2018 AMC 8 Problems/Problem 1

Problem 1

An amusement park has a collection of scale models, with ratio $1:20$, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica to the nearest whole number?

$\textbf{(A) }14\qquad\textbf{(B) }15\qquad\textbf{(C) }16\qquad\textbf{(D) }18\qquad\textbf{(E) }20$

Solution

You can set up a ratio: $\frac{1}{20}=\frac{x}{289}$. Cross multiplying, you get $20x=289$. You divide by 20 to get $x=14.45$. The closest integer is $\boxed{\textbf{(A)}14}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **First Problem**	Followed by **Problem 2**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 2

Problem 2

What is the value of the product

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdot\left(1+\frac{1}{5}\right)\cdot\left(1+\frac{1}{6}\right)?$$

$\textbf{(A) }\frac{7}{6}\qquad\textbf{(B) }\frac{4}{3}\qquad\textbf{(C) }\frac{7}{2}\qquad\textbf{(D) }7\qquad\textbf{(E) }8$

Solution

By adding up the numbers in each parentheses, we have: $\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\frac{7}{6}$.

Using telescoping, most of the terms cancel out diagonally. We are left with $\frac{7}{1}$ which is equivalent to 7. Thus the answer would be $\boxed{\textbf{(D) }7}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 1**	Followed by **Problem 3**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 3

Problem 3

Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?

$\textbf{(A) } \text{Arn}\qquad\textbf{(B) }\text{Bob}\qquad\textbf{(C) }\text{Cyd}\qquad\textbf{(D) }\text{Dan}\qquad \textbf{(E) }\text{Eve}$

Solution

The five numbers which cause people to leave the circle are $7, 14, 17, 21,$ and $27.$

Assuming the five people start with 1, Arn counts 7 so he leaves first. Then Cyd counts 14, as there are 7 numbers to be counted from this point. Then Fon, Bob, and Eve, count $17, 21,$ and 27 respectively, so last one standing is Dan. Hence the answer would be $\boxed{\text{(D) Dan}}$

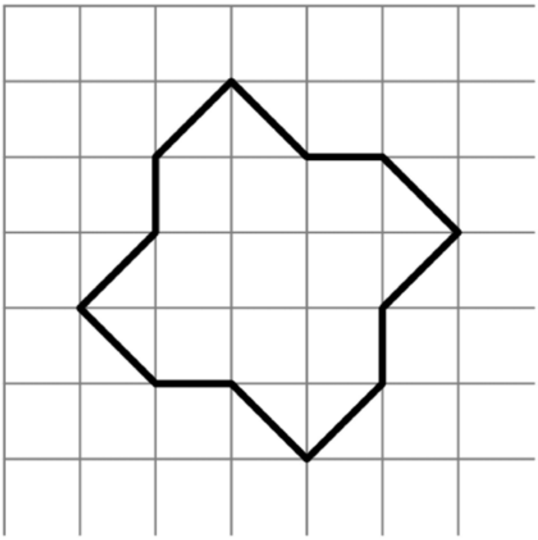
See Also

2018 AMC 8 (Problems • Answer Key • Resources)	
Preceded by **Problem 2**	Followed by **Problem 4**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 4

Problem 4

The twelve-sided figure shown has been drawn on $1\text{ cm}\times 1\text{ cm}$ graph paper. What is the area of the figure in cm^2?



$\textbf{(A) } 12 \qquad \textbf{(B) } 12.5 \qquad \textbf{(C) } 13 \qquad \textbf{(D) } 13.5 \qquad \textbf{(E) } 14$

Solution

We count $3 \cdot 3 = 9$ unit squares in the middle, and 4 small triangles each with an area of 1. Thus, the answer is $9 + 4 = \boxed{\textbf{(C) } 13}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 3**	Followed by **Problem 5**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 5

Problem 5

What is the value of $1 + 3 + 5 + \cdots + 2017 + 2019 - 2 - 4 - 6 - \cdots - 2016 - 2018$?

$\textbf{(A) }-1010\qquad\textbf{(B) }-1009\qquad\textbf{(C) }1008\qquad\textbf{(D) }1009\qquad \textbf{(E) }1010$

Solution

Rearranging the terms, we get $(1-2)+(3-4)+(5-6)+...(2017-2018)+2019$, and our answer is $-1009+2019=\boxed{1010}$, $\textbf{(E)}$

Solution 2

We can rewrite the given expression as $1+(3-2)+(5-4)+\cdots+(2017-2016)+(2019-2018)=1+1+1+\cdots+1$. The number of 1s is the same as the number of terms in $1,3,5,7...,2017,2019$. Thus the answer is $\boxed{\textbf{(E) }1010}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 4**	Followed by **Problem 6**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 6

Problem 6

On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take?

$\textbf{(A) }50\qquad\textbf{(B) }70\qquad\textbf{(C) }80\qquad\textbf{(D) }90\qquad \textbf{(E) }100$

Solution

Since Anh spends half an hour to drive 10 miles on the coastal road, her speed is $r=\frac{d}{t}=\frac{10}{0.5}=20$mph. Her speed on the highway then is 60mph. She drives 50 miles, so she also drives 50 minutes. The total amount of minutes spent on her trip is $30+50 \implies \boxed{\textbf{(C) }80}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 5**	Followed by **Problem 7**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 7

Problem 7

The 5-digit number $\underline{2}\,\underline{0}\,\underline{1}\,\underline{8}\,\underline{U}$ is divisible by 9. What is the remainder when this number is divided by 8?

$\textbf{(A) }1\qquad\textbf{(B) }3\qquad\textbf{(C) }5\qquad\textbf{(D) }6\qquad\textbf{(E) }7$

Solution

We use the property that the digits of a number must sum to a multiple of 9 if it are divisible by 9. This means $2+0+1+8+U$ must be divisible by 9. The only possible value for U then must be 7. Since we are looking for the remainder when divided by 8, we can ignore the thousands. The remainder when 187 is divided by 8 is $\boxed{\textbf{(B) } 3}$

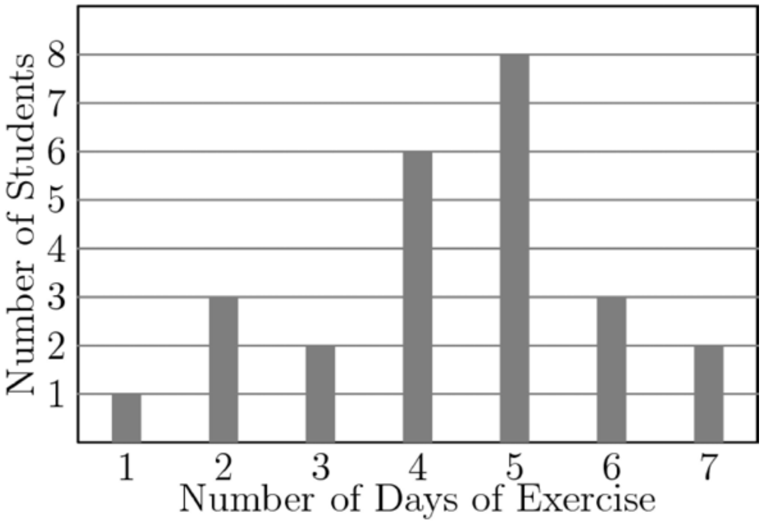
See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 6**	Followed by **Problem 8**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 8

Problem 8

Mr. Garcia asked the members of his health class how many days last week they exercised for at least 30 minutes. The results are summarized in the following bar graph, where the heights of the bars represent the number of students.



What was the mean number of days of exercise last week, rounded to the nearest hundredth, reported by the students in Mr. Garcia's class?

$\textbf{(A) } 3.50 \qquad \textbf{(B) } 3.57 \qquad \textbf{(C) } 4.36 \qquad \textbf{(D) } 4.50 \qquad \textbf{(E) } 5.00$

Solution

The mean number of days is the total number of days divided by the number of students. The total number of days is $1 \cdot 1 + 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 6 + 5 \cdot 8 + 6 \cdot 3 + 7 \cdot 2 = 109$. The total number of students is $1 + 3 + 2 + 6 + 8 + 3 + 2 = 25$. Hence, $\frac{109}{25} = \boxed{\textbf{(C) } 4.36}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 7**	Followed by **Problem 9**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 9

Problem 9

Tyler is tiling the floor of his 12 foot by 16 foot living room. He plans to place one-foot by one-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with two-foot by two-foot square tiles. How many tiles will he use?

$\textbf{(A) }48\qquad\textbf{(B) }87\qquad\textbf{(C) }91\qquad\textbf{(D) }96\qquad \textbf{(E) }120$

Solution

He will place $(12 \cdot 2) + (14 \cdot 2) = 52$ tiles around the border. For the inner part of the room, we have $10 \cdot 14 = 140$ square feet. Each tile takes up 4 square feet, so he will use $\frac{140}{4} = 35$ tiles for the inner part of the room. Thus, the answer is $52 + 35 = \boxed{\textbf{(B) }87}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 8**	Followed by **Problem 10**
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All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 10

Problem 10

The harmonic mean of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?

$\textbf{(A) }\frac{3}{7}\qquad\textbf{(B) }\frac{7}{12}\qquad\textbf{(C) }\frac{12}{7}\qquad\textbf{(D) }\frac{7}{4}\qquad \textbf{(E) }\frac{7}{3}$

Solution

The sum of the reciprocals is $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}$. Their average is $\frac{7}{12}$. Taking the reciprocal of this gives $\boxed{\textbf{(C) }\frac{12}{7}}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 9**	Followed by **Problem 11**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 11

Problem 11

Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

$$\begin{array}{ccc} \text{X} & \text{X} & \text{X} \\ \text{X} & \text{X} & \text{X} \end{array}$$

If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?

$\textbf{(A) } \frac{1}{3} \qquad \textbf{(B) } \frac{2}{5} \qquad \textbf{(C) } \frac{7}{15} \qquad \textbf{(D) } \frac{1}{2} \qquad \textbf{(E) } \frac{2}{3}$

Solution

There are a total of 6! ways to arrange the kids.

Abby and Bridget can sit in 3 ways if they are adjacent in the same column:

$$\begin{array}{ccc} \text{A} & \text{X} & \text{X} \\ \text{B} & \text{X} & \text{X} \end{array}$$

$$\begin{array}{ccc} \text{X} & \text{A} & \text{X} \\ \text{X} & \text{B} & \text{X} \end{array}$$

$$\begin{array}{ccc} \text{X} & \text{X} & \text{A} \\ \text{X} & \text{X} & \text{B} \end{array}$$

For each of these seat positions, Abby and Bridget can switch seats, and the other 4 people can be arranged in $4!$ ways which results in a total of $3 \times 2 \times 4!$ ways to arrange them.

By the same logic, there are 4 ways for Abby and Bridget to placed if they are adjacent in the same row, they can swap seats, and the other 4 people can be arranged in $4!$ ways, for a total of $4 \times 2 \times 4!$ ways to arrange them.

We sum the 2 possibilities up to get $\frac{14*4!}{6!} = \boxed{\frac{7}{15}}$ or $\textbf{(C)}$ - song2sons

Solution 2

We can ignore about the 4 other classmates because they aren't relevant. We can treat Abby and Bridget as a pair, so there are $\binom{6}{2} = 15$ total ways to seat them. If they sit in the same row, there are $2*2=4$ ways to seat them. If they sit in the same column, there are 3 ways to seat them. Thus our answer is $\frac{4+3}{15} = \boxed{\textbf{(C) } \frac{7}{15}}$

See also

2018 AMC 8 Problems/Problem 12

Problem 12

The clock in Sri's car, which is not accurate, gains time at a constant rate. One day as he begins shopping he notes that his car clock and his watch (which is accurate) both say 12:00 noon. When he is done shopping, his watch says 12:30 and his car clock says 12:35. Later that day, Sri loses his watch. He looks at his car clock and it says 7:00. What is the actual time?

$\textbf{(A) }5:50\qquad\textbf{(B) }6:00\qquad\textbf{(C) }6:30\qquad\textbf{(D) }6:55\qquad \textbf{(E) }8:10$

Solution

We see that every 35 minutes the clock passes, the watch passes 30 minutes. That means that the clock is $\frac{7}{6}$ as fast the watch, so when the car clock passes 7 hours, the watch has passed 6 hours, meaning that the time would be $\boxed{\textbf{(B) }6:00}$

See Also

2018 AMC 8 (Problems • Answer Key • Resources)	
Preceded by **Problem 11**	Followed by **Problem 13**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 13

Problem 13

Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test?

$\textbf{(A) }4\qquad\textbf{(B) }5\qquad\textbf{(C) }9\qquad\textbf{(D) }10\qquad \textbf{(E) }18$

Solution

Say Lalia gets a value of x on her first 4 tests, and a value of y on her last test. Thus, $4x+y=410.$

The value x has to be less than 82, because otherwise she would receive a lower score on her last test. Additionally, the greatest value for y is 100, so therefore the smallest value x can be is 78. As a result, only 4 numbers work, $78, 79, 80$ and 81. Thus the answer is $\boxed{\textbf{(A) }4}$. - song2sons

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 12**	Followed by **Problem 14**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 14

Problem 14

Let N be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of N?

$\textbf{(A) } 15\qquad\textbf{(B) } 16\qquad\textbf{(C) } 17\qquad\textbf{(D) } 18\qquad\textbf{(E) } 20$

Solution

If we start off with the first digit, we know that it can't be 9 since 9 is not a factor of 120. We scale down to the digit 8, which does work since it is a factor of 120. Now, we have to know what digits will take up the remaining four spots. To find this result, just divide $\frac{120}{8}=15$. The next place can be 5, as it is the largest factor, aside from 15. Consequently, our next three values will be $3, 1$ and 1 if we use the same logic! Therefore, our five-digit number is 85311, so the sum is $8+5+3+1+1=\boxed{\textbf{(D) } 18}$ -mathmaster010

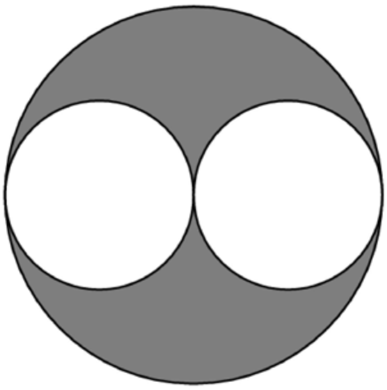
See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 13**	Followed by **Problem 15**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 15

Problem 15

In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?



$\textbf{(A) } \frac{1}{4} \qquad \textbf{(B) } \frac{1}{3} \qquad \textbf{(C) } \frac{1}{2} \qquad \textbf{(D) } 1 \qquad \textbf{(E) } \frac{\pi}{2}$

Solution

Let the radius of the large circle be R. Then the radii of the smaller circles are $\frac{R}{2}$. The areas the circles are directly proportional to the square of the radii, so the ratio of the area of the small circle to the large one is $\frac{1}{4}$. This means the combined area of the 2 smaller circles is $\frac{1}{2}$ the larger circle, therefore the shaded region is equal to the combined area of the 2 smaller circles, which is $\boxed{\textbf{(D) } 1}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 14**	Followed by **Problem 16**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 16

Problem 16

Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?

$\textbf{(A) }1440\qquad\textbf{(B) }2880\qquad\textbf{(C) }5760\qquad\textbf{(D) }182,440 \qquad \textbf{(E) }362,880$

Solution

Since the Arabic books and Spanish books have to by kept together, we can treat them both as just one book. That means we're trying to find the number of ways you can arrange one Arabic book, one Spanish book, and three German books, which is just 5 factorial. Now we multiply this product by $2!$ because there are 2 factorial ways to arrange just the Arabic books, and $4!$ ways to arrange just the Spanish books. Multiplying all these together, we have $2!\cdot4!\cdot5!=\boxed{\textbf{(C) }5760.}$ -xyab

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 15**	Followed by **Problem 17**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 17

Problem 17

Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is $10,560$ feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella?

$\textbf{(A) }704\qquad\textbf{(B) }845\qquad\textbf{(C) }1056\qquad\textbf{(D) }1760\qquad \textbf{(E) }3520$

Solution

Since Ella rides 5 times as fast as Bella, Ella rides at a rate of $\frac{25}{2}$ or $12\frac{1}{2}$. Together, they move 15 feet towards each other every unit. You divide 10560 by 15 to find the number of steps Ella takes, which results in the answer of $\boxed{\textbf{(A) }704}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 16**	Followed by **Problem 18**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 18

Problem 18

How many positive factors does $23,232$ have?

$\textbf{(A) }9\qquad\textbf{(B) }12\qquad\textbf{(C) }28\qquad\textbf{(D) }36\qquad\textbf{(E) }42$

Solution

We can first find the prime factorization of $23,232$, which is $2^6 \cdot 3^1 \cdot 11^2$. Now, we just add one to our powers and multiply. Therefore, the answer is $(1+6)*(1+1)*(1+2)=7*2*3=\boxed{42}$, $\textbf{(E)}$ -shreyasb

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 17**	Followed by **Problem 19**
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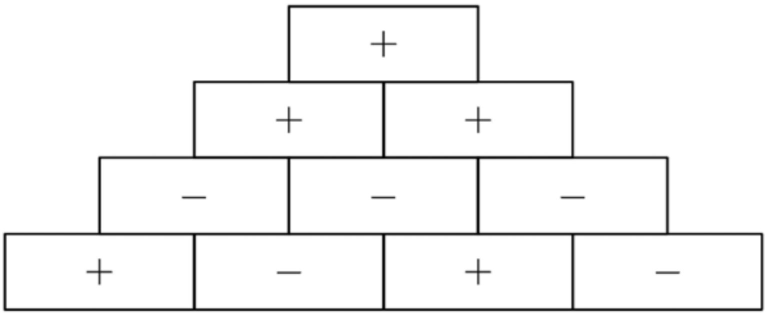
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 19

Problem 19

In a sign pyramid a cell gets a "+" if the two cells below it have the same sign, and it gets a "-" if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a "+" at the top of the pyramid?



$\textbf{(A) } 2 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } 8 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 16$

Solution 1

Instead of + and -, let us use 1 and 0, respectively. If we let a, b, c, and d be the values of the four cells on the bottom row, then the three cells on the next row are equal to $a+b$, $b+c$, and $c+d$ taken modulo 2 (this is exactly the same as finding $a \text{ XOR } b$, and so on). The two cells on the next row are $a+2b+c$ and $b+2c+d$ taken modulo 2, and lastly, the cell on the top row gets $a+3b+3c+d \pmod{2}$.

Thus, we are looking for the number of assignments of 0's and 1's for a, b, c, d such that $a+3b+3c+d \equiv 1 \pmod{2}$, or in other words, is odd. As $3 \equiv 1 \pmod{2}$, this is the same as finding the number of assignments such that $a+b+c+d \equiv 1 \pmod{2}$. Notice that, no matter what a, b, and c are, this uniquely determines d. There are $2^3=8$ ways to assign 0's and 1's arbitrarily to a, b, and c, so the answer is $\boxed{\textbf{(C) } 8}$.

Solution 2

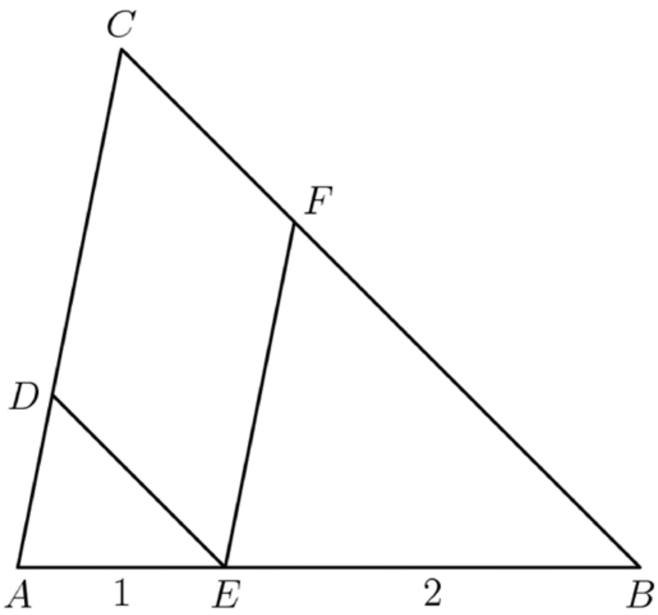
Each row is decided by the first cells (the other cells in the row are restricted from the cells above). Since we are given the first row already, we still need to decide the other 3 rows. The first cell in each row has only 2 possibilities (+ and -), so we have $2^3=\boxed{\textbf{(C) } 8}$ ways.

See Also

2018 AMC 8 Problems/Problem 20

Problem 20

In $\triangle ABC$, a point E is on $\overline{AB}$ with $AE = 1$ and $EB = 2$. Point D is on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$ and point F is on $\overline{BC}$ so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of $CDEF$ to the area of $\triangle ABC$?



$\textbf{(A) } \frac{4}{9} \qquad \textbf{(B) } \frac{1}{2} \qquad \textbf{(C) } \frac{5}{9} \qquad \textbf{(D) } \frac{3}{5} \qquad \textbf{(E) } \frac{2}{3}$

Solution

Looking at this diagram, we notice some similar triangles. $\angle ACB = \angle ADE$ since $\overline{DE} \parallel \overline{BC}$. Since $\triangle ABC$ and triangle $\triangle AED$ share $\angle A$, $\triangle ABC \sim \triangle AED$ by AA. Using similar logic, $\triangle ABC \sim \triangle EBF$. The ratio of the areas of two similar triangles is equivalent to the square of the ratio of the lenths, so $[AED]$ is $\frac{1}{9}[ABC]$ and $[EBF]$ is $\frac{4}{9}[ABC]$. This means that the area of quadrilateral $CDEF$ is is $1-(\frac{1}{9}+\frac{4}{9}) = \frac{4}{9}[ABC]$, so our answer is $\boxed{\textbf{(A) } \frac{4}{9}}$

See Also

2018 AMC 8 (**Problems** • **Answer Key** • Resources)	
Preceded by **Problem 19**	Followed by **Problem 21**
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

2018 AMC 8 Problems/Problem 21

Problem 21

How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }4\qquad \textbf{(E) }5$

Solution

Looking at the values, we notice that $11-7=4$, $9-5=4$ and $6-2=4$. This means we are looking for a value that is four less than a multiple of 11, 9, and 6. The least common multiple of these number is $11*3^2*2=198$, so the numbers that fulfill this can be written as $198k-4$, where k is a positive integer. This value is only a three digit integer when k is $1,2,3,4$ or 5, which gives $194, 392, 590, 788,$ and 986 respectively. Thus we have 5 values, so our answer is $\boxed{\textbf{(E) }5}$

See Also

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Preceded by **Problem 20**	Followed by **Problem 22**
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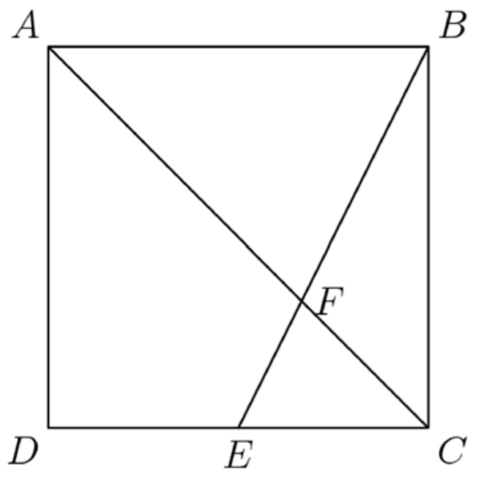
2018 AMC 8 Problems/Problem 22

Problem 22

Point E is the midpoint of side $\overline{CD}$ in square $ABCD,$ and $\overline{BE}$ meets diagonal $\overline{AC}$ at $F.$ The area of quadrilateral $AFED$ is $45.$ What is the area of $ABCD?$



$\textbf{(A) } 100 \qquad \textbf{(B) } 108 \qquad \textbf{(C) } 120 \qquad \textbf{(D) } 135 \qquad \textbf{(E) } 144$

Solution 1

Let the area of $\triangle CEF$ be x. Thus, the area of triangle $\triangle ACD$ is $45+x$ and the area of the square is $2(45+x) = 90+2x$.

By AAA similarity, $\triangle CEF \sim \triangle ABF$ with a 1:2 ratio, so the area of triangle $\triangle ABF$ is $4x$. Now consider trapezoid $ABED$. Its area is $45+4x$, which is three-fourths the area of the square. We set up an equation in x:

$$45+4x = \frac{3}{4}\left(90+2x\right)$$

Solving, we get $x = 9$. The area of square $ABCD$ is $90+2x = 90 + 2 \cdot 9 = \boxed{\textbf{(B) } 108}$.

Solution 1.5

Zooming Into Solution 1

We notice some similar triangles, though you want to know the ratios in order to begin using them.

45 is the area of half the triangle subtracted by a (half the triangle is thus $45+a$), so then the area of the whole square would be $90+2a$. Similar triangles come into use $here.$ $\triangle CEF \sim \triangle ABF$= $1/2$. From this we can deduce the area of triangle $\triangle ABF$ is $4x$. Now consider trapezoid $ABED$. Its area is $45+4x$, which is three-fourths the area of the square. We set up an equation in x:

$$45+4x = \frac{3}{4}\left(90+2x\right)$$

Solving, we get $x = 9$. The area of square $ABCD$ is $90+2x = 90 + 2 \cdot 9 = \boxed{\textbf{(B) } 108}$.

Solution 2

We can use analytic geometry for this problem.

Let us start by giving D the coordinate $(0,0)$, A the coordinate $(0,1)$, and so forth. $\overline{AC}$ and $\overline{EB}$ can be represented by the equations $y=-x+1$ and $y=2x-1$, respectively. Solving for their intersection gives point F coordinates $\left(\frac{2}{3},\frac{1}{3}\right)$.

Now, $\triangle EFC$'s area is simply $\frac{\frac{1}{2}\cdot\frac{1}{3}}{2}$ or $\frac{1}{12}$. This means that pentagon $ABCEF$'s area is $\frac{1}{2}+\frac{1}{12}=\frac{7}{12}$ of the entire square, and it follows that quadrilateral $AFED$'s area is $\frac{5}{12}$ of the square.

The area of the square is then $\frac{45}{\frac{5}{12}}=9\cdot 12=\boxed{\textbf{(B)}108}$.

See Also

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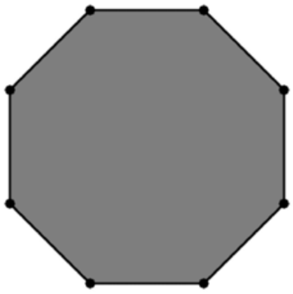
2018 AMC 8 Problems/Problem 23

From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



Solution

We will use constructive counting to solve this. There are 2 cases: Either all 3 points are adjacent, or exactly 2 points are adjacent. If all 3 points are adjacent, then we obviously have 8 choices. If we have exactly 2 adjacent points, then we will have 8 places to put the adjacent points and also 4 places to put the remaining point, so we have $8*4$ choices. The total amount of choices is $\binom{8}{3} = 8*7$. Thus our answer is $\frac{8+8*4}{8*7} = \frac{1+4}{7} = \boxed{\textbf{(D) } \frac{5}{7}}$

Solution 2 (Complementary)

We can decide 2 adjacent points with 8 choices. The remaining point will have 6 choices. However, we have counted the case with 3 adjacent points twice, so we need to subtract this case once. The case with the 3 adjacent points has 8 arrangements, so our answer is $\frac{8*6-8}{8*7} = \boxed{\textbf{(D) } \frac{5}{7}}$

See Also

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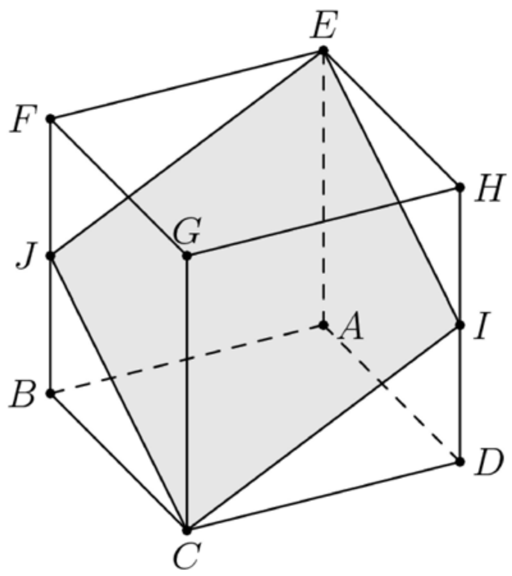
2018 AMC 8 Problems/Problem 24

Problem 24

In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is R^2?



$\textbf{(A) } \frac{5}{4} \qquad \textbf{(B) } \frac{4}{3} \qquad \textbf{(C) } \frac{3}{2} \qquad \textbf{(D) } \frac{25}{16} \qquad \textbf{(E) } \frac{9}{4}$

Solution

Note that $EJCI$ is a rhombus. Let the side length of the cube be s. By the Pythagorean theorem, $EC = \sqrt{3}s$ and $JI = \sqrt{2}s$. Since the area of a rhombus is half the product of it's diagonals, so the area of the cross section is $\frac{\sqrt{6}s^2}{2}$. $R = \frac{\sqrt{6}}{2}$. Thus $R^2 = \boxed{\textbf{(C) } \frac{3}{2}}$

Note

In the 2008 AMC 10A, Question 21 (https://artofproblemsolving.com/wiki/index.php?title=2008_AMC_10A_Problems/Problem_21) was nearly identical to this question, it's the same but for this question, you have to look for the square of the area, not the actual area.

See Also

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2018 AMC 8 Problems/Problem 25

Problem 25

How many perfect cubes lie between 2^8+1 and $2^{18}+1$, inclusive ?

$\textbf{(A) }4\qquad\textbf{(B) }9\qquad\textbf{(C) }10\qquad\textbf{(D) }57\qquad \textbf{(E) }58$

Solution

We compute $2^8+1=257$. We're all familiar with what 6^3 is, it's 216, which is too small. The smallest cube greater than it is $7^3=343$. $2^{18}+1$ is too large to calculate, but we notice that $2^{18}=(2^6)^3=64^3$ which is the largest cube less than $2^{18}+1$. Therefore, the amount of cubes is $64-7+1= \boxed{\textbf{(E) }58}$

See Also

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