Linear Programming Using Simplex Method

Logic & Problem Solving

Lecture Week 24

Linear Programming

Agenda:

- **Week 24 lecture coverage**
  - Introduction to LPP
  - Mathematical Formulation of LPP
  - Solving LPP using simplex method

Amazing Magic Tricks …

LPP Using Graphical Method :

Steps for graphical method :

1. **Formulate** the **mathematical model** of the given LPP.
2. Change the **inequalities** involved in the constraints to **equality**.
3. **Plot** each equation on the **graph** paper finding at least two points and also do the **origin test**.
4. Find the corners of **feasible region** or solution area and get the solution to the given LPP.

LPP Using Simplex Method :

Graphical method can solve LPP problems involving only **two decision variables** .

We must use **Simplex method** to solve LPP problems involving **Three or more decision variables** .

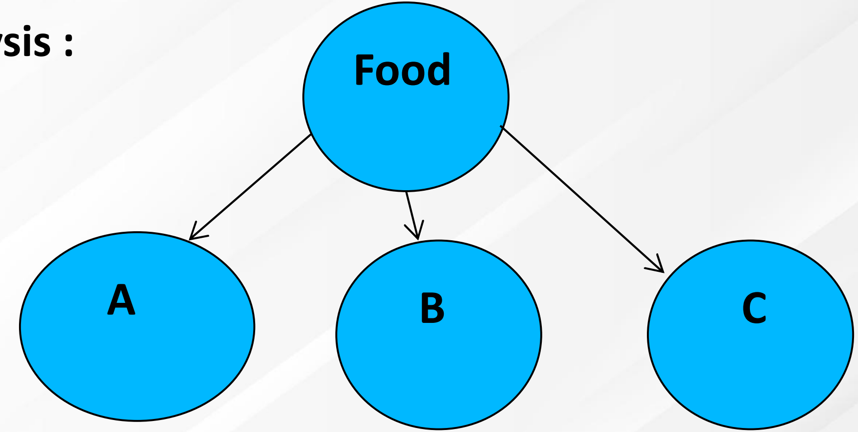
LPP Using Simplex Method :

For Example :

Suppose that **8, 12 and 9 units** of protein, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains **2, 6 and 1** units of protein, carbohydrate and fat respectively per kg. , food B contains **1, 1 and 3** units respectively per kg. and food C contains **2 ,3 and 2** units respectively per kg. If A costs **$ 85 per kg** , B costs **$ 40 per kg** and C costs **$30 per kg**; how many kg of each should he buy per week to minimize his cost and still meet his **minimum** requirements? Formulate the above problem as a linear programming problem.

LPP Using Simplex Method :

Problem Analysis :



Here there will be 3 decision variables x , y and z to represent quantity of Food A , Food B and Food C to minimize the cost and meet the protein requirements .This problem can't be solved graphically hence we have to use **Simplex method** .

LPP Using Simplex Method :

Variables used for Simplex Method

1. **Slack Variables :** This variable is used to change the $\leq$ inequality constraints to equality.
2. **Surplus variables:** This variable is used to change the $\geq$ inequality constraints to equality.
3. **Artificial variables:** This variable is used to change the $\geq$ and $=$ inequality constraints to equality.

LPP Using Simplex Method :

- **FOR CONSTRAINTS WITH ≤ SIGN:**
  Use **Slack variable** to change it into equality.

- **FOR CONSTRAINTS WITH ≥ SIGN**:
  Use **surplus variable with negative sign** and an **artificial variable** to change it into equality.

- **FOR CONSTRAINTS WITH = SIGN:**
  Use an **artificial variable** to change it into equality .

LPP Using Simplex Method :

For Example :

Maximize $Z = 2x + 3y + 2z$

Subjected to the constraints,

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

How many **slack ,surplus and artificial variables** are needed to solve the above LPP?

LPP Using Simplex Method :

Maximize $Z = 2x + 3y + 2z$

Subjected to the constraints,

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

How many slack ,surplus and artificial variables are needed to solve the above LPP?

Answer:

We need **2 slack variables** S1 and S2 for two ≤ inequalities . We also need **1 surplus variable** S3 and **1 artificial variable** A1 for ≥ inequality.

LPP Using Simplex Method :

Question:

Maximize $Z = 2x + 3y + 2z$

Subjected to the constraints,

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

Solution,

Let S_1 and S_2 be the slack variables , let S_3 be the surplus variable and let A_1 be the artificial variable.

$$\mathbf{x + y + z + S_1 = 60}$$

$$\mathbf{2x + 3y + 7z + S_2 = 150}$$

$$\mathbf{3x + 6y + 4z - S_3 + A_1 = 200}$$

$$\mathbf{x, y, z, S_1, S_2, S_3, A_1 \geq 0}$$

LPP Using Simplex Method :(Try it….)

Question :

Maximize $Z = 4x + 7y + 2z$

Subjected to the constraints,

$$x + y + z \leq 60$$
$$2x + 3y + 7z \geq 150$$
$$3x + 6y + 4z = 200$$
$$x, y, z \geq 0$$

Write the equation with all the variables required for simplex method .

LPP Using Simplex Method :

Question :

Maximize $Z = 4x + 7y + 2z$

Subjected to the constraints,

$$x + y + z \leq 60$$
$$2x + 3y + 7z \geq 150$$
$$3x + 6y + 4z = 200$$
$$x, y, z \geq 0$$

Write the equation with all the variables required for simplex method .

Solution,

Let S1 be the slack variable , S2 be the surplus variable and let A1 and A2 be the artificial variables.

$$\mathbf{x + y + z + S1 = 60}$$
$$\mathbf{2x + 3y + 7z - S2 + A1 = 150}$$
$$\mathbf{3x + 6y + 4z + A2 = 200}$$
$$\mathbf{x, y, z, S1, S2, A1, A2 \geq 0}$$

LPP Using Simplex Method : (Try it…)

Question :

Minimize $Z = 10x + 7y - 6z$

Subjected to the constraints,

$4x - y + 3z \geq 60$

$6x + 3y - 7z = 150$

$3x + 6y + 4z = 200$

$x, y, z \geq 0$

Write the equation with all the variables required for simplex method .

LPP Using Simplex Method :

Question :
Minimize $Z = 10x + 7y - 6z$
Subjected to the constraints,

$4x - y + 3z \geq 60$
$6x + 3y - 7z = 150$
$3x + 6y + 4z = 200$
$x, y, z \geq 0$

Write the equation with all the variables required for simplex method .

Solution,
Let S1 be the surplus variable and A1, A2 and A3 be the artificial variables.

$4x - y + 3z - S_1 + A_1 = 60$
$6x + 3y - 7z + A_2 = 150$
$3x + 6y + 4z + A_3 = 200$
$x, y, z, S_1, A_1, A_2, A_3 \geq 0$

Any Questions?

LPP Using Simplex Method :

Standard Equation for simplex table.

After defining all the variables and making the inequalities to equalities we need to write the **standard equations** required for the simplex table .

The standard equations **consists of all the variables** .

LPP Using Simplex Method :

Write the standard equation for the simplex table to solve the following LPP .

Maximize $Z' = 2x + 3y + 2z$

Subjected to the constraints,

$$x + y + z \leq 60$$
$$2x + 3y + 7z \leq 150$$
$$3x + 6y + 4z \geq 200$$
$$x, y, z \geq 0$$

Solution,

Let S_1 and S_2 be the slack variables and let S_3 be surplus variable.

Let A_1 be the artificial variables.

Now,

$$x + y + z + S_1 = 60$$
$$2x + 3y + 7z + S_2 = 150$$
$$3x + 6y + 4z - S_3 + A_1 = 200$$

LPP Using Simplex Method :

Standard equation for simplex table :

Maximize $Z' = 2x + 3y + 2z$

$x + y + z + S_1 = 60$

$2x + 3y + 7z + S_2 = 150$

$3x + 6y + 4z - S_3 + A_1 = 200$

$1Z' - 2x - 3y - 2z + 0S_1 + 0S_2 + 0S_3 + \mathbf{10\,A_1} = 0$

$0Z' + 1x + 1y + 1z + 1S_1 + 0S_2 + 0S_3 + 0A_1 = 60$

$0Z' + 2x + 3y + 7z + 0S_1 + 1S_2 + 0S_3 + 0A_1 = 150$

$0Z' + 3x + 6y + 4z + 0S_1 + 0S_2 - 1S_3 + 1A_1 = 200$

$x, y, z, S_1, S_2, S_3 \ \& \ A_1 \geq 0$

Note that all 4 equations contains all the variables.

LPP Using Simplex Method :

Standard Equation :

$1Z' - 2x - 3y - 2z + 0S_1 + 0S_2 + 0S_3 + \mathbf{10\ A_1} = 0$
$0Z' + 1x + 1y + 1z + 1S_1 + 0S_2 + 0S_3 + 0A_1 = 60$
$0Z' + 2x + 3y + 7z + 0S_1 + 1S_2 + 0S_3 + 0A_1 = 150$
$0Z' + 3x + 6y + 4z + 0S_1 + 0S_2 - 1S_3 + 1A_1 = 200$
$x, y, z, S_1, S_2, S_3 \ \& \ A_1 \geq 0$

The first equation is created from objective function by shifting all to left-hand side and making it equal to zero. Take note that in the first equation the coefficient of artificial variable is **+10** because for maximization problem we will always take Positive sign for artificial variable and the value 10 is assigned by seeing the coefficient of x, y and z in objective which are all single figure digit. If any one of the coefficient was double digit, then we must have to take 100 for the coefficient of artificial variable.

LPP Using Simplex Method :

Write the standard equation for the simplex table to solve the following LPP .

Minimize $Z' = 5x + 12y + 2z$

Subjected to the constraints,

$7x + 3y - z \leq 220$

$12x - 6y + 8z \geq 1500$

$8x + 16y - 4z = 2000$

$x, y, z \geq 0$

LPP Using Simplex Method :

Solution,

Let S_1 and S_2 be the slack variable and surplus variable respectively and let A_1 and A_2 be the artificial variables.

Minimize $Z' = 5x + 12y + 2z$

$$7x + 3y - z + S_1 = 220$$
$$12x - 6y + 8z - S_2 + A_1 = 1500$$
$$8x + 16y - 4z + A_2 = 2000$$
$$x, y, z, S_1, S_2, A_1 \,\&\, A_2 \geq 0$$

Standard equation for simplex table:

$$1Z' - 5x - 12y - 2z + 0S_1 + 0S_2 \mathbf{- 100\ A_1 - 100\ A_2} = 0$$
$$0Z' + 7x + 3y - 1z + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 220$$
$$0Z' + 12x - 6y + 8z + 0S_1 - 1S_2 + 1A_1 + 0A_2 = 1500$$
$$0Z' + 8x + 16y - 4z + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 2000$$
$$x, y, z, S_1, S_2, A_1 \,\&\, A_2 \geq 0$$

Note that all 4 equations contains all the variables.

LPP Using Simplex Method :

Standard equation for simplex table:

$1Z' - 5x - 12y - 2z + 0S_1 + 0S_2 - \mathbf{100\,A_1} - \mathbf{100\,A_2} = 0$

$0Z' + 7x + 3y - 1z + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 220$

$0Z' + 12x - 6y + 8z + 0S_1 - 1S_2 + 1A_1 + 0A_2 = 1500$

$0Z' + 8x + 16y - 4z + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 2000$

$x, y, z, S_1, S_2, A_1 \ \& \ A_2 \geq 0$

The first equation is created from objective function by shifting all to left-hand side and making it equal to zero .Take note that in the first equation the coefficient of artificial variables A_1 & A_2 are **-100** because for minimization problem we will always take negative sign for artificial variable and the value 100 is assigned by seeing the coefficient of x , y and z in objective function in which y has double digit coefficient. If any one of the coefficient was triple digit, then we must have to take 1000 for the coefficient of artificial variable.

LPP Using Simplex Method :

Write the standard equation for each of the following LPP defining all the necessary variables .

a. Maximize $Z = 3x_1 + 4x_2$
   Subjected to ,
   $x_1 + x_2 \leq 20$
   $2x_1 + 3x_2 \leq 50$
   $x_1, x_2 \geq 0$

b. Maximize $Z = 2x_1 + 3x_2$
   Subjected to ,
   $x_1 + 2x_2 \geq 50$
   $10x_1 + 20x_2 \leq 175$
   $x_1, x_2 \geq 0$

c. Minimize $Z = 3x_1 + 4x_2 + 5x_3$
   Subjected to,
   $x_1 + x_2 + x_3 \geq 30$
   $10x_1 + 15x_2 + 20x_3 \leq 600$
   $x_1, x_2, x_3 \geq 0$

LPP Using Simplex Method :

Question d:

Suppose that 8, 12 and 9 units of protein, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains 2, 6 and 1 units of protein, carbohydrate and fat respectively per kg. , food B contains 1, 1 and 3 units respectively per kg. and food C contains 2 ,3 and 2 units respectively per kg. If A costs $ 85 per kg , B costs $ 40 per kg and C costs $30 per kg; how many kg of each should he buy per week to minimize his cost and still meet his minimum requirements? **Write down the standard equations needed to solve the given LPP.**

LPP Using Simplex Method (Complete Solution) :

Question :

Solve the following LPP using simplex method.

Maximize $Z = 90x + 70y$

Subjected to ,

$125x + 100y \leq 25000$

$20x + 30y \leq 6000$

$x, y \geq 0$

LPP Using Simplex Method :

Question :

Solve the following LPP using simplex method.

Maximize $Z = 90x + 70y$

Subjected to ,

$125x + 100y \leq 25000$

$20x + 30y \leq 6000$

$x, y \geq 0$

Solution ,

Let S_1 and S_2 be the slack variables .

Now ,

$125x + 100y + S_1 = 25000$

$20x + 30y + S_2 = 6000$

$x, y, S_1, S_2 \geq 0$

LPP Using Simplex Method :

Standard equation for simplex table:

$$1\,Z - 90\,x - 70\,y + 0\,S_1 + 0\,S_2 = 0$$
$$0\,Z + 125\,x + 100\,y + 1\,S_1 + 0\,S_2 = 25000$$
$$0\,Z + 20\,x + 30\,y + 0\,S_1 + 1\,S_2 = 6000$$
$$x,\ y,\ S_1,\ S_2 \geq 0$$

Simplex table 1 :

Highest Negative (Key column)

Row	Z	X	Y	S_1	S_2	Constant	Ratio
R_0	1	-90	-70	0	0	0	_
R_1	0	125	100	1	0	25000	25000/125 = 200
R_2	0	20	30	0	1	6000	6000/20 =300

Minimum positive ratio (Key Row)

Key Element

LPP Using Simplex Method :

Row	Z	X	Y	S_1	S_2	Constant
R_0	1	-90	-70	0	0	0
R_1	0	125	100	1	0	25000
R_2	0	20	30	0	1	6000

Here X is the **key column** , R_1 is the **key row** and 125 is the **key element**.

Now, we must update key row (R_1) first using the formula,

New R_1 = Old R_1/ Key elements = Old R_1 / 125

0 , 1 ,4/5 ,1/125 ,0,200

New R_0 = Old R0 – (-90) x New R_1

New R_2 = Old R_2 – 20 x New R_1

We need to update rows until all the elements of R_0 are ≥ 0 for Maximization problems.

LPP Using Simplex Method :

Updating R_0 and R_2 using the formula :

New R_0 = Old R_0 – (-90) x New R_1

New R_2 = Old R2 – 20 x New R_1

Old R_0 -	(-90) x New R_1	New R_0	Old R_2 -	20 x New R_1	New R_2
1	0	1	0	0	0
-90	-90	0	20	20	0
-70	-72	2	30	16	14
0	-18/25	18/25	0	4/25	-4/25
0	0	0	1	0	1
0	-18000	18000	6000	4000	2000

LPP Using Simplex Method :

Simplex Table 2:

Row	Z	X	Y	S_1	S_2	Constant	Ratio
R_0	**1**	**0**	**2**	**18/25**	**0**	**18000**	
R_1	0	1	4/5	1/125	0	200	
R_2	0	0	14	-4/25	1	2000	

Here all the coefficient of variables in R_0 row are ≥ 0, so we reached the optimum solution .
Hence ,
Maximum Z= 18000
X= 200
Y = 0

Any Questions?

LPP Using Simplex Method (Another Problem) :

Question :

Solve the following LPP using simplex method.

Minimize $Z = 4x + 6y$

Subjected to ,

$$x + 2y \geq 80$$

$$3x + y \geq 75$$

$$x, y \geq 0$$

Solution ,

Let S_1 and S_2 be the surplus variables and let A_1 and A_2 be the artificial variables.

Now ,

$x + 2y - S_1 + A_1 = 80$

$3x + y - S_2 + A_2 = 75$

S_1, S_2, A_1 & $A_2 \geq 0$

LPP Using Simplex Method :

Standard equation for simplex table:

$1Z - 4x - 6y + 0S_1 + 0S_2 - 10A_1 - 10A_2 = 0$

$0Z + 1x + 2y - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 80$

$0Z + 3x + 1y + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 75$

$S_1, S_2, A_1 \ \& \ A_2 \geq 0$

Simplex Table 1 :

Row	Z	x	y	S_1	S_2	A_1	A_2	Constant
R_0	1	-4	-6	0	0	-10	-10	0
R_1	0	1	2	-1	0	1	0	80
R_2	0	3	1	0	-1	0	1	75

LPP Using Simplex Method :

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant
R_0	1	-4	-6	0	0	**-10**	**-10**	0
R_1	0	1	2	-1	0	1	0	80
R_2	0	3	1	0	-1	0	1	75

For Identity Matrix,

$$\textbf{New } R_0 = \textbf{Old } R_0 + 10\,(R_1+R_2)$$

LPP Using Simplex Method :

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant
R_0	1	-4	-6	0	0	-10	-10	0
R_1	0	1	2	-1	0	1	0	80
R_2	0	3	1	0	-1	0	1	75

New R_0 = Old R_0 + 10 (R_1+R_2)

Old R_0 +	10 $(R_1 + R_2)$	New R_0
1	0	1
-4	40	36
-6	30	24
0	-10	-10
0	-10	-10
-10	10	0
-10	10	0
0	1550	1550

LPP Using Simplex Method :

Simplex Table 2 :

Key Column (highest Positive)

Row	Z	x	y	S_1	S_2	A_1	A_2	Constant	Ratio
R_0	1	36	24	-10	-10	0	0	1550	-
R_1	0	1	2	-1	0	1	0	80	80/1 = 80
R_2	0	3	1	0	-1	0	1	75	75/3 = 25

Key Row (Minimum positive ratio)

Key Element

Here X is the **key column** ,R_2 is the **key row** and 3 is the **key element**.

New R_2 = Old R_2 / Key element = Old R_2 /3

0 , 1 , 1/3 , 0 , -1/3 , 0 , 1/3 ,25

New R_0 = Old R_0 – 36 x New R_2

New R_1 = Old R_1 – 1 x New R_2

We Need to update Rows until all the elements of $R_0 \leq 0$.

LPP Using Simplex Method :

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant
R_0	1	36	24	-10	-10	0	0	1550
R_1	0	1	2	-1	0	1	0	80
R_2	0	3	1	0	-1	0	1	75

New R_0 = Old R_0 – 36 x New R_2 , New R_1 = Old R_1 – 1 x New R_2

Old R_0 -	36 x New R_2	New R_0	Old R_1 -	1 x New R_2	New R_1
1	0	1	0	0	0
36	36	0	1	1	0
24	12	12	2	1/3	5/3
-10	0	-10	-1	0	-1
-10	-12	2	0	-1/3	1/3
0	0	0	1	0	1
0	12	-12	0	1/3	-1/3
1550	900	650	80	25	55

LPP Using Simplex Method :

Simplex Table 3 :

Key Column

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant	Ratio
R_0	1	0	12	-10	2	0	-12	650	-
R_1	0	0	5/3	-1	1/3	1	1/3	55	55/5*3 =33
R_2	0	1	1/3	0	-1/3	0	1/3	25	25/1*3 =75

Key Row

Key Element

Here Y is the **key column** , R_1 is the **key row** and 5/3 is the **key element**

Updating Key row using the formula ,

NewR_1 = Old R_1 / Key elements = Old R_1/(5/3)

0 , 0 , 1 , -3/5 , 1/5 , 3/5 ,1/5 ,33

New R_0 = Old R0 – 12 x New R_1

New R_2 = Old R_2 – 1/3 x New R_1

LPP Using Simplex Method :

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant	Ratio
R_0	1	0	12	-10	2	0	-12	650	-
R_1	0	0	5/3	-1	1/3	1	1/3	55	55/5*3 =33
R_2	0	1	1/3	0	-1/3	0	1/3	25	25/1*3 =75

New R_0 = Old R_0 – 12 x NewR_1 **New R_2 = Old R_2 – 1/3 x NewR_1**

Old R0 -	12 x NewR1	New R0	Old R2 -	1/3 x New R1	New R2
1	0	1	0	0	0
0	0	0	1	0	1
12	12	0	1/3	1/3	0
-10	-36/5	-14/5	0	-1/5	1/5
2	12/5	-2/5	-1/3	1/15	-2/5
0	36/5	-36/5	0	1/5	-1/5
-12	12/5	-72/5	1/3	1/15	4/15
650	396	254	25	11	14

LPP Using Simplex Method :

Simplex Table 4 :

Row	Z	x	y	S_1	S_2	A_3	A_2	Constant	Ratio
R_0	1	0	0	-14/5	-2/5	-36/5	-72/5	254	
R_1	0	0	1	-3/5	1/5	3/5	1/5	33	
R_2	0	1	0	1/5	-2/5	-1/5	4/15	14	

Here all the coefficient of variables in $\mathbf{R_0} \leq \mathbf{0}$,so we reached the optimum solution .

Hence,
Minimum Z = 254
x = 14
y = 33

LPP Using Simplex Method :

Question : (Past Year Course Work Question)

Martin and Son's company wants to manufacture a mixture containing three contents X, Y and Z. The cost of X, Y and Z are \$5, \$4 and \$3 respectively. The company prepares the mixture to meet out the demand of the costumers in the following manner.

The quantity of X cannot be more than 200 kgs in the mixtures.

The quantity of Y used should be at least 300 kgs.

The content of Z cannot be more than 400 kgs.

Find the optimal combination of the three contents for a mixture of 1000 kgs, so that the total cost is minimum.

LPP Using Simplex Method :

Question :

Martin and Son's company wants to manufacture a mixture containing three contents X, Y and Z. The cost of X, Y and Z are \$5, \$4 and \$3 respectively. The company prepares the mixture to meet out the demand of the costumers in the following manner.

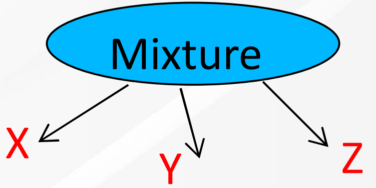
The quantity of X cannot be more than 200 kgs in the mixtures.

The quantity of Y used should be at least 300 kgs.

The content of Z cannot be more than 400 kgs.

Find the optimal combination of the three contents for a mixture of 1000 kgs, so that the total cost is minimum.

Problem Analysis:



Cost : \$ 5 \$4 \$3

X cannot be more than 200 kg ($X \le 200$)

Y should be at least 300 kg ($Y \ge 300$)

Z cannot be more that 400 kg ($Z \le 400$)

Total mixture must be 1000 kg ($X + Y + Z = 1000$)

LPP Using Simplex Method :

Mathematical Formulation :

<u>For Decision Variables</u>

Let x kg ,y kg and z kg of mixture X,Y and Z should be purchased and mix together to form 1000 kg of mixture in order to minimize the cost.

<u>For Objective Function</u>

Total cost = $5x + 4y + 3z$

Let $C = 5x + 4y + 3z$

Minimize $C = 5x + 4y + 3z$

<u>For Constraints</u>

$x \leq 200$ (Mixture X constraint)

$y \geq 300$ (Mixture Y constraint)

$z \leq 400$ (Mixture Z constraint)

$x + y + z = 1000$ (Total Mixture constraint)

$x, y, z \geq 0$

LPP Using Simplex Method :

Simplex Method Solution :

Let S_1 and S_2 be slack variables ,let S_3 be surplus variable and let A_1 and A_2 be artificial variables.

Now ,

$x + S_1 = 200$

$y - S_3 + A_1 = 300$

$z + S_2 = 400$

$x + y + z + A_2 = 1000$

Standard Equation for Simplex Table

$1\,C - 5\,x - 4\,y - 3\,z + 0S_1 + 0S_2 + 0S_3 - 10A_1 - 10A_2 = 0$

$0\,C + 1\,x + 0\,y + 0\,z + 1S_1 + 0S_2 + 0S_3 + 0A_1 + 0A_2 = 200$

$0\,C + 0\,x + 1\,y + 0\,z + 0S_1 + 0S_2 - 1S_3 + 1A_1 + 0A_2 = 300$

$0\,C + 0\,x + 0\,y + 1\,z + 0S_1 + 1S_2 + 0S_3 + 0A_1 + 0A_2 = 400$

$0\,C + 1\,x + 1\,y + 1\,z + 0S_1 + 0S_2 + 0S_3 + 0A_1 + 1A_2 = 1000$

LPP Using Simplex Method :

Simplex Table 1 :

Row	C	x	y	z	S_1	S_2	S_3	A_1	A_2	Constant
R_0	1	-5	-4	-3	0	0	0	-10	-10	0
R_1	0	1	0	0	1	0	0	0	0	200
R_2	0	0	1	0	0	0	-1	1	0	300
R_3	0	0	0	1	0	1	0	0	0	400
R_4	0	1	1	1	0	0	0	0	1	1000

For identity Matrix

$$\text{New } R_0 = \text{Old } R_0 + 10\,(R_2 + R_4)$$

LPP Using Simplex Method :

For identity Matrix : New R_0 = Old R_0 + 10 (R_2 +R_4)

Old R0 +	10(R2 +R4)	New R0
1	0	0
-5	10	5
-4	20	16
-3	10	7
0	0	0
0	0	0
0	-10	-10
-10	10	0
-10	10	0
0	13000	13000

LPP Using Simplex Method :

Simplex Table 2 :

Key Column

Row	C	x	y	z	S_1	S_2	S_3	A_1	A_2	Constant	Ratio
R_0	0	5	16	7	0	0	-10	0	0	13000	-
R_1	0	1	0	0	1	0	0	0	0	200	∞
R_2	0	0	1	0	0	0	-1	1	0	300	300
R_3	0	0	0	1	0	1	0	0	0	400	∞
R_4	0	1	1	1	0	0	0	0	1	1000	1000

Key Row

Key Element

Here we need to update R_2 first using : New R_2 = Old R_2 / 1

0 , 0 , 1 , 0 , 0 , 0 , -1 ,1 , 0 , 300

New R_0 = Old R_0 – 16 x New R_2

New R_1 = Old R_1 – 0 x New R_2

New R_3 = Old R_3 – 0 x New R_2

New R_4 = Old R_4 – 1 x New R_2

We need to update rows until all the coefficient of variables in R_0 are ≤ 0.

LPP Using Simplex Method :

Differences between Maximization and Minimization Problem

Maximization Problem	Minimization Problem
Artificial variable is written with +ve sign in standard equation . $\mathbf{1Z - 2x - 3y - 4z + 0S_1 + 0S_2 + 10A_1 + 10A_2 = 0}$	**Artificial variable** is written with -ve sign in standard equation . $\mathbf{1Z - 2x - 3y - 4z + 0S_1 + 0S_2 - 10A_1 - 10A_2 = 0}$
For **Key Column** we need to see **highest Negative** value in R_0 row .	For **Key Column** we need to see **highest positive** value in R_0 row .
For **Optimal Solution** all the coefficient of Variables in R_0 row should be ≥ **0** .	For **Optimal Solution** all the coefficient of Variables in R_0 row should be ≤ **0** .

Thank you