CSE 394 Circuits and Signals
Operations on Signals
Sampling

By

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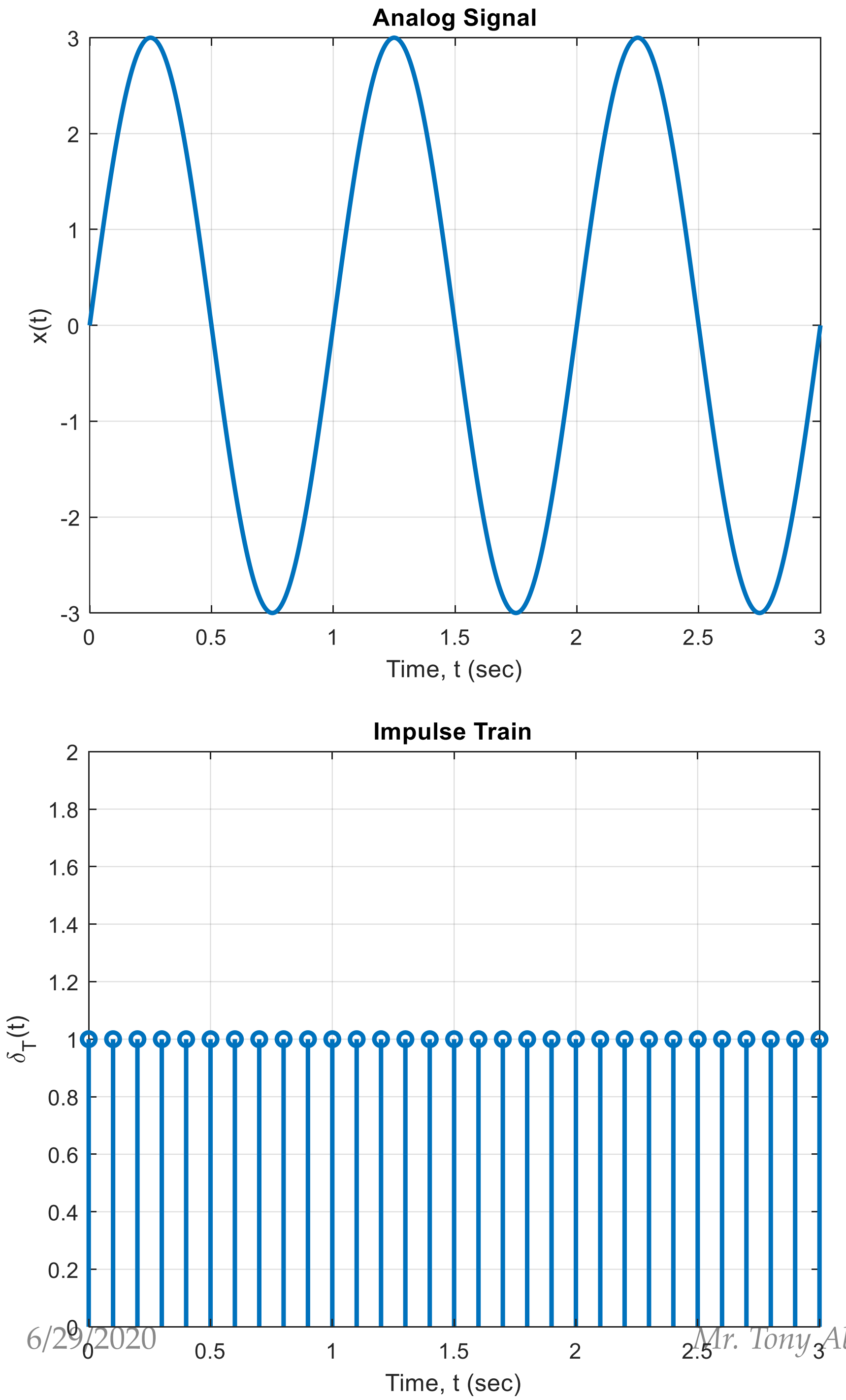
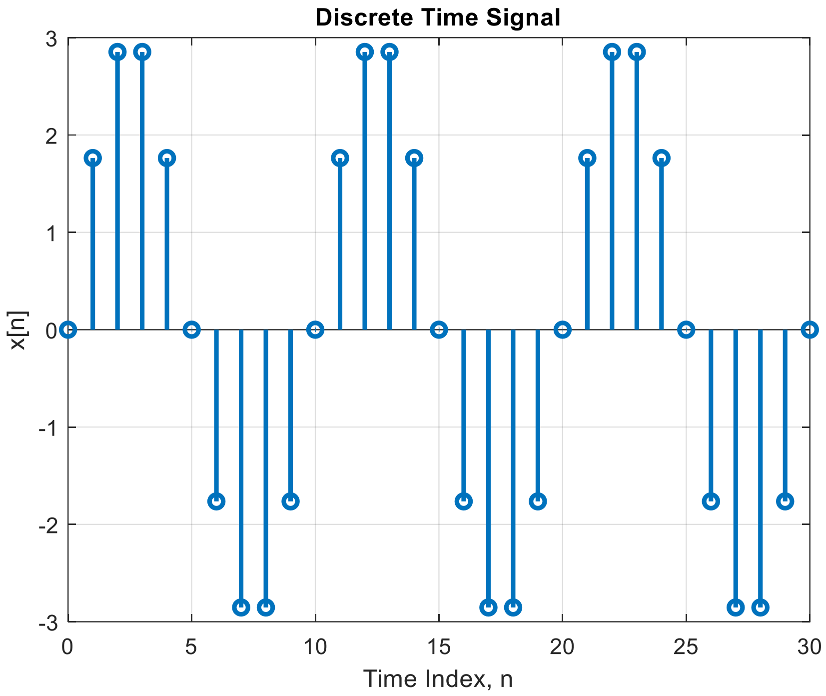
Sampling of Continuous Signals

Process of converting a continuous signal into a discrete time signal

Sampling



$\Rightarrow$



Nyquist Sampling Theorem

Nyquist Sampling Theorem states that for proper reconstruction of the analog signal from the sampled signal, the sampling frequency has to be atleast twice the maximum frequency present in the analog signal.

$$f_s \geq 2f_m$$

Here

$$f_s = Sampling\ Frequency$$

$$f_m = Maximum\ Frequency\ present\ in\ the\ analog\ signal$$

Nyquist Sampling Frequency is given as

$$f_N = 2f_m$$

Nyquist Sampling Theorem

Example: Consider a signal

$$\boldsymbol{x(t) = 4.sin(300\pi t) + 5.cos(800\pi t) - 3.sin(1200\pi t)}$$

What is the Nyquist Frequency? Write the discrete time signal produced, if the signal is sampled at 3 times the Nyquist Frequency

Answer:

What are the frequencies present in the signal?

$$2\pi f_1 = 300\,\pi \qquad f_1 = 150\,Hz$$

$$2\pi f_2 = 800\,\pi \qquad f_2 = 400\,Hz$$

$$2\pi f_3 = 1200\,\pi \qquad f_3 = 600\,Hz$$

So the maximum frequency is

$$f_m = 600\,Hz$$

So the Nyquist Frequency is

$$f_N = 2f_m$$

$$f_N = 1200\,Hz$$

Nyquist Sampling Theorem

Example: Consider a signal

$$\boldsymbol{x(t) = 4.sin(300\pi t) + 5.cos(800\pi t) - 3.sin(1200\pi t)}$$

What is the Nyquist Frequency? Write the discrete time signal produced, if the signal is sampled at 3 times the Nyquist Frequency

Answer:

$$f_N = 1200\ Hz$$

The sampling is done at a frequency of 3 times the Nyquist Frequency

$$f_s = 3f_N = 3 \times 1200 = 3600\ Hz$$

The sampling period will be

$$T_s = \frac{1}{f_s} = \frac{1}{3600}\, s$$

The sampled signal is obtained by substituting $t = nT_s$ in the analog signal

$$x(nT_s) = 4.sin(300\pi nT_s) + 5.cos(800\pi nT_s) - 3.sin(1200\pi nT_s)$$

Nyquist Sampling Theorem

Example: Consider a signal

$$\boldsymbol{x(t) = 4.sin(300\pi t) + 5.cos(800\pi t) - 3.sin(1200\pi t)}$$

What is the Nyquist Frequency? Write the discrete time signal produced, if the signal is sampled at 3 times the Nyquist Frequency

Answer:

$$x(nT_s) = 4.sin(300\pi nT_s) + 5.cos(800\pi nT_s) - 3.sin(1200\pi nT_s)$$

$$x(nT_s) = 4.sin\left(\frac{300\pi n}{3600}\right) + 5.cos\left(\frac{800\pi n}{3600}\right) - 3.sin\left(\frac{1200\pi n}{3600}\right)$$

$$x(nT_s) = 4.sin\left(\frac{\pi n}{12}\right) + 5.cos\left(\frac{2\pi n}{9}\right) - 3.sin\left(\frac{\pi n}{3}\right)$$

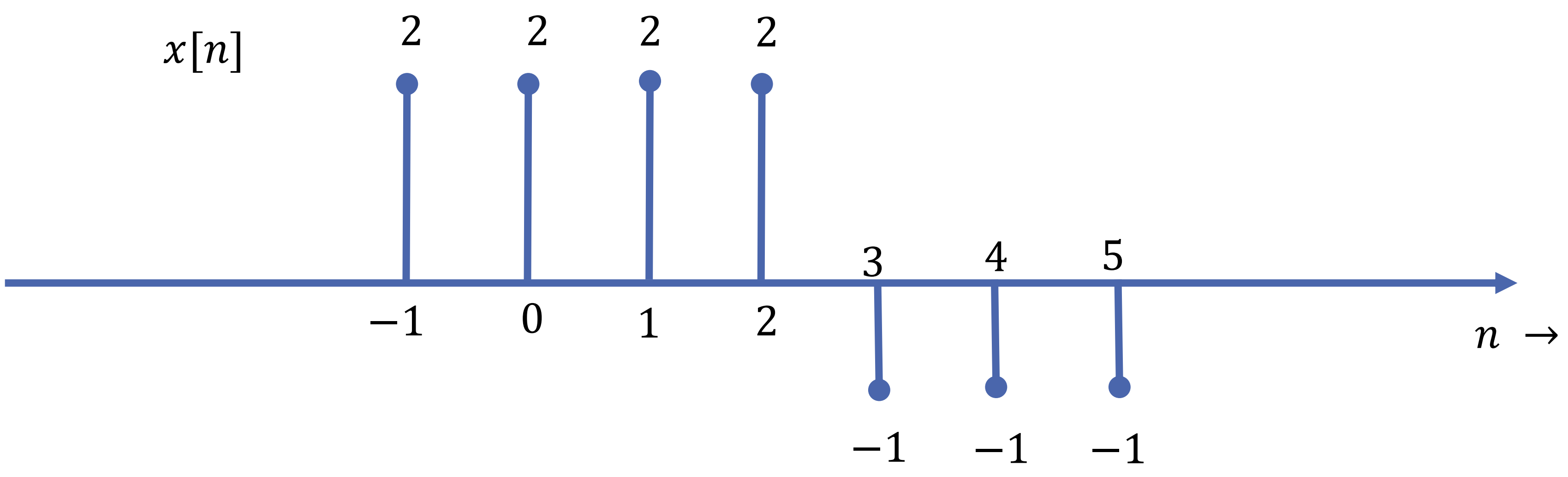
$$x[n] = 4.sin\left(\frac{\pi n}{12}\right) + 5.cos\left(\frac{2\pi n}{9}\right) - 3.sin\left(\frac{\pi n}{3}\right)$$

Representation of Discrete Time Signals[Sequence]

Consider a signal $x[n]$ given as

$$x[n] = \begin{cases} 2, & -1 \leq n \leq 2 \\ -1, & 3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$$

This can be represented graphically as follows



Mathematical Operations on Discrete Time Signals[Sequence]

Two types of operation are there

- Operations on the Independent Variable
- Operation on the Dependent variable

Operations on the Independent Variable

- Shifting
- Scaling
- Folding

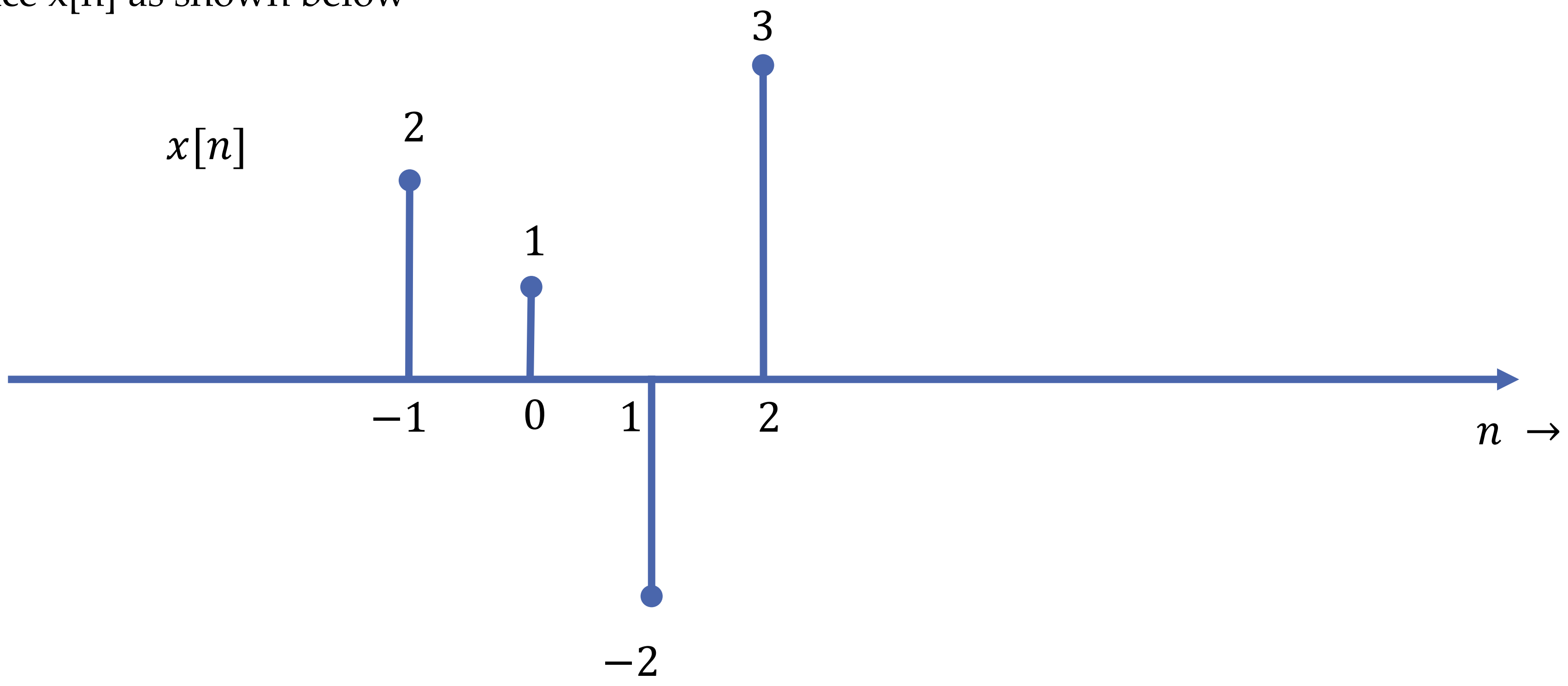
Operation on the Dependent variable

- Amplitude Scaling
- Addition
- Multiplication

Shifting

Shifting is the process of moving the sequence right or left from its original position

Consider a sequence x[n] as shown below



Shifting

From the signal

$$x[-1] = 2, \qquad x[0] = 1, \qquad x[1] = -2, \qquad x[2] = 3$$

Lets see what is

$$y[n] = x[n-2]$$

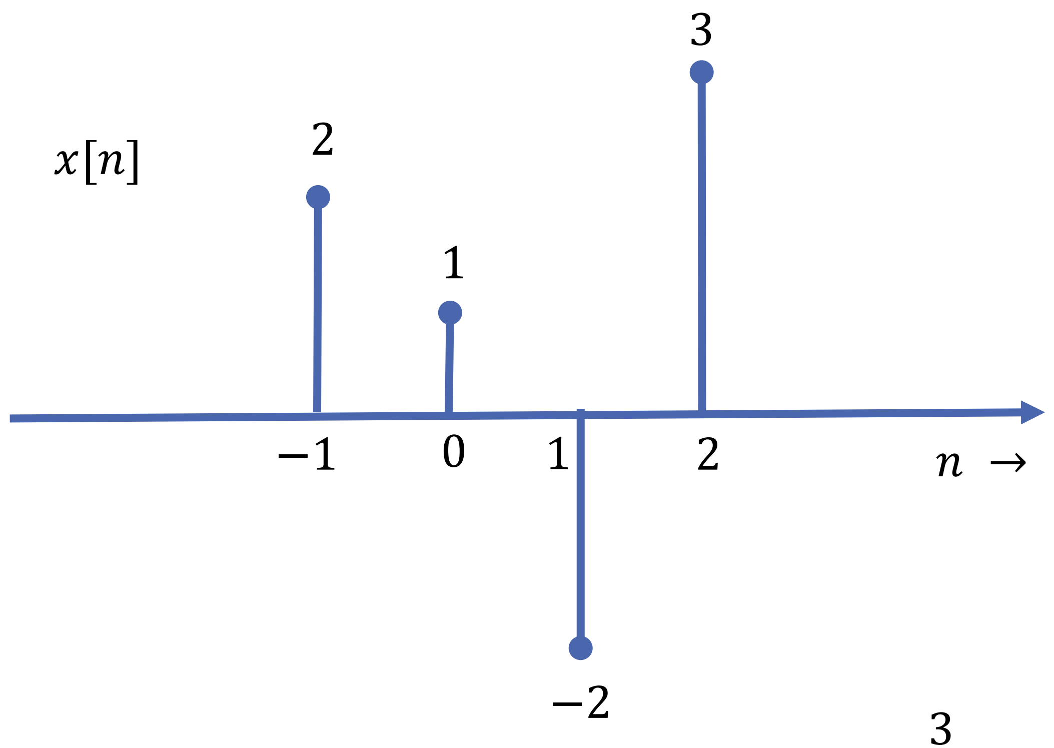
So

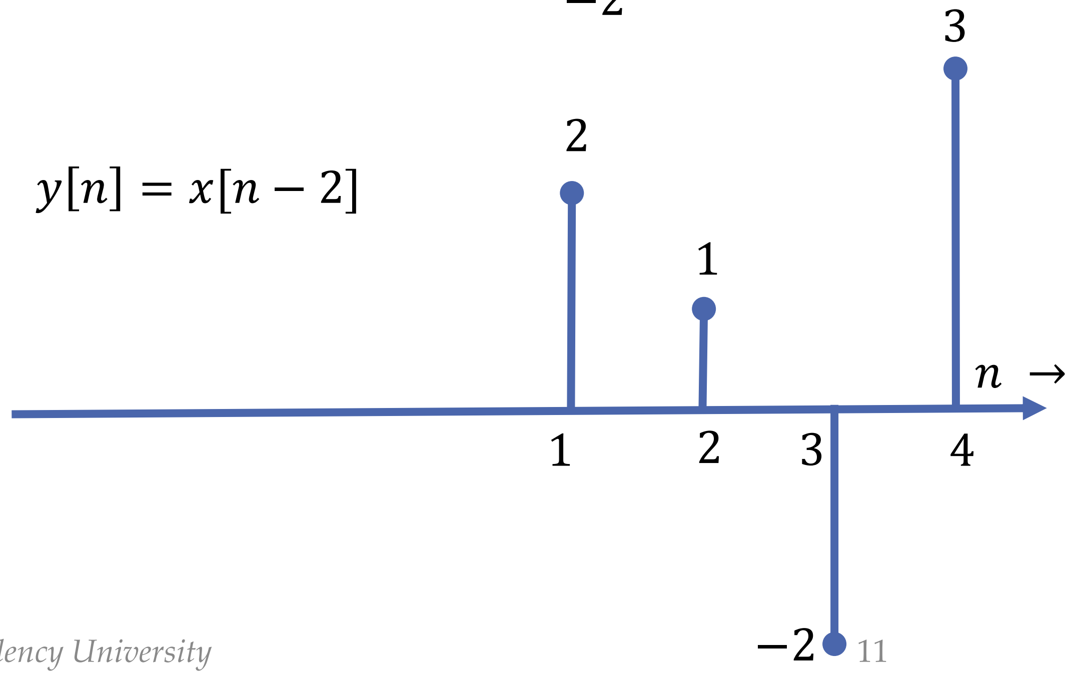
$$y[0] = x[0-2] = x[-2] = 0$$
$$y[1] = x[1-2] = x[-1] = 2$$
$$y[2] = x[2-2] = x[0] = 1$$
$$y[3] = x[3-2] = x[1] = -2$$
$$y[4] = x[4-2] = x[2] = 3$$
$$y[5] = x[5-2] = x[3] = 0$$

So the sequence $y[n] = x[n-2]$ becomes

What has happened?

The signal is shifted to the right by 2 times…..





Shifting

From the signal

$$x[-1] = 2, \qquad x[0] = 1, \qquad x[1] = -2, \qquad x[2] = 3$$

Lets see what is

$$y[n] = x[n+2]$$

So

$$y[0] = x[0+2] = x[2] = 3$$

$$y[1] = x[1+2] = x[3] = 0$$

$$y[-1] = x[-1+2] = x[1] = -2$$

$$y[-2] = x[-2+2] = x[0] = 1$$

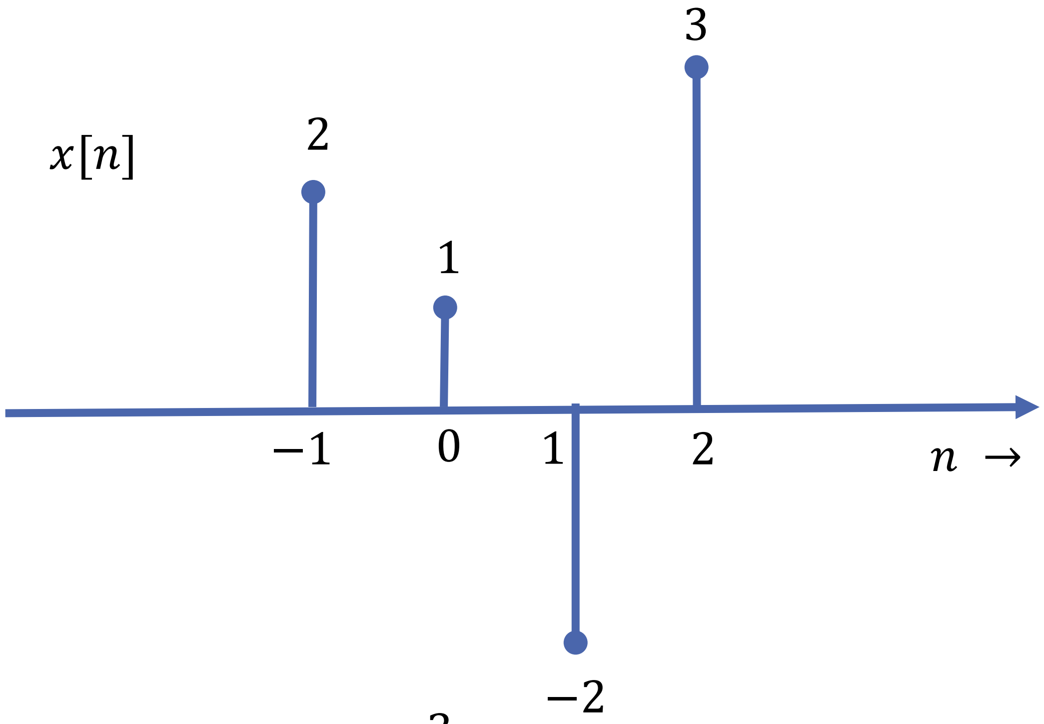
$$y[-3] = x[-3+2] = x[-1] = 2$$

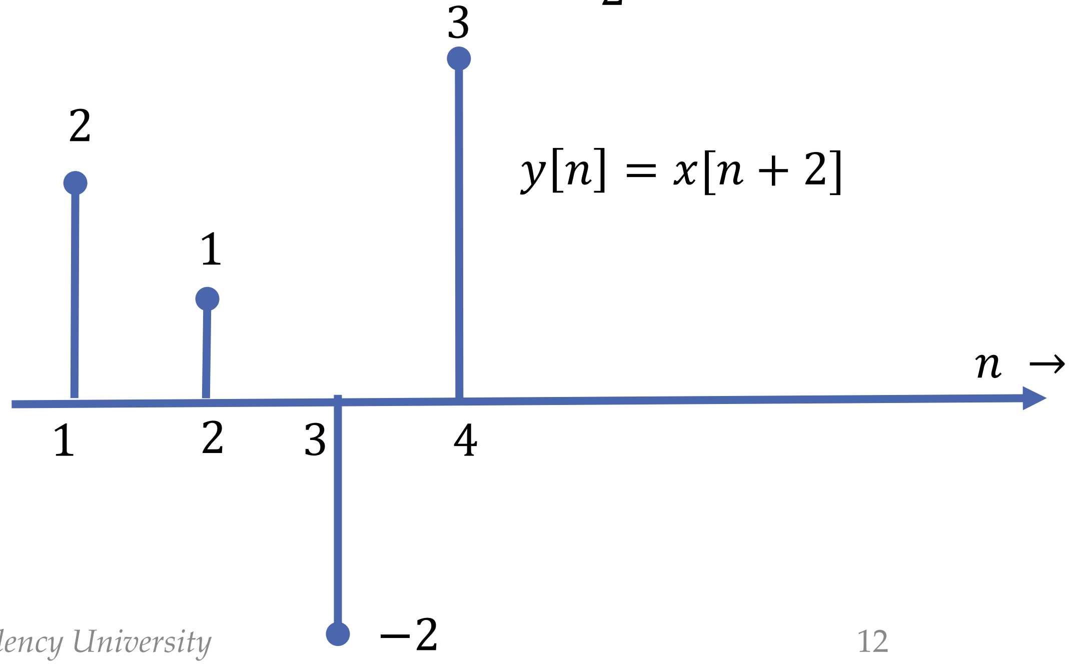
$$y[-4] = x[-4+2] = x[-2] = 0$$

So the sequence $y[n] = x[n+2]$ becomes

What has happened?

The signal is shifted to the left by 2 times…..





Shifting

So for the signal x[n] shifting is denoted as

$$x[n + n_0]$$

When n_0 is negative

The signal is shifted to right by n_0 times

When n_0 is positive

The signal is shifted to left by n_0 times

Folding or Time reversal

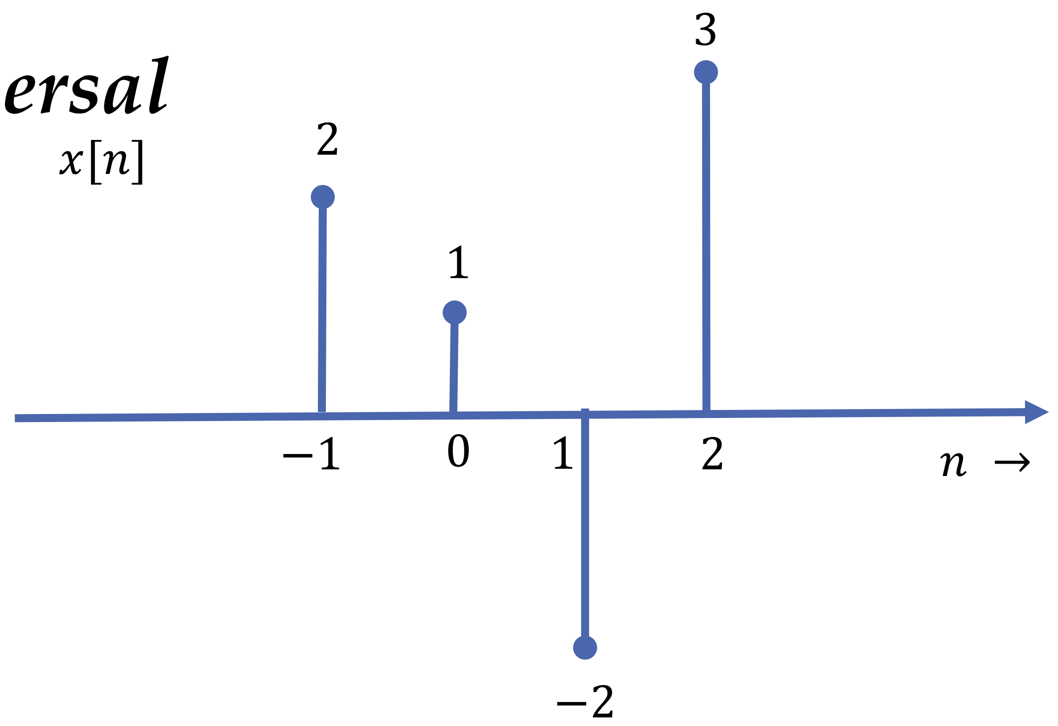
Time reversal is the operation

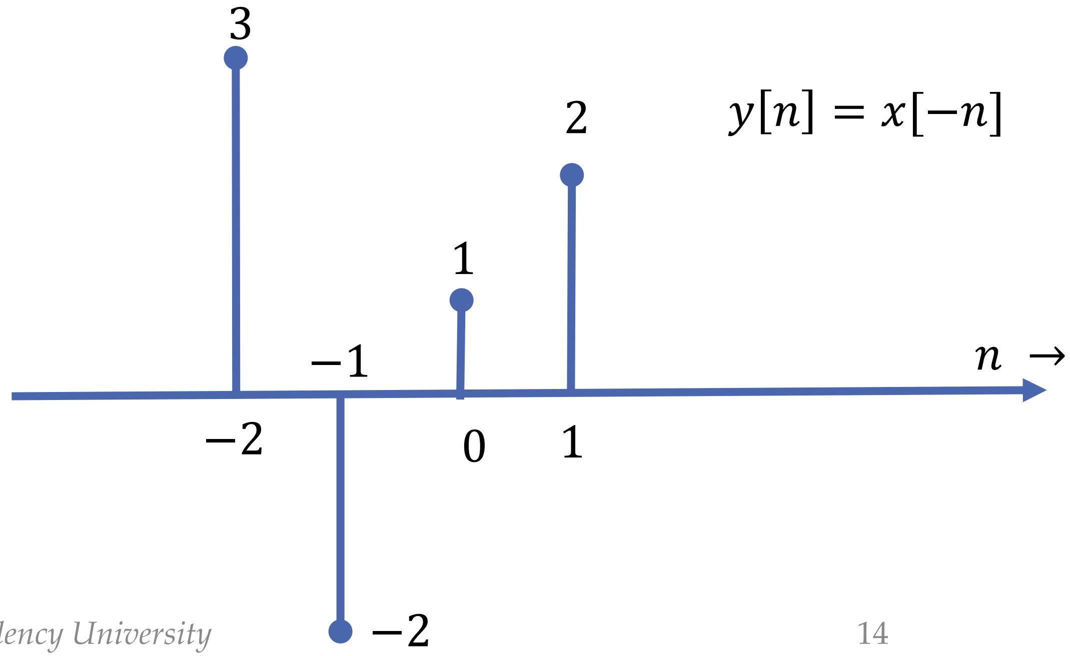
$$y[n] = x[-n]$$

So

$$y[0] = x[0] = x[0] = 1$$
$$y[1] = x[-1] = 2$$
$$y[2] = x[-2] = 0$$
$$y[-1] = x[1] = -2$$
$$y[-2] = x[2] = 3$$
$$y[-3] = x[3] = 0$$

So the sequence $y[n] = x[-n]$ becomes





What has happened?

The signal is folded…..

Thank You