

# Probability

# We will be covering...

- Concept of Probability
- Types of Events
  - Collectively Exhaustive
  - Mutually Exclusive
  - Independent
- Rules
  - Complement
  - Addition
  - Multiplicative

# Probability

- A probability in statistical theory is a number between 0 and 1 that measures the likelihood that some event will occur.
- A zero probability of an event signifies event not occurring and probability one signifies event being almost sure

# Collectively Exhaustive Events

- Events  $A_1, A_2, \dots, A_n$  are said to be collectively exhaustive events if at least one of the events must occur
- Example:
  - $A_1$ : Rain
  - $A_2$ : Sunny Weather
  - $A_3$ : Cloudy
  - $A_4$ : Snowfall

# Rules

- If A and B are events then:
  - Rule of Complement :  $P(\text{Non-occurrence of } A) = 1 - P(A)$
  - Addition Rule:  $P(\text{Occurrence of at least one of the events}) = P(A) + P(B) - P(\text{Occurrence of both } A \text{ and } B)$
  - Multiplication Rule:  $P(\text{Occurrence of } A \text{ given that } B \text{ has occurred}) = P(\text{Occurrence of both } A \text{ and } B) / P(B)$

# Addition Rule

- $P(\text{Occurrence of at least one of the events}) = P(A) + P(B) - P(\text{Occurrence of both A and B})$
- By Notation:  
$$P(A \text{ or } B \text{ or } A \text{ and } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- Example:
  - A: Ad Campaign will fail
  - B: Competitor launches a new product
  - $A \cap B$  : A and B both occur

Say  $P(A) = 0.23$ ,  $P(B) = 0.39$ ,  $P(A \cap B) = 0.18$  then

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) = 0.23 + 0.39 - 0.18 \\ &= 0.44 \end{aligned}$$

# Mutually Exclusive Events

- The two events A and B are said to be mutually exclusive (disjoint) if they both cannot occur simultaneously
- In case of mutually exclusive events A and B,
  - $P(A \cap B) = 0$
  - $P(A \cup B) = P(A) + P(B)$

Example: A: Snowfall, B: Temperature  $> 50$  degrees Celsius

$P(A) = 0.25$ ,  $P(B) = 0.20$  then  $P(A \text{ or } B) = 0.45$

# Multiplication Rule

- $P(\text{Occurrence of both A and B}) = P(\text{Occurrence of A given that B has occurred}) * P(B)$
- Or
- $P(\text{Occurrence of A given that B has occurred}) = P(\text{Occurrence of both A and B}) / P(B)$
- By Notations,  $P(A | B) = P(A \cap B) / P(B)$ ,  $P(A | B)$  is read as A given B
- Similarly,  $P(B | A) = P(A \cap B) / P(A)$
- $P(A|B)$  and  $P(B|A)$  are called conditional probabilities
- Example:
  - A: Ad Campaign will fail
  - B: Competitor launches a new product
  - $A \cap B$  : A and B both occur $P(A) = 0.25, P(B) = 0.32, P(A \cap B) = 0.17$  then

$$P(A|B) = P(A \cap B) / P(B) = 0.17 / 0.32 = 0.5312,$$

$$P(B|A) = P(A \cap B) / P(A) = 0.17 / 0.25 = 0.68$$



# Independence of Events

- If A and B are independent events then:
  - $P(\text{Occurrence of both A and B}) = P(A) P(B)$
- By Notations,  $P(A \cap B) = P(A) * P(B)$
- Example:
  - A: Ad Campaign will fail
  - B: Sensex goes up
  - $A \cap B$  : A and B both occur

Say,  $P(A) = 0.32$ ,  $P(B) = 0.64$ ,  $P(A \cap B) = P(A) * P(B) = 0.2048$