2017 AMC 8 Problems/Problem 1

Contents

Problem 1

Which of the following values is largest?

$\textbf{(A) }2+0+1+7\qquad\textbf{(B) }2\times 0+1+7\qquad\textbf{(C) }2+0\times 1+7\qquad\textbf{(D) }2+0+1\times 7\qquad\textbf{(E) }2\times 0\times 1\times 7$

Solution 1

We compute each expression individually according to the order of operations. We get $2+0+1+7=10$, $2\times 0+1+7=8$, $2+0\times 1+7=9$, $2+0+1\times 7=9$, and $2\times 0\times 1\times 7=0$. Since 10 is the greatest out of these numbers, $\boxed{\textbf{(A)}\ 2+0+1+7}$ is the answer.

Solution 2

We immediately see that every one of the choices, except for A and D, has a number multiplied by 0. This will only make the expression's value smaller. We are left with A and D, but in D, 1 is multiplied by 7 to get 7, whereas in answer choice A, we get 8 out of 7 and 1, instead of 7. Therefore, $\boxed{\textbf{(A)}\ 2+0+1+7}$ is your answer.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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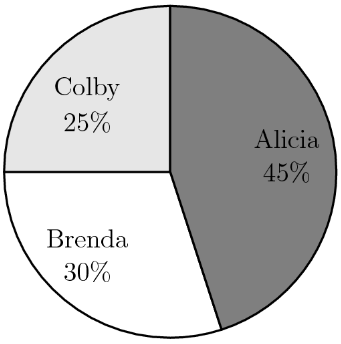
2017 AMC 8 Problems/Problem 2

Contents

Problem 2

Alicia, Brenda, and Colby were the candidates in a recent election for student president. The pie chart below shows how the votes were distributed among the three candidates. If Brenda received 36 votes, then how many votes were cast all together?



$\textbf{(A) }70\qquad\textbf{(B) }84\qquad\textbf{(C) }100\qquad\textbf{(D) }106\qquad\textbf{(E) }120$

Solution 1

Let x be the total amount of votes casted. From the chart, Brenda received 30% of the votes and had 36 votes. We can express this relationship as $\frac{30}{100}x=36$. Solving for x, we get $x=\boxed{\textbf{(E)}\ 120}$.

Solution 2

We're being asked for the total number of votes cast -- that represents 100% of the total number of votes (obviously). Brenda received 36 votes, which is $\frac{30}{100}=\frac{3}{10}$ of the total number of votes. Multiplying 36 by $\frac{10}{3}$, we get the total number of votes, which is $\boxed{\textbf{(E)}\ 120}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 3

Problem 3

What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$?

$\textbf{(A) }4\qquad\textbf{(B) }4\sqrt{2}\qquad\textbf{(C) }8\qquad\textbf{(D) }8\sqrt{2}\qquad\textbf{(E) }16$

Solution

$$\sqrt{16\sqrt{8\sqrt{4}}}=\sqrt{16\sqrt{8\cdot 2}}=\sqrt{16\sqrt{16}}=\sqrt{16\cdot 4}=\sqrt{64}=\boxed{\textbf{(C)}\ 8}.$$

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 4

Problem 4

When 0.000315 is multiplied by $7,928,564$ the product is closest to which of the following?

$\textbf{(A) }210\qquad\textbf{(B) }240\qquad\textbf{(C) }2100\qquad\textbf{(D) }2400\qquad\textbf{(E) }24000$

Solution

We can approximate $7,928,564$ to $8,000,000,$ and 0.000315 to $0.0003.$ Multiplying the two yields $2400.$ This gives our answer to be $\boxed{\textbf{(D)}\ 2400}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 5

Contents

Problem 5

What is the value of the expression $\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}{1+2+3+4+5+6+7+8}$?

$\textbf{(A) }1020\qquad\textbf{(B) }1120\qquad\textbf{(C) }1220\qquad\textbf{(D) }2240\qquad\textbf{(E) }3360$

Solution 1

Directly calculating:

We evaluate both the top and bottom: $\frac{40320}{36}$. This simplifies to $\boxed{\textbf{(B)}\ 1120}$.

Solution 2

It is well known that the sum of all numbers from 1 to n is $\frac{n(n+1)}{2}$. Therefore, the denominator is equal to $\frac{8\cdot 9}{2}=4\cdot 9=2\cdot 3\cdot 6$. Now we can cancel the factors of 2, 3, and 6 from both the numerator and denominator, only leaving $8\cdot 7\cdot 5\cdot 4\cdot 1$. This evaluates to $\boxed{\textbf{(B)}\ 1120}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 6

Problem 6

If the degree measures of the angles of a triangle are in the ratio $3:3:4$, what is the degree measure of the largest angle of the triangle?

$\textbf{(A) }18\qquad\textbf{(B) }36\qquad\textbf{(C) }60\qquad\textbf{(D) }72\qquad\textbf{(E) }90$

Solution

The sum of the ratios is 10. Since the sum of the angles of a triangle is 180°, the ratio can be scaled up to $54:54:72$ $(3\cdot 18:3\cdot 18:4\cdot 18)$. The numbers in the ratio $54:54:72$ represent the angles of the triangle. We want the largest, so the answer is $\boxed{\textbf{(D) } 72}$

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 7

Contents

Problem 7

Let Z be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must also be a factor of Z?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 19 \qquad \textbf{(C)}\ 101 \qquad \textbf{(D)}\ 111 \qquad \textbf{(E)}\ 1111$

Solution 1

Let $Z = \overline{ABCABC} = 1001 \cdot \overline{ABC} = 7 \cdot 11 \cdot 13 \cdot \overline{ABC}$. Clearly, Z is divisible by $\boxed{\textbf{(A)}\ 11}$.

Solution 2

We can see that numbers like 247247 can be written as $ABCABC$. We can see that the alternating sum of digits is $C-B+A-C+B-A$, which is 0. Because 0 is a multiple of 11, any number $ABCABC$ is a multiple of 11, so the answer is $\boxed{\textbf{(A)}\ 11}$.

-Baolan

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 8

Problem 8

Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

(1) It is prime.

(2) It is even.

(3) It is divisible by 7.

(4) One of its digits is 9.

This information allows Malcolm to determine Isabella's house number. What is its units digit?

$\textbf{(A) }4\qquad\textbf{(B) }6\qquad\textbf{(C) }7\qquad\textbf{(D) }8\qquad\textbf{(E) }9$

Solution

Notice that (1) cannot be true. Otherwise, the number would have to prime and either be even or divisible by 7. This only happens if the number is 2 or 7, neither of which are two-digit numbers, so we run into a contradiction. Thus, we must have (2), (3), and (4) true. By (2), the 2-digit number is even, and thus the digit in the tens place must be 9. The only even 2-digit number starting with 9 and divisible by 7 is 98, which has a units digit of $\boxed{\textbf{(D) }8}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 9

Problem 9

All of Macy's marbles are blue, red, green, or yellow. One third of her marbles are blue, one fourth of them are red, and six of them are green. What is the smallest number of yellow marbles that Macy could have?

$\textbf{(A) }1\qquad\textbf{(B) }2\qquad\textbf{(C) }3\qquad\textbf{(D) }4\qquad\textbf{(E) }5$

Solution

The 6 green marbles and yellow marbles form $1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$ of the total marbles. Now suppose the total number of marbles is x. We know the number of yellow marbles is $\frac{5}{12}x - 6$ and a positive integer. Therefore, 12 must divide x. Trying the smallest multiples of 12 for x, we see that when $x = 12$, we get there are -1 yellow marbles, which is impossible. However when $x = 24$, there are $\frac{5}{12} \cdot 24 - 6 = \boxed{\textbf{(D)}\ 4}$ yellow marbles, which must be the smallest possible.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 10

Problem 10

A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?

$\textbf{(A) }\frac{1}{10}\qquad\textbf{(B) }\frac{1}{5}\qquad\textbf{(C) }\frac{3}{10}\qquad\textbf{(D) }\frac{2}{5}\qquad\textbf{(E) }\frac{1}{2}$

Solution

There are $\binom{5}{3}$ possible groups of cards that can be selected. If 4 is largest card selected, then the other two cards must be either $1, 2,$ or $3,$ for a total $\binom{3}{2}$ groups of cards. Then the probability is just $\frac{\binom{3}{2}}{\binom{5}{3}} = \boxed{\textbf{(C) } \frac{3}{10}}$

See Also

2017 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 11

Problem 11

A square-shaped floor is covered with congruent square tiles. If the total number of tiles that lie on the two diagonals is 37, how many tiles cover the floor?

$\textbf{(A) }148\qquad\textbf{(B) }324\qquad\textbf{(C) }361\qquad\textbf{(D) }1296\qquad\textbf{(E) }1369$

Solution

Since the number of tiles lying on both diagonals is 37, counting one tile twice, there are $37=2x-1 \implies x=19$ tiles on each side. Hence, our answer is $19^2=361=\boxed{\textbf{(C) }361}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 12

Problem 12

The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?

$\textbf{(A) }2\text{ and }19\qquad\textbf{(B) }20\text{ and }39\qquad\textbf{(C) }40\text{ and }59\qquad\textbf{(D) }60\text{ and }79\qquad\textbf{(E) }80\text{ and }124$

Solution

Since the remainder is the same for all numbers, then we will only need to find the lowest common multiple of the three given numbers, and add the given remainder (No fancy Chinese Remainder Theorem) . The $\text{LCM}(4,5,6)$ is 60. Since $60+1=61$, and that is in the range of $\boxed{\textbf{(D) }60\text{ and }79}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 13

Problem 13

Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win?

$\textbf{(A) }0\qquad\textbf{(B) }1\qquad\textbf{(C) }2\qquad\textbf{(D) }3\qquad\textbf{(E) }4$

Solution

Given n games, there must be a total of n wins and n losses. Hence, $4+3+K=2+3+3$ where K is Kyler's wins. $K=1$, so our final answer is $\boxed{\textbf{(B)}\ 1}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 14

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Problem 14

Chloe and Zoe are both students in Ms. Demeanor's math class. Last night they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

$\textbf{(A) }89\qquad\textbf{(B) }92\qquad\textbf{(C) }93\qquad\textbf{(D) }96\qquad\textbf{(E) }98$

Solution 1

Let the number of questions that they solved alone be x. Let the percentage of problems they correctly solve together be a%. As given,

.

Hence, .

Zoe got problems right out of . Therefore, Zoe got percent of the problems correct.

Solution 2

Assume the total amount of problems is per half homework assignment, since we are dealing with percentages, and no values. Then, we know that Chloe got problems correct by herself, and got problems correct overall. We also know that Zoe had problems she did alone correct. We can see that the total amount of correct problems Chloe had when Zoe and she did the homework together is , which is the total amount of problems she got correct, subtracted by the number of correct problems she did alone. Therefore Zoe has problems out of problems correct. This is percent.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 15

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Problem 15

In the arrangement of letters and numerals below, by how many different paths can one spell AMC8? Beginning at the A in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.

	8	C	8	
8	C	M	C	8
C	M	A	M	C
8	C	M	C	8
	8	C	8	

$\textbf{(A) }8\qquad\textbf{(B) }9\qquad\textbf{(C) }12\qquad\textbf{(D) }24\qquad\textbf{(E) }36$

Solution

Notice that the A is adjacent to 4 Ms, each M is adjacent to 3 Cs, and each C is adjacent to 2 8s. Thus, the answer is $4\cdot 3\cdot 2=\boxed{\textbf{(D) }24}$.

Solution 2

There are three different kinds of paths that are on this diagram. The first kind is when you directly count A, M, C in a straight line. The second is when you count A, turn left or right to get M, then go straight to count M and C. The third is the one where you start with A, move forward to count M, turn left or right to count C, then move straight again to get 8.

There are 8 paths for each kind of path, making for $8\cdot 3=\boxed{24}$ paths.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 16

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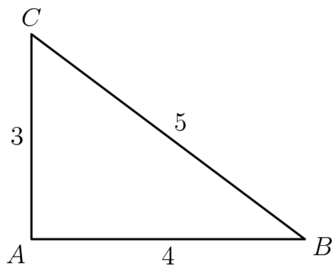
Problem 16

In the figure below, choose point D on $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?



$\textbf{(A) }\frac{3}{4}\qquad\textbf{(B) }\frac{3}{2}\qquad\textbf{(C) }2\qquad\textbf{(D) }\frac{12}{5}\qquad\textbf{(E) }\frac{5}{2}$

Solution 1

Essentially, we see that if we draw a line from point A to imaginary point D, that line would apply to both triangles. Let us say that x is the length of the line from B to D. So, the perimeter of $\triangle ABD$ would be $\overline{AD}+4+x$, while the perimeter of $\triangle ACD$ would be $\overline{AD}+3+(5-x)$. Notice that we can find out x from these two equations. We can find out that $x=2$, so that means that the area of $\triangle ABD = \frac{2\cdot 6}{5} = \boxed{\textbf{(D) } \frac{12}{5}}$

Solution 2

We know that the perimeters of the two small triangles are $3+CD+AD$ and $4+BD+AD$. Setting both equal and using $BD+CD = 5$, we have $BD = 2$ and $CD = 3$. Now, we simply have to find the area of $\triangle ABD$. Since $\frac{BD}{CD} = \frac{2}{3}$, we must have $\frac{[ABD]}{[ACD]} = 2/3$. Combining this with the fact that $[ABC] = [ABD] + [ACD] = \frac{3\cdot4}{2} = 6$, we get $[ABD] = \frac{2}{5}[ABC] = \frac{2}{5} \cdot 6 = \boxed{\textbf{(D) } \frac{12}{5}}$

Solution 3

Since point D is on line BC, it will split it into CD and DB. Let $CD = 5 - x$ and $DB = x$. Triangle CAD has side lengths $3, 5 - x, AD$ and triangle DAB has side lengths $x, 4, AD$. Since both perimeters are equal, we have the equation $3 + 5 - x + AD = 4 + x + AD$. Eliminating AD and solving the resulting linear equation gives $x = 2$. Draw a perpendicular from point D to AB. Call the point of intersection F. Because angle ABC is common to both triangles DBF and ABC, and both are right triangles, both are similar. The hypotenuse of triangle DBF is 2, so the altitude must be $6/5$ Because DBF and ABD share the same altitude, the height of ABD therefore must be $6/5$. The base of ABD is 4, so $[ABD] = \frac{1}{2} \cdot 4 \cdot \frac{6}{5} = \frac{12}{5} \implies \boxed{\textbf{(D) } \frac{12}{5}}$

Solution 4

Using any preferred method, realize $BD=2$. Since we are given a 3-4-5 right triangle, we know the value of $\sin(\angle ABC)=\frac{3}{5}$. Since we are given $AB=4$, apply the Sine Area Formula to get $\frac{1}{2}\cdot 4\cdot 2\cdot\frac{3}{5}=\boxed{\textbf{(D) } \frac{12}{5}}$.

See Also

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2017 AMC 8 Problems/Problem 17

Problem 17

Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 63 \qquad \textbf{(E)}\ 81$

Solution

We can represent the amount of gold with g and the amount of chests with c. We can use the problem to make the following equations:

$$9c - 18 = g$$

$$6c + 3 = g$$

Therefore, $6c + 3 = 9c - 18.$ This implies that $c = 7.$ We therefore have $g = 45.$ So, our answer is $\boxed{\textbf{(C)}\ 45}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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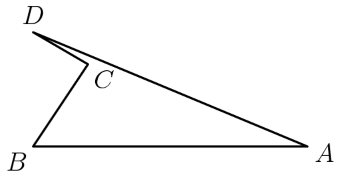
2017 AMC 8 Problems/Problem 18

Problem 18

In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$.

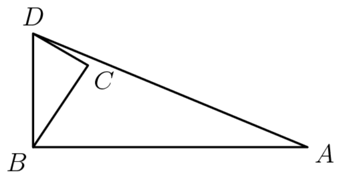


What is the area of quadrilateral $ABCD$?

$\textbf{(A) }12 \qquad \textbf{(B) }24 \qquad \textbf{(C) }26 \qquad \textbf{(D) }30 \qquad \textbf{(E) }36$

Solution

We first connect point B with point D.



We can see that $\triangle BCD$ is a 3-4-5 right triangle. We can also see that $\triangle BDA$ is a right triangle, by the 5-12-13 Pythagorean triple. With these lengths, we can solve the problem. The area of $\triangle BDA$ is $\frac{5 \cdot 12}{2}$, and the area of the smaller 3-4-5 triangle is $\frac{3 \cdot 4}{2}$. Thus, the area of quadrialteral $ABCD$ is $30 - 6 = \boxed{\textbf{(B)}\ 24}$.

See Also

2017 AMC 8 (**Problems** • **Answer Key** • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=2017))	
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2017 AMC 8 Problems/Problem 19

Contents

Problem 19

For any positive integer M, the notation $M!$ denotes the product of the integers 1 through M. What is the largest integer n for which 5^n is a factor of the sum $98!+99!+100!$?

$\textbf{(A)}\ 23 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 27$

Solution 1

Factoring out $98!+99!+100!$, we have $98!(10,000)$. Next, $98!$ has $\left\lfloor\frac{98}{5}\right\rfloor + \left\lfloor\frac{98}{25}\right\rfloor = 19 + 3 = 22$ factors of 5. Now $10,000$ has 4 factors of 5, so there are a total of $22 + 4 = \boxed{\textbf{(D)}\ 26}$ factors of 5.

Solution 2

The number of 5's in the factorization of $98! + 99! + 100!$ is the same as the number of trailing zeroes. The number of zeroes is taken by the floor value of each number divided by 5, until you can't divide by 5 anymore. Factorizing $98!+99!+100!$, you get $98!(1+99+9900)=98!(10000)$. To find the number of trailing zeroes in 98!, we do $\left\lfloor\frac{98}{5}\right\rfloor + \left\lfloor\frac{19}{5}\right\rfloor=19+3=22$. Now since 10000 has 4 zeroes, we add $22 + 4$ to get $\boxed{\textbf{(D)}\ 26}$ factors of 5.

See Also

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2017 AMC 8 Problems/Problem 20

Problem 20

An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?

$\textbf{(A) }\frac{14}{75}\qquad\textbf{(B) }\frac{56}{225}\qquad\textbf{(C) }\frac{107}{400}\qquad\textbf{(D) }\frac{7}{25}\qquad\textbf{(E) }\frac{9}{25}$

Solution

There are 5 options for the last digit, as the integer must be odd. The first digit now has 8 options left (it can't be 0 or the same as the last digit). The second digit also has 8 options left (it can't be the same as the first or last digit). Finally, the third digit has 7 options (it can't be the same as the three digits that are already chosen).

Since there are 9000 total integers, our answer is

$$\frac{8\cdot8\cdot7\cdot5}{9000}=\boxed{\textbf{(B) }\frac{56}{225}}.$$

See Also

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2017 AMC 8 Problems/Problem 21

Problem 21

Suppose a, b, and c are nonzero real numbers, and $a+b+c=0$. What are the possible value(s) for $\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}$?

$\textbf{(A) }0\qquad \textbf{(B) }1\text{ and }-1\qquad \textbf{(C) }2\text{ and }-2\qquad \textbf{(D) }0,2,\text{ and }-2\qquad \textbf{(E) }0,1,\text{ and }-1$

Solution

There are 2 cases to consider:

Case 1: 2 of a, b, and c are positive and the other is negative. WLOG, we can assume that a and b are positive and c is negative. In this case, we have that

$$\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}=1+1-1-1=0.$$

Case 2: 2 of a, b, and c are negative and the other is positive. WLOG, we can assume that a and b are negative and c is positive. In this case, we have that

$$\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}=-1-1+1+1=0.$$

In both cases, we get that the given expression equals $\boxed{\textbf{(A) }0}$.

See Also

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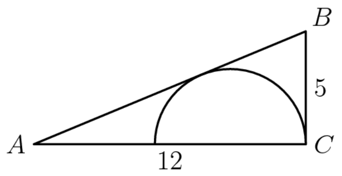
2017 AMC 8 Problems/Problem 22

Contents

Problem 22

In the right triangle ABC, $AC=12$, $BC=5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



$\textbf{(A) }\frac{7}{6}\qquad\textbf{(B) }\frac{13}{5}\qquad\textbf{(C) }\frac{59}{18}\qquad\textbf{(D) }\frac{10}{3}\qquad\textbf{(E) }\frac{60}{13}$

Solution 1

We can reflect triangle ABC over line $AC.$ This forms the triangle $AB'C$ and a circle out of the semicircle. Let us call the center of the circle $O.$

We can see that Circle O is the incircle of $AB'C.$ We can use the formula for finding the radius of the incircle to solve this problem. The area of $AB'C$ is $12\times5=60.$ The semiperimeter is $5+13=18.$ Simplifying $\dfrac{60}{18}=\dfrac{10}{3}.$ Our answer is therefore $\boxed{\textbf{(D)}\ \frac{10}{3}}$.

Solution 2

We immediately see that $AB=13$, and we label the center of the semicircle O And the point where the circle is tangent to the triangle D. Drawing radius OD with length x such that OD is perpendicular to AB, we immediately see that $ODB \cong OCB$ because of HL congruence, so $BD=5$ and $DA=8$. By similar triangles ODA and BCA, we see that $\frac{8}{12}=\frac{x}{5} \implies 12x=40 \implies x = \frac{10}{3} \implies \boxed{\textbf{(D)}\ \frac{10}{3}}$.

Solution 3

Let the center of the semicircle be O. Let the point of tangency between line AB and the semicircle be F. Angle BAC is common to triangles ABC and AFO. By tangent properties, angle AFO must be 90 degrees. Since both triangles ABC and AFO are right and share an angle, AFO is similar to ABC. The hypotenuse of AFO is $12-r$, where r is the radius of the circle. (See for yourself) The short leg of AFO is r. Because AFO ~ ABC, we have $r/(12-r)=5/13$ and solving gives $r=\boxed{\textbf{(D)}\ \frac{10}{3}}$

Solution 4

Let the tangency point on AB be D. Note

$$AD = AB - BD = AB - BC = 8$$

By Power of a Point,

$$12(12-2r) = 8^2$$

Solving for r gives

$$r = \boxed{\textbf{(D)}\ \frac{10}{3}}$$

See Also

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2017 AMC 8 Problems/Problem 23

Problem 23

Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

$\textbf{(A) }10\qquad\textbf{(B) }15\qquad\textbf{(C) }25\qquad\textbf{(D) }50\qquad\textbf{(E) }82$

Solution

It is well known that $\text{Distance} = \text{Speed} \cdot \text{Time}$. In the question, we want distance. From the question, we have that the time is 60 minutes or 1 hour. By the equation derived from $\text{Distance} = \text{Speed} \cdot \text{Time}$, we have $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$, so the speed is 1 mile per x minutes. Because we want the distance, we multiply the time and speed together yielding $60 \text{ mins} \cdot \frac{1 \text{ mile}}{x \text{ mins}}$. The minutes cancel out, so now we have $\frac{60}{x}$ as our distance for the first day. The distance for the following days are:

$$\frac{60}{x}, \frac{60}{x+5}, \frac{60}{x+10}, \frac{60}{x+15}$$

We know that $x, x+5, x+10, x+15$ are all factors of 60, therefore, $x=5$ because the factors have to be in an arithmetic sequence with the common difference being 5 and $x=5$ is the only solution.

$$\frac{60}{5}+\frac{60}{10}+\frac{60}{15}+\frac{60}{20} = 12+6+4+3 = \boxed{\textbf{(C)}\ 25}$$

See Also

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2017 AMC 8 Problems/Problem 24

Contents

Problem 24

Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

$\textbf{(A) }78\qquad\textbf{(B) }80\qquad\textbf{(C) }144\qquad\textbf{(D) }146\qquad\textbf{(E) }152$

Solution 1

In 360 days, there are

$$360\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}=144$$

days without calls. Note that in the last five days of the year, day 361 and 362 also do not have any calls, as they are not multiples of $3,4,$ or 5. Thus our answer is $144+2=\boxed{\textbf{(D) }146}$.

Alternatively, there are $365\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}=\boxed{146}$ days without calls. Multiplying the fractions in this order prevents partial days, as 365 is a multiple of 5, $365\cdot\frac{4}{5}$ is a multiple of 4 and $365\cdot\frac{4}{5}\cdot\frac{3}{4}$ is a multiple of 3.

Solution 2

We use Principle of Inclusion and Exclusion. There are 365 days in the year, and we subtract the days that she gets at least 1 phone call, which is

$$\left\lfloor\frac{365}{3}\right\rfloor+\left\lfloor\frac{365}{4}\right\rfloor+\left\lfloor\frac{365}{5}\right\rfloor$$

To this result we add the number of days where she gets at least 2 phone calls in a day because we double subtracted these days. This number is

$$\left\lfloor\frac{365}{12}\right\rfloor+\left\lfloor\frac{365}{15}\right\rfloor+\left\lfloor\frac{365}{20}\right\rfloor$$

We now subtract the number of days where she gets three phone calls, which is $\left\lfloor\frac{365}{60}\right\rfloor$. Therefore, our answer is

$$365-\left(\left\lfloor\frac{365}{3}\right\rfloor+\left\lfloor\frac{365}{4}\right\rfloor+\left\lfloor\frac{365}{5}\right\rfloor\right)+\left(\left\lfloor\frac{365}{12}\right\rfloor+\left\lfloor\frac{365}{15}\right\rfloor+\left\lfloor\frac{365}{20}\right\rfloor\right)-\left\lfloor\frac{365}{60}\right\rfloor$$

$$=365-285+72-6=\boxed{\textbf{(D) }146}.$$

See Also

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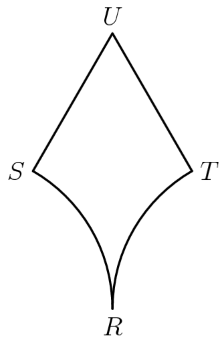
2017 AMC 8 Problems/Problem 25

Problem 25

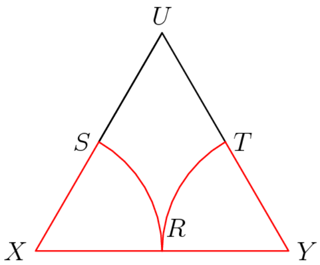
In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60^\circ$. Arcs $\overarc{TR}$ and $\overarc{SR}$ are each one-sixth of a circle with radius 2. What is the area of the region shown?



$\textbf{(A) } 3\sqrt{3}-\pi \qquad \textbf{(B) } 4\sqrt{3}-\frac{4\pi}{3} \qquad \textbf{(C) } 2\sqrt{3} \qquad \textbf{(D) } 4\sqrt{3}-\frac{2\pi}{3} \qquad \textbf{(E) } 4+\frac{4\pi}{3}$

Solution

Let the centers of the circles containing arcs $\overarc{SR}$ and $\overarc{TR}$ be X and Y, respectively. Extend $\overline{US}$ and $\overline{UT}$ to X and Y, and connect point X with point Y.



We can clearly see that $\triangle UXY$ is an equilateral triangle, because the problem states that $m\angle TUS = 60^\circ$. We can figure out that $m\angle SXR = 60^\circ$ and $m\angle TYR = 60^\circ$ because they are $\frac{1}{6}$ of a circle. The area of the figure is equal to $[\triangle UXY]$ minus the combined area of the 2 sectors of the circles(in red). Using the area formula for an equilateral triangle, $\frac{a^2\sqrt{3}}{4}$, where a is the side length of the equilateral triangle, $[\triangle UXY]$ is $\frac{\sqrt{3}}{4}\cdot 4^2 = 4\sqrt{3}$. The combined area of the 2 sectors is $2\cdot\frac{1}{6}\cdot\pi r^2$, which is $\frac{1}{3}\pi\cdot 2^2 = \frac{4\pi}{3}$. Thus, our final answer is $\boxed{\textbf{(B) } 4\sqrt{3}-\frac{4\pi}{3}}$.

See Also

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