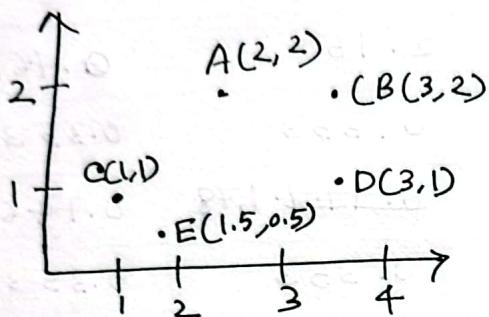


## K-means clustering

Use k-Means Algorithm to create two clusters



Assume  $A(2,2)$  and  $C(1,1)$  are centers of two clusters.

Iteration 1

Given Points	$c_1(2,2)$	$c_2(1,1)$	minDis	Select
$A(2,2)$	0	1.41	0	$c_1$
$B(3,2)$	1	2.24	1	$c_1$
$C(1,1)$	1.41	0	0	$c_2$
$D(3,1)$	1.41	2	1.41	$c_1$
$E(1.5,0.5)$	1.58	0.71	0.7	$c_2$

New cluster center,

$$c_1 = \left( \frac{2+3+3}{3}, \frac{2+2+1}{3} \right) = (2.67, 1.67)$$

$$c_2 = \left( \frac{1+1.5}{2}, \frac{1+0.5}{2} \right) = (1.25, 0.75)$$

## Iteration 2

Given Points	$C_1(2.67, 1.67)$	$C_2(1.25, 0.75)$	minDis	Select
A(2,2)	0.746	1.457	0.746	$C_1$
B(3,2)	0.467	2.15	0.467	$C_1$
C(1,1)	1.799	0.353	0.353	$C_2$
D(3,1)	0.746	1.78	0.746	$C_1$
E(1.5,0.5)	1.654	0.353	0.353	$C_2$

New Center,

$$C_1 = \left( \frac{2+3+3}{3}, \frac{2+2+1}{3} \right) = (2.67, 1.67)$$

$$C_2 = \left( \frac{1+1.5}{2}, \frac{1+0.5}{2} \right) = (1.25, 0.75)$$

This is completion of iteration 02.

The centers do not change anymore.

A(2,2), B(3,2), D(3,1) points belong to  $C_1$

C(1,1), E(1.5, 0.5) points belong to  $C_2$ .

## Practice Problems

Initial clusters are:  $A_1(2,10)$ ,  $A_4(5,8)$ ,  $A_7(1,2)$

I1

Given	$C_1(2,10)$	$C_2(5,8)$	$C_3(1,2)$	min Dis	Select
$A_1(2,10)$	0	3.61	8.06	0	$C_1$
$A_2(2,5)$	5	4.24	3.16	3.16	$C_3$
$A_3(8,4)$	8.48	5	7.28	5	$C_2$
$A_4(5,8)$	3.61	0	7.2	0	$C_2$
$A_5(7,5)$	7.07	3.61	6.71	3.61	$C_2$
$A_6(6,4)$	7.21	4.12	5.39	4.12	$C_2$
$A_7(1,2)$	8.06	7.2	0	0	$C_3$
$A_8(4,9)$	2.24	1.42	7.62	1.42	$C_2$

New Center,  $C_1 = (2, 10)$

$C_2 = (6, 6)$

$C_3 = (1.5, 3.5)$

I2

Given	$C_1(2,10)$	$C_2(6,6)$	$C_3(1.5,3.5)$	min Dis	Select
$A_1(2,10)$	0	5.65	6.52	0	$C_1$
$A_2(2,5)$	5	4.13	1.58	1.58	$C_3$
$A_3(8,4)$	8.48	2.83	6.52	2.83	$C_2$
$A_4(5,8)$	3.61	2.24	5.7	2.24	$C_2$
$A_5(7,5)$	7.07	1.41	5.7	1.41	$C_2$
$A_6(6,4)$	7.21	2	4.53	2	$C_2$
$A_7(1,2)$	8.06	0.4	1.58	1.58	$C_3$
$A_8(4,9)$	2.24	3.61	6.04	2.24	$C_1$

$$C_1 = \left( \frac{2+4}{2}, \frac{10+9}{2} \right) = (3, 9.5)$$

$$C_2 = \left( \frac{8+5+7+6}{4}, \frac{4+8+5+4}{4} \right) = (6.5, 5.25)$$

$$C_3 = \left( \frac{2+1}{2}, \frac{5+2}{2} \right) = (1.5, 3.5)$$

I3

Given	$C_1(3, 9.5)$	$C_2(6.5, 5.25)$	$C_3(1.5, 3.5)$	minDis	Select
A1(2, 10)	1.11	6.54	6.52	1.11	$C_1$
A2(2, 5)	4.61	4.51	1.58	1.58	$C_3$
A3(8, 4)	7.43	1.95	6.52	1.95	$C_2$
A4(5, 8)	2.5	3.13	5.7	2.5	$C_1$
A5(7, 5)	6.02	0.56	5.7	0.56	$C_2$
A6(6, 4)	6.26	1.35	4.53	1.35	$C_2$
A7(1, 2)	7.76	6.39	1.58	1.58	$C_3$
A8(4, 9)	1.12	4.51	6.04	1.12	$C_1$

ANSWER

# 1. Agglomerative Algorithm: Single Link



Sample No	X	Y
P1	0.4	0.53
P2	0.22	0.38
P3	0.35	0.32
P4	0.26	0.19
P5	0.08	0.41
P6	0.45	0.30

	P1	P2	P3	P4	P5	P6
P1	0					
P2	0.23	0				
P3	0.22	0.14	0			
P4	0.37	0.19	0.16	0		
P5	0.34	0.14	0.28	0.28	0	
P6	0.24	0.24	0.10	0.22	0.39	0

Here minimum value is 0.1. Update Distance Matrix

	P1	P2	P3, P6	P4	P5
P1	0				
P2	0.23	0			
P3, P6	0.22	0.14	0		
P4	0.37	0.19	0.16	0	
P5	0.34	0.14	0.28	0.28	0

Hence minimum value 0.14. Hence we combine  $P_2, P_5$

	$P_1$	$P_2, P_5$	$P_3, P_6$	$P_4$
$P_1$	0			
$P_2, P_5$	0.23	0		
$P_3, P_6$	0.22	0.14	0	
$P_4$	0.37	0.19	0.16	0

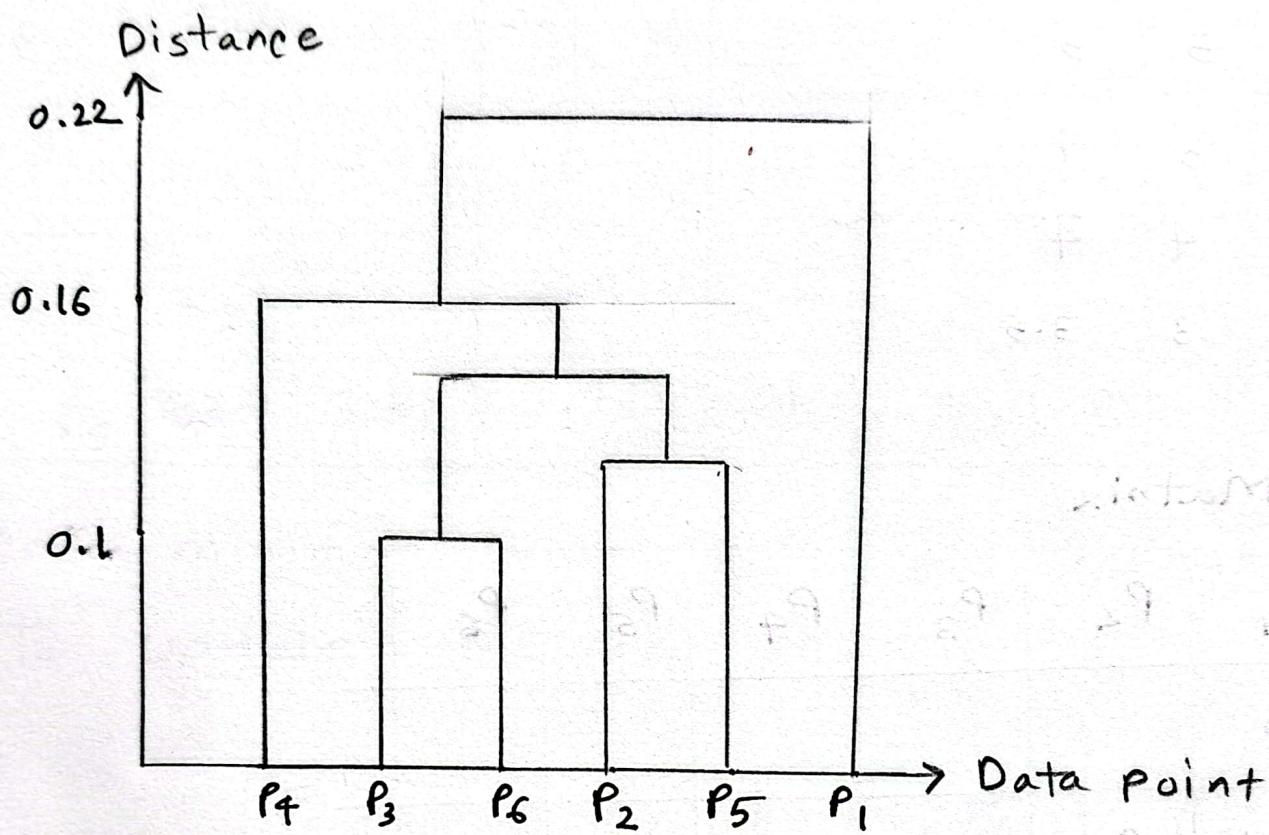
Hence minimum value 0.14. Hence we combine  
 $(P_2, P_5), (P_3, P_6)$

	$P_1$	$(P_2, P_5), (P_3, P_6)$	$P_4$
$P_1$	0		
$(P_2, P_5), (P_3, P_6)$	0.22	0	
$P_4$	0.37	0.16	0

Hence minimum value 0.16. Hence we combine  
 $[(P_2, P_5), (P_3, P_6)], P_4$

	$P_1$	$[(P_2, P_5), (P_3, P_6)], P_4$
$P_1$	0	
$[(P_2, P_5), (P_3, P_6)], P_4$	0.22	0

## Dendrogram



## 2. Agglomerative Algorithm: Complete Link

Sample      X      Y

P<sub>1</sub>      1      1

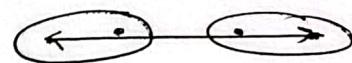
P<sub>2</sub>      1.5      1.5

P<sub>3</sub>      5      5

P<sub>4</sub>      3      4

P<sub>5</sub>      4      4

P<sub>6</sub>      3      3.5



Distance Matrix

	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
P <sub>1</sub>	0					
P <sub>2</sub>	0.71	0				
P <sub>3</sub>	5.66	4.95	0			
P <sub>4</sub>	3.61	2.92	2.24	0		
P <sub>5</sub>	4.24	3.54	1.41	1.0	0	
P <sub>6</sub>	3.20	2.5	2.5	(0.5)	1.12	0

Hence minimum value is 0.5. Merge (P<sub>4</sub>, P<sub>6</sub>). To update distance matrix

	$P_1$	$P_2$	$P_3$	$P_4, P_6$	$P_5$
$P_1$	0				
$P_2$	0.71	0			
$P_3$	5.66	4.95	0		
$P_4, P_6$	3.61	2.92	2.5	0	
$P_5$	4.24	3.54	1.41	1.12	0

Here minimum value 0.71 and merge ( $P_1, P_2$ ) .

To update distance Matrix

	$P_1, P_2$	$P_3$	$P_4, P_6$	$P_5$
$P_1, P_2$	0			
$P_3$	5.66	0		
$P_4, P_6$	3.61	2.5	0	
$P_5$	4.24	1.41	1.12	0

Here minimum value 1.12 and merge  $P_4, P_6$  and  $P_5$ .

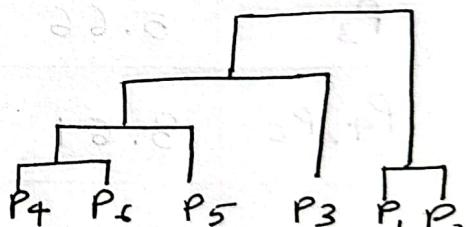
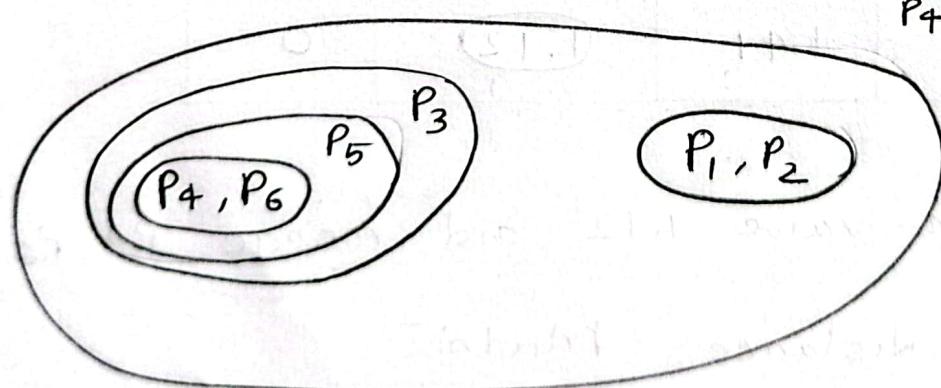
To update distance Matrix

	$P_1, P_2$	$P_3$	$(P_4, P_6) P_5$
$P_1, P_2$	0		
$P_3$	5.66	0	
$(P_4, P_6) P_5$	4.24	2.5	0

We get the minimum value as 2.5 and hence combine  $(P_4, P_6) P_5$  and  $P_3$ .

	$P_1, P_2$	$P_4, P_6, P_5, P_3$
$P_1, P_2$	0	
$P_4, P_6, P_5, P_3$	5.66	0

Dendrogram



### 3. Agglomerative : Average Link



	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
$P_1$	0					
$P_2$	0.71	0				
$P_3$	5.66	4.95	0			
$P_4$	3.61	2.92	2.24	0		
$P_5$	4.24	3.54	1.41	1.0	0	
$P_6$	3.20	2.5	2.5	0.5	1.12	0

↓

	$P_1$	$P_2$	$P_3$	$P_4, P_6$	$P_5$
$P_1$	0				
$P_2$	0.71	0			
$P_3$	5.66	4.95	0		
$P_4, P_6$	3.41	2.71	2.37	0	
$P_5$	4.24	3.54	1.41	1.06	0



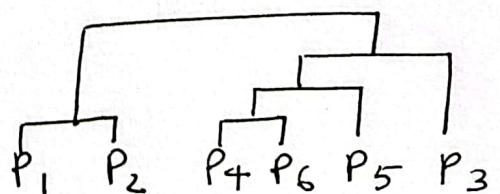
	$P_1, P_2$	$P_3$	$P_4, P_6$	$P_5$
$P_1, P_2$	0			
$P_3$	5.31	0		
$P_4, P_6$	3.06	2.37	0	
$P_5$	3.89	1.41	(1.06)	0

↓

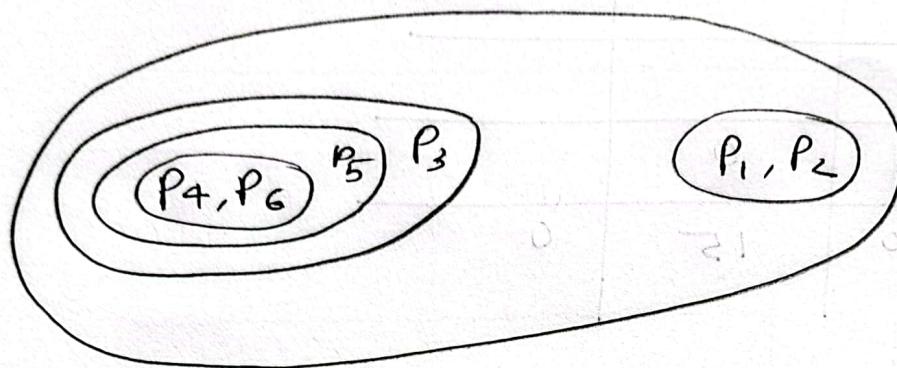
	$P_1, P_2$	$P_3$	$(P_4, P_6) P_5$
$P_1, P_2$	0		
$P_3$	5.31	0	
$(P_4, P_6) P_5$	3.48	(1.89)	0

↓

	$P_1, P_2$	$((P_4, P_6) P_5) P_3$
$P_1, P_2$	0	
$((P_4, P_6) P_5) P_3$	4.395	0



## Dendrogram



# Agglomerative Hienarchical clustering 6 one dimensional

18, 22, 25, 42, 27, 43

	18	22	25	27	42	43
18	0					
22	4	0				
25	7	3	0			
27	9	5	2	0		
42	24	20	17	15	0	
43	25	21	18	16	①	0



	18	22	25	27	42, 43
18	0				
22	4	0			
25	7	3	0		
27	9	5	2	0	
42, 43	24	20	17	15	0

18	22	25, 27	42, 43
18	0		
22	4	0	
25, 27	7	(3)	0
42, 43	24	20	15
			0



L'eliminazione uno è il primo turno. L'eliminazione due è il secondo turno.

18	22, (25, 27)	42, 43
18	0	
22, (25, 27)	(4)	0
42, 43	24	15
		0



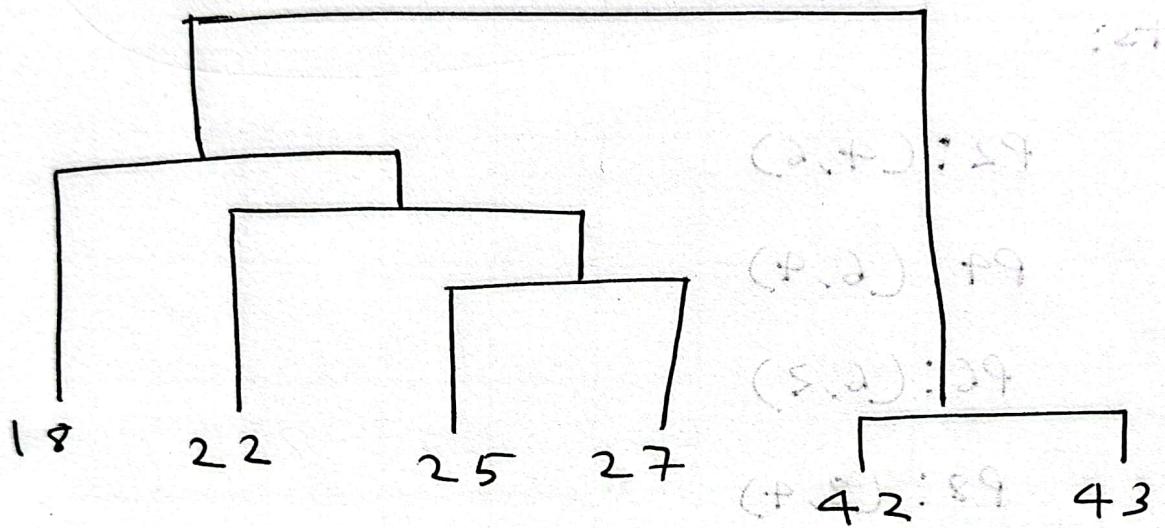
18, 22, 25, 27 | 42, 43

18, 22, 25, 27	0	15
42, 43	15	0



18, 22, 25, 27, 42, 43

18, 22, 25, 27, 42, 43	0
------------------------	---



(2, 2) : 22

(4, 2) : 19

(5, 2) : 29

(4, 2) : 21

(2, 5) : 19

(4, 2) : 21

(4, 3) : 19

(5, 2) : 29

(5, 2) : 21

(2, 5) : 19

(4, 2) : 21

(2, 5) : 19

## DBSCAN

Solved Example -1

min pts = 4 and epsilon ( $\epsilon$ ) = 1.9

$$\text{Distance}(A(x_1, y_1), B(x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Data Points:

P<sub>1</sub>: (3, 7)      P<sub>2</sub>: (4, 6)

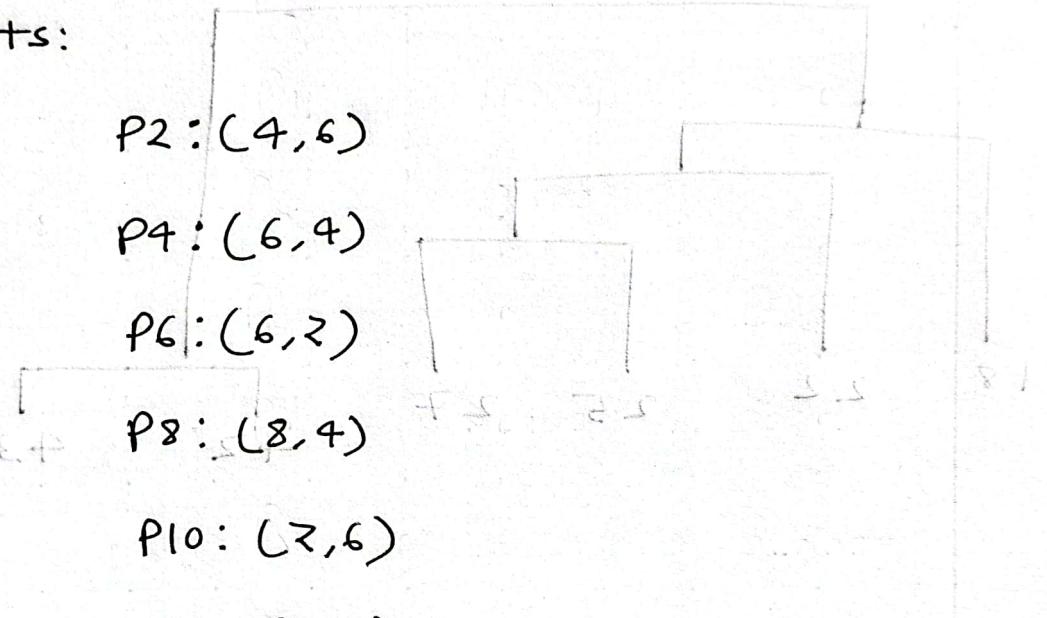
P<sub>3</sub>: (5, 5)      P<sub>4</sub>: (6, 4)

P<sub>5</sub>: (7, 3)      P<sub>6</sub>: (6, 2)

P<sub>7</sub>: (7, 2)      P<sub>8</sub>: (8, 4)

P<sub>9</sub>: (3, 3)      P<sub>10</sub>: (2, 6)

P<sub>11</sub>: (3, 5)      P<sub>12</sub>: (2, 4)



minpts = 4 and epsilon = 1.1

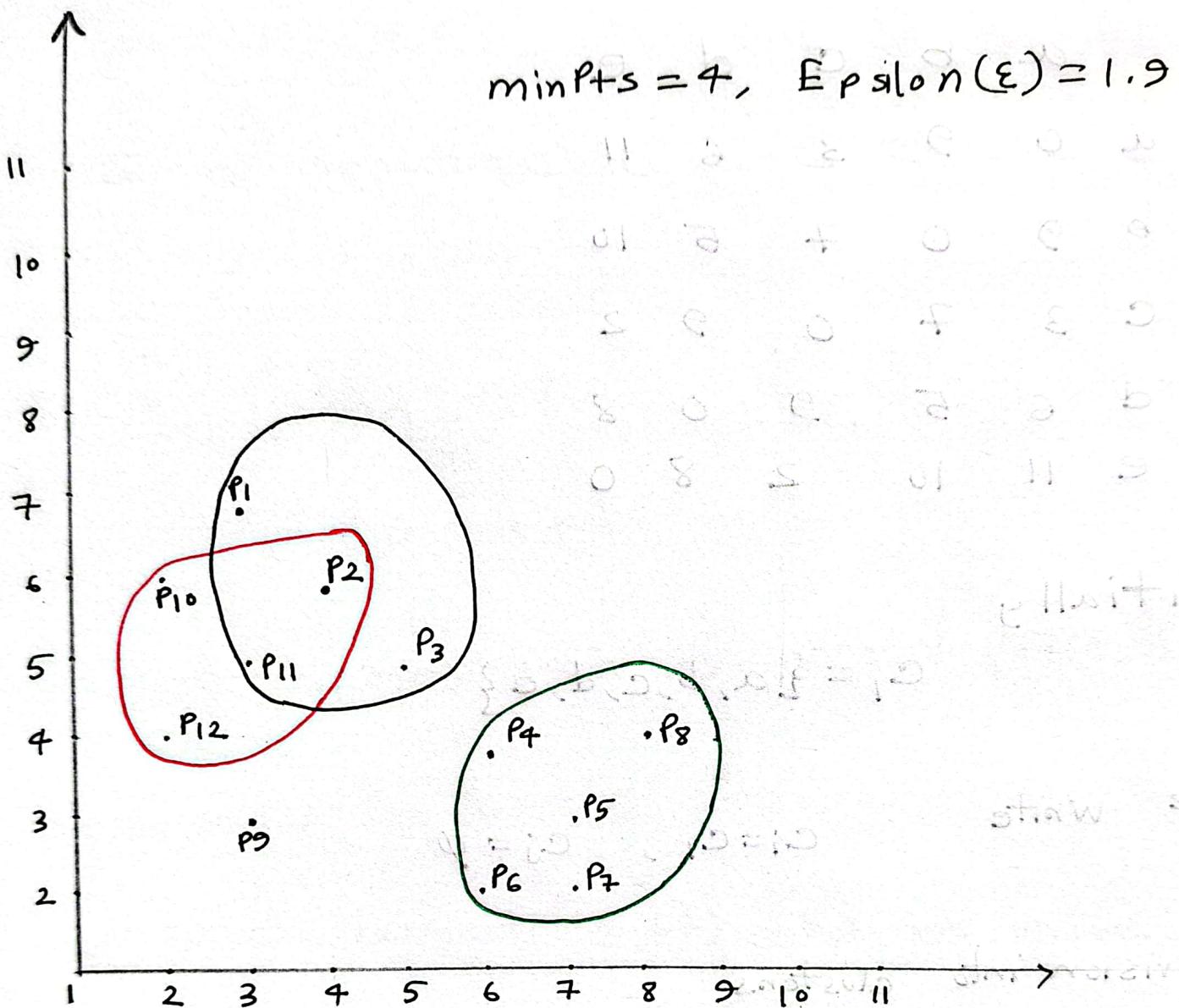
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>	P <sub>8</sub>	P <sub>9</sub>	P <sub>10</sub>	P <sub>11</sub>	P <sub>12</sub>
P <sub>1</sub>	0											
P <sub>2</sub>	1.41	0										
P <sub>3</sub>	2.83	1.41	0									
P <sub>4</sub>	4.24	2.83	1.41	0								
P <sub>5</sub>	5.66	4.24	2.83	1.41	0							
P <sub>6</sub>	5.83	4.44	3.16	2.00	1.41	0						
P <sub>7</sub>	6.40	5.00	3.61	2.24	1.00	0.00	0					
P <sub>8</sub>	5.83	4.44	3.16	2.00	1.41	2.83	2.24	0				
P <sub>9</sub>	4.00	3.16	2.83	3.16	4.00	3.16	4.12	5.10	0			
P <sub>10</sub>	1.41	2.00	3.16	4.44	5.83	5.66	6.40	6.32	3.16	0		
P <sub>11</sub>	2.00	1.41	2.00	3.16	4.44	4.24	5.00	5.10	2.00	1.41	0	
P <sub>12</sub>	3.16	2.83	3.16	4.00	5.10	4.44	5.39	6.00	1.41	2.00	1.41	0

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minPts = 4, Epsilon ( $\epsilon$ ) = 1.9

	Point	status	
P <sub>1</sub> : P <sub>2</sub> , P <sub>10</sub>	P <sub>1</sub>	Noise	Borden
P <sub>2</sub> : P <sub>1</sub> , P <sub>3</sub> , P <sub>10</sub>	P <sub>2</sub>	Core	
P <sub>3</sub> : P <sub>2</sub> , P <sub>4</sub>	P <sub>3</sub>	Noise	Borden
P <sub>4</sub> : P <sub>3</sub> , P <sub>5</sub>	P <sub>4</sub>	Noise	Borden
P <sub>5</sub> : P <sub>4</sub> , P <sub>6</sub> , P <sub>7</sub> , P <sub>8</sub>	P <sub>5</sub>	Core	
P <sub>6</sub> : P <sub>5</sub> , P <sub>7</sub>	P <sub>6</sub>	Noise	Borden
P <sub>7</sub> : P <sub>5</sub> , P <sub>6</sub>	P <sub>7</sub>	Noise	Borden
P <sub>8</sub> : P <sub>5</sub>	P <sub>8</sub>	Noise	Borden
P <sub>9</sub> : P <sub>12</sub>	P <sub>9</sub>	Noise	
P <sub>10</sub> : P <sub>1</sub> , P <sub>11</sub>	P <sub>10</sub>	Noise	Borden
P <sub>11</sub> : P <sub>2</sub> , P <sub>10</sub> , P <sub>12</sub>	P <sub>11</sub>	Core	
P <sub>12</sub> : P <sub>9</sub> , P <sub>11</sub>	P <sub>12</sub>	Noise	Borden

$$\min \text{pts} = 4, \text{ Epsilon}(\varepsilon) = 1.9$$



## DIANA (DIvisive ANALysis)

	a	b	c	d	e	Min
a	0	9	3	6	11	
b	9	0	7	5	10	
c	3	7	0	9	2	
d	6	5	9	0	8	
e	11	10	2	8	0	

① Initially

$$C_1 = \{a, b, c, d, e\}$$

② We write

$$C_i = C_1, \quad C_j = \emptyset$$

③ Division into clusters

④ initial iteration

$$\text{Average dissimilarity of } a = \frac{1}{4} (d(a,b) + d(a,c) + d(a,d) + d(a,e)) \\ = \frac{1}{4} [9 + 3 + 6 + 11] = 7.25$$

$$b = 7.75$$

$$c = 5.25$$

$$d = 7$$

$$e = 7.75$$

7.75 Highest.

Now, we move b to  $c_j$ .

$$c_i = \{a, c, d, e\}, c_j = \varnothing \cup \{b\} = \{b\}$$

(b) Remaining iterations

(i) 2nd iteration

$$\begin{aligned} D_a &= \frac{1}{3} [d(a,c) + d(a,d) + d(a,e)] - \frac{1}{1} [d(a,b)] \\ &= \frac{20}{3} - 9 = 11 - 2.33 = \end{aligned}$$

$$D_c = \frac{1}{3} [d(c,a) + d(c,d) + d(c,e)] - \frac{1}{1} [d(c,b)] = -2.33$$

$$D_d = 0.67 \quad D_e = 0$$

$D_d$  is the largest (0.67). So we move d to  $c_j$ .

We now have,

$$c_i = \{a, c, e\}, c_j = \{b\} \cup \{d\} = \{b, d\}$$

(ii) 3rd iteration

$$\begin{aligned} D_a &= \frac{1}{2} [d(a,c) + d(a,e)] - \frac{1}{2} [d(a,b) + d(a,d)] \\ &= \frac{1}{2} [3 + 11] - \frac{1}{2} [6 + 9] = -0.5 \end{aligned}$$

$$D_c = -13.5 \quad D_e = -2.5$$

All are negative. So we stop and form the clusters  $c_i$  and  $c_j$ .

- ④ To divide,  $c_i$  and  $c_j$ , we compute their diameters.

$$\text{diameter}(c_i) = \max\{d(a, c), d(a, e), d(c, e)\}$$
$$= \max\{3, 11, 6\} = 11$$

$$\text{diameter}(c_j) = \max\{d(b, d)\} = 5$$

The cluster with the largest diameter is  $c_j$ . So we now split  $c_i$ .

we now split  $c_i$ .

$$\{b, d\} = \{b\} \cup \{d\} \cup \{b, d\} = 12$$

$$\{a, c, e\} = \{a\} \cup \{c\} \cup \{e\} = 12$$

naturally, base (i)

$$[(b, d) + (a, c, e)] \rightarrow [(b, d) + (a, c, e)] = 24$$

$$\text{diam} = [24 + 3] \frac{1}{2} = [11 + 8] \frac{1}{2} = 9$$

$$2.5 = 2.5 \cdot 9 = 22.5 = 9$$

# Association Rules Exercise

① The support level 33%.

confidence level 50%.

$$\text{Rule: } X \rightarrow Y \quad \begin{aligned} \text{Support} &= \frac{\text{fnr}(X, Y)}{N} \\ \text{Confidence} &= \frac{\text{fnr}(X, Y)}{\text{fnr}(X)} \end{aligned}$$

①

1-item sets Frequency

Milk  $\rightarrow$  9

Bread  $\rightarrow$  10

Butter  $\rightarrow$  10

Egg  $\rightarrow$  3  $\frac{3}{12} = 0.25 < 0.33$

Cookies  $\rightarrow$  5  $\frac{5}{12} = 0.42 > 0.33$

Ketchup  $\rightarrow$  3

Frequent 1-item sets	Frequency
Milk	9
Bread	10
Butter	10
Cookies	5

②

2-item sets Frequency

Milk, Bread  $\rightarrow$  7

Milk, Butter  $\rightarrow$  7

Milk, Cookies  $\rightarrow$  3

Bread, Butter  $\rightarrow$  9

Bread, Cookies  $\rightarrow$  4

Butter, Cookies  $\rightarrow$  3

Frequent 2-item sets	Frequency
Milk, Bread	7
Milk, Butter	7
Bread, Butter	9
Bread, Cookies	4

Milk, Butter, Bread, Cookies

3-item sets Frequency

Milk, Butter, Bread → 6

Milk, Butter, Cookies → 1

Milk, Bread, Cookies → 2

Butter, Bread, Cookies → 3

Frequent 3-item sets	Frequency
Milk, Bread, Butter	6

Frequent 3-item set =  $\{ \text{Milk, Bread, Butter} \}$

Non-Empty subset are

$\{ \{\text{Milk}\}, \{\text{Bread}\}, \{\text{Butter}\}, \{\text{Milk, Bread}\}, \{\text{Milk, Butter}\}, \{\text{Bread, Butter}\} \}$

Min support = 30%

Min confidence = 60%

# Association Rule

$S \rightarrow (1-S)$

If  $\frac{\text{Support}(1)}{\text{Support}(S)} \geq \text{minimum confidence}$

Rule 1:  $\{\text{Milk}\} \rightarrow \{\text{Bread, Butter}\}$

$$\text{Support} = \frac{6}{12} = 50\%$$

$$\text{Confidence} = \frac{\text{Support}(\text{Milk, Bread, Butter})}{\text{Support}(\text{Milk})} = \frac{6/12}{9/12} = 0.67$$

Valid

Rule 2: {Bread} → {Milk, Butter}

$$\text{Support} = \frac{6}{12} = 50\%$$

$$\text{Confidence} = \frac{\text{Support}(\text{Milk, Bread, Butter})}{\text{Support}(\text{Bread})} = \frac{6/12}{10/12} = 0.6 = 60\%$$

Valid

Rule 3: {Butter} → {Milk, Bread}

$$\text{Support} = 50\%$$

$$\text{Confidence} = \frac{6/12}{\text{Support}(\text{Butter})} = \frac{6/12}{10/12} = 0.6 = 60\%$$

Valid

Rule 4: {Milk, Bread} → {Butter}

$$\text{Support} = 50\%$$

$$\text{Confidence} = \frac{6/12}{\text{Support}(\text{Milk, Bread})} = \frac{6/12}{7/12} = 85.7\%$$

Valid

Rule 5: {Milk, Butter} → {Bread}

$$\text{Support} = 50\%$$

$$\text{Confidence} = \frac{6/12}{7/12} = 85.7\%$$

Valid

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Rule 6:  $\{Bread, Butter\} \rightarrow \{Milk\}$

$$Support = 50\%$$

$$Confidence = \frac{6/12}{9/12} = 66.67\%$$

Valid.

### Solved Example 3

TID	List of Items IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

min Support = 2

Confidence = 50%

butter

butter

1-item sets	Frequency
I1	6
I2	7
I3	6
I4	2
I5	2

Frequent 1-item sets	Frequency
I1	6
I2	7
I3	6
I4	2
I5	2

2-item sets	Frequency
I1 I2 →	4
I1 I3 →	4
I1 I4 →	1
I1 I5 →	2
I2 I3 →	4
I2 I4 →	2
I2 I5 →	2
I3 I4 →	0
I3 I5 →	1
I4 I5 →	0

Frequent 2-item sets	Frequency
I1 I2	4
I1 I3	4
I1 I5	2
I2 I3	4
I2 I4	2
I2 I5	2

### 3-item Sets Frequency

I1 I2 I3 → 2

I1 I2 I4 → 1

I1 I2 I5 → 2

I1 I3 I4 → 0

I1 I3 I5 → 1

I1 I4 I5 → 0

I2 I3 I4 → 0

I2 I3 I5 → 1

I2 I4 I5 → 0

I3 I4 I5 → 0

I1 I2 I3 I4 I5

Frequent 3-item sets	Frequencies
I1 I2 I3	2
I1 I2 I5	2

Frequent 3-item set =  $I \Rightarrow \{1, 2, 3\}$  and  $\{1, 2, 5\}$

Non-empty subsets are

$\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

$\{\{1, 2\}, \{2\}, \{5\}, \{1, 2, 3\}, \{1, 2, 5\}, \{2, 3, 5\}\}$

Association Rule

$$S \rightarrow (1 - s)$$

if  $\text{Support}(I) / \text{Support}(S) \geq \text{min confidence}$

I = {1, 2, 3}

Rule 1:  $\{1\} \rightarrow \{2, 3\}$

$$\text{Support} = \frac{2}{9} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{\text{Support}\{1\}} = \frac{2/9}{6/9} = 33.33\% < 50\%$$

Invalid

Rule 2:  $\{2\} \rightarrow \{1, 3\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{\text{Support}\{2\}} = \frac{2/9}{7/9} = 28.57 < 50\%$$

Invalid

Rule 3:  $\{3\} \rightarrow \{1, 2\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{\text{Support}\{3\}} = \frac{2/9}{6/9} = 33.33\% < 50\%$$

Invalid

Rule 4:  $\{1, 2\} \rightarrow \{3\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{4/9} = 50\% \geq 50\%$$

Valid

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Rule 5:  $\{1, 3\} \rightarrow \{2\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{4/9} = 50\%$$

Valid

Rule 6:  $\{2, 3\} \rightarrow \{1\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{4/9} = 50\%$$

Valid

Now,

$$I = \{1, 2, 5\}$$

Rule 1:  $\{1\} \rightarrow \{2, 5\}$

$$\text{Support} = \frac{2}{9} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{6/9} = 33.33\% < 50\%$$

Invalid

Rule 2:  $\{2\} \rightarrow \{1, 5\}$

$$\text{Support} = 22.22\%$$

$$\text{Confidence} = \frac{2/9}{7/9} = 28.57\%$$

Rule 3:  $\{5\} \rightarrow \{1, 2\}$

Support = 22.22%

$$\text{Confidence} = \frac{2/9}{2/9} = 100\%$$

Valid

Rule 4:  $\{1, 2\} \rightarrow \{5\}$

Support = 22.22%

$$\text{Confidence} = \frac{2/9}{4/9} = 50\%$$

Valid

Rule 5:  $\{1, 5\} \rightarrow \{2\}$

Support = 22.22%

$$\text{Confidence} = \frac{2/9}{2/9} = 100\%$$

Valid

Rule 6:  $\{2, 5\} \rightarrow \{1\}$

Support = 22.22%

$$\text{Confidence} = \frac{2/9}{2/9} = 100\%$$

Valid

## Apriori Algorithm Example 1

Transaction ID	Items
T <sub>1</sub>	Hot Dogs, Buns, ketchup
T <sub>2</sub>	Hot Dogs, Buns
T <sub>3</sub>	Hot Dogs, Coke, chips
T <sub>4</sub>	chips, Coke
T <sub>5</sub>	chips, ketchup
T <sub>6</sub>	Hot Dogs, Coke, chips

minimum support threshold (c) = 33.33%

minimum confident threshold c = 60%

$$\text{Minimum Support Count} = \frac{33.33}{100} \times 6 = 2$$

Item Set	Sup. Count
Hot Dogs	4
Buns	2
Ketchup	2
Coke	3
chips	4



Itemset	Sup Count
Hot Dogs	4
Buns	2
Ketchup	2
Coke	3
chips	4

2 item set      Sup-count

Hot Dogs, Buns → 2

Hot Dogs, ketchup → 1

Hot Dogs, Coke → 2

Hot Dogs, chips → 2

Buns, ketchup → 1

Buns, Coke → 0

Buns, chips → 0

ketchup, Coke → 0

ketchup, chips → 1

Coke, chips → 3

Frequent 2 item sets	Sup Count
Hot Dogs, Buns	2
Hot Dogs, Coke	2
Hot Dogs, chips	2
Coke, chips	3

3 item sets      Sup-count

Hot Dogs, Buns, Coke      0

Hot Dogs, Buns, chips      0

Hot Dogs, Coke, Chips      2

Frequent -3	Sup-count
Hot Dogs, Coke, chips	2

Frequent itemset(1) = { Hot Dogs, Coke, chips }

Non-empty subsets are -

{ {Hot Dogs} , {Coke} , {chips} } { {Hot Dogs, Coke} , {Hot Dogs, chips} , {Coke, chips} }

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Rule 1:  $\{\text{Hot Dogs}\} \rightarrow \{\text{Coke, chips}\}$

$$\text{Support} = \frac{2}{6} = 33.33\%$$

$$\text{Confidence} = \frac{2/6}{4/6} = 50\% \quad \text{Rejected}$$

Rule 2:  $\{\text{Coke}\} \rightarrow \{\text{Hot Dogs, chips}\}$

$$\text{Support} = \frac{2}{6} = 33.33\%$$

$$\text{Confidence} = \frac{2/6}{3/6} = 66.67\% \quad \text{Selected}$$

Rule 3:  $\{\text{Chips}\} \rightarrow \{\text{Hot Dogs, Coke}\}$

$$S \rightarrow 33.33\%$$

$$C \rightarrow \frac{2/6}{4/6} = 50\% \quad \text{Rejected}$$

Rule 4:  $\{\text{Hot Dogs, Coke}\} \rightarrow \{\text{chips}\}$

$$S \rightarrow 33.33\%$$

$$C \rightarrow \frac{2/6}{2/6} = 100\% \quad \text{Selected}$$

Rule 5:  $\{\text{Hot Dogs, chips}\} \rightarrow \{\text{Coke}\}$

$$S \rightarrow 33.33\%$$

$$C \rightarrow \frac{2/6}{2/6} = 100\% \quad \text{Selected}$$

Rule 6:  $\{Coke, Chips\} \rightarrow \{Hot Dogs\}$

$$S \rightarrow 33.33\%$$

$$C \rightarrow \frac{2/6}{3/6} = 66.67\% > 60\%, Selected$$

### Apriori Algorithm Example - 2

TID	items
T1	Hot Dogs, Buns, ketchup
T2	Hot Dogs, Buns
T3	Hot Dogs, Coke, Chips
T4	Chips, Coke
T5	Chips, Ketchup
T6	Hot Dogs, Coke, Chips

minimum Support ( $S=3$ )

Frequent 1 item set	Sup-Count
Hot Dogs	4
Coke	3
Chips	4

Frequent 2 item set	Sup-Count
Coke, chips	3

Frequent item (1)  $\rightarrow \{\text{Coke, chips}\}$

Rule 1:  $\{\text{Coke}\} \rightarrow \{\text{chips}\}$

$$S \rightarrow \frac{3}{6} = 50\%$$

$$C \rightarrow \frac{3}{6} = 100\% \quad \text{Selected}$$

Rule 2:  $\{\text{chips}\} \rightarrow \{\text{Coke}\}$

$$S \rightarrow 50\%$$

$$C \rightarrow \frac{3/6}{4/6} = 75\% \quad \text{Selected.}$$