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1 Black Scholes Equation Check Solution Correctness

Cumulative normal distribution function, $\mathcal{N}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{-\infty} e^{\frac{-y^2}{2}} dy$. $\mathcal{N}'(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$.

$$\begin{aligned}
& d_2 = d_1 - \sigma\sqrt{T-t} \\
\implies\quad & \frac{d_2^2}{2} = \frac{d_1^2}{2} + \frac{\sigma^2(T-t)}{2} - d_1\sigma\sqrt{T-t} \\
\implies\quad & \frac{d_1^2}{2} - \frac{d_2^2}{2} = d_1\sigma\sqrt{T-t} - \frac{\sigma^2(T-t)}{2} = log(\frac{S}{K}) + r(T-t) \\
\implies\quad & log(\frac{S}{K}) = \frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t) \\
\implies\quad & \frac{S}{K} = exp(\frac{d_1^2}{2} - \frac{d_2^2}{2} - r(T-t)) = \frac{exp(\frac{-d_2^2}{2})}{exp(\frac{-d_1^2}{2} - r(T-t))} = \frac{\mathcal{N}'(d_2)}{\mathcal{N}'(d_1)e^{r(T-t)}} \\
\implies\quad & S\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)
\end{aligned}$$

$$\begin{aligned}
& d_1 = \frac{log(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}} \\
\implies\quad & \frac{\partial d_1}{\partial S} = \frac{1}{\sigma\sqrt{(T-t)}}\frac{\partial}{\partial S}(logS - logK + (r + \frac{\sigma^2}{2})(T-t)) = \frac{1}{\sigma S\sqrt{(T-t)}} \\
Similarly\quad & d_2 = d_1 - \sigma\sqrt{T-t} \implies \frac{\partial d_2}{\partial S} = \frac{\partial d_1}{\partial S} = \frac{1}{\sigma S\sqrt{(T-t)}}
\end{aligned}$$

$\frac{\partial c}{\partial S} = \mathcal{N}(d_1) \implies \frac{\partial^2 c}{\partial S^2} = \mathcal{N}(d_1)\frac{\partial d_1}{\partial S} = \mathcal{N}(d_1)\frac{1}{\sigma S\sqrt{(T-t)}}$