# University of south Asia

**Assignment on** : **01**

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**Course Title** : **Complex Variable & Laplace Transformation.**
**Course Code** : **MAT 315**
**Department Name** : **CSE**

**Submitted to**

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1.

Let, $f(z) = z^2$, Then by denfition,

$$f'(z) = \lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{(z+\Delta z)^2 - z^2}{\Delta z}$$

$$= \lim_{\Delta z \to 0} (2z + \Delta z)$$

$$= 2z$$

Hence $f(z)$ is differentiable at any point $z$. That is $f(z)$ is differential everywhare.

2 Along the straight line joining (0, 1) and (2 5) is

$$\frac{x-0}{0-2} = \frac{y-1}{1-5}$$

$$\Rightarrow \frac{x}{-2} = \frac{y-1}{-4}$$

$$\Rightarrow 2x = y-1$$

$$\Rightarrow y = 2x+1$$

$$\therefore dy = 2dx$$

when $y = 1$ then $x = 0$

$y = 5 \quad x = 2$

$$\therefore \int_0^2 \{(3x+2x+1)dx + (4x+2-x)2dx\}$$

$$= \int_0^2 (5x+1+6x+4)dx$$

$$= \int_0^2 (11x+5)\,dx$$

$$= 11\int_0^2 x\,dx + 5\int_0^2 dx$$

$$= 11\left|\frac{x^2}{2}\right|_0^2 + 5|x|_0^2$$

$$= \frac{11}{2}(2^2-0) + 5(2-0)$$

$$= 22 + 10$$

$$= 32 \text{ Ans.}$$

3 If $L\{F(t)\} = f(s)$ Them,

$$L\{e^{at}F(t)\} = f(s-a) \text{ for } s>a$$

Prove:

$$L\{e^{at}F(t)\} = \int_0^\infty e^{-st}e^{at}F(t)\,dt$$

$$= \int_0^\infty e^{-(s-a)t}F(t)\,dt$$

$$\therefore L\{e^{at}F(t)\} = f(s-a) \text{ for } s-a$$

(Proved)

4 If $L\{F(t)\} = f(s)$ them

$$L\{F'(t)\} = sf(s) - F(0)$$

$$L\{F''(t)\} = s^2 f(s) - sF(0) - F'(0)$$

And soon......

now for $n$ derivatives, we can write

$$L\{F^n(t)\} = s^n f(s) - s^{n-1}F(0) - s^{n-2}F'(0) - s^{n-3}F''(0)\ L\{F(t)\} = f(s)$$

$$L\{F'(t)\} = sf(s) - F(0)$$ (Proved)

5

$$L^{-1}\left\{\frac{1}{(s-2)(s+3)}\right\}$$

Solve:

$$L^{-1}\left\{\frac{1}{(s-2)(s+3)}\right\}$$

$$= L^{-1}\left\{\frac{1}{(s-2)(s+3)} + \frac{1}{(-3-2)(s+3)}\right\}$$

$$= L^{-1}\left\{\frac{1}{5(s-2)} + \frac{1}{-5(s+3)}\right\}$$

$$= \frac{1}{5}L^{-1}\left\{\frac{1}{(s-2)}\right\} - \frac{1}{5}L^{-1}\left\{\frac{1}{(s+3)}\right\}$$

$$= \frac{1}{5}e^{2t} - \frac{1}{5}e^{-3t}$$

Ans.

6 $L^{-1}\left\{\frac{1}{s^2-5s+6}\right\}$

$$= L^{-1}\left\{\frac{1}{s(s-2)-3(s-2)}\right\}$$

$$= L^{-1}\left\{\frac{1}{(s-3)} + \frac{1}{(s-2)}\right\}$$

$$= L^{-1}\left\{\frac{1}{(s-3)(3-2)} + \frac{1}{(2-3)(s-2)}\right\}$$

$$= L^{-1}\left\{\frac{1}{s-3} + \frac{-1}{(s-2)}\right\}$$

$$= L^{-1}\left\{\frac{1}{(s-3)}\right\} - L^{-1}\left\{\frac{1}{(s-2)}\right\}$$

$$= e^{3t} - e^{2t}$$

Ans

$\underline{\underline{7}}$

$$L^{-1}\left\{\frac{1}{\sqrt{2s+3}}\right\}$$

$$= L^{-1}\left\{\frac{1}{\sqrt{2(s+\frac{3}{2})}}\right\}$$

$$= L^{-1}\left\{\frac{1}{\sqrt{2}\sqrt{s-\frac{3}{2}}}\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{\frac{1}{(s-\frac{3}{2})^{\frac{1}{2}}}\right\}$$

$$= \frac{1}{\sqrt{2}} e^{-\frac{3}{2}t}\left[\frac{t^{\frac{1}{2}-1}}{\frac{1}{2}}\right.$$

$$= \frac{1}{\sqrt{2}} e^{-\frac{3}{2}t}\left[\frac{t^{-\frac{1}{2}}}{\frac{1}{2}}\right.$$

$$= \frac{1}{\sqrt{2}} e^{-\frac{3}{2}t} \cdot \frac{t^{-\frac{1}{2}}}{\sqrt{\pi}}$$

$\therefore \Gamma\frac{1}{2} = \sqrt{\pi}$

$$= \frac{1}{2\sqrt{\pi}} e^{-\frac{3}{2}t} \cdot t^{-\frac{1}{2}}$$

Hence, $L^{-1}\left\{\frac{1}{\sqrt{2s+3}}\right\} = \frac{1}{2\sqrt{\pi}} e^{-\frac{3}{2}t} \cdot t^{-\frac{1}{2}}$

(Ans)