Unit-Root Nonstatinarity, VAR(p) Model & Cointegration Analysis

Code

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The file `m_cofi_4rates.txt` contains the monthly rates of the 11th District Cost of Funds Index (COFI), the prime rate of U.S. banks, 1-year and 5-year U.S. Treasury constant maturity rates, and U.S. Treasury 3-month secondary market rates from September 1989 to June 2007. The COFI rates are obtained from the Federal Home Loan Bank of San Francisco, and the other rates are obtained from the Federal Reserve Bank of St. Louis. COFI is a weighted-average interest rate paid by savings institutions headquartered in Arizona, California, and Nevada and is one of the most popular adjustable-rate mortgage (ARM) indices. The prime rate is the interest rate at which banks lend to their most creditworthy customers.

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```
df <- read.table('m_cofi_4rates.txt', header=TRUE)
# this question's code was inspired by https://web.stanford.edu/~xing/statfinbook/_BookFun/ex9.4.5_unitrootCoint.txt
```

(a) Perform the augmented Dickey-Fuller test of the unit-root hypothesis for each of these rates.

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```
print('The augmented Dickey-Fuller test of the unit-root hypothesis for the monthly r
ates of COFI from September 1989 to June 2007 is:')
```

```
[1] "The augmented Dickey-Fuller test of the unit-root hypothesis for the monthly rat
es of COFI from September 1989 to June 2007 is:"
```

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```
adf.test(df$cofi)
```

```

    Augmented Dickey-Fuller Test

data:  df$cofi
Dickey-Fuller = -2.8745, Lag order = 5, p-value = 0.2092
alternative hypothesis: stationary
```

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```
print('The augmented Dickey-Fuller test of the unit-root hypothesis for the 1-year U.
S. Treasury constant maturity rates from September 1989 to June 2007 is:')
```

```
[1] "The augmented Dickey-Fuller test of the unit-root hypothesis for the 1-year U.S.
Treasury constant maturity rates from September 1989 to June 2007 is:"
```

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```
adf.test(df$X1ycmt)
```

```

    Augmented Dickey-Fuller Test

data:  df$X1ycmt
Dickey-Fuller = -2.0479, Lag order = 5, p-value = 0.5559
alternative hypothesis: stationary
```

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```
print('The augmented Dickey-Fuller test of the unit-root hypothesis for the 5-year U.
S. Treasury constant maturity rates from September 1989 to June 2007 is:')
```

```
[1] "The augmented Dickey-Fuller test of the unit-root hypothesis for the 5-year U.S.
Treasury constant maturity rates from September 1989 to June 2007 is:"
```

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```
adf.test(df$X5ycmt)
```

```
	Augmented Dickey-Fuller Test

data:  df$X5ycmt
Dickey-Fuller = -2.0488, Lag order = 5, p-value = 0.5555
alternative hypothesis: stationary
```

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```
print('The augmented Dickey-Fuller test of the unit-root hypothesis for the prime rat
es of U.S. banks from September 1989 to June 2007 is:')
```

```
[1] "The augmented Dickey-Fuller test of the unit-root hypothesis for the prime rates
of U.S. banks from September 1989 to June 2007 is:"
```

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```
adf.test(df$primeRate)
```

```
	Augmented Dickey-Fuller Test

data:  df$primeRate
Dickey-Fuller = -2.3052, Lag order = 5, p-value = 0.448
alternative hypothesis: stationary
```

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```
print('The augmented Dickey-Fuller test of the unit-root hypothesis for the U.S. Trea
sury 3-month secondary market rates from September 1989 to June 2007 is:')
```

```
[1] "The augmented Dickey-Fuller test of the unit-root hypothesis for the U.S. Treasu
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```

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```
adf.test(df$X3mTbill.2mkt)
```

```
	Augmented Dickey-Fuller Test

data:  df$X3mTbill.2mkt
Dickey-Fuller = -2.5454, Lag order = 5, p-value = 0.3472
alternative hypothesis: stationary
```

(b) Assuming the VAR(2) model for the multivariate time series of these five rates, perform Johansen’s test for the number of cointegration vectors.

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```
johansens <- ca.jo(df, type="eigen", K=2, season=NULL)
summary(johansens)
```

```
######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.28115617 0.18230111 0.12979406 0.07225424 0.03464075

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 4 |  7.47  6.50  8.18 11.65
r <= 3 | 15.90 12.91 14.90 19.19
r <= 2 | 29.47 18.90 21.07 25.75
r <= 1 | 42.67 24.78 27.14 32.14
r = 0  | 69.98 30.84 33.32 38.78

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                  cofi.l2  X1ycmt.l2  X5ycmt.l2 primeRate.l2 X3mTbill.2mkt.l2
cofi.l2          1.000000  1.0000000  1.0000000     1.000000        1.0000000
X1ycmt.l2       -5.351875  2.7399460  2.4996108     1.384167        2.9771100
X5ycmt.l2        1.534273 -0.9231277 -1.5278353    -1.699396       -4.6375957
primeRate.l2     1.908798  0.3260260  0.9516442    -3.724769       -1.7744334
X3mTbill.2mkt.l2 1.443581 -3.2981390 -3.0118510     2.822877       -0.6152458

Weights W:
(This is the loading matrix)

                   cofi.l2    X1ycmt.l2   X5ycmt.l2 primeRate.l2
cofi.d         -0.02811300 -0.028599110  0.00204150 -0.005949579
X1ycmt.d       -0.02835492  0.032656460 -0.02063724 -0.003506453
X5ycmt.d       -0.02717289  0.007413592  0.03393117  0.017832126
primeRate.d    -0.04463161  0.011492886 -0.04974080  0.009034110
X3mTbill.2mkt.d -0.04605080  0.116701546 -0.02176690 -0.009198674
               X3mTbill.2mkt.l2
cofi.d             2.823119e-05
X1ycmt.d           8.259722e-03
X5ycmt.d           9.537910e-03
primeRate.d        3.817692e-04
X3mTbill.2mkt.d    4.400697e-03
```

(c) Estimate the cointegration vectors and use them to describe the equilibrium relationship between the five rates.

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```
coin_vec_1 <- c(1, -5.351875, 1.534274, 1.908798, 1.443581)
coin_vec_2 <- c(1, 2.739946, -0.9231277, 0.3260260, -3.2981390)
coin_vec_3 <- c(1, 2.4996108, -1.5278353, 0.9516442, -3.0118510)
coin_vec_4 <- c(1, 1.384167, -1.699396, -3.724769, 2.822877)
coin_1 <- df$cofi * 1 - 5.351875 * df$X1ycmt + 1.534274 * df$X5ycmt + 1.908798 * df$p
rimeRate + 1.443581 * df$X3mTbill.2mkt
coin_2 <- df$cofi * 1 + 2.739946 * df$X1ycmt - 0.9231277 * df$X5ycmt + 0.3260260 * df
$primeRate - 3.2981390 * df$X3mTbill.2mkt
coin_3 <- df$cofi * 1 + 2.4996108 * df$X1ycmt - 1.5278353 * df$X5ycmt + 0.9516442 * d
f$primeRate - 3.0118510 * df$X3mTbill.2mkt
coin_4 <- df$cofi * 1 + 1.384167 * df$X1ycmt - 1.699396 * df$X5ycmt - 3.724769 * df$p
rimeRate + 2.822877 * df$X3mTbill.2mkt
series_model_1 <- as.matrix(df) %*% coin_vec_1
series_model_2 <- as.matrix(df) %*% coin_vec_2
series_model_3 <- as.matrix(df) %*% coin_vec_3
series_model_4 <- as.matrix(df) %*% coin_vec_4
plot(series_model_1, type = 'l')
```

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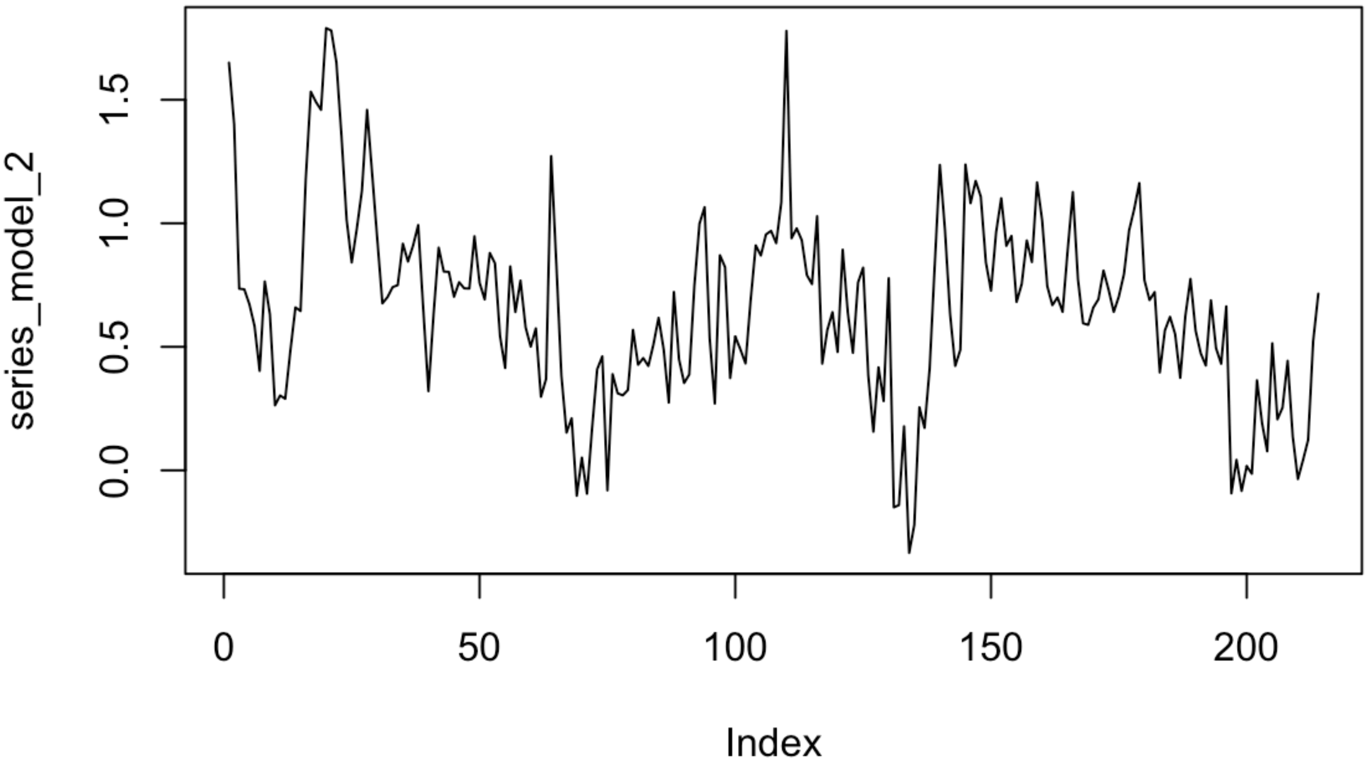
```
plot(series_model_2, type = 'l')
```



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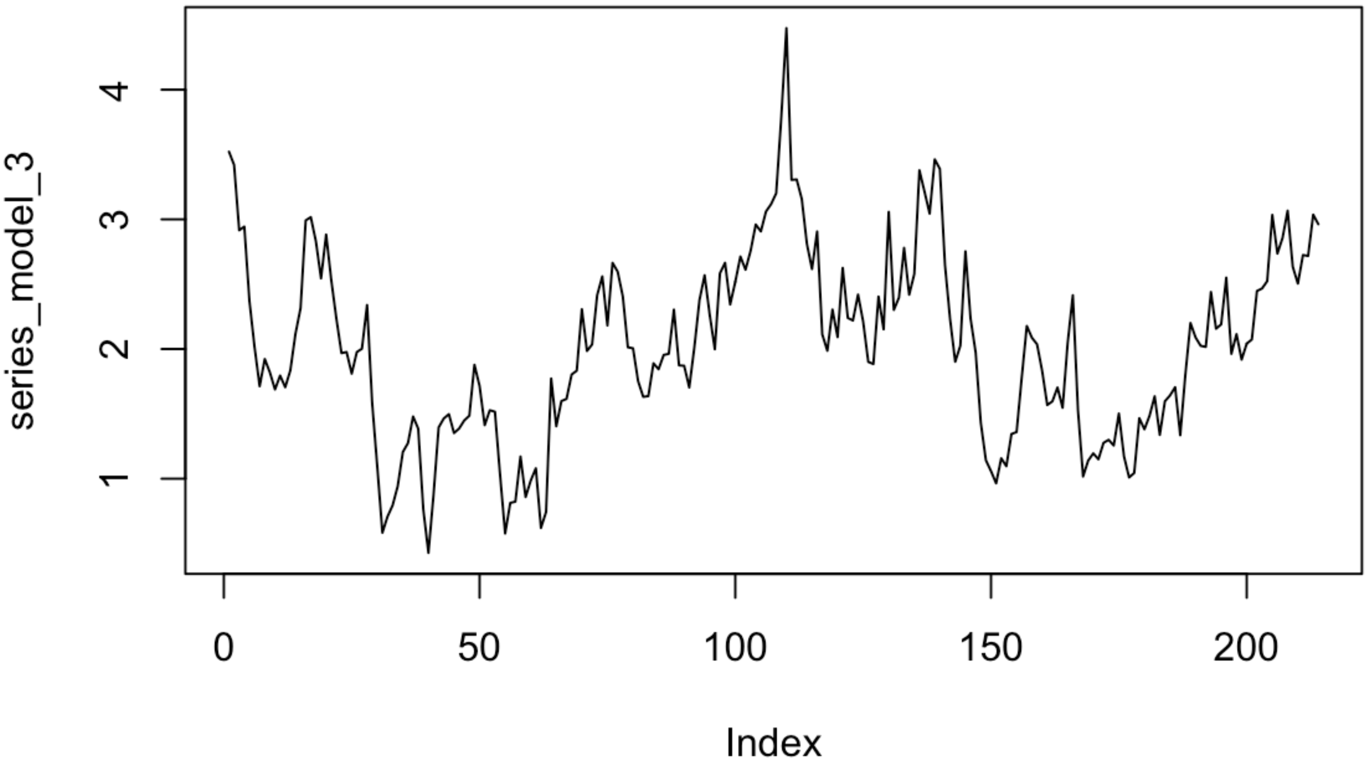
```
plot(series_model_3, type = 'l')
```



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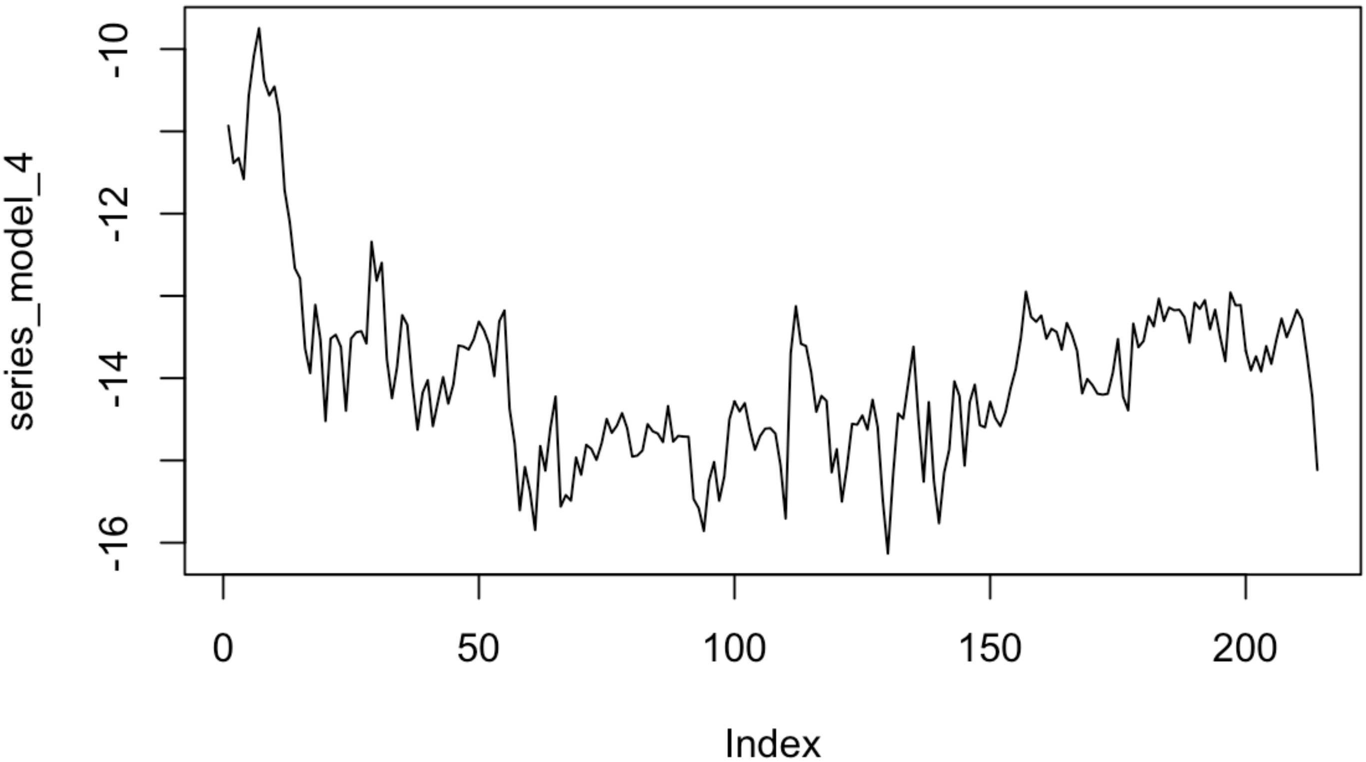
```
plot(series_model_4, type = 'l')
```



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```
print('Given our estimates of the cointegration vectors and their respective plots, w
e see that even though the indvidual components are not stationary and have variances
that diverge to infinity, they share common trends that result in beta_i^top y_t havi
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see that even though the indvidual components are not stationary and have variances t
hat diverge to infinity, they share common trends that result in beta_i^top y_t havin
g long-run equilibrium for i=1,...,4."
```

(d) Regress COFI on the four other rates. Discuss the economic meaning of this regression relationship and whether the regression is spurious.

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```
fit <- lm(df$cofi ~ df$X1ycmt + df$X5ycmt + df$primeRate + df$X3mTbill.2mkt)
summary(fit)
```

```
Call:
lm(formula = df$cofi ~ df$X1ycmt + df$X5ycmt + df$primeRate +
    df$X3mTbill.2mkt)

Residuals:
     Min       1Q   Median       3Q      Max
-0.71351 -0.21455 -0.03518  0.16148  1.26509

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)      -0.874633   0.338637  -2.583   0.0105 *
df$X1ycmt        -2.897190   0.116331 -24.905   <2e-16 ***
df$X5ycmt         1.236494   0.046559  26.558   <2e-16 ***
df$primeRate     -0.003372   0.087235  -0.039   0.9692
df$X3mTbill.2mkt  2.859960   0.140077  20.417   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3328 on 209 degrees of freedom
Multiple R-squared:  0.9607,    Adjusted R-squared:   0.96
F-statistic:  1279 on 4 and 209 DF,  p-value: < 2.2e-16
```

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```
print('Given the high R^2 we can conclude that the model explains the variability of
the response data around its mean fairly well. The COFI is a regional average of inte
rest expenses incurred by financial institutions, which in turn is used as a base for
calculating variable rate loans. It makes sense that these rates would vary according
to one another so the regression is unlikely to be spurious. Futhermore, from part (c
) since \beta_{ji}\neq 0, the linear regression of y_{tj} on the other components of
y_t is not spurious even though y_t is unit root stationary.')
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