Simulation

Covering…

- What is Simulation?
- Monte Carlo Technique
- Examples in Excel
- Examples in Python

The Essence of Computer Simulation

- A **stochastic system** is a system that evolves over time according to one or more probability distributions.
- **Computer simulation** imitates the operation of such a system by using the corresponding probability distributions to *randomly generate* the various events that occur in the system.
- Rather than literally operating a physical system, the computer just records the occurrences of the *simulated* events and the resulting performance of the system.
- Computer simulation is typically used when the stochastic system ivolved is too complex to be analyzed satisfactorily by analytical models.

Example 1: A Coin-Flipping Game

- Rules of the game:
  1. Each play of the game involves repeatedly flipping an unbiased coin until the *difference* between the number of heads and tails tossed is three.
  2. To play the game, you are required to pay $1 for each flip of the coin. You are not allowed to quit during the play of a game.
  3. You receive $8 at the end of each play of the game.
- Examples:

HHH	3 flips	You win $5
THTTT	5 flips	You win $3
THHTHTHTTTT	11 flips	You lose $3

Computer Simulation of Coin-Flipping Game

- A computer cannot flip coins. Instead it generates a sequence of *random numbers*.
- A number is a **random number** between 0 and 1 if it has been generated in such a way that *every* possible number within the interval has an equal chance of occurring.
- An easy way to generate random numbers is to use the RAND() function in Excel.
- To simulate the flip of a coin, let half the possible random numbers correspond to heads and the other half to tails.
  - 0.0000 to 0.4999 correspond to heads.
  - 0.5000 to 0.9999 correspond to tails.

Simulation Modeling

- One begins a simulation by developing a mathematical statement of the problem.
- The model should be realistic yet solvable within the speed and storage constraints of the computer system being used.
- Input values for the model as well as probability estimates for the random variables must then be determined.

Random Variables

- Random variable values are utilized in the model through a technique known as Monte Carlo simulation.
- Each random variable is mapped to a set of numbers so that each time one number in that set is generated, the corresponding value of the random variable is given as an input to the model.
- The mapping is done in such a way that the likelihood that a particular number is chosen is the same as the probability that the corresponding value of the random variable occurs.

Simulation Programs

- The computer program that performs the simulation is called a simulator.
- Flowcharts can be useful in writing such a program.
- While this program can be written in any general purpose language (e.g. BASIC, FORTRAN, C++, etc.) special languages which reduce the amount of code which must be written to perform the simulation have been developed.
- Special simulation languages include SIMSCRIPT, SPSS, DYNAMO, and SLAM.

Experimental Design

- Experimental design is an important consideration in the simulation process.
- Issues such as the length of time of the simulation and the treatment of initial data outputs from the model must be addressed prior to collecting and analyzing output data.
- Normally one is interested in results for the steady state (long run) operation of the system being modeled.
- The initial data inputs to the simulation generally represent a start-up period for the process and it may be important that the data outputs for this start-up period be neglected for predicting this long run behavior.

Example: Dynogen, Inc.

The price change of shares of Dynogen, Inc. has been observed over the past 50 trades. The frequency distribution is as follows:

Price Change	Number of Trades
-3/8	4
-1/4	2
-1/8	8
0	20
+1/8	10
+1/4	3
+3/8	2
+1/2	1
	Total = 50

Example: Dynogen, Inc.

- Relative Frequency Distribution and Random Number Mapping

Price Change	Relative Frequency	Rnd Numbers
-3/8	.08	00 - 07
-1/4	.04	08 - 11
-1/8	.16	12 - 27
0	.40	28 - 67
+1/8	.20	68 - 87
+1/4	.06	88 - 93
+3/8	.04	94 - 97
+1/2	.02	98 - 99
TOTAL	1.00	

Example: Dynogen, Inc.

If the current price per share of Dynogen is 23, use random numbers to simulate the price per share over the next 10 trades.

Use the following stream of random numbers:21, 84, 07, 30, 94, 57, 57, 19, 84, 84

Example: Dynogen, Inc.

- Simulation Worksheet

Trade Number	Random Number	Price Change	Stock Price
1	21	-1/8	22 7/8
2	84	+1/8	23
3	07	-3/8	22 5/8
4	30	0	22 5/8
5	94	+3/8	23
6	57	0	23
7	57	0	23
8	19	-1/8	22 7/8
9	84	+1/8	23
10	84	+1/8	23 1/8

Example: Dynogen, Inc.

- Spreadsheet for Stock Price Simulation

Lower	Upper		Trade	Price	Stock
Random	Random	Price	Number	Change	Price
Number	Number	Change	1	0.125	23.125
0.00	0.08	-0.375	2	0.375	23.500
0.08	0.12	-0.250	3	0.000	23.500
0.12	0.28	-0.125	4	0.000	23.500
0.28	0.68	0.000	5	0.000	23.500
0.68	0.88	0.125	6	0.000	23.500
0.88	0.94	0.250	7	0.125	23.625
0.94	0.98	0.375	8	0.125	23.750
0.98	1.00	0.500	9	0.000	23.750
			10	0.125	23.875

Example: Dynogen, Inc.

- Theoretical Results and Observed Results

Based on the probability distribution, the expected price change per trade can be calculated by:

$$(.08)(-3/8) + (.04)(-1/4) + (.16)(-1/8) + (.40)(0)$$

$$+ (.20)(1/8) + (.06)(1/4) + (.04)(3/8) + (.02)(1/2) = +.005$$

The expected price change for 10 trades is (10)(.005) = .05. Hence, the expected stock price after 10 trades is 23 + .05 = 23.05.

Compare this ending price with the spreadsheet simulation and "manual" simulation results on the previous slides.

Sick drivers problem

- At a bus terminal every bus should leave with the driver. At the terminus they keep 2 drivers as reserved if any one on scheduled duty is sick and could not come. Following is the probability distribution that driver becomes sick:

No. of Absent Drivers	0	1	2	3	4	5
Probability	0.30	0.20	0.15	0.10	0.13	0.12

Simulate the data for a week and find utilization of reserved drivers. Also find how many days and how many buses cannot run because of non-availability of drivers.

Purchase - sale

- A trader deals in a perishable commodity, the daily demand and supply of which are random variables. Records of the past 500 trading days are shown below:

Supply		Demand	
Availability(Kg)	No. of days	Demand (kg)	No. of days
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The trader buys the commodity at Rs. 20 per kg and sells it at Rs.30 per kg. Any commodity remaining at the end of a day results in a loss of Rs.8 per kg (after resale). Simulate the supply-demand data for 30 days. Calculate corresponding purchases, sales and profit/loss. Also calculate total profit/loss for those 30 days.

References used

- Introduction to Management Science
  - By Hillier and Hillier
- Introduction to Management Science Quantitative approaches in decision making
  - By Anderson Sweeney Williams
- Statistical and Quantitative Methods
  - By Ranjeet Chitale

Thank You