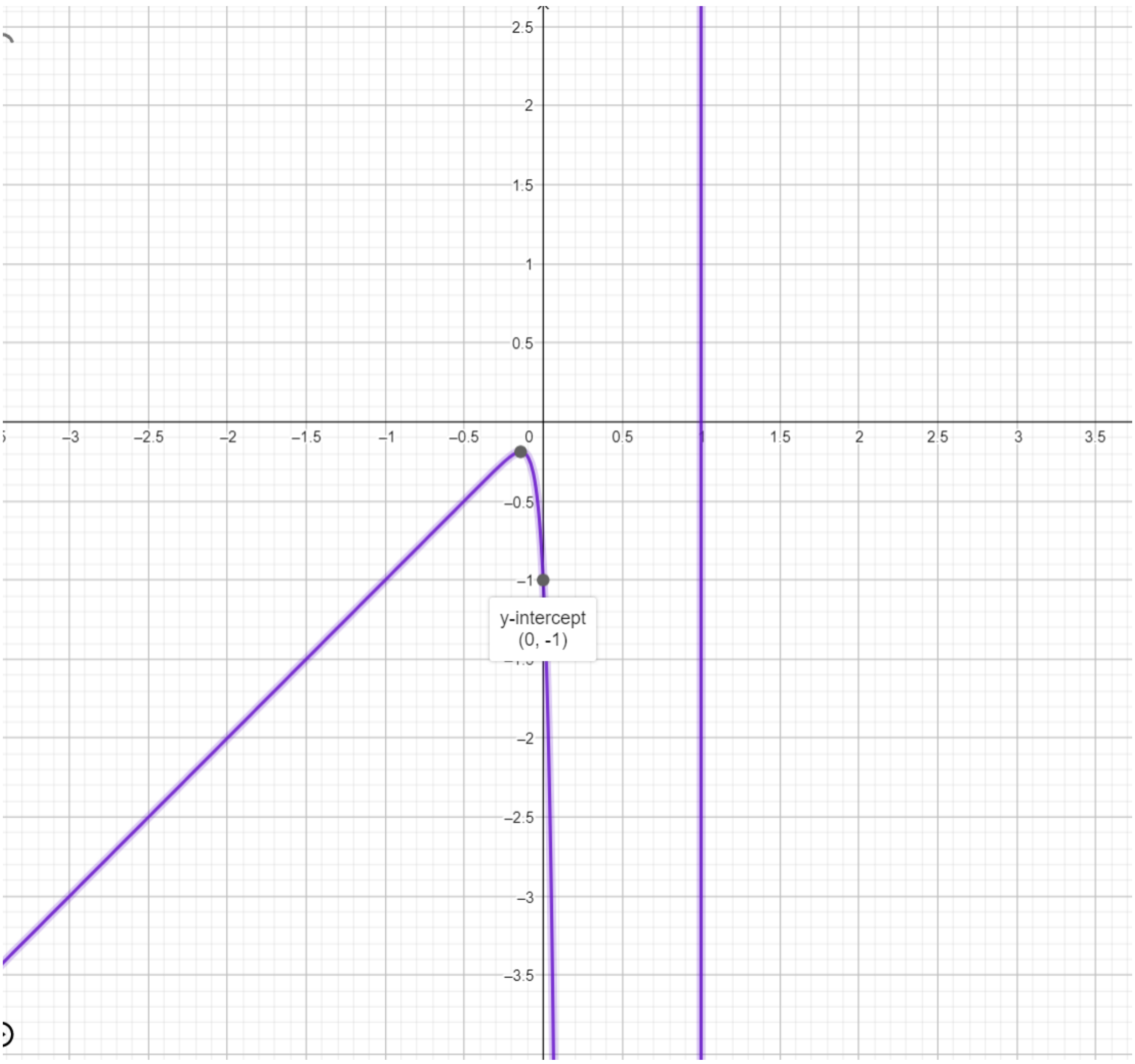


Figure 36. adding Q(X) and R(X) together S(X) = Q(X) + R(X); S(X) = 1 at X =1 and S(X) = -1 at X=0

Table 9. values for all three fucntion Q(X) and R(X) together S(X) = Q(X) + R(X).

x	q(x)	r(x)	s(x)
-4.5	-4.5	0	-4.5
-4	-4	0	-4
-3.5	-3.5	0	-3.5
-3	-3	0	-3
-2.5	-2.5	0	-2.5
-2	-2	0	-2
-1.5	-1.5	0	-1.5
-1	-1.0000000001692	-0.0000000001692	-1.0000000003384
-0.5	-0.5000130072977	-0.0000065036488	-0.5000195109465
0	-1	0	-1
0.5	-76879.41976467772	38439.95988233886	-38439.45988233886
1	-5910522062.023283	5910522063.023283	1
1.5	-454400461972585...	681600692958881.1	227200230986295.2
2	-349342710574850...	698685421149700...	349342710574850...
2.5	-268574395593695...	671435988984237...	402861593390542...

4- To make the graph back to 22.5 degrees which is Y = X/2
By dividing S(X) by two
T(X) = ½ * S(X) = ½ * (R(X) + Q(X))
If X = 0 then T(X) = -0.5 and If X =1 then T(X) = 0.5

$$t(x) = \frac{q(x) + r(x)}{2}$$

$$\rightarrow \frac{x - e^{22.5x} + x\, e^{22.5x}}{2}$$

$$f_1(x) = \frac{x}{2}$$

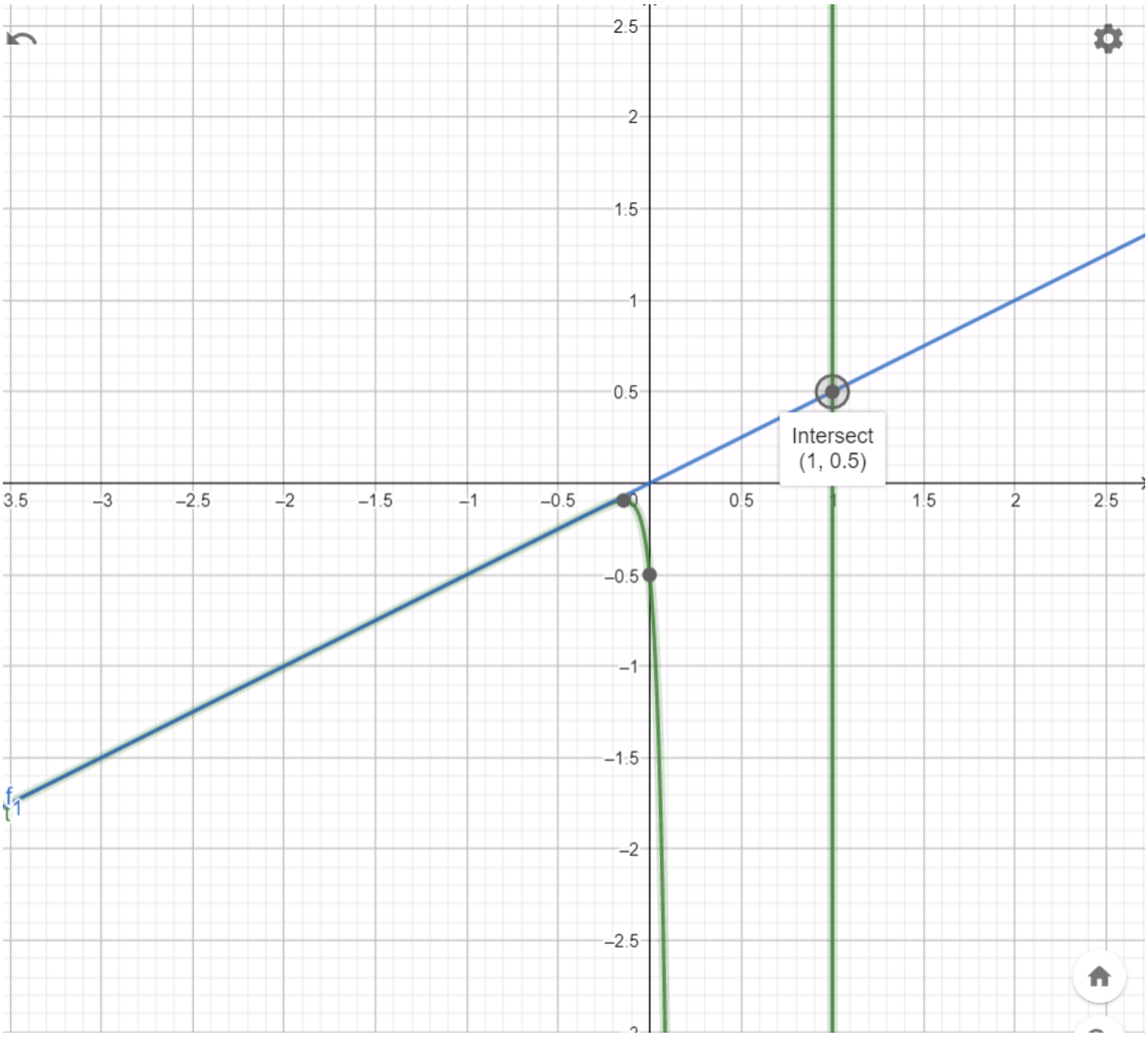


Figure 37. T(X) = ½ * S(X) so T(X) = -0.5 at X =0 and T(X) = 0.5 at X =1.

Table 10. T(X) = -0.5 at X =0 and T(X) = 0.5 at X =1.

x	q(x)	r(x)	s(x)	t(x)
-5.5	-5.5	0	-5.5	-2.75
-5	-5	0	-5	-2.5
-4.5	-4.5	0	-4.5	-2.25
-4	-4	0	-4	-2
-3.5	-3.5	0	-3.5	-1.75
-3	-3	0	-3	-1.5
-2.5	-2.5	0	-2.5	-1.25
-2	-2	0	-2	-1
-1.5	-1.5	0	-1.5	-0.75
-1	-1.0000000001692	-0.0000000001692	-1.0000000003384	-0.5000000001692
-0.5	-0.5000130072977	-0.0000065036488	-0.5000195109465	-0.2500097554732
0	-1	0	-1	-0.5
0.5	-76879.41976467772	38439.95988233886	-38439.45988233886	-19219.72994116943
1	-5910522062.023283	5910522063.023283	1	0.5
1.5	-454400461972585...	681600692958881.1	227200230986295.2	113600115493147.6

4.1 using Our Odd number identity unit Circle f(Z) in combine with [e].

$$v(x) = e^{\left(\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{x}\right)}$$

$$eq2 : x = e$$

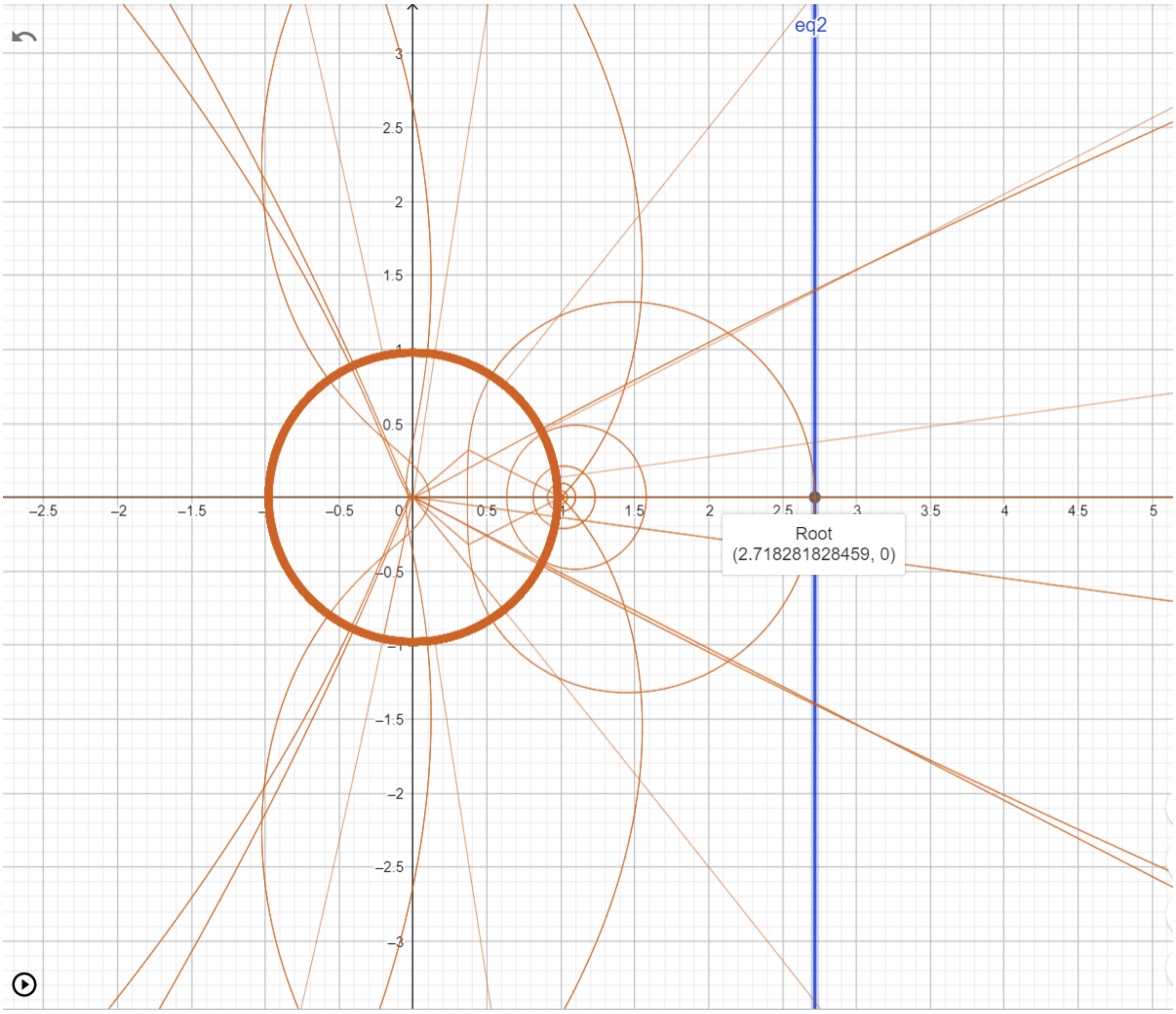


Figure 38. graph shows identity unit Circle when using [e] to the power of f(Z) Odd identity function. And show there is root at [1] and intersection at [e].

5- We are going to Set X = X/2 -1/2
   If X is odd number; then X/2 will be even number + 0.5; and by subtracting this 0.5 we are using even number, i.e., we are setting X = X-1 where X-1 is the number before X where X is an odd number. And to keep X as an odd number we are going to add one again.
   So, this will be for odd numbers.

$$n(x) = e^{\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{\frac{x}{2}-0.5}+1}$$

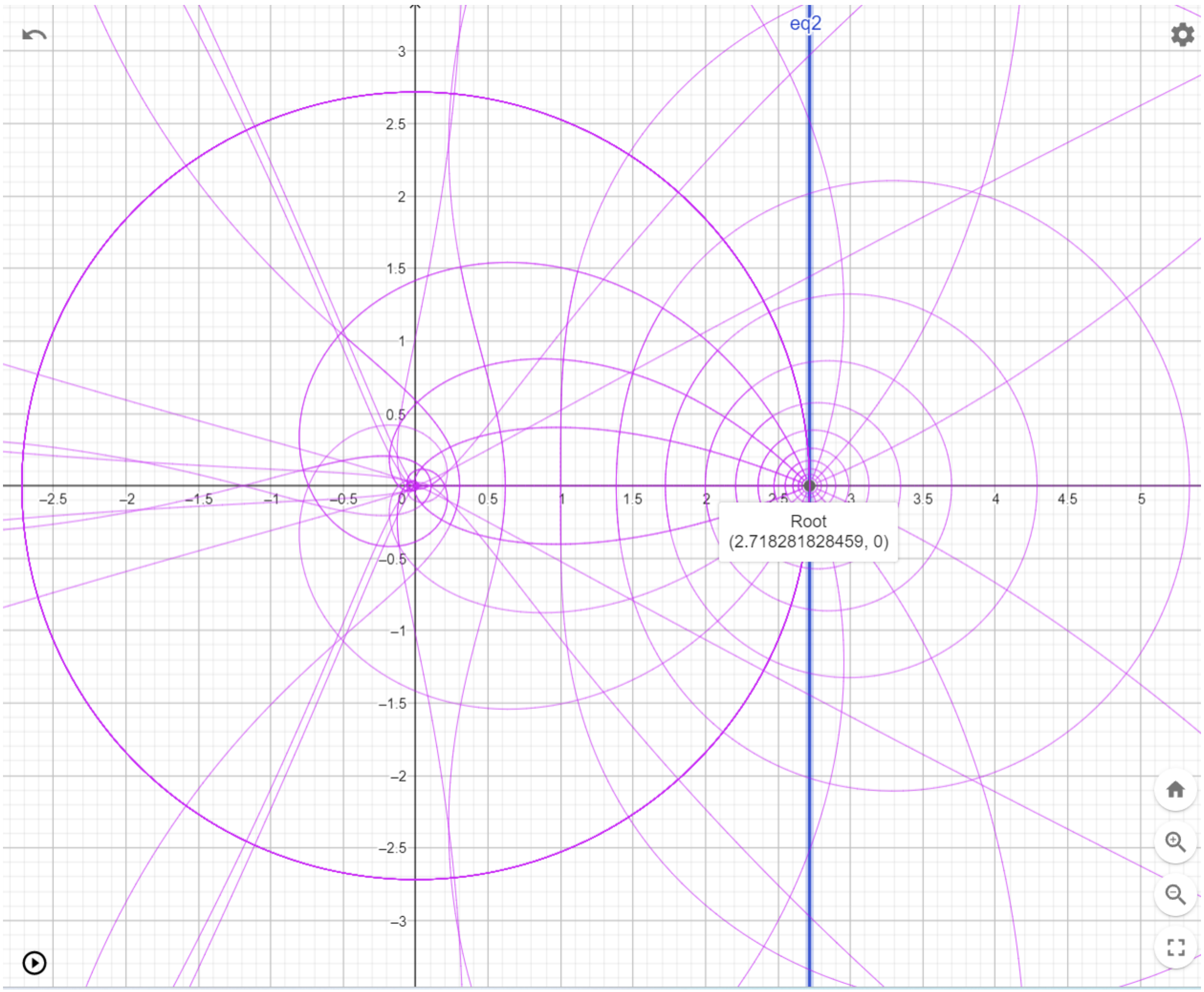
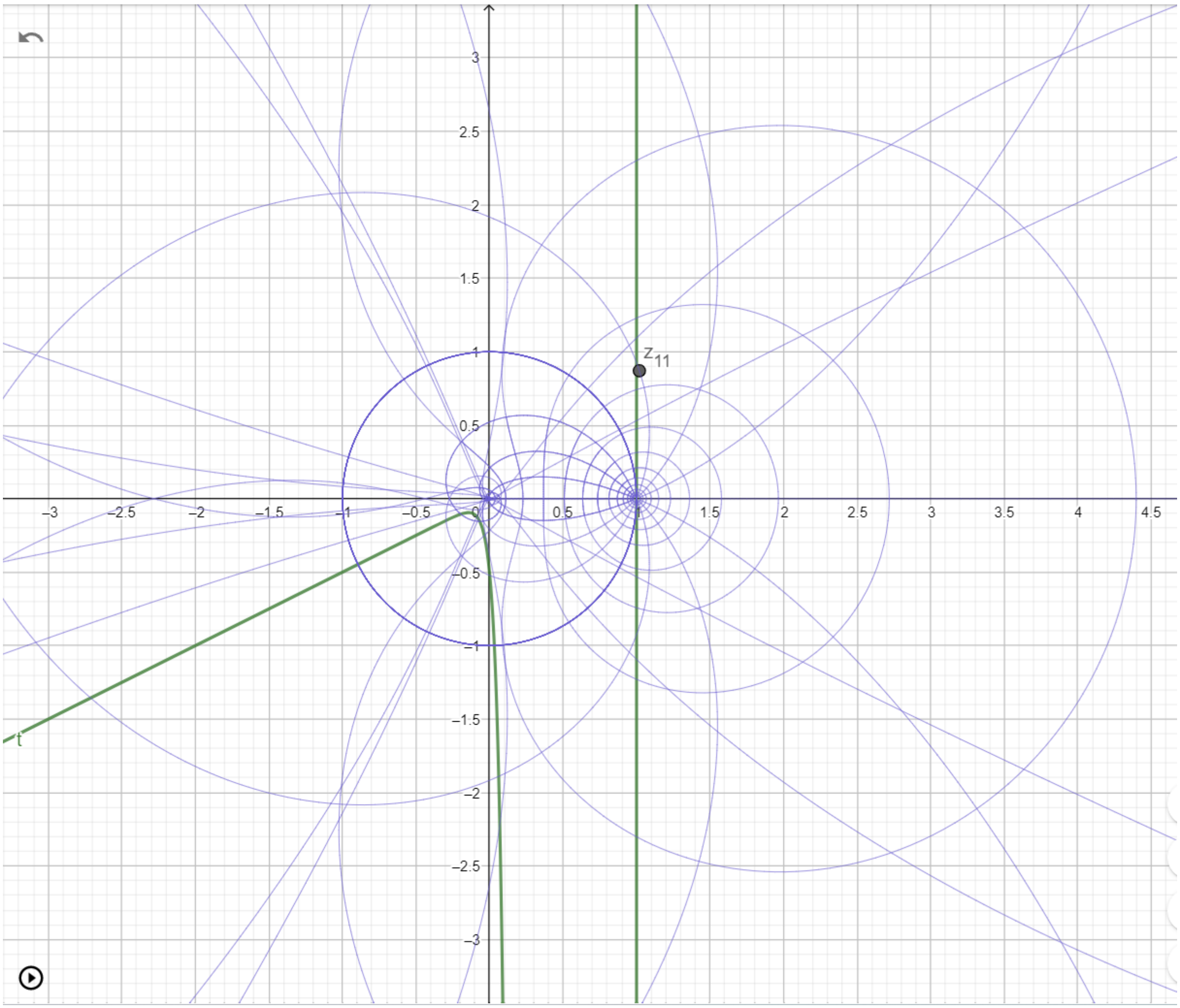


Figure 39. graph show Zero location at [e].

But for even numbers as if we assumed X was odd at the beginning

$$u(x) = e^{\left(\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{\frac{x}{2}-\frac{1}{2}}\right)}$$



This means if S is odd, we can get the same angel as even numbers if we used S = S-1= S/2 -1/2 if S is odd number.

Z10; is complex number on complex plane and will move it value on the odd number Identity circle between = { 1, -1, i, -i} as S changed its value between odd numbers.

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$$\longrightarrow -1 + 0\iota$$

1- For Odd numbers in set { 1, 5, 9 , 13 , 17 , 21 , ……} ; the complex number Z10 will changes values between {1,1-} with this order {1,-1,1,-1,1,-1,…..}

2- For Odd numbers in set {3, 7, 11 ,15 ,19,…….}; the complex number Z10 will changes values between {i,-i} with this oder {i,-i,i,-i,i,-i,…..}

3- For Odd numbers in set {-3,-7,-11,-15,-19,…..} ; the complex number Z10 will changes values between {1,1-} with this order {1,-1,1,-1,1,-1,…..}

4- And this is why the cycle of values resets after 8 and not 4 values.

5- to restrict values between only two values {1,-1} we are going to use the negative value for S; so we are goin to use this function instead.

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$\longrightarrow$ 1 - 0i

I) for S= S-1 ; half odd numbers will have Z10 = {1, -1} ; and the other half will have Z10={i,-i}

$$k(x) = e^{\left(\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{\frac{x}{2}-0.5}\right)}$$

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\longrightarrow$ -1 + 0i

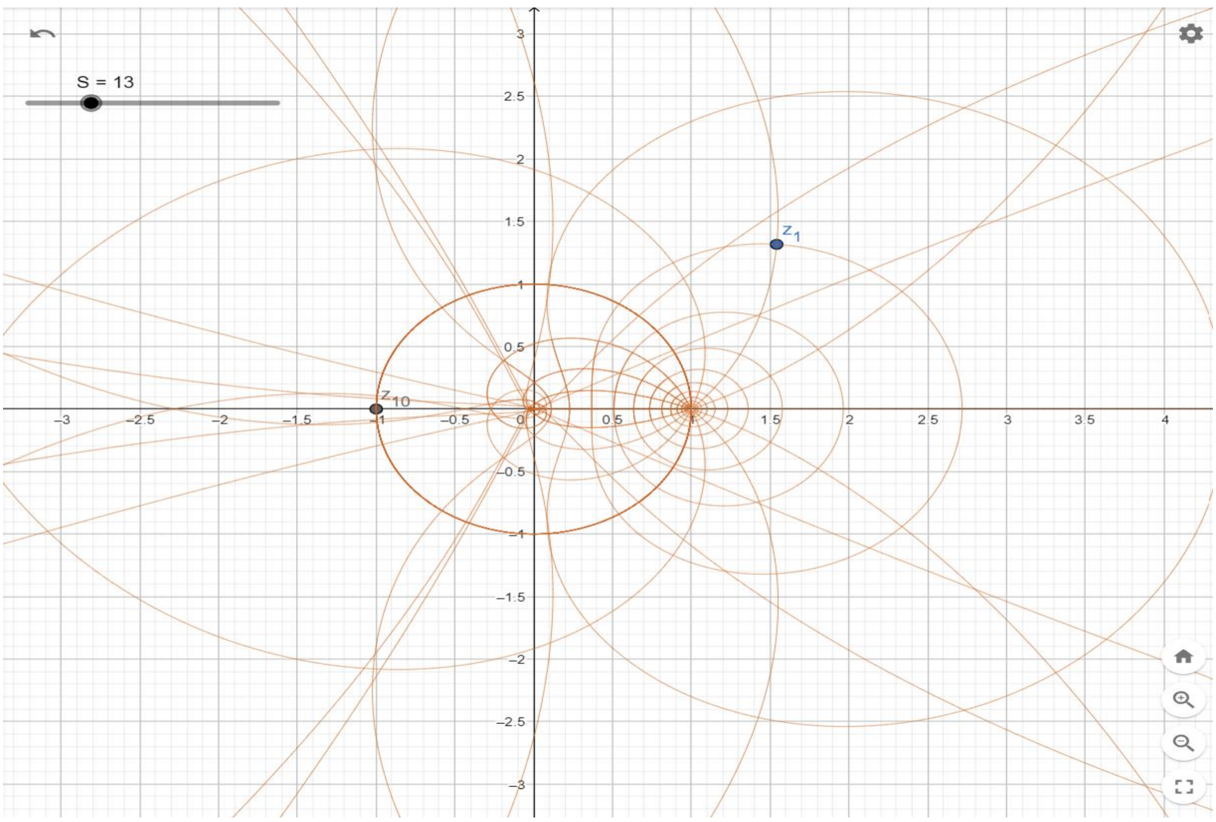


Figure 39. Z10 = -1 at S = 13 and S-1 = 12

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\longrightarrow$ 1 - 0ί

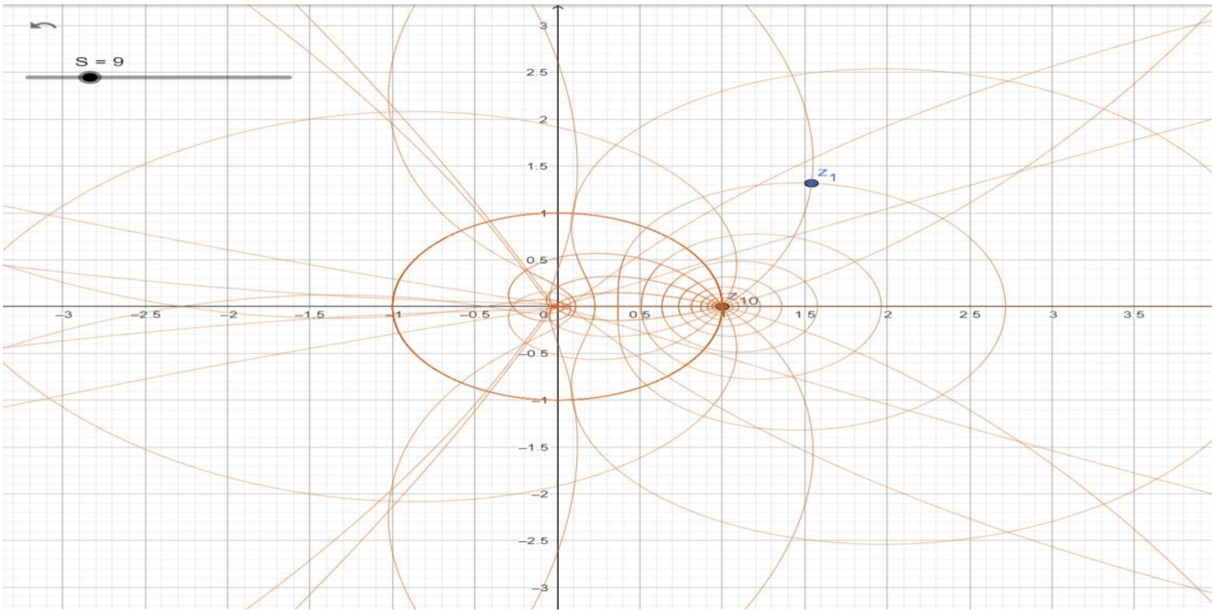


Figure 40. Z10 = 1 at S = 9 and S-1 = 8

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\longrightarrow$ -1 + 0ί

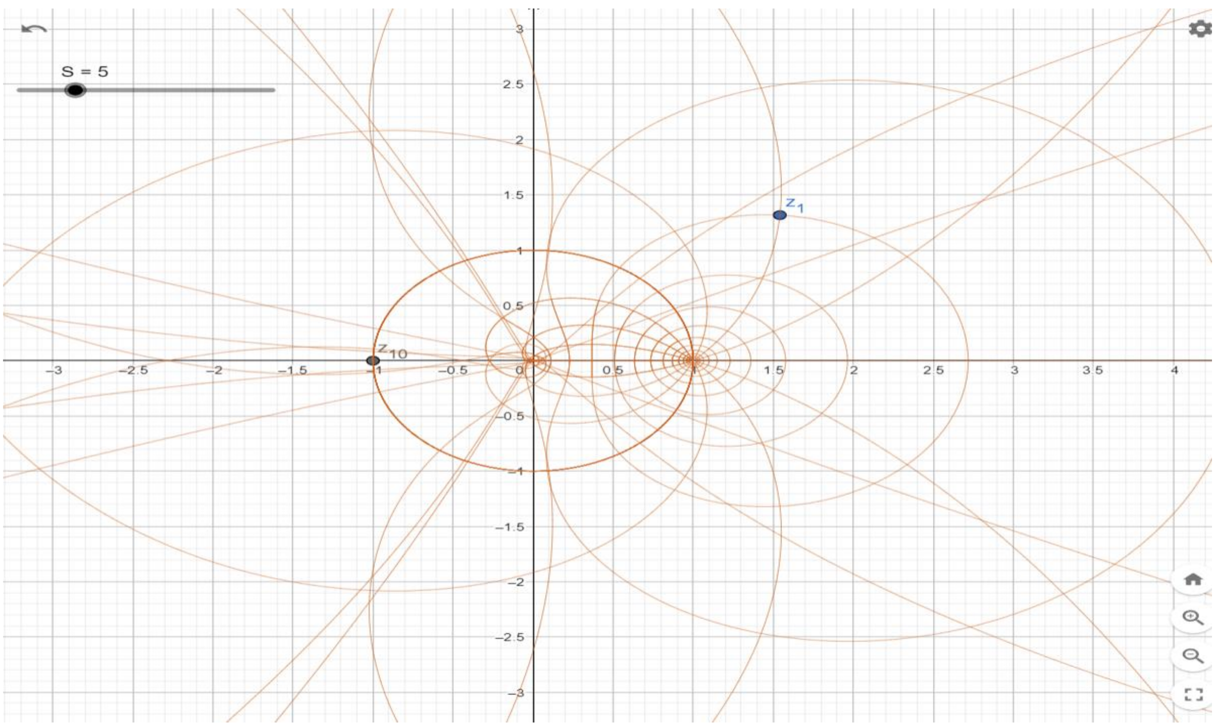


Figure 41. Z10 = -1 at S = 5 and S-1 = 4

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\rightarrow 1 - 0i$

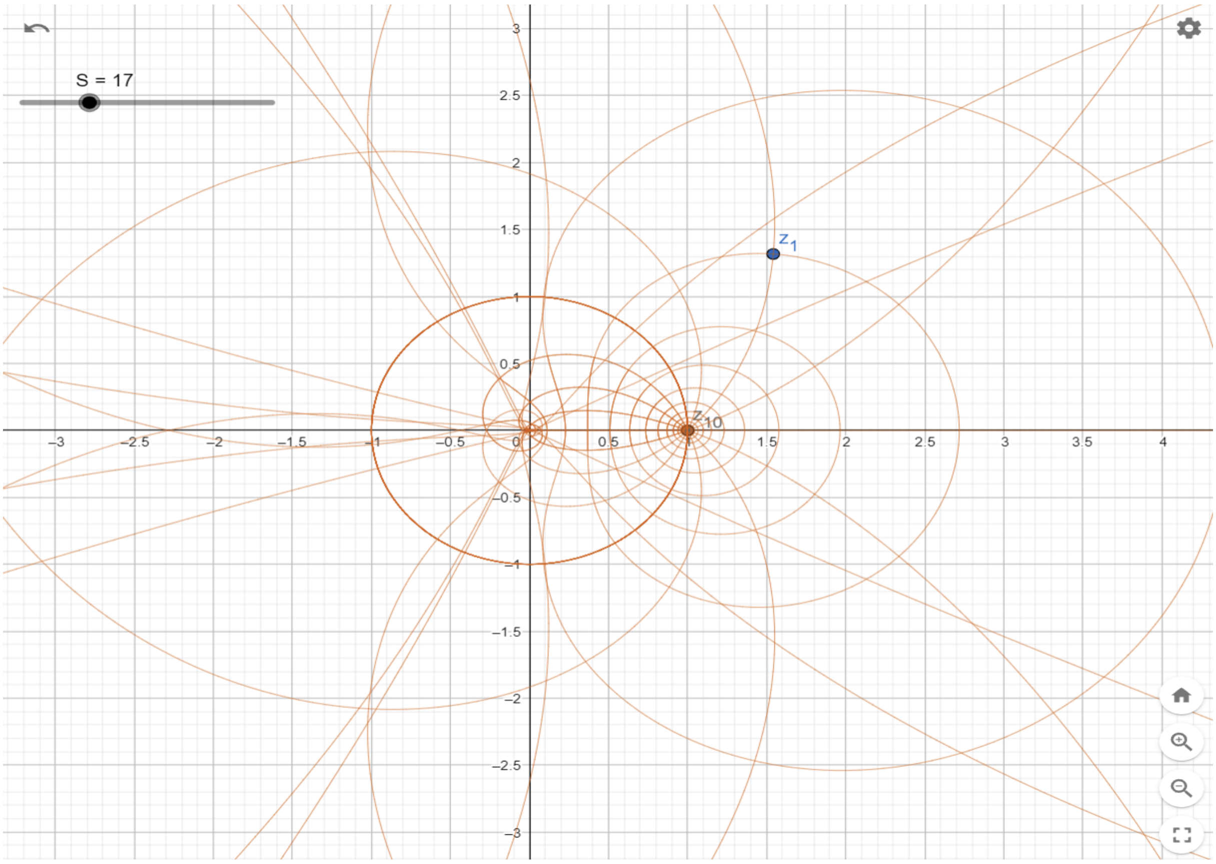


Figure 42. Z10 = 1 at S = 17 and S-1 = 16

II) for S= 2S-2 ; all odd numbers will have Z10 = {1, -1}

But if we used the new formula for the complex number for odd numbers

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$\rightarrow 1 - 0i$

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$\longrightarrow 1 + 0i$

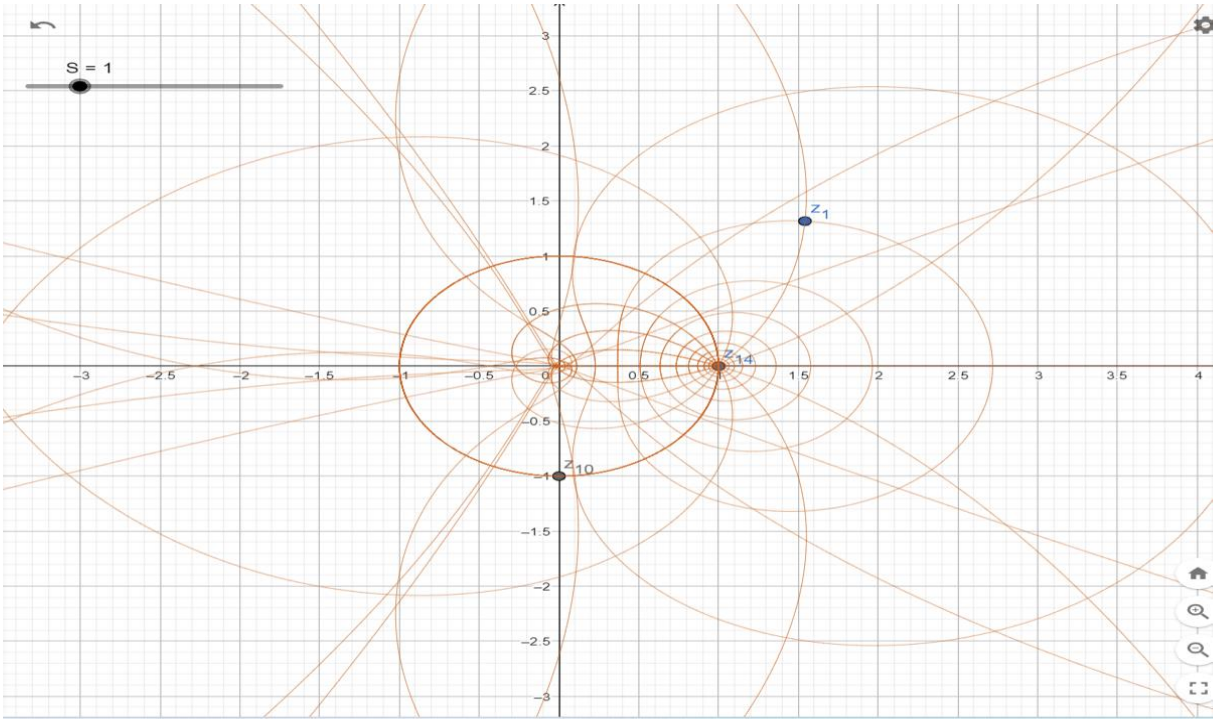


Figure 43. Z14 = 1 at S = 1 and 2S-2 = 0

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$\longrightarrow -1 + 0i$

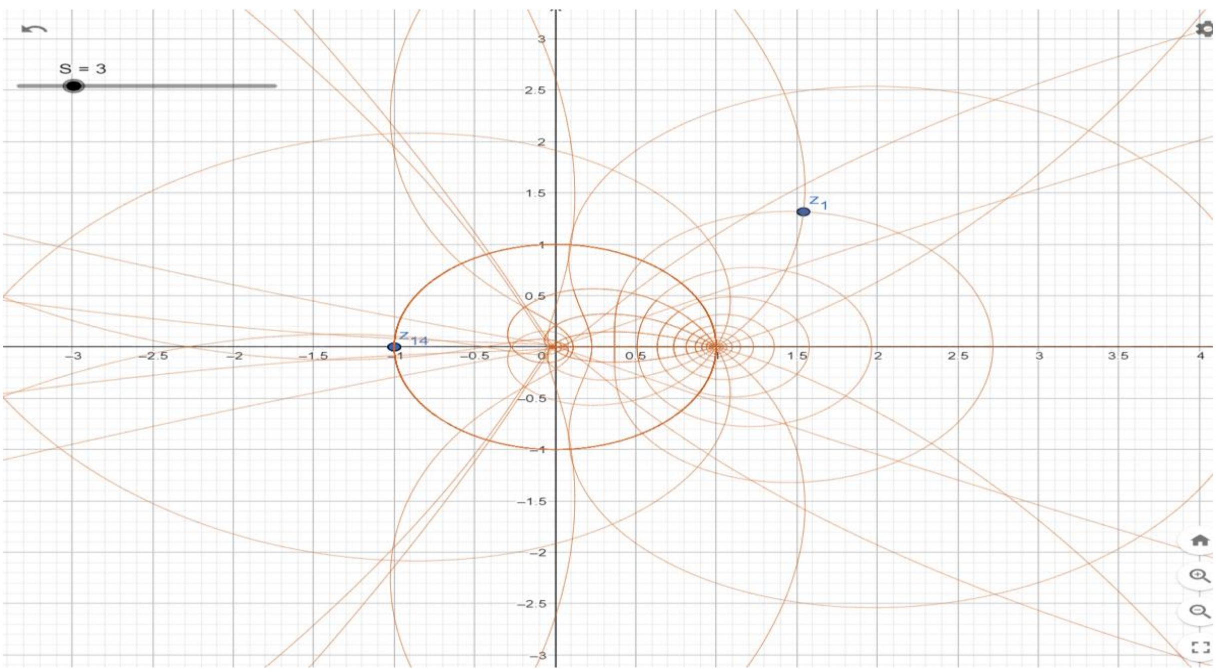


Figure 44. Z14 = -1 at S = 3 and 2S-2 = 4

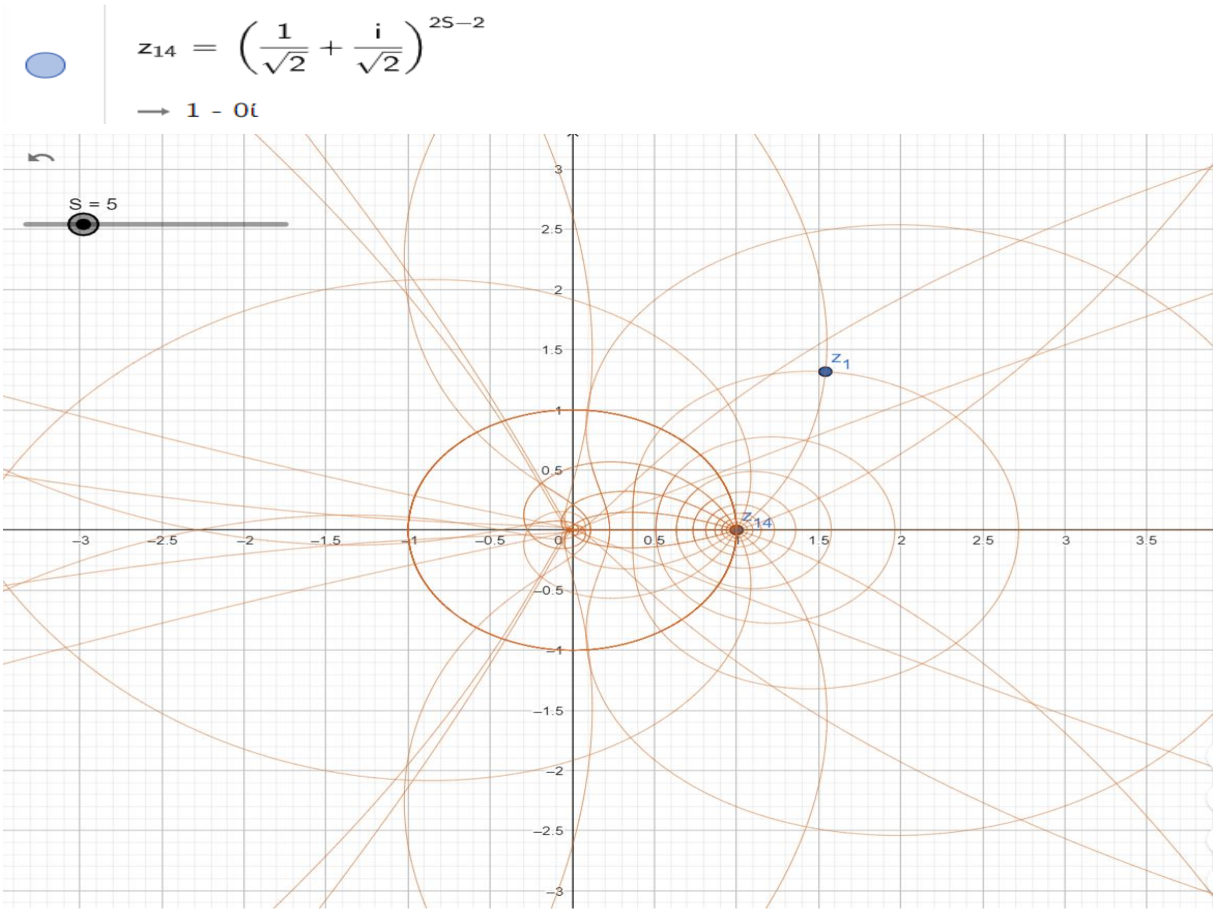


Figure 45. Z14 = 1 at S = 5 and 2S-2 = 8

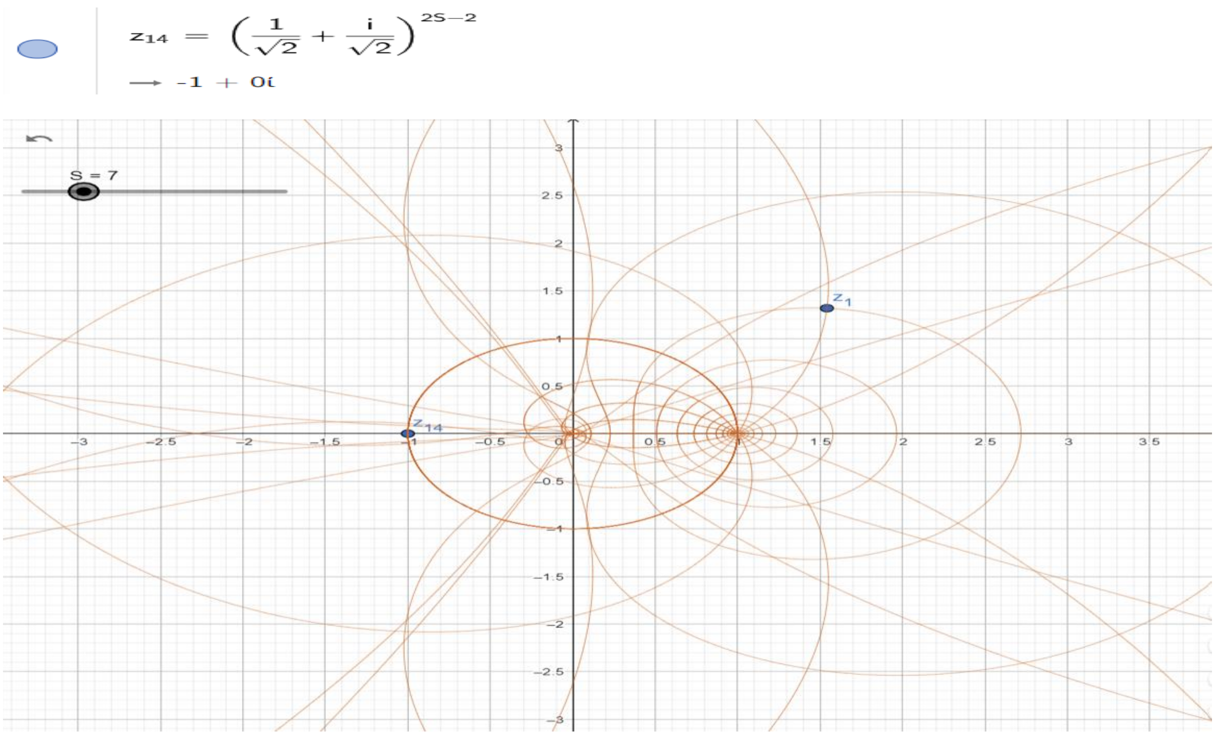


Figure 46. Z14 = -1 at S = 7 and 2S-2 = 12

Conclusion

Using our new Identity unit function in complex plane, helped in explaining the distribution of odd numbers and even numbers in complex plane. Also using exponential function in combine with our Identity function helped in determine that S = 2S + 2 can be used. And how when we used this form of transformation in combine with our new Identity function get us all the odd numbers on values = {1, -1}. Then we can say that

$$f(x) = z^{x} = \left(\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}\right)^{x} ;where\ x = 2x + 2$$

Then

$$\pm 1 = \left(\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}\right)^{2X+2}$$

$$e^{i\left(\pm\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}\right)^{2X+2}} = e^{-2^{x}\left(\pm\frac{1}{2} \pm i\frac{1}{2}\right)^{2X}}$$

$$e^{i\left(\pm\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}\right)^{2X+2}} = e^{\pm i}$$

$$-i * 2^{x}\left(\pm\frac{1}{2} \pm i\,\frac{1}{2}\right)^{2X} = \left(\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}\right)^{2X-2}$$

Also, we showed that.

$$\zeta\left(S + \frac{1}{2}\right) = \begin{cases} 2^{S}(\pi)^{S-\frac{1}{2}} * \dfrac{Sin\left(\frac{\pi}{2}\left(S + \frac{1}{2}\right)\right)}{Sin\left(\frac{\pi}{4}\right)} * \Gamma\left(\frac{1}{2} - S\right)\zeta\left(\frac{1}{2} - S\right) \\ 2^{S}(\pi)^{S-\frac{1}{2}} * \dfrac{Cos\left(\frac{\pi}{2}\left(S - \frac{1}{2}\right)\right)}{Sin\left(\frac{\pi}{4}\right)}\ \Gamma\left(\frac{1}{2} - S\right)\zeta\left(\frac{1}{2} - S\right) \end{cases} \qquad \rightarrow EQ(D)$$

By This Equation EQ(D), Zeta function formula at S = S +0.5, will have Sin wave with a Root at S = -0.5 and even negative Roots and Odd positive Roots. And at S = S -0.5, will have Sin wave with a Root at S=0.5 and even positive roots and odd negative roots. (For each Natural number).

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[2] G. Marsaglia, On the randomness of pi and other decimal expansions, Interstat 5, 2005, http://www.yaroslavvb.com/papers/marsaglia-on.pdf.

[3] G. Pólya, Mathematics and Plausible Reasoning (vol. I, Induction and Analogy in Mathematics, and vol. II, Patterns of Plausible Inference, Princeton University Press, Princeton, 1954).