Cardanic Gyroscope

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1 Objective

In this experiment, we sought to determine the precession frequency as a function of the torque and the angular velocity of the gyroscope as well as the nutational frequency as function of the angular velocity and the moment of inertia. For the measurement of the precession frequency, we used three combination of masses that were added to the gyroscope; $m_1 = 0.03696$kg, $m_2 = 0.1643$kg and $m_2 - m_1 = 0.12734$kg . The masses are $m_1 = 0.03696$kg and $m_2 = 0.1643$kg. A stroboscope is used to determine the gyroscope frequency. When the red sticker on the gyroscope is no longer moving, then the frequency of the gyroscope and stroboscope matches. For the nutation frequency, we used the following mass combinations: 0kg, $2m_1 = 0.07392$kg and $2m_2 = 0.3286$kg. For the gyroscope to undergo nutation, a sharp tap is applied to it.

We were able to get an experimental value for B_1, B_2 and B_3 for the relationship between precession frequency against gyro frequency to be -0.974 ± 0.037, -0.706 ± 0.076 and -0.533 ± 0.011 respectively. For a theoretical value of -1, the percentage discrepancy are as follow: 2.6%, 29.4% and 46.7%.

Similarly, we were able to get an experimental value for B_1, B_2 and B_3 for the relationship between nutation frequency against gyro frequency to be 0.68 ± 0.10, 0.783 ± 0.088 and 0.407 ± 0.082 respectively. For a theoretical value of 1.34, 1.11 and 0.55, the percentage discrepancy are as follow: 47.8%, 29.5% and 26.4%. These theoretical values were obtained from the graph provided.

2 Introduction

The fundamental equation for the gyroscope theory is:

$$\frac{d'\vec{L}_{ns}}{dt} + \vec{\Omega} \times \vec{L}_s = \vec{T} \tag{1}$$

, where $\vec{L}_{ns}$ and $\vec{L}_s$ are the two components that makes up the angular momentum of the gyroscope, Ω is the angular velocity of the moved reference system with the origin at the center of gravity and axis defined by the inner gimbal frame and figure axis and $\vec{T}$ is the torque applied to the gyroscope.

The precession frequency of the gyroscope, assuming that it was not perturbed from its axis of rotation, can be calculated using the equation:

$$\omega_{pr} = \frac{d'\theta}{dt} = \frac{mgr}{I_s^z} \cdot \frac{1}{\omega_z} \tag{2}$$

, where m is the added mass, r is the distance from origin which produces the torque due to gravitational acceleration, g is the gravitational strength and I_s^z is the moment of inertia of the rotor along the figure axis.

The above equation is then linearized by taking its logarithm. Equation 2 then becomes:

$$ln[\omega_{pr}] = -ln[\omega_z] + ln[\frac{mgr}{I_s^z}] \tag{3}$$

If the figure axis of the rotating gyroscope experience a sharp push, the figure axis undergoes a periodic movement with the nutation frequency to be:

$$\omega_{nu} = \frac{I_s^z}{\sqrt{I_x I_y}} \cdot \omega_z \tag{4}$$

, where ω_{nu} is the nutation frequency.

We also referred to the graph provided to us to calculate the theoretical gradient of the relationship between nutation frequency and gyroscope frequency.

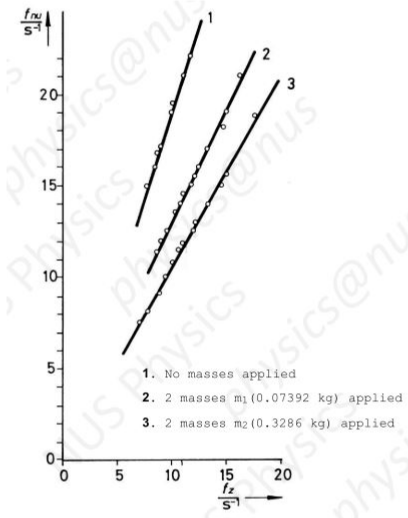

Figure 1: Theoretical graph of nutation frequency against gyroscope frequency

3 Experimental Results

Throughout the experiment, effort was made to measure the matching stroboscope frequency ω_z to the precession frequency ω_{pr}. However, the spinning of the gyroscope disk would slow down significantly before the gyroscope could complete one full precession. Therefore, this would cause our measured values to be lower than what is expected as the frequencies are not matched.

Similarly, the strength of the tap used to begin the nutation of the gyroscope affects the nutation frequency. Since there was no way to ensure that the tap was consistent throughout every measurement, this further adds uncertainties to our readings.

Lastly, there is the possibility of missed or double counting by the light gate due to how fast the wire (which was used as a form of marker) passes through the light gate. While repeated measurements were done to reduce this deviation, there would still be additional uncertainty involved in the experiment.

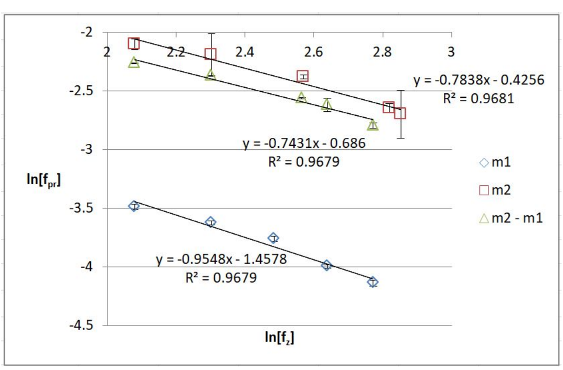

Figure 2: Graph of precession frequency against gyroscope frequency

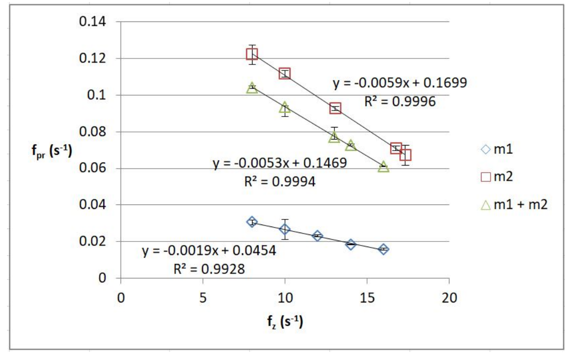

Figure 3: Graph of ln[precession frequency] against ln[gyroscope frequency]

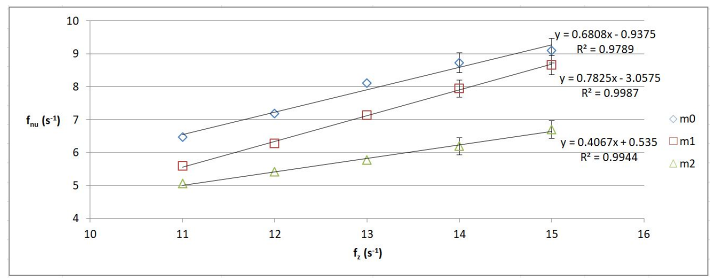

Figure 4: Graph of nutation frequency against gyroscope frequency

4 Conclusion

In this experiment, we sought to determine the precession frequency as a function of the torque and the angular velocity of the gyroscope as well as the nutational frequency as function of the angular velocity and the moment of inertia. We were able to get an experimental value for B_1, B_2 and B_3 for the relationship between precession frequency against gyro frequency to be -0.974 ± 0.037, -0.706 ± 0.076 and -0.533 ± 0.011 respectively. For a theoretical value of -1, the percentage discrepancy are as follow: 2.6%, 29.4% and 46.7%.

Similarly, we were able to get an experimental value for B_1, B_2 and B_3 for the relationship between nutation frequency against gyro frequency to be 0.70 ± 0.10, 0.782 ± 0.088 and 0.405 ± 0.082 respectively. For a theoretical value of 1.34, 1.11 and 0.55, the percentage discrepancy are as follow: 47.8%, 29.5% and 26.4%. Since our percentage discrepancy is much larger than 5%, our data does not lie within experimental uncertainty. However, this large discrepancy was mainly due to fact that we were not able to get a fix gyroscope frequency as well as control the tap that was used to nutate the gyroscope. Despite so, the general linear relationship were proven.