CSE 394 Circuits and Signals
Classification of Systems
Convolution Sum

By

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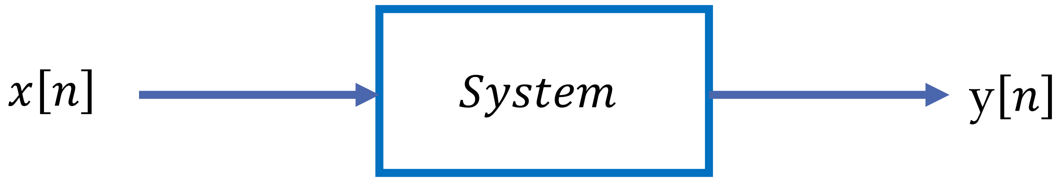
System

A device or an algorithm, that according to some rules, operate on a signal called the input signal to produce another signal as output signal or response



A discrete time system is represented mathematically as an input output relationship or as a difference equation.

Example

$$y[n] = n.x[n]$$

$$y[n] = 2.x[n] - 3x[n-1] - 5y[n-1] + 4y[n-2]$$

Linearity of System

A system is linear if the system satisfies the principle of homogeneity and the principle of superposition

Principle of Homogeneity

If

$$x[n] \longrightarrow \boxed{\;System\;} \longrightarrow y[n]$$

Then

$$c.x[n] \longrightarrow \boxed{\;System\;} \longrightarrow c.y[n]$$

Linearity of System

Principle of Superposition

If

$$x_1[n] \longrightarrow \boxed{System} \longrightarrow y_1[n] \quad \& \quad x_2[n] \longrightarrow \boxed{System} \longrightarrow y_2[n]$$

Then

$$x_1[n] + x_2[n] \longrightarrow \boxed{System} \longrightarrow y_1[n] + y_2[n]$$

Linearity of System

Combining both principles

If

$$x_1[n] \longrightarrow \boxed{System} \longrightarrow y_1[n] \quad \& \quad x_2[n] \longrightarrow \boxed{System} \longrightarrow y_2[n]$$

Then

$$x_{ws}[n] = c_1 . x_1\ [n] + c_2 . x_2[n] \longrightarrow \boxed{System} \longrightarrow y_{ws}[n] = c_1 y_1[n] + c_2 y_2[n]$$

Linearity

Example: Check whether the system

$$\boldsymbol{y[n] = n.x[n]}$$

is linear or not

Answer:

Response of the system to an input $x_1[n]$

$$y_1[n] = n.x_1[n]$$

Response of the system to an input $x_2[n]$

$$y_2[n] = n.x_2[n]$$

Response of the system to the weighted sum of inputs $x_{ws}[n] = c_1x_1[n] + c_2x_2[n]$

$$y_w[n] = n.x_{ws}[n]$$

$$y_w[n] = n.\left[c_1x_1[n] + c_2x_2[n]\right]$$

$$y_w[n] = n.c_1x_1[n] + n.c_2x_2[n]$$

Weighted Sum of outputs

$$y_{ws}[n] = c_1y_1[n] + c_2y_2[n]$$

$$y_{ws}[n] = c_1n.x_1[n] + c_2n.x_2[n]$$

Here we can see that the response of the system to a weighted sum of input is equal to the weighted sum of outputs. Hence the system is LINEAR

Linearity

Example: Check whether the system

$$\boldsymbol{y[n] = e^{x[n]}}$$

is linear or not

Answer:

Response of the system to an input $x_1[n]$

$$y_1[n] = e^{x_1[n]}$$

Response of the system to an input $x_2[n]$

$$y_2[n] = e^{x_2[n]}$$

Response of the system to the weighted sum of inputs $x_{ws}[n] = c_1x_1[n] + c_2x_2[n]$

$$y_w[n] = e^{x_{ws}[n]}$$

$$y_w[n] = e^{c_1x_1[n]+c_2x_2[n]}$$

$$y_w[n] = e^{c_1x_1[n]}e^{c_2x_2[n]}$$

Weighted Sum of outputs

$$y_{ws}[n] = c_1y_1[n] + c_2y_2[n]$$

$$y_{ws}[n] = c_1e^{x_1[n]} + c_2e^{x_2[n]}$$

Here we can see that the response of the system to a weighted sum of input is NOT equal to the weighted sum of outputs. Hence the system is NONLINEAR

Time Invariance/ Shift Invariance

A system is said to be time invariant or shift invariant if the input output relationship of the system does not change with respect to time.

If

$$x[n] \longrightarrow \boxed{System} \longrightarrow y[n]$$

Then

$$x[n-n_0] \longrightarrow \boxed{System} \longrightarrow y[n-n_0]$$

Shift Invariance

Example: Check whether the system

$$\boldsymbol{y[n] = n.x[n]}$$

is shift invariant or not

Answer:

Response of the system to a delayed input $x_d[n] = x[n - n_0]$

$$y_0[n] = n.x_d[n]$$

$$y_0[n] = n.x[n - n_0]$$

The delayed output

$$y_d[n] = (n - n_0).x[n - n_0]$$

Here we can see that the response of the system to delayed input is NOT equal to the delayed output. Hence the system is NOT SHIFT INVARIANT. The system is SHIFT VARIANT

Shift Invariance

Example: Check whether the system

$$\boldsymbol{y[n] = e^{x[n]}}$$

is shift invariant or not

Answer:

Response of the system to a delayed input $x_d[n] = x[n - n_0]$

$$y_0[n] = e^{x_d[n]}$$

$$y_0[n] = e^{x[n-n_0]}$$

The delayed output

$$y_d[n] = e^{x[n-n_0]}$$

Here we can see that the response of the system to delayed input is equal to the delayed output. Hence the system is SHIFT INVARIANT.

Causality

A system is said to causal if the output depends only on the past and the present inputs. If the output depends on future inputs, the system is non – causal.

Example

Consider the system described by

$$y[n] = 2.x[n-3]$$

This is a causal system

Consider the system described by

$$y[n] = x[n+3]$$

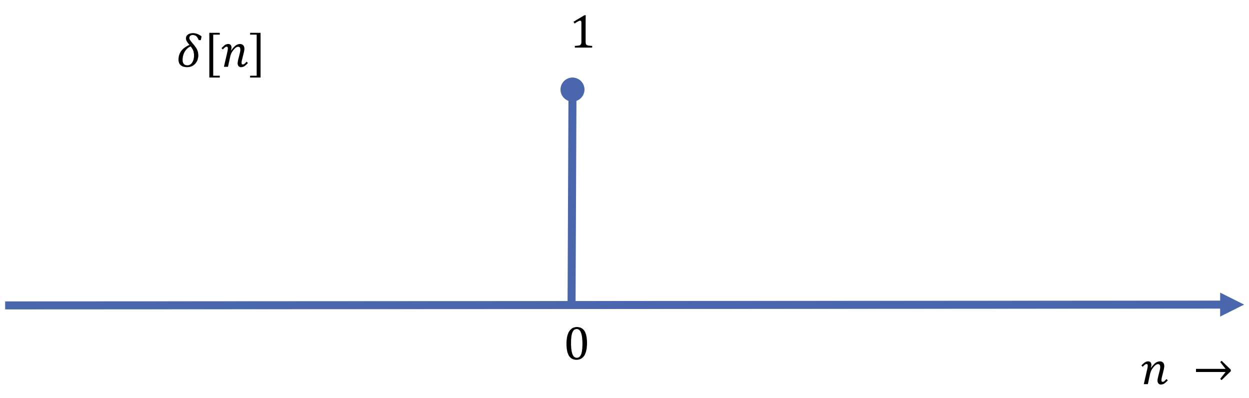
This is a non causal system

Standard Signals

Unit Impulse Sequence

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

The signal is shown graphically as follows

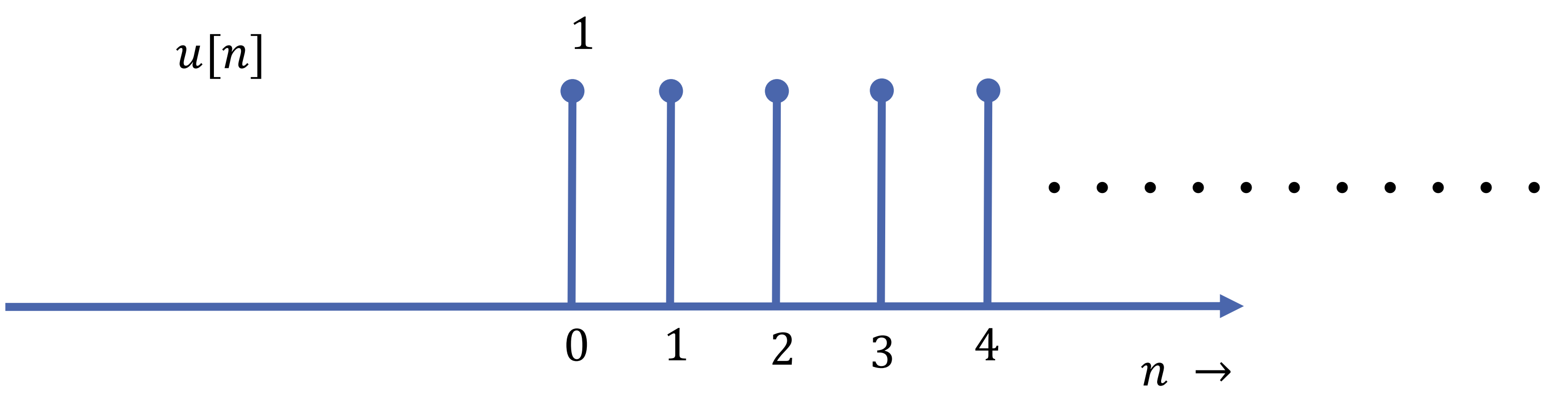


Standard Signals

Unit Step Sequence

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The signal is shown graphically as follows



Impulse Response of a System

Impulse response of a system is the response of the system to a unit impulse input $\delta[n]$

It is denoted using $h[n]$

$$x[n] = \delta[n] \longrightarrow \boxed{System} \longrightarrow h[n]$$

Response of Linear Shift/Time Invariant Systems

Consider a Linear Shift Invariant System (LSI/LTI) shown below

Let the impulse response of the system be $h[n]$. Then the output of the system to any input $x[n]$ can be found out as

$$y[n] = x[n] * h[n]$$

The * operation is called the Linear Convolution. This can be written as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

OR

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Response of Linear Shift/Time Invariant Systems

The impulse response of an LSI system is $(0.25)^n u[n]$. Find the output of the system when the input is

a. $u[n]$

b. $(0.5)^n u[n-2]$

Answer(a) :

$$h[n] = (0.25)^n u[n]$$

$$x[n] = u[n]$$

The output of the system is given as the linear convolution of $x[n]$ and $h[n]$

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

OR

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Response of Linear Shift/Time Invariant Systems

The impulse response of an LSI system is $(0.25)^n u[n]$. Find the output of the system when the input is

a. $u[n]$

b. $(0.5)^n u[n-2]$

Answer(a) :

$$h[n] = (0.25)^n u[n]$$

$$x[n] = u[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$h[k] = (0.25)^k u[k]$$

$$x[n-k] = u[n-k]$$

So

$$y[n] = \sum_{k=-\infty}^{\infty} (0.25)^k u[k]u[n-k]$$

Response of Linear Shift/Time Invariant Systems

$$y[n] = \sum_{k=-\infty}^{\infty} (0.25)^k u[k]u[n-k]$$

What is u[k] ?

$$u[k] = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

So the summation becomes

$$y[n] = \sum_{k=0}^{\infty} (0.25)^k u[n-k]$$

What is u[n-k] ?

$$u[n-k] = \begin{cases} 1, & n-k \geq 0 \\ 0, & n-k < 0 \end{cases}$$

$$u[n-k] = \begin{cases} 1, & n \geq k \\ 0, & n < k \end{cases}$$

$$u[n-k] = \begin{cases} 1, & k \leq n \\ 0, & k > n \end{cases}$$

So

$$y[n] = \sum_{k=0}^{n} (0.25)^k$$

Expanding we get

$$y[n] = 1 + (0.25) + (0.25)^2 + (0.25)^3 + \ldots\ldots + (0.25)^n$$

This is the sum of finite GP

$$S = 1 + r + r^2 + r^3 + \ldots\ldots + r^n$$

$$S = \frac{1 - r^{n+1}}{1 - r}$$

Response of Linear Shift/Time Invariant Systems

$$y[n] = 1 + (0.25) + (0.25)^2 + (0.25)^3 + \ldots\ldots + (0.25)^n$$

Using

$$S = 1 + r + r^2 + r^3 + \ldots\ldots + r^n$$

$$S = \frac{1 - r^n}{1 - r}$$

We get

$$y[n] = \frac{1 - (0.25)^{n+1}}{1 - 0.25}$$

$$y[n] = \frac{4}{3}[1 - (0.25)^{n+1}]u[n]$$

Thank You