Pre-Course: Probability exercise

1. You toss a fair coin until you get head twice. What is the probability that you made k tosses?

2. Each person in a group of n people is requested to select a number between 1 to k. Describe the probability that at least 2 people chose the same number.

3. The frequency of the *Malum Indentum* disease in the population is 1 in 10,000. A test that checks if one is infected with the disease is 99% accurate. One takes the test and gets a positive response (test says she is infected). What's the probability that she is infected?

4. Let X and Y be discrete random variables, Z be a continuous random variable, and α and β constants. Prove the following equalities:
    a. $E(X+Y) = E(X)+E(Y)$
    b. $E(\alpha Z) = \alpha E(Z)$
    c. If X and Y are independent then $E(XY) = E(X)E(Y)$
    d. $V(\alpha X+\beta) = \alpha^2 V(X)$
    e. If X and Y are independent then $V(X+Y) = V(X)+V(Y)$

5. Let $X_i \sim Unif(0, 1)$ for $1 <= i <= n$ be IID (independent identically distributed) random variables. Let $Y = \max(X_1, \ldots, X_n)$. What is E(Y)?

6. A drunken point hops on the number line, making jumps sized 1. The probability to jump to the right is fixed: P(right) = p. Let X_n be the position of the point after n jumps.
    a. What is $E(X_n)$?
    b. What is $V(X_n)$?