COMP6212 Assignment 2 : Shakib-Bin Hamid 25250094 sh3g12

1 Black Scholes Equation Check Solution Correctness

Cumulative normal distribution function, $\mathcal{N}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{-\infty} e^{\frac{-y^2}{2}} dy$. $\mathcal{N}'(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$.

$$
\begin{aligned}
& d_2 = d_1 - \sigma\sqrt{T-t} \\
\implies \quad & \frac{d_2^2}{2} = \frac{d_1^2}{2} + \frac{\sigma^2(T-t)}{2} - d_1\sigma\sqrt{T-t} \\
\implies \quad & \frac{d_1^2}{2} - \frac{d_2^2}{2} = d_1\sigma\sqrt{T-t} - \frac{\sigma^2(T-t)}{2} = log(\frac{S}{K}) + r(T-t) \\
\implies \quad & log(\frac{S}{K}) = \frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t) \\
\implies \quad & \frac{S}{K} = exp(\frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t)) = \frac{exp(\frac{-d_2^2}{2})}{exp(\frac{-d_1^2}{2} - r(T-t))} = \frac{\mathcal{N}'(d_2)}{\mathcal{N}'(d_1)e^{r(T-t)}} \\
\implies \quad & S\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)
\end{aligned}
$$

$$
\begin{aligned}
& d_2 = d_1 - \sigma\sqrt{T-t} \\
\implies \quad & \frac{d_2^2}{2} = \frac{d_1^2}{2} + \frac{\sigma^2(T-t)}{2} - d_1\sigma\sqrt{T-t} \\
\implies \quad & \frac{d_1^2}{2} - \frac{d_2^2}{2} = d_1\sigma\sqrt{T-t} - \frac{\sigma^2(T-t)}{2} = log(\frac{S}{K}) + r(T-t) \\
\implies \quad & log(\frac{S}{K}) = \frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t) \\
\implies \quad & \frac{S}{K} = exp(\frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t)) = \frac{exp(\frac{-d_2^2}{2})}{exp(\frac{-d_1^2}{2} - r(T-t))} = \frac{\mathcal{N}'(d_2)}{\mathcal{N}'(d_1)e^{r(T-t)}} \\
\implies \quad & S\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)
\end{aligned}
$$