

ITC Calculus Pre-Course Task

August 21, 2017

1 Instructions

Answers should be submitted in a text file. Each line should have one answer corresponding to the respective question in the order that they appear in this document. Only the letter of the answer should be in the line e.g. "e". Submit your calculations in a separate file.

2 Question Set 1

- Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that
 - $f(0) = 0$
 - $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6
 - $-1 \leq f(x) \leq 3$ for all x between -3 and 6
 - $f(c) = 1$ for at least one c between -3 and 6
 - $f(c) = 0$ for at least one c between -1 and 3
- If $f(x) = 8 - x^2$ for $-2 \leq x \leq 2$ and $f(x) = x^2$ otherwise, then $\int_{-1}^3 f(x)dx$ is a number between
 - 8 and 16
 - 16 and 24
 - 24 and 32
 - 32 and 40
- The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is
 - 9
 - 12
 - 14
 - 21
 - 40
- If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x , then $\int_1^3 f(2x)dx =$
 - $2F(3) - 2F(1)$
 - $0.5F(3) - 0.5F(1)$
 - $2F(6) - 2F(2)$
 - $F(6) - F(2)$
 - $0.5F(6) - 0.5F(2)$

5. Population y grows according to the equation $\frac{dy}{dt} = ky$ where k is a constant and t is measured in years, If the population doubles every 10 years, then the value of k is:
- (a) 0.069
 - (b) 0.2
 - (c) 0.301
 - (d) 3.322
 - (e) 5

3 Question Set 2

1. Find $\lim_{x \rightarrow b} \frac{\sqrt{x} - \sqrt{b}}{x - b}$ for $b > 0$
- (a) ∞
 - (b) $\frac{1}{2\sqrt{b}}$
 - (c) 0
 - (d) $2\sqrt{b}$
 - (e) b
2. Let f be a differentiable function, where all derivatives exist, such that $f(0) = 0$, $f'(0) = 0$ and $\forall_x |f''(X)| \leq M$. Which of the following is not necessarily true?
- (a) $f(1) \leq M/2$
 - (b) 0 is neither a maximum nor a minimum.
 - (c) $\forall_{\epsilon > 0} \exists_{\delta > 0}$ s.t. if $x \in (\delta, \delta)$ then $|f(x)| < \epsilon$
 - (d) If $\lim s_n = 0$, then $\lim f(sn) = 0$.
 - (e) None of the above.

4 Question Set 3

1. Evaluate $\int (3 + 4x - 2e^x) dx$
- (a) $2x^2 - 2e^x + C$
 - (b) $2x^2 - 2x + \frac{e^{x+1}}{x+1} + C$
 - (c) $3x + 2x^2 - 0.5 * (e^x)^2 + C$
 - (d) $3x + 2x^2 - 2 * \ln|x| + C$
 - (e) $3x + 2x^2 - 2e^x + C$
2. Evaluate $\int \frac{4}{x^2} dx$
- (a) $\frac{4x}{\frac{1}{3}x^3} + C$
 - (b) $4\ln|x^2| + C$
 - (c) $-\frac{4}{3}x^{-3} + C$
 - (d) $\frac{-4}{x} + C$
 - (e) $\frac{4}{x^2} + C$
3. Evaluate $\int_0^\infty e^{-2x} dx$
- (a) e^2

- (b) $\frac{3}{4}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{2}$

4. Find the power series representation for the function $\frac{x^2}{1-3x}$

- (a) $x^2 + x^3 + x^4 + x^5 + \dots$
- (b) $x^2 - x^3 + x^4 - x^5 + \dots$
- (c) $1 + 3x + 9x^2 + 27x^3 + \dots$
- (d) $x^2 + 3x^3 + 9x^4 + 27x^5 + \dots$
- (e) $x^2 - 3x^3 + 9x^4 - 27x^5 + \dots$

5. Use the trapezoidal rule with $n = 3$ to estimate the value of $\int_2^8 \frac{1}{1+2x} dx$ and calculate the maximum error.

- (a) 0.6348919 with error 0.128
- (b) 0.128 with error 0.25
- (c) 1.713457 with error 0.05
- (d) 3π with error 0.0128
- (e) 0.7348 with error 0.0128