DEPARTMENT OF STATISTICS

Jahangirnagar University

Institute of Information Technology

Subject: Probability & Stochastic Processes

Code: IT-5102
Assignment-1
Full Marks: 10

Course Instructor: Prof. Dr. Md Rezaul Karim

Objective

The purpose of this assignment is to understand the concept of Markov chains and transition probability matrices. By applying these matrices, you will also learn to calculate the stationary distribution, which provides insights into the long-term behavior of the system (the grades of students over multiple semesters).

Instructions

You will collect data on the grades of students over several semesters. The grades are typically represented as:

- A+, A, A-, B+, B, B-, C+, C, C-, etc.

Each student's grade for each semester should be recorded. Your task is to create a transition probability matrix that models the changes in grades from one semester to the next.

Tasks

1. Data Collection

- Collect data on the grades of students over at least **8 semesters** (you can collect more semesters if necessary).
- For each student, record the grade in each semester. The grades should follow a clear order, such as A+, A, A-, B+, B, B-, etc.
- Make sure to collect enough data from a variety of students so that the transitions between grades can be meaningfully analyzed.

Please go on to the next page…

2. Prepare Data for Transition Matrix

- Organize the grades of all students by semester (Semester 1 to Semester 8).
- For each student, calculate the change in grade from Semester n to Semester $n+1$ (e.g., from Semester 1 to Semester 2, from Semester 2 to Semester 3, etc.).
- The set of grades in the data forms the *state space* for your system. Label each grade from 1 to n (where n is the total number of distinct grades).

3. Construct the Transition Probability Matrix

- Construct the transition matrix P, where p_{ij} represents the probability of a student transitioning from grade i in Semester n to grade j in Semester $n+1$.
- To calculate p_{ij}, you need to:
  1. Count how many times grade i is followed by grade j.
  2. Divide this count by the total number of times grade i appears in the data.

4. Check the Nature of the State

- Analyze whether the system represented by the transition matrix is:
  1. *Irreducible*: All states can be reached from any state.
  2. *Aperiodic*: There is no fixed period for transitions.
- To check for *irreducibility*, verify that for every pair of grades, there is a non-zero probability of transitioning between them across multiple semesters.
- To check for *aperiodicity*, ensure that for any state, the greatest common divisor of the number of steps to return to the same state is 1.

5. Compute the Stationary Distribution

- The stationary distribution is a probability vector π such that:

$$\pi P = \pi$$

  This represents the long-term distribution of grades across semesters.

- Solve the equation $\pi P = \pi$ with the condition that the sum of all elements in π is 1:

$$\sum_{i=1}^{n} \pi_i = 1$$

- You can solve this equation using linear algebra techniques or computational tools such as Python or R.

6. Interpretation and Analysis

- Once you have computed the stationary distribution, interpret the results. What does the stationary distribution tell you about the long-term behavior of student grades?
- For example, does it suggest that most students will eventually have an average grade (like a B or C), or does it indicate more variability between grades?
- Discuss the real-world implications of your findings, such as how the transition matrix can be used by educational institutions to forecast student performance in future semesters.

Deliverables

- The transition probability matrix P showing the probabilities of grade transitions between semesters.
- An analysis of whether the Markov chain is irreducible and aperiodic.
- The stationary distribution vector π.
- A report discussing the interpretation of the stationary distribution, including insights on the nature of grade transitions over time.