hetFL

shamsiiat.abdurakhmanova

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1 Numerical experiments

1.1 Datasets

1.1.1 Synthetic Dataset

Experiments were performed on a synthetic dataset whose empirical graph $\mathcal{G}$ is partitioned into 3 equal-sized clusters $\mathcal{P} = \{\mathcal{C}^{(1)}, \mathcal{C}^{(2)}, \mathcal{C}^{(3)}\}$, with $|\mathcal{C}^{(1)}| = |\mathcal{C}^{(2)}| = |\mathcal{C}^{(3)}|$. We denote the cluster assignment of node $i \in \mathcal{V}$ by $c^{(i)} \in \{1, 2, 3\}$. The edges in $\mathcal{G}$ are generated via realizations of independent binary random variables $b_{i,i'} \in \{0, 1\}$. These random variables are indexed by pairs $i, i^{'}$ of nodes that are connected by an edge $\{i, i^{'}\} \in \mathcal{E}$ if and only if $b_{i,i'} = 1$.
Two nodes in the same cluster are connected with probability $Prob\{b_{i,i'} = 1\} := p_{in}$ if nodes $i, i^{'}$ belong to the same cluster. In contrast, $Prob\{b_{i,i'} = 1\} := p_{out}$ if nodes $i, i^{'}$ belong to different clusters. Every edge in $\mathcal{G}$ has the same weight, $A_e = 1$ for all $e \in \mathcal{E}$.

Each node $i \in \mathcal{V}$ of the empirical graph $\mathcal{G}$ holds a local dataset $\mathcal{D}^{(i)}$ of the form $\mathcal{D}^{(i)} := \{(x^{(i,1)}, y^{(i,1)}), ..., (x^{(i,m_i)}, y^{(i,m_i)})\}$. Thus, dataset $\mathcal{D}_{(i)}$ consist of m_i data points, each characterized by a feature vector $\mathbf{x}^{(i,r)} \in \mathbb{R}^d$ and scalar label $y^{(i,r)}$, for $r = 1, ..., m_i$. The feature vectors $\mathbf{x}^{(i,r)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{d\times d})$, are drawn i.i.d. from a standard multivariate normal distribution.

The labels of the data points are generated by a noisy linear model

$$y^{(i,r)} = (\mathbf{w}^{(i)})^T \mathbf{x}^{(i,r)} + \varepsilon^{(i,r)} \tag{1}$$

The noise $\varepsilon^{(i,r)} \sim \mathcal{N}(0, 1)$, for $i \in \mathcal{V}$ and $r = 1, .., m_i$, are i.i.d. realizations of a normal distribution. The true underlying vector $\mathbf{w}^{(i)} \sim \mathcal{N}(0, 1)$ is drawn from a standard normal distribution and is the same for nodes from the same cluster, i.e. $\mathbf{w}^{(i)} = \mathbf{w}^{(i^{'})}$ if $c^{(i)} = c^{(i^{'})}$.

Datasets were divided into training and validation subsets by using resampling with replacement. The size of the validation subset was $m_i^{(val)} = 100$.

1.1.2 Shared Dataset

Dataset $\mathcal{D}^{(test)}$, which predictions are shared across all nodes was formed as follows: the feature, weight and noise vectors are drawn i.i.d. from a standard normal distribution and labels are generated by a noisy linear model. The size of the dataset was $m' = 100$.

1.2 Experiments

In these experiments empirical graph $\mathcal{G}$ consist of 15 nodes partitioned into three clusters. Two nodes in the same cluster are connected with probability $p_{in} = 0.8$ if nodes $i, i^{'}$ belong to the same cluster and $p_{out} = 0.2$ if nodes $i, i^{'}$ belong to different clusters.

Each node $i \in \mathcal{V}$ of the empirical graph $\mathcal{G}$ holds a local dataset $\mathcal{D}^{(i)}$ consisting of m_i data points, each characterized by a feature vector $\mathbf{x}^{(i,r)} \in \mathbb{R}^d$ and scalar label $y^{(i,r)}$, for $r = 1, ..., m_i$, where $d = 10$. The sample size of the shared dataset $\mathcal{D}^{(test)}$ is $m^{'} = 100$.

To learn the local parameters $\mathbf{w}^{(i)}$, we use Algorithm 2 with local loss

$$L_{(i)}(h^{(i)}) = \frac{1}{m_i} \sum_{r=1}^{m_i} \left(y^{(i,r)} - h^{(i)}(\mathbf{x}^{(i,r)}) \right)^2 \tag{2}$$

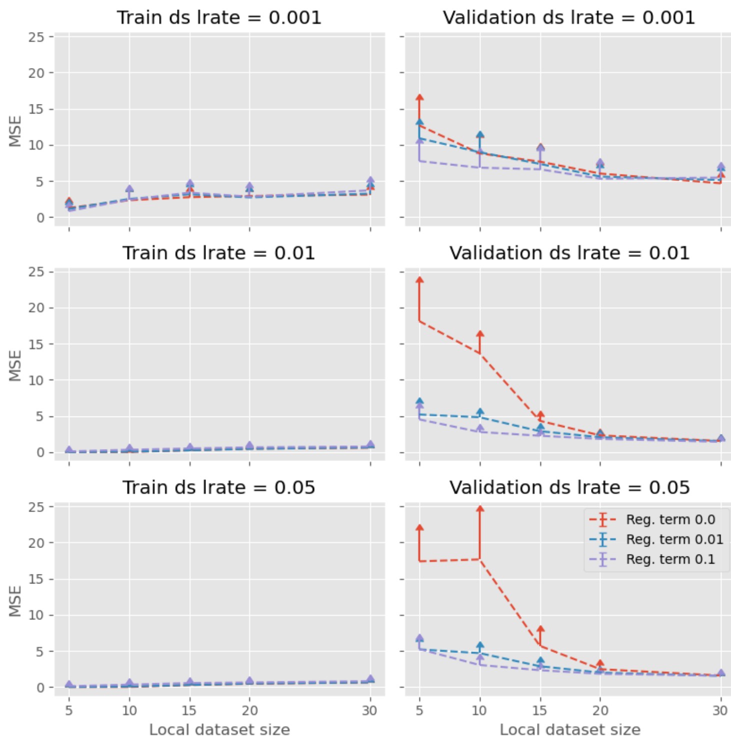
and regularizer

$$\frac{\lambda}{2m^{'}} \sum_{i^{'} \in \mathcal{V}} A_{i,i'} \sum_{r=1}^{m^{'}} \left(h^{(i)}(\mathbf{x}^{(r)}) - h^{(i^{'})}(\mathbf{x}^{(r)}) \right)^2 \tag{3}$$

As stopping criterion in Algorithm 2, we use a fixed number of R = 1000 iterations.

Below we plot average training and validation MSE of all nodes over 10 runs. On each run new local $\mathcal{D}^{(i)}$ and shared $\mathcal{D}^{(test)}$ datasets were generated. The error bar is one standard deviation (lower limit is omitted for clarity).

Results on the synthetic datasets:


Train ds lrate = 0.001
Validation ds lrate = 0.001
Train ds lrate = 0.01
Validation ds lrate = 0.01
Train ds lrate = 0.05
Validation ds lrate = 0.05
MSE
Local dataset size
Reg. term 0.0
Reg. term 0.01
Reg. term 0.1