Optimization Techniques

Optimization Techniques

- Need for Optimization
- Setting up Optimization Problem
  - Formulation
- Different Optimization Techniques:
  - Linear Programming
  - Transportation Problem
  - Assignment Problem
  - Queuing Systems

Need for Optimization

- Controlling the excess usage of resources
- Maximising the profit
- Minimising the cost

Setting up Optimization Problem

- Formulation
- Construction of a mathematical model
- Acquiring the data

Different Optimization Techniques – Linear Programming

- LPP is a mathematical technique for allotting the limited resources of a firm in an optimum manner.

Linear Programming (LP) Problem

- The maximization or minimization of some quantity is the objective in all linear programming problems.
- All LP problems have constraints that limit the degree to which the objective can be pursued.
- A feasible solution satisfies all the problem's constraints.
- An optimal solution is a feasible solution that results in the largest possible objective function value when maximizing (or smallest when minimizing).

Linear Programming (LP) Problem

- If both the objective function and the constraints are linear, the problem is referred to as a <u>linear programming problem</u>.
- <u>Linear functions</u> are functions in which each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0).
- <u>Linear constraints</u> are linear functions that are restricted to be "less than or equal to", "equal to", or "greater than or equal to" a constant.

Problem Formulation

- <u>Problem formulation or modeling</u> is the process of translating a verbal statement of a problem into a mathematical statement.

Guidelines for Model Formulation

- Understand the problem thoroughly.
- Describe the objective.
- Describe each constraint.
- Define the decision variables.
- Write the objective in terms of the decision variables.
- Write the constraints in terms of the decision variables.

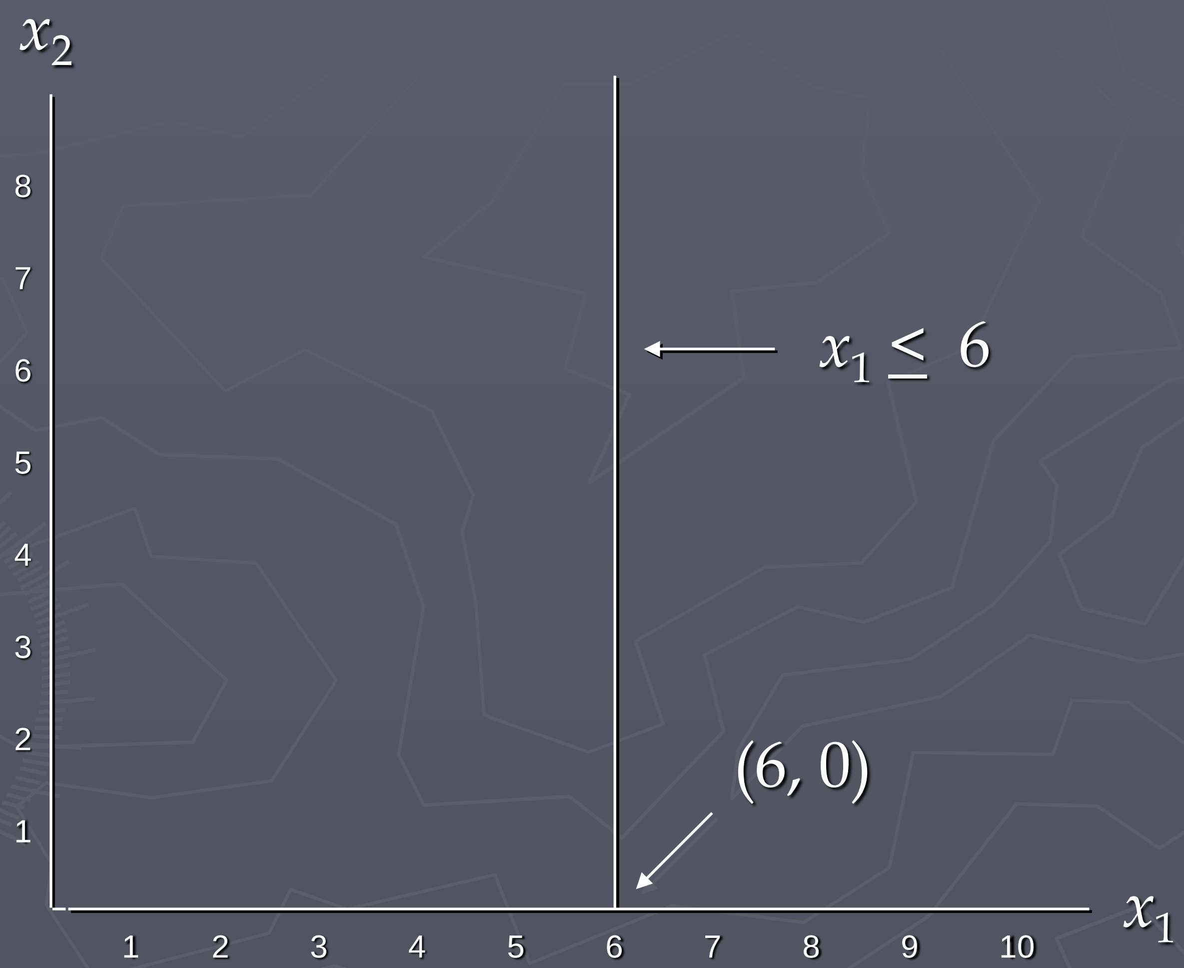
Example 1: A Maximization Problem

- LP Formulation

$$
\begin{array}{ll}
\text{Max} & 5x_1 + 7x_2 \\
\\
\text{s.t.} & \begin{aligned} x_1 \quad\quad\quad\; &\le 6 \\ 2x_1 + 3x_2 &\le 19 \\ x_1 + \;\; x_2 &\le 8 \end{aligned} \\
\\
 & x_1, x_2 \ge 0
\end{array}
$$

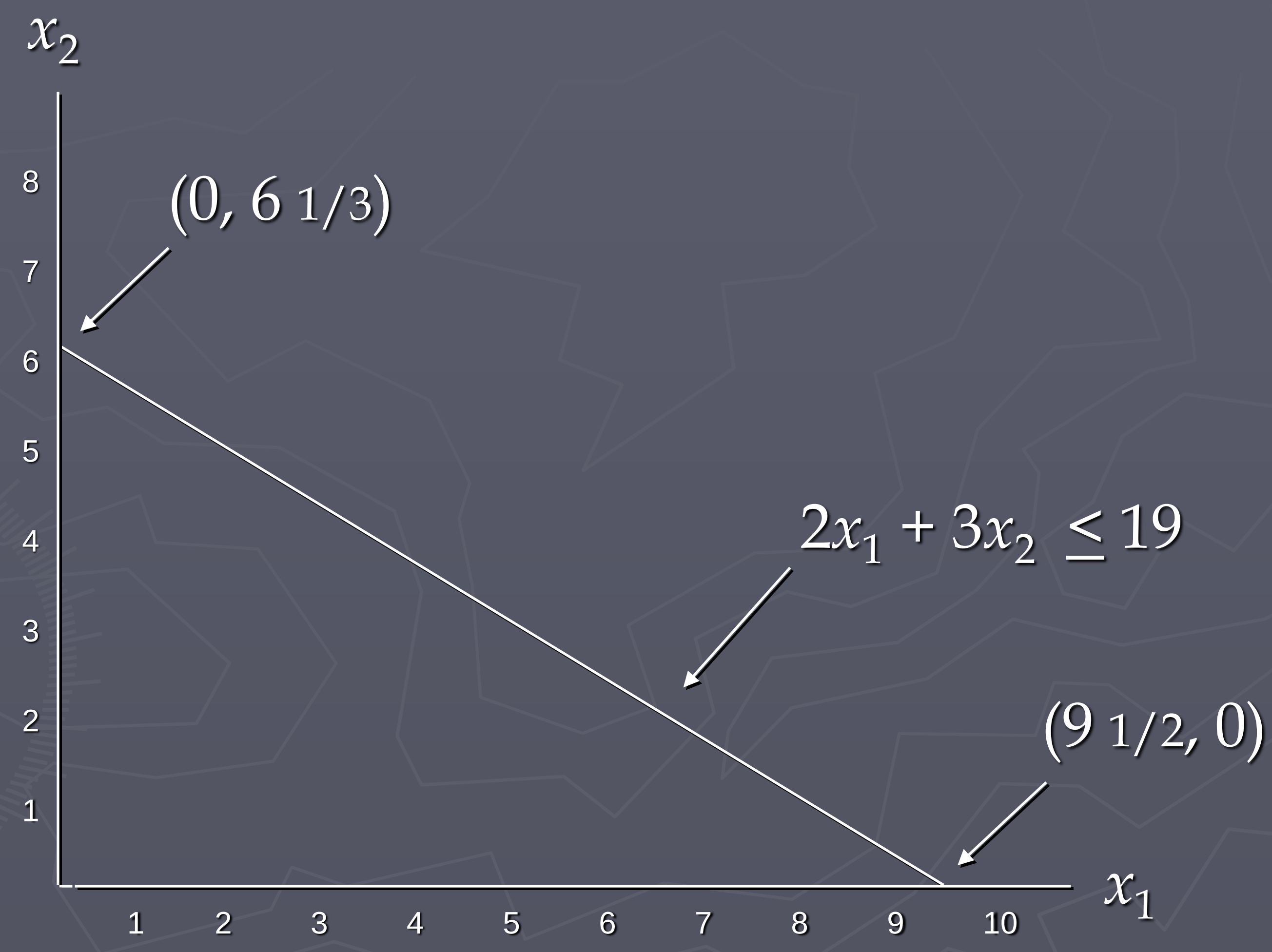
Example 1: Graphical Solution

- Constraint #1 Graphed



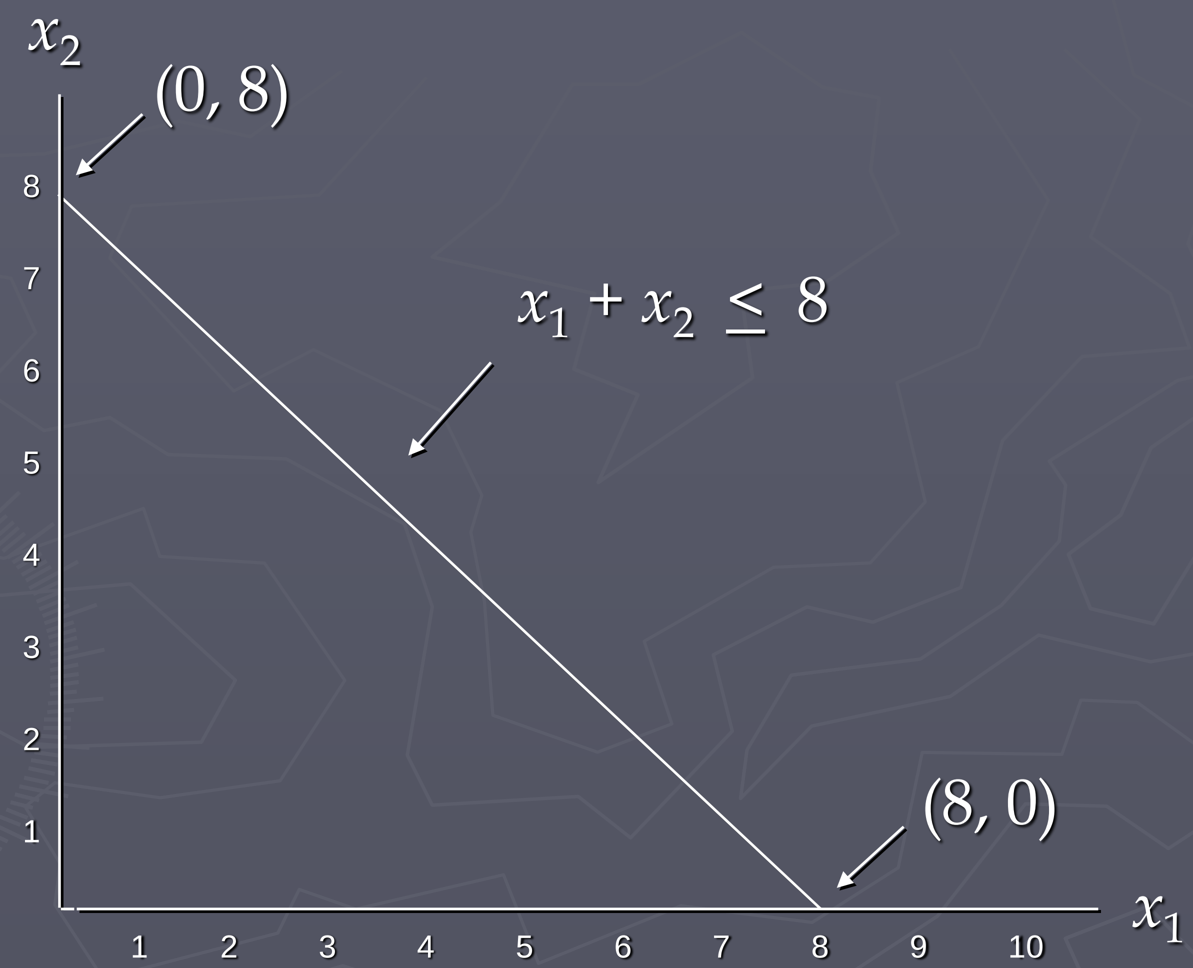
Example 1: Graphical Solution

- Constraint #2 Graphed



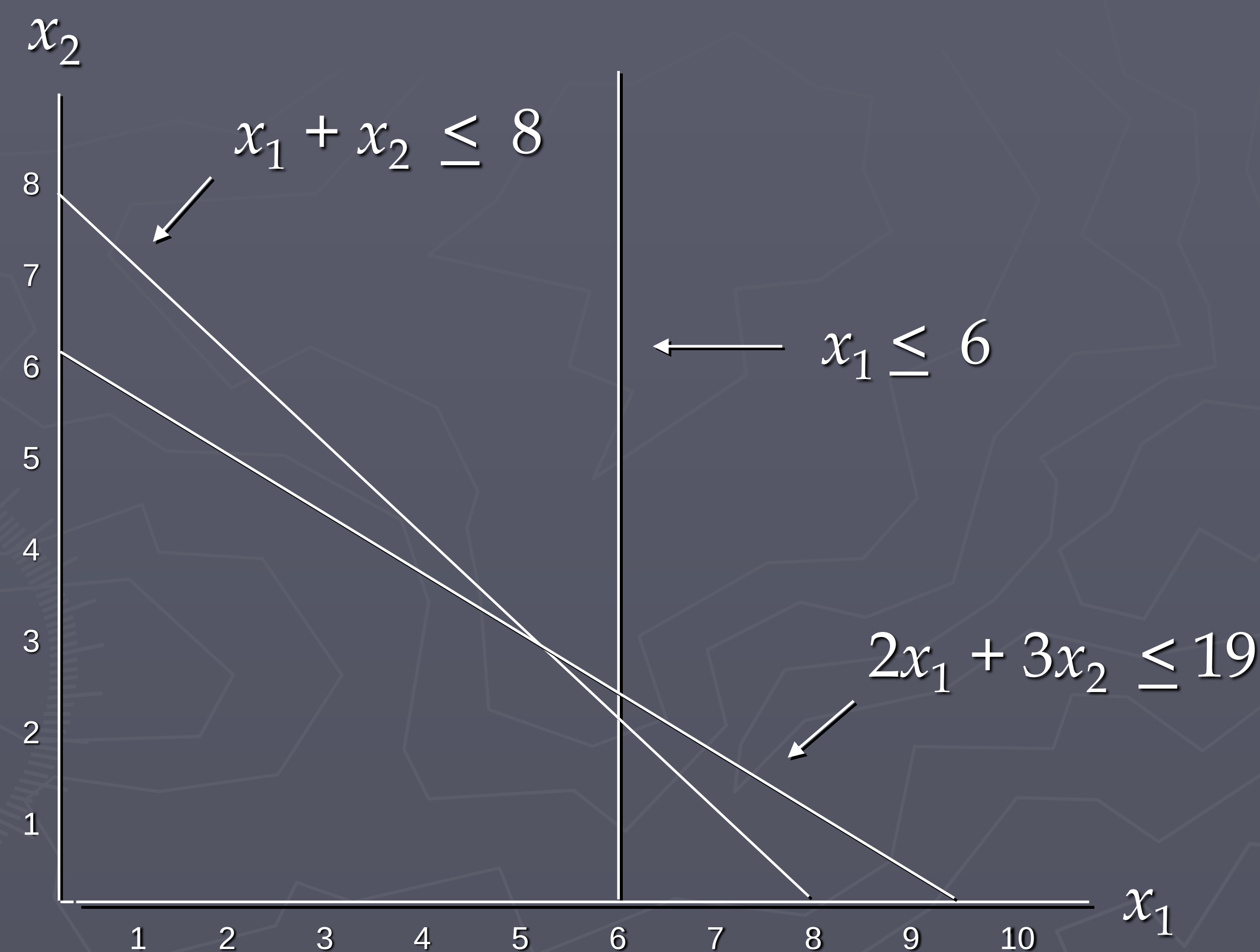
Example 1: Graphical Solution

- Constraint #3 Graphed



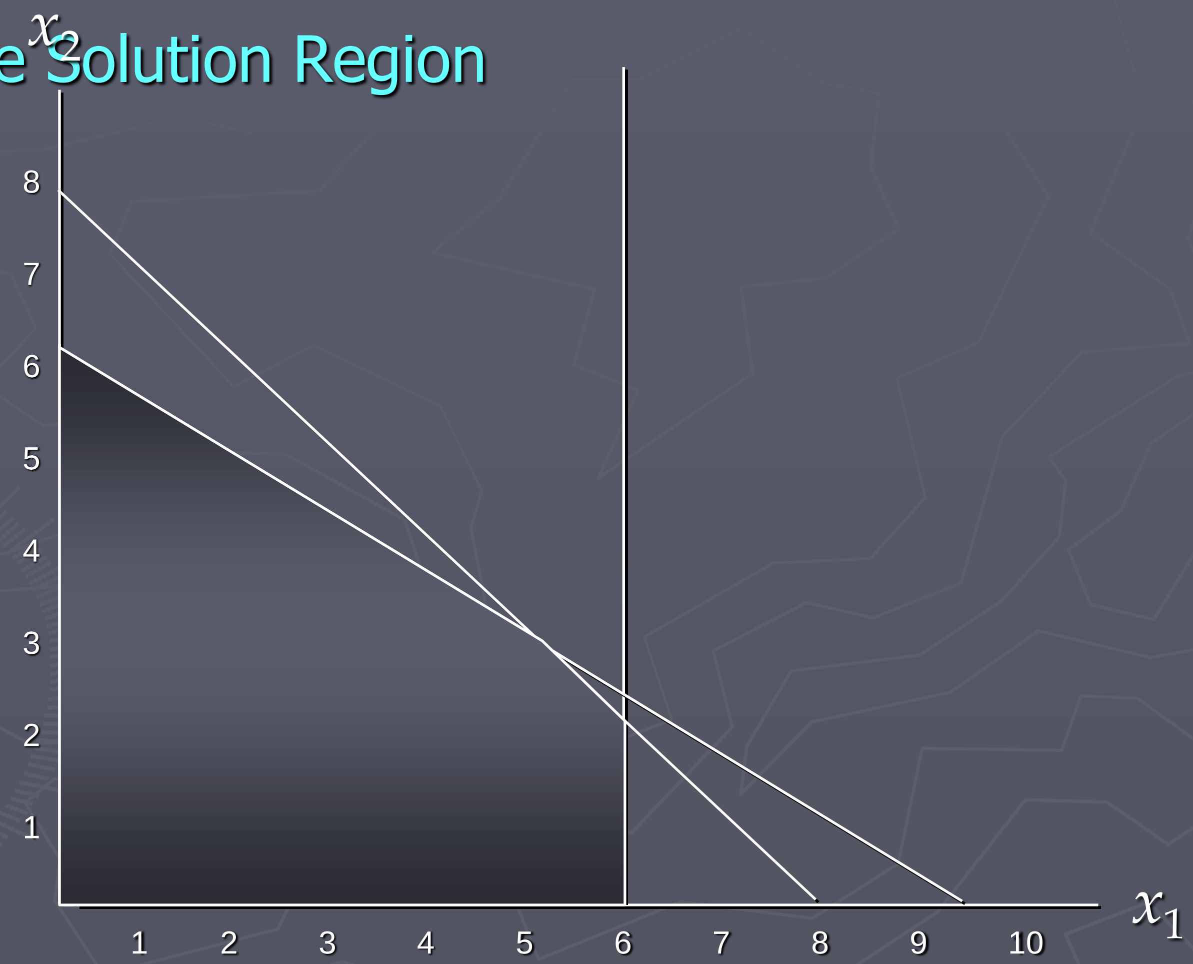
Example 1: Graphical Solution

- Combined-Constraint Graph



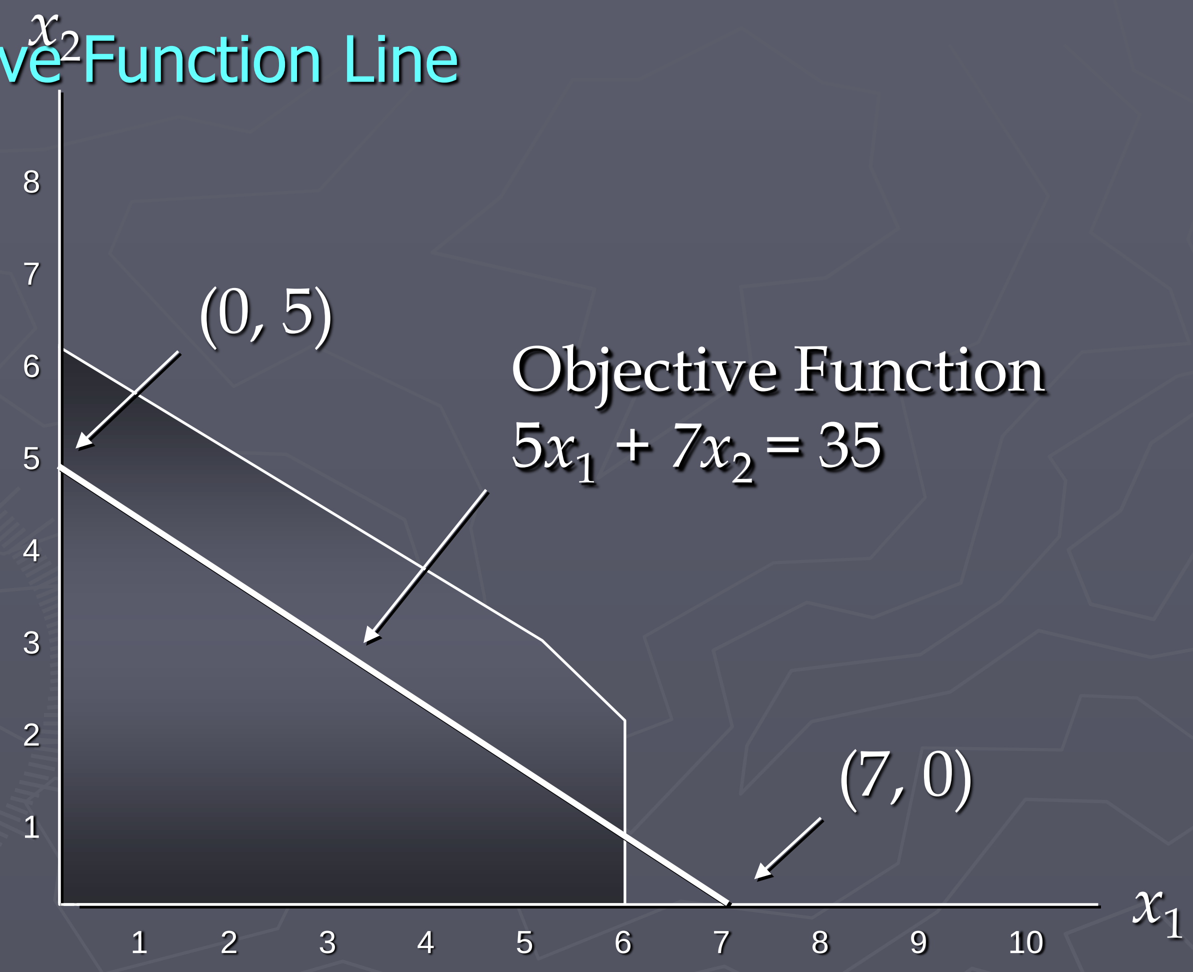
Example 1: Graphical Solution

- Feasible Solution Region



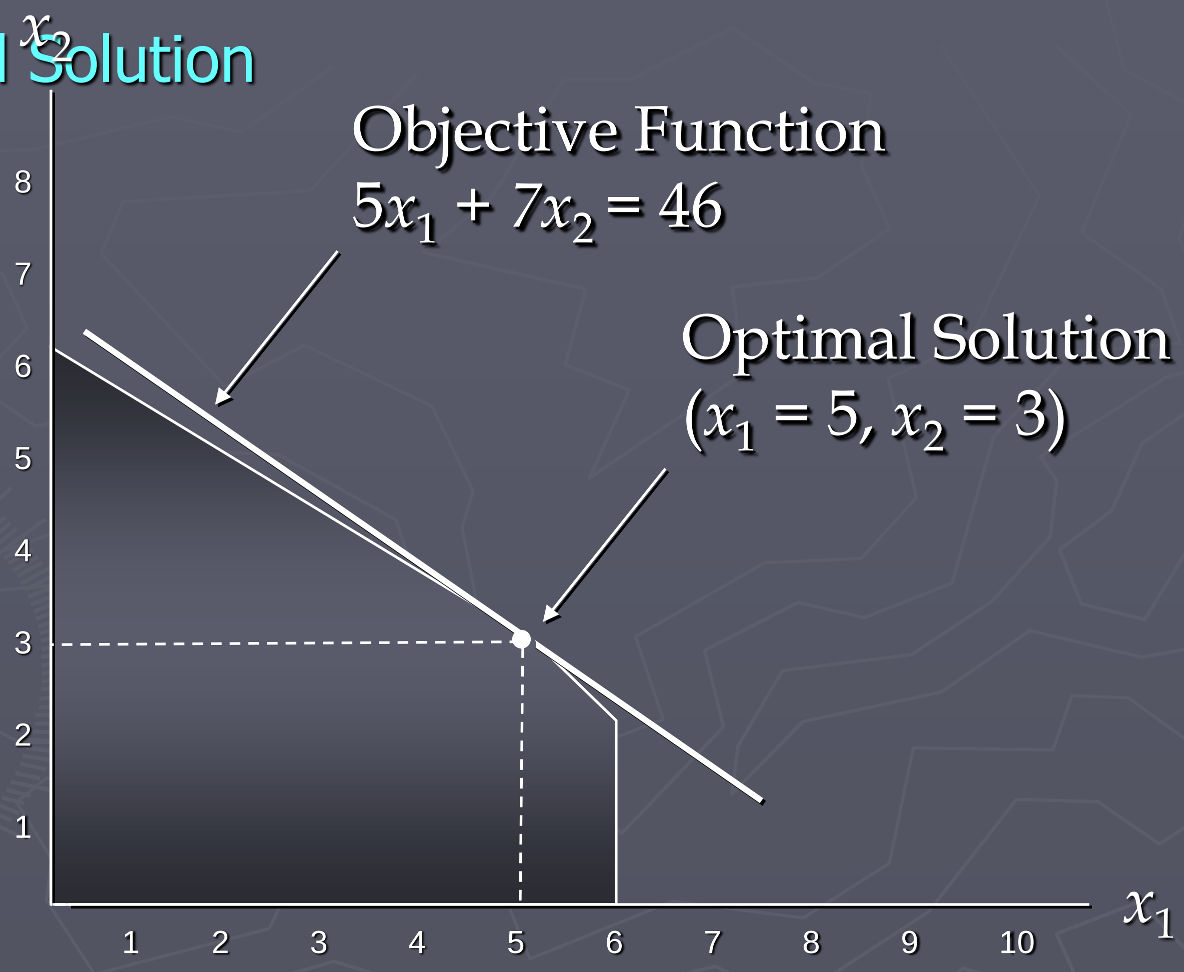
Example 1: Graphical Solution

- Objective Function Line



Example 1: Graphical Solution

- Optimal Solution



Feasible Region

- The feasible region for a two-variable linear programming problem can be nonexistent, a single point, a line, a polygon, or an unbounded area.
- Any linear program falls in one of three categories:
  - is infeasible
  - has a unique optimal solution or alternate optimal solutions
  - has an objective function that can be increased without bound
- A feasible region may be unbounded and yet there may be optimal solutions. This is common in minimization problems and is possible in maximization problems.

Linear Programming

EXAMPLES

A company that operates 10 hours a day manufactures two products on three sequential processes. The following table summarizes the data of the problem

	Minutes per unit			
Product	Process 1	Process 2	Process 3	Unit Profit
1	10	6	8	Rs. 2/-
2	5	20	10	Rs. 3/-

Determine the optimal mix of the two products.

Problem 2:

Solve this linear programming problem.

Maximize

$$Z = 20x_1 + 10x_2 + 15x_3$$

Subject to:

$$3x_1+2x_2+5x_3 \leq 55$$
$$2x_1+x_2+x_3 \leq 26$$
$$x_1+x_2+3x_3 \leq 30$$
$$5x_1+2x_2+4x_3 \leq 57$$
$$x_1,x_2,x_3 \geq 0$$

Problem 3:

A company makes three models of desks, an executive model, an office model and a student model. Each desk spends time in the cabinet shop, the finishing shop and the crating shop as shown in the table:

Type of desk	Cabinet shop (in hrs.)	Finishing shop (in hrs.)	Crating shop (in hrs.)	Profit (in Rs.)
Executive	2	1	1	150
Office	1	2	1	125
Student	1	1	.5	50
Available hours	16	16	10	

How many of each type of model should be made to maximize profits?