DIFFERENTIAL EQUATIONS
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COMPUTATIONAL PRACTICUM
REPORT

Link To GitHub

- https://github.com/sham1lk/DEgraphs

Analytical Solution

$$\frac{d\,y(x)}{d\,x} = \frac{4}{x^2} - y(x)^2 - \frac{y(x)}{x}$$

$$y(x) = -v(x)$$

$$-\frac{d\,v(x)}{d\,x} = -v(x)^2 + \frac{v(x)}{x} + \frac{4}{x^2} \qquad v(x) = -\frac{\frac{du(x)}{dx}}{u(x)}$$

$$-\frac{\frac{d^2u(x)}{dx^2}}{u(x)} + \frac{\left(\frac{du(x)}{dx}\right)^2}{u(x)^2} = \frac{\left(\frac{du(x)}{dx}\right)^2}{u(x)^2} + \frac{\frac{du(x)}{dx}}{x\,u(x)} - \frac{4}{x^2}$$

$$x^2\,\frac{d^2\,u(x)}{d\,x^2} + x\,\frac{d\,u(x)}{d\,x} - 4\,u(x) = 0$$

$$\lambda^2 - 4 = 0$$

$$u(x) = u_1(x) + u_2(x) = \frac{c_1}{x^2} + c_2\,x^2 \qquad v(x) = \frac{2\,c_1 - 2\,c_2\,x^4}{c_1\,x + c_2\,x^5}$$

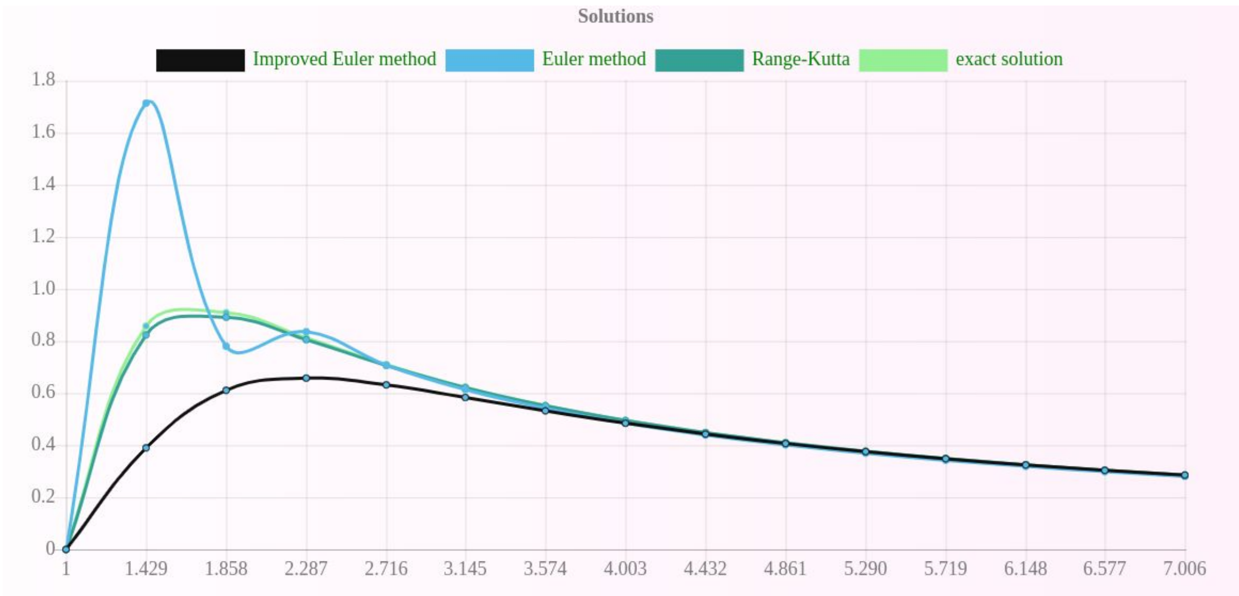
$$\frac{c_2\left(-2\,x^4 + \frac{2\,c_1}{c_2}\right)}{c_2\left(x^5 + \frac{c_1\,x}{c_2}\right)} \qquad v(x) = \frac{2}{x} - \frac{4\,x^3}{x^4 + c_1} \qquad y(x) = -\frac{2}{x} + \frac{4\,x^3}{x^4 + c_1}$$

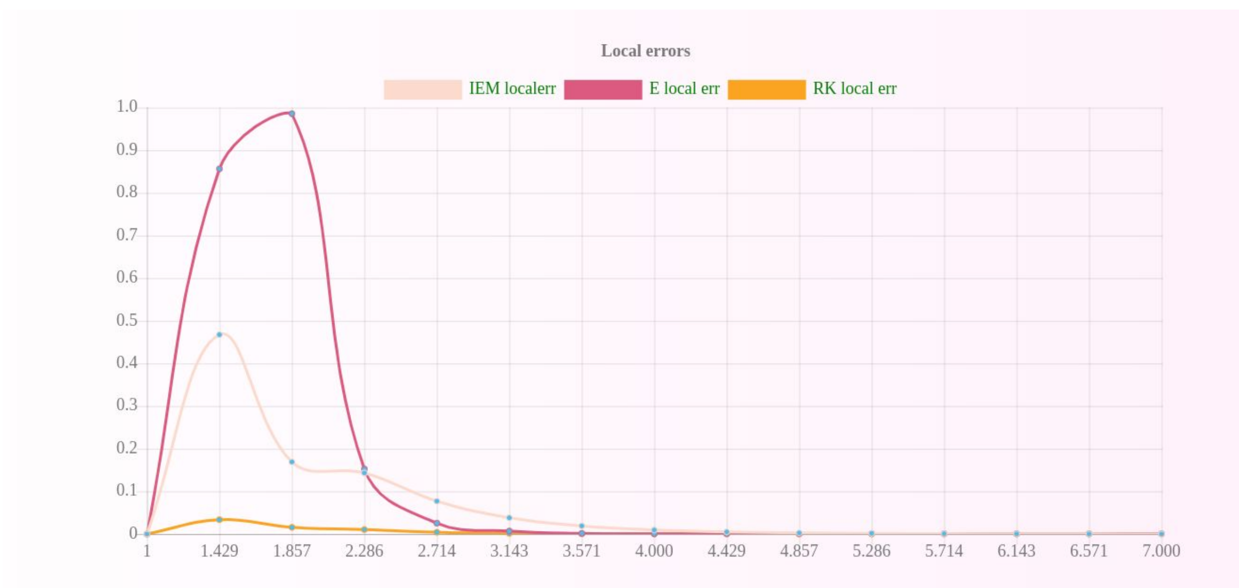
IVP

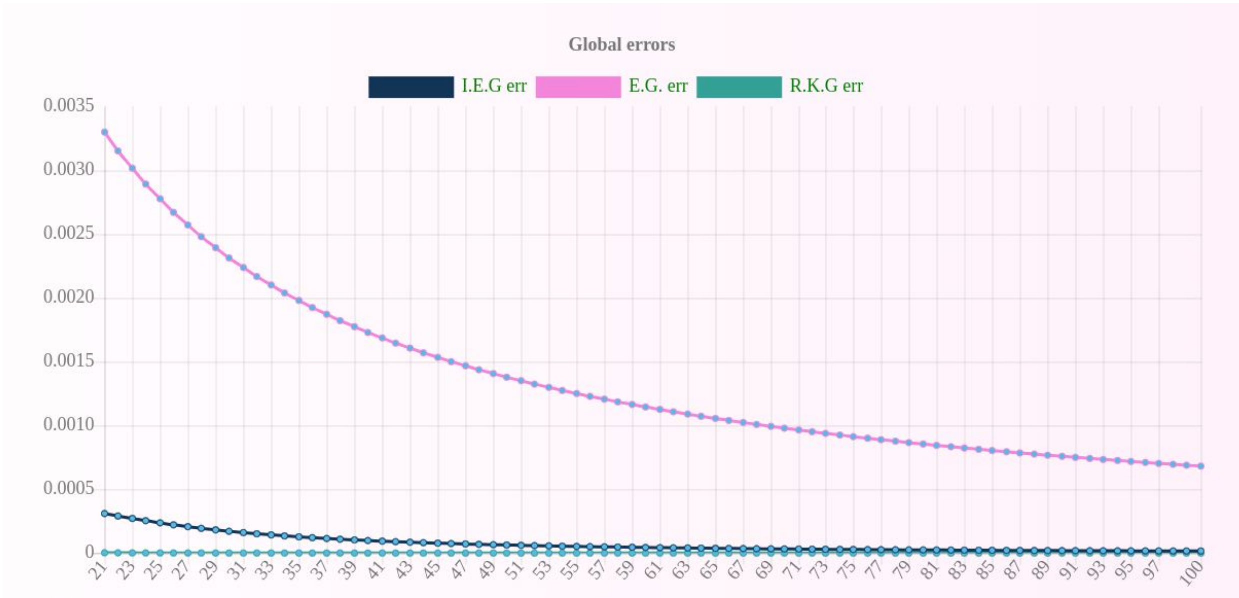
c = (2*x0^ 4 - y0 *x0^ 5) / (2 + y0*x0);

c = 1.

Screenshots







Results

The first image shows 4 graphs of solving the differential equation by different methods. The second figure shows local errors. Local error is an error on each step specifically, regardless of the previous step's error. For finding local errors first I find two global errors at the point at which I want to find a local error and at the points in front of it, then I subtract from first point second one. From the first and second screenshot, we see that the Euler method is the worst one, Runge-Kutta is the best and Improved Euler is in the middle.

In the last chart, we can see the dependency on the quality of the approximations on the number of points. We see that the more points the more accurately the methods work. To build a graph, I solved equations with a different number of points (from N0 to N) and calculated the global error at the last point.

Analysis of the code

The whole project contains 9 files. graph.html and graph.css are markup and styles, respectively rest for js files. In the project I used the principles of OOP and MVC.

OOP

From the parent class which contains 3 methods ("myfunction", which stores the differential equation, "solution", which stores the solution equation and "local_err", which calculates the graph of error locales), 4 classes(Exact,Euler,IEuler and RK methods) are inherited that contain the methods for solving equation and finding global errors.

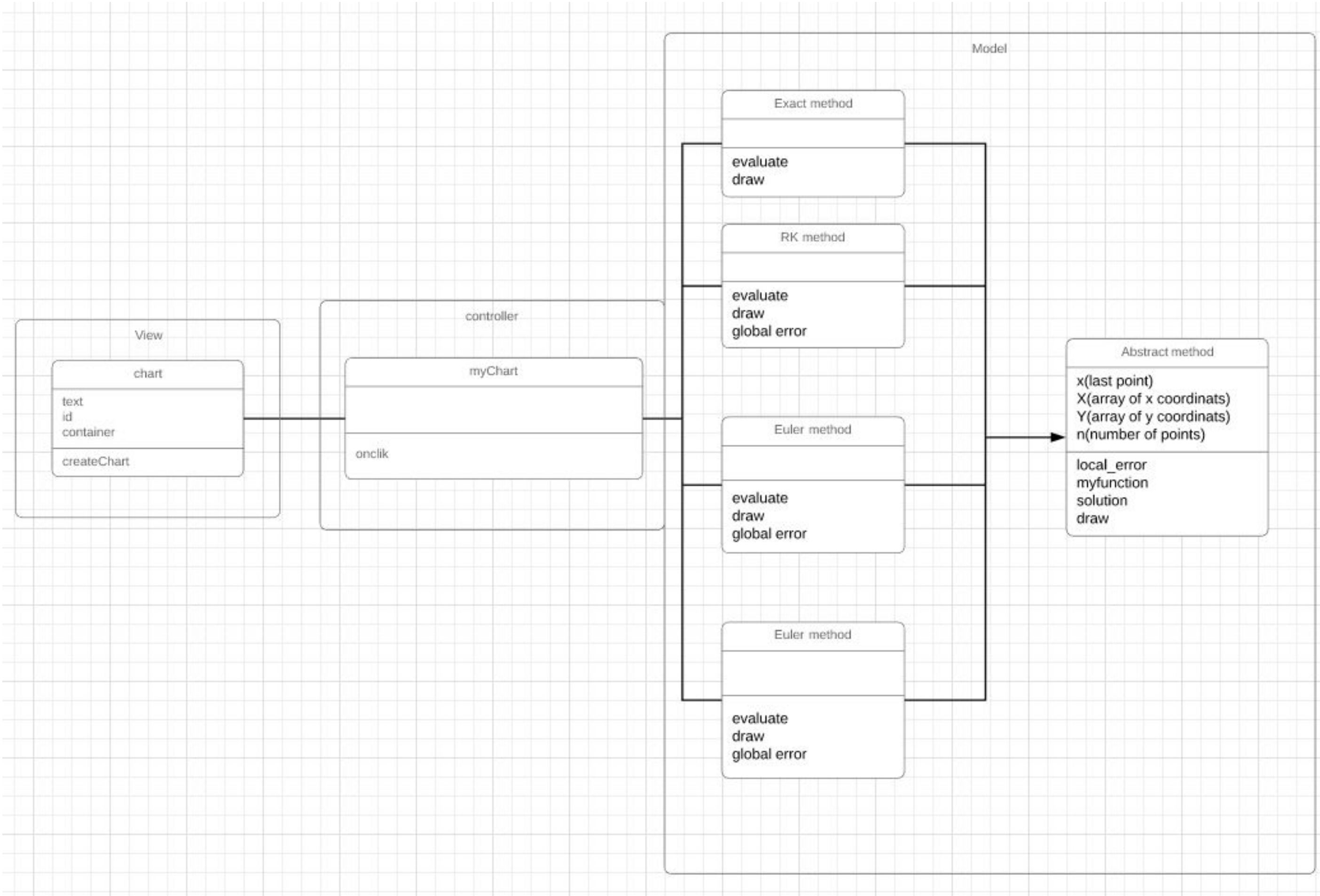
MVC

Model: Classes described in OOP.

View: Class Chart that contains some information about chart as attributes and has a method “createChart” that creates Chart.

Controller: function that tracks value changes and controls the plotting process.

UML


Model
Exact method
evaluate
draw
RK method
evaluate
draw
global error
View
chart
text
id
container
createChart
controller
myChart
onclik
Euler method
evaluate
draw
global error
Euler method
evaluate
draw
global error
Abstract method
x(last point)
X(array of x coordinats)
Y(array of y coordinats)
n(number of points)
local_error
myfunction
solution
draw