PCA for Implied Volatility

Code

Casey Tirshfield

The file `impvol_sp500_atm_tom.txt` contains at-the-money implied volatilities (i.e., $I_t(1, \tau)$) with different times to maturity $(\tau = T - t)$ of European calls on the S&P500 index for the period from January 3, 2005 to April 10, 2006.

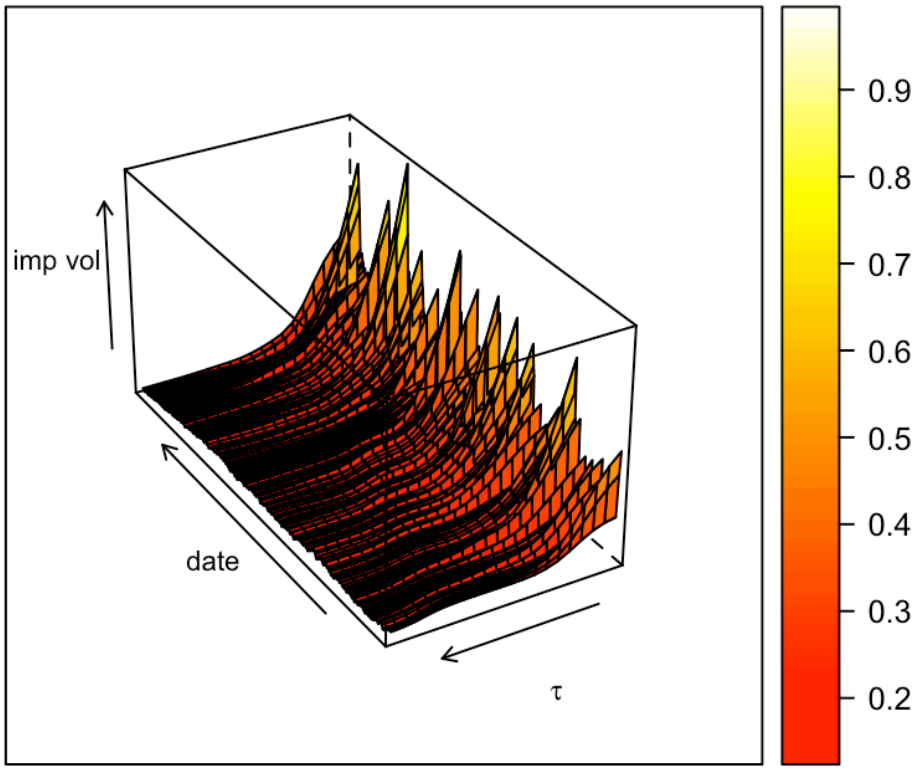
Hide

```
df <- read.table('impvol_sp500_atm_tom.txt', header=FALSE)
# make row of times to maturity the column names
tau <- t(df[1,])
colnames(df) <- df[1,]
df <- df[-1,]
# make rownames sequential numbers to simulate time
rownames(df) <- c(1:320)
# make dataframe a matrix
df_matrix <- as.matrix(df)
```

(a) Plot the implied volatility surface versus different dates and different times to maturity. You can use wireframe in the R package lattice or surf in MATLAB to plot functions of two variables as surfaces.

Hide

```
wireframe(df_matrix, main='Implied Vol Surface v. Dates and Time to Maturity', drape=
TRUE, aspect=c(.5, .5), screen = list(z=120, x=-60), zlab=list('imp vol', cex=0.7), x
lab=list('date', cex=0.7), ylab=list(expression(tau), cex=0.7), col.regions=heat.colo
rs(100))
```



(b) Perform PCA for the differenced series $\Delta_t(1,\tau) = \log_t(1,\tau) - \log I_{t-1}(1,\tau)$. Plot the first three eigenvectors versus τ.

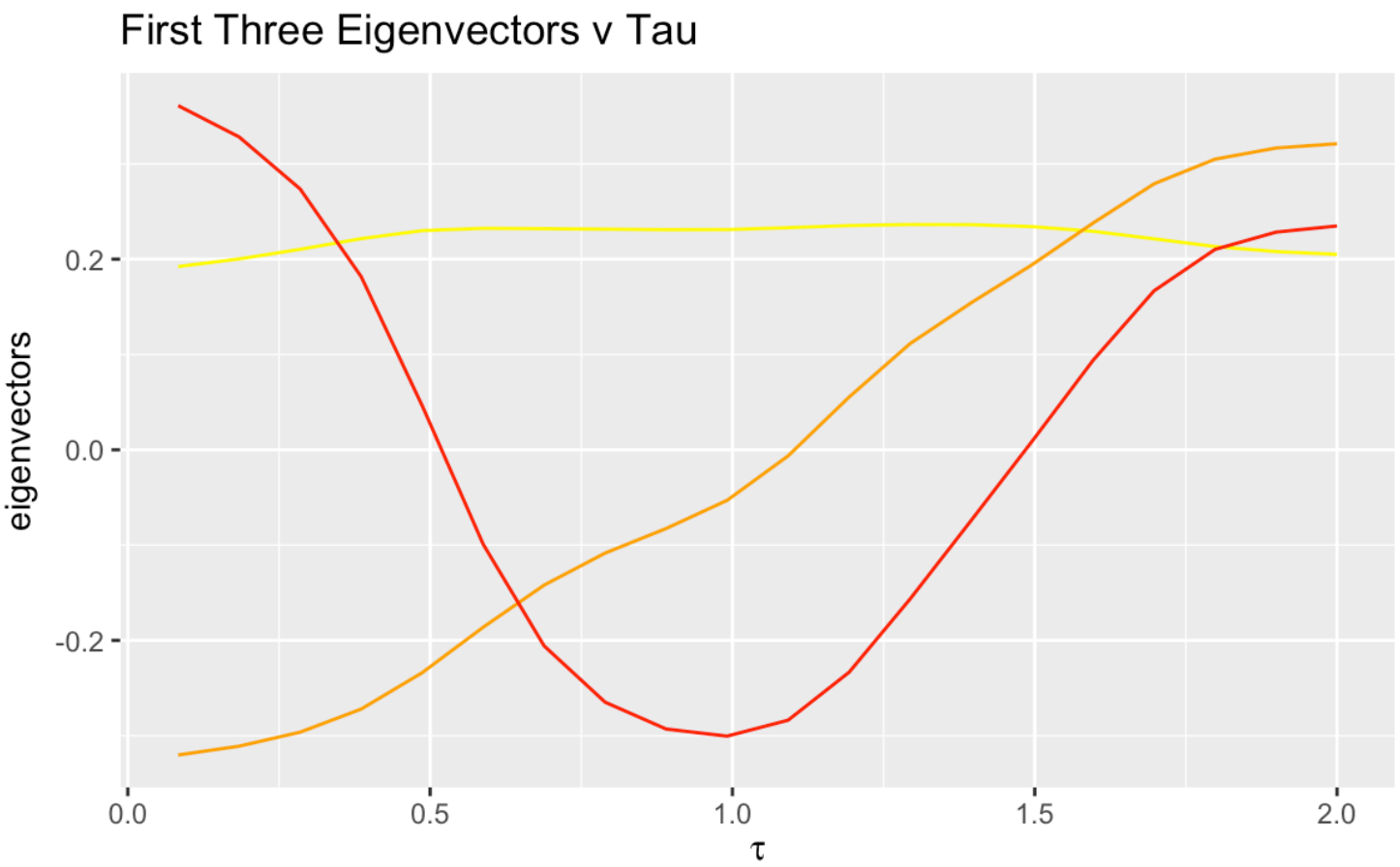
Hide

```
log_df <- log(as.matrix(df))
diff_log <- diff(log_df, lag=1, differences=1)
PCA <- prcomp(diff_log, center= TRUE, scale=TRUE)
summary(PCA)
```

```
Importance of components:
                          PC1    PC2     PC3     PC4     PC5     PC6     PC7
Standard deviation     4.0268 1.4592 1.11098 0.49672 0.32420 0.19776 0.13162
Proportion of Variance 0.8107 0.1065 0.06171 0.01234 0.00526 0.00196 0.00087
Cumulative Proportion  0.8107 0.9172 0.97891 0.99125 0.99650 0.99846 0.99932
                           PC8     PC9    PC10    PC11    PC12    PC13
Standard deviation     0.08514 0.06282 0.03755 0.02310 0.01503 0.01013
Proportion of Variance 0.00036 0.00020 0.00007 0.00003 0.00001 0.00001
Cumulative Proportion  0.99969 0.99988 0.99995 0.99998 0.99999 1.00000
                           PC14     PC15     PC16     PC17     PC18      PC19
Standard deviation     0.006216 0.004601 0.003259 0.001891 0.001423 0.0005899
Proportion of Variance 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000
Cumulative Proportion  1.000000 1.000000 1.000000 1.000000 1.000000 1.0000000
                            PC20
Standard deviation     0.0002333
Proportion of Variance 0.0000000
Cumulative Proportion  1.0000000
```

Hide

```
eigenvec1 <- as.data.frame(PCA$rotation[,1])
eigenvec2 <- as.data.frame(PCA$rotation[,2])
eigenvec3 <- as.data.frame(PCA$rotation[,3])
df_plot <- data.frame(tau, eigenvec1, eigenvec2, eigenvec3)
ggplot(df_plot, aes(x=tau)) + geom_line(aes(y=eigenvec1), color='yellow') + geom_line
(aes(y=eigenvec2), color='orange') + geom_line(aes(y=eigenvec3), color='red') + ylab(
'eigenvectors') + xlab(expression(tau)) + ggtitle('First Three Eigenvectors v Tau')
```


First Three Eigenvectors v Tau
eigenvectors
0.2
0.0
-0.2
0.0
0.5
1.0
1.5
2.0
τ