CONTROL SYSTEMS - ASSIGNMENT

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Section: T-4

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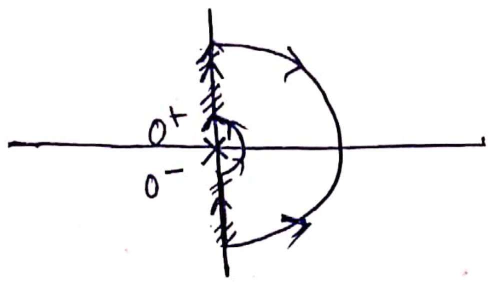
Set: B

Q1) $G(s)H(s)$: open loop TF $= \dfrac{k(s+7)}{s^2(s+2)(s+11)}$

- No of poles $=4$
- No of zeroes $=1$

So, here pole is present at $s=0$, we choose a contour as shown:



$$TF(j\omega) = \frac{k(j\omega+7)}{-\omega^2(j\omega+2)(j\omega+11)} \quad (0^+ < \omega < \infty)$$

(i) For $\omega = 0^+$

$$TF = \frac{k(0^+ + 7)}{-(0^+)^2(0^+ + 2)(0^+ + 11)} = \infty \angle -180^\circ$$

(ii) Similarly, for $\omega = 0^-$

$$TF(j\omega) = \infty \angle -180^\circ$$

(iii) For $\omega = \infty$

$$TF = \frac{k(\infty+7)}{-\infty^2(\infty+2)(\infty+11)} = 0^\circ \angle -270^\circ$$

(iv) For $\omega = -\infty$

$$TF = 0^\circ \angle 270^\circ$$

For smaller arc around origin

$$TF(\sigma + j\omega) = \frac{K(\epsilon e^{j\theta} + 7)}{\epsilon^2 e^{j2\theta}(\epsilon e^{j\theta}+2)(e^{j\theta}\epsilon + 11)}$$

$$\lim_{\epsilon \to \infty} TF = \frac{K}{22 \lim_{\epsilon \to 0} \epsilon^2 e^{j2\theta}} = \infty \angle 2\theta$$

as θ changes from $-\pi/2$ to $\pi/2$, 2θ will change from $-\pi$ to 0 to π

- For $\omega = \infty \angle \theta$ semicircle, TF revolves around origin in a circle, but will not produce any circulation around -1.

- Now, we need to check whether the TF crosses real axis.

$$TF = \frac{K(j\omega+7)}{-\omega^2(j\omega+2)(j\omega+11)} \Rightarrow \frac{K}{-\omega^2}\frac{(j\omega+7)(2-j\omega)(11-j\omega)}{(4+\omega^2)(121+\omega^2)}$$

$$= \frac{-K}{\omega^2}\frac{1}{4+\omega^2}\frac{1}{121+\omega^2}\left[(7+j\omega)(22-\omega^2-13(j\omega))\right]$$

$$\cancel{\frac{-K(1)(1)}{\omega^2(4+\omega^2)(121+\omega^2)}\left[7(22-\omega^2)+\omega^2(13)+(22-\omega^2)j\omega\right]}$$

$$= \frac{-K(1)(1)}{\omega^2(4+\omega^2)(121+\omega^2)}\left[7(22-\omega^2-13j\omega) + j\omega(22-\omega^2-13j\omega)\right]$$

$$= \frac{-K(1)(1)}{\omega^2(4+\omega^2)(121+\omega^2)}\left[154-7\omega^2-91j\omega+22j\omega - j\omega^3+13\omega^2\right]$$

$$\cancel{\frac{-K(1)(1)}{\omega^2(4+\omega^2)(121+\omega^2)}\left[154 - 68j\omega + 6\omega^2\right]}$$

$$TF = \frac{-K(1)(1)}{\omega^2(4+\omega^2)(121+\omega^2)}\left[7(22-\omega^2)+\omega^2(13) + (22-\omega^2)j\omega - 91j\omega\right]$$

it will cross real axis when img part $=0$

$\therefore j\omega(-68-\omega^2)=0$ (ω is a real no)

$\therefore \omega = j\sqrt{\cancel{80}68} \rightarrow$ not ~~a~~ real no.

$\therefore$ It never crosses real axis when travelling along $j\omega$ axis

Nyquist plot

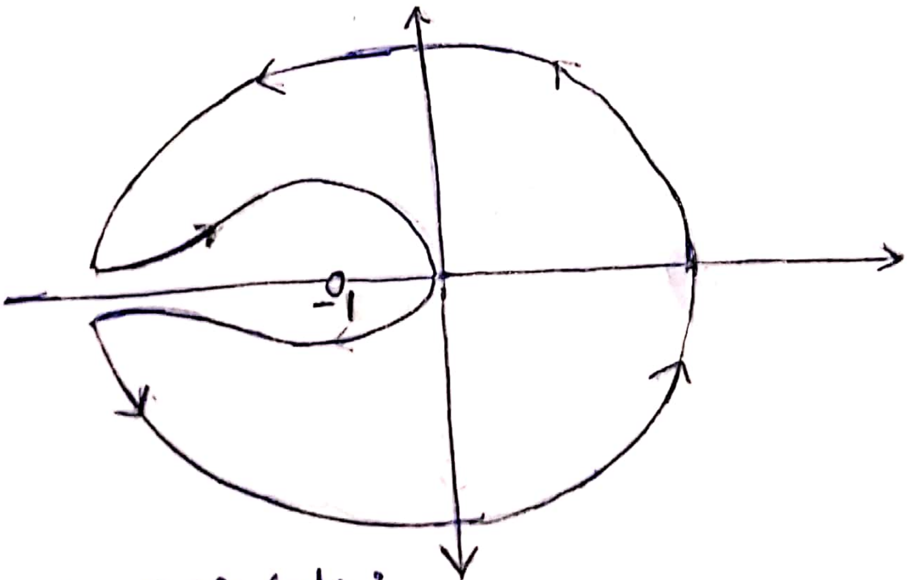
no of circulation around -1, is ~~N(=0)~~ $N=-2$ (in CW)

$\therefore P = 0$

$\therefore$ ~~RE~~ $Z=2$

Therefore, the system is unstable for all K.

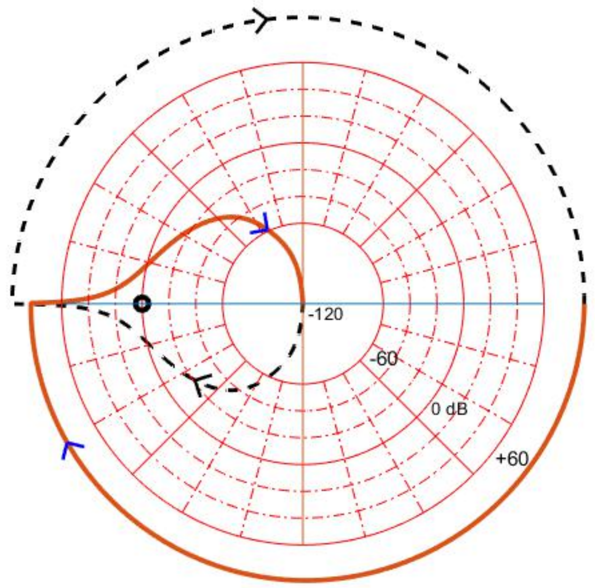
Gain margin $= -\infty$



MATLAB Code:

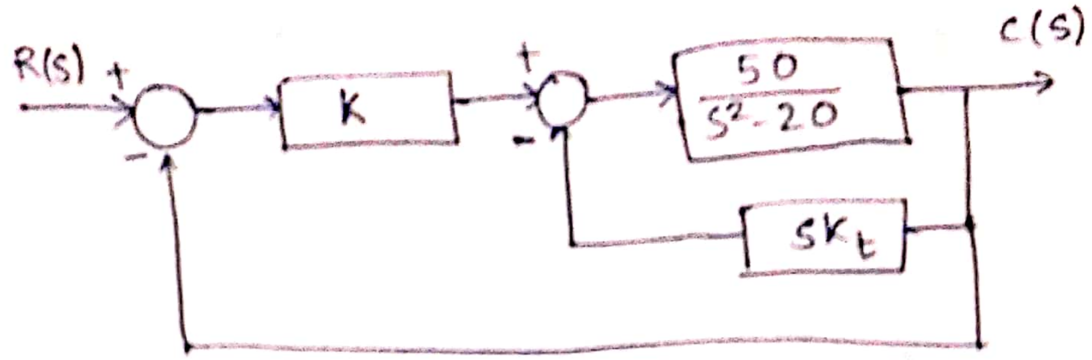
```
>> num = [1 7];
>> den1 = [1 0 0];
>> den 2 = [1 2];
>> den 3 = [1 11];
>> den12 = conv (den1, den2);
>> den = conv (den12, den 3);
>> sys = tf (num, den);
>> nyquist (sys)
```

MATLAB plot



This plot was obtained by using nyqlog.m
This is because nyquist(sys) produces an incomplete plot as the values tend to infinity.

Q2)



$$G(s)H(s) = \frac{50K}{s^2+50K_t s-20} \quad \text{and} \quad \frac{C(s)}{R(s)} = \frac{50K}{s^2+50K_t s+50K-20}$$

Specifications reqd.:

(i) t_s (5% tolerance band) ≤ 75 ms

(ii) $M_p \leq 16\%$

(i) $t_s(5\%) = 3/\zeta\omega_n$

$2\zeta\omega_n = 50K_t$ & $\omega_n^2 = 50K-20 \Rightarrow \omega_n = \sqrt{50K-20}$

$\zeta = \dfrac{25K_t}{\omega_n} = \dfrac{25K_t}{\sqrt{50K-20}}$ where $K > 2/5$

& $t_s = \dfrac{3}{25K_t} \leq 75/1000$ s $\Rightarrow$ $\boxed{K_t \geq 8/5}$

(ii) $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \leq 0.16$

$\Rightarrow \dfrac{\zeta^2\pi^2}{1-\zeta^2} \geq (1.832)^2$

$\Rightarrow \zeta^2\pi^2 \geq (1.832)^2(1-\zeta^2)$

$\Rightarrow \zeta^2(\pi^2+1) \geq (1.832)^2 \Rightarrow \zeta^2 \geq 0.30905$

$\Rightarrow \cancel{\dfrac{(25)^2K_t^2(\pi^2+1)}{50K-20} \geq (1.832)^2}$

$$\therefore \frac{25 \times 25 K_t^2}{(50K-20)} \geq 0.30905$$

$$\Rightarrow \frac{25 \times 25 K_t^2}{0.30905} \geq (50K-20)$$

(using $K_t = 8/5$)

$\therefore K \leq 103.943$

However $\omega_n > 0 \Rightarrow K > 2/5$

$$\therefore \boxed{0.4 < K \leq 103.943}$$

$$\boxed{K_t \geq 1.6}$$

For equality,

$K = 103.943 \quad \Rightarrow \zeta = 0.556$

$K_t = 1.6 \quad \Rightarrow \omega_n = 71.952$

(3) OPEN Loop TF $= \dfrac{K}{s(s^2+2s+7)}$

OL zeroes = 0

poles @ $s = 0, \; -1 \pm j\sqrt{6}$

No of poles = 3. So, there would be three branches.

- One ~~steadiness~~ starting from zero will go to negative infinity.

The other 2 branches from complex poles will cross imaginary axis

Angle of asymptotes

$3\theta = (2n+1)180^\circ$

$\therefore \theta = 60^\circ, 180^\circ, 300^\circ$

$\Rightarrow$ There are no breakaway or break in points.

Centroid:

$$\sigma_a = \frac{0-1-1}{3} = -2/3$$

Angle of departure:

$$-\phi_1 - \phi_2 + \phi = -180^\circ$$

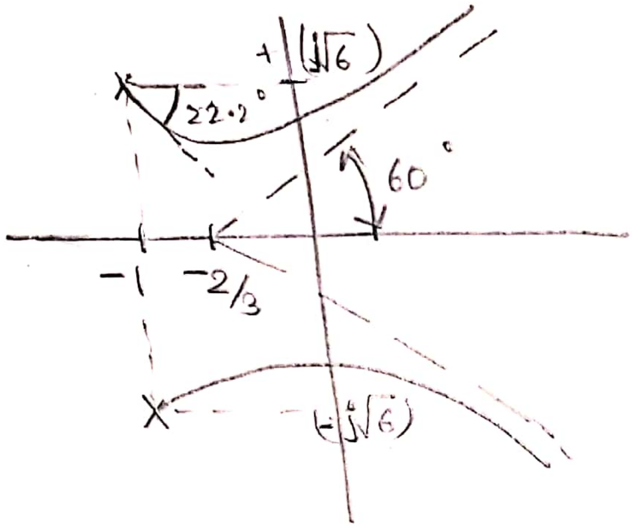
$$\therefore (-90) + \phi = -180^\circ + \phi_2$$

$$= -180^\circ + \tan^{-1}(\sqrt{6}) + 90^\circ$$

$$= -90 + 69.7923$$

$$= \underline{-22.2076}$$

Root Locus plot:



Stability Analysis:

Characteristic equation

$$s^3 + 2s^2 + 7s + K = 0$$

Routh criteria

s^3	1	7
s^2	2	K
s	$\frac{14-K}{2}$	0
s^0	K	

$14 - K > 0 \Rightarrow \underline{K < 16 \text{ and } K > 0}$

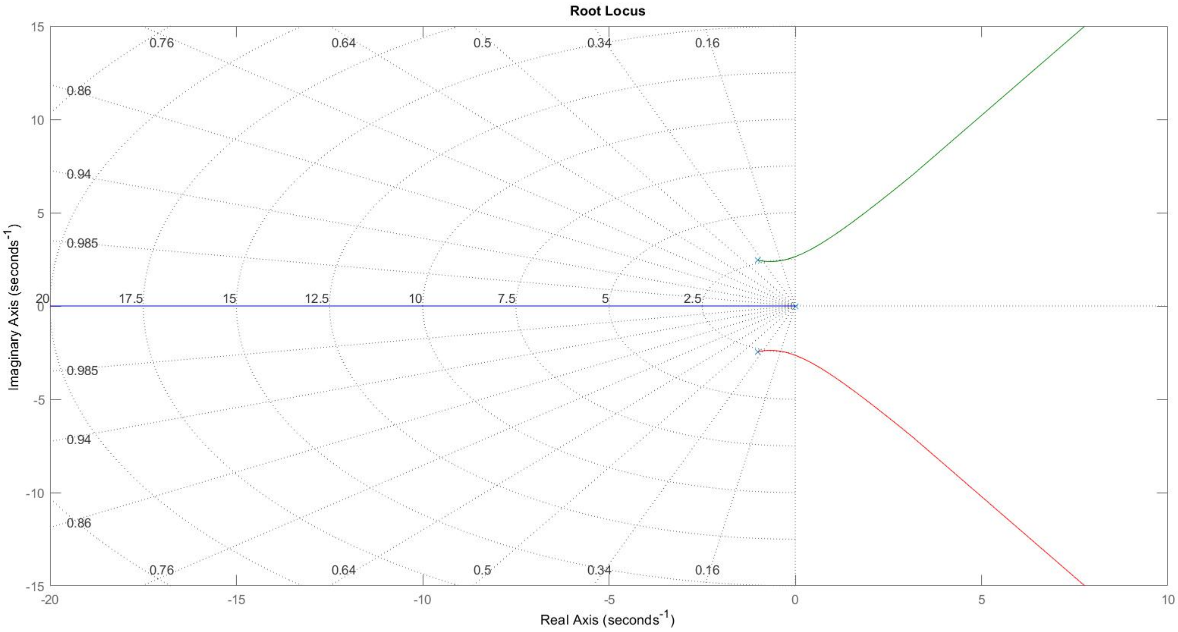
The system is stable for $0 < K < 16$

MATLAB Code:

```
sys = tf([1], [1 2 7 0]);
rlocus(sys);
```

Scanned with CamScanner

MATLAB PLOT


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.76
0.64
0.5
0.34
0.16
0.86
0.94
0.985
20
17.5
15
12.5
10
7.5
5
2.5
15
10
5
0
-5
-10
-15
-20
-15
-10
-5
0
5
10

Q4

$$TF = \frac{k(s+1)}{(5s+1)(s^2+2s+4)}$$

$$e_{ss} = 0.16 = \frac{1}{1+K_p}$$

$$K_p = \lim_{s\to 0} \frac{K(s+1)}{(5s+1)(s^2+2s+4)} = K/4$$

$$\therefore\ 0.16 = \frac{4}{4+K} \Rightarrow K = 21$$

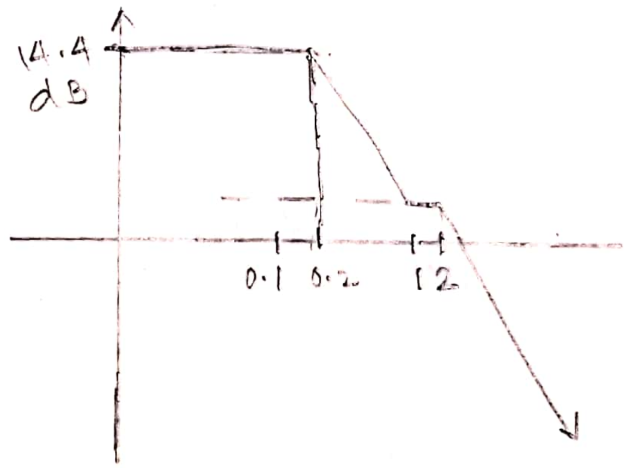
$$TF = \frac{21(s+1)}{(5s+1)(s^2+2s+4)} = \frac{21/4\,(s+1)}{(s/_{1/5}+1)(s^2/4 + s/2 + 1)}$$

zero @ s = 1 rad/s

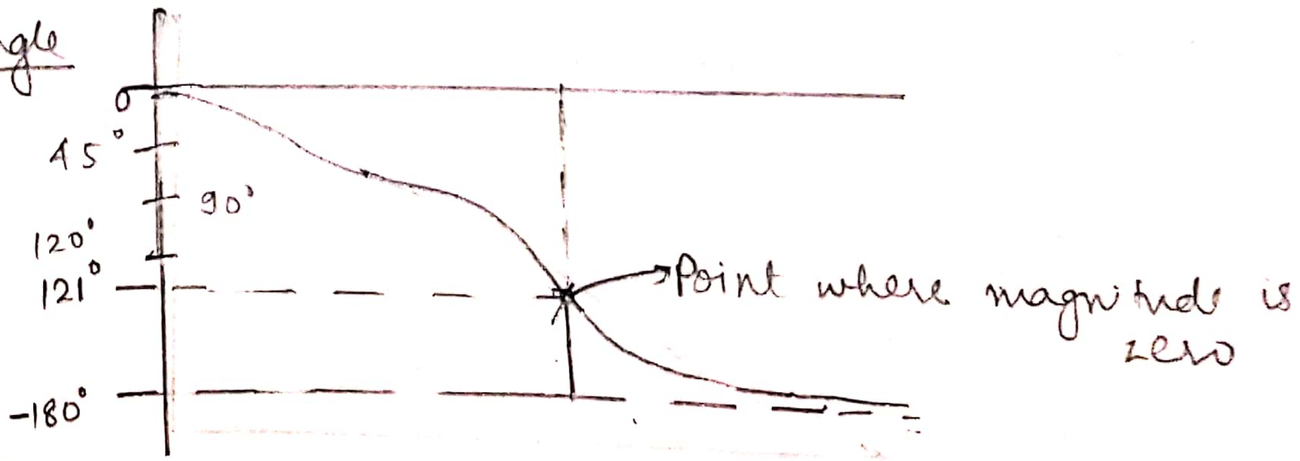
poles @ s = 0.2 rad/s, single order

s = 2 rad/s, second order pole

Magnitude

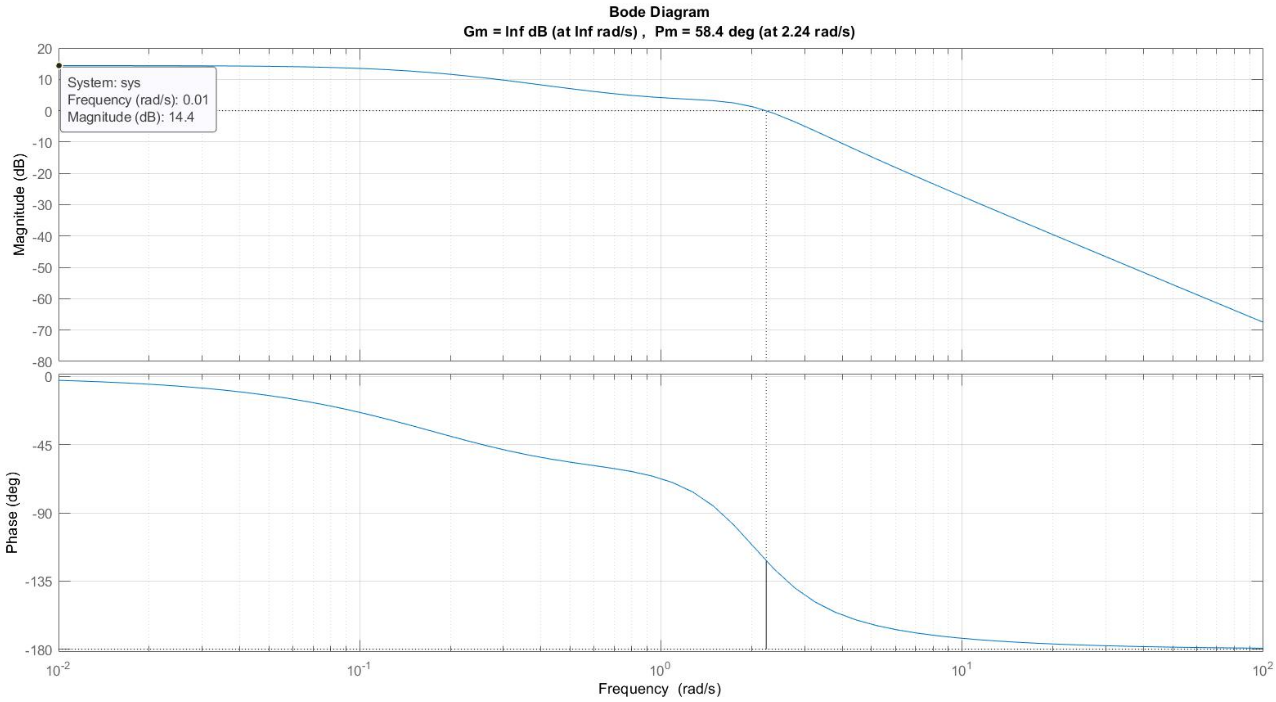


Angle



MATLAB plot & code

```
sys = tf([21 21], [5 11 22 4]);
bode(sys)
grid on
margin(sys)
```



From plot

$20 \log K_p = 14.2$

$\Rightarrow K_p = 5.12 \Rightarrow \text{error} = \dfrac{1}{1+5.12} = 0.16$ verified

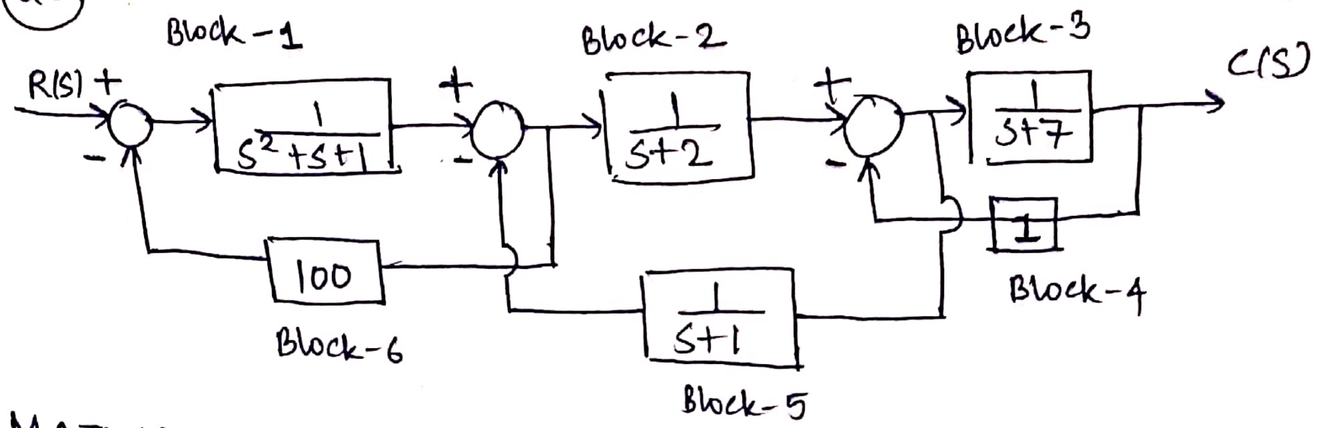
Gain margin $= \infty$

Phase margin $= 58.4^\circ$ (at 2.24 rad/s)

$\because$ both gain and phase margins are +ve,

Hence system is stable.

Q5



MATLAB Code :

```
n1 =1; d1=[1 1 1];
n2 =1; d2=[0 1 2];
n3=1 ;d3=[0 1 7];
n4=1 ;d4=1;
n5=1; d5=[0 11]
n6= 100; d6 =1;
nblocks=6;
blkbuild;
q= [1 0 -6 0 0
2 1 -5 0 0
3 2 -4 0 0
4 3 0 0 0
5 2-4 0 0
6 1-5 0 0];
input =1;
output = 3;
[aa, bb, cc,dd] = connect (a,b,c,d,q,input,output)
[num,den] = ss2tf (aa,bb,cc,dd);
printsys (num,den)
```

$$\text{num/den} = \frac{s+1}{s^5+12s^4+139s^3+1161s^2+2650s+1623}$$

$$TF = (s+1)/(s^5+12s^4+139s^3+1161s^2+2650s+1623)$$