High Frequency Analysis

Code

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The file `ibm_intrastrade_200306.txt` contains the transaction data on the New York Stock Exchange for IBM stock in June 2003. The data are obtained from Wharton Research Data Services.

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```
df <- read.table('ibm_intraday_200306.txt', header=TRUE)
# first we change the format of the date column to use the code inspired by https://s
tackoverflow.com/questions/15573468/split-or-subset-data-into-30-minute-intervals
df$DATE <- ymd(df$DATE)
```

(a) Let x_i denote the number of trades in the ith 5-minute interval. Ignoring the time gaps between trading days, this gives the time series x_t of the number of trades on IBM stock in 5-minute intervals on the NYSE in June, 2003. Plot the time series and its ACF. Determine if there are intraday period patterns in the series.

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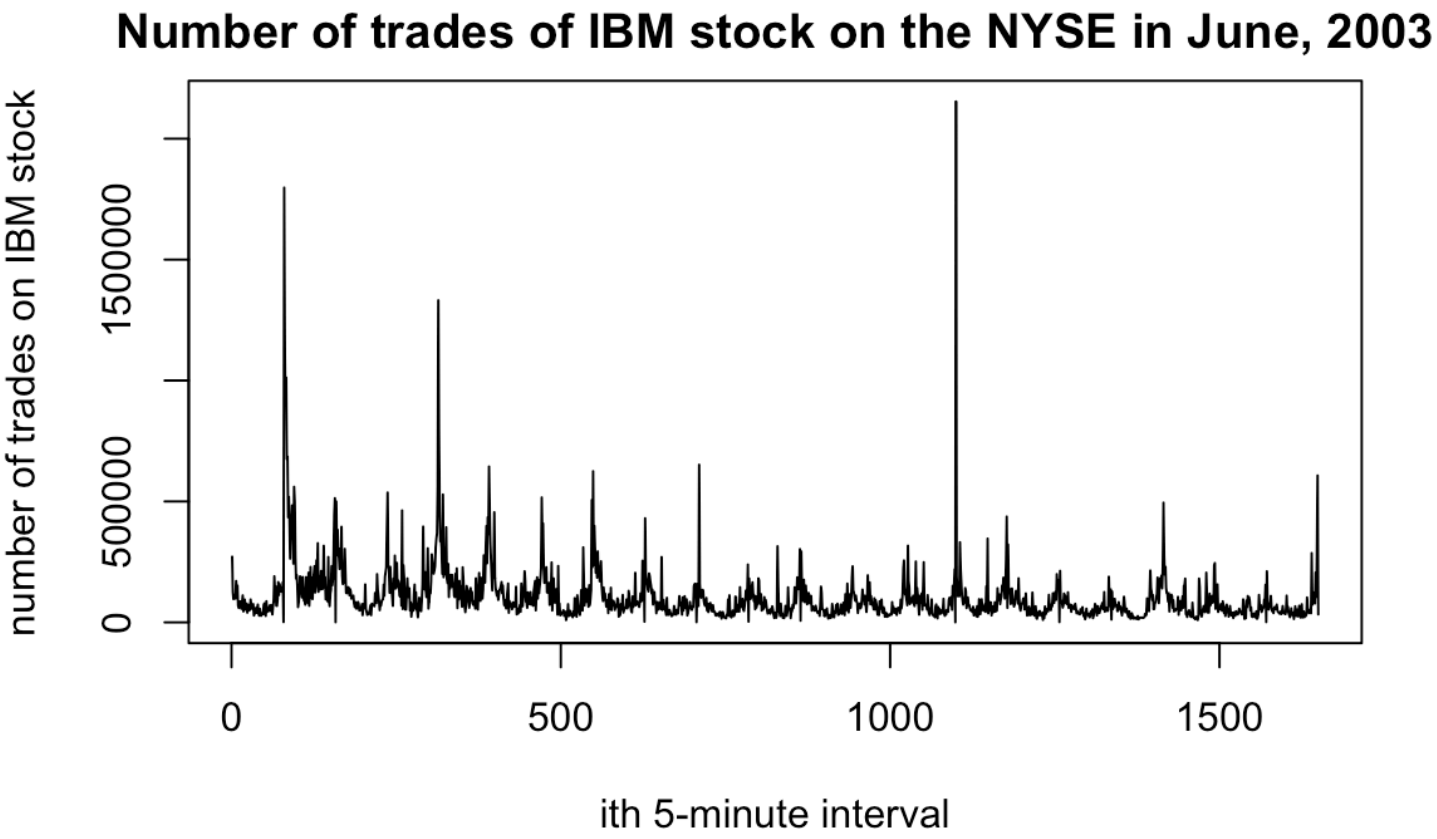
```
# this is the inspired code that stores our data in separate data frames of 5-min per
iods
store_df <- split(df, cut(strptime(paste(df$DATE, df$TIME), format="%Y-%m-%d %H:%M:%S
"), "5 mins"))
# here we take the sum of the number of trades per 5-min interval
list_freq <- lapply(store_df, function(x)sum(x$SIZE))
# in order to plot more easily we make the list a matrix
matrix_freq_w_0s <- matrix(unlist(list_freq), nrow=length(list_freq), byrow=TRUE)
# now we remove all rows with an entry of 0, thus ignoring the time gaps between trad
ing days
freq_matrix <- matrix_freq_w_0s[apply(matrix_freq_w_0s!=0, 1, all),]
# plot series
plot(freq_matrix, type='l', xlab='ith 5-minute interval', ylab='number of trades on I
BM stock', main='Number of trades of IBM stock on the NYSE in June, 2003')
# plot ACF
par(mar = c(4, 4, 4, 4))
```



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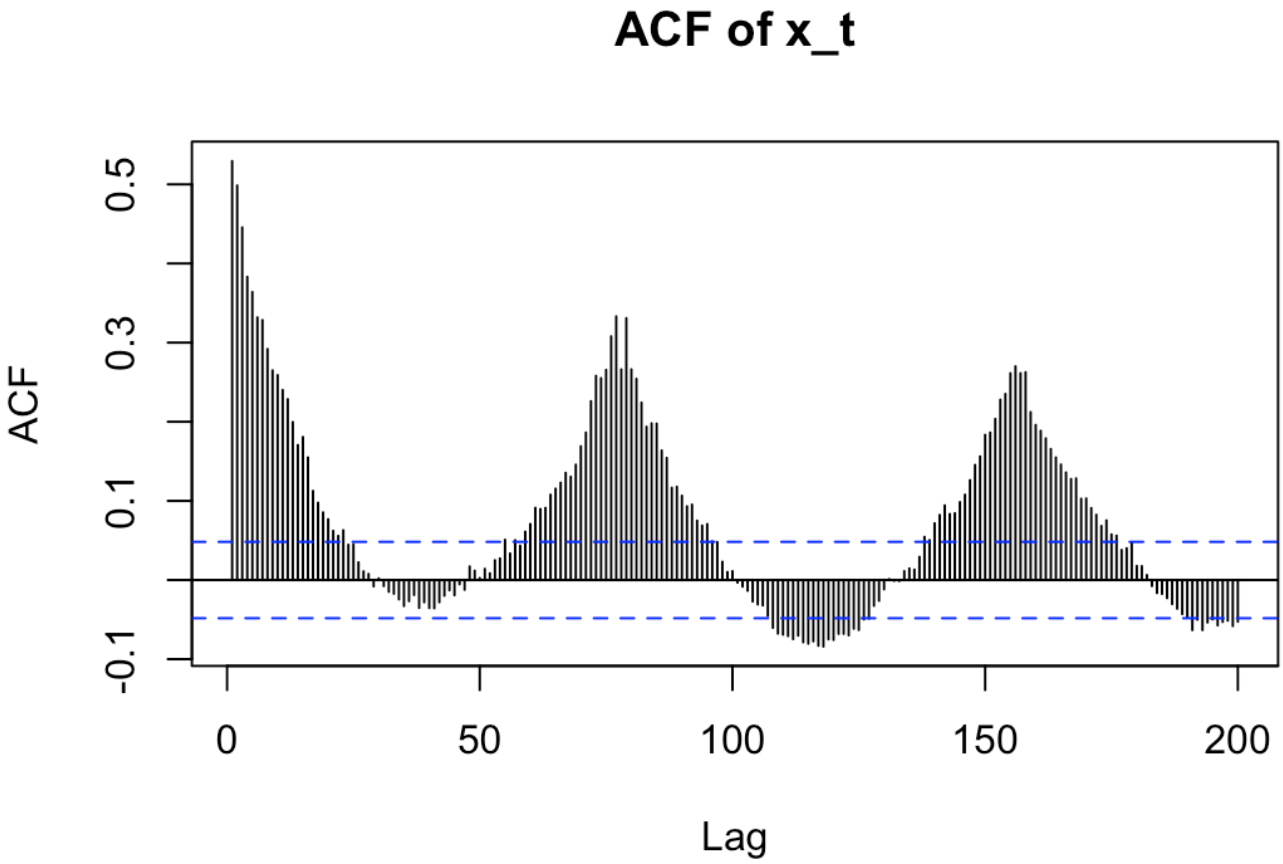
```
Acf(freq_matrix, lag.max = 200, type = "correlation", plot = TRUE, na.action = na.con
tiguous, demean = TRUE, main='ACF of x_t')
```



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```
# discussion
print('From the plot of both the series and its ACF, it is easy to see that there are
intraday period patterns characterized by spikes in the number of trades in the first
seconds that the market is open and near the close. This seasonality is called U-Shap
ed (see page 289 of Tze Leung Lai and Haipeng Xings book, "Statistical Models and Met
hods for Financial Markets").')
```

```
[1] "From the plot of both the series and its ACF, it is easy to see that there are i
ntraday period patterns characterized by spikes in the number of trades in the first
seconds that the market is open and near the close. This seasonality is called U-Shap
ed (see page 289 of Tze Leung Lai and Haipeng Xings book, \"Statistical Models and Me
thods for Financial Markets\")."
```

(b) Using the last transaction price in the ith 5-minute interval as the stock price in that interval, plot the time series y_t of 5-minute log returns during the period and the corresponding ACF.

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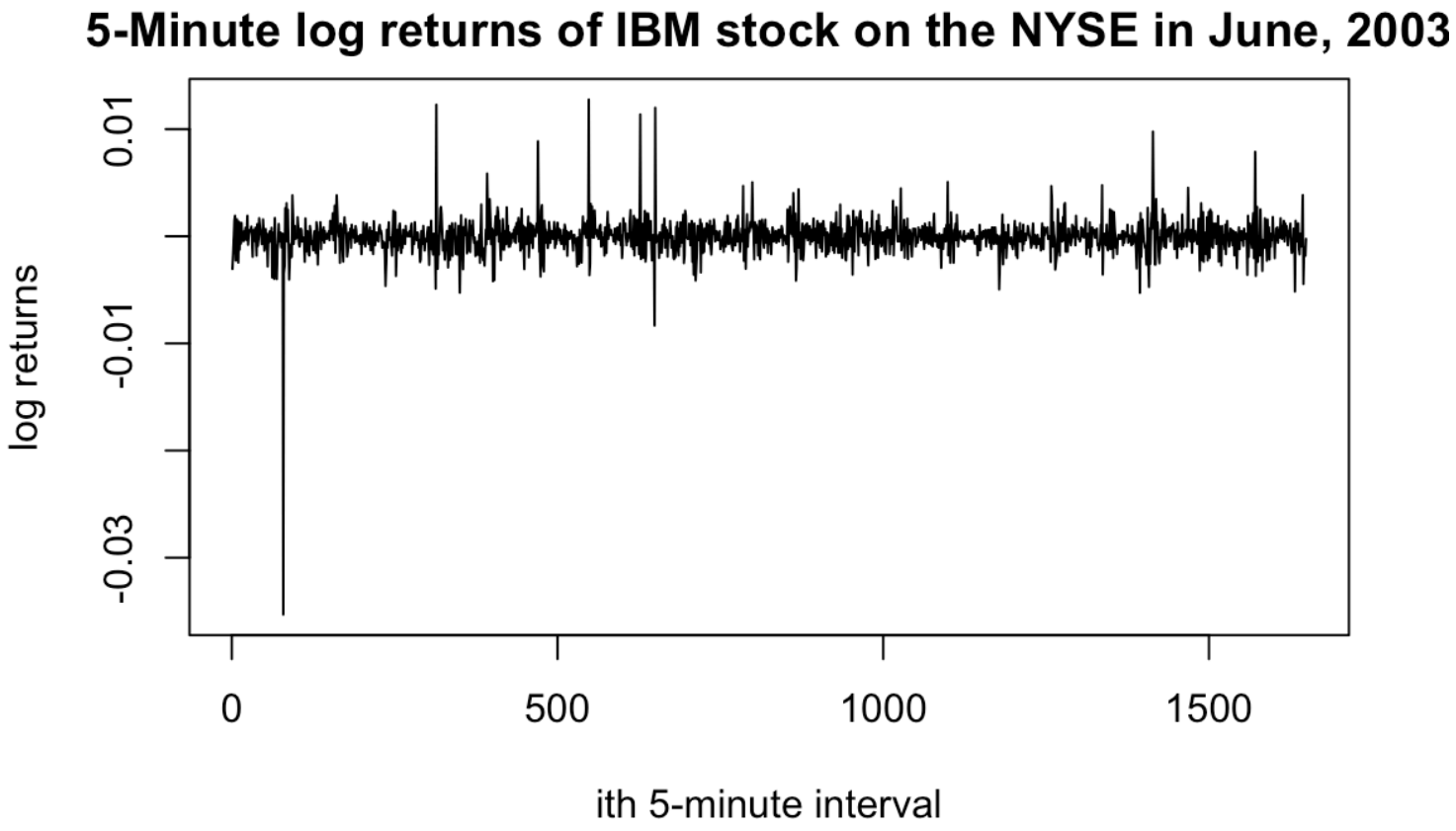
```
# here we take the last transaction price per 5-min interval
list_price <- lapply(store_df, function(x)tail(x$PRICE, n=1))
# now we remove all rows with an entry of 0, thus ignoring the time gaps between trad
ing days
list_price <- list_price[lapply(list_price, length)>0]
# in order to plot more easily we make the list a matrix
price_matrix <- matrix(unlist(list_price), nrow=length(list_price), byrow=TRUE)
# here we compute the time series of 5-minute log returns
log_returns <- diff(log(price_matrix), lag=1, differences=1)
# plot series
plot(log_returns, type='l', xlab='ith 5-minute interval', ylab='log returns', main='5
-Minute log returns of IBM stock on the NYSE in June, 2003')
# plot ACF
par(mar = c(4, 4, 4, 4))
```



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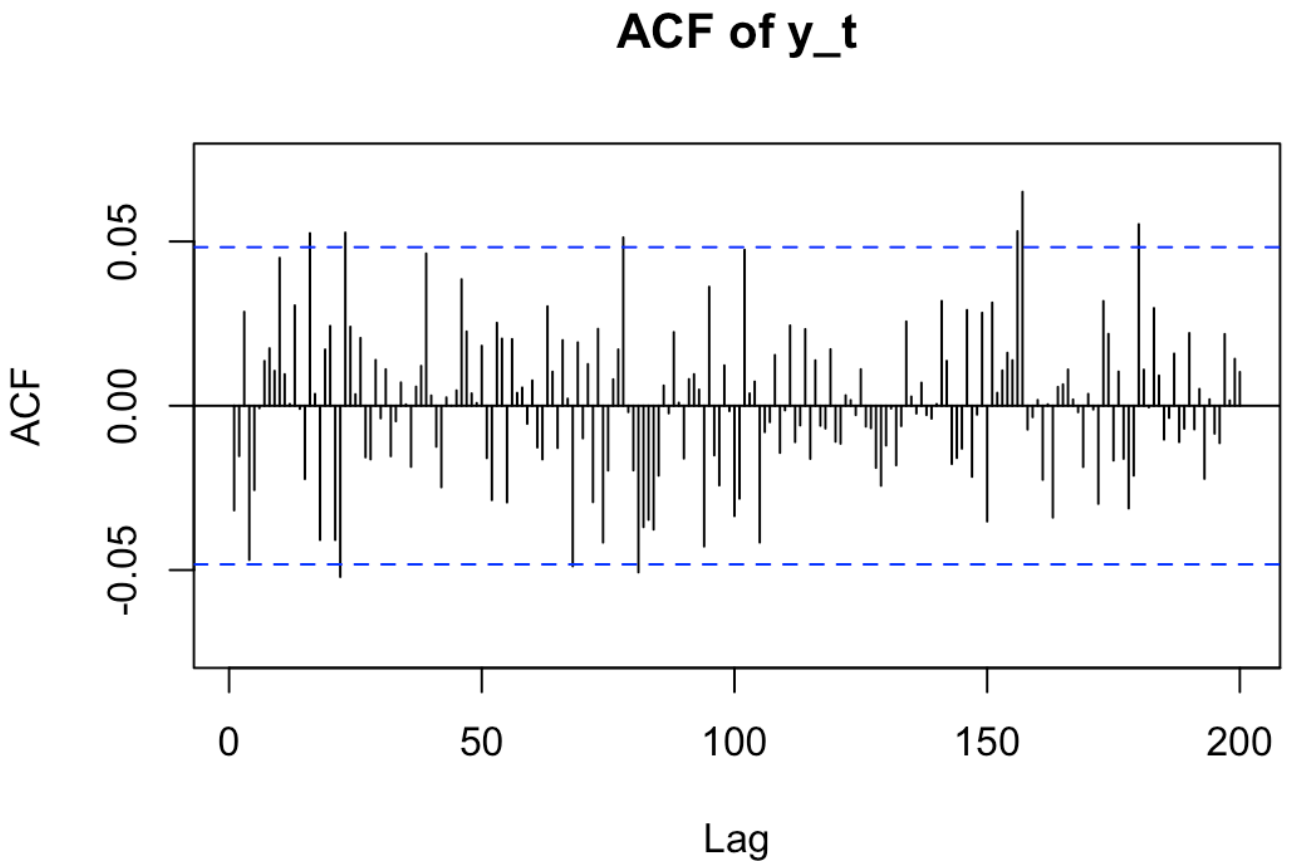
```
Acf(log_returns, lag.max=200, type="correlation", plot=TRUE, na.action=na.contiguous,
demean=TRUE, main='ACF of y_t')
```



(c) Consider the bivariate time series (x_t, y_t). How does y_t vary with x_t? Are there intraday periodic patterns in (x_t, y_t)?

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```
# plot the two time series with the same x-axis and different y-axes
par(mar = c(4, 4, 4, 4))
plot(freq_matrix, col='orange', type='l', xlab='ith 5-minute interval', ylab=NA, main
='x_t v. y_t')
legend(-50, 2200000, legend=c("x_t", "y_t"), col=c("orange", "red"), lty=1:1, cex=0.8
)
```

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```
par(new=TRUE)
plot(log_returns, col='red', type='l', axes=FALSE, xlab=NA, ylab=NA)
```

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```
axis(side=4)
mtext('log returns', col='red', side = 4, line=2.5)
```

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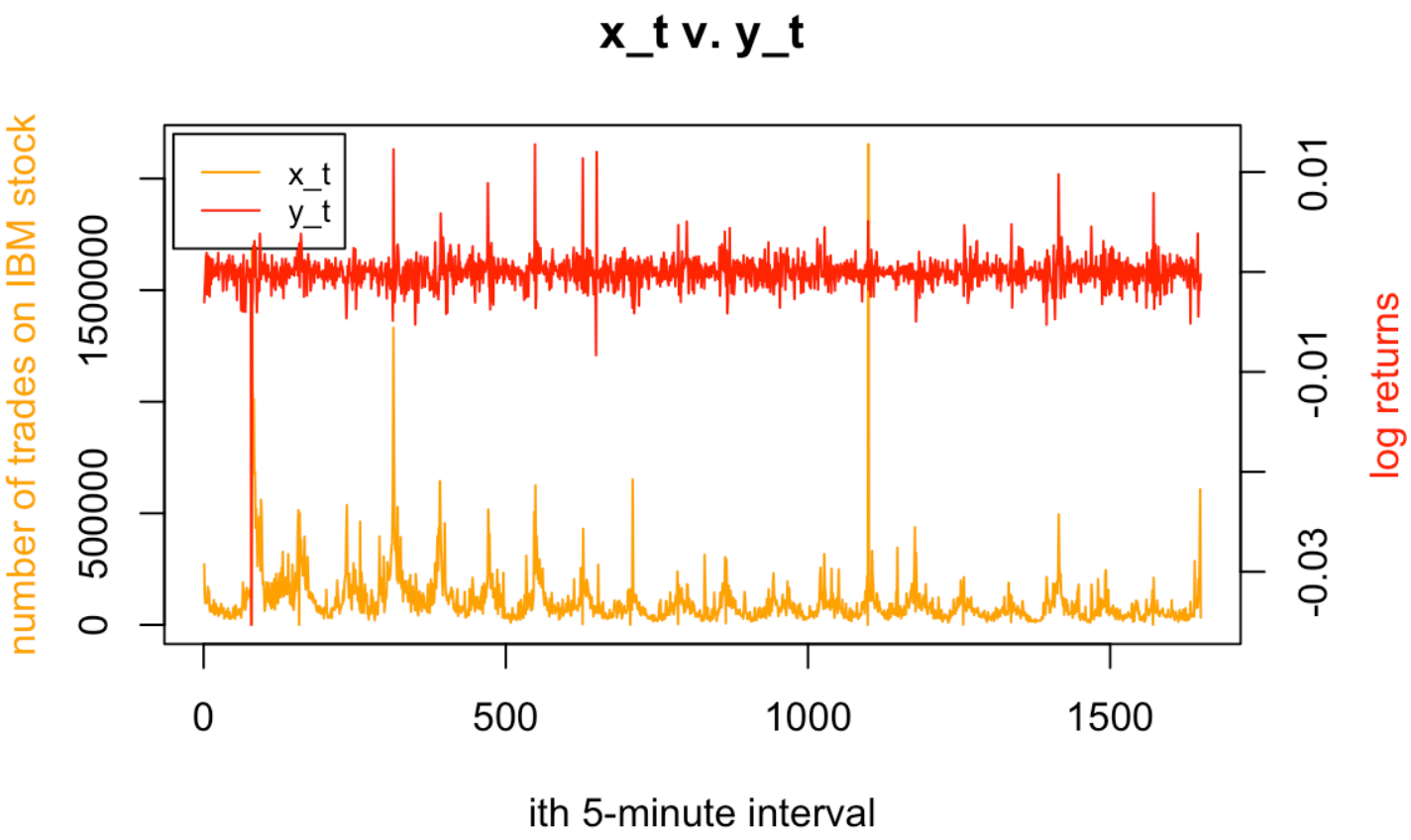
```
mtext('number of trades on IBM stock', col='orange', side=2, line=2.5)
# this code was inspired by https://www.youtube.com/watch?v=-ImppGbVpXI
x_t <- as.ts(freq_matrix)
y_t <- as.ts(log_returns)
# plot the ACF
par(mar = c(4, 4, 4, 4))
```



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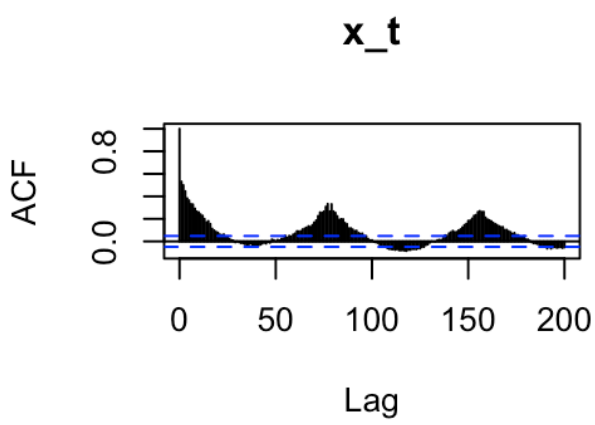
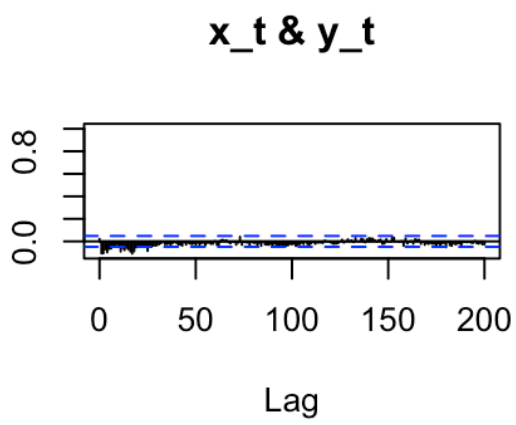
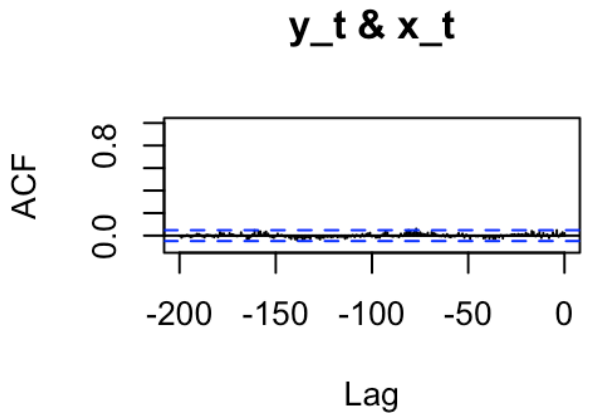
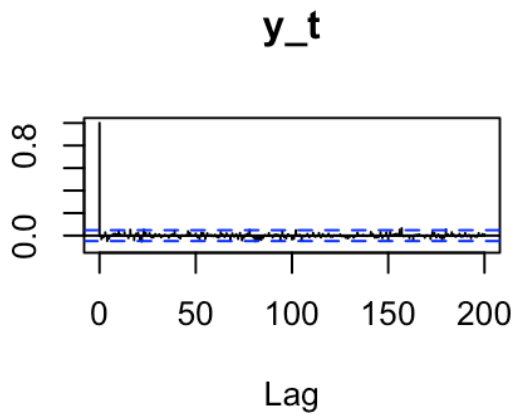
```
acf(ts.intersect(x_t, y_t), lag.max=200)
```









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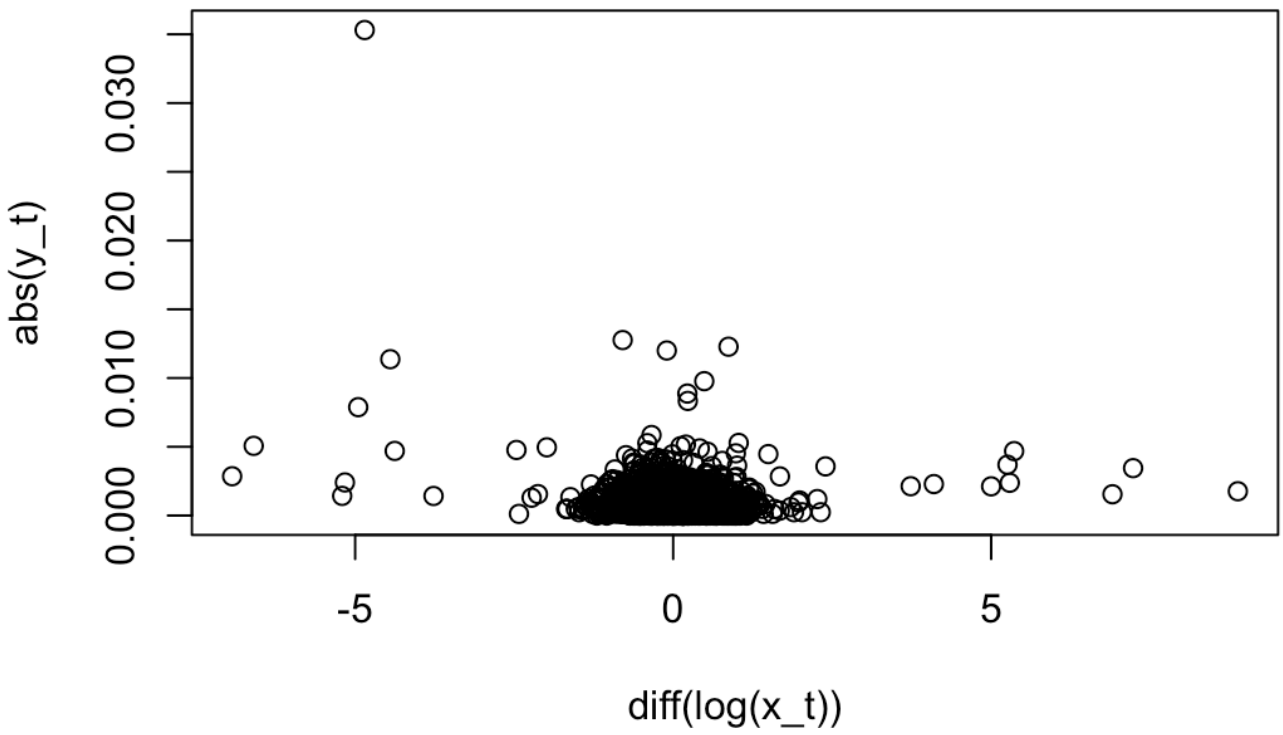
```
# we make a scatter plot to find correlation
plot(diff(log(x_t)), abs(y_t))
```



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```
# here we compute the correlation between the volumne and the absolute value of the l
og returns
corr <- cor(freq_matrix[-1], abs(log_returns))
print(corr)
```

```
          [,1]
[1,] 0.4979111
```

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```
# here we regress the volume on the squared returns to find R^2
regres <- lm(freq_matrix[-1] ~ log_returns^2)
summary(regres)
```

```
Call:
lm(formula = freq_matrix[-1] ~ log_returns^2)

Residuals:
    Min      1Q  Median      3Q     Max
-106681  -59174  -31057   19804 2086462

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   104357       2968  35.159  < 2e-16 ***
log_returns -7402963    1712077  -4.324 1.62e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 120500 on 1647 degrees of freedom
Multiple R-squared:  0.01122,   Adjusted R-squared:  0.01062
F-statistic:  18.7 on 1 and 1647 DF,  p-value: 1.624e-05
```

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```
print('The ACF plots in the top left and bottom right show that there is little to no statistically significant correlation between the time series x_t and y_t, this is also supported by the regression. We also see that there is no intraday periodic pattern in (x_t, y_t). This makes sense since returns do not tend to spike at the beginning and end of the trading day like volume does. That having been said, when we compute the correlation between the volume and the absolute value of the log_returns, we get some correlation.')
```

```
[1] "The ACF plots in the top left and bottom right show that there is little to no statistically significant correlation between the time series x_t and y_t, this is also supported by the regression. We also see that there is no intraday periodic pattern in (x_t, y_t). This makes sense since returns do not tend to spike at the beginning and end of the trading day like volume does. That having been said, when we compute the correlation between the volume and the absolute value of the log_returns, we get some correlation."
```

(d) Tabulate the relative frequencies of price changes in multiples of the tick size $\$0.0625$.

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```
df_ts <- ts(df$PRICE)
price_changes <- diff(df_ts, lag=1, differences=1)
# we tabulate the relative frequencies of price changes in multiples of the tick size
freq_price_changes <- price_changes/0.0625
tabulated <- table(freq_price_changes)
print(tabulated)
```

```
freq_price_changes
              -131.2              -129.44    -48.1600000000001
                   1                    1                    1
   -47.8399999999999               -23.52    -23.3600000000001
                   1                    1                    1
   -21.1199999999999    -19.8399999999999    -18.5600000000002
                   1                    1                    1
   -17.4400000000001    -17.2800000000002    -17.1200000000001
                   1                    1                    1
   -16.3200000000002                  -16    -15.8399999999999
                   1                    3                    2
             -15.52    -15.3599999999999                -15.2
                   2                    2                    1
   -15.1999999999998    -14.8800000000001    -14.7583999999999
                   2                    1                    1
             -14.72               -14.24    -14.0799999999999
                   1                    1                    2
   -13.9200000000001    -13.9199999999998               -13.76
                   1                    1                    3
   -13.6000000000001    -13.5999999999999               -13.28
                   1                    1                    1
   -13.0559999999998               -12.96                -12.8
                   1                    1                    4
   -12.6400000000001    -12.6399999999999               -12.48
                   2                    1                    2
   -12.3999999999999    -12.0320000000002                  -12
                   1                    1                    1
             -11.52    -11.0143999999998    -10.9744000000001
                   1                    1                    1
   -10.3999999999999               -10.24    -9.68000000000006
                   1                    1                    1
   -9.59999999999991    -9.44000000000005    -9.27999999999997
                   2                    1                    3
   -9.24800000000005    -9.13599999999997    -9.11999999999989
                   1                    1                    1
   -8.79999999999995     -8.7360000000001     -8.6400000000001
                   2                    1                    2
   -8.63999999999987    -8.54080000000022    -8.32000000000016
                   1                    1                    2
   -8.31200000000013    -8.25119999999993    -8.16000000000008
                   1                    1                    1
                  -8    -7.89440000000013    -7.83999999999992
```

3	1	1
-7.7503999999999	-7.67999999999984	-7.64799999999991
1	1	1
-7.51999999999998	-7.3599999999999	-7.20000000000005
3	1	7
-7.03999999999996	-6.88000000000011	-6.72000000000003
2	1	1
-6.56479999999988	-6.56000000000017	-6.55999999999995
1	1	2
-6.55199999999991	-6.40000000000009	-6.39999999999986
1	1	1
-6.32639999999992	-6.2976000000001	-6.27840000000015
1	1	1
-6.24000000000001	-6.11040000000003	-6.07680000000005
2	1	1
-6.00160000000005	-5.97759999999994	-5.92000000000007
1	1	1
-5.75999999999999	-5.59999999999991	-5.58400000000006
4	1	2
-5.44000000000005	-5.27999999999997	-5.12000000000012
2	1	3
-5.11999999999989	-4.96000000000004	-4.95999999999981
4	3	2
-4.88000000000011	-4.87200000000007	-4.83680000000004
1	1	1
-4.80960000000005	-4.80000000000018	-4.79999999999995
1	4	9
-4.6400000000001	-4.63999999999987	-4.48000000000002
4	2	2
-4.32000000000016	-4.31999999999994	-4.16000000000008
2	4	5
-4.15999999999985	-4.07999999999993	-4.05759999999987
1	1	1
-4.04320000000007	-4	-3.98399999999992
1	39	1
-3.98080000000004	-3.9344000000001	-3.85600000000022
1	1	1
-3.85599999999999	-3.84000000000015	-3.83999999999992
1	3	7
-3.74399999999991	-3.71519999999987	-3.68319999999994
1	1	1
-3.68000000000006	-3.67039999999997	-3.66399999999999
8	1	1
-3.65599999999995	-3.61760000000004	-3.51999999999998
1	1	12
-3.36320000000001	-3.36000000000013	-3.3599999999999
2	4	3
-3.27999999999997	-3.24800000000005	-3.22080000000005
2	1	1
-3.20320000000015	-3.20000000000005	-3.19999999999982

1	16	1
-3.1232	-3.11680000000001	-3.03999999999996
1	1	14
-2.98080000000004	-2.96000000000004	-2.88000000000011
1	1	10
-2.87999999999988	-2.83999999999992	-2.83200000000011
9	1	1
-2.79999999999995	-2.78240000000005	-2.76800000000003
5	1	1
-2.72000000000003	-2.7199999999998	-2.62080000000014
27	5	2
-2.57600000000002	-2.56000000000017	-2.55999999999995
1	8	19
-2.54880000000003	-2.52800000000002	-2.50240000000008
1	1	1
-2.49600000000009	-2.46399999999994	-2.43200000000002
1	1	1
-2.40000000000009	-2.39999999999986	-2.38719999999989
57	19	1
-2.38400000000001	-2.36799999999994	-2.35200000000009
2	1	1
-2.31359999999995	-2.27199999999993	-2.24000000000001
1	1	61
-2.23999999999978	-2.21759999999995	-2.16640000000007
1	1	1
-2.16000000000008	-2.1536000000001	-2.14400000000001
1	1	2
-2.10879999999997	-2.09120000000007	-2.08000000000015
1	1	33
-2.07999999999993	-2.07680000000005	-2.06400000000008
47	1	1
-2.048	-2.03200000000015	-2.03199999999993
1	2	1
-2.01760000000013	-2.01600000000008	-2
1	1	3
-1.9856000000002	-1.98399999999992	-1.97760000000017
1	1	1
-1.96799999999985	-1.9552000000001	-1.94400000000019
2	1	1
-1.93280000000004	-1.92799999999988	-1.92000000000007
1	1	94
-1.91999999999985	-1.91200000000003	-1.904
26	1	3
-1.88800000000015	-1.87999999999988	-1.87199999999984
1	1	1
-1.85599999999999	-1.85439999999994	-1.84480000000008
1	1	2
-1.84000000000015	-1.83999999999992	-1.82400000000007
1	1	2
-1.8112000000001	-1.77600000000007	-1.77599999999984

1	1	1
-1.76000000000022	-1.75999999999999	-1.74400000000014
1	141	2
-1.74399999999991	-1.73759999999993	-1.72800000000007
2	1	1
-1.72799999999984	-1.72319999999991	-1.72160000000008
1	1	1
-1.7152000000001	-1.68000000000006	-1.67999999999984
1	3	3
-1.66879999999992	-1.65280000000007	-1.63200000000006
1	1	5
-1.61600000000021	-1.61599999999999	-1.60000000000014
1	3	108
-1.59999999999991	-1.59680000000003	-1.58400000000006
183	1	8
-1.58399999999983	-1.56800000000021	-1.56799999999998
1	1	3
-1.5616	-1.53600000000006	-1.53440000000001
1	2	1
-1.51999999999998	-1.51519999999982	-1.5136
5	1	1
-1.50399999999991	-1.48799999999983	-1.47679999999991
1	1	1
-1.47199999999998	-1.45600000000013	-1.4559999999999
1	2	2
-1.44000000000005	-1.43999999999983	-1.42400000000021
262	64	1
-1.42399999999998	-1.41759999999999	-1.40800000000013
12	1	2
-1.4079999999999	-1.40160000000014	-1.39840000000004
3	1	1
-1.39200000000005	-1.38400000000001	-1.36159999999995
1	1	1
-1.36000000000013	-1.3599999999999	-1.3456000000001
4	5	1
-1.3280000000002	-1.3248000000001	-1.31999999999994
1	1	1
-1.31200000000013	-1.30720000000019	-1.30560000000014
3	1	1
-1.29600000000005	-1.2847999999999	-1.2800000000002
3	1	72
-1.27999999999997	-1.27359999999999	-1.26559999999995
388	2	1
-1.26400000000012	-1.2639999999999	-1.24800000000005
6	10	2
-1.24639999999999	-1.24480000000017	-1.24000000000001
1	1	1
-1.23199999999997	-1.22239999999988	-1.21600000000012
2	1	1
-1.21120000000019	-1.20959999999991	-1.20320000000015

1	1	1
-1.20000000000005	-1.19999999999982	-1.19839999999999
9	2	1
-1.19359999999983	-1.1840000000002	-1.18399999999997
1	2	6
-1.16959999999995	-1.16000000000008	-1.15520000000015
1	2	1
-1.1536000000001	-1.15200000000004	-1.15199999999982
1	2	1
-1.15039999999999	-1.14879999999994	-1.14720000000011
1	1	1
-1.13600000000019	-1.13599999999997	-1.13439999999991
2	4	1
-1.12000000000012	-1.11999999999989	-1.10400000000004
298	319	10
-1.10399999999981	-1.08800000000019	-1.08799999999997
7	1	4
-1.0752	-1.07200000000012	-1.07199999999989
1	3	1
-1.05600000000004	-1.03999999999996	-1.03359999999998
3	10	1
-1.02400000000011	-1.02399999999989	-1.00800000000004
3	2	2
-1.00799999999981	-1.00479999999993	-1.00160000000005
1	1	1
-0.995200000000068	-0.992000000000189	-0.991999999999962
2	1	8
-0.98239999999987	-0.980800000000045	-0.976000000000113
1	1	11
-0.975999999999885	-0.963199999999915	-0.960000000000036
5	1	826
-0.959999999999809	-0.948799999999892	-0.944000000000187
175	1	8
-0.94399999999996	-0.940800000000081	-0.939200000000028
29	1	1
-0.928000000000111	-0.927999999999884	-0.920000000000073
8	6	2
-0.915200000000141	-0.912000000000035	-0.911999999999807
1	3	1
-0.903999999999996	-0.896000000000186	-0.895999999999958
1	1	10
-0.892800000000079	-0.889599999999973	-0.880000000000109
1	1	14
-0.879999999999882	-0.878399999999829	-0.865600000000086
11	1	2
-0.864000000000033	-0.863999999999805	-0.851200000000063
6	2	1
-0.848000000000184	-0.847999999999956	-0.839999999999918
2	5	2
-0.832000000000107	-0.83199999999988	-0.822400000000016

11	3	1
-0.816000000000031	-0.815999999999804	-0.807999999999993
10	1	1
-0.800000000000182	-0.799999999999955	-0.798399999999901
331	1583	1
-0.784000000000106	-0.783999999999878	-0.782400000000052
27	17	1
-0.77760000000012	-0.774400000000014	-0.768000000000029
1	1	21
-0.767999999999802	-0.755200000000059	-0.751999999999953
4	1	5
-0.747200000000021	-0.745600000000195	-0.736000000000104
1	1	6
-0.735999999999876	-0.732799999999997	-0.723199999999906
3	2	1
-0.72160000000008	-0.720000000000027	-0.7199999999998
1	23	6
-0.704000000000178	-0.703999999999951	-0.700799999999845
2	12	1
-0.688000000000102	-0.687999999999874	-0.683199999999943
3	2	1
-0.680000000000064	-0.679999999999836	-0.672000000000025
2	1	20
-0.671999999999798	-0.670399999999972	-0.668799999999919
2	1	1
-0.66560000000004	-0.656000000000176	-0.655999999999949
1	8	14
-0.644800000000032	-0.6400000000001	-0.639999999999873
1	1678	1284
-0.632000000000062	-0.628800000000183	-0.624000000000024
1	1	68
-0.623999999999796	-0.619200000000092	-0.617600000000039
18	1	1
-0.61440000000016	-0.611200000000053	-0.611199999999826
1	1	1
-0.6096	-0.608000000000175	-0.607999999999947
1	7	18
-0.601599999999962	-0.592000000000098	-0.591999999999871
1	5	7
-0.585600000000113	-0.576000000000022	-0.560000000000173
1	13	12
-0.559999999999945	-0.55840000000012	-0.544000000000096
38	1	8
-0.543999999999869	-0.536000000000058	-0.52800000000002
5	1	17
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 [ reached getOption("max.print") -- omitted 174 entries ]
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Hide

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# histogram
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, ylab='relative frequency', xlab='price change in multiples of $0.0625')
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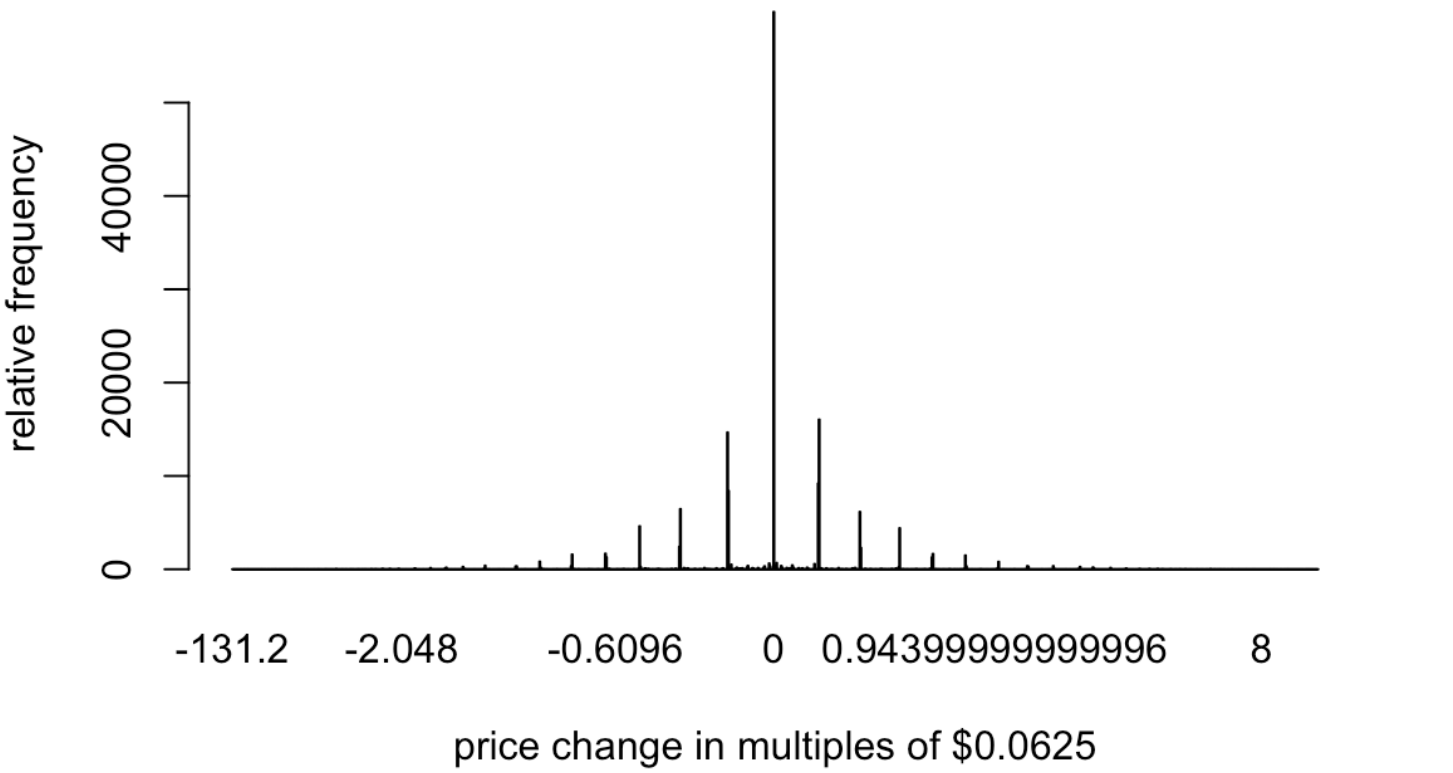
Realative frequency of price change in multiples of $0.0625
relative frequency
0
20000
40000
-131.2
-2.048
-0.6096
0
0.94399999999996
8
price change in multiples of $0.0625