# LIBOR Transition

Theoretical Insights and Numerical Applications in Interest Rate Modeling

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# Contents

# Abstract

The transition from LIBOR to alternative risk-free reference rates has triggered one of the most significant shifts in global financial markets. This project explores the theoretical frameworks underpinning LIBOR-based instruments and the adjustments required in valuation models due to the shift to benchmarks such as SOFR, SONIA, and €STR. Detailed analyses of yield curve interpolation techniques including Nelson-Siegel, Svensson, and cubic splines are presented, along with numerical simulations of interest rate derivatives. The results provide insights into the practical implications and challenges of the LIBOR transition, particularly in pricing, risk management, and the overall stability of financial markets.


**Contribution :** In this project, all three group members collaborated effectively to deliver a comprehensive analysis of the LIBOR transition. Shakil Rohimun was responsible for retrieving real market data from FRED and developed both the MATLAB code and the Latex section for the Mathematical Deep Dive. Chaker Meraihi contributed to the coding and prepared the content for the Implications for Financial Markets section. Charles Yang handled the MATLAB coding and Latex preparation for the Transition from LIBOR to New Reference Rates section. Additionally, all team members participated in the literature review and supported each other throughout the project.

**Keywords :** LIBOR Transition, Yield Curve, Nelson-Siegel, Svensson, Cubic Splines, Interest Rate Modeling, Financial Derivatives, SOFR, SONIA, Risk-Free Rates

# Introduction

LIBOR has long served as a key benchmark for short-term interest rates worldwide. Established in the 1980s to reflect the average rate at which major banks borrow from each other, LIBOR became fundamental for pricing a wide range of financial products. However, following the 2008 crisis and subsequent manipulation scandals, confidence in LIBOR diminished as its determination increasingly relied on subjective estimates rather than actual market transactions. Regulators have thus pushed for a transition to alternative risk-free rates such as the Secured Overnight Financing Rate (SOFR) in the United States, the Euro Short-Term Rate (€STR) in the Eurozone, and the Sterling Overnight Index Average (SONIA) in the United Kingdom. This transition presents significant challenges, including the need to update valuation models and modify financial contracts.

All the real data used in this project were obtained from the FRED website (`https://fred.stlouisfed.org/`). Data were retrieved using the following codes, corresponding to these financial indicators:

- 3-Month USD LIBOR (LIOR3M): LIBOR for U.S. dollars with a 3-month maturity.
- 3-Month GBP LIBOR (LIOR3MUKM): LIBOR for British pounds with a 3-month maturity.
- Secured Overnight Financing Rate (SOFR): The cost of overnight borrowing collateralized by U.S. Treasury securities.
- Sterling Overnight Index Average (SONIA) (IUDSOIA): The overnight interest rate for unsecured transactions in British pounds.
- Euro Short-Term Rate: Volume-Weighted Trimmed Mean Rate (ECBESTRVOLWGTTRMDMNRT): The euro short-term rate calculated as a volume-weighted average of overnight borrowing costs.
- Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity (DGS1): Yield on U.S. Treasury securities with a 1-year maturity.
- Market Yield on U.S. Treasury Securities at 2-Year Constant Maturity (DGS2): Yield on U.S. Treasury securities with a 2-year maturity.
- Market Yield on U.S. Treasury Securities at 5-Year Constant Maturity (DGS5): Yield on U.S. Treasury securities with a 5-year maturity.
- Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity (DGS10): Yield on U.S. Treasury securities with a 10-year maturity.

The aim of this project is to analyze the implications of the LIBOR transition by exploring the mathematical frameworks behind LIBOR-based instruments, assessing the impact of switching to alternative reference rates, and applying numerical methods to model this transition in the context of interest rate derivatives.

# 1 Definitions and Context

## 1.1 Interest Rates and Benchmark Rates

### 1.1.1 Definition of Reference Interest Rates

Interest rates play a fundamental role in financial markets, influencing everything from central bank policies to corporate financing decisions. In mathematical terms, an interest rate $r(t)$ at time $t$ represents the cost of borrowing or the return on lending per unit of currency. Given an initial capital $P_0$, the value $P(T)$ at a future time $T$ is determined by the compounding structure of interest rates.

The general formula for continuously compounded interest is:

$$P(T) = P_0 e^{rT} \tag{1}$$

where:

- $P_0$ is the initial principal,
- $r$ is the continuous interest rate,
- $T$ is the time to maturity.

In contrast, for discrete compounding, the future value is given by:

$$P(T) = P_0 \left(1 + \frac{r}{m}\right)^{mT} \tag{2}$$

where $m$ is the number of compounding periods per year.

Reference interest rates, often referred to as benchmark rates, serve as standard measures to determine borrowing costs across financial markets. These rates are used in various financial instruments such as loans, derivatives, and bonds.

### 1.1.2 Fixed vs Floating Interest Rates

An important distinction in financial markets is between fixed and floating interest rates.

**Fixed interest rates** remain constant over the life of a financial contract. The cash flow at time $t$ is given by:

$$C_t = C_0 e^{r_{\text{fixed}} t} \tag{3}$$

where $r_{\text{fixed}}$ is a predetermined rate.

**Floating interest rates** vary according to a benchmark, such as LIBOR or SOFR. The cash flow is determined dynamically:

$$C_t = C_0 e^{r_{\text{floating}}(t) t} \tag{4}$$

where $r_{\text{floating}}(t)$ is a time-dependent rate linked to market conditions.

Floating rates are widely used in derivative contracts and loans, where periodic adjustments ensure that the rate reflects current market conditions.

### 1.1.3 The Role of Benchmark Rates

Benchmark interest rates are essential for pricing financial instruments and serve as reference points for variable-rate lending. Commonly used benchmarks include:

- **LIBOR (London Interbank Offered Rate)**: Historically the most used reference rate.
- **EURIBOR (Euro Interbank Offered Rate)**: The benchmark for the Eurozone.
- **SOFR (Secured Overnight Financing Rate)**: A risk-free rate based on US Treasury repurchase agreements.
- **€STR (Euro Short-Term Rate)**: A replacement for EONIA in the Eurozone.

Each of these benchmarks has unique properties based on the markets they represent. LIBOR, for instance, was calculated based on interbank lending rates, while SOFR is derived from secured repo transactions, making it a nearly risk-free rate.

To concretely illustrate the historical evolution of interest rates discussed earlier, Figure 1 displays raw market data retrieved from the FRED database. This figure highlights the historical behavior of LIBOR rates and their alternative benchmarks, supporting the context described in this introductory section and setting the stage for subsequent analysis.

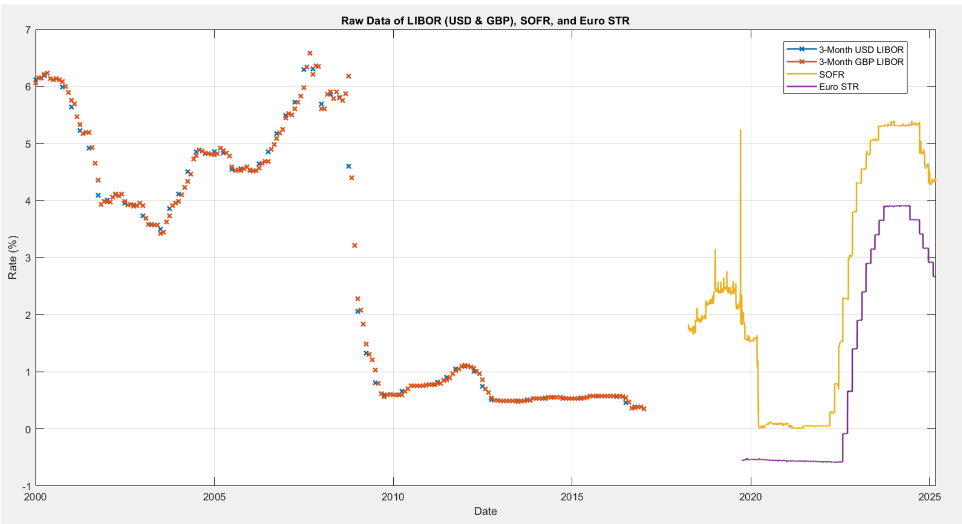


Figure 1: Raw data obtained from FRED.

## 1.2 What is LIBOR?

### 1.2.1 Definition and Calculation of LIBOR

LIBOR, or the London Interbank Offered Rate, was the most widely used benchmark for short-term interest rates globally. It was set daily for multiple currencies and maturities, reflecting the average rate at which major banks could borrow unsecured funds in the interbank market.

Mathematically, LIBOR for a given maturity $T$ is defined as:

$$r_{\text{LIBOR}}(T) = \frac{1}{T}\left(\frac{P_0}{P(T)} - 1\right) \tag{5}$$

where:

- $P_0$ is the present value of a zero-coupon bond maturing at $T$,
- $P(T)$ is its market price at time $T$.

LIBOR was published for different maturities: overnight, 1 week, 1 month, 3 months, 6 months, and 12 months.

#### 1.2.2 Markets and Instruments Impacted by LIBOR

LIBOR served as the benchmark rate for a vast array of financial products, including:

- Interest rate swaps : Floating-leg payments were often tied to LIBOR.
- Futures and options : LIBOR futures contracts allowed traders to hedge against short-term interest rate changes.
- Corporate and mortgage loans : Many adjustable-rate loans were pegged to LIBOR.
- Floating-rate bonds : Interest payments adjusted periodically based on LIBOR.

LIBOR's role as a global benchmark meant that fluctuations in its rate had widespread implications for financial stability.

#### 1.2.3 Criticism and Scandals: Manipulations and Liquidity Issues

Despite its importance, LIBOR suffered from serious flaws. The most notable concerns were:

**1. Manipulation Scandals** In the aftermath of the 2008 financial crisis, it was discovered that banks had been manipulating LIBOR submissions to project financial strength or influence derivative contracts. Mathematically, banks reported artificially low or high borrowing rates, impacting the calculated LIBOR:

$$r_{\text{LIBOR, manipulated}} = r_{\text{LIBOR, true}} + \epsilon \tag{6}$$

where $\epsilon$ represents the bias introduced by manipulation.

**2. Decline in Interbank Lending** LIBOR was based on unsecured lending between banks, but post-crisis, interbank lending volume decreased significantly. This meant that the rate was derived from fewer actual transactions and relied heavily on estimations rather than market-driven data.

**3. Transition to Alternative Rates** Due to these issues, regulators recommended a transition to risk-free rates such as SOFR and €STR. These alternatives are based on real transactions rather than subjective estimates.

Figure 2 compares the historical trends and 30-day rolling volatility of the 3-month LIBOR rates (USD and GBP) against SOFR. This visual comparison clearly highlights the relatively lower volatility and increased reliability of SOFR, directly addressing LIBOR's susceptibility to manipulation discussed in this subsection.

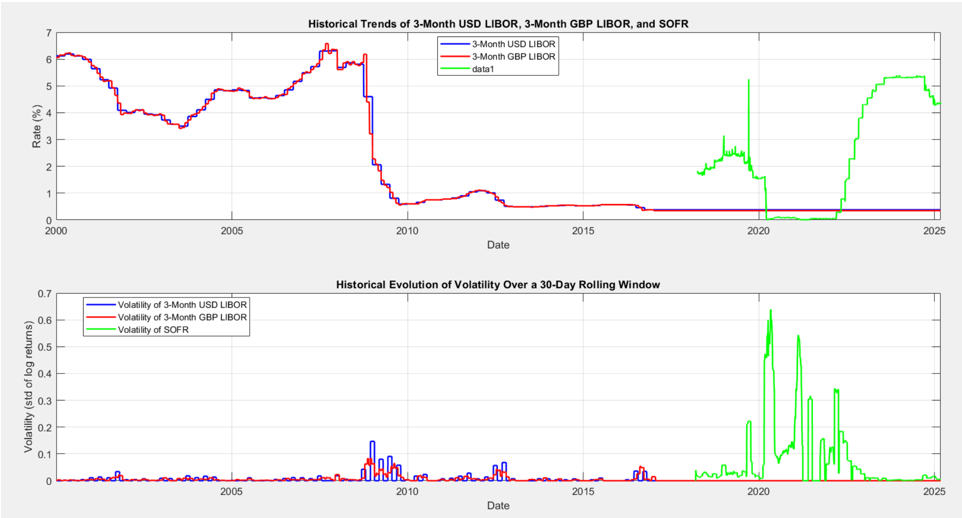


Figure 2: Comparison of 3-Month USD LIBOR, 3-Month GBP LIBOR, and SOFR: Historical Trends and 30-Day Rolling Volatility

LIBOR's decline is a consequence of both its susceptibility to manipulation and a shift in market dynamics. As global finance moves toward more transparent and transaction-based benchmarks, it is essential to understand the mathematical and financial implications of this transition. The next chapters will delve deeper into the modeling of alternative risk-free rates and their impact on financial derivatives.

# 2 Transition from LIBOR to New Reference Rates

## 2.1 Why Abandon LIBOR?

The transition away from LIBOR is driven by concerns about its reliability, a declining volume of underlying transactions, and regulatory initiatives aimed at ensuring financial stability. The following subsections outline the fundamental reasons behind this shift.

### 2.1.1 Loss of Reliability and Lack of Underlying Transactions

LIBOR was originally designed to reflect the average rate at which major banks borrow from one another in the interbank market. However, since the 2008 financial crisis, the volume of unsecured interbank lending has significantly declined. As a result, LIBOR rates have increasingly relied on expert judgment rather than actual transactions, making the benchmark susceptible to manipulation.

Mathematically, the LIBOR rate at a given maturity $T$ is traditionally defined as:

$$r_{\text{LIBOR}}(T) = \frac{1}{T} \sum_{i=1}^{N} \omega_i r_i(T) \tag{7}$$

where:

- $r_i(T)$ represents the interbank borrowing rate submitted by bank $i$,
- $\omega_i$ is the weight assigned to bank $i$ in the LIBOR panel,
- $N$ is the number of panel banks.

However, due to the lack of actual transactions, the estimation of $r_{\text{LIBOR}}(T)$ was largely based on subjective inputs rather than market-driven rates. This lack of robustness led to discrepancies between reported LIBOR and actual borrowing costs.

#### 2.1.2 Regulatory Reforms and FSB Recommendations

In response to LIBOR's deficiencies, the Financial Stability Board (FSB) initiated global efforts to transition toward alternative benchmarks that are based on observable transactions. The key recommendations from the FSB include:

- Reducing reliance on expert judgment and ensuring rates are based on verifiable market activity.
- Promoting risk-free rates (RFRs) that reflect actual borrowing costs in secured or overnight lending markets.
- Encouraging financial institutions to adapt their valuation models and contracts to incorporate these new benchmarks.

The regulatory shift has been supported by major central banks and financial authorities, including the U.S. Federal Reserve, the European Central Bank (ECB), and the Bank of England.

### 2.2 New Benchmark Rates

The new reference rates replacing LIBOR aim to provide more accurate and transparent measures of borrowing costs. Unlike LIBOR, these rates are based on transactions in liquid markets, making them more resilient to manipulation.

#### 2.2.1 SOFR: Secured Overnight Financing Rate

The Secured Overnight Financing Rate (SOFR) is the designated replacement for USD LIBOR in the United States. SOFR is based on the cost of borrowing cash overnight collateralized by U.S. Treasury securities in the repurchase agreement (repo) market.

Mathematically, SOFR at time $t$ is computed as:

$$\text{SOFR}(t) = \frac{\sum_{i=1}^{N} V_i R_i}{\sum_{i=1}^{N} V_i} \tag{8}$$

where:

- $V_i$ is the transaction volume of repo $i$,
- $R_i$ is the interest rate for transaction $i$,
- $N$ is the number of repo transactions.

SOFR differs from LIBOR in that it is based on secured transactions, making it a nearly risk-free rate (RFR). However, since it is an overnight rate, market participants have developed term SOFR rates to approximate longer maturities.

### 2.2.2 €STR: Euro Short-Term Rate

The Euro Short-Term Rate (€STR) was introduced by the European Central Bank (ECB) to replace EONIA and serve as an alternative to EURIBOR. €STR is calculated based on unsecured overnight borrowing between euro-area banks.

The rate at time $t$ is computed as:

$$\text{€STR}(t) = \frac{\sum_{i=1}^{N} V_i R_i}{\sum_{i=1}^{N} V_i} \tag{9}$$

where $V_i$ and $R_i$ are the volume and interest rate of each eligible transaction.

Compared to LIBOR, €STR is based on real transactions rather than bank-submitted estimates, making it a more reliable indicator of euro-area funding costs.

### 2.2.3 SONIA: Sterling Overnight Index Average

The Sterling Overnight Index Average (SONIA) is the primary replacement for GBP LIBOR. Managed by the Bank of England, SONIA reflects the average interest rate paid on overnight unsecured borrowing in the sterling market.

SONIA is calculated as:

$$\text{SONIA}(t) = \frac{\sum_{i=1}^{N} V_i R_i}{\sum_{i=1}^{N} V_i} \tag{10}$$

where the summation is taken over eligible unsecured lending transactions in the overnight market.

SONIA has been in use since 1997 and has been widely adopted for financial contracts transitioning from GBP LIBOR.

Figure 3 illustrates historical trends and the 30-day rolling volatility of GBP 3-month LIBOR compared with SONIA. This visualization emphasizes the reduced volatility associated with SONIA, reinforcing the discussion on the transition to more transparent and reliable benchmarks described in this subsection.

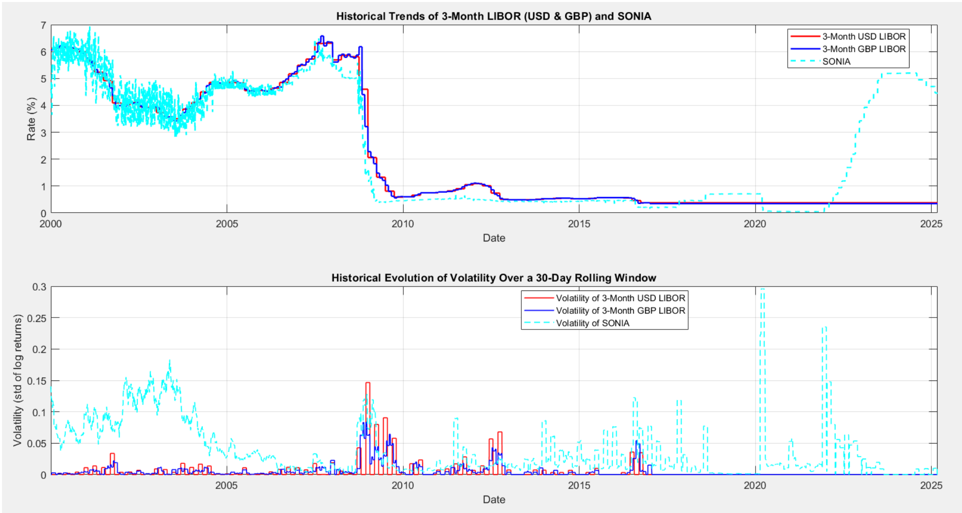


Figure 3: Historical Trends and 30-Day Rolling Volatility of 3-Month LIBOR (USD & GBP) and SONIA

### 2.2.4 Other Alternative Rates: TONA, SARON, and More

In addition to SOFR, €STR, and SONIA, other jurisdictions have developed their own risk-free reference rates:

- TONA (Tokyo Overnight Average Rate): Japan's replacement for JPY LIBOR.
- SARON (Swiss Average Rate Overnight): The Swiss National Bank's alternative to CHF LIBOR, based on secured transactions in the Swiss repo market.
- HONIA (Hong Kong Overnight Index Average): Used in Hong Kong as a transition from HIBOR.

Each of these rates follows a methodology similar to SOFR and €STR, ensuring that they are grounded in actual market transactions rather than estimates.

Figure 4 provides a comprehensive overview of various global LIBOR replacements, summarizing their key characteristics. This graphical representation complements the textual description in this subsection by clearly displaying the methodological and regional differences among the new risk-free rates.

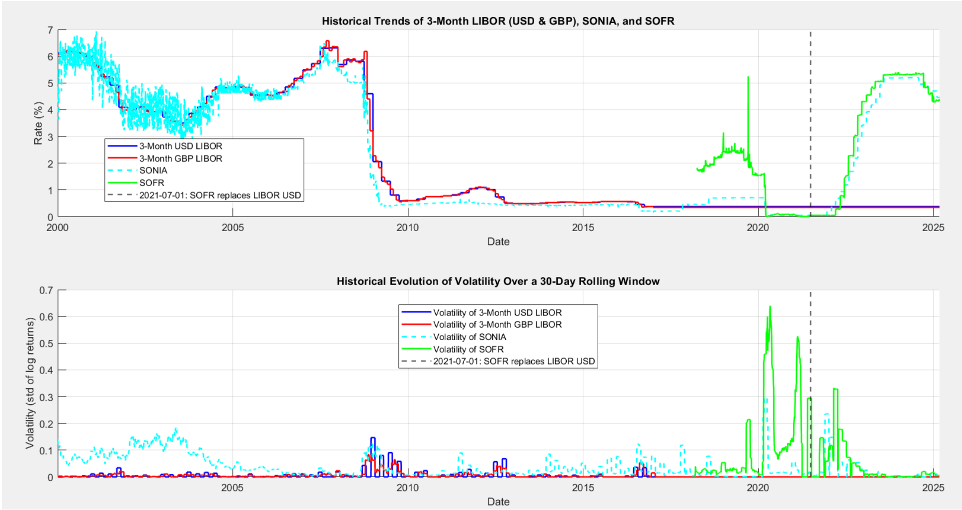


Figure 4: Overview of Global LIBOR Replacements and Their Characteristics

The transition from LIBOR to alternative risk-free rates represents one of the most significant shifts in financial markets in recent history. The move is necessary due to LIBOR's declining reliability and the need for more transparent, transaction-based benchmarks. Each new rate SOFR, €STR, SONIA, and others provides a more robust measure of borrowing costs, reducing systemic risk and enhancing market stability.

# 3 Implications for Financial Markets

The transition from LIBOR to new reference rates has far-reaching consequences across financial markets, particularly in the pricing and risk management of interest rate derivatives, bonds, and structured products. The following sections examine the impact on key financial instruments and the necessary adjustments in risk modeling.

## 3.1 Impact on Interest Rate Derivatives

### 3.1.1 Interest Rate Swaps

Interest rate swaps (IRS) are among the most affected instruments in the LIBOR transition, as they traditionally involve the exchange of fixed interest payments for floating-rate payments tied to LIBOR. The valuation of a vanilla interest rate swap with notional $N$, fixed rate $r_f$, floating rate $r_{\text{LIBOR}}(t)$, and payment dates $t_1, t_2, ..., t_n$ is given by:

$$V_{\text{swap}} = N \sum_{i=1}^{n} e^{-r(t_i)t_i} \left[ r_{\text{LIBOR}}(t_i) - r_f \right] \Delta t_i \tag{11}$$

where:

- $r_{\text{LIBOR}}(t_i)$ is the floating rate at time $t_i$,
- $r_f$ is the fixed swap rate,
- $\Delta t_i$ is the accrual period,
- $r(t_i)$ is the discount rate.

With the transition to SOFR, SONIA, and €STR, floating legs must now reference these new overnight rates. Since these rates are compounded in arrears rather than determined in advance like LIBOR, swap valuation models must be adjusted to account for the accrual-based calculation:

$$r_{\text{SOFR}}(t_i) = \frac{1}{\Delta t_i} \left( \prod_{j=1}^{m} (1 + r_j \Delta t_j) - 1 \right) \tag{12}$$

where $r_j$ are the daily overnight SOFR rates within the accrual period.

### 3.1.2 Application : Impact of the LIBOR Transition on Interest Rate Swaps

To illustrate the impact of the transition from LIBOR to SONIA, we apply the valuation formulas to a concrete example using real market data. We consider a vanilla interest rate swap with a notional of $N = 1000000$ GBP, a fixed rate of $rf = 1.5\%$, and quarterly payments ($\Delta t_i = 0.25$).

The floating leg of the swap is initially referenced to the 3-month LIBOR GBP rate. Following the transition, the floating leg is referenced to the SONIA GBP rate. We use historical data for both rates from January 2020 to April 2024.

The data used for this analysis includes:

- LIBOR 3M GBP: The historical 3-month LIBOR rates for GBP.
- SONIA GBP: The historical SONIA overnight rates for GBP.

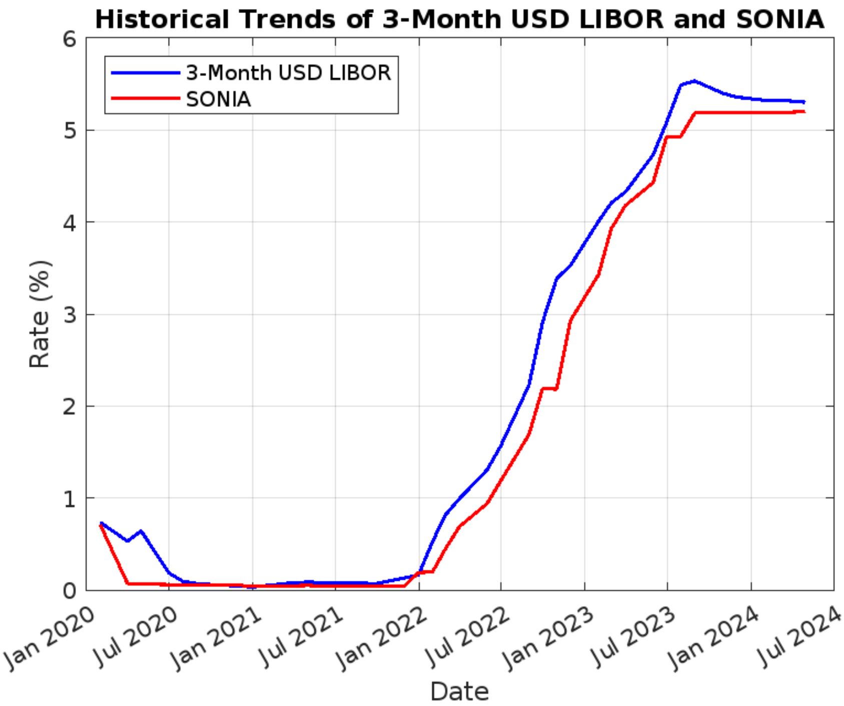


Figure 5: Historical Trends of 3-Month USD LIBOR and SONIA

**Valuation of the LIBOR-based Swap :** Using the valuation formula for LIBOR-based swaps (Equation 11), where r(ti) is the discount rate (assumed to be the SONIA rate), we calculate the value of the LIBOR-based swap as: $V_{\text{swap, LIBOR}} = 979,636.49$ GBP

**Valuation of the SONIA-based Swap :** For the SONIA-based swap, the floating rate is compounded in arrears as described in Equation (12). The value of the SONIA-based swap is calculated as: $V_{\text{swap, SONIA}} = 495,019.55$ GBP

**Results and Analysis :** The results of the swap valuations are summarized in the following table.

Table 1: Valuation of LIBOR-based and SONIA-based Swaps

| **Type of Swap** | **Valuation (GBP)** |
|---|---|
| LIBOR-based Swap | 979,636.49 |
| SONIA-based Swap | 495,019.55 |

This difference is primarily due to the fact that LIBOR rates are generally higher than SONIA rates, leading to larger floating payments in the LIBOR-based swap. The transition to SONIA thus reduces the value of the swap, reflecting the lower risk and volatility associated with SONIA.

The transition from LIBOR to SONIA has a substantial impact on the valuation of interest rate swaps. Market participants must adjust their valuation models and risk management strategies to account for the differences between these rates. This example highlights the importance of understanding the implications of benchmark rate transitions for financial derivatives.

### 3.1.3 Interest Rate Options

Interest rate options such as caps, floors, and swaptions have traditionally relied on LIBOR as the floating reference rate. The pricing of a European caplet, which pays out when the floating rate exceeds a strike $K$, is given by:

$$V_{\text{caplet}} = e^{-rT}\mathbb{E}\left[\max(r_{\text{LIBOR}} - K, 0)\right] \tag{13}$$

The challenge in transitioning to SOFR-based options arises due to differences in rate behavior. LIBOR is a forward-looking term rate, while SOFR is compounded overnight. As a result, pricing models must adapt to account for SOFR's lower volatility and different distributional properties.

Figure 6 displays the impact of transitioning from LIBOR to SOFR on the valuation of interest rate caps and floors. The chart emphasizes the adjustments required in option pricing models due to fundamental differences in volatility and structure between these two rates, as discussed previously.

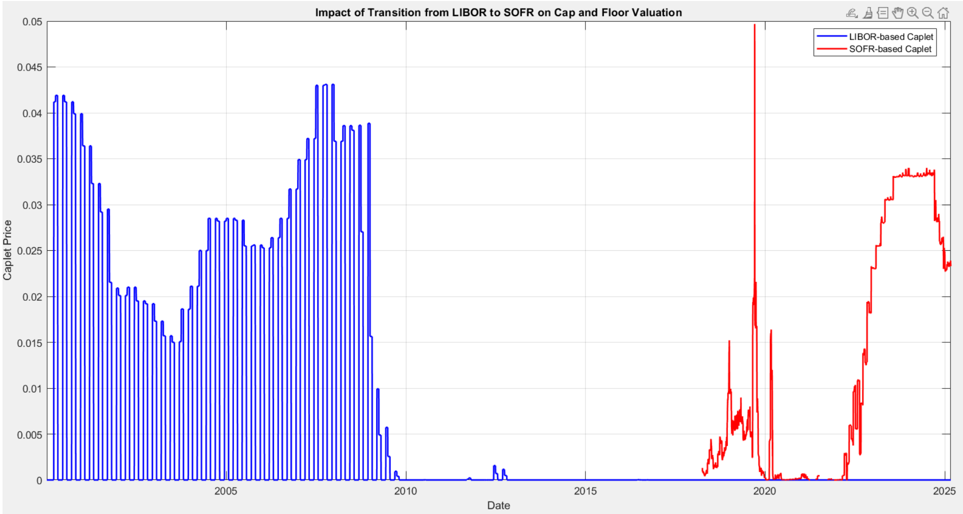


Figure 6: Impact of Transition from LIBOR to SOFR on Cap and Floor Valuation

#### 3.1.4 Structured Products and Cross-Currency Swaps

Structured products, including callable range accrual notes and autocallables, have been heavily linked to LIBOR. The transition requires adjustments in coupon calculations and risk management.

Cross-currency swaps, where counterparties exchange principal and interest payments in different currencies, must also migrate to alternative risk-free rates. The valuation of a cross-currency swap under LIBOR was:

$$V_{\mathrm{CCS}} = \sum_{i=1}^{n} e^{-r_{\mathrm{USD}}(t_i)t_i} \left(r_{\mathrm{LIBOR,USD}}(t_i) - r_{\mathrm{LIBOR,EUR}}(t_i)\right) \Delta t_i \tag{14}$$

With the transition, LIBOR terms must be replaced with the appropriate overnight rates, such as SOFR for USD and €STR for EUR.

## 3.2 Impact on Bond and Loan Markets

### 3.2.1 Floating-Rate Bonds

Floating-rate bonds (FRNs) typically reset interest payments based on LIBOR. The coupon payment at time $t$ for a LIBOR-linked FRN was:

$$C_t = N \times (r_{\mathrm{LIBOR}}(t) + \text{spread}) \times \Delta t \tag{15}$$

With the shift to SOFR-based FRNs, coupons now reference a compounded overnight rate:

$$C_t = N \times \left( \frac{1}{\Delta t} \left( \prod_{j=1}^{m} (1 + r_j \Delta t_j) - 1 \right) + \text{spread} \right) \times \Delta t \tag{16}$$

These modifications impact pricing, risk sensitivity, and investor demand.

### 3.2.2 Syndicated Loans and Mortgages

Many corporate loans and mortgages have historically been tied to LIBOR. The shift to alternative rates like SOFR and €STR necessitates contract amendments, with fallback provisions specifying how loans will transition if LIBOR ceases to be published.

## 3.3 Risk Management and Model Adjustments

### 3.3.1 Fallback Rates for Existing Derivatives

Fallback rates define the methodology for transitioning existing LIBOR-linked contracts to new rates. A common approach involves using an adjusted risk-free rate plus a spread:

$$r_{\text{Fallback}}(t) = r_{\text{RFR}}(t) + S_{\text{adjustment}} \tag{17}$$

where $S_{\text{adjustment}}$ compensates for the economic differences between LIBOR and the new rate.

### 3.3.2 Spread Adjustments Between LIBOR and New Rates

The difference between LIBOR and risk-free rates is non-trivial, requiring spread adjustments to ensure economic equivalence. This spread is often calculated as the median historical difference between LIBOR and the alternative rate over a predefined period:

$$S_{\text{adjustment}} = \text{Median}\left(r_{\text{LIBOR}} - r_{\text{RFR}}\right) \tag{18}$$

### 3.3.3 Changes in Interest Rate Risk Modeling

The transition necessitates significant updates in risk modeling approaches:

- Yield Curve Construction : Moving from LIBOR-based discounting to OIS-based discounting.
- Volatility Models : LIBOR exhibited term structure volatility, while SOFR rates are overnight-based, requiring model recalibration.
- Monte Carlo Simulations : Adjustments to stochastic processes used in pricing derivatives.

For instance, under the Hull-White model, the evolution of LIBOR rates was given by:

$$dr_{\text{LIBOR}} = \theta(t)dt + \sigma dW_t \tag{19}$$

where $\sigma$ represents volatility and $W_t$ is a Wiener process. For SOFR, which lacks a term structure, models must adapt accordingly.

The transition from LIBOR affects all major asset classes, including derivatives, bonds, and loans. While the new risk-free rates provide increased transparency and reliability, they also introduce structural changes in pricing, valuation models, and risk management frameworks.

# 4 Mathematical Deep Dive

The transition from LIBOR to alternative risk-free rates (RFRs) requires a significant shift in interest rate modeling. This section explores the mathematical frameworks for LIBOR modeling, short-rate models, yield curve construction, and the impact of the transition on financial instruments.

## 4.1 Modeling LIBOR and the Transition

### 4.1.1 The Black Model for LIBOR

LIBOR derivatives, such as caps and floors, have traditionally been priced using the Black model. Given a forward LIBOR rate $F(T)$ at maturity $T$, the price of a caplet with strike $K$ and volatility $\sigma$ is:

$$V_{\text{caplet}} = P(0,T)\left[F(T)N(d_1) - KN(d_2)\right] \tag{20}$$

where:

$$d_1 = \frac{\ln(F(T)/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T} \tag{21}$$

LIBOR's transition to SOFR introduces complications because SOFR is an overnight rate, and term rates must be derived using compounding methodologies rather than direct forward rates.

Figure 7 depicts caplet prices as a function of strike rates computed using the Black model. This figure illustrates the price sensitivity relative to strike selection, which serves as a reference point for understanding the challenges faced in adapting traditional LIBOR-based pricing models to SOFR, a central focus of this subsection.

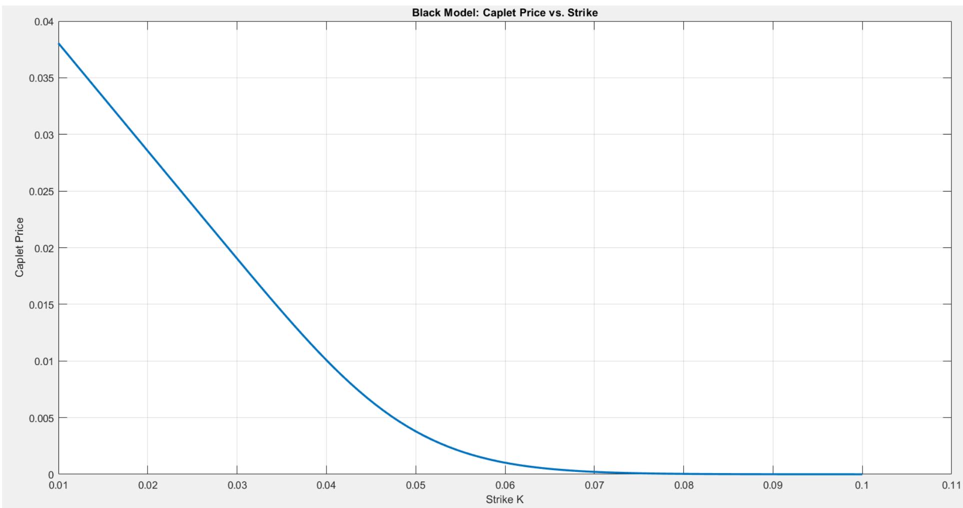


Figure 7: Caplet price as a function of the strike using the Black model.

### 4.1.2 Short-Rate Models: Vasicek and Cox-Ingersoll-Ross (CIR)

Short-rate models provide a fundamental framework for interest rate modeling. Two common models are:

**Vasicek Model:**

$$dr_t = \kappa(\theta - r_t)dt + \sigma dW_t \tag{22}$$

where:

- $\kappa$ is the speed of mean reversion,
- $\theta$ is the long-term mean level,
- $\sigma$ is the volatility,
- $W_t$ is a Wiener process.

Figure 8 presents a simulated short-rate trajectory based on the Vasicek model, clearly showing mean-reversion behavior. This simulation provides practical insight into the Vasicek model's key properties, which are essential for modeling interest rate dynamics in the context of new benchmark rates discussed earlier.

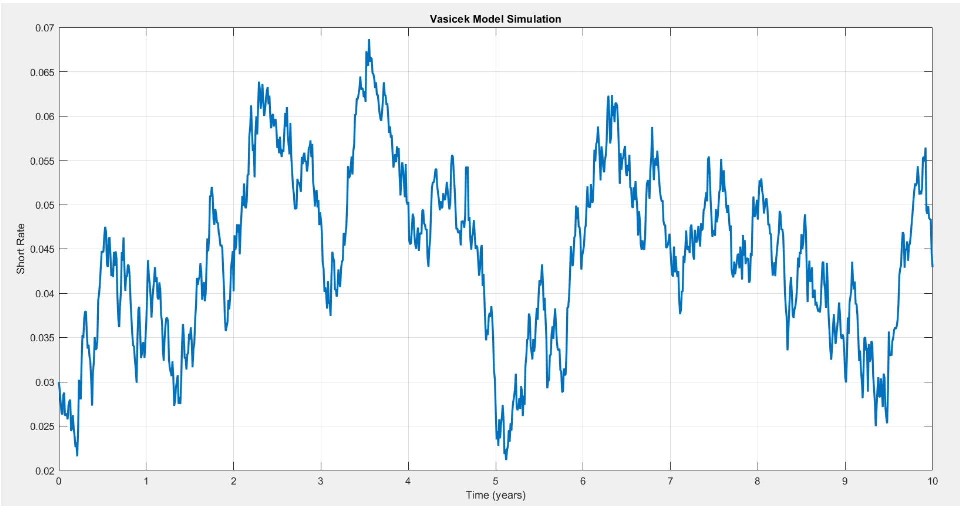


Figure 8: Simulation of a short-rate trajectory using the Vasicek model.

**Cox-Ingersoll-Ross (CIR) Model:**

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t \tag{23}$$

The CIR model ensures non-negative interest rates, making it more realistic for modeling short-term rates like SOFR.

Figure 9 depicts a simulated trajectory of the short-rate according to the Cox-Ingersoll-Ross (CIR) model. This visualization demonstrates how the CIR model ensures positive interest rates and incorporates volatility proportional to the square root of the rate, highlighting its practical suitability for modeling short-term rates like SOFR.

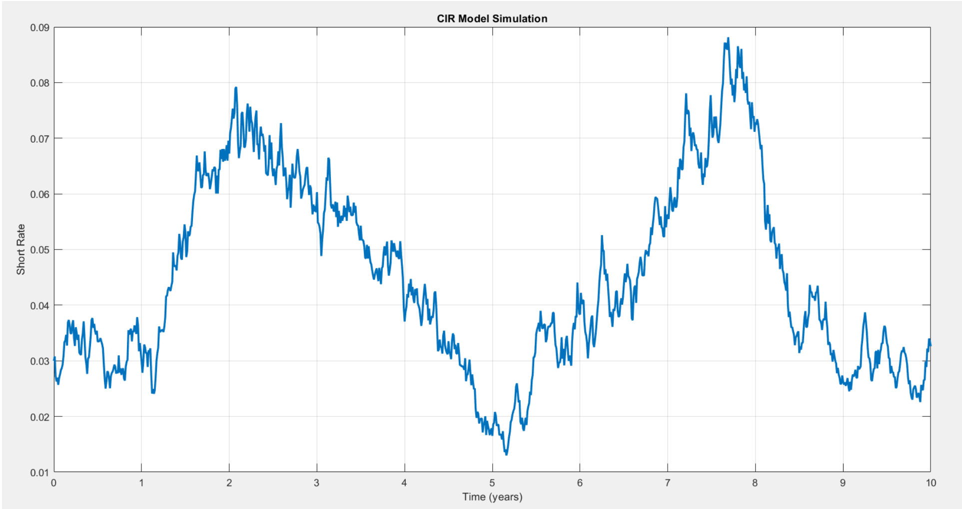


Figure 9: Simulation of a short-rate trajectory using the CIR model.

### 4.1.3 Hull-White Model and Transition Impact

The Hull-White model extends Vasicek by introducing a time-dependent mean-reversion level:

$$dr_t = [\theta(t) - \alpha r_t]dt + \sigma dW_t \tag{24}$$

where $\theta(t)$ is calibrated to match observed yield curves. The transition from LIBOR to SOFR requires recalibrating this model since SOFR lacks a term structure, impacting derivative pricing.

## 4.2 Yield Curve Construction and Arbitrage-Free Interpolation

### 4.2.1 Yield Curve Construction under LIBOR and SOFR

The construction of a yield curve requires bootstrapping from market instruments. Given a set of discount factors $P(0,T)$, the spot rate $R(T)$ is defined as:

$$P(0,T) = e^{-R(T)T} \tag{25}$$

For SOFR, yield curve estimation involves compounding overnight rates:

$$R_{\text{SOFR}}(T) = \frac{1}{T}\sum_{i=1}^{N} \ln(1 + r_{\text{SOFR}}(t_i)\Delta t_i) \tag{26}$$

### 4.2.2 Interpolation Techniques: Nelson-Siegel, Svensson, and Cubic Splines

U.S. Treasury securities serve as a benchmark for risk-free interest rates, reflecting the borrowing costs of the U.S. government over various maturities. Shorter maturities, such as the 1-year and 2-year (DGS1, DGS2), are typically more sensitive to monetary policy changes and short-term economic conditions. Longer maturities, like the 5-year and 10-year (DGS5, DGS10), tend to incorporate market expectations about inflation, growth, and other macroeconomic factors over a broader horizon. By examining these yields simultaneously, one can observe the shape of the yield curve and derive insights into market sentiment and future rate movements.

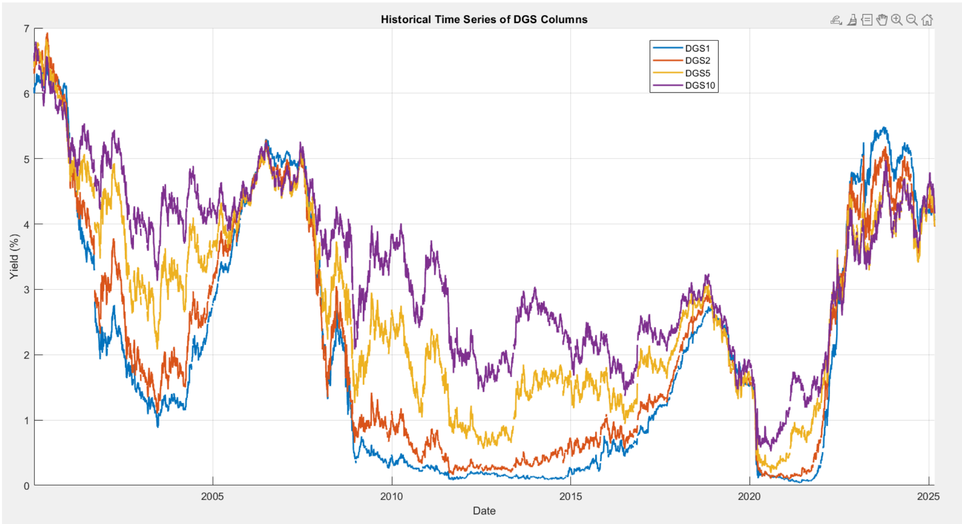


Figure 10: Historical time series of U.S. Treasury yields for 1-year (DGS1), 2-year (DGS2), 5-year (DGS5), and 10-year (DGS10) maturities

Yield curve interpolation methods include:

**Nelson-Siegel Model:**

$$R(T) = \beta_0 + \beta_1 \frac{1 - e^{-T/\tau}}{T/\tau} + \beta_2 \left( \frac{1 - e^{-T/\tau}}{T/\tau} - e^{-T/\tau} \right) \tag{27}$$

**Svensson Extension:**

$$R(T) = \beta_0 + \beta_1 \frac{1 - e^{-T/\tau_1}}{T/\tau_1} + \beta_2 \left( \frac{1 - e^{-T/\tau_1}}{T/\tau_1} - e^{-T/\tau_1} \right) + \beta_3 \left( \frac{1 - e^{-T/\tau_2}}{T/\tau_2} - e^{-T/\tau_2} \right) \tag{28}$$

Figure 11 shows the yield curve estimated using the Svensson model, while Figure 12 presents an overview of the calibrated data. The calibration process involves determining the parameter values that minimize the differences between the model-implied yields and the observed market data. By incorporating additional parameters, the Svensson model extends the basic Nelson-Siegel framework, enabling it to capture more complex yield curve shapes, especially over longer maturities.

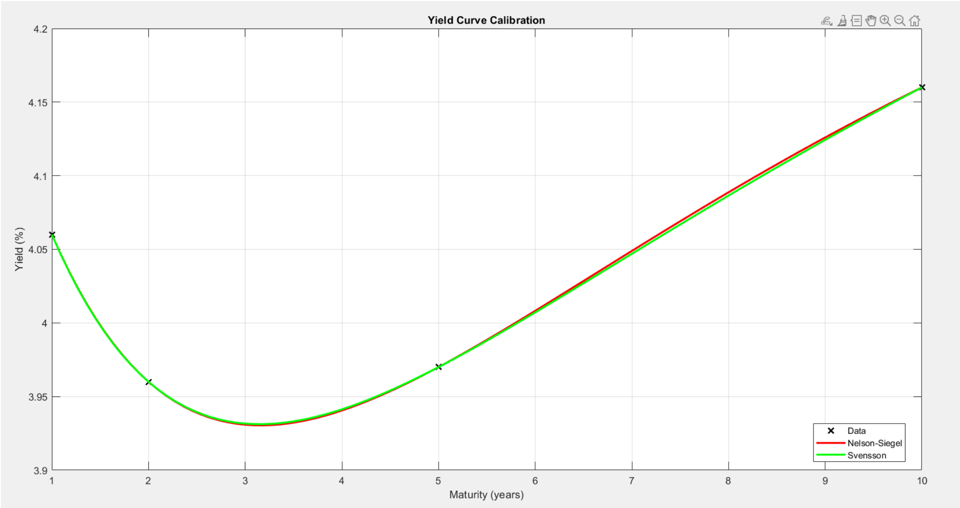


Figure 11: Yield Curve Estimated Using the Svensson Model

--- Calibrated Nelson-Siegel Parameters ---

| beta0 | beta1 | beta2 | tau |
|---|---|---|---|
| 4.5679 | -0.28389 | -1.6624 | 2.2131 |

--- Calibrated Svensson Parameters ---

| beta0 | beta1 | beta2 | beta3 | tau1 | tau2 |
|---|---|---|---|---|---|
| 11.541 | -7.2508 | -5.1208 | -1.3401 | 87.846 | 1.9979 |

Figure 12: Overview of Calibrated Data

**Cubic Splines:**

$$S(T) = a + bT + cT^2 + dT^3 \tag{29}$$

Each method impacts forward rate estimation, influencing derivative pricing.

Figure 13 presents the yield curve interpolation using cubic splines. This graph demonstrates clearly how cubic spline interpolation captures subtle variations in short-term interest rates without imposing a specific functional form, a significant point discussed within this subsection.

## 4.3 Simulation of Transition Impact

### 4.3.1 Pricing an Interest Rate Swap: LIBOR vs. SOFR

For a LIBOR-based interest rate swap, the value of the floating leg is:

$$V_{\text{floating}} = N \sum_{i=1}^{n} e^{-r(t_i)t_i} r_{\text{LIBOR}}(t_i) \Delta t_i \tag{30}$$

Under SOFR, the floating leg must be computed using compounded rates:

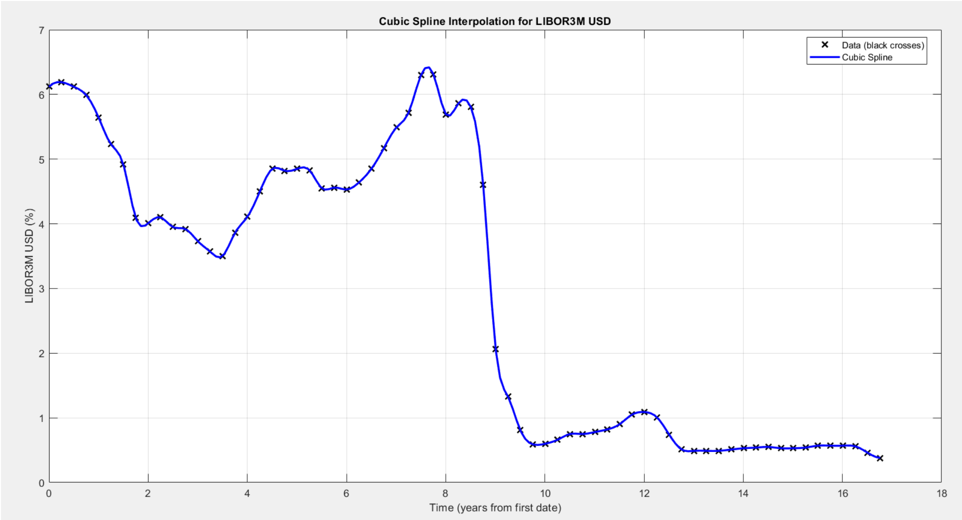


Figure 13: Yield curve interpolation using cubic splines.

$$r_{\text{SOFR}}(t_i) = \frac{1}{\Delta t_i} \left( \prod_{j=1}^{m} (1 + r_j \Delta t_j) - 1 \right) \tag{31}$$

The fixed leg remains:

$$V_{\text{fixed}} = N \sum_{i=1}^{n} e^{-r(t_i)t_i} r_f \Delta t_i \tag{32}$$

### 4.3.2 Impact on a Fixed Income Portfolio

The transition affects portfolio valuation due to different discounting methodologies. The price of a bond under LIBOR discounting:

$$P_{\text{LIBOR}} = \sum_{i=1}^{n} C_i e^{-r_{\text{LIBOR}}(t_i)t_i} \tag{33}$$

Under SOFR discounting:

$$P_{\text{SOFR}} = \sum_{i=1}^{n} C_i e^{-r_{\text{SOFR}}(t_i)t_i} \tag{34}$$

Differences in volatility and market liquidity impact portfolio risk.

# 5 Conclusion

The transition from LIBOR to alternative reference rates represents one of the most significant shifts in global financial markets in recent decades. This project has examined both the theoretical foundations and the practical implications of this transition by exploring advanced yield curve construction techniques, including the Nelson-Siegel and Svensson models as well as cubic spline interpolation. These methods enable us to derive discount factors and construct yield curves from market data, which are essential for accurate pricing and risk management of interest rate derivatives.

Our numerical applications illustrate the challenges in recalibrating traditional LIBOR-based models to accommodate risk-free rates such as SOFR, SONIA, and €STR. The analysis shows that each interpolation technique offers its own advantages, but also underscores the importance of careful calibration to capture the unique dynamics of the new benchmarks accurately. In particular, our simulations of interest rate swaps and fixed income portfolios highlight the adjustments required in valuation models, as well as the evolving market liquidity and volatility characteristics associated with the LIBOR transition.

Overall, this study reinforces the need for transparent and robust benchmark rates in maintaining financial stability. As market conditions continue to evolve, further research and model refinement will be critical to optimize risk management strategies and ensure a smooth transition in the post-LIBOR era.

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