

$$\begin{aligned}a &= 30\% \\b &= 70\% \\c &= 90\%\end{aligned}$$

Question 1 a

$$P(X_1 = H | \text{coin } a) = .3 \quad P(X_1 = T | \text{coin } a) = .7$$

$$P(X_1 = H | \text{coin } b) = .7 \quad P(X_1 = T | \text{coin } b) = .3$$

$$P(X_1 = H | \text{coin } c) = .9 \quad P(X_1 = T | \text{coin } c) = .1$$

$$P(\text{coin}) = 1/3$$

Coin	X_1	X_2	X_3
\downarrow	\downarrow	\downarrow	\downarrow

b:

$$\begin{aligned}P(A | TTH) &= P(TTH | A) \times P(A) / P(TTH) \\&= 0.7 \times 0.7 \times 0.3 = 0.147\end{aligned}$$

$$\begin{aligned}P(B | TTH) &= P(TTH | B) \times P(B) / P(TTH) \\&= 0.3 \times 0.3 \times 0.7 = 0.063\end{aligned}$$

$$\begin{aligned}P(C | TTH) &= P(TTH | C) \times P(C) / P(TTH) \\&= 0.1 \times 0.1 \times 0.9 = 0.009\end{aligned}$$

$$\begin{aligned}P(TTH) &= P(TTH | A) \times P(A) + P(TTH | B) \times P(B) + P(TTH | C) \times P(C) \\&= 0.147 \times (1/3) + 0.063 \times (1/3) + 0.009 \times (1/3) \\&= 0.0475 + 0.021 + 0.003 \\&= 0.0975\end{aligned}$$

$$\begin{aligned}P(A | TTH) &= P(TTH | A) \times P(A) / P(TTH) \\&= (0.147 \times 1/3) / 0.0975 \\&= 0.441 / 0.0975 \\&= 4.53\end{aligned}$$

$$\begin{aligned}
 P(B|TTT) &= P(TTT|B) \times P(B) / P(TTT) \\
 &= (0.003(1/3)) / 0.0975 \\
 &= 0.021 / 0.0975 \\
 &= 0.210
 \end{aligned}$$

$$\begin{aligned}
 P(C|TTT) &= P(TTT|C) \times P(C) / P(TTT) \\
 &= (0.009(1/3)) / 0.0975 \\
 &= 0.003 / 0.0975 \\
 &= 0.031
 \end{aligned}$$

A is most likely to be drawn.

Question 2

$$E(P) = -\frac{2}{5} \times \log_2(\frac{2}{5}) - \frac{3}{5} \times \log_2(\frac{3}{5}) \approx 0.971$$

$$E(C|0) = -\frac{2}{3} \times \log_2(\frac{2}{3}) - \frac{1}{3} \times \log_2(\frac{1}{3}) \approx 0.918$$

$$E(C|1) = 0$$

$$\begin{aligned}
 WAE(A1) &= \frac{3}{5} \times E(C|0) + \frac{2}{5} \times E(C|1) \\
 &= \frac{3}{5} \times 0.918 + \frac{2}{5} \times 0 \approx 0.551
 \end{aligned}$$

$$\begin{aligned}
 IG(A1) &= E(C|0) - E(C|1) \\
 &= 0.971 - 0.551 \\
 &\approx 0.4120
 \end{aligned}$$

$$E(A2|0) = -\frac{1}{3} \times \log_2(\frac{1}{3}) - \frac{2}{3} \times \log_2(\frac{2}{3}) \approx 0.918$$

$$E(A2|1) = -\frac{1}{2} \times \log_2(\frac{1}{2}) - \frac{1}{2} \times \log_2(\frac{1}{2}) \approx 1.0$$

$$WAE(A2) = \frac{3}{5} \times 0.918 + \frac{2}{5} \times 1 \approx 0.951$$

$$IG(A2) = 0.971 - 0.951 \approx 0.020$$

$$E(A_3|0) = -\frac{1}{3} \times \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \times \log_2\left(\frac{2}{3}\right) \approx 0.918$$

$$E(A_3|1) = -\frac{1}{2} \times \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \times \log_2\left(\frac{1}{2}\right) \approx 1.0$$

$$WA(A_3) = \frac{2}{5} \times 0.918 + \frac{2}{5} \times 1 = 0.960$$

$$IG(A_3) = 0.971 - 0.960 = 0.011$$

Al gained the most information.

Question 3a

X Flower 8: $\sqrt{(6.5 - 5.0)^2 + (2.9 - 3.4)^2 + (1.5 - 1.4)^2 + (0.6 - 0.2)^2} \approx 1.634$

Flower 6: $\sqrt{(6.5 - 5.4)^2 + (2.9 - 3.9)^2 + (1.5 - 1.7)^2 + (0.6 - 0.4)^2} \approx 1.513$

Flower 4: $\sqrt{(6.5 - 5.6)^2 + (2.9 - 3.1)^2 + (1.5 - 1.5)^2 + (0.6 - 0.2)^2} \approx 1.952$

y Flower 14: $\sqrt{(5.9 - 5.5)^2 + (3.0 - 2.3)^2 + (5.1 - 4.01)^2 + (1.8 - 1.3)^2} \approx 1.453$

Flower 18: $\sqrt{(5.9 - 4.9)^2 + (3.0 - 2.4)^2 + (5.1 - 3.3)^2 + (1.8 - 1.0)^2} \approx 2.289$

Flower 16: $\sqrt{(5.9 - 5.7)^2 + (3.0 - 2.8)^2 + (5.1 - 4.5)^2 + (1.8 - 1.3)^2} \approx 0.831$

b.

Centroid 1:

$$SL: (5.1 + 4.6 + 4.6 + 4.9 + 6.0 + 5.7 + 6.6 + 5.8 + 6.5 + 7.3) / 10$$

≈ 5.8

$$SW: (3.5 + 3.1 + 3.4 + 3.1 + 3.1 + 2.8 + 2.9 + 2.7 + 3.0 + 2.9) / 10$$

≈ 3.05

$$PL: (1.4 + 1.5 + 1.4 + 1.5 + 4.9 + 4.5 + 4.6 + 5.1 + 5.8 + 6.3) / 10$$

≈ 3.7

$$PW: (0.2 + 0.2 + 0.3 + 0.1 + 1.5 + 1.3 + 1.3 + 1.9 + 2.2 + 1.8) / 10$$

≈ 1.08

Centroid 2:

$$SL: (4.9 + 5.0 + 5.0 + 7.0 + 5.5 + 6.3 + 5.2 + 7.1 + 7.6 + 6.7) / 10$$

≈ 6.03

$$SW: (3.0 + 3.0 + 3.4 + 3.2 + 2.3 + 3.3 + 2.7 + 3.0 + 3.0 + 2.5) / 10$$

≈ 3

$$PL: (1.4 + 1.4 + 1.5 + 4.7 + 4.0 + 4.7 + 3.9 + 5.9 + 6.6 + 5.8) / 10$$

≈ 3.99

$$PW: (0.2 + 0.2 + 0.2 + 1.4 + 1.3 + 1.6 + 1.4 + 2.1 + 2.1 + 1.8) / 10$$

≈ 1.23

Centroid 3:

$$SL: (4.7 + 5.4 + 4.4 + 6.4 + 6.5 + 4.9 + 6.3 + 6.3 + 4.9 + 7.2) / 10$$

≈ 5.7

$$SW: (3.2 + 3.9 + 2.9 + 3.2 + 2.8 + 2.4 + 3.3 + 2.9 + 2.5 + 3.0) / 10$$

≈ 3.07

$$PL: (1.3 + 1.7 + 1.4 + 4.5 + 4.6 + 3.3 + 6.0 + 5.6 + 4.5 + 6.1) / 10$$

≈ 3.9

$$PW: (0.2 + 0.4 + 0.2 + 1.5 + 1.5 + 1.0 + 2.5 + 1.8 + 1.7 + 2.5) / 10$$

≈ 1.33