New Odd Numbers Identity and The None-trivial Zeros of Zeta Function

Shaimaa Said Soltan

This Book introduce a new identity unit circle function for complex plane and specific for odd numbers.
All visualization in this book done using GeoGebra mathematics software, great tool to visualize ideas.

Table of Contents

New Odd Numbers Identity and The None-trivial Zeros of Zeta Function

Abstract

Odd number distribution has been a focal point for multiple exploratory research papers and will continue to be, it is very reach topic to explore, and each exploratory research have contributed something new to this subject.

In this book we are conducting exploratory research on odd number distribution in complex plane and tackling this distribution from different points of views. During this research we are going to introduce new function to visualize the localization difference between even and odd number distribution in complex plane.

One of the most popular formulas in math is Zeta function, and Riemann's functional equation. If we looked at this functional equation from an abstract point of view, it is mainly a Geometric wave function multiplied by some magnitude (the value of rest of terms in this functional equation).

This why in this research are going to focus on exploring geometric functions (Sin and Cos) including their inverse domain. We are going to explore both domains first in term of its zero's distribution, which reflects the distribution of the numbers in general, then we are going to show some expletory visualization in the inverse domain for the Geometric function (Sin) in Riemann's functional equation which is a visualization that highlights an explanation for the localization of none-trivial Zeros for Zeta function.

Riemann's functional equation

$$\zeta(s) = 2^s \pi^{s-1} \ \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s).$$

The Riemann zeta function on the critical line can be written

$$\zeta\left(\frac{1}{2}+it\right) = e^{-i\theta(t)} Z(t),$$

$$Z(t) = e^{i\theta(t)} \zeta\left(\frac{1}{2}+it\right).$$

Then Zeta function will be zero

1- At $\sin\left(\frac{\pi s}{2}\right)$ is Zero for any complex number S.

2- If exponential term is zero also when S = S + 0.5 where S is any complex number.

Our new proposed Odd Identity unit circle function is looking at complex plane from the frame of reference point of view in complex plane and its rotation and scaling and shifting operations in the complex plane relaying on a power function. First, we explore the new Odd Identity function without using Euler number then we will see how this new Identity will operate and its characteristics then see if it is combined with Euler number.

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^x = \pm \mathrm{Cos}(2x\theta) \pm i\,\mathrm{Sin}(2x\theta) = \pm\mathrm{Cos}((2 * x * 22.5)°) \pm i\,\mathrm{Sin}((2 * x * 22.5)°)$$

Keywords: zeta function, Riemann hypothesis, complex plane, none-trivial zeros, critical strip, gamma function

1. Introduction

A) $f(x) = z^x = (\pm\frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^x$ new Identity function for odd number in a complex plane.

our objective in this research is to show how this new Identity function f(x) shows odd number distribution, which is the same as imaginary unit Identity but with angel θ = π/4 = 180°/4.

$$\theta * 2 * x = \frac{\pi}{8} * 2 *\ x = 22.5° * 2\ x = 45° * x$$

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^x = \pm \mathrm{Cos}(2x\theta) \pm i\ \mathrm{Sin}(2x\theta) = \pm\mathrm{Cos}((2 * x * 22.5)°) \pm i\ \mathrm{Sin}((2 * x * 22.5)°)$$

First, we will explore eigen characteristics of Sin and Cos waves then we will visualize these characteristics more using this newly introduced Identity for odd numbers in complex plane.

2. Eigen characteristics for geometric functions $Cos(\pi * \sqrt{X}\ )and\ Sin(\pi * \sqrt{X}\ )$ **roots distributions.**

We will see here that each of these two geometric functions has its own characteristic in terms of the distribution of its Zeros in its wave signals, regardless of scaling or shifting transformations.

2.1 Eigen characteristics for $Cos(\pi * \sqrt{X}\ )$ **roots distribution**

These geometric functions of Cos wave have roots distributed in specific values for X. Starting from a starting point with specific jumping steps up and until the root.
{+2, +4, +6, +8, + 10, +12, +14, +16….}

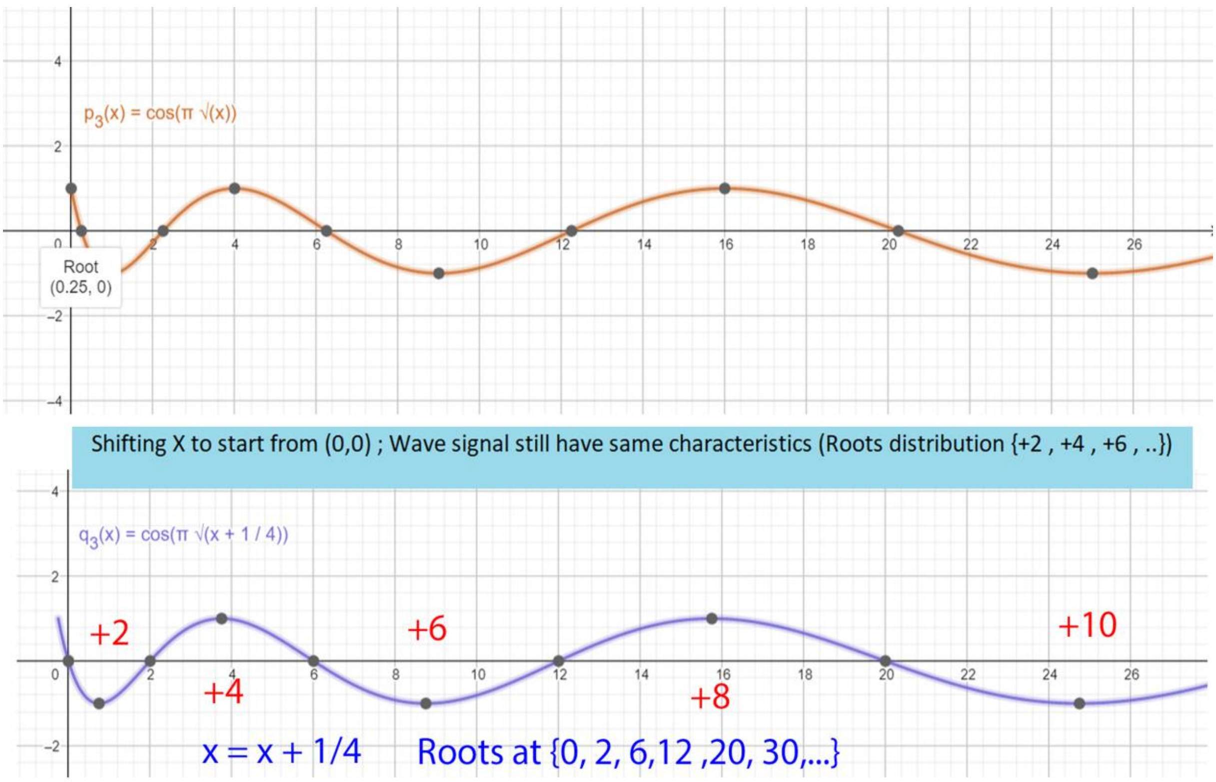


Figure 1. Roots distribution characteristics for $Cos(\pi * \sqrt{X}\ )$and $Cos\left(\pi * \sqrt{X + \frac{1}{4}}\right)$

2.2 Eigen characteristics for $Sin(\pi * \sqrt{X})$ roots distribution

These geometric functions of Sin wave have roots distributed in specific values for X. Starting from a starting point with specific jumping steps up and until to the root. {+1, +3, +5, +7, +9, +11, +13, +16…….}

** For every natural value in {1, 2, 3, 4, 5, 6, 7, 8…}

***We get roots at its square {1, 4, 9, 16, 25, 36, 49, 64, 81…}

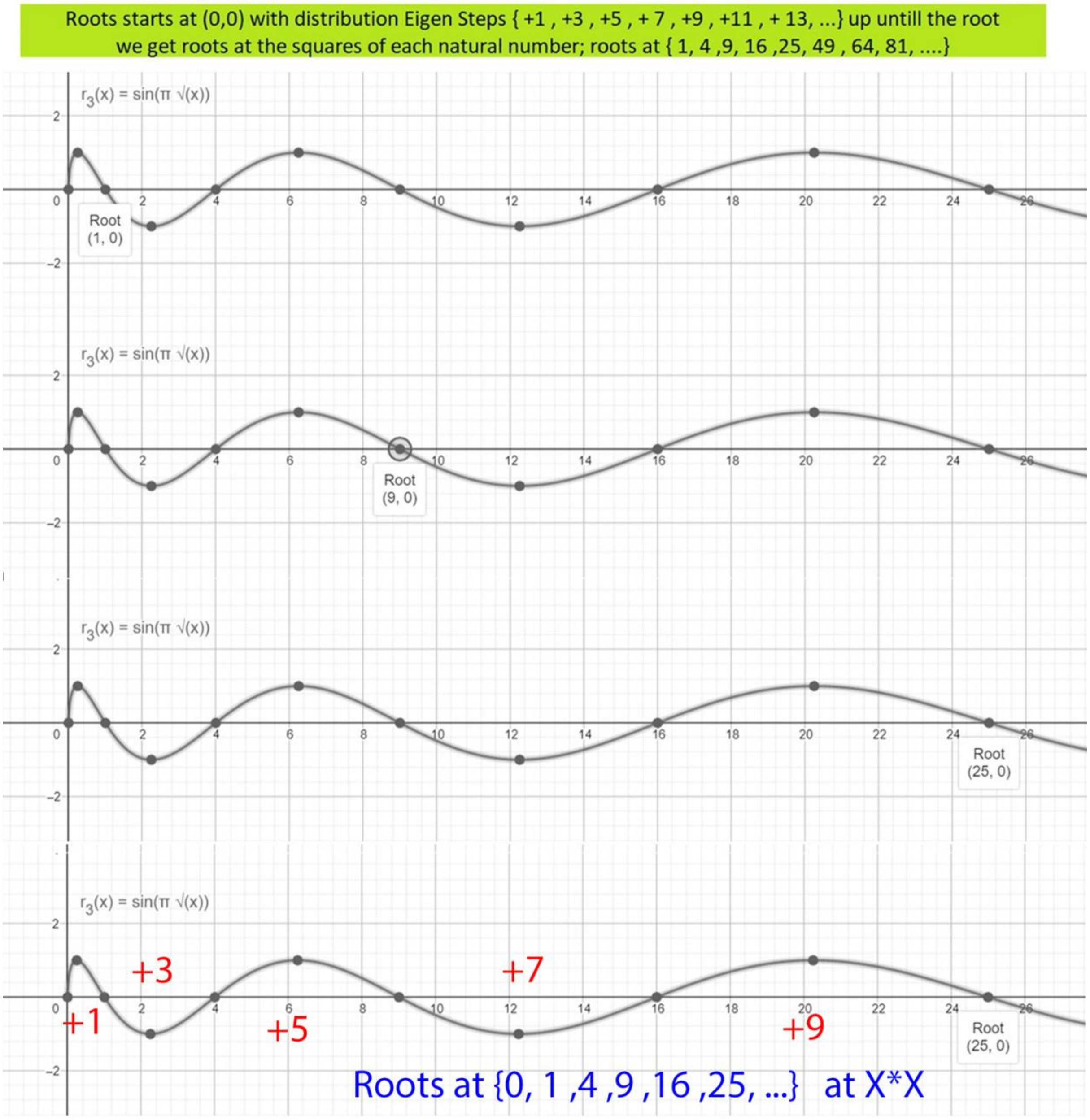


Figure 2. Roots distribution characteristics for $Sin(\pi * \sqrt{X})$

2.3 Eigen characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$ roots distribution

These geometric functions of Sin wave have roots distributed in specific values for X. Starting from a start point with specific jumping steps up and until to the root.

{+1, +3, +5, +7, +9, +11, +13, +16…….}

** For every natural value in {1π, 2π, 3π, 4, 5, 6, 7, 8…}

***We get roots at its square {1π, 4π, 9π, 16π, 25π, 36π, 49π, 64π, 81π…}

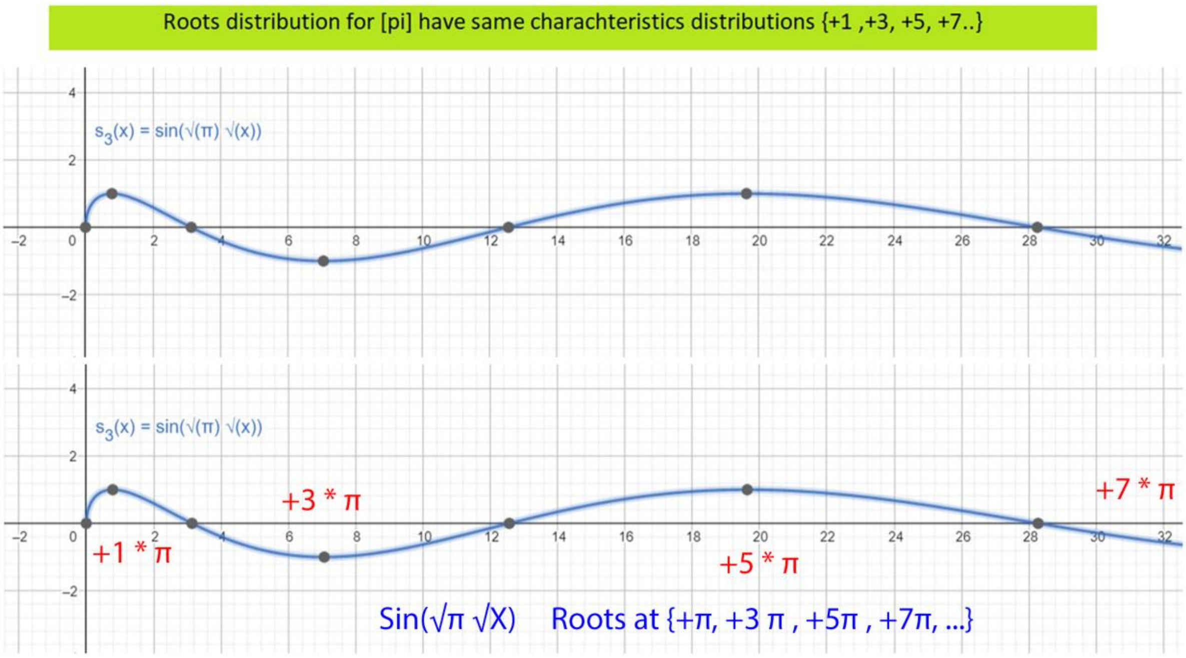


Figure 3. Root distribution characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$

2.4 Characteristics for $Sin(\sqrt{\pi} * \sqrt{X})$ and $Cos(\pi * \sqrt{x})$

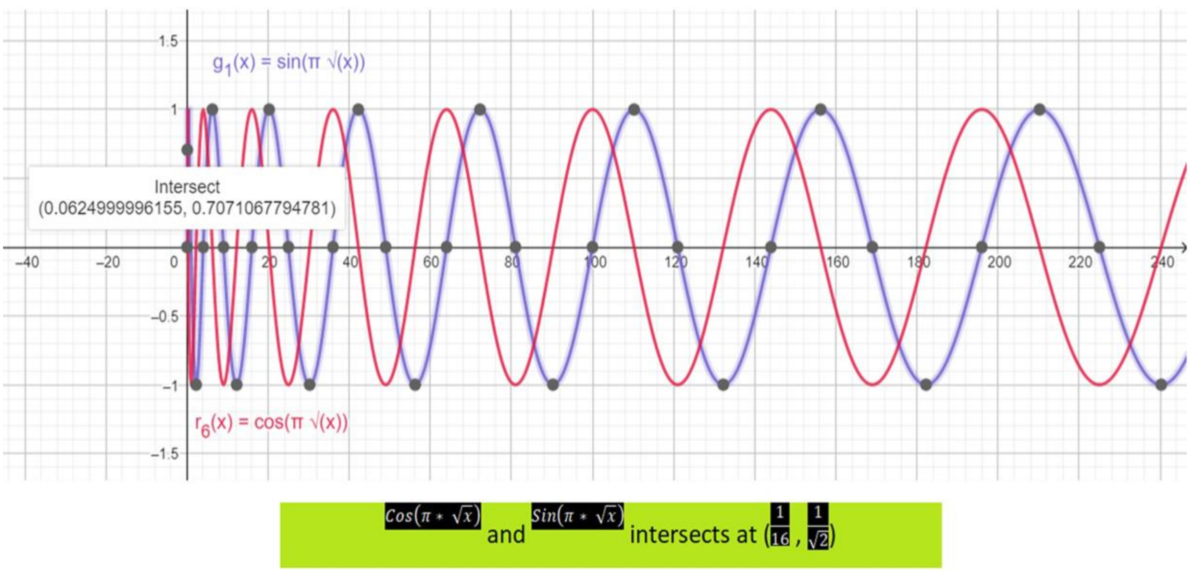


Figure 4. $Cos(\pi * \sqrt{x})$ and $Sin(\pi * \sqrt{x})$ intersects at (1/16, $\frac{1}{\sqrt{2}}$)

2.5 Eigen characteristics for $Sin\left(\frac{\pi\, S}{2}\right)$ in Zeta function formula

First, we are going to summaries the synchronization between the two geometric functions.

$$Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right) = Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)$$

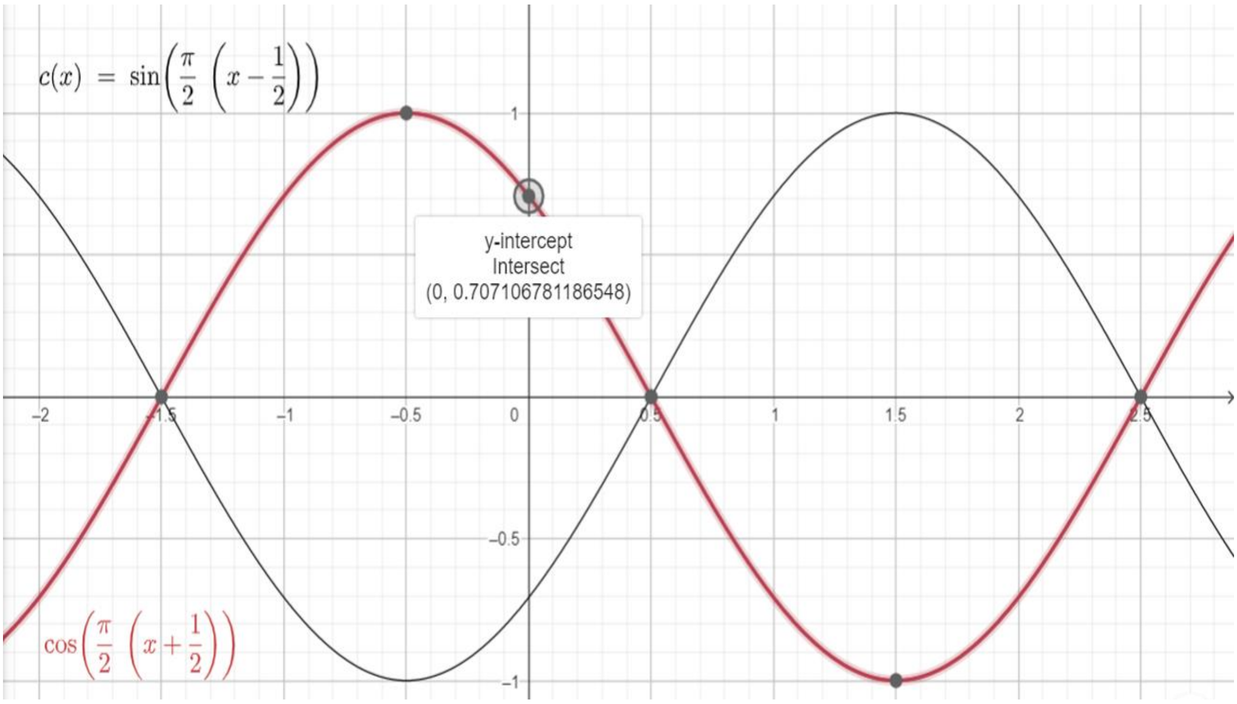


Figure 5. Synchronizations between Sin Geometric function and Cos Geometric function at X=0.5

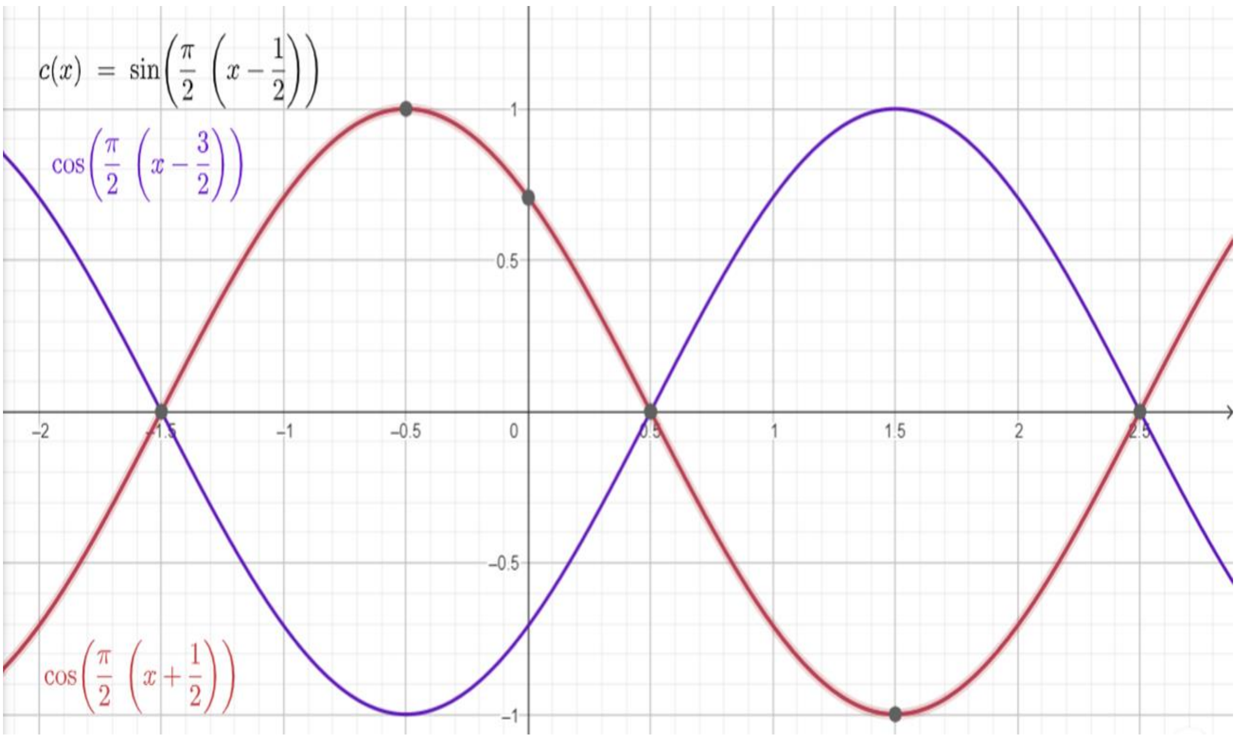


Figure 6. Identical Synchronizations between Sin and Cos Geometric functions at X=0.5 with a difference of one full cycle.

2.5.1 Identical Geometric wave signals $Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right) = Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)$

$\zeta(1-S) = \frac{2}{(2\,\pi)^S}\,Cos\left(\frac{\pi\,S}{2}\right)\,\Gamma(S)\,\zeta(S)$ → EQ [1]

$\zeta(S) = 2^S\,\pi^{S-1}\,Sin\left(\frac{\pi\,S}{2}\right)\,\Gamma(1-S)\,\zeta(1-S)$ → EQ [2]

$Cos\left(\frac{\pi}{8}*2*X\right) = Sin\left(\frac{\pi}{8}*(2*X+4)\right)$ → EQ [3]

$Cos\left(\frac{\pi}{2}*\frac{X}{2}\right) = Sin(\frac{\pi}{2}*(\frac{X}{2}+1))$ → EQ [4]

$Cos\left(\frac{\pi}{2}*S\right) = Sin(\frac{\pi}{2}*(S+1))$ → EQ [5]

$Cos\left(\frac{\pi}{2}*(S-\frac{1}{2})\right) = Sin(\frac{\pi}{2}*(S+\frac{1}{2}))$ →EQ [6]

$Cos\left(\frac{\pi}{2}*(S-1)\right) = Sin(\frac{\pi}{2}*S))$ → EQ [7]

$Cos\left(\frac{\pi}{2}*(S-\frac{3}{2})\right) = Sin(\frac{\pi}{2}*(S-\frac{1}{2}))$ → EQ [8]

$\Gamma(S)\,\Gamma(1-S) = \frac{\pi}{Sin(\pi S)}$ → EQ [9]

From EQ [1] and EQ [2]

$$\zeta(S) = 2^S\,\pi^{S-1}\,Sin\left(\frac{\pi\,S}{2}\right)\,\Gamma(1-S) * \frac{2}{(2\,\pi)^S}\,Cos\left(\frac{\pi\,S}{2}\right)\,\Gamma(S)\,\zeta(S)$$

$$1 = 2^S\,\pi^{S-1}\,Sin\left(\frac{\pi\,S}{2}\right)\,\Gamma(1-S) * \frac{2}{(2\,\pi)^S}\,Cos\left(\frac{\pi\,S}{2}\right)\,\Gamma(S)$$

$$1 \ = 2^S \ \pi^{S-1} \ Sin\left(\frac{\pi \ S}{2}\right) \ * \ \frac{2}{(2 \ \pi)^S} \ Cos\left(\frac{\pi \ S}{2}\right) \ \Gamma(S) \ \Gamma(1-S)$$

From EQ [5]

$$1 \ = \ Sin\left(\frac{\pi \ S}{2}\right) * \ Cos\left(\frac{\pi \ S}{2}\right) \ * \ \frac{2^S \ \pi^{S-1} * 2}{(2 \ \pi)^S} \ \Gamma(S) \ \Gamma(1-S)$$

$$1 \ = \ Sin\left(\frac{\pi \ S}{2}\right) * \ Sin(\frac{\pi}{2} * (S+1)) \ * \ \frac{2}{\pi} * \ \Gamma(S) \ \Gamma(1-S)$$

From EQ [9]

$$1 \ = \ Sin\left(\frac{\pi \ S}{2}\right) * \ Sin(\frac{\pi}{2} * (S+1)) \ * \ \frac{2}{\pi} * \ \Gamma(S) \ \Gamma(1-S)$$

$$1 \ = \ Sin\left(\frac{\pi \ S}{2}\right) * \ Sin\left(\frac{\pi}{2} * (S+1)\right) * \ \frac{2}{\pi} * \frac{\pi}{Sin(\pi \ S)}$$

$$\frac{Sin\left(\frac{\pi \ S}{2}\right) * \ Sin\left(\frac{\pi}{2} * (S+1)\right)}{Sin(\pi \ S)} = \frac{1}{2}$$

$$2 * \ Sin\left(\frac{\pi \ S}{2}\right) * \ Sin\left(\frac{\pi}{2} * (S+1)\right) = \ Sin(\pi \ S)$$

$2 * \ Sin\left(\frac{\pi \ S}{2}\right) * \ \ cos\left(\frac{\pi \ S}{2}\right) \ = \ Sin(\pi \ S)$➔ EQ [10]

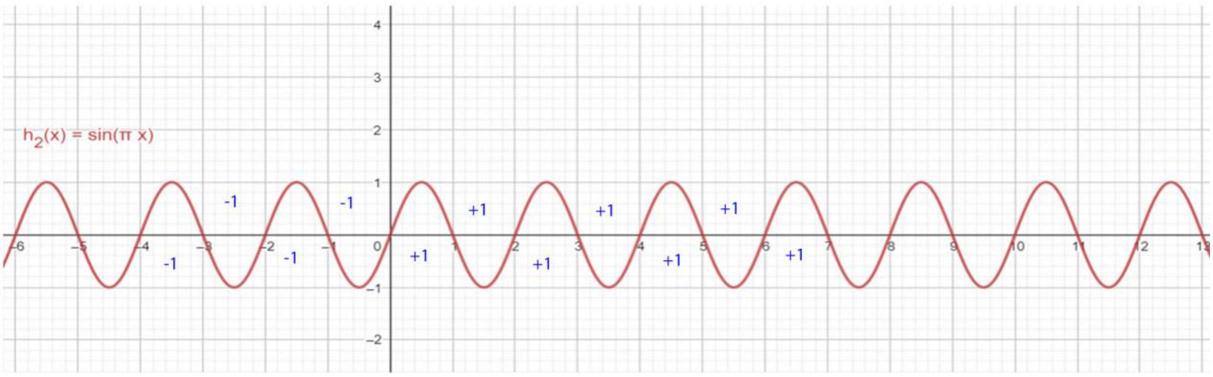


Figure 7. $Sin(\pi \ S)$➔ Have roots for any natural number value for ±S.

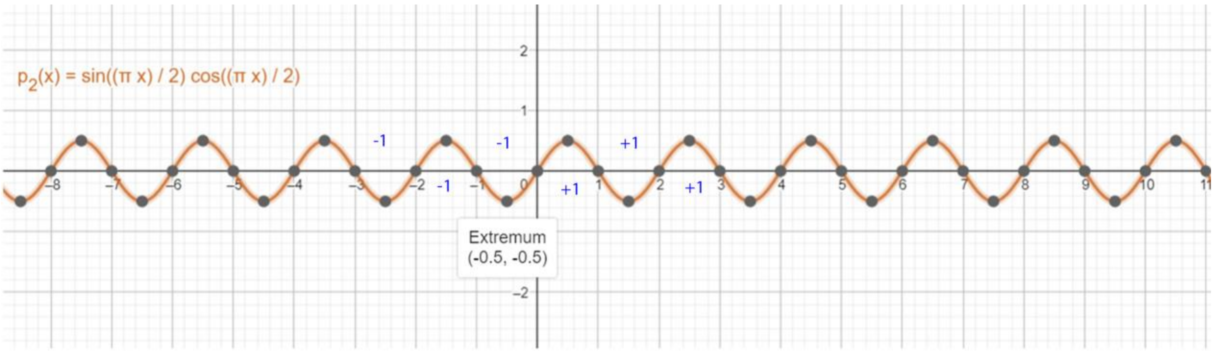


Figure 8. $Sin\left(\frac{\pi S}{2}\right) * \ cos\left(\frac{\pi S}{2}\right)$ ➔ have roots for any natural number value for ±S.

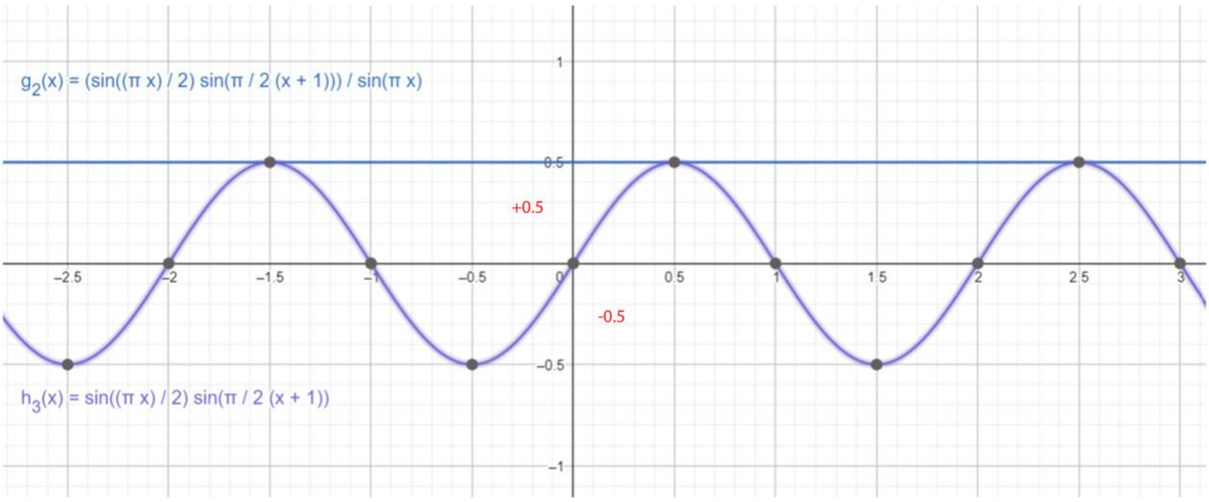


Figure 9. $\frac{Sin\left(\frac{\pi S}{2}\right) * \ Sin\left(\frac{\pi}{2} * (S+1)\right)}{Sin(\pi S)} = \frac{1}{2}$ ➔blue line; equal $\frac{1}{2}$ for any natural number value for ±S.

Back substitution from EQ [5]

$Sin\left(\frac{\pi S}{2}\right) = \frac{Sin(\pi S)}{2 * Cos(\frac{\pi S}{2})}$ ➔ EQ [11]

$Cos\left(\frac{\pi S}{2}\right) = \frac{Sin(\pi S)}{2 * Sin(\frac{\pi S}{2})}$ ➔ EQ [12]

From EQ [6] Substitute back in EQ [2]

$$\zeta(S) = 2^S \ \pi^{S-1} \ Cos\left(\frac{\pi}{2} \ (S-1)\right) \ \Gamma(1-S) \ \zeta(1-S)$$

Let S = S + 0.5

$$\zeta\left(S+\frac{1}{2}\right)=(2*\pi)^{\,S}*\sqrt{\frac{2}{\pi}}*\ Cos\left(\frac{\pi}{2}\ (S-\frac{1}{2})\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)$$

From EQ [6]

$Cos\left(\frac{\pi}{2}*(S-\frac{1}{2})\right)=Sin(\frac{\pi}{2}*(S+\frac{1}{2}))$ →EQ [6]

$$\zeta\left(S+\frac{1}{2}\right)=(2*\pi)^{\,S}*\sqrt{\frac{2}{\pi}}*\ Sin\left(\frac{\pi}{2}\ (S+\frac{1}{2})\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)$$

$$\zeta\left(S+\frac{1}{2}\right)=\begin{cases}(2*\pi)^{\,S}*\sqrt{\frac{2}{\pi}}*\ Sin\left(\frac{\pi}{2}\ (S+\frac{1}{2})\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)\\(2*\pi)^{\,S}*\sqrt{\frac{2}{\pi}}*\ Cos\left(\frac{\pi}{2}\ (S-\frac{1}{2})\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)\end{cases}$$

$$\zeta\left(S+\frac{1}{2}\right)=\begin{cases}(2*\pi)^{S}*\sqrt{\frac{2}{\pi}}*\ Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)=0\,;\ when\ S\ odd\\(2*\pi)^{S}*\sqrt{\frac{2}{\pi}}*\ Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\ \Gamma\left(\frac{1}{2}-S\right)\ \zeta\left(\frac{1}{2}-S\right)=0;when\ S\ odd\end{cases}\qquad \rightarrow EQ(A)$$

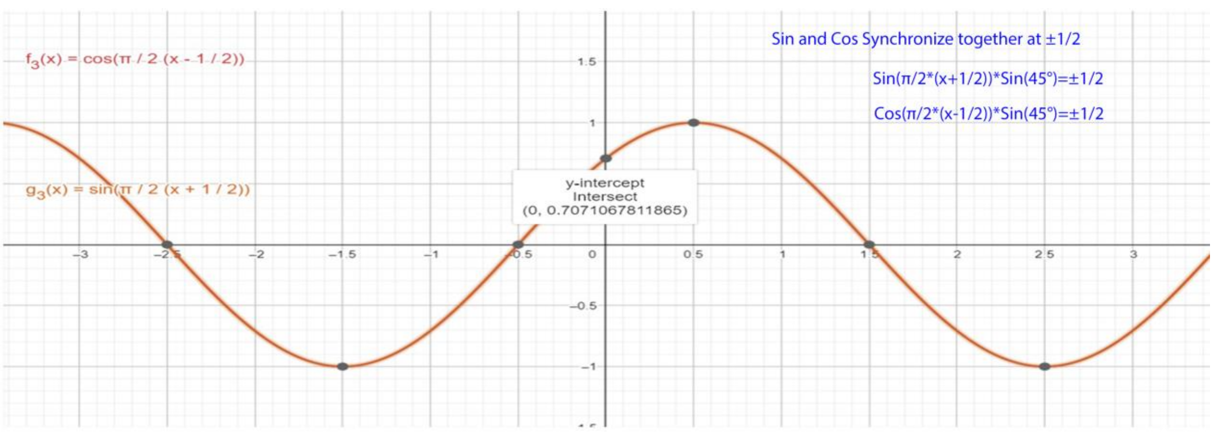


Figure 10. $Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\ and\ Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)$ → Identical with roots at $[\frac{-1}{2}]$ and Y-intercept = $\frac{1}{\sqrt{2}}$; for any ±S

For S = S -1

$$\zeta\left(S-\frac{1}{2}\right)=\begin{cases}(2*\pi)^{S-1}*\sqrt{\frac{2}{\pi}}*\ Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{3}{2}-S\right)\zeta\left(\frac{3}{2}-S\right)=0\ ;\ when\ S\ odd\\(2*\pi)^{S-1}*\sqrt{\frac{2}{\pi}}*\ Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right)\Gamma\left(\frac{3}{2}-S\right)\zeta\left(\frac{3}{2}-S\right)=0;when\ S\ odd\end{cases}\quad \rightarrow EQ(B)$$

From EQ [8]; $Cos\left(\frac{\pi}{2}\left(S-\frac{3}{2}\right)\right)=\ Sin\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)$; $Identical\ wave\ signals$

Until now from EQ (A) and EQ (B) we showed that Zeta function will have Root at (S= -0.5)
And equal Zero if (S = S ± 0.5) for any odd number S.

Next, we will show how all these Zeros will have imaginary unit value by introducing a new Identity function for odd numbers in a complex plane.

2.5.2 New Odd Numbers Identity $f(x)=z^x\ =\ (\pm\frac{1}{\sqrt{2}}\pm i\ \frac{1}{\sqrt{2}})^x$ in a complex plane.

our objective in this part is to show how this new Identity function f(x) shows odd number distribution, which is the same as imaginary unit Identity but with angel = π/4 = 180º/4.

$$\theta*2*x=\frac{\pi}{8}*2*\ x=22.5°*2\ x=45°*x$$

$$e^{i\theta x}=Cos(\theta x)+i\ Sin(\theta x)\qquad\rightarrow EQ(13)$$

$$z=\frac{1}{\sqrt{2}}+i\ \frac{1}{\sqrt{2}}\qquad\rightarrow EQ(14)$$

$$f(x)=z^x\ =\ (\pm\frac{1}{\sqrt{2}}\pm i\ \frac{1}{\sqrt{2}})^x\qquad\rightarrow EQ(15)$$

$$Cos\left(\frac{\pi}{8}*2*X\right)=Sin\left(\frac{\pi}{8}*(2*X+4)\right)\qquad\rightarrow EQ(16)$$

$$f(x)=\begin{cases}(\pm\frac{1}{\sqrt{2}}\pm\frac{i}{\sqrt{2}})^x\\\pm Cos(\theta*2x)\pm i\ Sin(\theta*2\ x)\end{cases}\qquad\rightarrow EQ(18)$$

At X = ½

$$f(x)=\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{\frac{1}{2}}=\mathrm{Cos}(22.5°)+i\ \mathrm{Sin}(22.5°)$$

$$=\ 0.9238795325113+0.3826834323651i$$

$$Cos\left(\frac{\pi}{2}*\frac{x}{2}\right)=Sin(\frac{\pi}{2}*(\frac{x}{2}+1)) \rightarrow \text{EQ [19]}$$

$$Cos\left(\frac{\pi}{2}*(S-\frac{1}{2})\right)=Sin(\frac{\pi}{2}*(S+\frac{1}{2})) \rightarrow \text{EQ [20]}$$

$$Cos\left(\frac{\pi}{8}\right)=Sin(\frac{\pi}{4}+\frac{\pi}{8})$$

$$Cos\left(\frac{\pi}{2}*\frac{x}{4}\right)=Sin\left(\frac{\pi}{2}\left(\frac{x}{4}+1\right)\right)$$

$$Cos\left(\frac{\pi}{2}*\frac{x+0.5}{2}\right)=Cos\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right)=Sin\left(\frac{\pi}{4}\left(x+\frac{5}{2}\right)\right) \rightarrow EQ(22)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-0.5}{2}\right)=Cos\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=Sin\left(\frac{\pi}{4}\left(x+\frac{3}{2}\right)\right) \rightarrow EQ(21)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-1.5}{2}\right)=Cos\left(\frac{\pi}{4}*\left(x-\frac{3}{2}\right)\right)=Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right) \rightarrow EQ(23)$$

$$Cos\left(\frac{\pi}{2}*\frac{x-2.5}{2}\right)=Cos\left(\frac{\pi}{4}*\left(x-\frac{5}{2}\right)\right)=Sin\left(\frac{\pi}{4}\left(x-\frac{1}{2}\right)\right) \rightarrow EQ(24)$$

$$Cos\left(\frac{\pi}{2}*\left(\frac{x}{4}+\frac{1}{2}+1\right)\right)=Sin\left(\frac{\pi}{2}\left(\frac{x}{4}-\frac{1}{2}-1\right)\right)$$

$$Cos\left(\frac{\pi}{2}*\left(\frac{x}{4}+\frac{3}{2}\right)\right)=Sin\left(\frac{\pi}{2}\left(\frac{x}{4}-\frac{3}{2}\right)\right) \rightarrow EQ(25)$$

$$Cos\left(\frac{\pi}{8}*(x+6)\right)=Sin\left(\frac{\pi}{8}\,(x-6)\right) \rightarrow EQ(26)$$

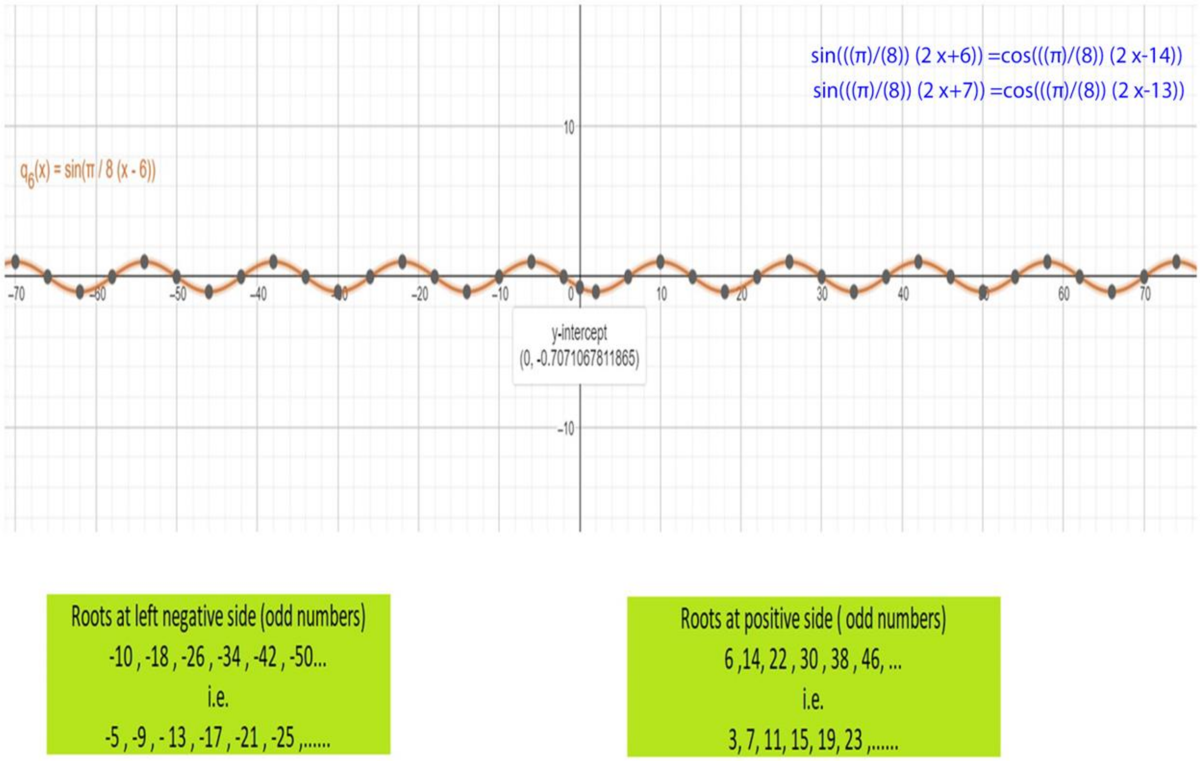


Figure 11. odd numbers Root distribution for $Sin\left(\frac{\pi}{8}\,(x-6)\right)$

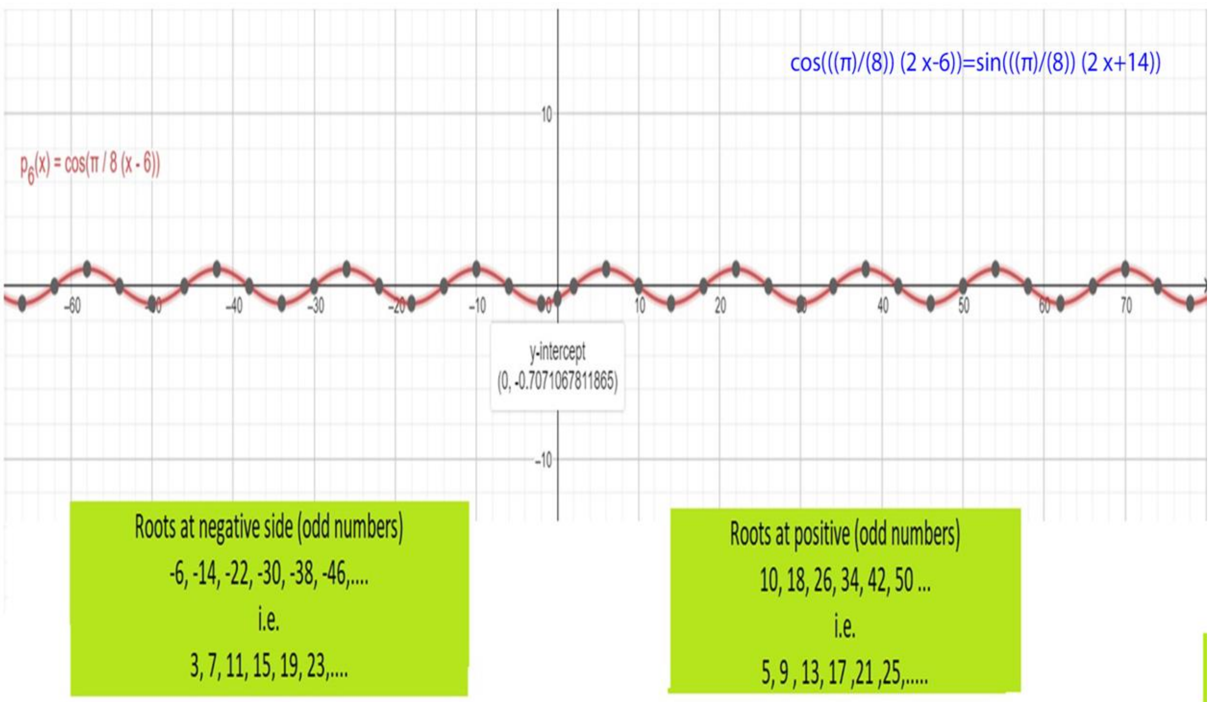


Figure 12. odd numbers Root distribution for $Cos\left(\frac{\pi}{8} * (x-6)\right)$

I) If we used degrees (π = 180°), and X = X ± 0.5 and θ = 22.5° THEN $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$; then wave signal will have Root at (±0.5,0)

II) $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$ When X = X - 0.5 all roots will be Odd negative numbers.

III) $Sin\left(\frac{\pi}{8}\left(x \pm \frac{1}{2}\right)\right)$ When X = X + 0.5 all roots will be Odd positive numbers.

IV) $For\ Sin(22.5 * \ (X + 0.5)\ );\ there\ will\ be\ Y = \frac{\pm 1}{\sqrt{2}}\ ;\ for\ X = \pm 0.5$

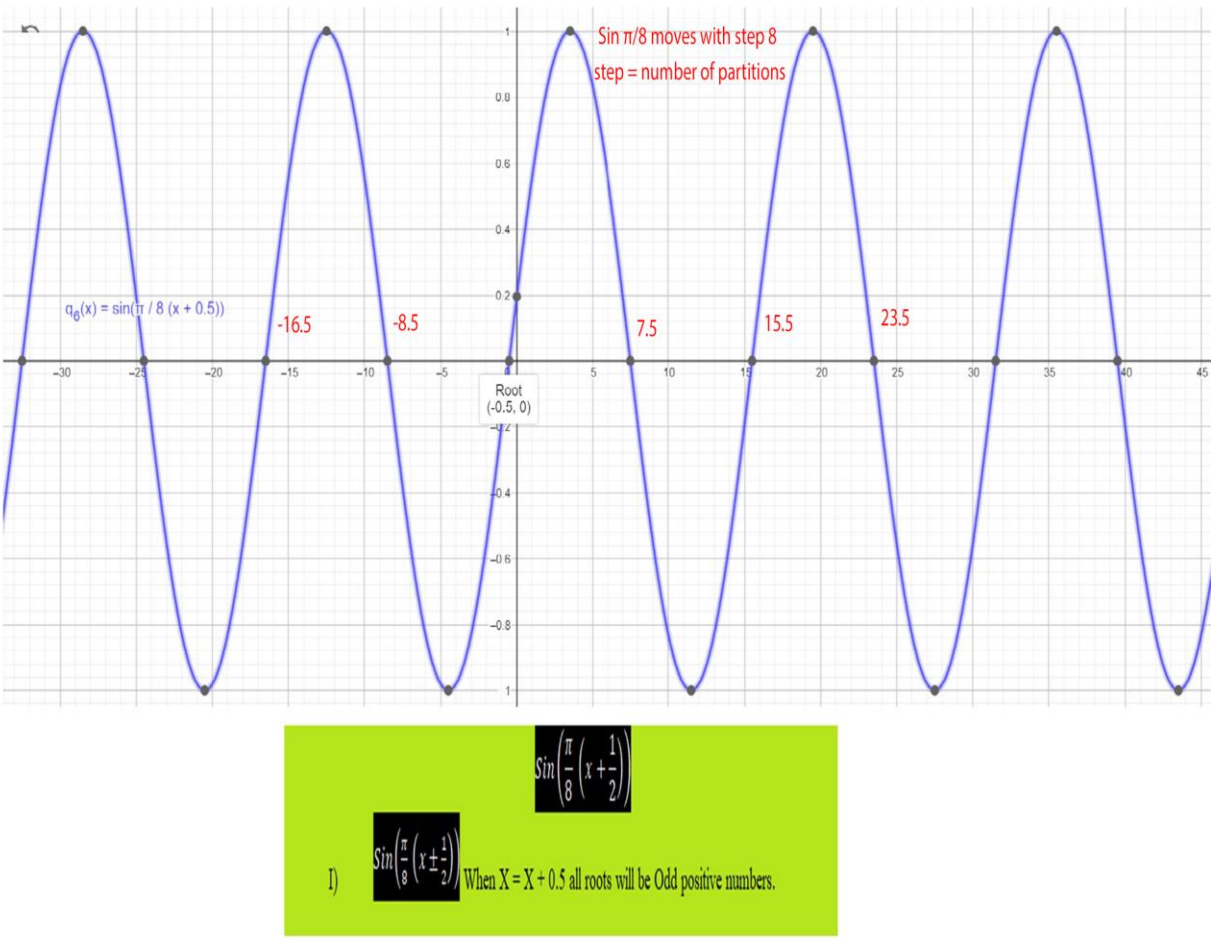


Figure 13. using $\frac{\pi}{8}$ instead of 22.5 decrease the frequency so it is easier to see the roots.

Please note here in Figure 12. we are using 22.5 as number not degrees. This will increase the sign wave frequency, but we will still have root at -0.5.

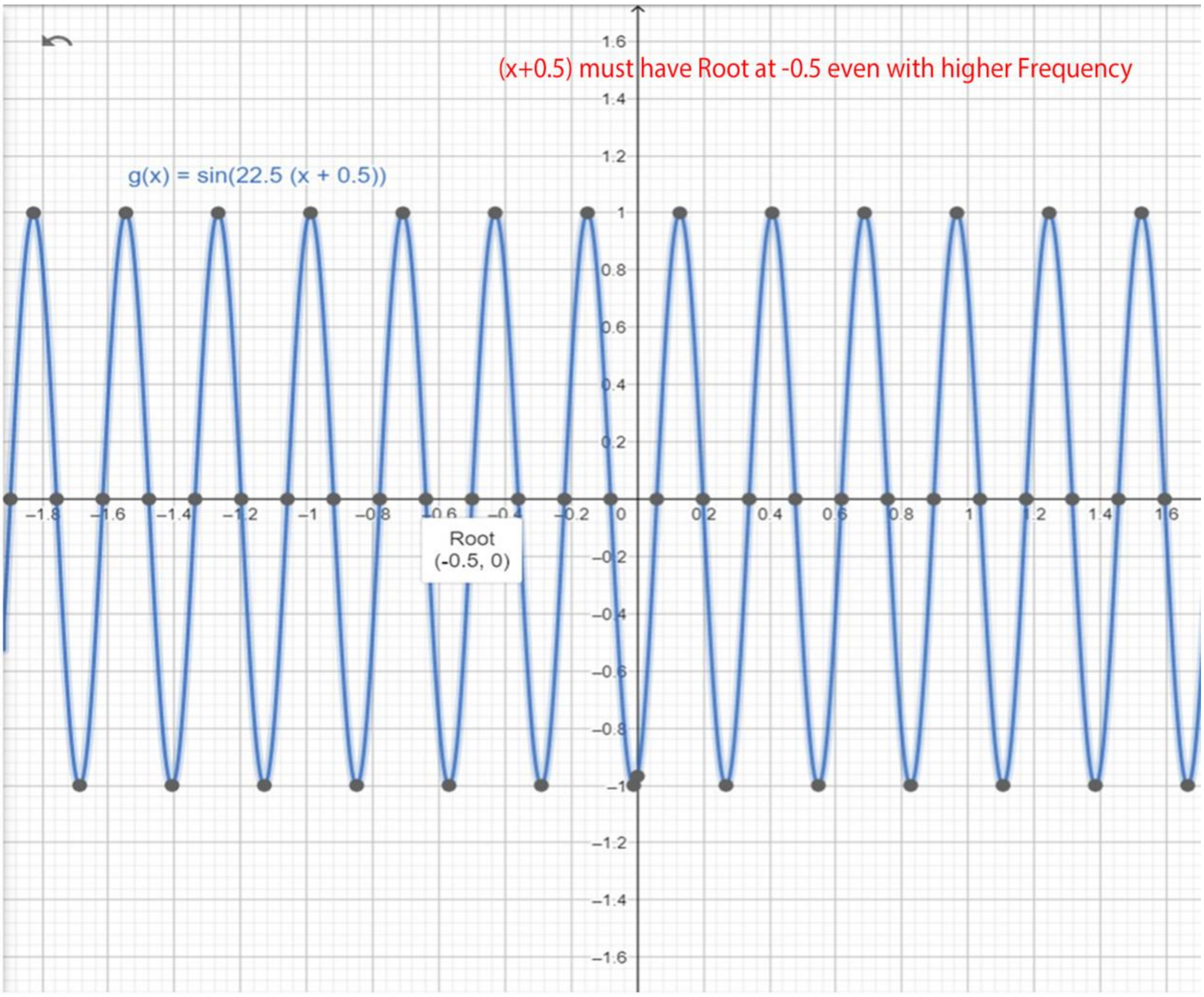


Figure 14. using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at -0.5.

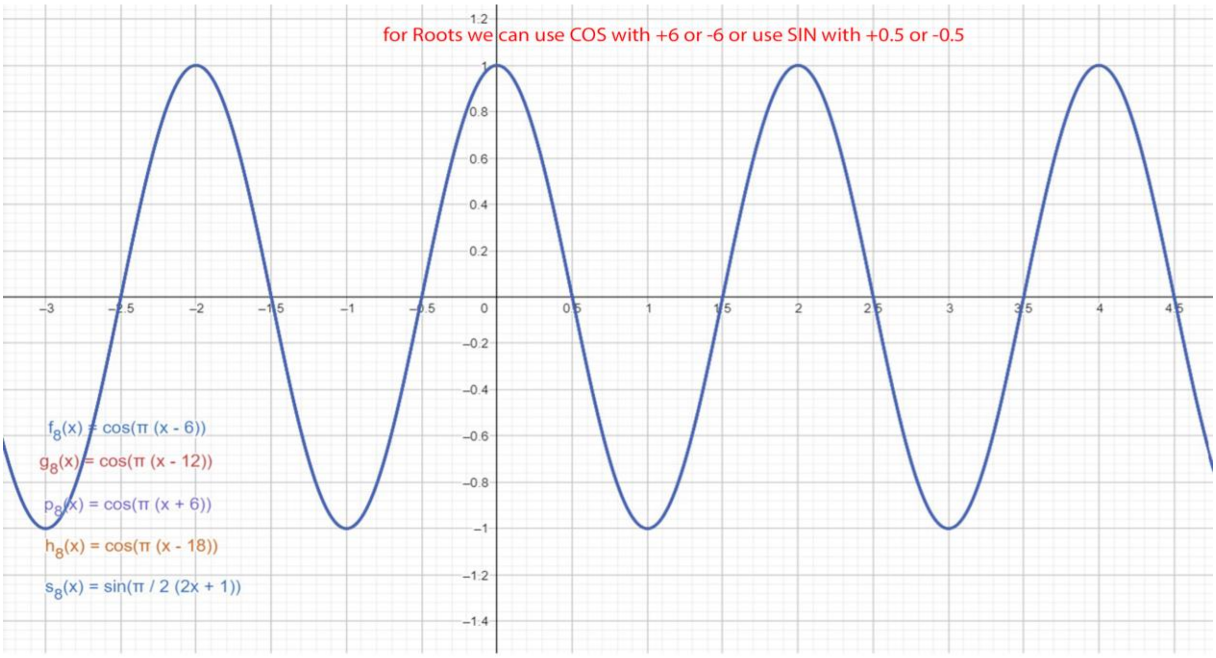


Figure 15. for Roots we can use COS with {+6, -6} or use SIN with {+0.5, -0.5}.

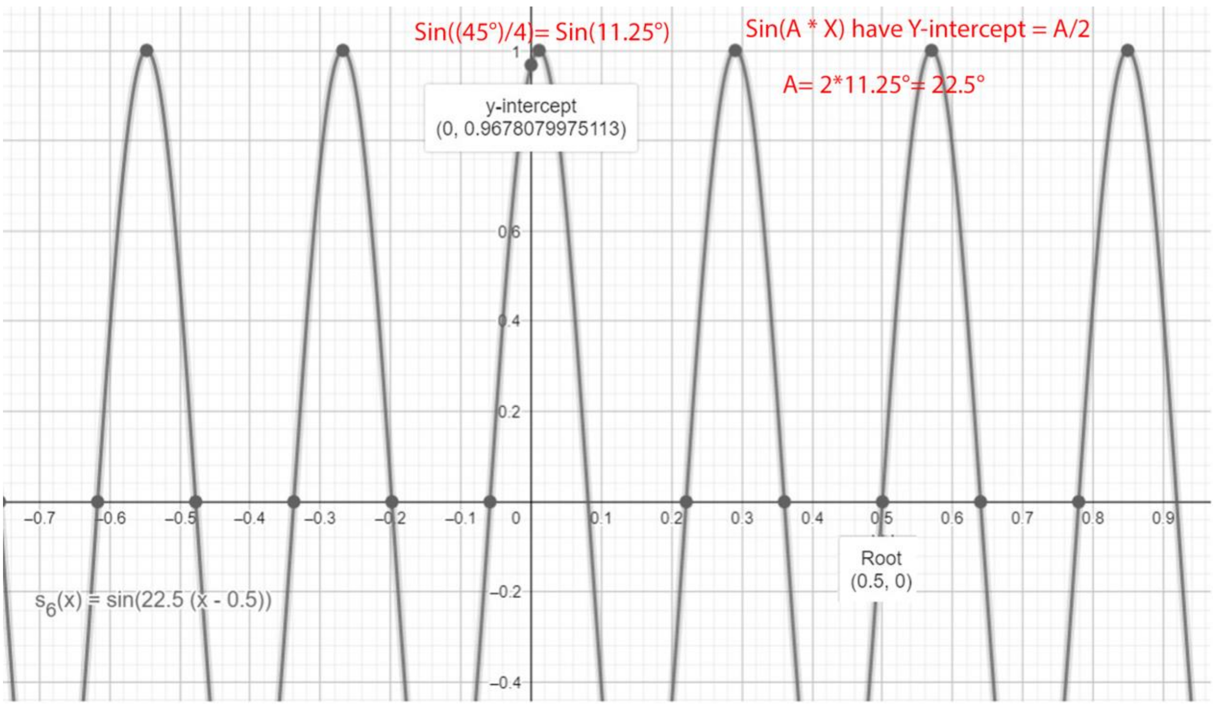
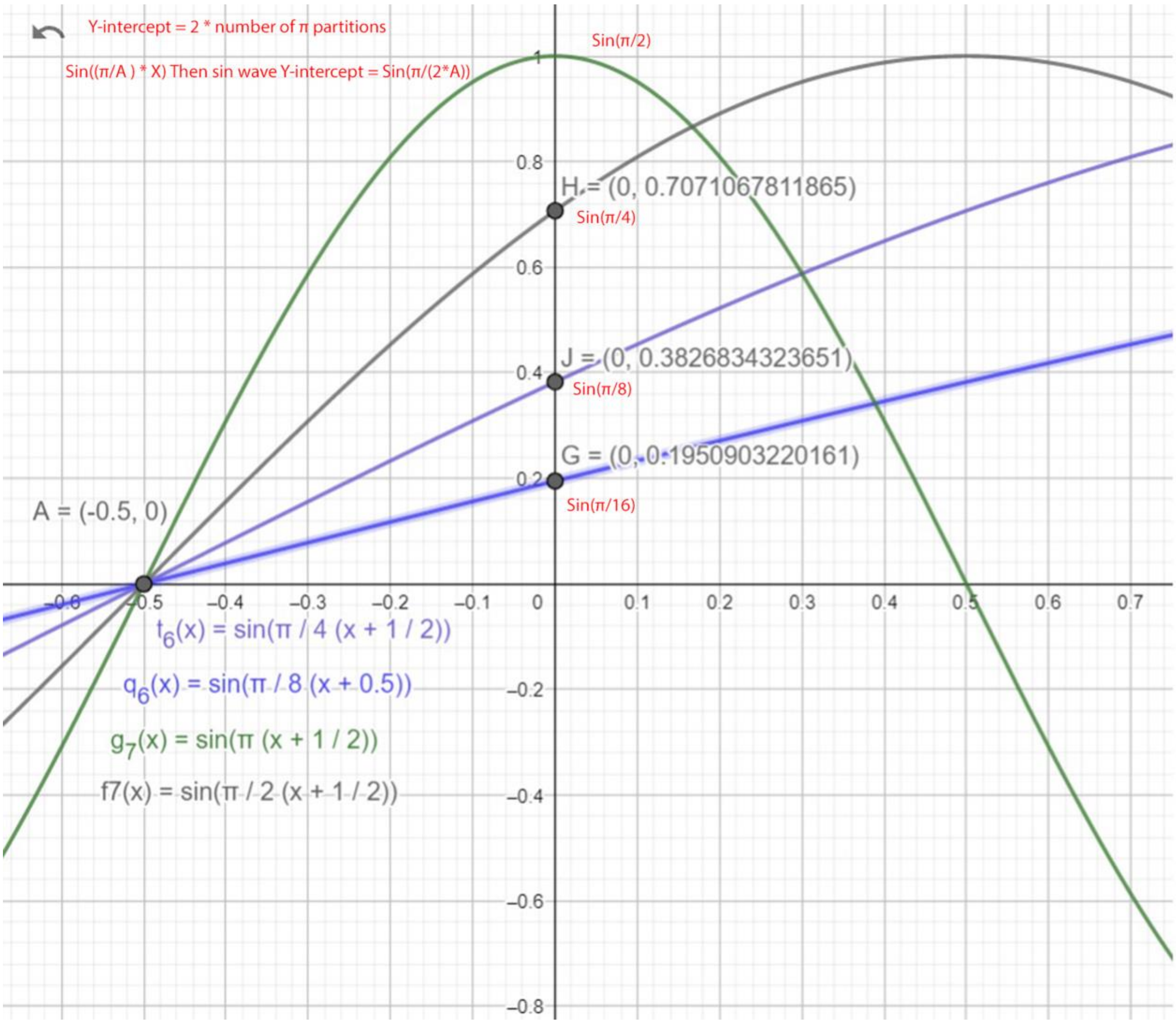
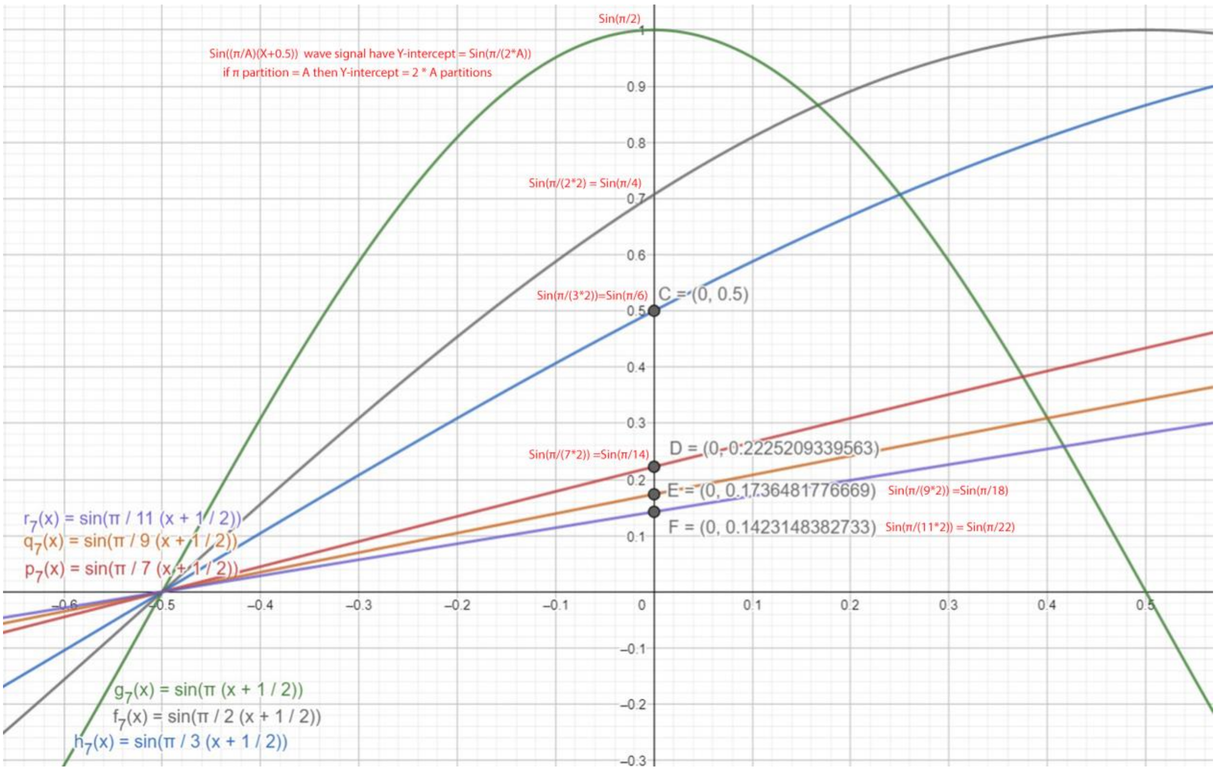


Figure 16. $Sin\left(22.5\left(x-\frac{1}{2}\right)\right)$ using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at 0.5.



$Sin\left(A * \left(x + \frac{1}{2}\right)\right)$ for any A = { $\frac{\pi}{1}$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, $\frac{\pi}{8}$, $\frac{\pi}{16}$,....} will have root at -0.5 and at X = 0 then Y = $Sin\left(\frac{A}{2}\right)$
in order to keep Sin wave characterstics, the width of the half Sin wave needs to be adjusted by the same ratio
and this is why it keeps intersecting on Y axis because the slope keeps changing each time we change A
each time we change A we change the number of partitions of pi by factor of 1/2.

Figure 17. $Sin\left(A * \left(x + \frac{1}{2}\right)\right)$ for any A even partitons for pi; the Sin wave will keep its charateriscts by adjusting its width and slope with the same ratio we partion pi with.



$Sin\left(A*\left(x+\frac{1}{2}\right)\right)$ for any A = { $\frac{\pi}{1}$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, $\frac{\pi}{8}$, $\frac{\pi}{16}$,....} will have root at -0.5 and at X = 0 then Y = $Sin\left(\frac{A}{2}\right)$
in order to keep Sin wave characterstics, the width of the half Sin wave needs to be adjusted by the same ratio
and this is why it keeps intersecting on Y axis because the slope keeps changing each time we change A
each time we change A we change the number of partitions of pi by factor of 1/2.

Figure 18. $Sin\left(A*\left(x+\frac{1}{2}\right)\right)$ wave signal have Y-intercept $Sin\left(\frac{A}{2}\right)$

Sin wave will keep its charateriscts by adjusting its width and slope with the same ratio we partion pi with.

In the previous graph A can take any partition value { $\frac{\pi}{1}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{5}, \frac{\pi}{6}, \frac{\pi}{7}, \ldots\ldots$ }

To keep Sin wave characteristics, changing A in $A*(x+\frac{1}{2})$ will adjust Sin wave width to keep root -1/2.

$$for\ A = \frac{\pi}{3}\ THEN\ A*\left(x+\frac{1}{2}\right) = \frac{\pi}{3}\left(x+\frac{1}{2}\right) = \frac{\pi}{3}*x + \frac{\pi}{6}$$

$$for\ A = \frac{\pi}{5}\ THEN\ A*\left(x+\frac{1}{2}\right) = \frac{\pi}{5}\left(x+\frac{1}{2}\right) = \frac{\pi}{5}*x + \frac{\pi}{10}$$

Y intercept will be = $\frac{A}{2}$ for any A. (A is partition segments for π)

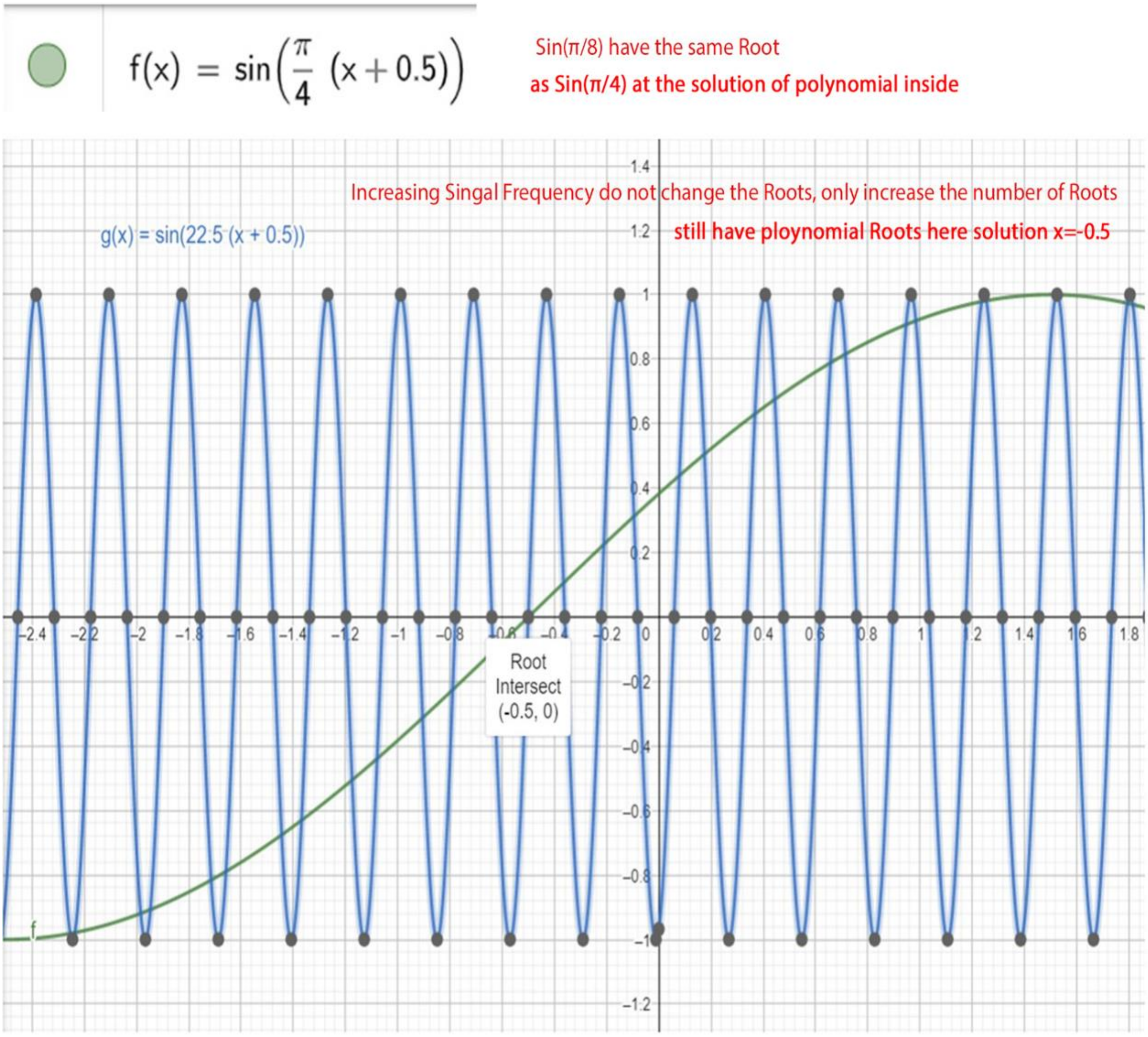


Figure 19. $Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right)$ and $Sin\left(22.5\left(x+\frac{1}{2}\right)\right)$ both intersects at same roots even with different frequency.

Table 1. f(x) = $Sin\left(\frac{\pi}{4}\left(x+\frac{1}{2}\right)\right)$ = { $\frac{\pm 1}{\sqrt{2}}$} for any X = X + 0.5 as X={0,1,2,3,4,5,6,7…} increases distribution divide numbers into odd and even numbers on both sides of x = 0.5 for f(x) = 1/sqrt(2).

Sin(π/4 (X+0.5) have Root ±1/√2 at specific locaions
same root value at same step or same partitions
from a specific start point (here start point = 0.5)

x	f(x)
-19.5	-0.707106781...
-17.5	-0.707106781...
-15.5	0.7071067811...
-13.5	0.7071067811...
-11.5	-0.707106781...
-9.5	-0.707106781...
-7.5	0.7071067811...
-5.5	0.7071067811...
-3.5	-0.707106781...
-1.5	-0.707106781...
0.5	0.7071067811...
2.5	0.7071067811...
4.5	-0.707106781...
6.5	-0.707106781...

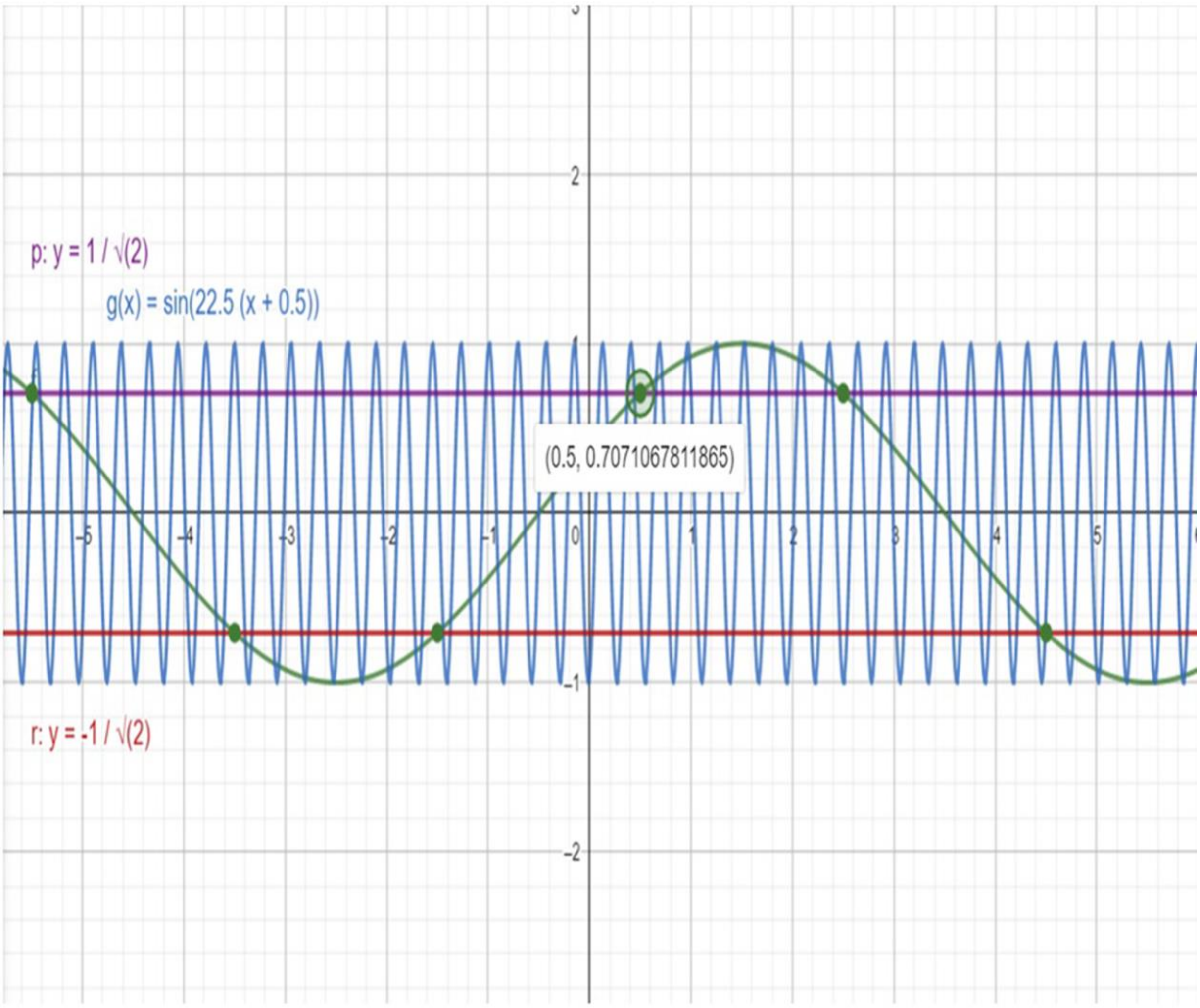


Figure 20. Shows how Y = 1/sqrt(2) intersects with $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ (green wave) at (X+0.5 , 1/sqrt(2)) and intersects with Y = -1/sqrt(2) at (X-1.5 , -1/sqrt(2))

Therefore; if we mulitply f(x)= $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ by $\pm\frac{1}{\sqrt{2}}$ i.e. multipy by Sin(45) or Sin(225); then Q(X)= {0.5 , -0.5} all the time for odd numbers.

$$Q(x) = Sin\left(\frac{\pi}{2} * \left(x + \frac{1}{2}\right)\right) * Sin\left(\frac{\pi}{4}\right) = Sin\left(\frac{\pi}{2} * \left(x + \frac{1}{2}\right)\right) * Sin(45°) = \pm\frac{1}{2}\ for\ any\ x$$

**And we can use this method to get visualization for Sin^{-1} function for (X = X +0.5) **

2.6 Visualization for of Geometric function Sin^{-1}

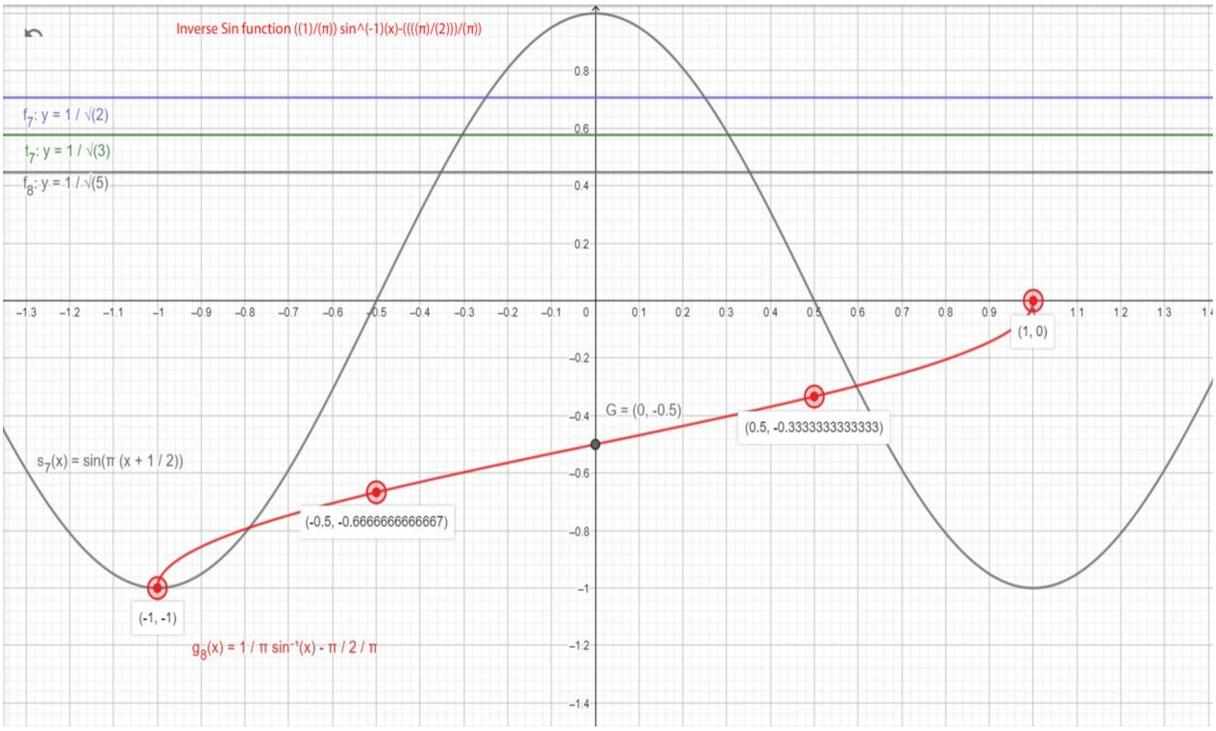


Figure 21. (Red free line) Is the Visualization for Sin^{-1} function [$\frac{1}{\pi} * Sin^{-1}(X) - \frac{1}{A}$ and here A=2]

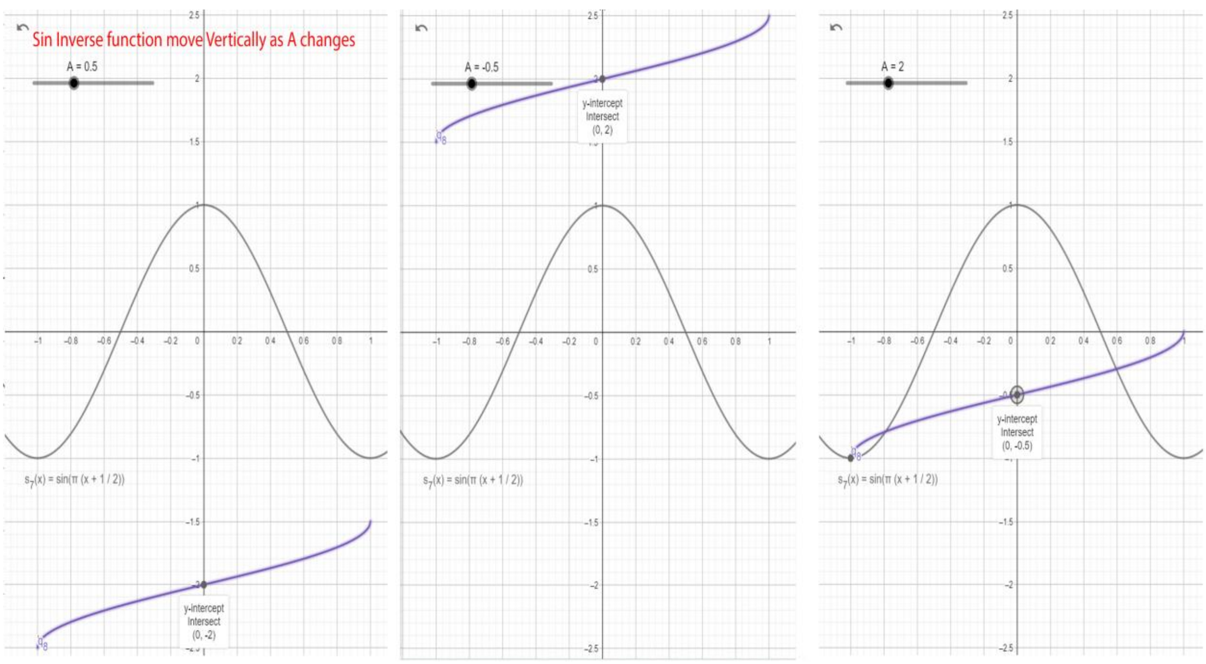


Figure 22. General Inverse function for any Value of A, Inverse function $\frac{1}{\pi} * Sin^{-1}(X) - \frac{1}{A}$moves vertically

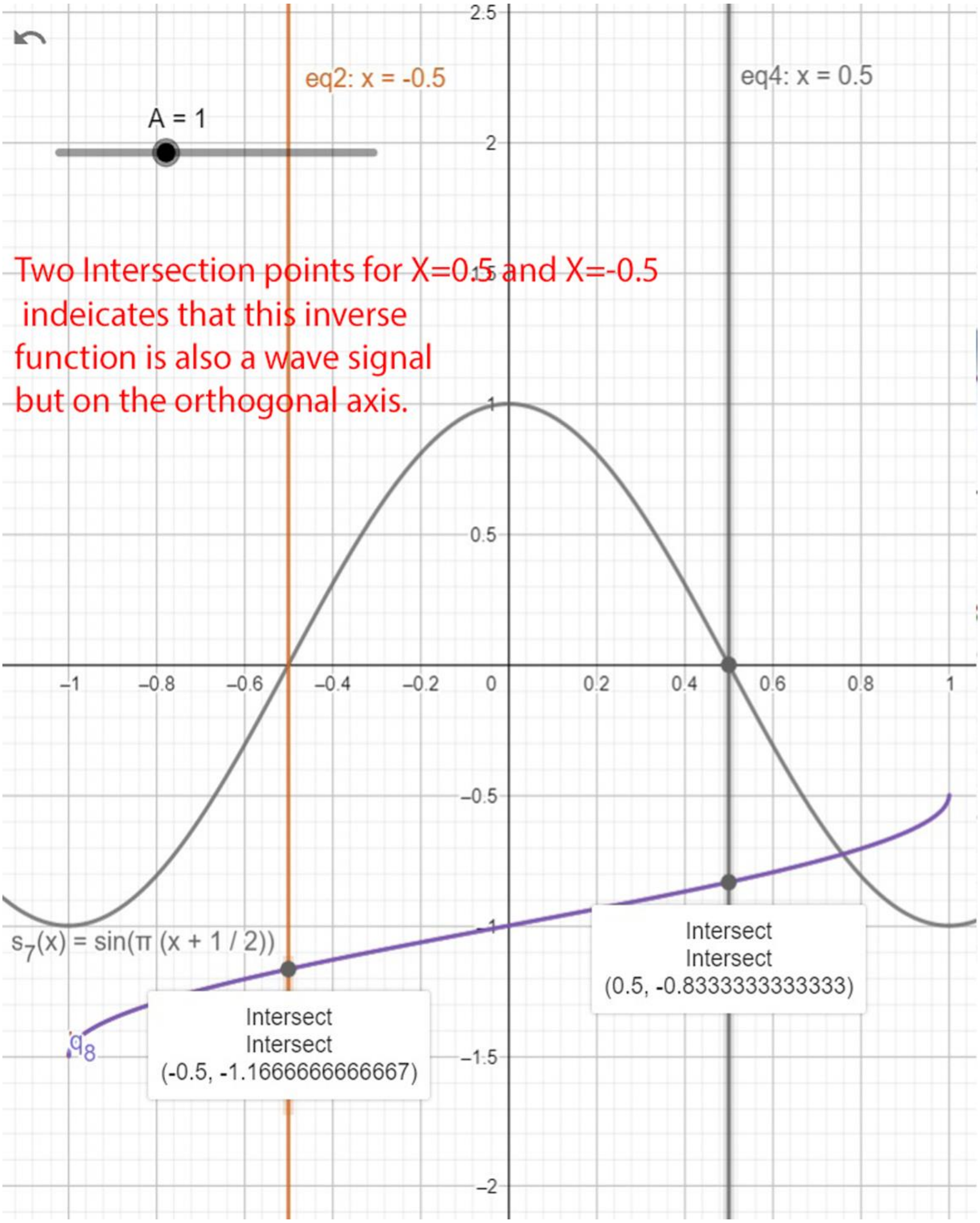


Figure 23. Inverse function $\frac{1}{\pi} * Sin^{-1}(X) - \frac{1}{A}$ at A = 1 is a nother wave on the vertical orthognal axis.

Table 2. Inverse of Sin function = $\frac{1}{\pi} * Sin^{-1}(X) - \frac{1}{A}$

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{-0.5}$
-1.5	
-1	1.5
-0.5	1.833333333...
0	2
0.5	2.166666666...
1	2.5
1.5	

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{1}$
-1.5	
-1	-1.5
-0.5	-1.166666666...
0	-1
0.5	-0.833333333...
1	-0.5
1.5	

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{2}$
-1.5	
-1	-1
-0.5	-0.666666666...
0	-0.5
0.5	-0.333333333...
1	0
1.5	

Table 3. Inverse of Sin function = $\frac{1}{\pi} * Sin^{-1}(X) - \frac{1}{A}$ orthogonal zeros

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{3}$
-1.5	
-1	-0.833333333...
-0.5	-0.5
0	-0.333333333...
0.5	-0.166666666...
1	0.1666666666...
1.5	

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{5}$
-1.5	
-1	-0.7
-0.5	-0.366666666...
0	-0.2
0.5	-0.033333333...
1	0.3
1.5	

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A} \rightarrow \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{3.1415926535898}$
-1.5	
-1	-0.818309886...
-0.5	-0.484976552...
0	-0.318309886...
0.5	-0.151643219...
1	0.1816901138...
1.5	

x	$q_8(x) = \frac{1}{\pi}\sin^{-1}(x) - \frac{1}{A}$
-1.5	
-1	-1/A-3/6
-0.5	-1/A-1/6
0	-1/A
0.5	-1/A+1/6
1	-1/A +3/6
1.5	

Table 4. shows $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right) * Sin(45°) = \pm\frac{1}{2}$; for each negative odd natural number x = x+0.5.

And for each positive even natural numebr x = x + 0.5. (Sin wave transformation)

$a = \sin(45°) - \frac{1}{\sqrt{2}}$

$\rightarrow 0$

$b = \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{4}\right)$

$\rightarrow 0.5$

$q(x) = \sin\left(\frac{\pi}{4} (x + 0.5)\right) \sin(45°)$

$f(x) = \sin\left(\frac{\pi}{4} (x + 0.5)\right)$

x	f(x)	s(x)	q(x)
-11.5	-0.707106781...	0	-0.5
-9.5	-0.707106781...	-1	-0.5
-7.5	0.7071067811...	0	0.5
-5.5	0.7071067811...	1	0.5
-3.5	-0.707106781...	0	-0.5
-1.5	-0.707106781...	-1	-0.5
0.5	0.7071067811...	0	0.5
2.5	0.7071067811...	1	0.5
4.5	-0.707106781...	0	-0.5
6.5	-0.707106781...	-1	-0.5
8.5	0.7071067811...	0	0.5
10.5	0.7071067811...	1	0.5
12.5	-0.707106781...	0	-0.5
14.5	-0.707106781...	-1	-0.5
16.5	0.7071067811	0	0.5

2.7 Visualization for Geometric function in Geometric inverse domain

Multiply any geometric function by an amount will be interpreted visually as an amplitude for the wave signal of this geometric function.

Here we are using an amount equal $\frac{A}{\pi}$ $where\ A = 1$

The blue signal is for $p(x) = \frac{A}{\pi} * Sin(\pi * x);\ where\ A = 1$

The red signal is for $g(x) = \frac{A}{\pi} * Sin(x);\ where\ A = 1$

Therefore, the amplitude of this blue wave = $\frac{A}{\pi} = \frac{1}{\pi}$

And the wavelength still the same not changed one full wave is between [-1,1], only the amplitude changing which is an increase in the height of the wave on the Y axes or in complex plane will be on increase in Imaginary axes value only, but the full wavelength will still the same between [-1,1].

And what we see here as red signal is a full one cycle of sin inverse function in the inverse domain projected on 2D complex plane. Similarly, the wavelength for the red signal stays the same between [1, -1] only the amplitude will change which will be a change in the amplitude also which is a change in the height of the signal which is change in the imaginary axes only. And due to symmetry, the amplitude will be = A/2 = ½ at A = 1.

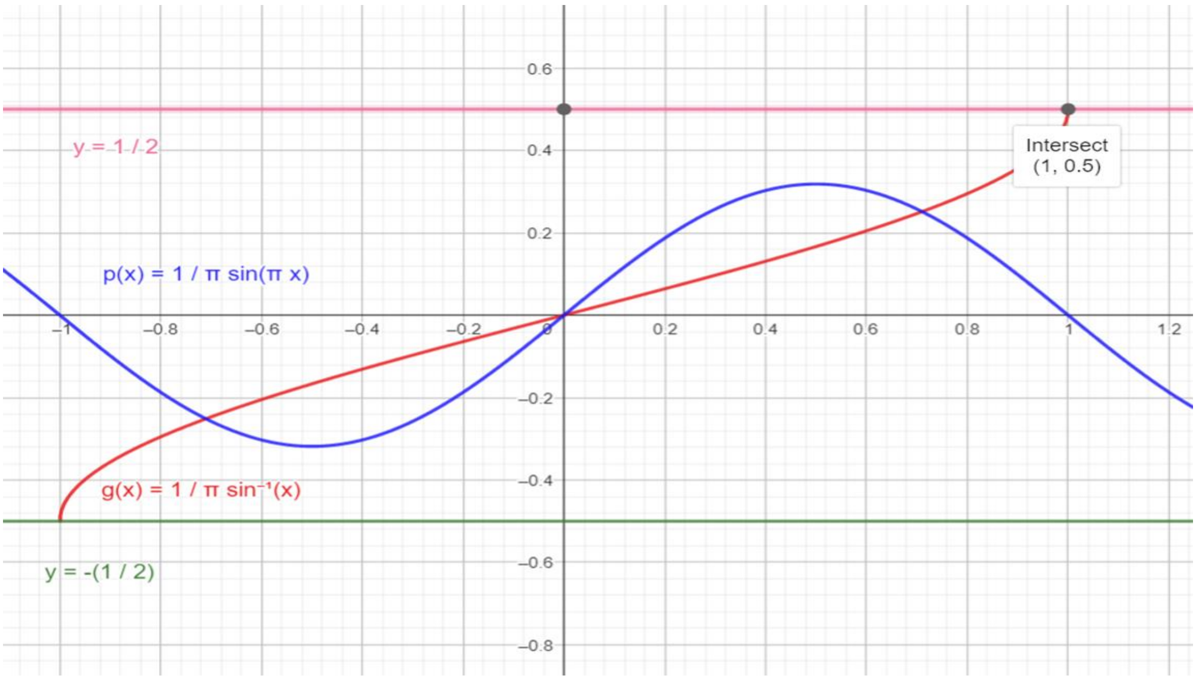


Figure 24. Geometric inverse Sin function one full Cycle in both domains with similar wavelength and A=1

At A = 3 as we here in this figure the red wave signal wavelength still bounded between [-1,1] but the height on the imaginary axes = A/2 = 1.5.

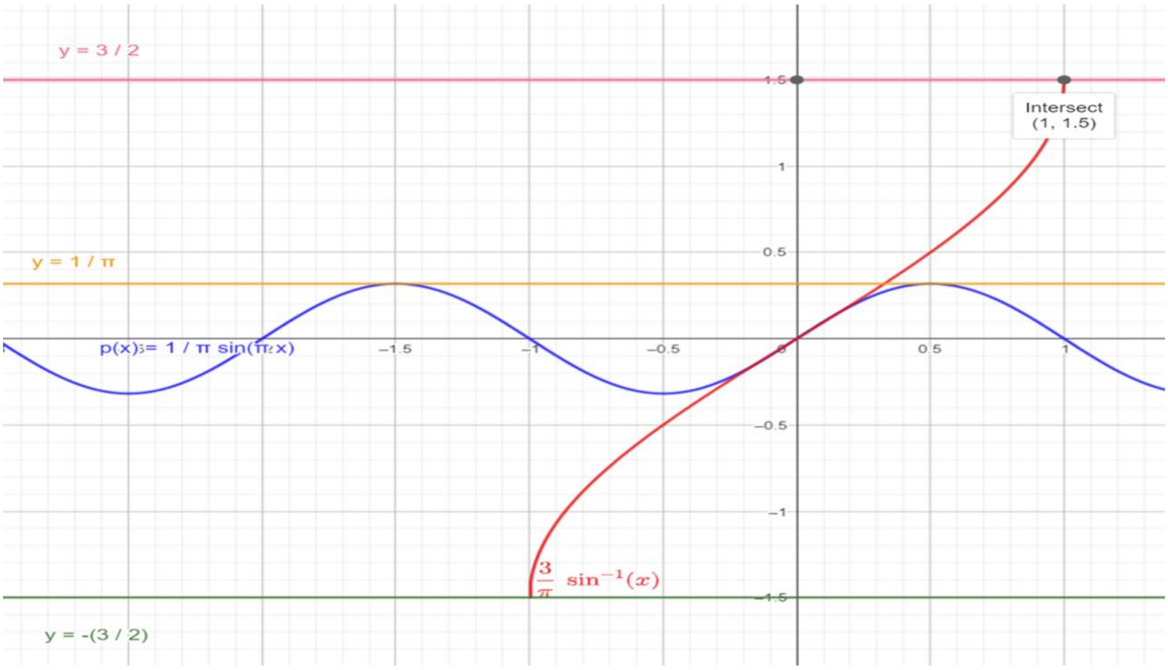


Figure 25. Geometric inverse Sin function one full Cycle in both domains with similar wavelength and A =3.

As we see with each value the value length the same only the angel of the wave changing but still bounded between [-1,1] with amplitude =A/2 = 13/2 = 6.5.

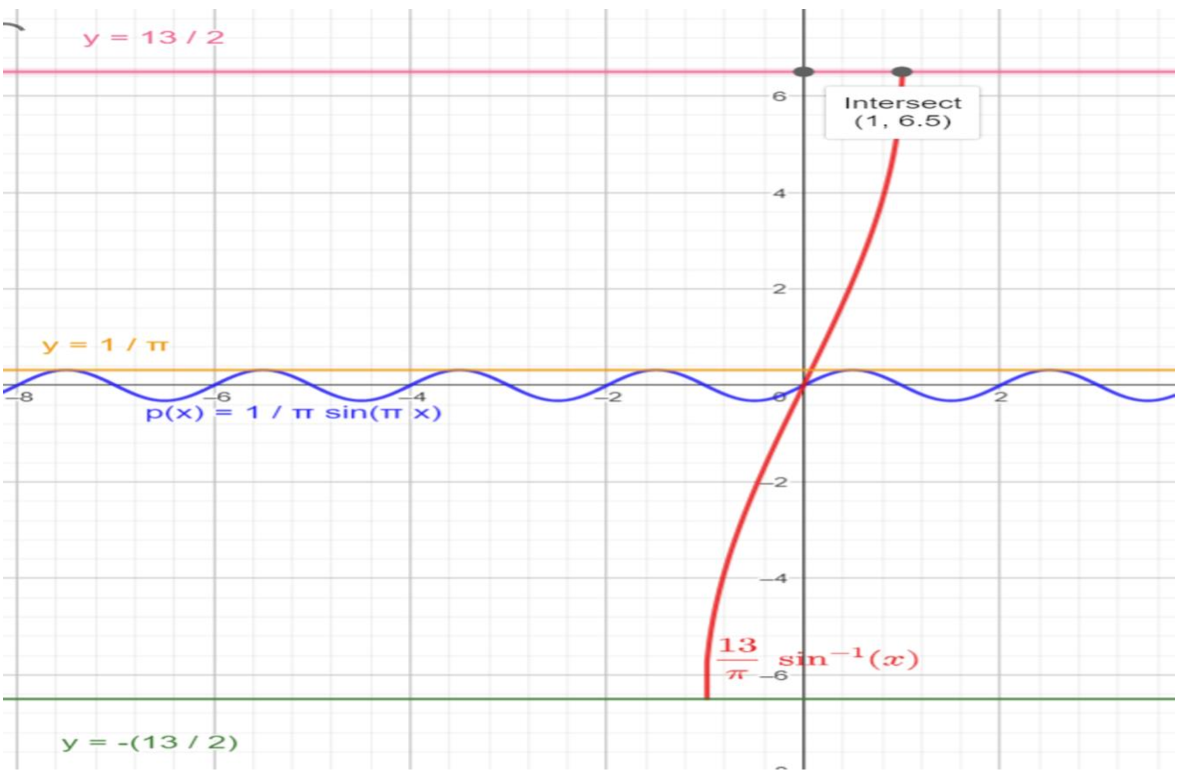


Figure 26. Geometric inverse Sin function one full Cycle in both domains with similar wavelength and A =13.

Very interesting note here is that the angel of the inverse Sin wave signal is very close to the amount we use as magnitude $\frac{A}{\pi}$, only if we use the degree values for π instead of the real value for $\pi = 3.14$.

Therefore $Sin^{-1}\left(\frac{A}{180}\right) \cong \frac{A}{\pi}$

$\alpha = \sin^{-1}\left(\frac{1}{180}\right)$ $= 0.318311523603067°$	$\alpha = \sin^{-1}\left(\frac{3}{180}\right)$ $= 0.954973873784914°$	$\alpha = \sin^{-1}\left(\frac{13}{180}\right)$ $= 4.141634350687623°$
$a = \frac{1}{\pi}$ $= 0.318309886183791$	$a = \frac{3}{\pi}$ $= 0.954929658551372$	$a = \frac{13}{\pi}$ $= 4.138028520389279$

2.8 Geometric inverse domain and Electromagnetic waves

I am going to borrow the visualization of the electromagnetic wave here to give a better visualization understanding for this two domains Sin geometric function and its inverse.

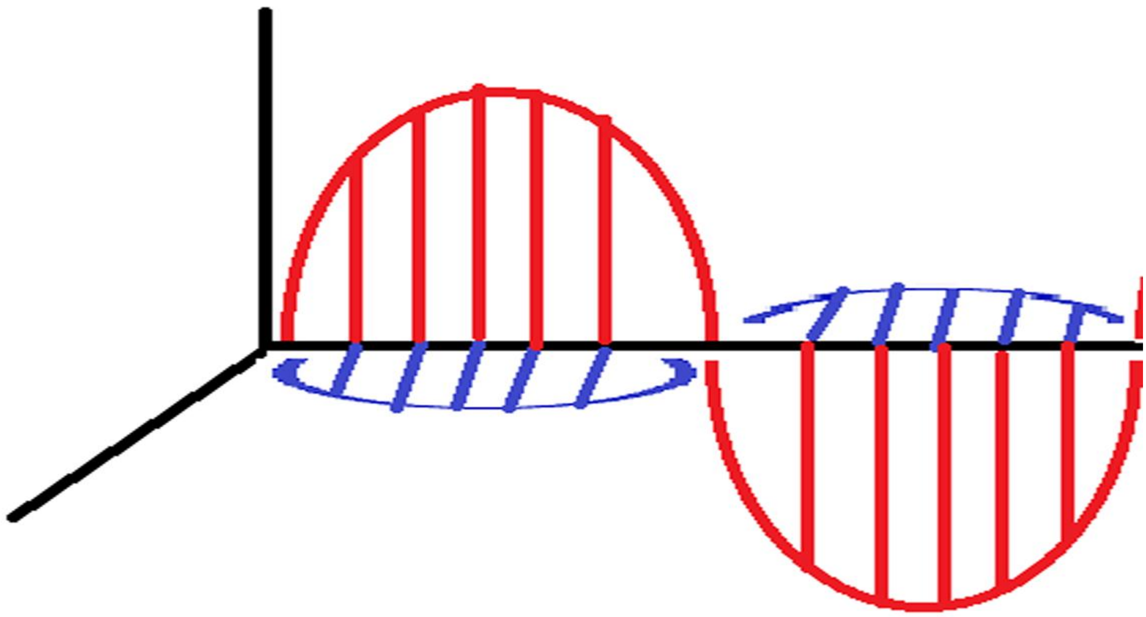


Figure 27. One full Cycle for electromagnetic wave visualization.

How we introduce the concept of sin inverse wave is similar but without using the orthogonal concept yet next we are going to see how the orthogonal axes is used in inverse domain.

As we see here the orthogonal wave in Sin inverse domain is Cos inverse

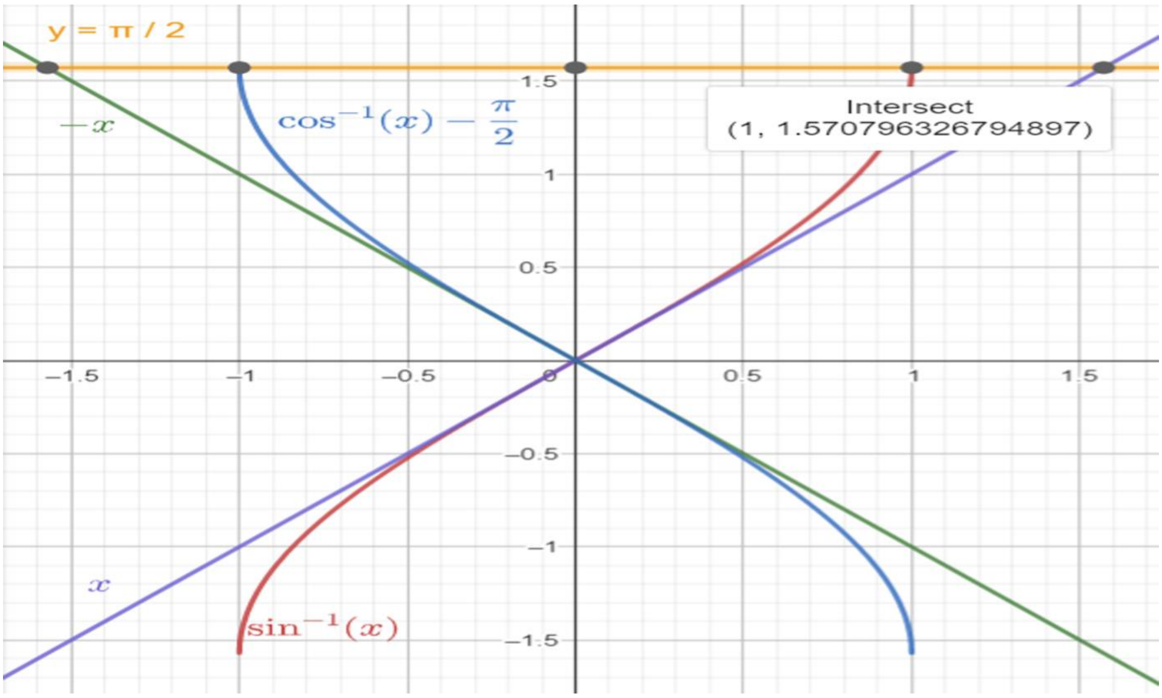


Figure 28. Orthogonal one full cycle of Geometric functions in the inverse domain.

2.9 Geometric inverse domain at (X, Y) = (0.5, 0.5)

Similarly in the geometric inverse domain we can have the transformation at the

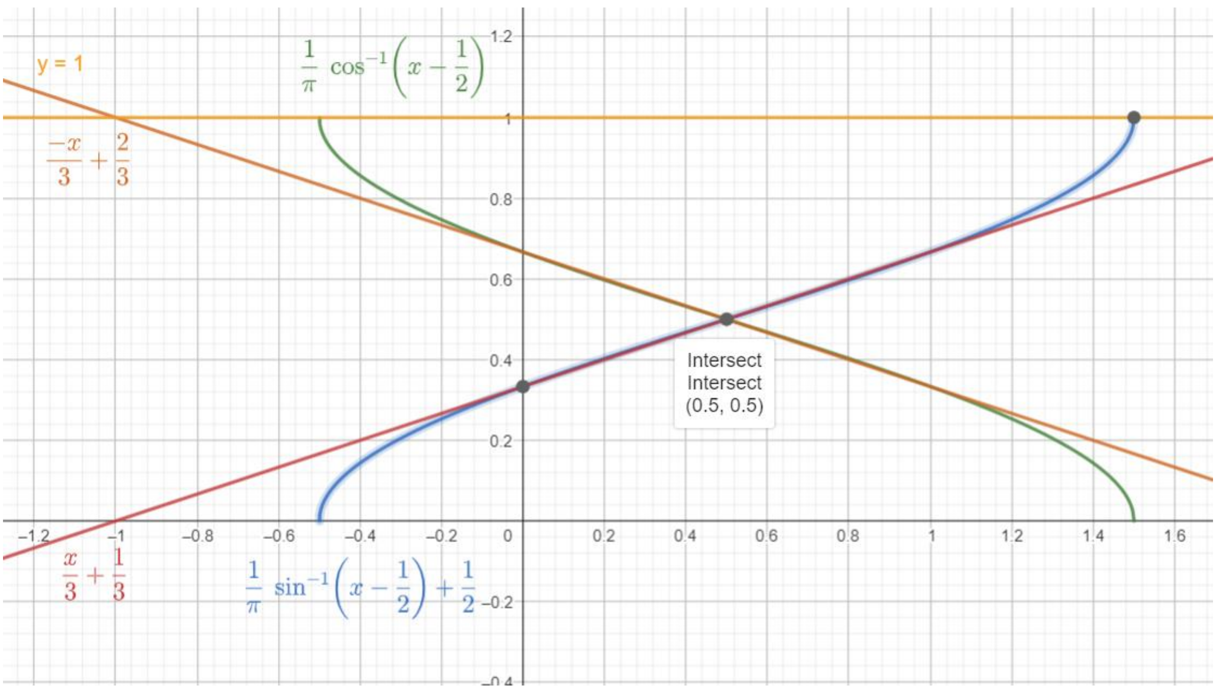


Figure 29. Orthogonal one full cycle of Geometric functions Projection in the inverse domain at X =X-0.5.

As we see here both geometric functions inverse will be equal to 0 at 0.5. and the wavelength still the same but from [-0.5 until 1.5] instead of from [-1,1]. Next, we will see how these two geometric functions are used alternatively in Zeta function.

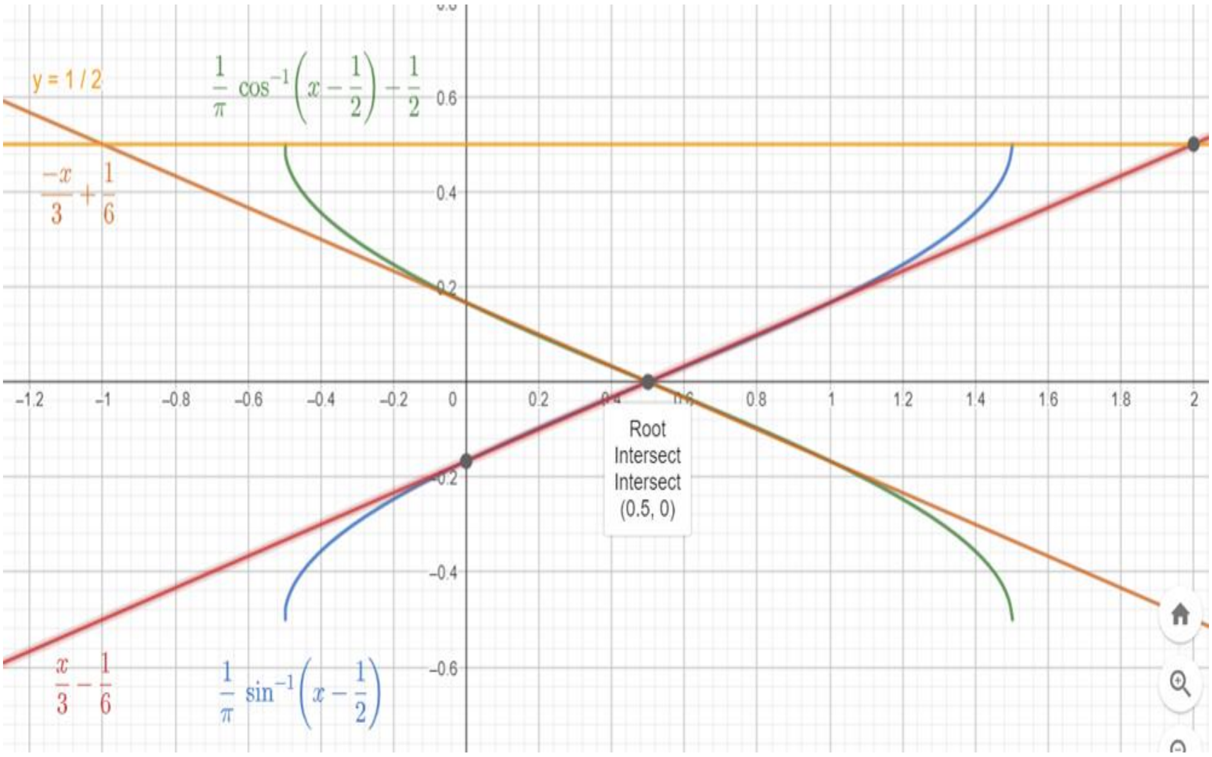


Figure 30. Orthogonal one full cycle of Geometric functions Projection in the inverse domain at X =0.5 and Y=0.

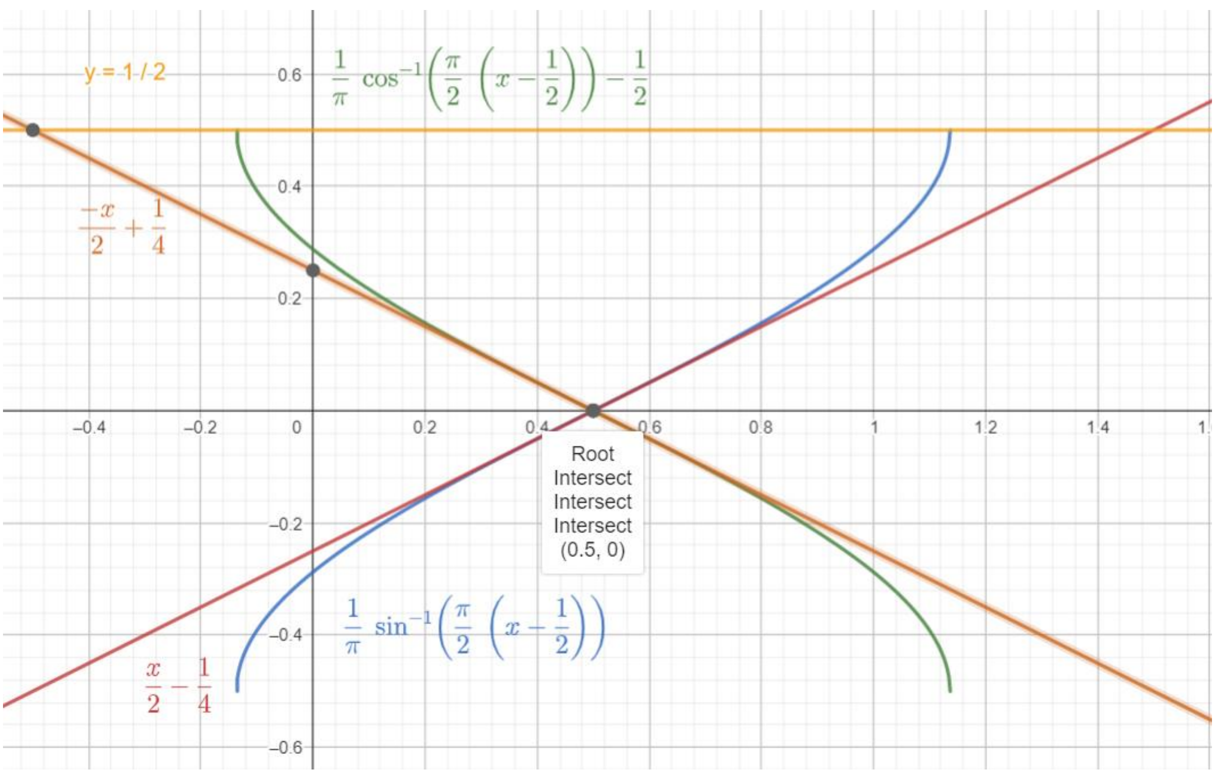


Figure 31. Orthogonal ½ cycle of Geometric functions Projection in the inverse domain at X =0.5 and Y=0.

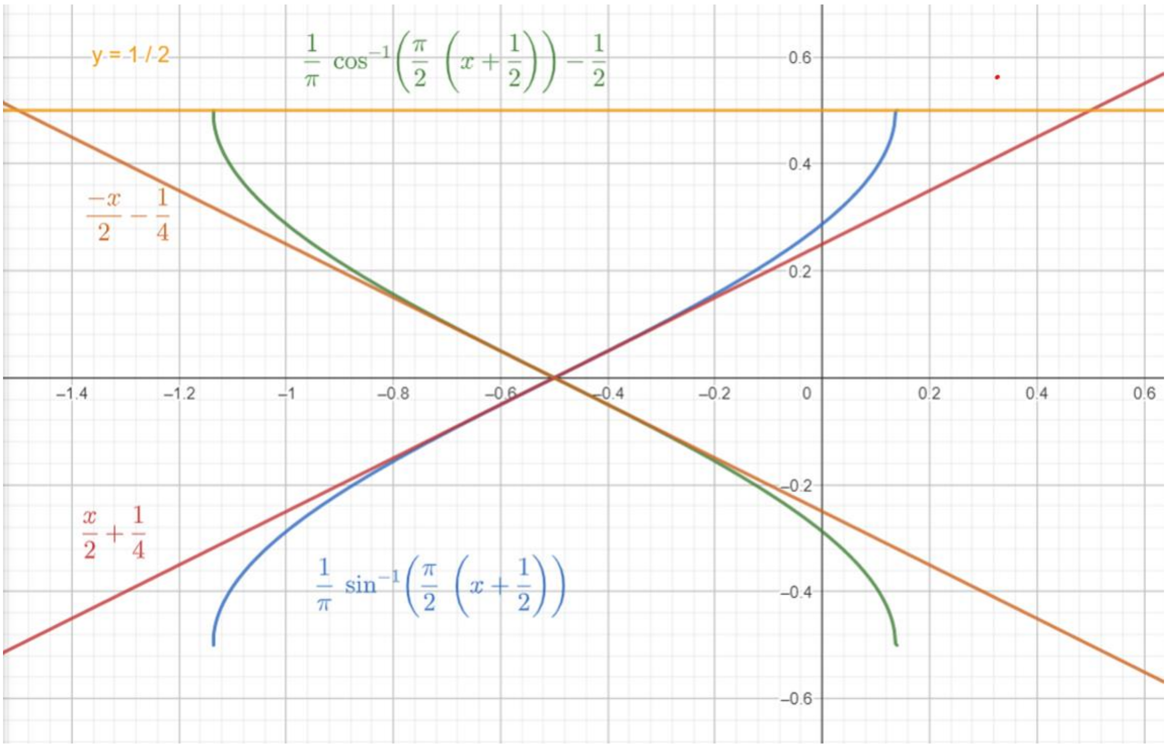


Figure 32. Geometric functions Terms in Zeta function in the inverse domain at X =-0.5 and Y=0.

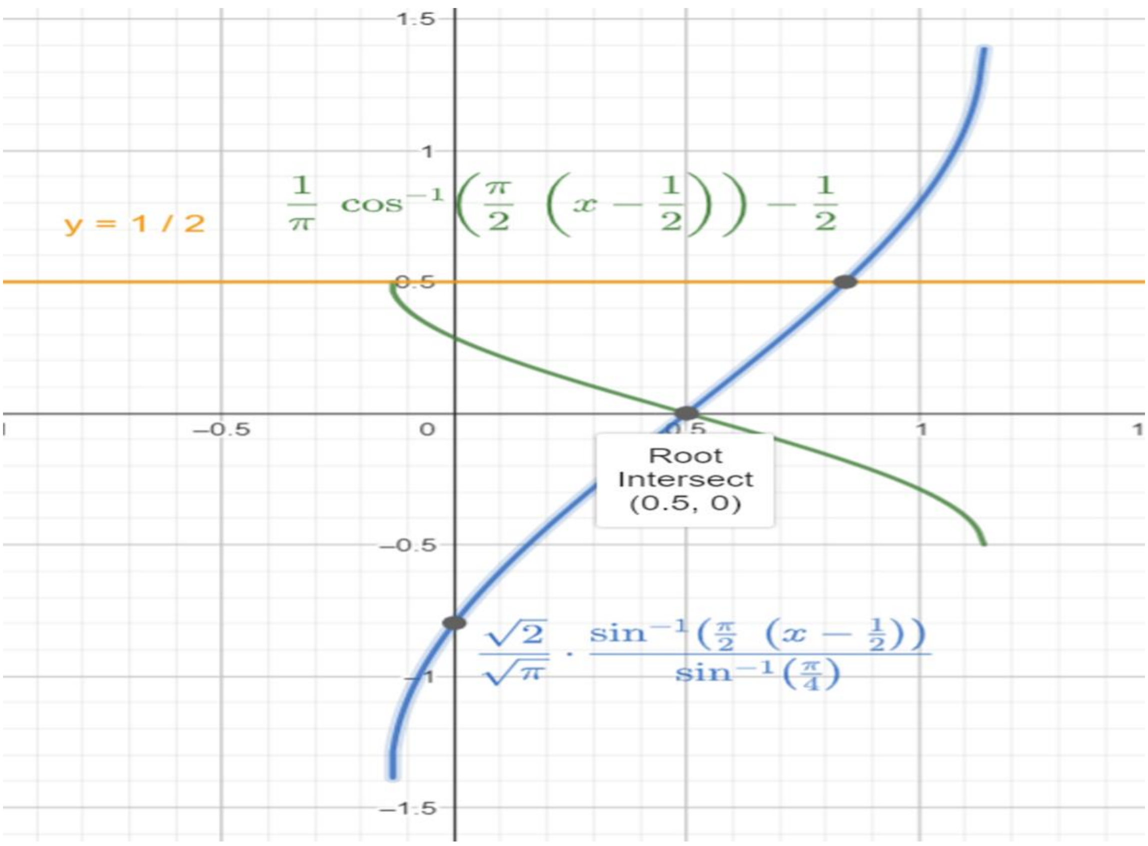


Figure 33. Geometric functions Terms in Zeta function in the inverse domain at Odd Identity.

3. Zeta Function none-trivial Zeros.

If we used the same trick we used here when we multiplied by Sin(45)
And do the same in Zeta function formula knowing that

$$\sqrt{2} = \frac{1}{Sin(45)} = \frac{1}{Sin(\frac{\pi}{4})} \quad \rightarrow EQ(C)$$

And from EQ(A)

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} (2*\pi)^S * \sqrt{\frac{2}{\pi}} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) = 0\,;\ when\ S\ odd \\ (2*\pi)^S * \sqrt{\frac{2}{\pi}} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) = 0; when\ S\ odd \end{cases} \quad \rightarrow EQ(A)$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \sqrt{2} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \sqrt{2} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases}$$

If we substitute from EQ(C) into EQ(A)

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \frac{1}{Sin(\frac{\pi}{4})} * \; Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \frac{1}{Sin(\frac{\pi}{4})} * \; Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases}$$

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \frac{Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} * \; \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \frac{Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} \; \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases} \quad \rightarrow EQ(D)$$

This Equation EQ(D) have Sin wave that has Root at S = -0.5 and even negative Roots at S = S +0.5 and Odd positive Roots at S = S +0.5.

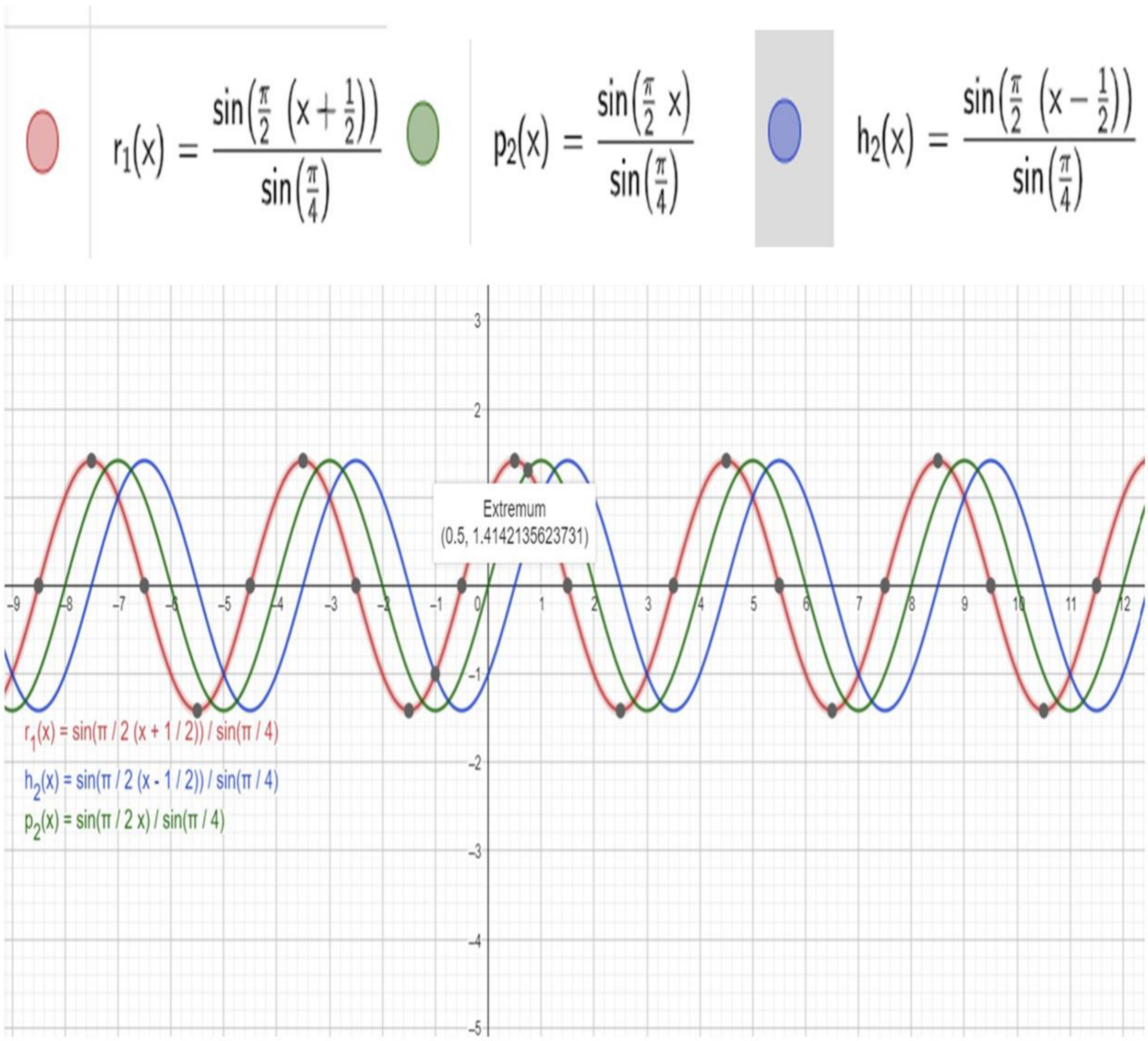


Figure 34. Green Sin wave (original Zeta Sin wave), have Roots only at even numbers and by adding or subtracting 0.5 from an even number we get { odd number ±0.5 , even number ±0.5}

Roots still starts at the solution of the polynomial inside Sin function

Here our polynomial is y = X+0.5 or y = X-0.5

So roots start point will be at 0.5 or -0.5 and then the location of the rest of the roots depends on sin wave frequency or on the number of partitions we divide pi with.

Table 5. even and odd roots flipping bwteen positive side and negative side depends on adding 0.5 or subtractig 0.5. x= { ±odd number ±0.5 , ±even number ±0.5}

Here Start point at polynomial solution at x=0.5 and steps = 2 as we devide pi by 2.

$$r_1(x) = \frac{\sin\left(\frac{\pi}{2}\ \left(x + \frac{1}{2}\right)\right)}{\sin\left(\frac{\pi}{4}\right)}$$

x	**$r_1(x)$**
-12.5	0
-10.5	0
-8.5	0
-6.5	0
-4.5	0
-2.5	0
-0.5	0
1.5	0
3.5	0
5.5	0
7.5	0
9.5	0
11.5	0
13.5	0

$$h_2(x) = \frac{\sin\left(\frac{\pi}{2}\ \left(x - \frac{1}{2}\right)\right)}{\sin\left(\frac{\pi}{4}\right)}$$

x	**$h_2(x)$**
-13.5	0
-11.5	0
-9.5	0
-7.5	0
-5.5	0
-3.5	0
-1.5	0
0.5	0
2.5	0
4.5	0
6.5	0
8.5	0
10.5	0
12.5	0

Table 6. shwos root shifting between $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ and $Sin\left(\frac{\pi}{4} * \left(x - \frac{1}{2}\right)\right)$ by 1 between both.

$f(x) = \sin\left(\frac{\pi}{4}(x + 0.5)\right)$ $s(x) = \sin\left(\frac{\pi}{4}(x - 0.5)\right)$

x	f(x)	s(x)
-9.5	-0.707106781...	-1
-8.5	0	-0.707106781...
-7.5	0.7071067811...	0
-6.5	1	0.7071067811...
-5.5	0.7071067811...	1
-4.5	0	0.7071067811...
-3.5	-0.707106781...	0
-2.5	-1	-0.707106781...
-1.5	-0.707106781...	-1
-0.5	0	-0.707106781...
0.5	0.7071067811...	0
1.5	1	0.7071067811...
2.5	0.7071067811...	1
3.5	0	0.7071067811...
4.5	-0.707106781	0

F(x) and S(x) = $\{0, \pm 1, \pm 0.7071067811865, \pm 0.3826834323651, \pm 0.9238795325113\}$

F(x) and S(x) = $\{0, \pm 1, \pm \frac{1}{\sqrt{2}}, \pm Sin(\frac{\pi}{8}), \pm Sin(\frac{3 * \pi}{8})\}$, both functions are the same but thier roots are sliding by 1 for any x = x+0.5.

Roots set for these two functions.

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\{0\ ,\pm 1\ ,\pm 0.7071067811865\ ,\pm 0.3826834323651\ ,\pm 0.9238795325113\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{\ 0\ ,\pm 1\ ,\pm\frac{1}{\sqrt{2}},\pm Sin\left(\frac{\pi}{8}\right),\pm Sin\left(\frac{3*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{0\ ,\pm 1\ ,\pm Sin\left(\frac{\pi}{4}\right),\pm Sin\left(\frac{\pi}{8}\right),\pm Sin\left(\frac{3*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)=\{0\ ,\pm 1\ ,\pm Sin\left(\frac{5*\pi}{4}\right),\pm Sin\left(\frac{9*\pi}{8}\right),\pm Sin\left(\frac{11*\pi}{8}\right)\}$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0\ ,\pm 1\ ,\pm Sin\left(\frac{1}{1}*\frac{\pi}{4}\right),\pm Sin\left(\frac{1}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{3}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(26)$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0\ ,\pm 1\ ,\pm Sin\left(\frac{2}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{7}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{5}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(27)$$

$$Sin\left(\frac{\pi}{4}*\left(x+\frac{1}{2}\right)\right) and\ Sin\left(\frac{\pi}{4}*\left(x-\frac{1}{2}\right)\right)$$
$$=\left\{0\ ,\pm 1\ ,\pm Sin\left(\frac{2}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{9}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{11}{2}*\frac{\pi}{4}\right)\right\} \rightarrow EQ(28)$$

From EQ(26) and EQ(27) and EQ(28) these two functions have 5 steady roots $\{\ 0\ ,\pm 1\ ,\pm\frac{1}{\sqrt{2}}\}$ and 4 other

moving roots starting from $Sin\left(\frac{1}{2}*\frac{\pi}{4}\right)$

like { $\pm Sin\left(\frac{1}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{3}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{5}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{7}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{9}{2}*\frac{\pi}{4}\right),\pm Sin\left(\frac{11}{2}*\frac{\pi}{4}\right)\dots$ }

3.1 Zeta Function none-trivial Zeros at X = X -0.5

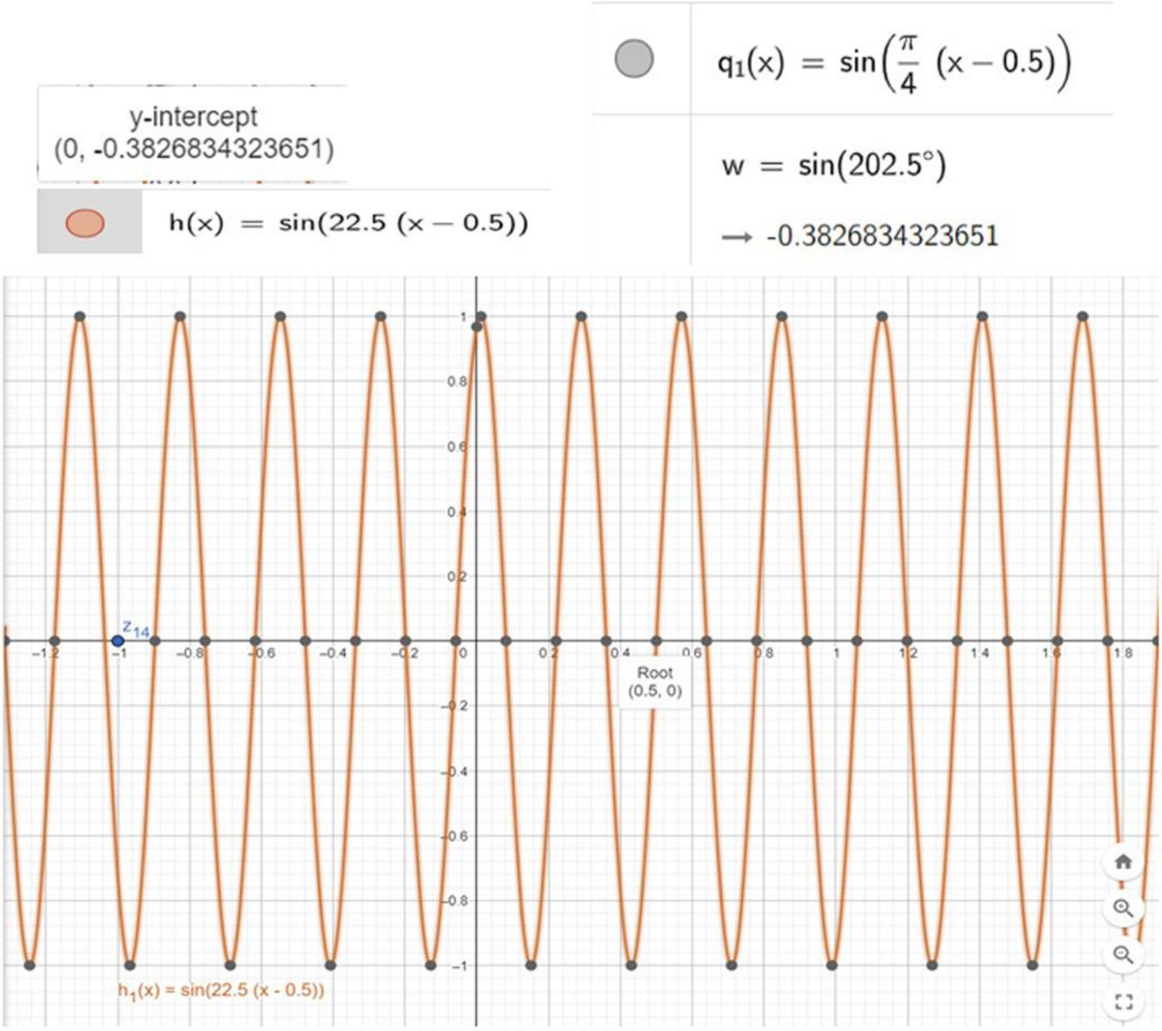


Figure 35. using 22.5 instead of $\frac{\pi}{8}$ increases the frequency but still have root at 0.5.

Here in Figure 35. The Inside polynomial y =X-0.5 so start point is 0.5 and frequency 8 roots between start point and [1]

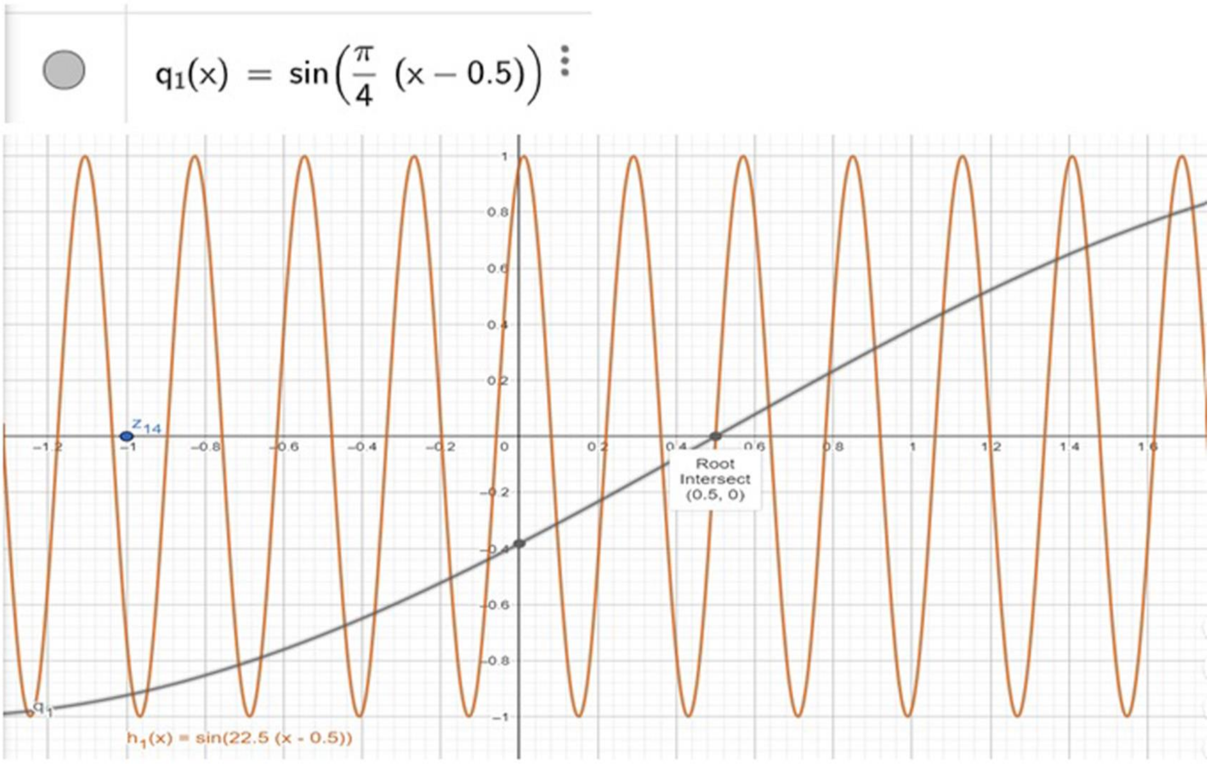


Figure 36. waves with different frequencies still intersects at roots of polynomial inside Sin. (Here at 0.5).

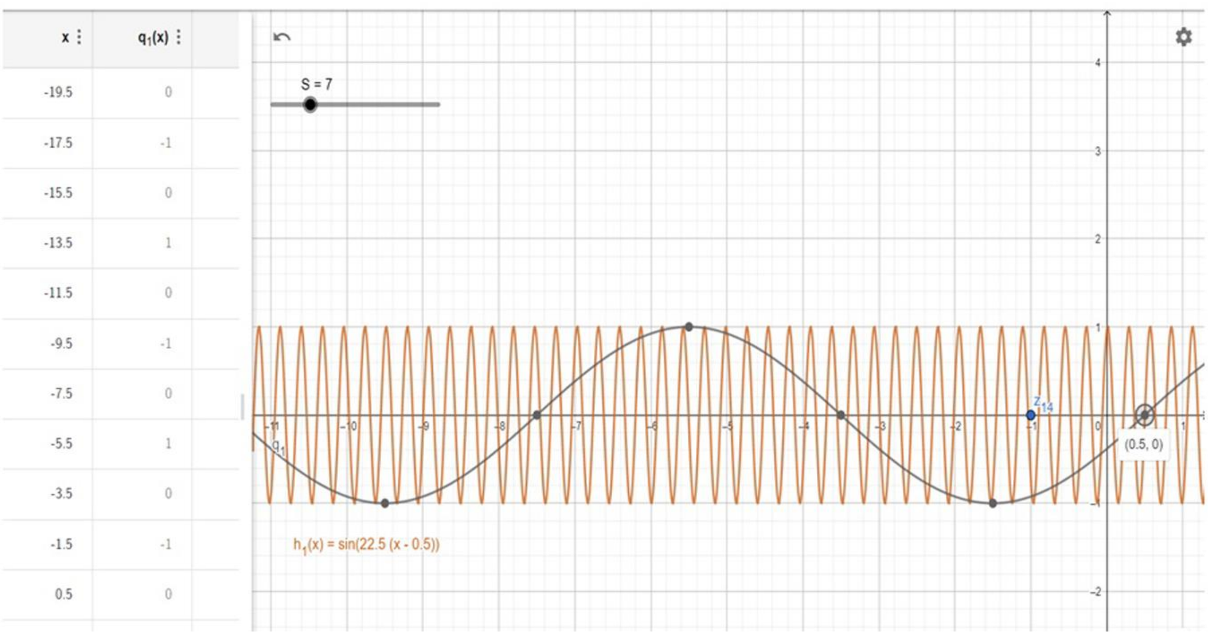


Figure 37. using $\frac{\pi}{4}$ and start point at 0.5 gives roots every -4 or +4 (number of partitions of pi)

- Q1(X) = 0 at X = { 0.5,-3.5 , -7.5 , -11.5, -15.4,…..} ; with step = 4
- And Q1(X) = { i , -i} at X={-1.5 , -5.5 , -9.5, -13.5, -17.5, …..} with step = 4

  We will see later how to make Q1(X) = 0 for all Odd numbers.

1- For any X = X ±0.5
Then zeta functional Sin() term, will be moving in term of θ = 22.5º and will hav rootse for all odd numbers with value = 1/sqrt(2)
Zeta functional sin() term will intersect with Y at point sin(θ = 22.5º) = 0.38268343236509

2- Sin() term in zeta function at X = X + 0.5 will equal to = { 0.5 , -0.5} if we multiply it by by $= \frac{\pm 1}{\sqrt{2}}$

or Sin(45º). And in complex plane multiplication means rotation. Which means if we rorate our complex plane axis by 45º

3- Because sin() term in Zeta function have 90º with S; but we are going to replace it by S = S + 0.5

we converted the engel into 45º which made all odd numbers sin() are with value $= \frac{\pm 1}{\sqrt{2}}$ which is

Acutally due to 45º rotation done when we tranfered S = S + 0.5. by this angel all odd numbers landed on rotated axis by 45º.

4- We can do the oposite transformation by normalizing this rotation back into the original complex

plane axis by replaceing $\sqrt{2}$ by $\frac{1}{Sin(45)}$ then we will get all the roots back to the orginal complex

plane axis. (this $\sqrt{2}$ came zeta formula when replacing each S = S +0.5 in 2^S term)

$$\zeta\left(S+\frac{1}{2}\right)=\begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} * \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)}{Sin(\frac{\pi}{4})} \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases} \rightarrow EQ(D)$$

$$Cos\left(\frac{\pi}{8} * 2 * X\right) = Sin\left(\frac{\pi}{8} * (2 * X + 4)\right)$$

$$Cos\left(\frac{\pi}{2} * \frac{X}{2}\right) = Sin(\frac{\pi}{2} * (\frac{X}{2} + 1))$$

$$Sin\left(\frac{\pi}{2} * \left(\frac{x+0.5}{2}\right)\right) = Sin\left(\frac{\pi}{4} * \left(x+\frac{1}{2}\right)\right)$$

In the rest of the document we are goin to see the distribution odd roots on our new odd Identity function and how it explaines the distribution of the roots for Sin() term in Zeta function.

4. Odd Identity unit Circle function Properties $f(x) = z^x \; = \; (\pm\frac{1}{\sqrt{2}} \pm i\ \frac{1}{\sqrt{2}})^x$

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^x = \pm \mathrm{Cos}(2x\theta) \pm i\,\mathrm{Sin}(2x\theta) = \pm\mathrm{Cos}((2 * x * 22.5)^\circ) \pm i\,\mathrm{Sin}((2 * x * 22.5)^\circ)$$

This f(x) is like Euler's Identity but for odd numbers. We will call it odd Identity unit Circle.

1- equivalent to the complex plane unit circle. And Euler's Identity $e^{i\pi} + 1 = 0$
2- odd Identity unit Circle f(X) axis, rotates 45 degrees from the original complex plane axis X, Y.
3- this f(Z) Odd Identity unit circle axis is Y =X and Y =-X, which means if X = e then Y = e or Y =-e
4- this odd Identity unit circle function intersects with Y = X and Y =-X at square root of two.
5- This Odd Identity unit circle function intersects with 4 axis (2 original and 2 rotated), in 8 points. {1, -1, i, -i, sqrt (2), -sqrt (2)}
6- every cycle of 8 integer values for x, we start new cycle of same values of f(x). first cycle starts at X =0, second cycle starts at X =8.

Table 7. Odd Identity Unit Circle Rotation Values at [x] Natural Numbers (Cycle every 8)

We have 4 axes with total 8 unique complex numbers $\left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right) and\ (\pm 1)\ and\ (\pm i)$.

x	$f(z) = z^x \; = \; (\frac{1}{\sqrt{2}} + i\ \frac{1}{\sqrt{2}})^x$
0	1+0i
1	$\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
2	i
3	$\frac{-1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
4	-1+0i
5	$\frac{-1}{\sqrt{2}} + i\,\frac{-1}{\sqrt{2}}$
6	-i
7	$\frac{1}{\sqrt{2}} + i\,\frac{-1}{\sqrt{2}}$
8 -→ end of one cycle	1+0i

9	$\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}$
10	….

7- For any even integer values for X; f(X) value will be on original complex plane axis. And any odd integer values of X; f(X) value will be a complex number on the odd Identity unit circle axis which are the new rotated axis (45 degrees).

$$f(x) = z^x \;=\; (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$$

$$for\ all\ odd\ values\ of\ x, \qquad f(x) = \pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}$$

Which means all values of f(X) will be on the new Odd Identity unit Circle; where Cos (45) = Sin (45) = 1/sqrt (2).

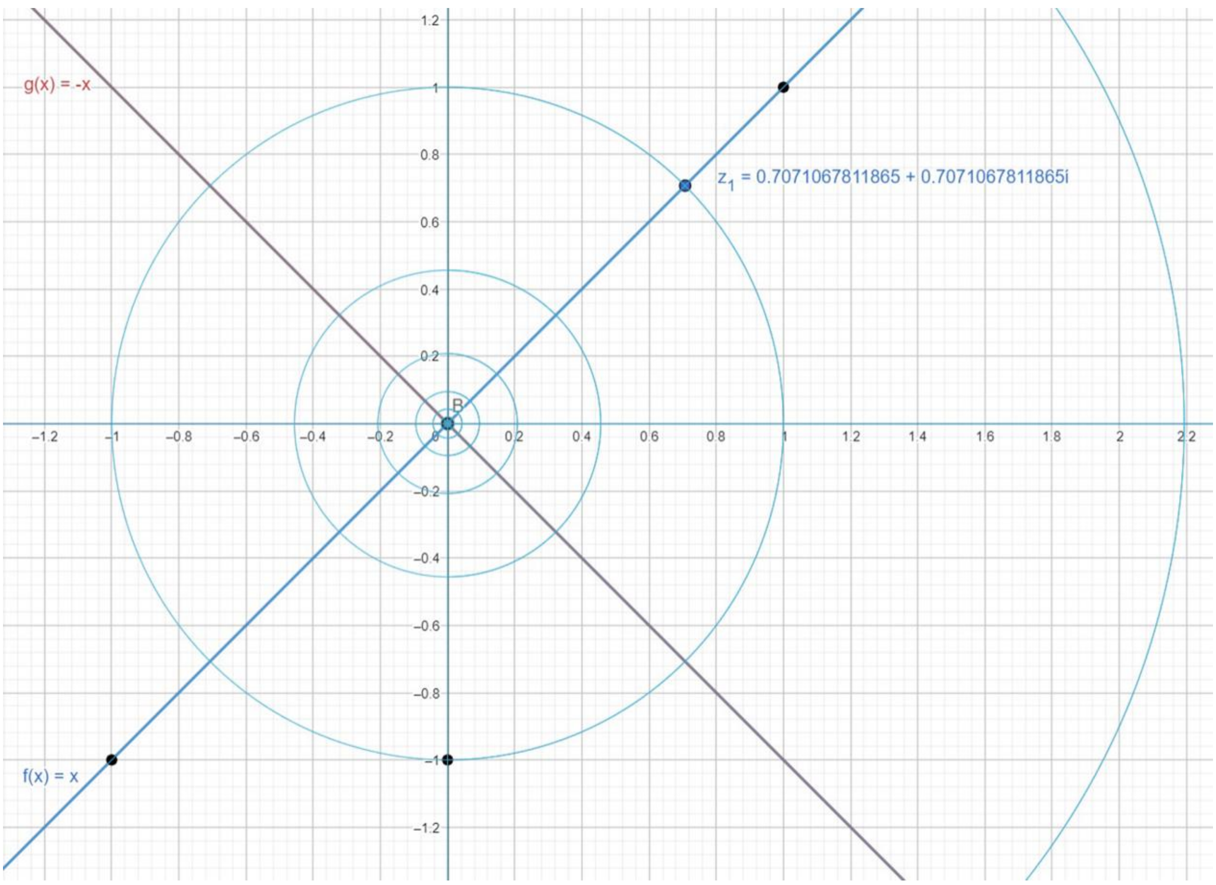


Figure 38. Shows our Odd Identity 4 axis and how [e] intersects with the New Odd Identity axis at [e].

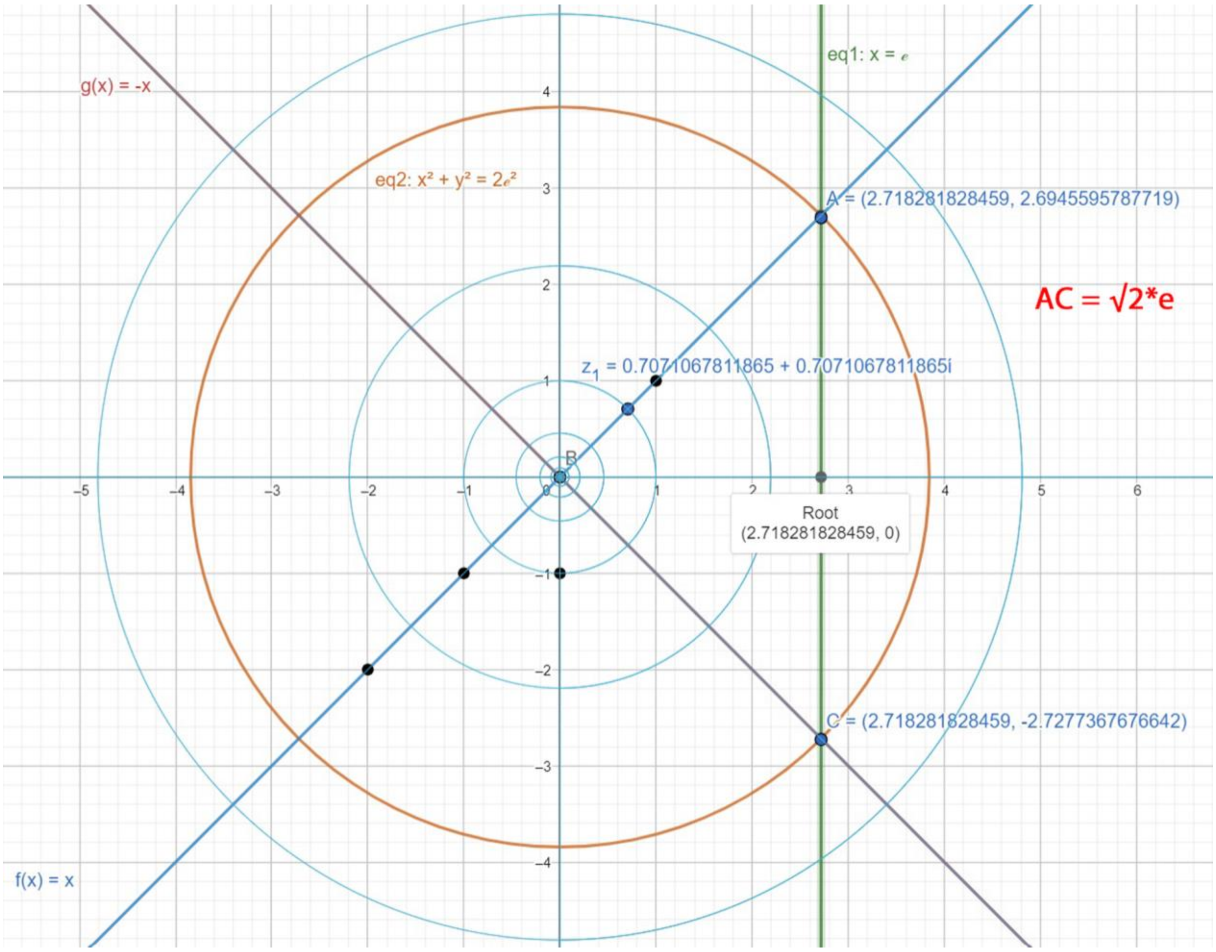


Figure 39. Δ(ABC) triangle with two equal sides =[e] intersects with $x^2 + y^2 = 2 * e^2$ at [e].

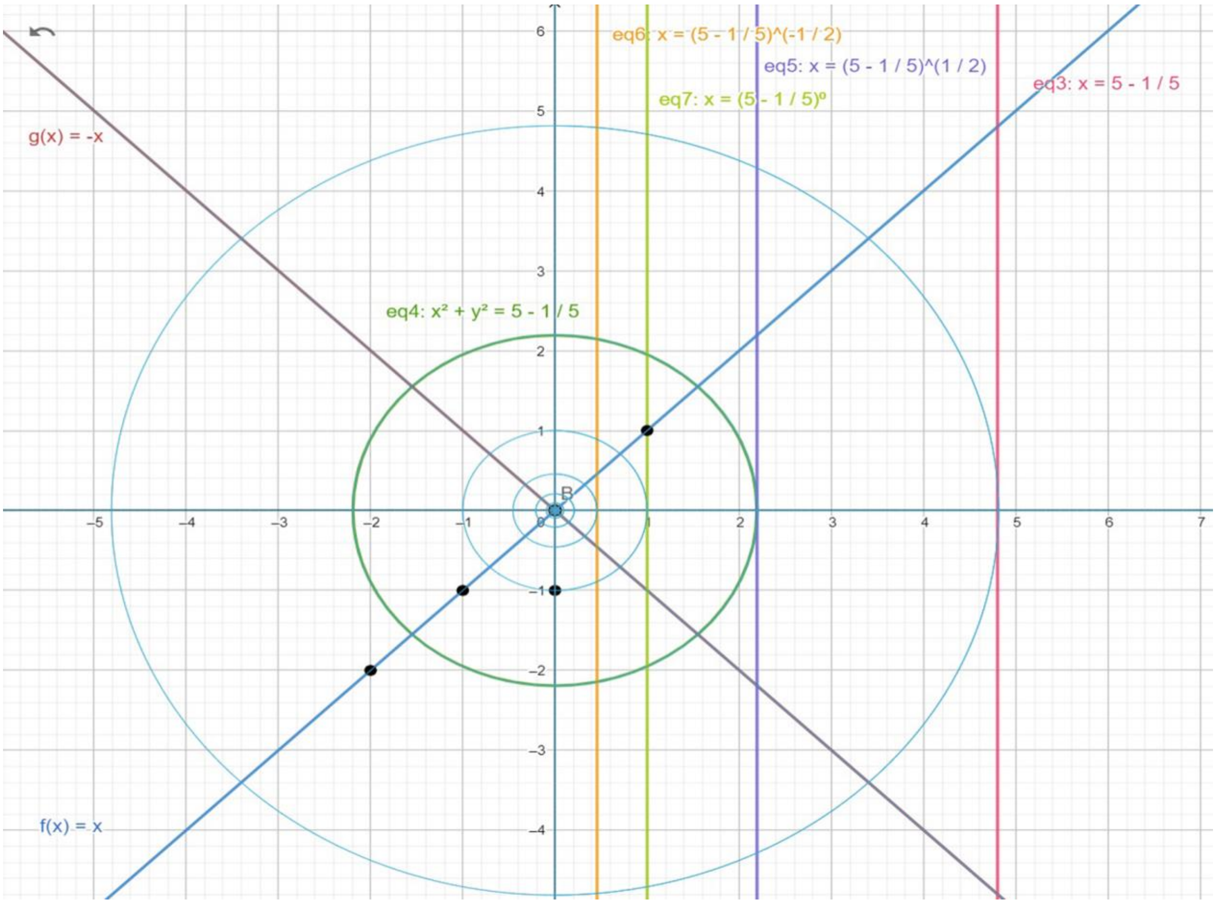


Figure 40. our new Odd Identity Circle Root Circles (First Circle after Circle 1 is Circle [$x^2 + y^2 = (5 - \frac{1}{5})$])

4.1 Odd Identity unit Circle function at natural number values for x

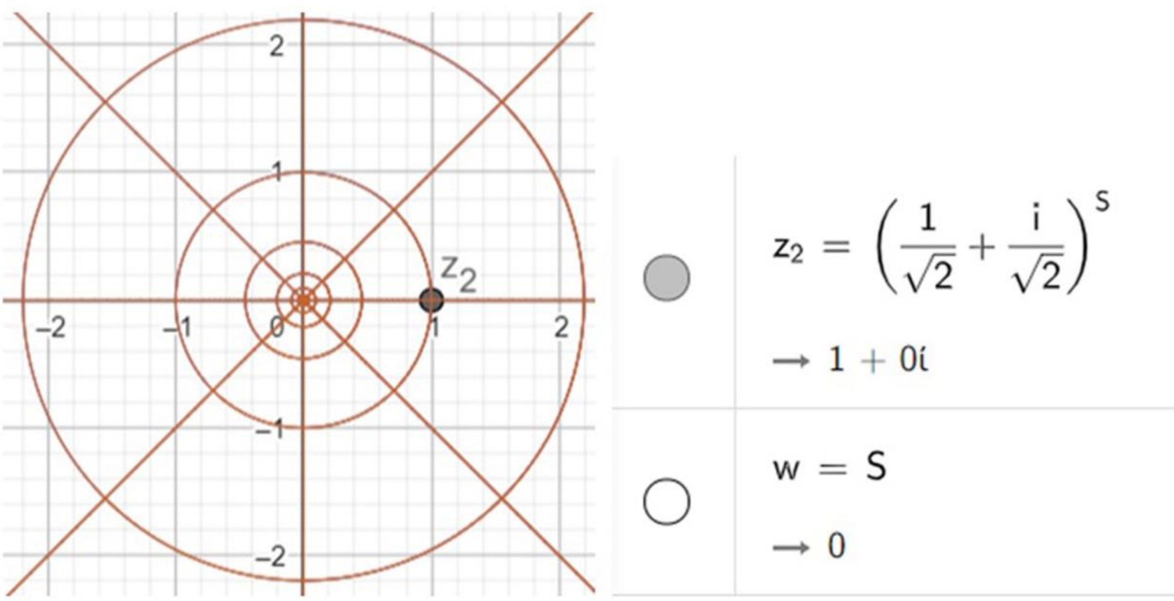


Figure 41. New F(x) Odd numbers unit identity, at x = S = 0, start point at noraml complex plane x axis.

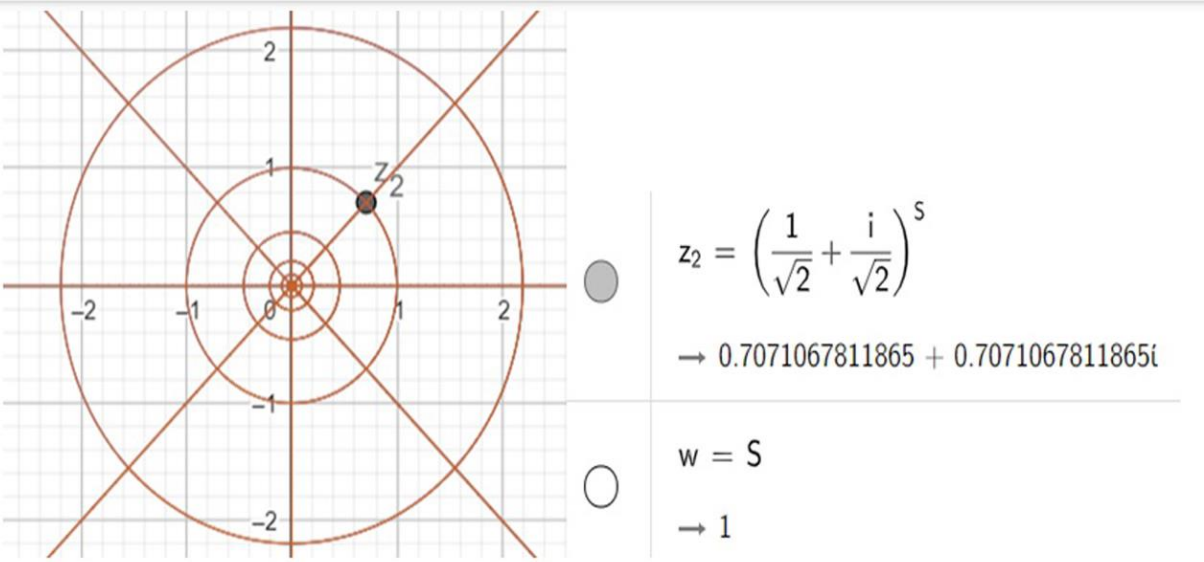


Figure 42. New F(x) Odd numbers unit identity, at x = S = 1, [Z2] at new Odd Idnetity unit axis at (45) degrees from start point at 1 on noraml complex plane X axis.

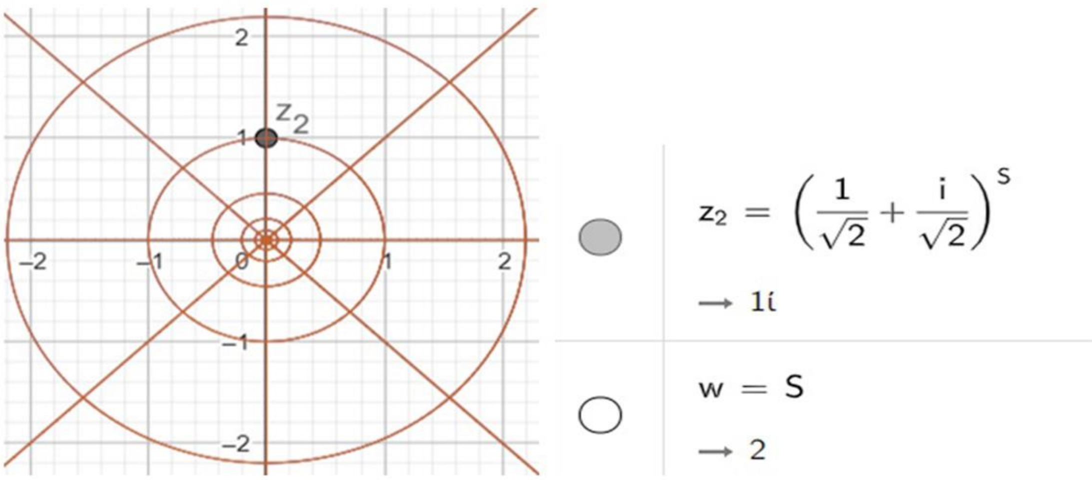


Figure 43. New F(x) Odd numbers unit identity, at x = S = 2, [Z2] at noraml complex plane Y axis.

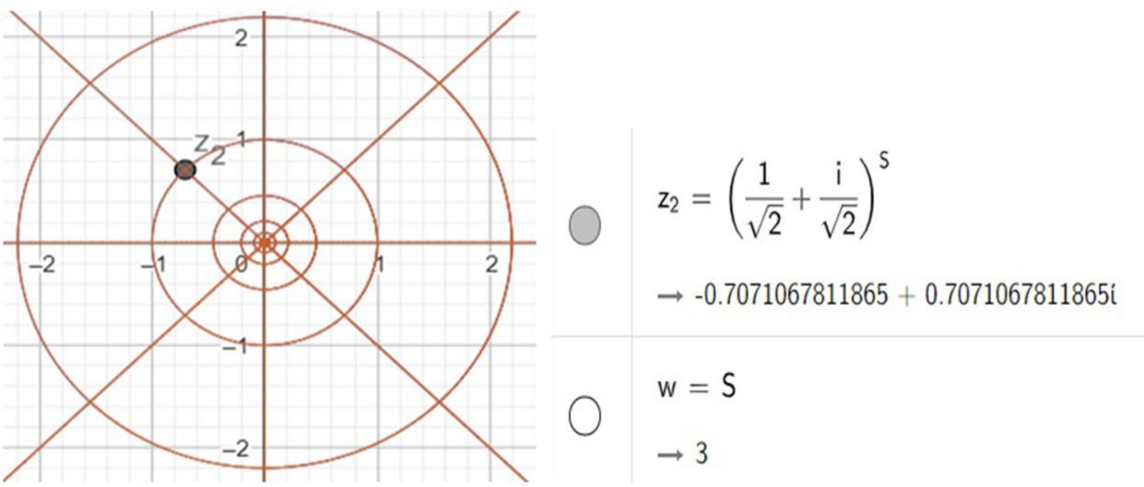


Figure 44. New F(x) Odd numbers unit identity, at x = S = 3, [Z2] at new Odd Idnetity unit axis at (135) degrees from start point at 1 on noraml complex plane X axis.

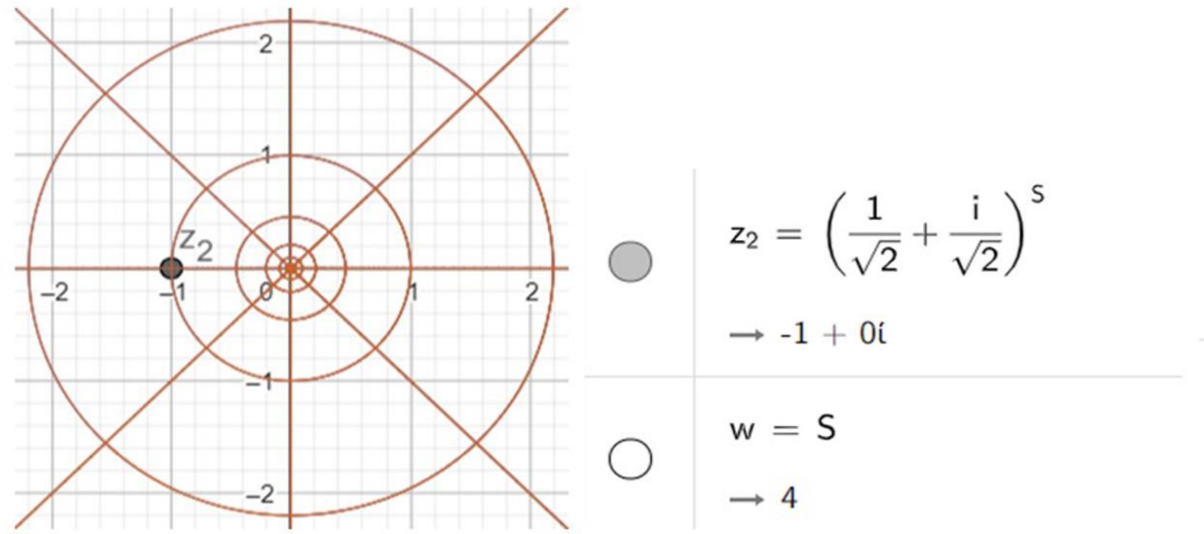


Figure 45. New F(x) Odd numbers unit identity, at x = S = 4, [Z2] at noraml complex plane X axis.

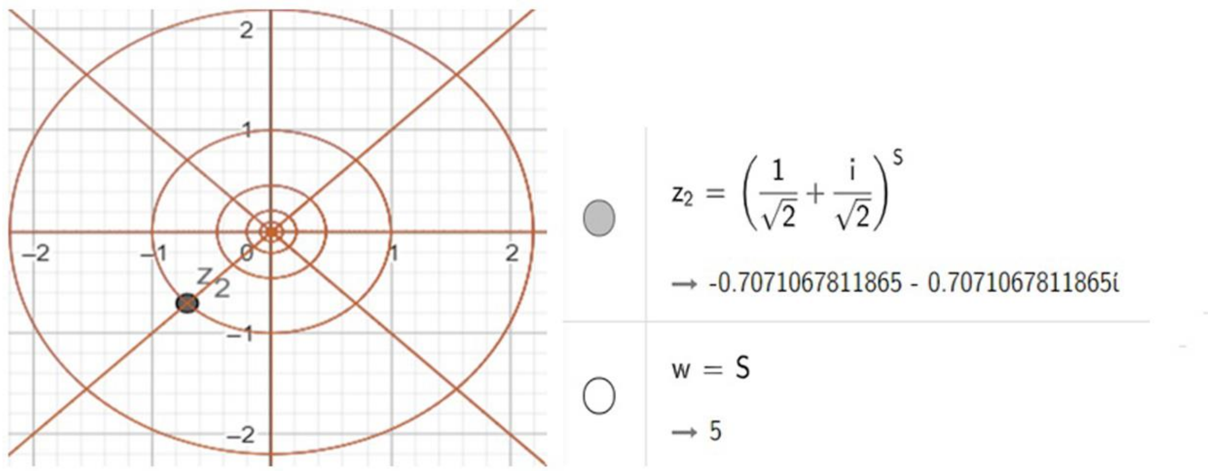


Figure 46. New F(x) Odd numbers unit identity, at x = S = 5, [Z2] at new Odd Idnetity unit axis at (225) degrees from start point at 1 on noraml complex plane X axis.

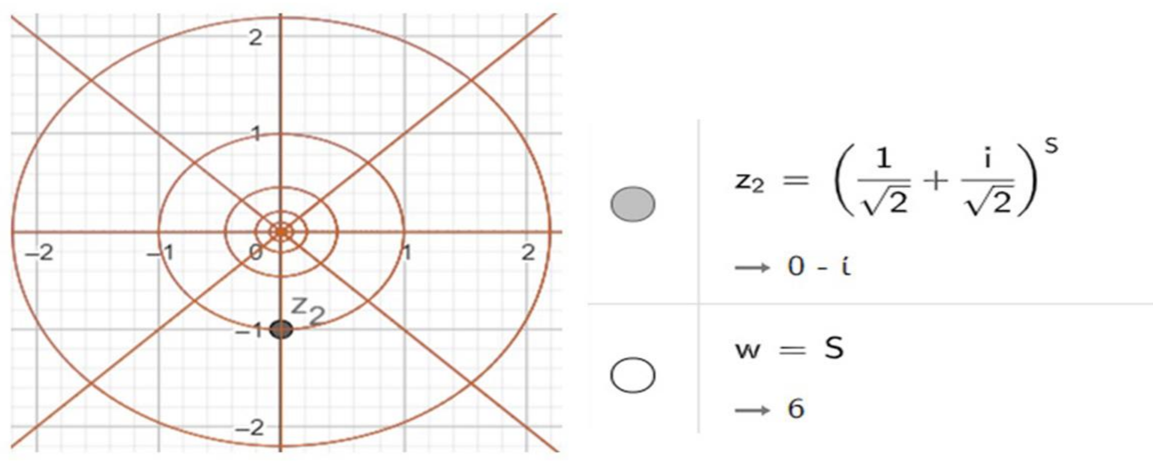


Figure 47. New F(x) Odd numbers unit identity, at x = S = 6, [Z2] at noraml complex plane Y axis.

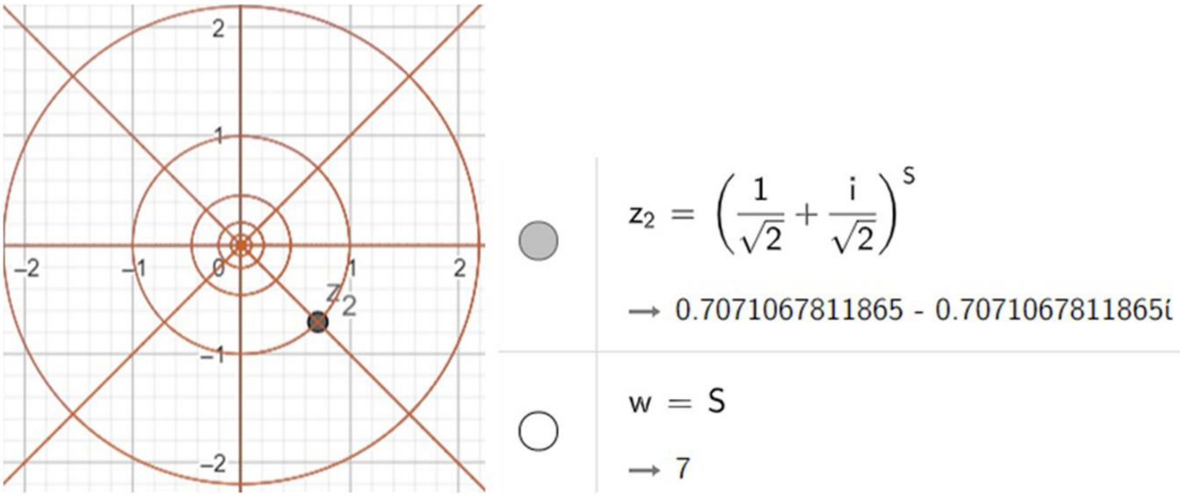


Figure 48. New F(x) Odd numbers unit identity, at x = S = 7, [Z2] at new Odd Idnetity unit axis at (315) degrees from start point at 1 on noraml complex plane X axis.

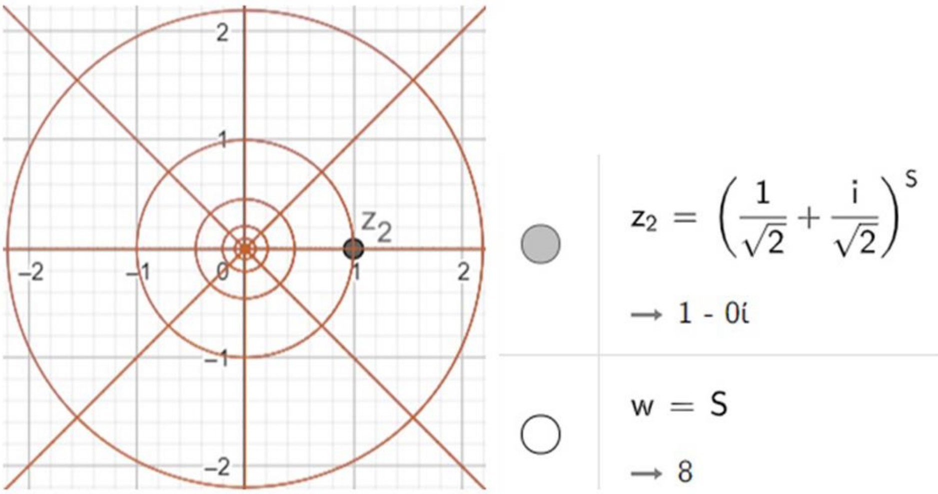


Figure 49. New F(x) Odd numbers unit identity, at x = S = 8, [Z2] completes a full cycle and goes back to start point on noraml complex plane x axis at 1.

4.2 Odd Identity unit Circle function at any real number values for x

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^{x} = \pm\,\mathrm{Cos}(2x\theta) \pm i\,\mathrm{Sin}(2x\theta) = \pm\mathrm{Cos}((2 * x * 22.5)°) \pm i\,\mathrm{Sin}((2 * x * 22.5)°)$$

With natural number values for x we used (45) degrees, and we got full cycle every 8 values. Here in real values for x we are going to use (22.5) degrees, starting at square root at X = 0.5. Or at X = X+0.5 and X =0.

1- for all real values of x, then f(X) value will be any point on the odd Identity unit Circle. Where odd Identity unit Circle origin (0,0) but the axis is rotated by 22.5 degrees.

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^{x} = \pm\,\mathrm{Cos}(x\theta) \pm i\,\mathrm{Sin}(x\theta)$$

2- for x = S = 0.5, THEN θ = 22.5°, because at x = 1, was θ = 45°
so here with real numbers values we are going to use a cycle with start point at x = S = 0.5, THEN θ = 22.5°

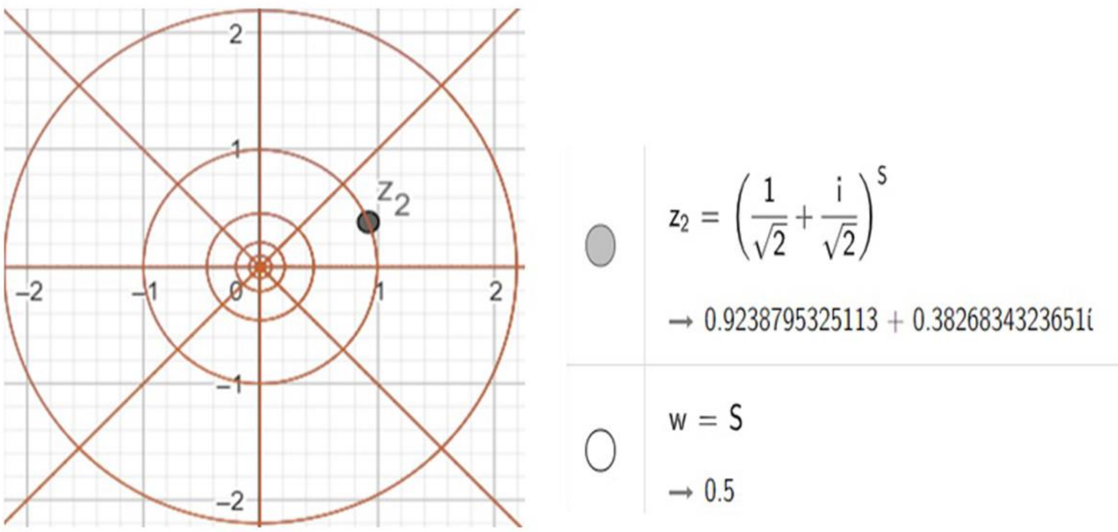


Figure 50. New F(x) Odd numbers unit identity, at x = S = 0.5, for real values for x, [Z2] is a start Cycle point, starts at (22.5) degrees.

$$f(x) = \left(\pm\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)^{\frac{1}{2}} = \pm\mathrm{Cos}(22.5) \pm i\,\mathrm{Sin}(22.5)$$

3- one property for this θ = 22.5° and Sin (22.5) and Cos (22.5)

$$Cos\left(\frac{\pi}{8} * 2 * X\right) = Sin\left(\frac{\pi}{8} * (2 * X + 4)\right)$$

$$Cos\left(\frac{\pi}{2} * \frac{X}{2}\right) = Sin(\frac{\pi}{2} * (\frac{X}{2} + 1))$$

$$Cos\left(\frac{\pi}{2} * \frac{x}{2}\right) \text{ in complex number have same value as } Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$$

$$\text{in another complex number in the same cycle of } 22.5 \text{ degree partitions}$$

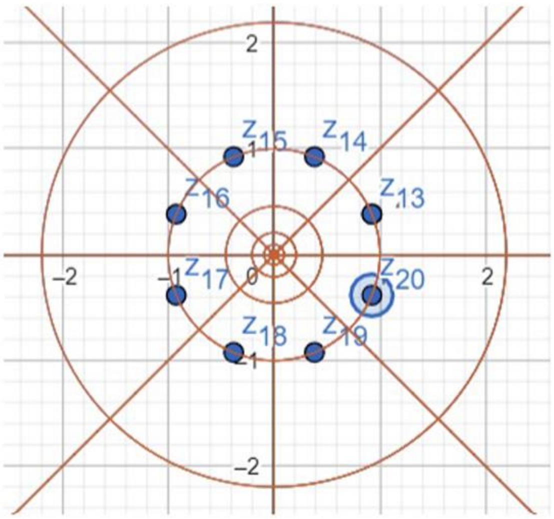
Table 8. shows how starting a Cycle at (22.5) degrees we will have $Sin\left(\frac{\pi}{4} * \left(x + \frac{1}{2}\right)\right)$ for x =2 will equal

$Cos\left(\frac{\pi}{2} * \frac{x}{2}\right)$ at x = 1 as it show in matching pair colours in the table {Cos(22.5) = Sin(112.5)}

x	θ	$f(x) = z^x = (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$
0.5	22.5^{0}	$Cos(22.5^{0}) + i\,Sin(22.5^{0}) = 0.9238795325113 + 0.3826834323651i$
1.5	67.5^{0}	$Cos(67.5^{0}) + i\,Sin(67.5^{0}) = 0.3826834323651 + 0.9238795325113i$
2.5	112.5^{0}	$Cos(112.5^{0}) + i\,Sin(112.5^{0}) = -0.3826834323651 + 0.9238795325113i$
3.5	157.5^{0}	$Cos(135^{0}) + i\,Sin(135^{0}) = -0.9238795325113 + 0.3826834323651i$
4.5	202.5^{0}	$Cos(202.5^{0}) + i\,Sin(202.5^{0}) = -0.9238795325113 - 0.3826834323651i$
5.5	247.5^{0}	$Cos(247.5^{0}) + i\,Sin(247.5^{0}) = -0.3826834323651 - 0.9238795325113i$
6.5	292.5^{0}	$Cos(292.5^{0}) + i\,Sin(292.5^{0}) = 0.3826834323651 - 0.9238795325113i$
7.5	337.5^{0}	$Cos(337.5^{0}) + i\,Sin(337.5^{0}) = 0.9238795325113 - 0.3826834323651\,i$
8	360^{0}	1-0 i
8.5	382.5^{0}	$Cos(382.5^{0}) + i\,Sin(382.5^{0}) = 0.9238795325113 + 0.3826834323651\,i$
9.5	427.5^{0}	…….

Table 9. every cycle cover 8 values for x. one cycle starts at X= 0.5 and θ=22.5 and ends at X =8, θ=360 new cycle of same values of f(x).

x	θ	$f(x) = z^x = (\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}})^x$
0.5	22.5°	$Cos(22.5°) + i\,Sin(\,22.5°) = 0.9238795325113 + 0.3826834323651i$
1.5	67.5°	$Cos(67.5°) + i\,Sin(\,67.5°) = 0.3826834323651 + 0.9238795325113i$
2.5	112.5°	$Cos(112.5°) + i\,Sin(\,112.5°) = -0.3826834323651 + 0.9238795325113i$
3.5	157.5°	$Cos(135°) + i\,Sin(\,135°) = -0.9238795325113 + 0.3826834323651i$
4.5	202.5°	$Cos(202.5°) + i\,Sin(\,202.5°) = -0.9238795325113 - 0.3826834323651i$
5.5	247.5°	$Cos(247.5°) + i\,Sin(\,247.5°) = -0.3826834323651 - 0.9238795325113i$
6.5	292.5°	$Cos(292.5°) + i\,Sin(\,292.5°) = 0.3826834323651 - 0.9238795325113i$
7.5	337.5°	$Cos(337.5°) + i\,Sin(\,337.5°) = 0.9238795325113 - 0.3826834323651\,i$
8	360°	1-0 i
8.5	382.5°	$Cos(382.5°) + i\,Sin(\,382.5°) = 0.9238795325113 + 0.3826834323651\,i$
9.5	427.5°	…….



$z_{13} = \cos(22.5°) + i \sin(22.5°)$

$\rightarrow 0.9238795325113 + 0.3826834323651i$

$z_{14} = \cos(67.5°) + i \sin(67.5°)$

$\rightarrow 0.3826834323651 + 0.9238795325113i$

$z_{15} = \cos(112.5°) + i \sin(112.5°)$

$\rightarrow -0.3826834323651 + 0.9238795325113i$

$z_{16} = \cos(157.5°) + i \sin(157.5°)$

$\rightarrow -0.9238795325113 + 0.3826834323651i$

$z_{17} = \cos(202.5°) + i \sin(202.5°)$

$\rightarrow -0.9238795325113 - 0.3826834323651i$

$z_{18} = \cos(247.5°) + i \sin(247.5°)$

$\rightarrow -0.3826834323651 - 0.9238795325113i$

$z_{19} = \cos(292.5°) + i \sin(292.5°)$

$\rightarrow 0.3826834323651 - 0.9238795325113i$

$z_{20} = \cos(337.5°) + i \sin(337.5°)$

$\rightarrow 0.9238795325113 - 0.3826834323651i$

Figure 51. positions of complex numbers for first cycles of F(x) on Odd numbers unit identity circle, starts at (22.5) degrees.

4.3 Using e and θ = 22.5 to represent our odd new Identity function in a complex plane

1- $e^{\theta x} = e^{22.5\,x}$; intersect Y at point (0,1) and start from X =1.5 Y =0 for any X

And $e^{\theta x} = 2\,e^{22.5\,x}$; intersect Y at point (0,2) and start from X =1.5 Y =0 for any X

And $e^{\theta x} = 3\,e^{22.5\,x}$; intersect Y at point (0,3) and start from X =1.5 Y =0 for any X

THEN we are going to use 22.5 as a number in order explore residuals in limited operational machines. Because this will not be going to change the characteristics of the graph itself.

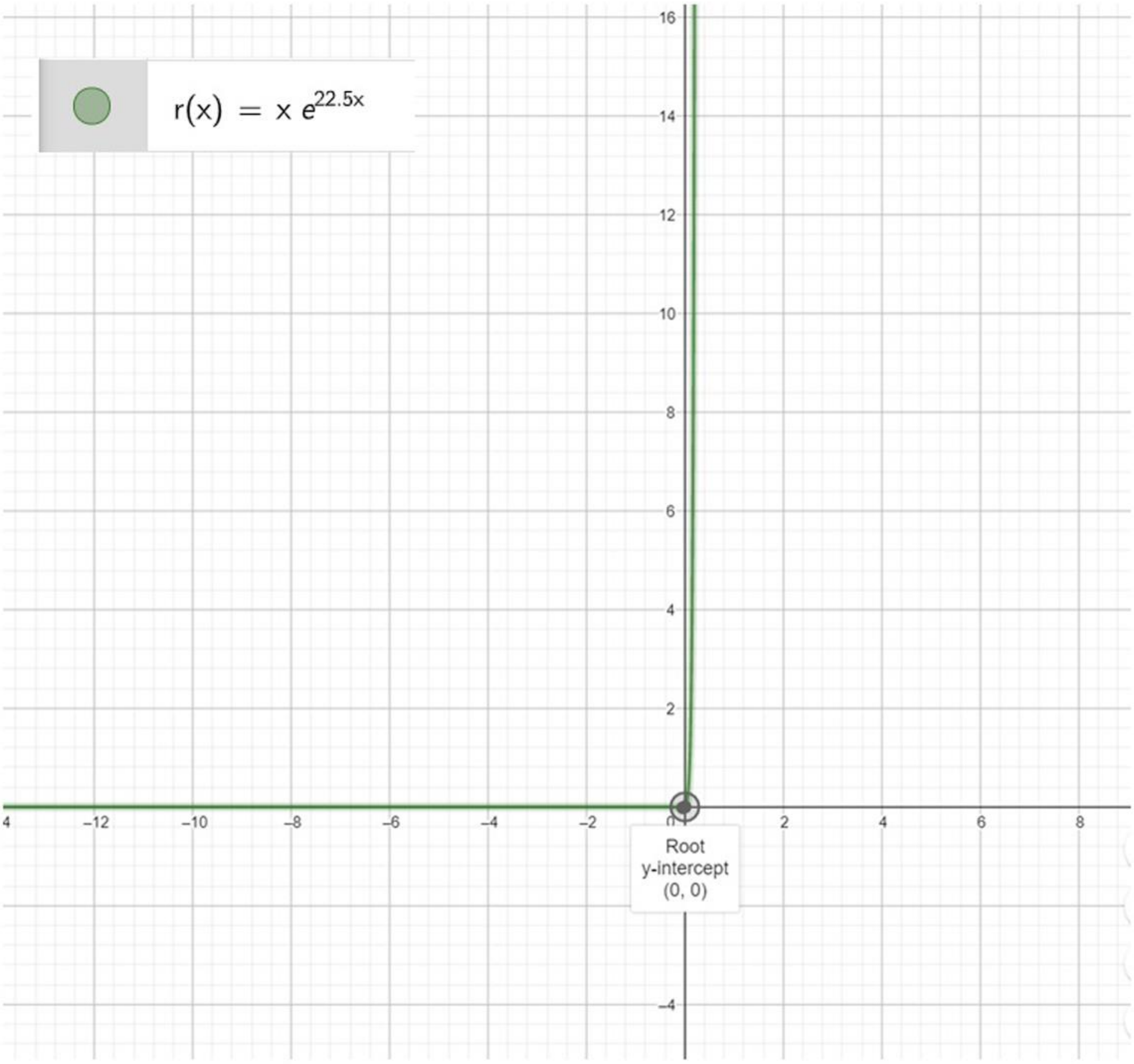


Figure 52. using (22.5) as number not as degree (pi/8), gave us scaled version of the graph.

2- Now we are going to remove this 22.5 degrees from f(x) = X

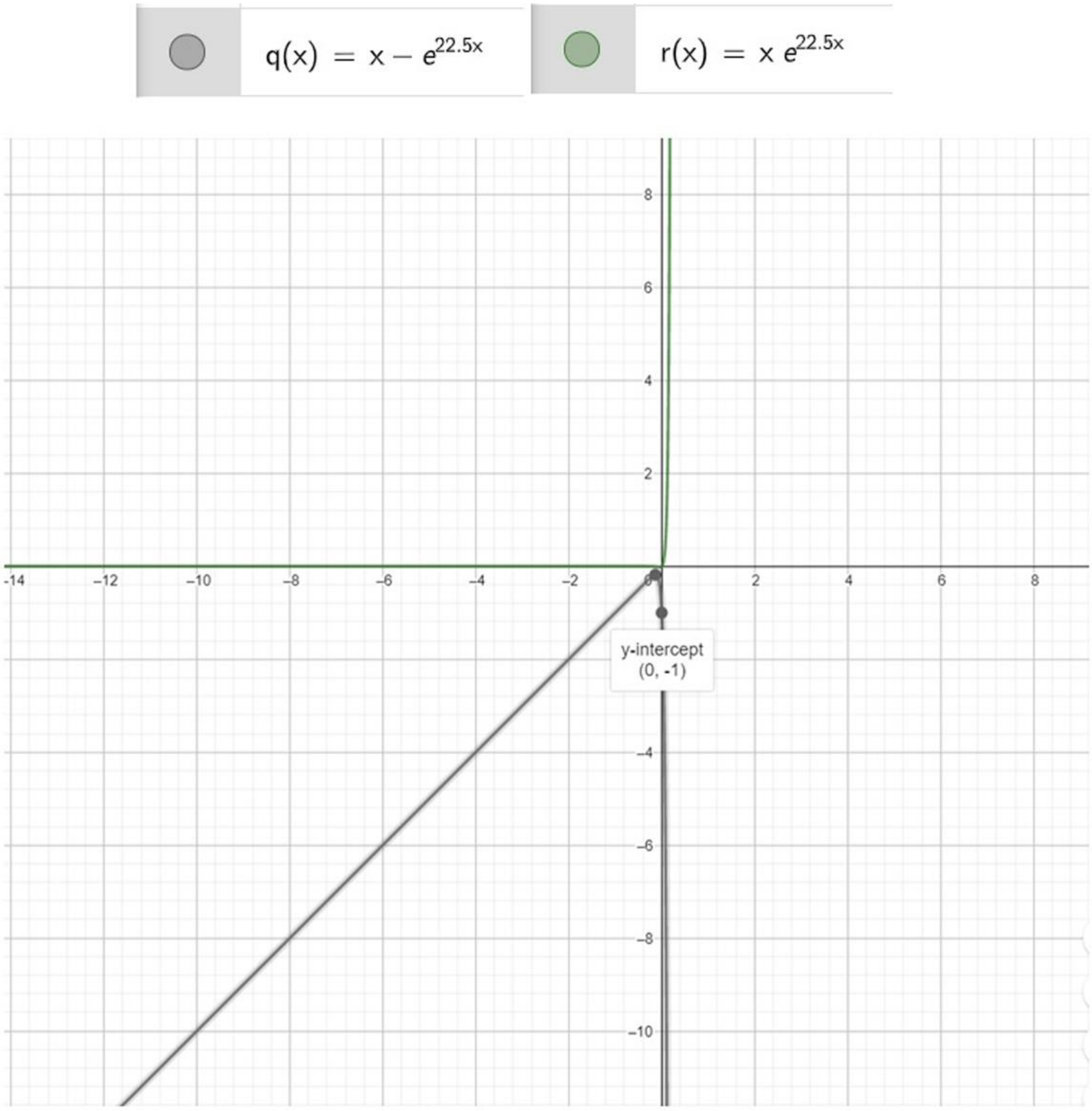


Figure 53. Removing 22.5 degrees from f(x) = X from scaled version of the graph with (pi/8).

Table 10. Q(X) = X for all x <= -1.5 and Q(X) = -1 at X = 0 AND R(X) = 0 for X <= -1.5 and R(X) = 0 at X =0

$q(x) = x - e^{22.5x}$

$r(x) = x\, e^{22.5x}$

x	q(x)	r(x)
-4.5	-4.5	0
-4	-4	0
-3.5	-3.5	0
-3	-3	0
-2.5	-2.5	0
-2	-2	0
-1.5	-1.5	0
-1	-1.0000000001692	-0.0000000001692
-0.5	-0.5000130072977	-0.0000065036488
0	-1	0
0.5	-76879.41976467772	38439.95988233886
1	-5910522062.023283	5910522063.023283
1.5	-454400461972585...	681600692958881.1
2	-349342710574850...	698685421149700...
2.5	-268574395593695...	671435988984237...

3- We are going to add both functions together.
S(X) = Q(X) + R(X); this will make S(X) = 1 at X =1 and S(X) = -1 at X=0
Half the graph is at Y =X i.e., at 45 degrees

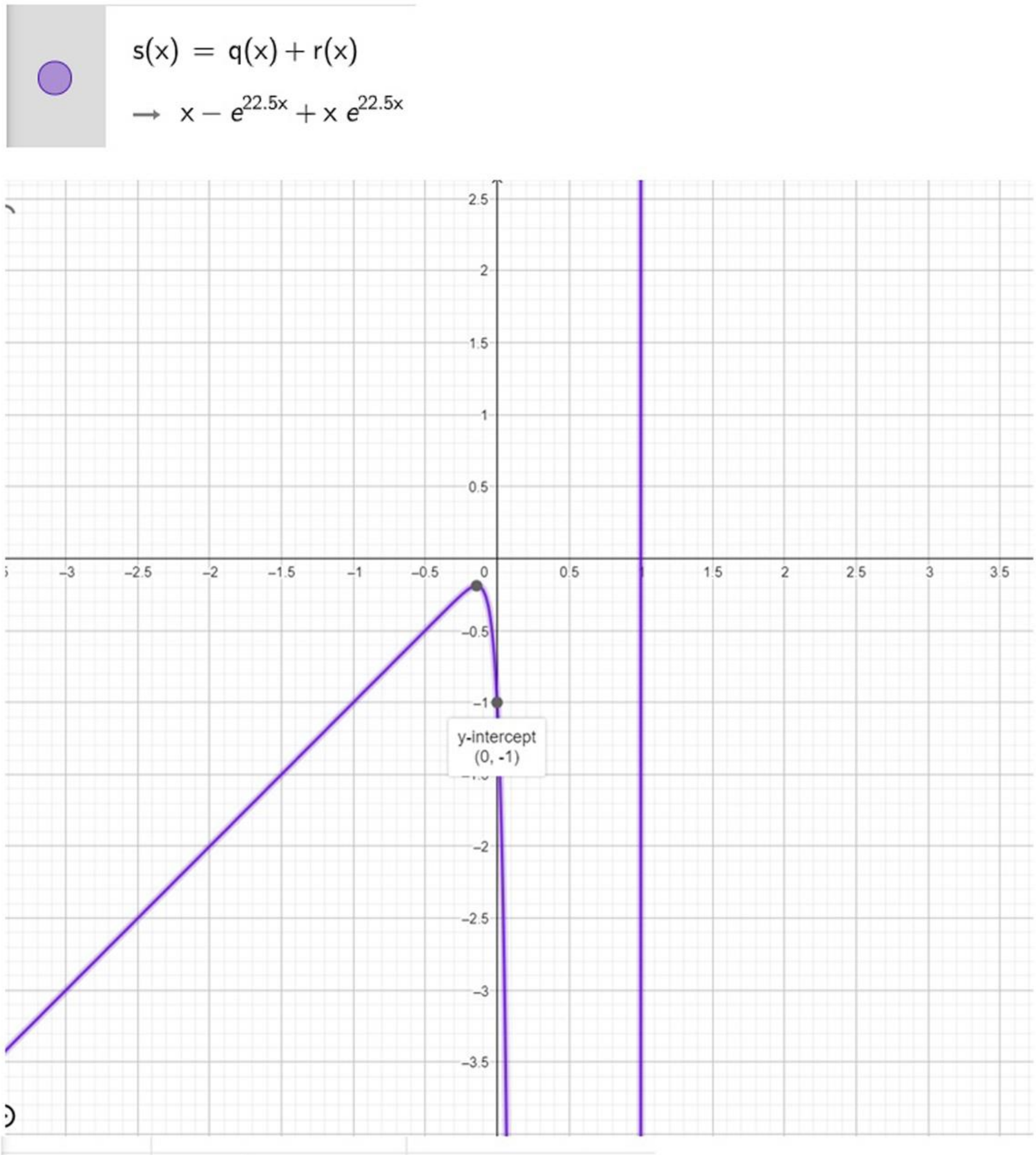


Figure 54. adding Q(X) and R(X) together S(X) = Q(X) + R(X); S(X) = 1 at X =1 and S(X) = -1 at X=0

Table 11. values for all three fucntion Q(X) and R(X) together S(X) = Q(X) + R(X).

x	q(x)	r(x)	s(x)
-4.5	-4.5	0	-4.5
-4	-4	0	-4
-3.5	-3.5	0	-3.5
-3	-3	0	-3
-2.5	-2.5	0	-2.5
-2	-2	0	-2
-1.5	-1.5	0	-1.5
-1	-1.0000000001692	-0.0000000001692	-1.0000000003384
-0.5	-0.5000130072977	-0.0000065036488	-0.5000195109465
0	-1	0	-1
0.5	-76879.41976467772	38439.95988233886	-38439.45988233886
1	-5910522062.023283	5910522063.023283	1
1.5	-454400461972585...	681600692958881.1	227200230986295.2
2	-349342710574850...	698685421149700...	349342710574850...
2.5	-268574395593695...	671435988984237...	402861593390542...

4- To make the graph back to 22.5 degrees which is Y = X/2
By dividing S(X) by two
T(X) = ½ * S(X) = ½ * (R(X) + Q(X))
If X = 0 then T(X) = -0.5 and If X =1, then T(X) = 0.5

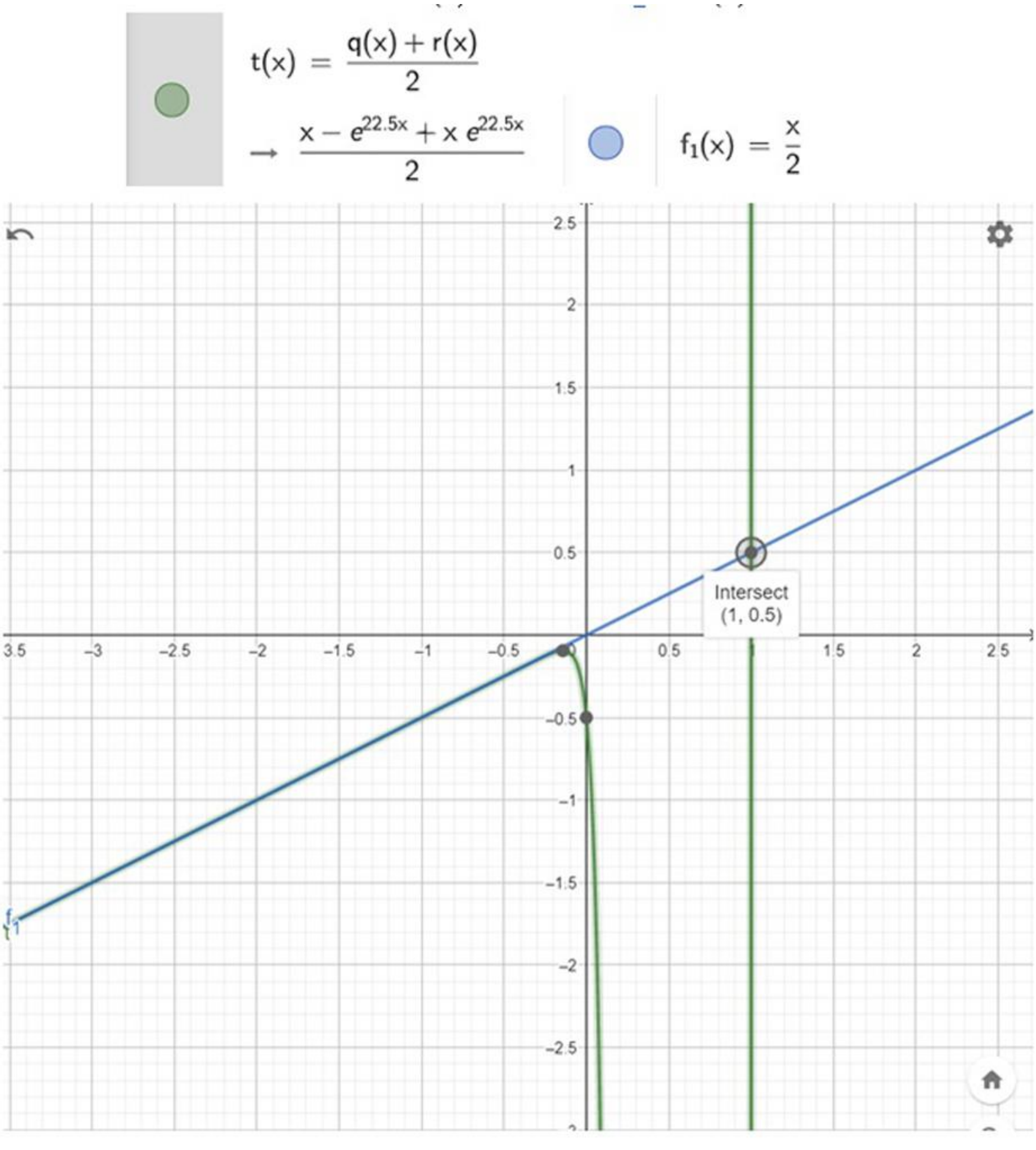


Figure 55. T(X) = ½ * S(X) so T(X) = -0.5 at X =0 and T(X) = 0.5 at X =1.

Table 12. T(X) = -0.5 at X =0 and T(X) = 0.5 at X =1.

x	q(x)	r(x)	s(x)	t(x)
-5.5	-5.5	0	-5.5	-2.75
-5	-5	0	-5	-2.5
-4.5	-4.5	0	-4.5	-2.25
-4	-4	0	-4	-2
-3.5	-3.5	0	-3.5	-1.75
-3	-3	0	-3	-1.5
-2.5	-2.5	0	-2.5	-1.25
-2	-2	0	-2	-1
-1.5	-1.5	0	-1.5	-0.75
-1	-1.0000000001692	-0.0000000001692	-1.0000000003384	-0.5000000001692
-0.5	-0.5000130072977	-0.0000065036488	-0.5000195109465	-0.2500097554732
0	-1	0	-1	-0.5
0.5	-76879.41976467772	38439.95988233886	-38439.45988233886	-19219.72994116943
1	-5910522062.023283	5910522063.023283	1	0.5
1.5	-454400461972585...	681600692958881.1	227200230986295.2	113600115493147.6

4.4 using Our Odd number identity unit Circle f(Z) combined with Euler Number [e].

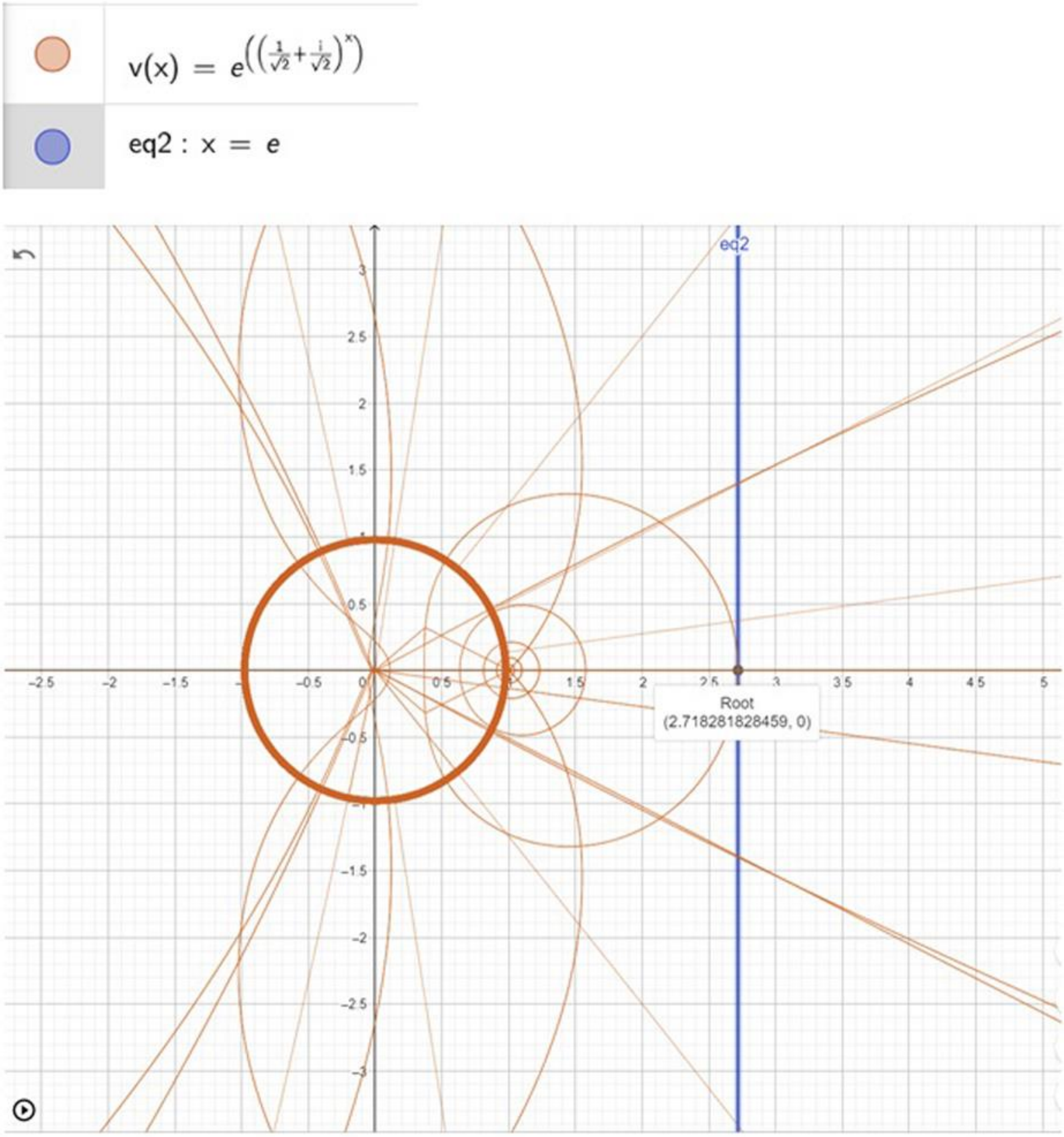


Figure 56. graph shows identity unit Circle when using [e] to the power of f(Z) Odd identity function. And show there is root at [1] and intersection at [e].

5- We are going to Set X = X/2 -1/2
If X is odd number; then X/2 will be even number + 0.5; and by subtracting this 0.5 we are using even number, i.e., we are setting X = X-1 where X-1 is the number before X where X is an odd number. And to keep X as an odd number we are going to add one again.
So, this will be for odd numbers.

$$n(x) = e^{\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)^{\frac{x}{2}-0.5}+1}$$

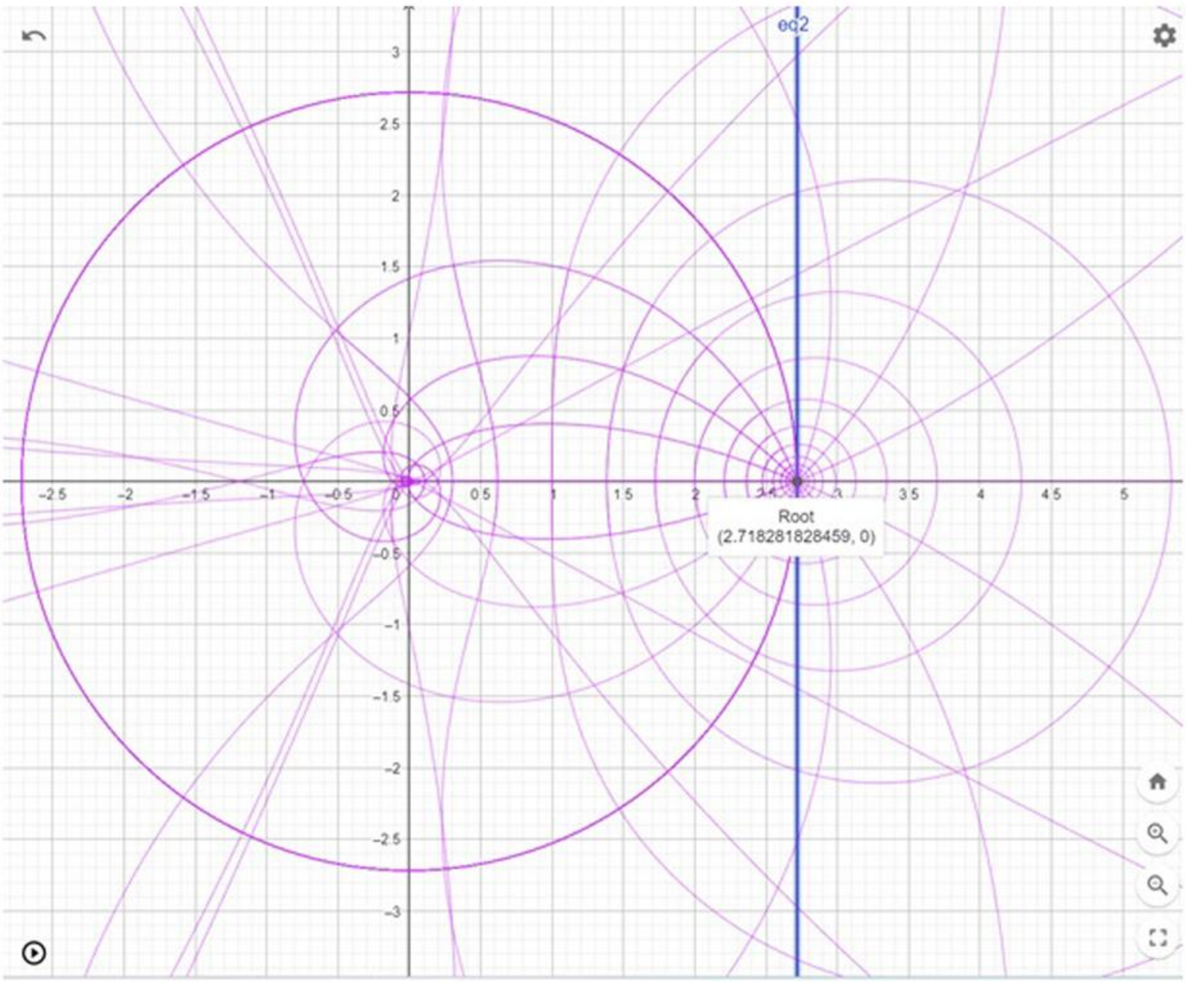


Figure 57. graph show Zero location at [e].

But for even numbers as if we assumed X was odd at the beginning

$$u(x) = e^{\left(\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{\frac{x}{2}-\frac{1}{2}}\right)}$$

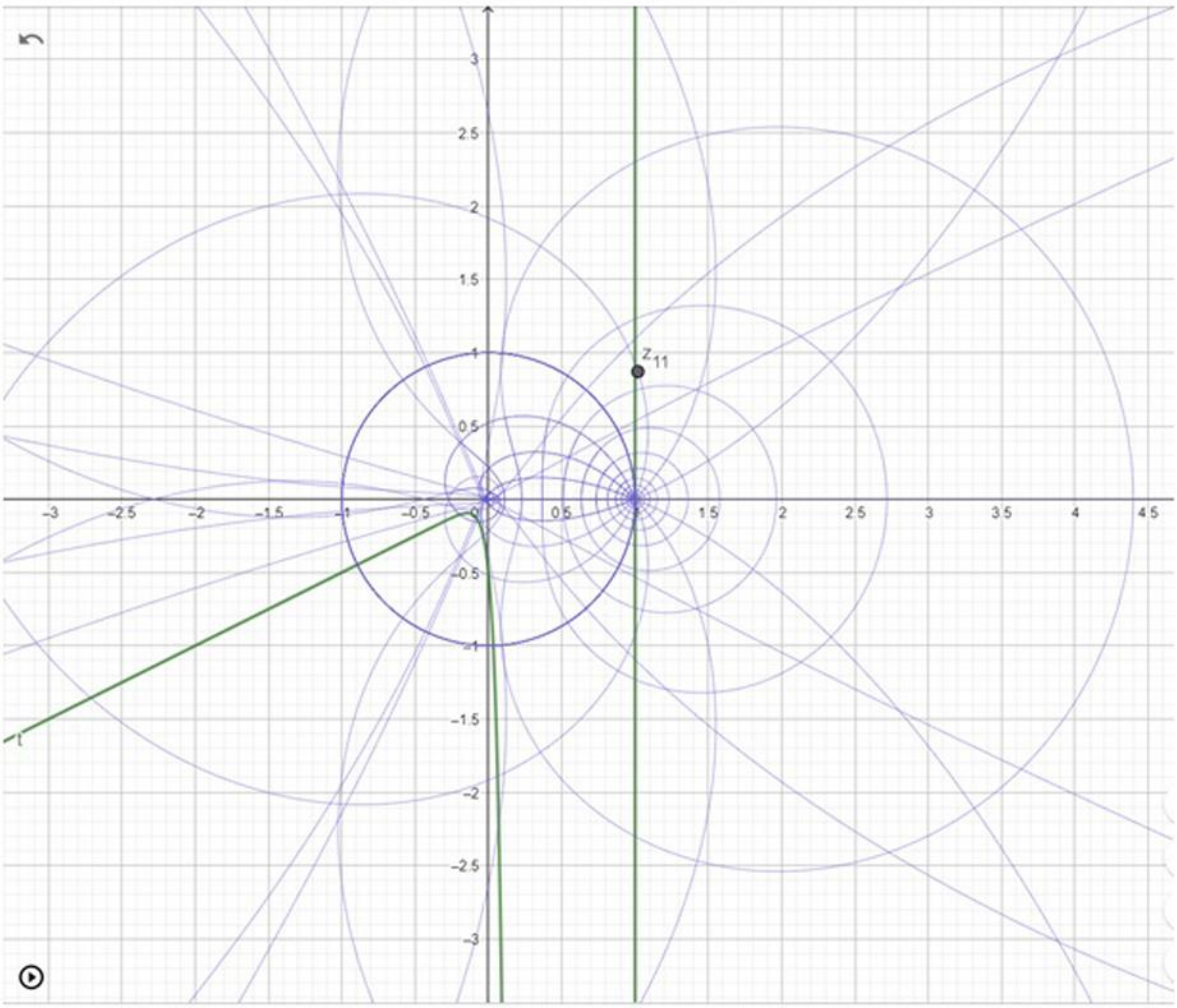


Figure 58. graph show Zero location at unit Circle [1].

This means if S is odd, we can get the same angel as even numbers if we used S = S-1= S/2 -1/2 if S is odd number.

Z10; is complex number on complex plane and will move it value on the odd number Identity circle between = { 1, -1, i, -i} as S changed its value between odd numbers.

$$z_{10} = \left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{S-1}$$

$\rightarrow$ -1 + 0ί

1- For Odd numbers in set { 1, 5, 9 , 13 , 17 , 21 , ……} ; the complex number Z10 will changes values between {1,1-} with this order {1,-1,1,-1,1,-1,…..}

2- For Odd numbers in set {3, 7, 11 ,15 ,19,…….}; the complex number Z10 will changes values between {i,-i} with this oder {i,-i,i,-i,i,-i,…..}

3- For Odd numbers in set {-3,-7,-11,-15,-19,…..} ; the complex number Z10 will changes values between {1,1-} with this order {1,-1,1,-1,1,-1,…..}

4- And this is why the cycle of values resets after 8 and not 4 values.

5- to restrect values between only two values {1,-1} we are going to use the negative value for S; so we are goin to use this function instead.

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$$\longrightarrow 1 - 0i$$

I) for S= S-1 ; half odd numbers will have Z10 = {1, -1} ; and the other half will have Z10={i,-i}

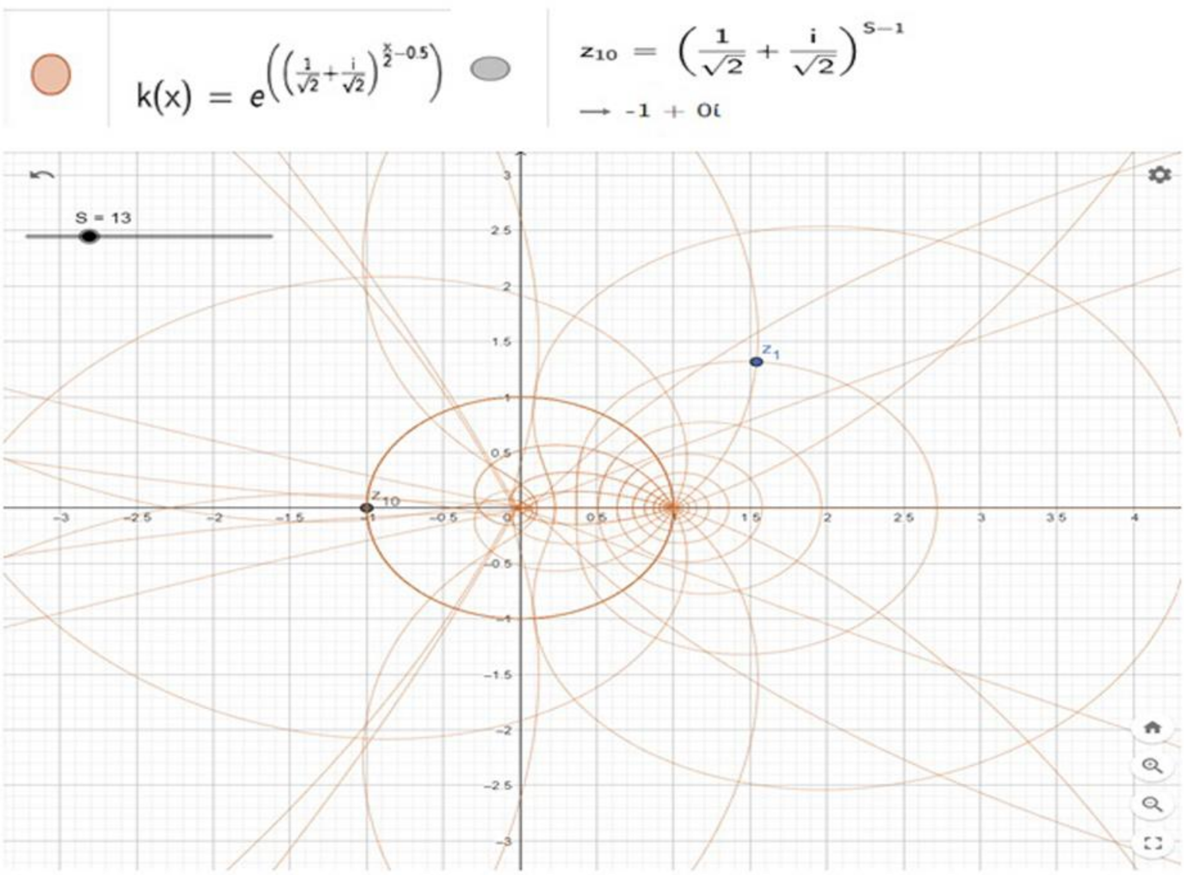


Figure 59. Z10 = -1 at S = 13 and S-1 = 12

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\rightarrow 1 - 0i$

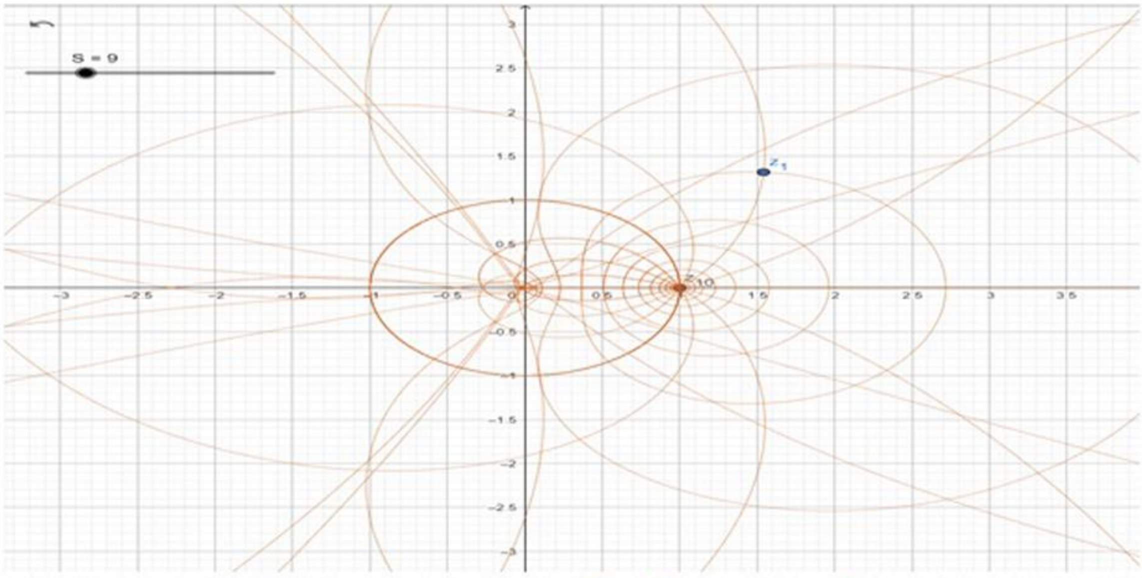


Figure 60. Z10 = 1 at S = 9 and S-1 = 8

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$\rightarrow -1 + 0i$

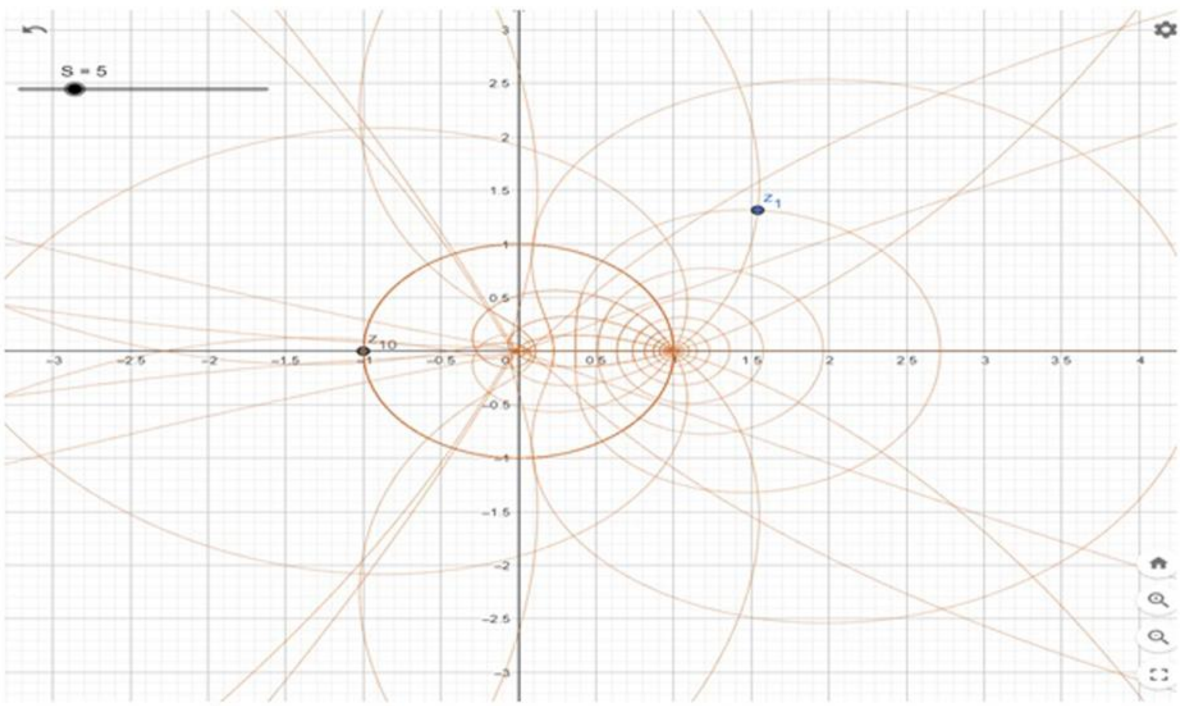


Figure 61. Z10 = -1 at S = 5 and S-1 = 4

$$z_{10} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{S-1}$$

$$\rightarrow 1 - 0i$$

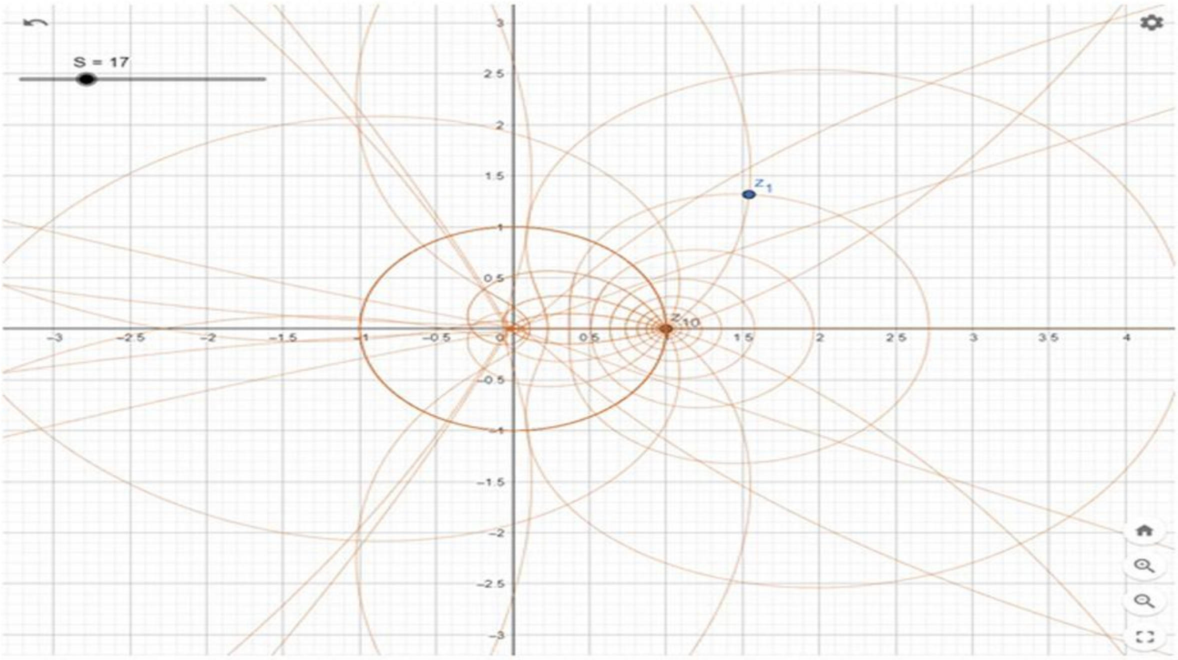


Figure 62. Z10 = 1 at S = 17 and S-1 = 16

II) for S= 2S-2 ; all odd numbers will have Z10 = {1, -1}

But if we used the new formula for the complex number for odd numbers

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$$\rightarrow 1 - 0i$$

$$z_{14} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2S-2}$$

$$\rightarrow 1 + 0i$$

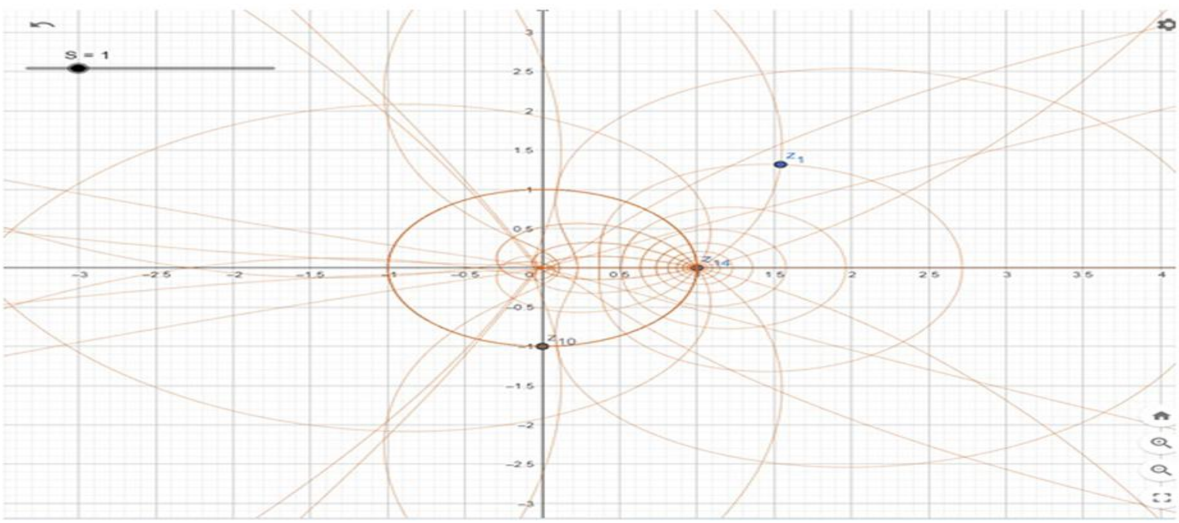


Figure 63. Z14 = 1 at S = 1 and 2S-2 = 0

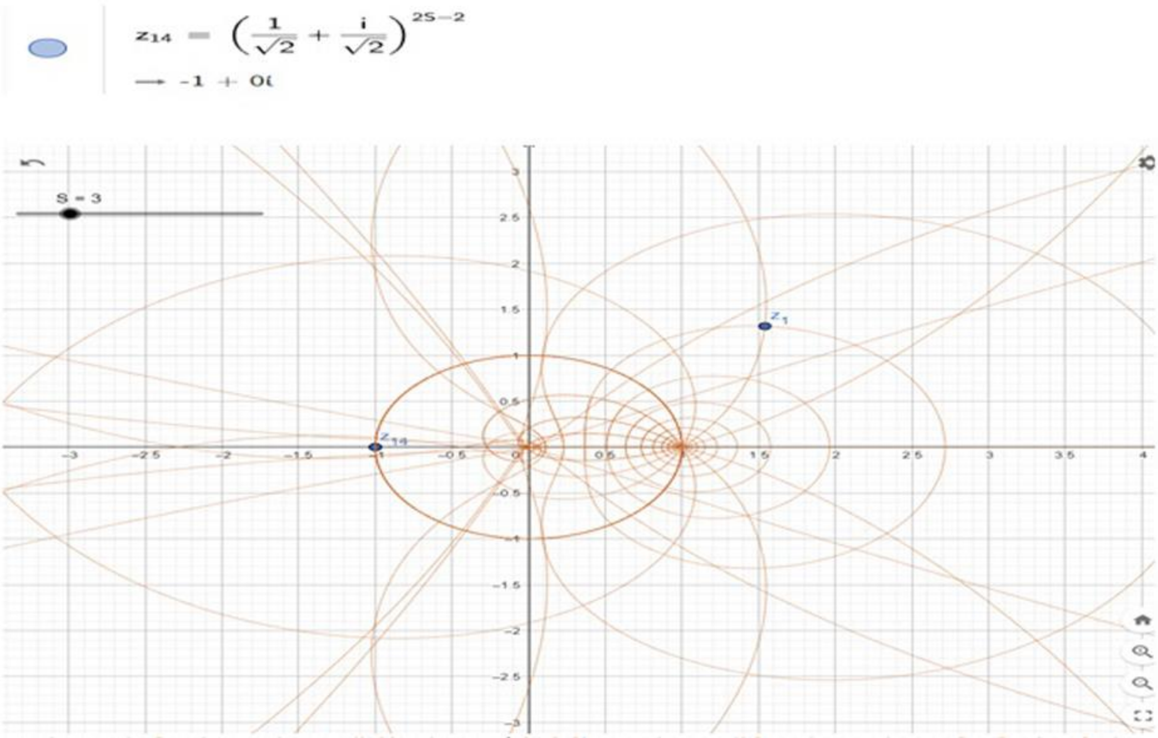


Figure 64. Z14 = -1 at S = 3 and 2S-2 = 4

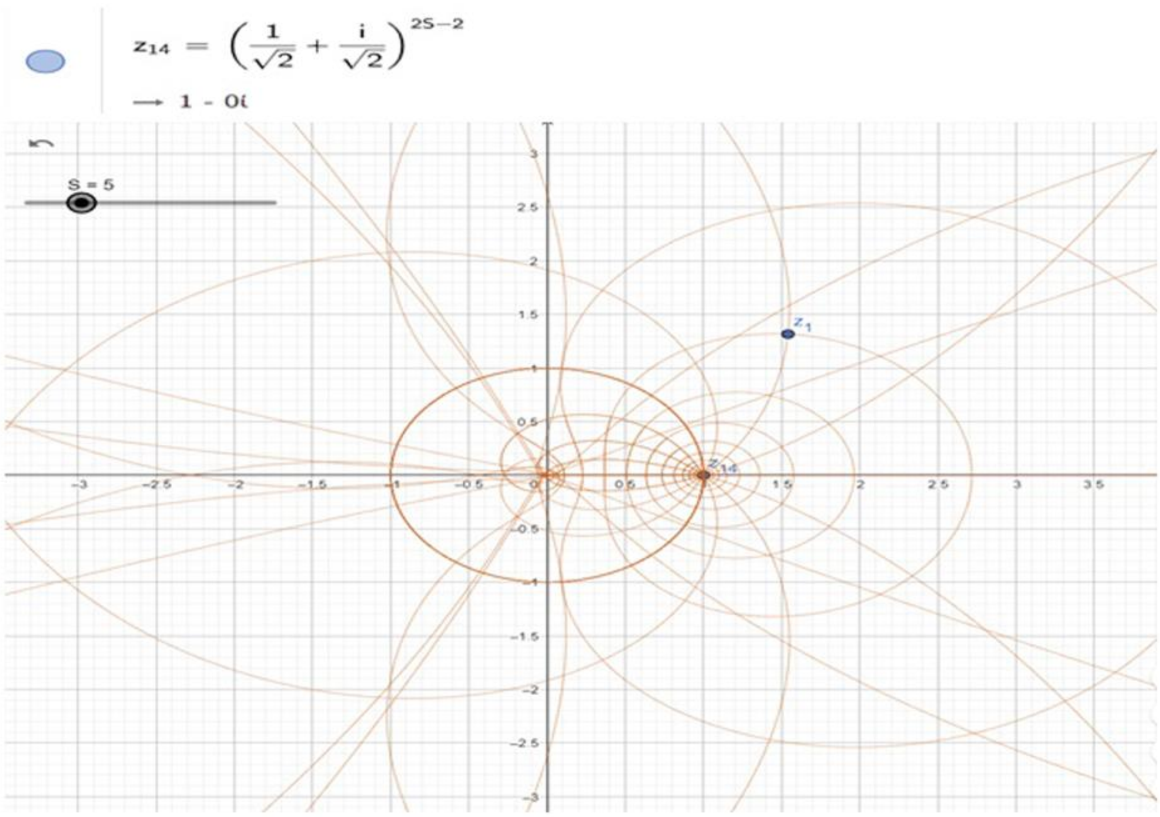


Figure 65. Z14 = 1 at S = 5 and 2S-2 = 8

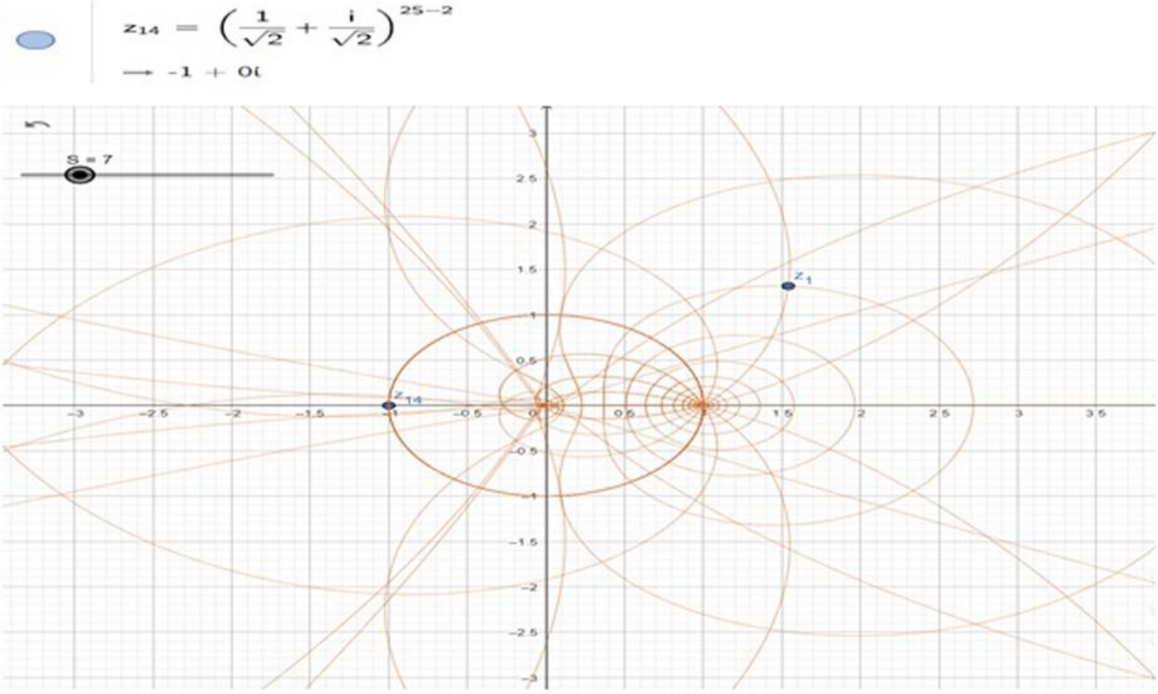


Figure 66. Z14 = -1 at S = 7 and 2S-2 = 12

Conclusion

In an introductory exploratory analysis for Sin and Cos wave characteristics in a complex plane, we showed that, if a geometric functions Sin or Cos are working on a polynomial as its inputs, we will get roots for Sin and Cos at these polynomial solutions. And even if both Sin waves operates in a different frequency but on the same polynomial, both will intersect at roots on the solution of the polynomial function. Also, the steps between roots (frequency) mainly depends on the partition used to partition pi on. In this paper on Sin function are working on polynomial function (X + 0.5) or (X -0.5) as inputs for Sin or Cos. Also, we were able to visualize the inverse of gematric Sin function for any X for a general polynomial (X +A) or (X-A). And based on this we introduced new Odd Identity unit function for complex plane. Using our new Identity unit function in complex plane, helped in explaining the distribution of odd numbers and even numbers in complex plane. Also using exponential function in combine with our Identity function helped in determine that S = 2S + 2 can be used. And how when we used this form of transformation in combine with our new Identity function get us all the odd numbers on values = {1, -1}. Then we can say that

$$f(x) = z^{x} = \left(\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}\right)^{x} ; where\ x = 2x + 2$$

Then at X Odd number

$$\pm 1 = \left(\pm\frac{1}{\sqrt{2}} \pm i\,\frac{1}{\sqrt{2}}\right)^{2X+2}$$

$$e^{i\left(\pm\frac{1}{\sqrt{2}}\pm i\frac{1}{\sqrt{2}}\right)^{2X+2}} = e^{-2^x\left(\pm\frac{1}{2}\pm i\frac{1}{2}\right)^{2X}}$$

$$e^{i\left(\pm\frac{1}{\sqrt{2}}\pm i\frac{1}{\sqrt{2}}\right)^{2X+2}} = e^{\pm i}$$

$$-i * 2^x\left(\pm\frac{1}{2}\pm i\,\frac{1}{2}\right)^{2X} = \left(\pm\frac{1}{\sqrt{2}}\pm i\,\frac{1}{\sqrt{2}}\right)^{2X-2}$$

Also, we showed that.

$$\zeta\left(S+\frac{1}{2}\right) = \begin{cases} 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Sin\left(\frac{\pi}{2}\left(S+\frac{1}{2}\right)\right)}{Sin\left(\frac{\pi}{4}\right)} * \Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \\ 2^S(\pi)^{S-\frac{1}{2}} * \dfrac{Cos\left(\frac{\pi}{2}\left(S-\frac{1}{2}\right)\right)}{Sin\left(\frac{\pi}{4}\right)}\Gamma\left(\frac{1}{2}-S\right)\zeta\left(\frac{1}{2}-S\right) \end{cases} \rightarrow EQ(D)$$

By This Equation EQ(D), Zeta function formula at S = S +0.5, the Sin wave will have a Root at S = -0.5, even negative Roots and Odd positive Roots. And at S = S -0.5, the Sin wave will have a Root at S=0.5, even positive roots and odd negative roots. (For each Natural number).

References

[1] Soltan, S. (2023). New Odd Numbers Identity and the None-trivial Zeros of Zeta Function | soltan | Journal of Mathematics Research | CCSE. Journal of Mathematics Research, Vol. 15, No. 2 (2023). DOI: 10.5539/jmr.v15n2p74.

[2] J. Cheeger and D. Ebin, Comparison theorems in Riemannian geometry.

[3] G. Marsaglia, On the randomness of pi and other decimal expansions, Interstat 5, 2005, http://www.yaroslavvb.com/papers/marsaglia-on.pdf.

[4] G. Pólya, Mathematics and Plausible Reasoning (vol. I, Induction and Analogy in Mathematics, and vol. II, Patterns of Plausible Inference, Princeton University Press, Princeton, 1954).

Copyrights