1 Efficient Frontier

Let a portfolio of N assets be $\boldsymbol{\pi}$, whose expected return is $\boldsymbol{\mu}$ and the co-variance is $\boldsymbol{\Sigma}$.

1.1 Efficient Frontier with 3 Assets

According to the paper, the expected return of the portfolio, $E = \sum_{i=1}^{N} \pi_i \mu_i = \boldsymbol{\pi}^t \boldsymbol{\mu}$. The risk is analogous to the variance of the returns, i.e. $V = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} \pi_i \pi_j = \boldsymbol{\pi}^t \boldsymbol{\Sigma} \boldsymbol{\pi}$.

Given $\boldsymbol{\mu} = \boldsymbol{m}$ and $\boldsymbol{\Sigma} = \boldsymbol{C}$ for a 3 assets, we can generate 100 random portfolios, where each portfolio $\boldsymbol{\pi} = (\pi_1 \pi_2 \pi_3)^t$ s.t. $\mathbf{1}^t \boldsymbol{\pi} = 1$ by `y=randn(3,1); y=y/norm(y,1)`. Then we can calculate $E - V$ for each of the portfolios by `E=y'*m; V=sqrt(y'*C*y)`.

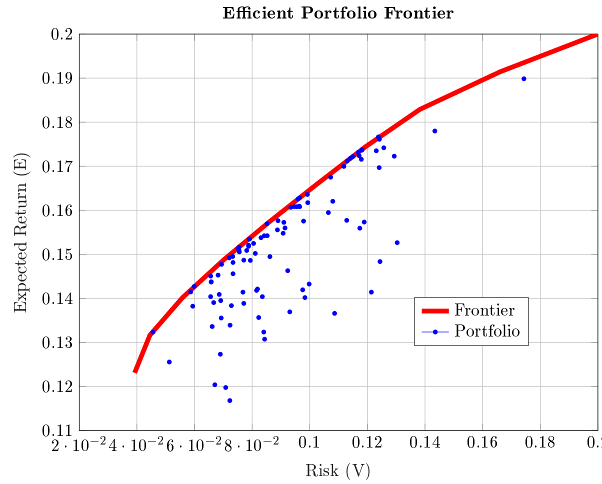


Figure 1: Efficient Portfolio

Finally I make the scatter plot and on the same figure I plot the efficient frontier using `estimateFrontier` function. As expected all the random portfolios were on the correct one side of the frontier.

1.2 Efficient Frontier with 2 Assets

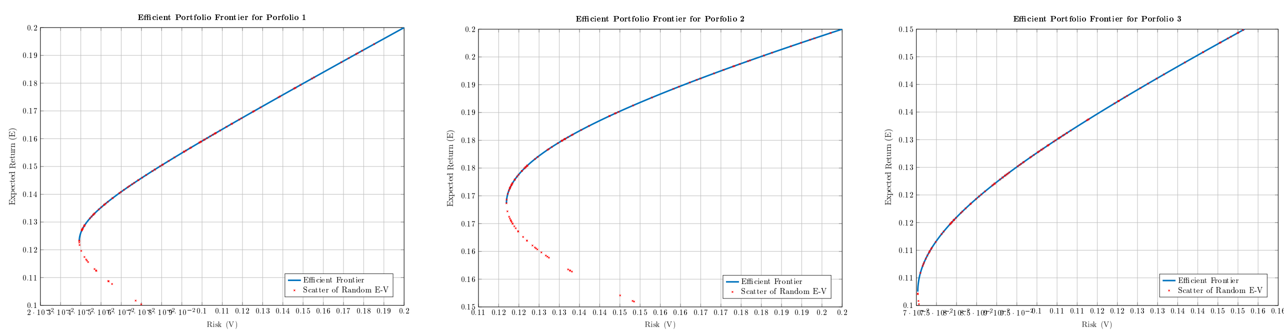


Figure 2: Efficient Frontier for 2 Asset Portfolios

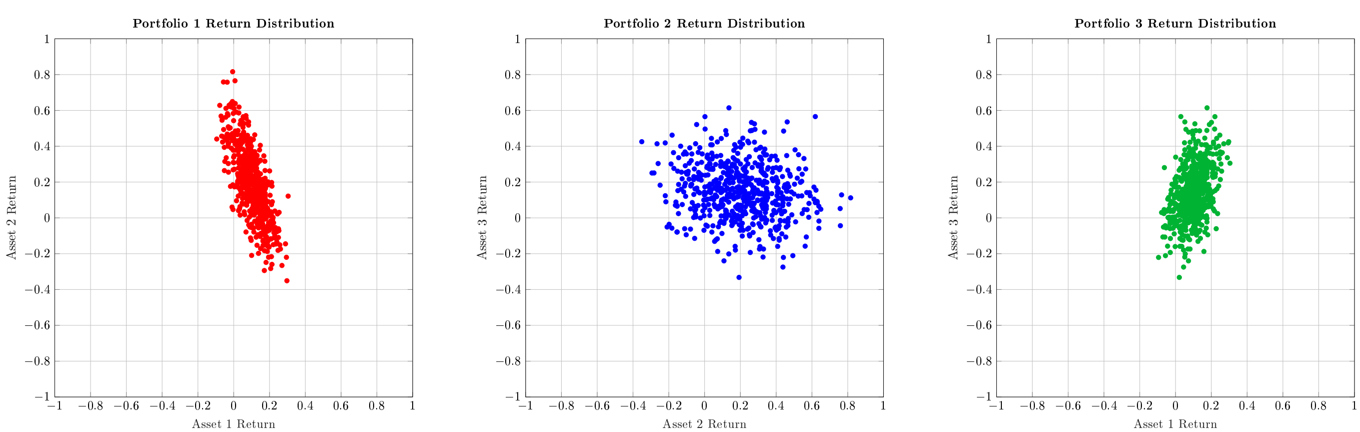
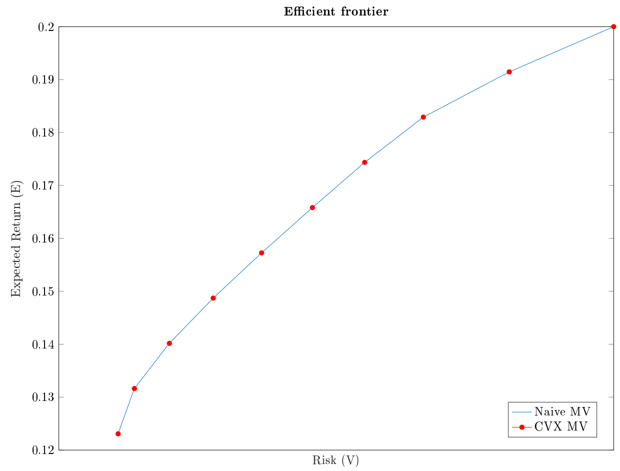


Figure 3: Distribution of 2 Asset Returns

1.3 Use of `linprog` in NaiveMV

1.4 Efficient Frontier : NaiveMV vs CVX



(a) Similarity