Special Cases in Simplex Method

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SAU/AM(M)/2014/14

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Simplex Method

Step 1 : Determine a starting basic feasible solution.

Step 2 : Determine the entering basic variable by selecting the non-basic variable with the most negative value (in case of maximization) or with the most positive (in case of minimization) in the Z-row. Stop if there is no entering variable, the last solution is optimal.
Else, go to Step 3.

Step 3 : Select a leaving variable using the feasibility condition

Step 4 : Determine the new basic solution by using the appropriate Gauss-Jordan computations.
Repeat Step 2.

Simplex Table

A format of Simplex Table for some given LPP, here X_j are Decision Variables, S_j are Slack Variables, a_{ij} is the coefficient of X_j corresponding to i^{th} constraint.

Non-Basic Variable

Coefficient of Objective Function, decides Entering Variable

	Basic Variable	X_1	X_2	S_1	S_2	S_3	Solution	Min Ratio
Z-row	Z	C_1	C_2	C_3	C_4	C_5	**z**	-
S_1-row	S_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	b_{11}	b_{11}/a_{1j}
S_2-row	S_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	b_{21}	b_{21}/a_{2j}
S_3-row	S_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	b_{31}	b_{31}/a_{3j}

Objective Function Value

Decides the Leaving Variable

Right Hand Side Value of Constraints

Special Cases in Simplex

Special Cases that arise in the use of Simplex Method :

1. **Degeneracy**
2. **Alternative Optima**
3. **Unbounded Solution**
4. **Infeasible Solution**

Degeneracy

A solution of the problem is said to be **degenerate solution if the value of at least one basic variable becomes zero**.

In the simplex table, **a tie for the minimum ratio** occurs which is broken arbitrarily.

It can cause the solution to **cycle indefinitely**.

Graphically, it occurs due to **redundant constraint** (i.e. a constraint that can be removed from system without changing the feasible solution).

Degeneracy

For example, consider the problem

Max z $= 3x_1 + 9x_2$; **Subject to :** $x_1 + 4x_2 \leq 8$; $x_1 + 2x_2 \leq 4$; $x_1, x_2 \geq 0$

Basic Variable	X_1	X_2	S_1	S_2	Solution	Min Ratio
Z_0	-3	-9	0	0	**0**	-
S_1	1	4	1	0	8	2
S_2	1	2	0	1	4	2
Z_2	-3/4	0	9/4	0	**18**	-
X_2	1/4	1	1/4	0	2	8
S_2	1/2	0	-1/2	1	0	0
Z_3	0	0	3/2	3/2	**18**	-
X_2	0	1	1/2	-1/2	2	-
X_1	1	0	-1	2	0	-

X_2 enters S_1 leaves

Tie in Min Ratio (Degeneracy)

X_1 enters S_2 leaves

S_2 becomes zero in next iteration because of degeneracy

Optimum Solution : X_1=0, X_2=2, Max Z=18

Alternative Optima

If the **z-row value for one or more non-basic variable is zero** in the optimal tableau, **alternate optimal solution exists.**

Graphically, it happens when the **objective function is parallel to a non-redundant binding constraint** (i.e. a constraint that is satisfied as an equation at the optimal solution).

The **zero coefficient of non-basic variable x_i indicates that it can be made basic**, altering the value of basic variable **without changing the value of z.**

Alternative Optima

For example, consider the problem

Max $z = 2x_1 + 4x_2$; **Subject to :** $x_1 + 2x_2 \le 5$; $x_1 + x_2 \le 4$; $x_1, x_2 \ge 0$

Basic Variable	X_1	X_2	S_1	S_2	Solution	Min Ratio
Z_0	-2	-4	0	0	**0**	-
S_1	1	2	1	0	5	5/2
S_2	1	1	0	1	4	4/1
Z_2	0	0	2	0	**10**	-
X_2	1/2	1	1/2	0	5/2	5
S_2	1/2	0	-1/2	1	3/2	3
Z_3	0	0	2	0	**10**	-
X_2	0	1	1	-1	1	-
X_1	1	0	-1	2	3	-

X_2 enters
S_1 leaves

X_1 enters
S_2 leaves

Optimum Solution, but coefficient of non-basic variable X_1 is 0, indicates that it can be made basic.

Alternate Optimum Solution X_1 , X_1 are basic variables.

Optimum Solution : X_1=3, X_2=1 Max Z=10

Unbounded Solution

When determining the leaving variable of any tableau, if there is no positive minimum ratio or **all entries of pivot column are negative or zero.**

For example, consider the problem

Max $z = 2x_1 + x_2$; **Subject to :** $-x_1 + x_2 \le 10$; $-2x_1 \le 40$; $x_1, x_2 \ge 0$

X1 is the entering variable but all the constraint coefficients under X1 are ≤ 0 , meaning that X1 can be increased indefinitely, which gives unbounded solution.

Basic Variable	X_1	X_2	S_1	S_2	Solution	Min Ratio
Z_0	-2	-1	0	0	**0**	-
S_1	-1	1	1	0	10	-
S_2	-2	0	0	1	40	-

Infeasible Solution

If **at least one artificial variable is positive** in the optimum iteration, then the LPP has **no feasible solution**.

This situation obviously does not occur when all the constraints are type ≤ with non-negative right-hand side because all slack variables provide an obvious feasible solution.

So, this situation occurs in Big-M Method & Two-Phase Method in which artificial variable are used.

Infeasible Solution

For example, consider the problem

Max $z = 3x_1 + 2x_2$; **Subject to :** $2x_1 + x_2 \leq 2$; $3x_1 + 4x_2 \geq 12$; $x_1, x_2 \geq 0$

Basic Variable	X_1	X_2	S_1	S_2	R_1	Solution	Min Ratio
Z_0	-3	-2	0	0	100	**0**	-
S_1	2	1	1	0	0	2	-
R_1	3	4	0	-1	1	12	-
Z_0	-303	-402	100	0	0	**-1200**	-
S_1	2	1	1	0	0	2	2
R_1	3	4	0	-1	1	12	3
Z_1	501	0	402	100	0	**-396**	-
X_2	2	1	1	0	0	2	-
R_1	-5	0	-4	-1	1	4	-

X_2 enters
S_1 leaves

Pseudo Optimum Solution, but artificial variable has a positive value.

References

- Operations Research - An introduction (9th edition), Hamdy A. Taha, 2013
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THANK YOU