

Session 9

Assignment 1

Task 1:

1. You survey households in your area to find the average rent they are paying. Find the standard deviation from the following data:

\$1550, \$1700, \$900, \$850, \$1000, \$950

Solution:

Step 1: Calculate Mean of sample

$$\text{Mean} = (1550 + 1700 + 900 + 850 + 1000 + 950) / 6 = \mathbf{\$1,158.33}$$

Step 2: Calculate $X_i - \text{Mean}$

Step 3: Calculate $(X_i - \text{Mean})^2$

Step 4: Calculate Sum $([X_i - \text{Mean}]^2)$

Step 5: Calculate Sum $([X_i - \text{Mean}]^2) / [n - 1]$

$$s^2 = \frac{\sum (X - \bar{X})^2}{N - 1}$$

$s^2 \rightarrow \text{Variance}$

(This is the **Variance**)

Step 6: Calculate SQRT $\{\text{Sum } ([X_i - \text{Mean}]^2) / [n - 1]\}$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$s \rightarrow \text{Standard Deviation}$

(This is the **Standard Deviation**)

As per the sample,

Mean = \$1,158.33

Standard Deviation = \$367.99

X_i (Rent)	X_i – Mean	(X_i – Mean)²
\$1,550.00	\$391.67	\$153,405.39
\$1,700.00	\$541.67	\$293,406.39
\$900.00	– \$258.33	\$66,734.39
\$850.00	– \$308.33	\$95,067.39
\$1,000.00	– \$158.33	\$25,068.39
\$950.00	– \$208.33	\$43,401.39

Mean	\$1,158.33
Sum((X _i – Mean) ²)	\$677,083.34
Sum((X_i – Mean)²)/(n-1)	\$135,416.67
Variance	
SQRT(Sum((X_i – Mean)²)/(n-1))	\$367.99
Standard Deviation	

2. Find the variance for the following set of data representing trees in California (heights in feet):

3, 21, 98, 203, 17, 9

Solution:

Step 1: Calculate Mean for sample data
 $(3+21+98+203+17+9) / 6 = \mathbf{58.5}$

Step 2: Calculate X_i – Mean

Step 3: Calculate (X_i – Mean)²

Step 4: Calculate Sum ([X_i – Mean]²)

Step 5: Calculate Sum ([X_i – Mean]²) / [n – 1])

$$s^2 = \frac{\sum (X - \bar{X})^2}{N-1}$$

$S^2 = \text{Variance}$

(This is the **Variance** of the sample)

As per the sample,

Mean = 58.5

Variance = 6219.9

X_i (Height)	$X_i - \text{Mean}$	$(X_i - \text{Mean})^2$
3	-55.5	3080.25
21	-37.5	1406.25
98	39.5	1560.25
203	144.5	20880.25
17	-41.5	1722.25
9	-49.5	2450.25

Mean	58.5
$\text{Sum}((X_i - \text{Mean})^2)$	31099.5
$\text{Sum}((X_i - \text{Mean})^2) / (n-1)$	6219.9
Variance	

3. In a class on 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in two subjects and 3 failed in three subjects. Find the probability distribution of the variable for number of subjects a student from the given class has failed in.

Solution:

For a random student,

The probability of failing in 0 subjects, $P(X=0) = 0.8$

The probability of failing in 1 subjects, $P(X=1) = 0.1$

The probability of failing in 2 subjects, $P(X=2) = 0.07$

The probability of failing in 3 subjects, $P(X=3) = 0.03$

The probability distribution can be shown as:

X	0	1	2	3
P(X)	0.8	0.1	0.07	0.03

Task 2:

1. A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution:

$$\begin{aligned}n &= 20 \\k &= 20 - 5 = 15 \\n - k &= 5\end{aligned}$$

The probability of success = probability of giving a right answer = $s = 1/4$

$$\begin{aligned}\text{Hence, probability of failure} &= \text{probability of giving a wrong answer} \\&= 1 - s \\&= 1 - 1/4 \\&= 3/4\end{aligned}$$

Substitute the values in the formula for Binomial distribution,

$$\text{So, } P(\text{exactly 5 out of 20 answers incorrect}) = C(20, 5) * (1/4)^5 * (3/4)^{15}$$

$$\begin{aligned}P(5 \text{ out of } 20) &= ((20 * 19 * 18 * 17 * 16) / (5 * 4 * 3 * 2 * 1)) * (1/4)^5 * (3/4)^{15} \\&= 0.0000034\end{aligned}$$

Therefore, the required probability is **0.0000034** approximately.

2. A die marked A to E is rolled 50 times. Find the probability of getting a “D” exactly 5 times.

Solution:

$$n = 50, k = 5, n - k = 45$$

The probability of success = probability of getting a “D” = $s = 1/5$

Hence, the probability of failure = probability of not getting a “D” = $1 - s = 4/5$

$$C(50, 5) * (1/5)^5 * (4/5)^{45} = \mathbf{0.0295} \text{ times approximately}$$

**3. Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls.
Find the probabilities of all the possible outcomes.**

Solution:

Given,

Total number of black balls = 6

Total number of red balls = 4

$P(\text{drawing a black ball}) = 6/10 = 3/5$

$P(\text{drawing a red ball}) = 4/10 = 2/5$

Probability of all possible outcomes = $3/5 * 2/5 + 3/5 * 3/5 + 2/5 * 2/5 = \mathbf{0.76}$