App:4 Solar Energy by Using Secant's Method

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1. Introduction

The search for new technologies is a never-ending journey. Scientists throughout the years have built up upon each other's work and developed alternatives for all kinds of devices. For instance, understanding solar power was a significant step in obtaining countless new machines and introducing new fields of study. Furthermore, our primary purpose of this paper is to get familiar with how solar energy is collected. Renewable energy uses a distinct collector with a field filled with mirrors that reflect the power into one spot. In our study, we explore the importance of the given variables in our application and learn to obtain an angle using the point where the power is concentrated.

The application's domain is addressed in subsection 2.2. Our application benefits physicists and engineers in calculations and solar power usage; it shows how energy is collected in high amounts and then converted. Then, we first introduce and define solar power in subsection 2.1.1, followed by subsection 2.1.2 on how the energy is processed, its advantages and disadvantages of the technology, and lastly, a brief look at collectors.

In subsection 2.2, we get a view of some methods and specify our chosen method, which is the secant method. Examining how the method is used, its algorithm, and give examples on its application in subsection 2.2.3. Finally, we apply the secant method to the problem and provide a precise description of it, in addition to calculating the solution of the angle on MATLAB using the given variables, which is all discussed in subsection 2.3.

2. Literature Review

This section will thoroughly present the problem's domain, explore some essential terms, and analyze the Numerical method used to solve it, including the algorithm. Furthermore, this section will cover some examples of our chosen method and its application to our problem in a gradual process. Lastly, we examine all the needed computations and the outcome.

2.1. The Domain

In this section, we demonstrate the domain of our problem, which is exclusively scientific and technological. Also, we explore some relevant terminologies along with the advantages and disadvantages of using solar energy. Finally, we give a brief look at renewable energy collectors and their connection to our application.

2.1.1 What is Solar Energy?

Solar energy is the energy radiating from the sun that is usually converted into thermal or electricity. Solar energy is one of the sufficient renewable energy sources available, and the United States of America owns part of the richest solar power resources in the whole world. (Seia, 2000) Solar technologies can provide this energy for various applications, including generating electricity, providing lighting or a comfortable indoor environment, and boiling water for home, industrial or commercial use. There are three ways to use or harness solar energy: photovoltaics, solar heating, and concentrating solar energy. Photovoltaics generate electricity directly from sunlight via a process and can be used in emergencies. Solar-powered applications (SHC) and concentrated solar power(CSP) applications are used on the heat generated by the sun during certain periods (Seia, 2000). It is crucial to understand the way solar power work to interpret our application which we will explain in the following subsection 2.1.2

2.1.2 How dose solar energy work?

Initially, a solar panel (also known as a solar module) consists of a layer of silicon cells, a metal frame, a glass sleeve, and wires to transmit electrical current from silicon. Silicon (atomic number 14 on the periodic table) is a material that allows it to absorb and convert the sun into usable electricity. When it triggers the electric current. (Robert, 2012)This is known as "photoelectric law," and it describes the general function of solar panel technology.

The PV process works through the following broad steps (Robert, 2012):

1. The solar cells absorb the silicon photovoltaic (PV) from the solar radiation.
2. When sunlight interacts with the silicon cell, the electrons begin to move, causing an electric current to flow.
3. The wires pick up and feed this direct current (DC) into a solar inverter to be converted into alternating current (AC).

Now that we took a brief outlook on how it works, it is worth mentioning that our problem is solely used scientifically. Mechanical engineers and physicists can benefit from this application to find the concentration of solar collectors at some angle. We get more into this in the application section.

2.1.3 Solar Energy Collectors

The basis of our problem and practical application is the calculation of solar energy concentration(C), which can lead to finding the rim Angle(A) at the specified concentration. First of all, we need to define what a solar collector is. Solar collector is a device intended to collect heat by absorbing sunlight (Figure 1). The solar collector aims to convert the thermal energy in sunlight or solar radiation into a more usable and storable form. **(Godfrey,2004).**

Figure 1**:** Solar energy collector

These collectors are commonly installed on the roof and must be very firm as they are exposed to many climate conditions **(Godfrey,2004).** These devices are primarily used for solar heating. The use of these solar collectors provides numerous alternatives for heating and electrical systems (**J.M.K.C. Donev,2018).**

These devices are primarily used for solar heating. The use of these solar collectors provides numerous alternatives for heating and electrical systems. (**J.M.K.C. Donev,2018)** There are many different solar collectors, but all of them have the same goal and purpose. In general, some material is used to collect and focus energy from the sun into a focal point which we then can find the concentration of and compute all the wanted values. How collectors work in our application is explained in the upcoming subsection 2.3.

2.1.4 Advantages & Disadvantage of Solar Energy

Advantages	Disadvantages
Renewable and Safe Energy Source Solar energy does not end as long as the sun exists, and it is an indispensable source. Solar energy is an environmentally safe source, and it is environmentally friendly energy that does not cause any form of air and air pollution.	**High Cost** Solar energy cost includes solar panels, inverter, batteries, wiring, and the installation Nevertheless
Reduces Electricity Bills Solar panels operate at no financial cost after completion, because sunlight is available at no cost, as it does not require any processing or mining operations once a solar panel is built, as the use of solar panels can reduce the cost of electricity bills.	**Weather-Dependent** The efficiency of the solar system drops during cloudy and rainy days. Solar panels are dependent on sunlight to effectively gather solar energy**.**
Diverse Applications Solar energy used in its direct form in many applications, for example drying agricultural crops, heating water and houses.	**Solar Energy Storage Is Expensive** Solar energy has use directly, or it can be stored in large batteries. These batteries, used in off-the-grid solar systems, can be charged during the day so that the energy is used at night.
Low Maintenance Costs Solar energy systems generally don't require a lot of maintenance. You only need to keep them relatively clean, so cleaning them a couple of times per year will do the job.	**Uses a Lot of Space** Solar panels require a lot of space to collect as much sunlight as possible, and some roofs are not big enough to fit the number of solar panels that you would like to have.

Table 1 :advantages and disadvantage of solar-energy (Aris Vourvoulias, 2020)

2.1.5 Summary

At the end of the first section, we now have an overview of solar energy, and how this solar energy works through the photovoltaic process in certain steps. It also pointed out some advantages and disadvantages of solar energy and explained what a solar collector is.

In this project, we aim to use the methods that we studied in applied mathematics to solve the problem of finding an angle where the energy is centered, using methods for finding the root of equations. In the next section we will explain in detail the method chosen for solving our solar energy problem.

2.2. The Secant Method

2.2.1. Numerical Analysis

Here we will talk broadly about the best way to apply for solar-energy collection by solve the equation for the geometrical concentration factor, which is the secant method. Let's take an overview at the basis of this method which is the numerical analysis.

Numerical analysis is the study of method for the numerical and approximate solution of mathematically posed problems (Linz, 2019). The method used is among many methods of solving problems of nonlinear equations, nonlinear equations methods including the following:

- The Bisection Method
- The Secant Method
- Newton's Method
- Muller's Method
- Fixed-Point Iteration Method

2.2.2. Linear Interpolation Method

Most functions can be approximated by a straight line over a small interval. There are a couple of nonlinear equations methods (iterative methods) are based on doing just that, are Newton’s method and the secant method (Levy, 2010).

2.2.3. The Secant Method

The simplest method for solving nonlinear equations is the Secant method. The secant method is using the secant line for solving a nonlinear equation is a known iterative process. An important feature of this method is that it does not use derivatives when applied, unlike Newton's Method (Hernández, 2002). The results in this method are for single equations f(x) = 0, where f is a real-valued function of one real variable. the secant method will is locally super linearly convergent if the standard assumptions hold (Kelley, 1995).

In order to start, one needs two guesses n and n-1. Let n+1 be the current iterate and n the previous iterate. The secant method is thus:

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right]$$

we see in Figure 2 the secant line through these two points and find where it intersects the x-axis. The two points may both be on one side of the root as, but they could also be on opposite sides.

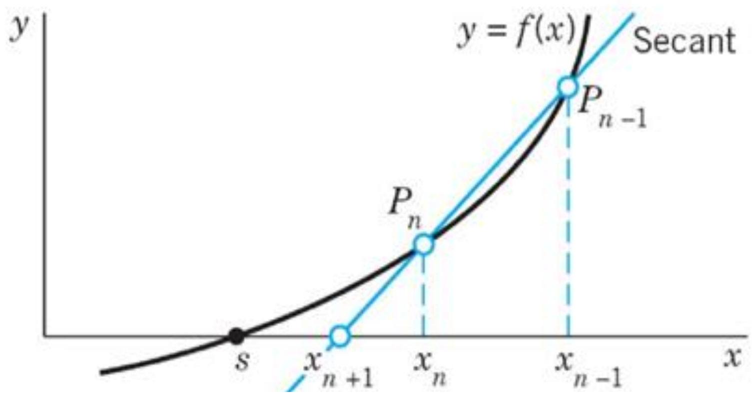

Figure 2: Graph of secant method

2.2.4Algorithm:

The secant's method is based on two linear approximation points of the function using the secant line. as Figure 3.

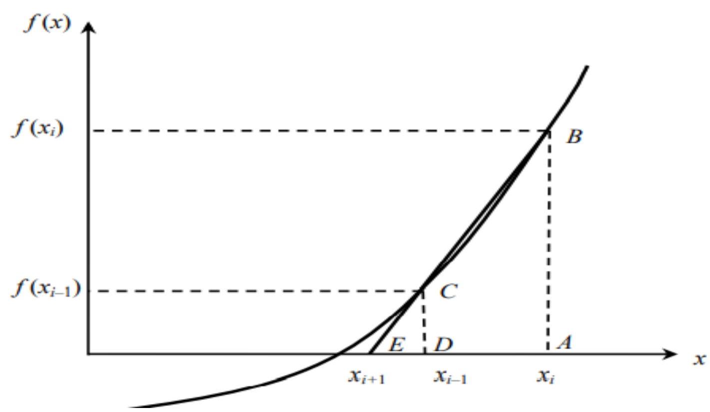

Figure 3:Graph of how secant's method works according to its secant line

This method’s algorithm consists of three short steps, first step is to use the given two initial guesses, xi-1, and xi, to calculate the estimate of the next value's root, xi+1. As shown in Figure 4 discuss the equation used to approximate the root of the next value. To get the new root substitute the two initial guesses into the function, f(xi-1) and f(xi). Then use the equation shown in Figure 4 to get the root of the next value by subtracting xi-1 from xi and multiply it with f(xi), then divide the result value with the result of subtraction f(xi-1) from f(xi). (Egwu Eric Kaw & Kalu, Jan 2010)

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Figure 4:Description of the equation used to calculate the root of the next value

The second step of the secant's method is to Find the absolute relative approximate error. It is the way to calculate the error size of the result (Gerald & Wheatley, 2003) by using this expression shown in Figure 5.

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right|$$

Figure 5:Description of the equation used to find the absolute

The third step is to Compare the absolute relative approximate error with the pre-specified error tolerance. That is by checking if the new absolute relative approximate error value is larger than the pre-specified error tolerance, then back to step one. But if the new absolute relative approximate error value is less than the pre-specified error tolerance, then stop the algorithm (Gerald & Wheatley, 2003). Note that you cannot do this step after the first iteration because you do not have a previous absolute relative approximate error.

2.2.5 Advantages and Disadvantages of Secant's Method:

- **Advantages**:

  - Secant's method does not need to evaluate the derivative of the function as newton's method does, which is more practical for many functions. With the availability of implementing the equation on MATLAB, it will become more convenient (Egwu Eric Kaw & Kalu, Jan 2010).

  - Requires two guesses that do not need to bracket the root (Gerald & Wheatley, 2003).

  - When secant method converges, it will typically converge faster than a liner rate (Egwu Eric Kaw & Kalu, Jan 2010).

  - It requires only one function evaluation per iteration as compared to Newton's method which requires two.

- **Disadvantages**:

  - " The secant method is an open method and may or may not converge." (Egwu Eric Kaw & Kalu, Jan 2010).

  - Secant's methods need to two initial guesses not like the newton's method (Gerald & Wheatley, 2003).

  - Newton's method generalizes more easily to new methods for solving simultaneous systems of nonlinear equations.

2.2.6 Examples:

To explain the secant method more clearly, here are some examples:
Use secant's method to **determine** $e^{-x} - x = 0$ if x_0= 0 and x_1= 1. Find the *absolute relative approximate error of 0.5*% or less.

<u>Iteration 1:</u>

The estimate of the root:

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= 1 - \frac{f(e^{-1} - 1)(1 - 0)}{f(e^{-1} - 1) - f(e^{-0} - 0)}$$

$$x_2 = 0.61270$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_2 - x_1}{x_2}\right| \times 10$$

$$= \left|\frac{0.61270 - 1}{0.61270}\right| \times 100$$

$$= 63.21201$$

Iteration 2:

The estimate of the root: $x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$

$$= 0.61270 - \frac{f(e^{-0.61270} - 0.61270)(0.61270 - 1)}{f(e^{-0.61270} - 0.61270) - f(e^{-1} - 1)}$$

$$x_3 = 0.56384$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_3 - x_2}{x_3}\right| \times 10$$

$$= \left|\frac{0.56384 - 0.61270}{0.56384}\right| \times 100$$

$$= 8.66557$$

Iteration 3:

The estimate of the root:

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$$= 0.56384 - \frac{f(e^{-0.56384} - 0.56384)(0.56384 - 0.61270)}{f(e^{-0.56384} - 0.56384) - f(e^{-0.61270} - 0.61270)}$$

$$x_4 = 0.56717$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_4 - x_3}{x_4}\right| \times 10$$

$$= \left|\frac{0.56717 - 0.56384}{0.56717}\right| \times 100$$

$$= 0.58712$$

<u>Iteration 4:</u>

The estimate of the root: $x_5 = x_4 - \frac{f(x_4)(x_4 - x_3)}{f(x_4) - f(x_3)}$

$$= 0.56717 - \frac{f(e^{-0.56717} - 0.56717)(0.56717 - 0.56384)}{f(e^{-0.56717} - 0.56717) - f(e^{-0.56384} - 0.56384)}$$

$$x_3 = 0.567143$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_3 - x_2}{x_3}\right| \times 10$$

$$= \left|\frac{0.567143 - 0.56717}{0.567143}\right| \times 100$$

$$=0.00476$$

The absolute relative approximate error is less than 0.5% then stop the algorithm.

(Faires, 2005)

Another example:

Use secant's method to determine $x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$

assuming that $x_{-1} = 0.02$ and $x_0 = 0.05$

Find the *absolute relative approximate error of 0.5*% or less.

Iteration 1:

The estimate of the root:

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$= x_0 - \frac{\left(x_0^3 - 0.165x_0^2 + 3.993\times10^{-4}\right)\times(x_0 - x_{-1})}{\left(x_0^3 - 0.165x_0^2 + 3.993\times10^{-4}\right) - \left(x_{-1}^3 - 0.165x_{-1}^2 + 3.993\times10^{-4}\right)}$$

$$= 0.05 - \frac{\left[0.05^3 - 0.165(0.05)^2 + 3.993\times10^{-4}\right]\times[0.05 - 0.02]}{\left[0.05^3 - 0.165(0.05)^2 + 3.993\times10^{-4}\right] - \left[0.02^3 - 0.165(0.02)^2 + 3.993\times10^{-4}\right]}$$

$$= 0.06461$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_1 - x_0}{x_1}\right|\times100$$

$$= \left|\frac{0.06461 - 0.05}{0.06461}\right|\times100$$

$$= 22.62\%$$

Iteration 2:

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= x_1 - \frac{\left(x_1^3 - 0.165x_1^2 + 3.993\times10^{-4}\right)\times(x_1 - x_0)}{\left(x_1^3 - 0.165x_1^2 + 3.993\times10^{-4}\right) - \left(x_0^3 - 0.165x_0^2 + 3.993\times10^{-4}\right)}$$

$$= 0.06461 - \frac{\left[0.06461^3 - 0.165(0.06461)^2 + 3.993\times10^{-4}\right]\times(0.06461 - 0.05)}{\left[0.06461^3 - 0.165(0.06461)^2 + 3.993\times10^{-4}\right] - \left[0.05^3 - 0.165(0.05)^2 + 3.993\times10^{-4}\right]}$$

$$= 0.06241$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_2 - x_1}{x_2}\right|\times100$$

$$= \left|\frac{0.06241 - 0.06461}{0.06241}\right|\times100$$

$$= 3.525\%$$

Iteration 3:

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= x_2 - \frac{\left(x_2^3 - 0.165x_2^2 + 3.993\times10^{-4}\right)\times(x_2 - x_1)}{\left(x_2^3 - 0.165x_2^2 + 3.993\times10^{-4}\right) - \left(x_1^3 - 0.165x_1^2 + 3.993\times10^{-4}\right)}$$

$$= 0.06241 - \frac{\left[0.06241^3 - 0.165(0.06241)^2 + 3.993\times10^{-4}\right]\times(0.06241 - 0.06461)}{\left[0.06241^3 - 0.165(0.06241)^2 + 3.993\times10^{-4}\right] - \left[0.06461^3 - 0.165(0.06461)^2 + 3.993\times10^{-4}\right]}$$

$$= 0.06238$$

The absolute relative approximate error is:

$$|\in_a| = \left|\frac{x_3 - x_2}{x_3}\right|\times100$$

$$= \left|\frac{0.06238 - 0.06241}{0.06238}\right|\times100$$

$$= 0.0595\%$$

The absolute relative approximate error is less than 0.5% then stop the algorithm.

(Egwu Eric Kaw & Kalu, Jan 2010).

Iteration Number, i	x_{i-1}	x_i	x_{i+1}	$\|\in_a\|\%$	$f(x_{i+1})$
1	0.02	0.05	0.06461	22.62	-1.9812×10^{-5}
2	0.05	0.06461	0.06241	3.525	-3.2852×10^{-7}
3	0.06461	0.06241	0.06238	0.0595	2.0252×10^{-9}

Table 2:Secant method results as a function of iterations

2.3 Problem description:

Our problem focuses on the concentration (C) and the rim angle(A) of a solar-energy collector. These two highly affect the application and the output. In our application, they are multiple parameters related to nanotechnology and physics, such as the solar collector's Diameter(D), the height(H), and lastly, the fractional coverage of the field(F). By changing the input parameters, we end up with different outcomes until the user gets to the desired solution. We are looking for an instance where the rim angle (A) is 0. Ultimately, we get the root of the function benefiting from our numerical analysis study; we decided to use the secant method as our method of choice.

2.3.1 Application:

Following the introductory points above, this section will dive more into some details of combining our application with the method of choice. We will illustrate the secant method's use and its application on solar-energy collectors given the prespecified parameters to help. Moreover, we will get to a rim angle(A) solution step-by-step in the next subsection.

Applying the method to the application:

As previously discussed, to apply our method, we need to explore the definition of both the concentration(C) of a solar collector and the rim angle(A) that we are looking to solve.

Our two main parameters are obtained when a parabolic-shaped collector with a surface of mirrors reflects the sun's rays into a focal point (Supa, 2014). This small area will contain a high concentration of energy that is later on converted to electricity. An angle is created between the light rays and reflected rays into the focal region; this angle is called the rim angle(A) (Supa, 2014).

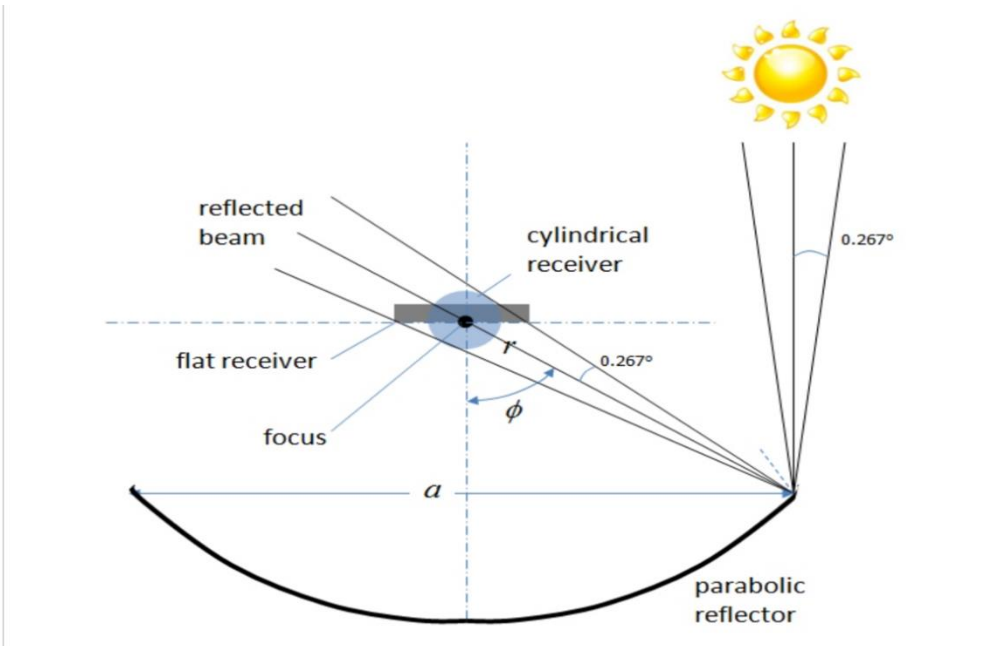

Figure 6:shows the parabolic collector's focus point

As mentioned in our problem description, the rim angle is determined by applying physics as well as other technologies. We will obtain it using the secant iterative method. First, we need to find our two initial guesses, which both can be approximated using our problem's plot (look figure 6). Moreover, after letting the user choose the range, we now use the given variables to get the root.

first and for most, to solve the application using the secant method, we have to rearrange our equation

$$C = \frac{\pi(\frac{h}{\cos \mathrm{A}})^2 \mathrm{F}}{0.5\pi D^2 \,(1 + sin\mathrm{A} - 0.5 \cos A)}$$

Given that: C = 1200, h= 300, F = 0.8, and D=14

In this case, simplifying the expression more by plugging the given values could help to understand the problem more and derive a solution.

$$C = \frac{\pi \left(\frac{h}{\cos \mathrm{A}}\right)^2 (F)}{0.5\pi(D)^2 \,(1 + sin\mathrm{A} - 0.5 \cos A)}$$

$$1200 = \frac{\pi \left(\frac{300}{\cos \mathrm{A}}\right)^2 (0.8)}{0.5\pi(14)^2 \,(1 + sin\mathrm{A} - 0.5 \cos A)}$$

$$1200 = \frac{72000/(cosA)^2}{98(1 + sin\mathrm{A} - 0.5 \cos A)}$$

$$\frac{1200 \times 98}{72000} = \frac{1}{(1 + sin\mathrm{A} - 0.5 \cos A)(cosA)^2}$$

$$\frac{49}{30} = \frac{1}{(1 + sin\mathrm{A} - 0.5\cos A)(cosA)^2}$$

$$\frac{(49)}{(30)}(1 + \sin(A) - 0.5\cos(A))(\cos(A)))^2 - 1 = 0$$

We conclude that the final expression for our function is

$$\frac{(0.5D^2\,C)}{(h^2\,F)}(1 + \sin(A) - 0.5\cos(A))(\cos(A)))^2 - 1 = 0$$

This expression now is in the form of finding the root.

2.3.2 Solution

$$\frac{(0.5D^2\,C)}{(h^2\,F)}(1 + \sin(A) - 0.5\cos(A))(\cos(A)))^2 - 1 = 0$$

Given that: C = 1200, h= 300, F = 0.8, and D=14

$$\left(\frac{49}{30}\right)(1 + \sin(A) - 0.5\cos(A))(\cos(A))^2 - 1 = 0$$

We will use the secant method to solve the above equation to find roots f(x)=0, secant method requires two initial guesses x_{-1} and $x_0$the optimal guess should lay between 0 and $\frac{\pi}{2}$, let us assume the initial guesses of the root as x_{-1}= 0.7 and x_0=0.9

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}, \qquad n = 1,2,3, \dots \dots$$

Iteration number **1**:

x_{-1}= 0.7 and x_0=0.9

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}, \qquad x_1 = 0.3 - \frac{f(0.9)(0.9 - 0.7)}{f(0.9) - f(0.7)}$$

$$x_1 = 0.9 - \frac{\left[\left(\left(\frac{49}{30}\right)(1 + \sin(0.9) - 0.5\cos(0.9)\right)(\cos(0.9))^2 - 1\right](0.9 - 0.7)}{\left[\left(\frac{49}{30}\right)(1 + \sin(0.9) - 0.5\cos(0.9))(\cos(0.9))^2 - 1\right] - \left[\left(\frac{49}{30}\right)(1 + \sin(0.7) - 0.5\cos(0.7))(\cos(0.7))^2 - 1\right]}$$

$$x_1 = 0.3 - \frac{(-0.07066)(0.2)}{(-0.06066) - (0.20561)}$$

$$\boldsymbol{x_1 = 0.84885}$$

The absolute relative approximate error $|\in_a|$ at the end of iteration 1:

$|\in_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100$

$$|\in_a| = \left|\frac{0.84885 - 0.9}{0.84885}\right| \times 100, \; |\in_a| = \mathbf{6.03}\%$$

Iteration number **2**:

x_0= 0.9 and x_1=0.84885

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}\,, \qquad x_2 = 0.13543 - \frac{f(0.84885)(0.84885 - 0.9)}{f(0.84885) - f(0.9)}$$

$$x_2 = 0.84885 - \frac{\left[\left(\left(\frac{49}{30}\right)(1 + \sin(0.84885) - 0.5\cos(00.84885)\right)(\cos(0.84885))^2 - 1\right](0.84885 - 0.3)}{\left[\left(\frac{49}{30}\right)(1 + \sin(0.84885) - 0.5\cos(0.84885))(\cos(0.84885))^2 - 1\right] - \left[\left(\frac{49}{30}\right)(1 + \sin(0.9) - 0.5\cos(0.9))(\cos(0.9))^2 - 1\right]}$$

$$x_2 = 0.84885 - \frac{(0.01297)(-0.05115)}{(0.01297) - (-0.07066)}$$

$$\boldsymbol{x_2 = 0.85678}$$

The absolute relative approximate error $|\in_a|$ at the end of iteration **2**:

$|\in_a| = \left|\frac{x_{2-}\, x_1}{x_2}\right| \times 100$

$$|\in_a| = \left|\frac{0.85678 \quad - 0.84885}{0.85678}\right| \times 100, \;\; |\in_a| = \mathbf{0.93}\%$$

Iteration number **3**:

x_1= 0.84885 and x_2=0.85678

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}\,, \qquad x_3 = 0.85678 - \frac{f(0.85678)(0.85678 - 0.84885)}{f(0.85678) - f(0.84885)}$$

$$x_3 = 0.85678 - \frac{\left[\left(\left(\frac{49}{30}\right)(1+\sin(0.85678) - 0.5\cos(0.85678))(\cos(0.85678))^2 - 1\right](0.85678 - 0.84885)}{\left[\left(\frac{49}{30}\right)(1+\sin(0.85678) - 0.5\cos(0.85678))(\cos(0.85678))^2 - 1\right] - \left[\left(\frac{49}{30}\right)(1+\sin(0.84885) - 0.5\cos(0.84885))(\cos(0.84885))^2 - 1\right]}$$

$$x_3 = 0.85678 - \frac{(0.00048)(0.00793)}{(0.00048) - (0.01297)}$$

$$\boldsymbol{x_3 = 0.85709}$$

The absolute relative approximate error $|\in_a|$ at the end of iteration 3:

$$|\in_a| = \left|\frac{x_{2-}x_1}{x_2}\right| \times 100$$

$$|\in_a| = \left|\frac{0.85709 - 0.11363}{0.85709}\right| \times 100, \quad |\in_a| = \mathbf{0.04}\%$$

Iteration number 4:

x_2= 0.85678 and x_3= 0.85709

$$x_4 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(1)}, \qquad x_4 = 0.85709 - \frac{f(0.85709)(0.85709 - 0.85678)}{f(0.85709) - f(0.85678)}$$

$$x_4 = 0.85709 - \frac{\left[\left(\left(\frac{49}{30}\right)(1+\sin(0.85709) - 0.5\cos(0.85709))(\cos(0.85709))\right)^2 - 1\right](0.85709 - 0.85678)}{\left[\left(\frac{49}{30}\right)(1+\sin(0.85709) - 0.5\cos(0.85709))(\cos(0.85709))^2 - 1\right] - \left[\left(\frac{49}{30}\right)(1+\sin(0.85678) - 0.5\cos(0.85678))(\cos(0.85678))^2 - 1\right]}$$

$$x_2 = 0.85709 - \frac{(-0.000007)(0.0003)}{(-0.000007) - (0.00048)}$$

$$x_2 = \mathbf{0.85709}$$

The absolute relative approximate error $|\in_a|$ at the end of iteration **4**:

$|\in_a| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100$

$$|\in_a| = \left|\frac{0.85709 - 0.85709}{0.85709}\right| \times 100, \quad |\in_a| = \mathbf{0.00}\%$$

The root is 0.85709

we will solve the equation for the root, to make sure that is the optimal rim angle for the given collector.

$$1200 = \frac{\pi\left(\frac{300}{\cos \mathrm{A}}\right)^2 (0.8)}{0.5\pi(14)^2\,(1 + sin\mathrm{A} - 0.5\cos A)}$$

$$C = \frac{\pi\left(\frac{300}{\cos(0.85709)}\right)^2 (0.8)}{0.5\pi(14)^2\,(1+\sin(0.85709) - 0.5\cos(0.85709))}, \qquad C \approx 1200.0$$

2.3.3 Solution using MATLAB:

In this section, we will find the optimal angle of the collector by using code on MATLAB. This program designed to find the Rim Angle of the solar energy collector and let the user prespecified the other parameters.

Input :

Geometrical concentration (C)

Fractional coverage (F)

Collector Diameter (D)

Collector Hight (H)

The two guesses x0, x1

Accuracy e

Output :

A root of the function by given inputs

Assumptions: The inputs in the right range will produce the correct angle, otherwise will produce an error message if the root does not found in 20 iteration

Steps:

If $|f(x_0)| < |f(x_1)|$ $then\ swap\ x_0\ and\ x_1\ , to\ avoid\ nigative\ \ numbers$

Repeat

$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$

$x_0 = x_1$

$x_2 = x_0$

Until $|\boldsymbol{f}(\boldsymbol{x_2})| < \boldsymbol{e}\ (\boldsymbol{Accuracy})$

Figure 7:Overview: inputs, outputs, assumptions, and steps for the MATLAB code.

```
function  rimAngle = rimAngleWithSecant()


%This program is designed and used to find the Rim Angle(A) of a Solar-Energy
collector with some prespecified parameters.

% Using Secant Method

% Concerntration factor (C)= 1200

% Fractional Coverage (F)=0.8

% Diameter of the collector's field (D)= 14

% Height of the collector(H)= 300


fprintf('----------welcome to rim angle calculator using secant method-------
----------- \n\n');


% We decided to make our program user-friendly + interactive

%let the user enter the required arguments to calculate the Rim Angle(A)


fprintf(' ------------------- Entering required input ------------------
\n');

concentration_factor = input('Please enter the geometrical concentration
factor (C): '); %the needed concentration_factor

fractional_converage = input('Please enter the fractional converge (F): ');

diameter = input('Please enter the diameter of collector (D): ');

height = input('Please enter the height of collector (H): ');

%let the user choose the two guesses

fprintf('the two guesses prefer to be in the range [0, pi/2]\n');

x1 = input('Please enter the first guess x0: ');

x2 = input('Please enter the second guess x1: ');

%let the user enter the accuracy

e= input("Enter the accuracy: ");


% calculate the equation at x0, x1

fx1=((0.5*diameter^2*concentration_factor)/(height^2*fractional_converage))*(
1+sin(x1)-0.5*cos(x1))*(cos(x1))^2-1;
```

```
fx2=((0.5*diameter^2*concentration_factor)/(height^2*fractional_converage))*(
1+sin(x2)-0.5*cos(x2))*(cos(x2))^2-1;

% if fx1< fx2 swap the values , to avoid negative number

if abs(fx1) < abs (fx2)

    temp = x1; x1 = x2; x2 = temp;

    temp = fx1; fx1 = fx2; fx2 = temp;

end


%check if solution found by error tolerance

%check if one of the guesses is the root

if abs(fx1)<e

    rimAngle=x1;

    fprintf('\n\n------------------The root is: %f----------------------
\n',x1);

    return;%solution is found

elseif abs(fx2)<e

    rimAngle=x2;

    fprintf('\n\n------------------The root is: %f----------------------
\n',x2);

    return;%solution is found

end



%start calculation the root

fprintf('--------------------------------------------------------------------
---------\n');

fprintf('                      START FINDING THE ROOT\n');

fprintf('--------------------------------------------------------------------
---------\n');

fprintf('%-21s%-20s%-15s%-15s\n','iteration :','A','f(A)','relative error
|,àà a|% ');

% variable to count the iteration

iteration_number=0;

%put the max iteration as 20
```

```
MAX_iteration =20;

while iteration_number<MAX_iteration
    iteration_number=iteration_number+1;
    %calculate new x
    x=x2-fx2*(x2-x1)/(fx2-fx1);
    % find f(x)
fx=((0.5*diameter^2*concentration_factor)/(height^2*fractional_converage))*(1
+sin(x)-0.5*cos(x))*(cos(x))^2-1;
    fprintf('\t%2d%18.5f\t\t%8.5f\t\t%8.2f %%\n',iteration_number,x,fx,
abs((x-x2)/x)*100);
    % check if its less than error tolerance
    if (abs(fx)<e) || abs((x-x2)/x)< e
         %if yes , print it's found and return
         fprintf('\n---------------------------------------------------------
\n');
         fprintf('\t\tthe root is: %.5f\n',x);
         fprintf('---------------------------------------------------------
\n');
         fprintf('A graph will appear to illustrate C(A) ,thank you for using
the program.');
         % plot functio by given inputs in the range [0 pi/2]
         t =[0 : .00001 :pi/2];
         f =(49/30).*(1+sin(t)-0.5*cos(t)).*(cos(t)).^2-1;
         plot(t,f,'-k','LineWidth',2);
         hold on
         %plot a point to indicate the root
         plot(x,fx,'ro','LineWidth',4)
         hold off
         grid
         title('the Zero of the given function');
         xlabel('Rim Angle');
         ylabel ('f(A)');
```

```
         rimAngle=x;
         return; %the solution is founded
    else %otherwies,update the values of x1,x2
        x1 = x2;
        x2 = x;

fx1=((0.5*diameter^2*concentration_factor)/(height^2*fractional_converage))*(
1+sin(x1)-0.5*cos(x1))*(cos(x1))^2-1;

fx2=((0.5*diameter^2*concentration_factor)/(height^2*fractional_converage))*(
1+sin(x2)-0.5*cos(x2))*(cos(x2))^2-1;
    end


end
% if the root does not found after 20 iteration an error message appear
warning('secant: maximum number of iterations exceeded.');
x= []; MAX_iteration = 'No zeros in given interval';
rimAngle =x;
end
```

Script Implementation *See rimAngleWithSecant.m*

```
----------welcome to rim angle calculator using secant method------------------

 ------------------- Entering required input ------------------
Please enter the geometrical concentration factor (C): 1200
Please enter the fractional converge (F): 0.8
Please enter the diameter of collector (D): 14
Please enter the height of collector (H): 300
the two guesses prefer to be in the range [0, pi/2]
Please enter the first guess x0: 0.7
Please enter the second guess x1: 0.9
Enter the accuracy: 0.000003
--------------------------------------------------------------------------
                START FINDING THE ROOT
--------------------------------------------------------------------------
iteration :      A            f(A)        relative error |∈a|%
       1       0.84885        0.01297             6.03 %
       2       0.85678        0.00049             0.93 %
       3       0.85709       -0.00000             0.04 %
       4       0.85709        0.00000             0.00 %

-----------------------------------------------------
                 the root is: 0.85709
-----------------------------------------------------
A graph will appear to illustrate C(A), thank you for using the program.
```

Figure 8:Implementation of the code for finding the root of rim angle equation

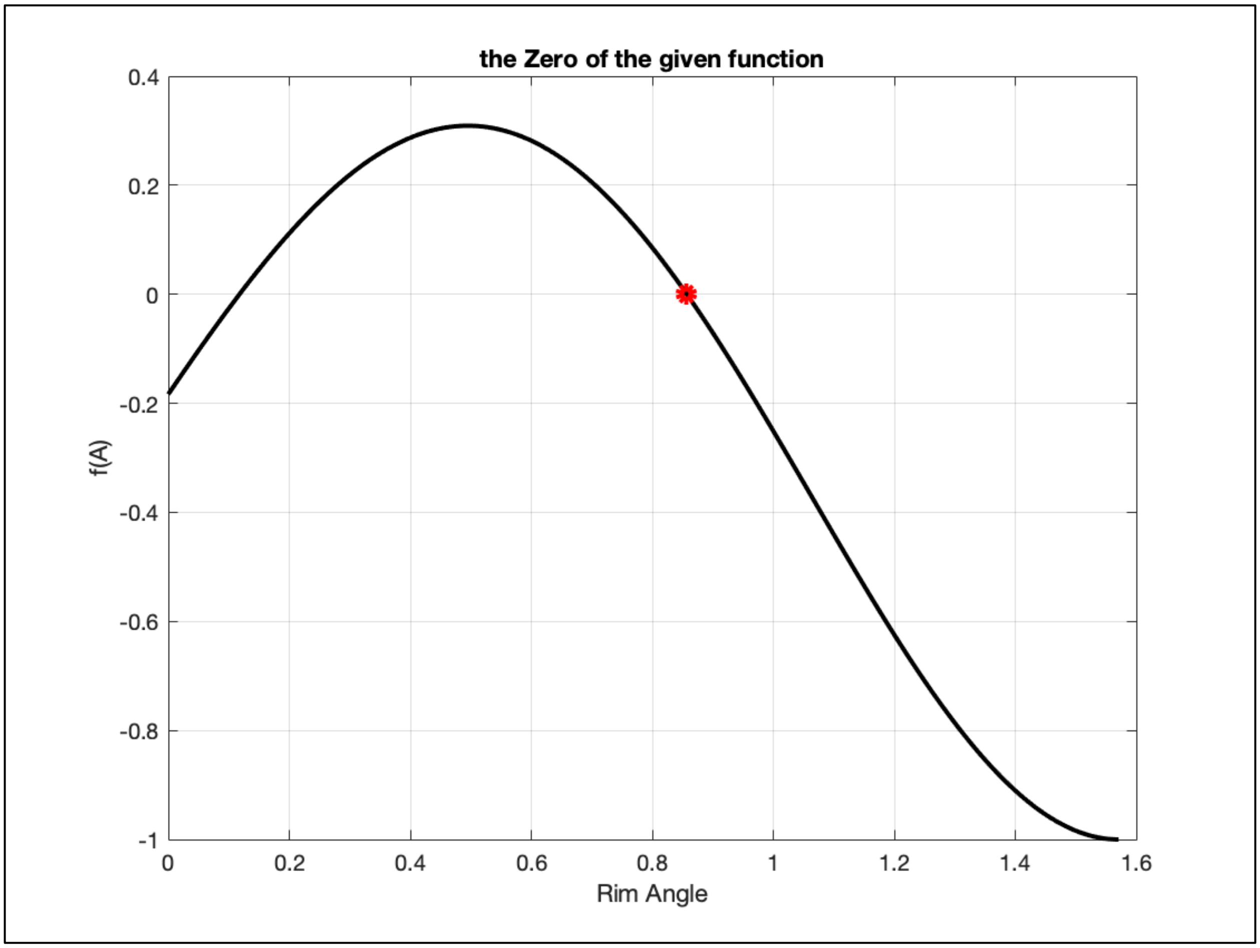

Figure 9:Graph for the rim angle (A) in the range 0 and π/2

Note: The red point in the graph represents the root of the equation C(A)=0, which is the rim angle of the given inputs

We observe from our solution that depending on the given inputs we will get an angle where C(A)=0, if we plug back the value of the rim angle (A) in our problem it is guaranteed to get the input C. Furthermore, the secant's method does not converge every time but in the case of our application it gave a solution to whatever value of input we chose this indicated that it is most suitable for the problem.

3. Conclusion

To conclude, through the application given to us, we presented this paper on collecting solar energy and our analysis of the equation and demonstrating the crucial variables and angle. We began by defining solar energy, how the power is processed, its benefits and drawbacks, and gave a brief introduction to collectors. We looked for the best method to solve the problem accurately, and we found that the secant method is the most appropriate. Furthermore, we explored the basic terms, the definition of the method, algorithm, and examples. Similarly, the problem's description was explained extensively, and we tested different values to ensure convergence. Lastly, after implementing the equation in MATLAB, the result came to 0.85709 for the given inputs, and depending on the inputs provided by the user, the outcome should be accurate.

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